research paper series

Theory and Methods

Research Paper 2006/10

Bilateral Predictions in a Multilateral World?
Revisiting the Theory

by

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Abstract

This paper addresses Choi and Krishna's (2004) recent empirical test of a bilateral factor content of trade prediction, originally developed by Helpman (1984). I revisit the theory and show that, contrary to common belief, Helpman's paper does not generalize Brecher and Choudhri's 2-country prediction on the factor content of trade to multiple countries. I reconsider Helpman's proof and show that bilateral factor price differences restrict an arbitrary number of bilateral and multilateral factor flows. This implies that the basic lesson from the comparative advantage commodity trade literature applies also to the factor content of trade: bilateral comparisons are not relevant in a multi-country world.

JEL Code: F11.

Outline

1. Introduction

In a recent paper in this journal, Choi and Prishna (2004) claim to provide a significant advancement in testing the Heckscher-Ohlin theory of international trade. The authors provide strong empirical support for a prediction on the bilateral factor content of trade, originally developed by Helpman (1984). Choi and Krishna's "claim of significance" is rooted in the empirical attractiveness of Helpman's prediction. First, by being a bilateral prediction, it accommodates an empirical inquiry for any subset of trading countries for which high quality data is available. Second, it requires no assumptions about consumer preferences. Third, as it is a multi-cone model, the framework dispenses with the empirically embarrassing factor price equalization property. Finally, and most importantly, it is a refutable hypothesis about the pattern of trade using available free-trade factor prices.

I revisit Helpman (1984) and claim that his widely-cited bilateral Heckscher-Ohlin prediction is not a hypothesis about the direction of international specialization.[1] I provide several arguments for this claim. Although the claim may be surprising to some readers, it will be of no surprise to those who recall the pioneering general equilibrium trade literature from the 1950s, in particular the seminal work of Lionel McKenzie (1954, 1955). Introducing activity analysis as a tool to analyze international specialization in what is now known as the "multi-cone Heckscher-Ohlin framework", McKenzie (1954, p. 180) has stressed that "*…it is not possible through merely bilateral comparison to develop a…theory of efficient multilateral specialization*". Given that this is true for commodity trade, it must also be true for the factor content of trade. Second, McKenzie's (1955) formal analysis of the factor price equalization problem has brought to light the central role of factor price equalization in a general equilibrium system in which countries are permitted to produce different sets of goods. Here the central message is that (McKenzie, p. 245) "*in the set of goods price vectors which do not permit equalization of factor prices no assured statement about specialization can be made without stronger assumptions* [on production and demand]".

The paper is organized as follows. In section 2, I show that, contrary to its claim, Helpman's prediction does not generalize Brecher and Choudhri's (1982) n-good, 2-factor and 2-country formulation of the factor content of trade to multiple countries. Section 3 revisits Helpman's proof and shows that the logic leads to an arbitrary number of

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[1] Helpman and Krugman (1985, pp. 24-27) and Feenstra (2004, pp.58-60) provide detailed discussions of Helpman (1984); Staiger (1986) extends Helpman's analysis to deal with intermediates.

restrictions on bilateral and multilateral trade flows. Section 4 discusses Choi and Krishna's empirical findings. Section 5 concludes with a discussion on the informational role of prices.

2. Helpman (1984) versus Brecher and Choudhri (1982)

Helpman's theoretical prediction builds on Deardorff (1979) and Brecher and Choudhri (1982). The central theme in these papers is to provide predictions in the spirit of Heckscher-Ohlin, but in the absence of factor price equalization. All three papers investigate the property of a competitive free trade equilibrium with two key characteristics. First, all countries possess identical production functions. Second, countries' factor endowments are assumed to be sufficiently dissimilar so that countries' free trade factor prices are different.

Formally, consider a competitive equilibrium with m countries, n goods, l factors and a common technology matrix, $A(.)=<a_{v\tau}(.)>$, where $a_{v\tau}$ are the units of factor v necessary to produce 1 unit of good τ. Although identical technologies imply the same functional forms for $a_{v\tau}$, the equilibrium least-cost input coefficients will depend on country specific factor prices.

If T^{ij} denotes the vector of gross imports of country j from country i, F^{ij} denotes the factor content of T^{ij} evaluated at the exporter's input techniques, i.e. $F^{ij}=A(w^i)T^{ij}$, where w^i is the free-trade factor price vector of the exporting country i. For two countries, i and j, who are engaged in bilateral trade, Helpman derives the following prediction on the net bilateral factor content of trade:

$$(w^j-w^i)' (F^{ij}-F^{ji}) \geq 0. \tag{1}$$

Inequality (1) has been commonly interpreted as saying that factors embodied in trade should flow towards the country with the higher factor price. If factor v has a higher absolute price in country j, $w^j_v-w^i_v >0$, then j will, 'on average', be a net importer of that factor relative to country i, i.e. $F^{ij}_v- F^{ji}_v> 0$.

Helpman claims that (1) is a direct generalization of Brecher and Choudhri (1982) to multiple countries. Helpman derives his intuition for (1) from the Lerner-Pearce diagram. Figure 1 considers the case of 3 countries, 6 goods and 2 factors. Countries are ranked according to their relative factor endowments: $(K/L)^1> (K/L)^2> (K/L)^3$. Since countries' factor endowments are assumed to be in different cones of diversification, the three

countries will specialize in the production of different goods. The most capital-abundant country 1 will produce the most capital-intensive goods 1 and 2; country 2 will produce goods 3 and 4 and the least capital- abundant country 3 will produce the least capital-intensive goods 5 and 6. The intuition for (1) stems from the implicit assumption that there is a one-to-one correspondence between this factor endowment ranking and the ranking of free trade equilibrium factor price ratios ω^i $(=(w/r)^i)$: $(K/L)^1 > (K/L)^2 > (K/L)^3$ <=> $(w/r)^1 > (w/r)^2 > (w/r)^3$. In any pair-wise comparison, the more capital-abundant country will also have the higher equilibrium wage-rental ratio. In reference to Figure 1, Helpman (1984, p. 90) writes:

"It is now a simple matter to observe that the more capital-rich a country is, the more capital and less labour is uses per dollar output in all lines of production (more generally, it never uses less capital and more labour). Hence, whatever trade there may exist between two countries, exports of the relatively capital rich country will embody a higher capital-labour ratio than exports of the relatively labour rich country. This describes a clear bilateral factor content pattern of trade (see Brecher and Choudhri, 1982)".

Figure 1: Lerner-Pearce Diagram
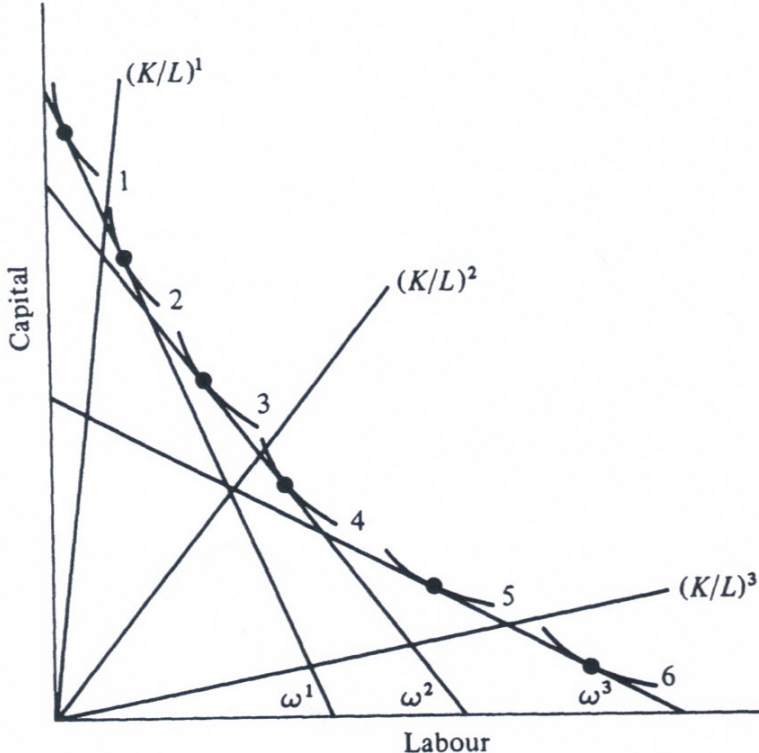
Brecher and Choudhri (1982) use a 2-country version of this diagram to prove that the factor content version of the Heckscher-Ohlin theorem holds also in the absence of factor price equalization: the relatively capital abundant country will be a net exporter of capital and a net importer of labour. Employing the price-definition of relative factor abundance, Brecher and Choudhri show that if $(w/r)^1 > (w/r)^2$, then country 1 will be a net exporter of capital and a net importer of labour. Alternatively, if $(w/r)^1 < (w/r)^2$, then the trading pattern will be just the opposite. To ease the comparison with (1), let us express this algebraically. If K^{ij} (L^{ij}) denotes the capital (labour) content of the gross import vector of country j from country i, (i,j =1,2), Brecher and Choudhri's (1982) prediction can be written algebraically as:

$$\left[\left(\frac{w}{r} \right)^1 - \left(\frac{w}{r} \right)^2 \right] \left[\begin{array}{c} K^{12} - K^{21} \\ L^{21} - L^{12} \end{array} \right] \geq \left[\begin{array}{c} 0 \\ 0 \end{array} \right]. \tag{2}$$

If Helpman's prediction in (1) were a true generalization of (2) to multiple countries, then (1) should coincide with (2) for the countries and two factors. For the case of two dimensions, (1) becomes then

$$\left[r^1 - r^2, \quad w^1 - w^2 \right] \left[\begin{array}{c} K^{21} - K^{12} \\ L^{21} - L^{12} \end{array} \right] \geq 0. \tag{3}$$

The predictions (2) and (3) are quite different. Inequality (2) provides sign predictions on *individual factors* based on country-specific differences in *relative* factor prices. In contrast, inequality (3) makes a sign prediction on the entire net factor content of trade vector F^{21}-F^{12}: $(r^1-r^2)(K^{21}-K^{12})+(w^1-w^2)(L^{21}-L^{12}) \geq 0$. The latter inequality, however, does not appear to have a meaningful economic interpretation. It pertains to a weighted average of the net factor content of trade with the weights being the difference in *absolute* factor prices. But we know that it is the difference in *relative* not absolute factor prices that determines the pattern of international specialization in neoclassical trade theory.

3. Revisiting Helpman's proof

Helpman arrives at (1) by using the property of the GDP function and a thought experiment on factor reallocation.[2] In a free trade equilibrium a country's GDP can be

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[2] In what follows, I revisit Helpman's proof by adopting the user-friendly notation used by Feenstra (2004, p. 58-59).

written as $G(p,V^j) = p' Y^j = w^{j\prime} V^j$, where V^j denotes the country's endowment vector, Y^j its production vector and p the free trade equilibrium goods price vector. Helpman derives then the following relationships:

$$G(p,V^j) + p' T^{ij} = p' Y^j + p' T^{ij} \leq G(p, V^j + F^{ij})$$
$$\leq G(p,V^j) + [\partial G(p,V^j)/\partial V^j]' F^{ij} \qquad (4)$$
$$= p' Y^j + w^{j\prime} F^{ij}.$$

The second inequality and the last line in (4) stem from the concavity of the GDP function with respect to factor endowments and the envelope theorem. Helpman's justification for the first inequality is based on the following thought experiment: If country j were given a factor endowment gift of F^{ij}, then the assumption of identical technologies implies that it would be feasible for country j to produce T^{ij} itself. However, since factor prices in country j are different than in i, country j could do 'potentially better' than that. Specifically, country j's GDP from its 'gift-augmented' endowment $G(p, V^j + F^{ij})$ will be at least as large as the sum of its pre-gift GDP $G(p,V^j)$ and the market value of the corresponding imports $p' T^{ij}$. Combining the zero profit condition, $p' T^{ij} = w^{i\prime} F^{ij}$, with (4), one obtains $w^{i\prime} F^{ij} \leq w^{j\prime} F^{ij}$ or

$$(w^j - w^i)' F^{ij} \geq 0. \qquad (5)$$

Applying the same reasoning to country i, one obtains $(w^j - w^i)' F^{ji} \leq 0$, which gives us then (1). If there are m countries, Helpman's logic implies a total of $m(m-1)/2$ bilateral predictions.

However, there are two issues with regard to Helpman's logic which seemed to have been unnoticed in the literature. First, (5) results from Helpman's implicit assumption that the equilibrium price vector p is the same when comparing country j's revenue function under V^j and $V^j + F^{ij}$. If all countries were in the same cone of diversification a factor reallocation F^{ij} from country i to country j will not change the equilibrium goods price vector p, given that the factor reallocation is not too large.
(Dixit and Norman, p. 121-122). However, in a multi-cone framework a reshuffling of resources can lead to changes in the equilibrium price vector p.[3] As a result, the second inequality in (4) is questionable as it assumes that the price vector p in the GDP function is unaltered; especially, since the logic considers $m(m-1)$ different reallocations.

But also if one were to assume that factor reallocations do not affect the equilibrium price vector, there is a more serious issue behind the first inequality in (4). Specifically, the

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[3] The magnitude of the price change will depend on consumer preferences and the sizes of the economies.

logic behind (4) is not specific to F^{ij}. It can be applied to any endowment gift for country j associated with the gross trade flow of i to any third country or, in fact, any subgroup of trading partners. To keep the notation simple, let k index a destination. If T^{ik} denotes the gross trade flow from country i to destination k, where k could pertain to just a single country or a any subgroup of countries, the factor content of trade is then defined as $F^{ik}= A(w^i)T^{ik}$. The reasoning from above leads then to the following:[4]

$$(w^j-w^i)' \, F^{ik} \geq 0. \tag{6}$$

Inequality (6) implies that the factor price difference between countries i and j does not only restrict the bilateral factor content of trade between this country pair but also restricts the bilateral factor content of trade between i and any other third country k or subgroup of countries. As a result, (w^j-w^i) restricts the factor content of any bilateral and multilateral exports by country i.

Alternatively, (6) implies that there are m-1 different restrictions for a given bilateral factor content of trade F^{ij}:

$$(w^k-w^i)' \, F^{ij} \geq 0, \text{ for any } k=1,\dots,m, \, k \neq i. \tag{7}$$

It is difficult to provide a meaningful interpretation of (7) since it implies that each bilateral factor price difference (w^k-w^i) provides a prediction on the gross factor content of trade F^{ij}.

4. Revisiting Choi and Krishna

Choi and Krishna (2004) test (1) for a sample of 8 countries which results into 28 bilateral comparisons.[5] Employing a variety of factor price measures, their data generate signs compatible with the theoretical predictions in about 80% of the cases. However, as argued above, the theory yields many more restrictions. Leaving aside how to interpret them, how many restrictions could one test with Choi and Krishna's data? Applying (7) to two-way trade flows F^{ij} and F^{ji} one obtains:

$$(w^k-w^i)' \, F^{ij} - (w^j-w^l)' \, F^{ji} \geq 0, \text{ for } k \neq i \text{ and } l \neq j. \tag{8}$$

It can be easily seen that (1) is a special case of (8) for $k=j$ and $l=i$. For 8 countries the theory implies 49 different restrictions on the bilateral trade flow on each country pair, leading to a total of 1372 (=28 x 49) restrictions. Consequently, Choi and Krishna have tested only about 2% of all the restrictions that are suggested in (8).

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[4] (6) follows from $G(p,V^j)+p' \, T^{ik} \leq G(p,V^j+F^{ik}) \leq G(p,V^j)+w^{j\prime} \, F^{ik}$ and the zero profit condition $p' \, T^{ik}=w^{i\prime} \, F^{ik}$.
[5] The country sample consists of the US, Canada, Denmark, France, Germany, UK, Netherlands and Korea.

However, as mentioned above, bilateral factor-price differences restrict not only bilateral but also multi-lateral trade flows. Alternatively, we can ask ourselves how many factor flows are restricted by a given factor-price difference vector $(w^j\text{-}w^i)$. Applying (6) to a destination index κ which identifies a subset of trading partners, one obtains:

$$(w^j\text{-}w^i)'\, F^{i\kappa} \geq 0, \text{ for any } \kappa \subset \{1,\dots,8\}\backslash\{i\}. \qquad (9)$$

For a fixed j and i, (9) implies that $(w^j\text{-}w^i)$ restricts 127 different gross exports of country i.[6] Varying j and i across the sample yields a total of 7112 (=8x7x127) restrictions! This implies that Choi and Krishna investigated only about 0.3% of all the restrictions that could have been investigated with their country sample. Would it make sense to go back and employ Choi and Krishna's data set to test the sign of the 8456 restrictions that haven't been investigated? I don't think so. Since these inequalities stem from the concavity property of the GDP function, it would provide just evidence for or against an implication of this property. However, it would not provide a test of a prediction about the direction of international specialization in the spirit of Heckscher-Ohlin.

5. Conclusion: Hayek, autarky and the informational role of prices

In conclusion, I provide an informational argument why (1) does not constitute a prediction about the direction of international trade. The argument is rooted in Hayek's (1945) fundamental insight that in a market economy goods prices contain all the relevant information about underlying fundamentals. In the theoretical trade literature, the most general statements about the direction of international specialization are based on autarky prices. The reason for this is that an economy's autarky prices contain all the relevant information about the country's fundamentals (e.g. preferences, technologies or tastes) in the absence of international specialization. This information is used to evaluate, or restrict, an economy's vector of international specialization with its trading partners. If the focus is on commodity trade, the theory of comparative advantage implies that the commodity autarky price vector p^a restricts the economy's net commodity import vector T, predicting that $p^a T > 0$ (Deardorff, 1980).[7] Alternatively, if the focus is on the the factor content of trade, defined as $F=A(.)T$, the theory implies that the corresponding autarky factor price vector w^a restricts F, predicting that $w^a F > 0$ (Deardorff, 1982). In both cases, information

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[6] As each trade flow corresponds to a subset of trading partners, the number of trade flows equals the total number of subsets of a set of size 7, which is 2^7-1.

[7] Confirmative evidence for this prediction is given in Bernhofen and Brown (2004).

from outside the trading regime (i.e. autarky) is used to predict the direction of international specialization in a trading regime.

In contrast, (1) is based on free trade factor price differences between two trading partners. Since free trade factor prices embody information about a regime where international specialization has already taken place, they do not provide enough information about country-specific fundamentals to predict which trading pattern should or should not occur in such a regime. In the absence of autarky information,

predicting the factor content of multilateral trade requires the strong assumption that all countries have the same homothetic preference structures (Vanek, 1968). Obtaining a prediction on the direction of the factor content of bilateral trade is expected to require assumptions that are too restrictive to be empirically meaningful.

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