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Market Size and the Survival of Foreign-owned Firms

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Abstract

We develop a general equilibrium model with heterogeneous firms and Foreign Direct Investment (FDI) cost uncertainty and investigate the survival of foreign-owned firms. The survival probabilities of foreign-owned firms depend on firm-level characteristics such as productivity and host country characteristics such as market size. We show that a foreign-owned firm will be less likely to be shut down when its parent firm's productivity is higher and its indigenous competitors are less productive. Whilst a larger market size will always reduce the survival probability of indigenous firms, it can lead to a higher survival probability for foreign-owned firms if their parent firms are sufficiently productive.

JEL classification: F12, L11.

Keywords: FDI, heterogeneous firms, market size.

Outline

Non-Technical Summary

The global presence of multinationals has increased dramatically in the last two decades and sales by foreign subsidiaries of Multinational Enterprises (MNEs) have grown at a much higher rate than exports of national firms. Recent evidence suggests that MNEs may face substantial *local* production uncertainty when investing abroad, especially in emerging markets. There is also robust empirical evidence that MNEs are “footloose” in that their subsidiaries are more likely to shut down than the indigenous firms in the host country. This raises issues over how foreign subsidiaries’ survival probabilities are affected by characteristics of the host country, such as market size, market entry costs and the underlying determinants of firm heterogeneity.

To explore these issues, we develop a general equilibrium model where MNEs’ foreign subsidiaries face cost uncertainty in the local markets. Following the heterogeneous firm trade literature, we suppose that the unit production cost of an indigenous entrant to any market is determined by a random draw from a known cost distribution that is specific to that market. Successful entrants then have the option of serving other markets by either exporting from their home plant or by setting up a foreign subsidiary (FDI). Our key assumption is that the unit cost of the subsidiary is jointly determined by its parents’ unit cost (determined by its draw in the source market) and a random draw from the cost distribution that is common to all indigenous entrants in the *host* country. After entry, MNEs may chose to shut down the foreign subsidiary (and therefore “fail” in their investment) if the realized cost outcome is sufficiently high.

We show that the survival probabilities of subsidiaries will therefore depend on the efficiency of the parent firm, the market size of the host country, the competitiveness of the indigenous firms, trade costs and entry barriers. In particular, the higher the productivity of the parent firm, the less likely the foreign subsidiary will be to shut down. But technological improvement in the host country, reflecting falling average unit costs of indigenous firms, will reduce a given subsidiary’s chances of survival. Most interestingly, whilst the probability of survival for all indigenous entrants is lower in a larger market, foreign entrants may have a greater (lower) survival probability in a *larger* host country if their parent firms are sufficiently productive (unproductive). Furthermore, while foreign investment liberalization tends to increase the quantity of FDI, it also raises the overall failure rate of foreign-owned firms in the host country. Falling FDI entry barriers will lead to entry of subsidiaries of low-productivity parents, which are less likely to survive after entry.

1. Introduction

The global presence of multinationals has increased dramatically in the last two decades and sales by foreign subsidiaries of multinational enterprises (MNEs) have grown at a much higher rate than exports of national firms. The general equilibrium “proximity-concentration” approach (Markusen 1984, Brainard 1993, Markusen and Venables 1998, 2000) considers multinational firms that produce and sell the same product in multiple countries, serving each market by the local production of their subsidiaries. The dominant assumption of this strand of models is that all types of firms have identical productivities - in terms of marginal output per unit input, which are predetermined prior to market entry. Therefore MNEs and NEs (national enterprises) have the same productivities or market shares in their home markets, although they may differ in their shares in the world market due to different modes of serving foreign customers. This, however, is at odds with the prevailing evidence that multinationals are typically more productive and sell more than national firms in the same industry, even in their home markets. This highlights the importance of firm heterogeneity – in terms of productivity differences - as a key determinant of the structure of international commerce[1]. Helpman, Melitz and Yeaple (2004) (HMY hereafter) emphasize the role of productivity differences across firms in determining the sales of MNEs relative to the exports of NEs. In their model only the most productive firms can afford the high fixed costs of setting up foreign subsidiaries, whilst the less productive firms remain NEs and may serve foreign markets by exporting.

However, even allowing for heterogeneous firms, the standard assumption is that the productivity of an MNEs’ subsidiary is the same as that of the parent firm. This assumption is called into question by emerging evidence suggesting that MNEs may face substantial *local* production uncertainty when investing abroad, especially in emerging markets such as Brazil, Russia, India and China[2]. There is also robust empirical evidence that foreign firms are “footloose” and more likely to exit than their indigenous counterparts in the host country [Gorg and Strobl 2003, Bernard and Sjoholm 2003]. This raises issues over how foreign firms’ survival probabilities are affected by characteristics of the host country such as market size, entry barriers and the underlying determinants of firm heterogeneity.

[1] For empirical studies on the link between productivity and firms’ entry modes of serving foreign markets, see Head and Ries (2003), Girma, Kneller and Pisu (2005).
[2] See for example Aizeman and Marion (2004), Hausman and Gavin (1995).

To explore these issues, we develop a general equilibrium heterogeneous firm model where MNEs' foreign subsidiaries face productivity uncertainty in the local markets. Building on the work of Melitz and Ottaviano (2005), we consider a monopolistic competitive industry where firms differ in their unit costs. Following the heterogeneous firm trade literature, we model the cost of an indigenous entrant to any market as if it were a random draw from a known cost distribution that is specific to that market. Successful entrants then have the option of serving any other market by either exporting from their home plant or setting up a foreign subsidiary (FDI). Our key assumption is that the unit cost of the subsidiary is jointly determined by the parent firms' cost in its *home* country and a random cost draw from a distribution that is common to all indigenous entrants in the *host* country. After entry, MNEs may chose to shut down the foreign subsidiary (and therefore "fail" in their investment) if the realized cost outcome is sufficiently high. The survival probabilities of subsidiaries will therefore depend on the cost of its parent firm, the market size of the host country, the competitiveness of the indigenous firms, trade costs and entry barriers. In particular, the higher the productivity of the parent firm, the less likely the foreign subsidiary will be to shut down. But technological improvement in the host country, reflecting falling average unit costs of indigenous firms, will reduce a subsidiarys' chances of survival. Most interestingly, whilst the probability of survival for all indigenous entrants is lower in a larger market, foreign entrants may have a greater (lower) survival probability in a larger host country if their parent firms are sufficiently productive (unproductive). This is because a larger host market has two effects on foreign entrants. First, it leads to more competitors and a lower price mark-up that tends to have a negative "competition" effect on foreign firms' survival. Second, access to a larger number of consumers tends to have a positive "sales" effect on their survival. Foreign subsidiaries with more productive parents will be less affected by the competition effect and can take greater advantage of the sales effect. Hence high (low)-productivity MNEs may indeed find their subsidiaries more likely to be profitable and to survive in a larger (smaller) host market. Furthermore, while foreign investment liberalization tends to increase the quantity of FDI, it also raises the overall failure rate of foreign entrants in the host country. Falling FDI entry barriers will lead to entry of subsidiaries of low-productivity parents, which are less likely to survive after entry.

To our knowledge the only paper that considers the role of productivity uncertainty in FDI is Aizeman and Marion (2004). They investigate the role of productivity and demand uncertainty on vertical versus horizontal FDI, and show that an increase in the volatility of productivity uncertainty will reduce the incentive for vertical investment while increasing that of horizontal FDI. For the purpose of their study, they analyze the investment decision of a single monopolist firm and do not consider the possibility of subsidiary shut down. Both their model and ours assume productivity draws for foreign production. But the layout of our model is based on the general equilibrium specifications in Melitz and Ottaviano (2005) that endogenize firm level price markups to market size and openness. Hence, our model may be considered as combining elements of the heterogeneous firm trade literature and the investment uncertainty models.

The rest of this paper is organized as follows. Section II sets out the model and section III examines MNEs' investment decisions. The survival of foreign firms is investigated in section IV. Section V concludes.

2. The model

There are N countries that produce goods in two sectors, one provides a homogeneous product chosen as numeraire and the other differentiated goods. Labor is the only input. and country n is endowed with L^n units of labor. Following Melitz and Ottaviano (2005), we assume that in country n individuals' preferences across goods are identical and characterised by a quasi-linear utility with a quadratic subutility that is symmetric in all varieties

$$u^n = q_0^n + \alpha\int_0^{M^n} q_i^n di - \frac{\gamma}{2}\int_0^{M^n}\left(q_i^{\ n}\right)^2 di - \frac{\eta}{2}\left(\int_0^{M^n} q_i^{\ n} di\right)^2 \qquad (1)$$

where $q_0^{\ n}$, $q_i^{\ n}$ and M^n are respectively, the representative consumption of the homogeneous good and variety i in the differentiated sector and the number of varieties available. These preferences lead to a linear demand function for each variety i for a representative consumer in country n

$$q_i^n = \frac{\alpha}{\eta M^n + \gamma} - \frac{1}{\gamma} p_i^n + \frac{\eta M^n}{(\eta M^n + \gamma)\gamma} \cdot \overline{P}^n \qquad (2)$$

where $\overline{P}^n = \frac{\int_0^{M^n} p_i di}{M^n}$ represents the average price of the differentiated varieties [3]. Parameters α , η and γ are all positive and identical across countries; γ indexes the degree of product differentiation across varieties, the larger γ the more differentiated the varieties; α and η index the degree of substitution between the numeraire good and the

[3] See Melitz and Ottaviano (2005) for more details of the derivation.

differentiated goods (the higher α or lower η the more the consumer's demand is biased toward differentiated goods relative to the numeraire). For each variety there exists a common price ceiling above which there will be zero demand:

$$p_i^n \leq \hat{P}^n = \frac{\eta M^n \overline{P}^n + \alpha\gamma}{\eta M^n + \gamma} \tag{3}$$

We adopt the standard assumption that there are no transport costs on the homogeneous good, which is assumed to be produced under constant returns to scale with unit labor input per unit output. All countries are assumed to have positive production in that sector, so that countries have a common wage of unity. In contrast the markets for differentiated goods are segmented and trade between them is costly. In country *n* there exists a continuum of potential indigenous entrants who are *identical* prior to entry and have representative marginal cost $\overline{a}^n$. To enter the industry a firm has to undertake a sunk fixed entry cost of f_E [4]. Immediately after entry, firms make an idiosyncratic cost draw (denoted as ϕ) with zero mean and cumulative density function $\Omega^n(\phi)$ over support $[-\phi_L, \infty]$ where $0 < \phi_L < \overline{a}^n / 2$. The marginal production cost of firm i after entry is then given by

$$a_i^n = \overline{a}^n + \phi_i^n \tag{4}$$

Let $G^n(a)$ denote the cumulative distribution over entrants' costs in country *n*. Then

$$G^n(a) = \Omega^n(a - \overline{a}^n) \ , \quad a \in [a_L^n, \infty] \tag{5}$$

where $a_L^n = \overline{a}^n - \phi_L$ is the lower boundary of entrants' costs. It follows that $E(a) = \overline{a}^n$ and $Var(a) = Var(\phi)$. Since there are no fixed costs of production, an indigenous entrant will decide either to serve its home market or to exit immediately, depending on whether its marginal cost (a_i^n) is less or greater than its home price ceiling ($\hat{P}^n$). With respect to foreign markets the successful entrant has the option of either exporting or FDI. If it chooses to export to country *j*, a per unit trade cost τ^{nj} will be incurred. If instead it takes the FDI option, it pays an entry cost f_I^j to establish a foreign subsidiary. The literature has typically assumed that the marginal cost of this subsidiary (a_i^{nj}) will be the same as that of the parent (a_i^n). But, as noted in the introduction, this makes entry by an established foreign firm radically different from that of an indigenous entrant. While it is reasonable to expect

[4] Such fixed costs may include research and development associated with a new variety and standard startup costs, which must be "sunk" in nature and are incurred before the "true" marginal production costs are revealed.(see further discussion on this in Baldwin 2005).

that the costs of a subsidiary will be partly determined by its parent, it is likely to reflect some element of the risk associated with entering this new and unfamiliar market[5]. We therefore assume that

$$a_i^j = a_i^n + \phi_i^j \qquad (6)$$

where ϕ_i^j is an idiosyncratic stochastic cost draw from the same distribution as faced by indigenous entrants to market j $(\Omega^j(\phi))$. By this assumption, the production cost of a subsidiary in market *j* of a parent from market *n* is determined in the same way as that of an indigenous entrant to market *j*, except that the cost of the parent (a_i^n) replaces the representative cost of that market ($\bar{a}^j$). Furthermore, producing abroad incurs additional fixed costs (f_c^{nj}) due to the costs of communication between parent and subsidiary[6]. This structure will enable us to analyse the probability of survival or failure of foreign-owned firms.

Let $p^n(a)$ and $q^n(a)$ denote the profit-maximising price and per-consumer sales in market *n* of a firm with cost *a*. Its profits are then $\pi_D^n = [p^n(a) - a]q^n(a)L^n$. Using (2), profit maximization leads to price, quantity and mark up of:

$$p^n(a) = \frac{\hat{P}^n + a}{2} \;;\; q^n(a) = \frac{\hat{P}^n - a}{2\gamma} \;;\; m^n(a) = p^n - a = \frac{\hat{P}^n - a}{2} \qquad (7)$$

Then (3) and (7) imply that $a_D^n = \hat{P}^n$ is the cost ceiling in market *n*, and only firms with costs below this ceiling will face positive demand. Profits are given by :

$$\pi_D^n(a) = \frac{L^n(a_D^n - a)^2}{4\gamma} = \frac{L^n\left[m^n(a)\right]^2}{\gamma} \qquad a < a_D^n \qquad (8)$$

Those entrants with cost draws above the ceiling will exit immediately without producing[7]. Note that $\pi_D^n(a)$ is monotonically increasing in a_D^n and the markup $m^n(a)$ [8]. For a given

[5] One would expect that the sources of such productivity uncertainty such as labour force quality and local managerial efficiency are likely to affect both foreign and indigenous firms in similar ways.
[6] Grossman , Helpman and Szeidl (2003) make a similar assumption.
[7] Such immediate exit behaviour of firms is assumed for simplicity. In reality it may take some period for firms to realize and learn their true costs and then decide whether to continue production or exit.
[8] The price-cost mark up is decreasing in a, hence a more productive firm sets a higher markup. This result can also be obtained under the CES preferences assumption. However, Melitz and Ottaviano (2005) show that an important feature of [8] is that *m* is increasing in the cost ceiling which responds to the number of firms in the market, and therefore avoids the well-known limitation of the Dixit-Stiglitz (1977) set up that firms' markups are exogeneously fixed by the common elasticity of substitution between varieties and thus unresponsive to the degree of market competition.

firm cost, a higher cost ceiling leads to a larger price mark-up and higher firm profits. In other words, when firms find it easier to survive, they tend to charge higher prices and earn greater profits. Recall that the source of cost differences across firms is their random cost draws, so using (4) the condition for firm survival in the domestic market can be expressed as

$$\phi < \hat{\phi}_D^n \equiv a_D^n - \bar{a}^n \tag{9}$$

where $\hat{\phi}_D^n$ represents the "ease of survival" for indigenous firms in country *n*.

Next consider the profits surviving domestic firm with unit labour cost a in market *n* would obtain by serving a foreign market *j*. Let $\pi_X^{nj}(a) = \left(p_X^{nj}(a) - a - \tau^{nj}\right) q_X^{nj}(a) L^j$ denote the firm's export profit in market *j*, where $p_X^{nj}(a)$ and $q_X^{nj}(a)$ are the unit price and per consumer export sales. By analogy with the domestic profit function, the optimal exporting profit is given by

$$\pi_X^{nj}(a) = \frac{L^j(\hat{P}^j - a - \tau^{nj})^2}{4\gamma} = \frac{L^j(a_D^j - a - \tau^{nj})^2}{4\gamma}, \quad a < a_D^j - \tau^{nj} \tag{10a}$$

This implies a corresponding cost ceiling for exports from country *n* to *j* of:

$$a_X^{nj} = a_D^j - \tau^{nj} \tag{10b}$$

Alternatively, the firm may serve the foreign market via FDI, which would yield operating profit:

$$\pi_I^{nj}(a, \phi^j) = \frac{L^j(a_D^j - a^{nj})^2}{4\gamma} - f_c^{nj} = \frac{L^j(a_D^j - a - \phi^j)^2}{4\gamma} - f_c^{nj} \tag{11}$$

Unlike export profits, FDI profits depend not only on the parent's marginal cost (a), but also the stochastic cost draw in the host country (ϕ^j). Indeed, $\pi_I^{nj} > 0$ if and only if the cost draw at the foreign subsidiary is sufficiently low (i.e. $\phi^j < a_D^j - a - \sqrt{\frac{4\gamma f_c^{nj}}{L^j}}$). But unlike indigenous firms, the subsidiary may be closed even if it can earn a positive profit, because its parent has the option of serving country *j* by exporting from the home plant. Therefore the subsidiary will survive if and only if $\pi_I^{nj}(a_i^j) > \pi_X^{nj}(a_i^n)$, which is equivalent to

$$\phi^j < a_D^j - a - \sqrt{\left(a_D^j - a - \tau^{nj}\right)^2 + \frac{4\gamma \cdot f_C^{nj}}{L^j}} \tag{12a}$$

Hence for a subsidiary with parent cost a_i^n, there exists a FDI survival *cost draw ceiling:*

$$\hat{\phi}^{nj}(a) \equiv \sup\left\{a_D^j - a - \sqrt{\left(a_D^j - a - \tau^{nj}\right)^2 + \frac{4\gamma \cdot f_C^{nj}}{L^j}}, -\phi_L^j\right\} \qquad (12b)$$

The subsidiary will survive if and only if its local cost draw is sufficiently low i.e. $\phi^j < \hat{\phi}^{nj}(a)$. Otherwise the parent will prefer to serve this market by exporting. We can show that the easier it is for an indigenous entrant to survive in the host country and the more productive the parent firm, the more likely the subsidiary will survive after entry.

Lemma 1. $\hat{\phi}^{nj}(a)$, *is increasing in* a_D^j, *but decreasing in* a.

Due to the productivity and profit uncertainty associated with FDI , a foreign firm's market entry decision has to take into account of the possibility of FDI failure. HMY (2004) show that in the absence of FDI productivity uncertainty, there is clear-cut self-selection in the mode of those firms serving the foreign market: the lowest cost firms undertake FDI whilst higher cost firms export. Because the subsidiary has the same productivity as its parent, foreign firms know ex ante which mode of entry will be most profitable. Does the same ranking occur when there is FDI cost uncertainty? We first consider the FDI *entry* decision for market *j* of a firm in country *n* with cost a. Whether the firm will undertake FDI depends on its *expected* FDI profit net of entry cost (f_I^j). We can write the firm's expected net operating profit from FDI as follows

$$\Pi_I^{nj}(a) \equiv E\left[\pi_I^{nj}(a) \middle| \phi < \hat{\phi}^{nj}(a)\right] = \int_{-\phi_L^j}^{\hat{\phi}^{nj}(a)} \left[\frac{L^j}{4\gamma}\left(a_D^j - a - \phi\right)^2 - f_C^{nj}\right] d\Omega^j(\phi) - f_I^j \quad (13)$$

Since $\hat{\phi}^{nj}(a)$ is decreasing in a as shown in Lemma 1, we have $\partial\Pi_I^{nj}(a)/a < 0$. This implies that there exists a cost cut off a_I^{nj} such that $\Pi_I^{nj}(a_I^{nj}) = 0$. Firms with cost $a < a_I^{nj}$ will expect a positive net profit from FDI, whilst those with cost $a > a_I^{nj}$ will not. Following HMY, we further assume that f_I^j is sufficiently high that:

$$f_I^j > \int_{-\phi_L^j}^{\tau - \sqrt{\frac{4\gamma f_C^{nj}}{L^j}}} \left[\frac{L^j}{4\gamma}\left(\tau^{nj} - \phi\right)^2 - f_C^{nj}\right] d\Omega^j(\phi) \qquad (14)$$

This condition ensures that $a_I^{nj} < a_X^{nj}$ [9] i.e. those firms that have positive expected profits from FDI also find exporting profitable. Therefore firms with costs $a \in \left[a_L^n, a_I^{nj}\right]$ will

[9] Equation (14) also ensures that $\Pi_I(a_X^{nj}) < \Pi_I(a_I^{nj}) = 0$, so that $\Pi_I(a) < 0$ if $a > a_X^{nj}$. This means that firms with marginal costs above the exporting cut off will have negative net expected profit from FDI as well, so that not serving market *j* is their preferred option. Hence firms that find it profitable to undertake FDI will also find it profitable to export.

undertake FDI in country *j*, while those with $a \in \left[a_I^j, a_X^j\right]$ will choose to serve market *j* by exporting. However, not all subsidiaries will survive to produce. This will depend on the local cost draw, specifically on whether condition [12] is fulfilled. A subsidiary with cost $a < a_I^j$ will be closed down and the parent will revert to exporting to country *j*. But if the local cost draw is sufficiently low ($\phi < \phi^{nj}\left(a_i^j\right)$), the subsidiary will survive to serve the market.

The above configurations are summarized in Figure 1, which describes the distribution of different types of firms in country *n* according to their modes of serving a particular foreign market *j*. The horizontal axis represents firm marginal and the vertical axis is the probability density function. There are five types of firms[10]: (1) L type firms with $a \in [a_D^n, \infty]$ that are the least productive entrants and exit the industry. The remaining groups, which all serve the domestic market are: (2) D type firms with $a \in [a_D{}^n, a_X{}^{nj}]$ that do not sell in country *j*; (3) X type firms with $a \in [a_X{}^{nj}, a_I{}^{nj}]$ that export to country *j*; (4) XF type firms with $a \in [a_L{}^n, a_I{}^{nj}]$ whose subsidiaries in country *j* failed in their investment due to a high local cost draw, and the parent reverted to exporting; and (5) I type firms with $a \in [a_L{}^n, a_I{}^{nj}]$ whose subsidiaries realized low local production costs and therefore produce and sell in market *j*.

It is clear from Figure 1 that in the observed *home* productivity distribution multinational firms are not only *on average* more productive than other firms, but also have a productivity distribution that stochastically dominates that of exporters[11]. This result replicates those in HMY and Head and Ries (2003) where there is no FDI productivity certainty. But in contrast to HMY, we show that exporters and multinationals can *co-exist* within a certain range of firm productivity, which explains why even though multinationals are *on average* more productive than those who export, we still observe firms with similar productivities or sales *in the home market* taking divergent paths (exporting or FDI) in serving a given foreign market.

We close the model by characterizing the steady state equilibrium. Under free entry, the expected total profit of entrants in each country should be zero in equilibrium. These profits include potential profits from exporting or FDI. The export and FDI cost cut offs

[10] Our labeling of different types of firms parallels that in Baldwin (2005).
[11] See Girma, Kneller and Pisu (2005) for an empirical test on exporter and multinational productivities that finds that the latter stochastically dominates the former.

(a_X^{nj}, a_I^{nj}) are determined by domestic and the target market's domestic survival cutoffs, as shown in equations (10) and (13). Let $E(\pi_D^n)$ and $E(\pi_{IX}^n)$ denote the expected profit of serving the domestic market and all foreign markets (either by exporting or FDI) by potential entrants in country *n*, respectively. Then the free entry condition can be expressed as[12]:

$$E(\pi^n) = E\left[\pi_D^n\left(a_D^n\right)\right] + \sum_{j \neq n} E\left[\pi_{IX}^{nj}\left(a_D^j\right)\right] - f_E^n = 0 \quad , \quad \text{n=1, ….,N} \qquad (15)$$

This is a system of N equations and N cost ceilings $a_D^1,..., a_D^N$. Its solutions will determine all the country level equilibrium domestic cost ceilings as well as the exporting and FDI entry cost ceilings (a_X^{nj} and $a_I^{nj}, j \neq n$) from (10b) and (13). In the following section we derive testable cross-sectional predictions on firm survival across countries based on this equilibrium condition.

3. Survival of foreign-owned firms

3.1 *Determinants of firm survival*

We now consider the pre-entry probability of survival for individual firms. Consider a subsidiary in host country *j* of a parent from source country *n*. Let Γ_D^j and $\Gamma^{nj}(a^n)$ denote the survival probability of an indigenous firm and this subsidiary, respectively, where the parent of the subsidiary has cost a. From (9), the survival probability of an indigenous entrant in country *m* is given by

$$\Gamma_D^j = \Pr(\phi < \phi_D^j) = \Omega^j(a_D^j - \bar{a}^j) \qquad (16)$$

Using (12), the survival probability of the subsidiary is given by

$$\Gamma^{nj}(a) = \text{Prob}(\phi < \hat{\phi}^{nj}) = \Omega^n\left(\hat{\phi}^{nj}\right) = \Omega^n\left(a_D^j - a - \sqrt{\left(a_D^j - a - \tau^{nj}\right)^2 + \frac{4\gamma \cdot f_C^{nj}}{L^j}}\right) \qquad (17)$$

Both firms are subject to draws from the same cost distribution after entry, and therefore share the common determinants a_D^j and $\Omega^j(.)$. A shift of the distribution of the cost draw or a change in the survival cost ceiling will affect the survival probability of both indigenous firms and foreign subsidiaries in the same direction.

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[12] Further details are provided in the Appendix. The equilibrium condition is based on the assumption that trade costs are "non prohibitive" . That is τ^{nj} is sufficiently low so that there will be positive number of firms from *n* that find it profitable to export to country *j*.

Foreign-ownership does matter for a firm's survival, however, as revealed by comparing [16] and [17]. If a parent has the same cost as a indigenous firm, then the subsidiary of the former has a lower survival probability than the latter since $\Gamma^{nj}(\bar{a}^j) < \Gamma_D^j$. This leads to our first proposition:

Proposition 1: *A subsidiary is less likely to survive than an indigenous firm with the same productivity as its parent.*

This proposition corresponds to the popular view that multinationals are more "footloose". It is also supported by recent empirical studies at firm and plant level. When plant size and productivity are controlled for, Bernard and Sjoholm (2003) found that foreign firms are significantly *more likely* to shut down than *comparable* domestic plants in Indonesia. Gorg and Strobl (2003) show similar results for Irish plants.

The survival of a subsidiary also depends crucially on the cost of its parent. Lemma 1 shows that ϕ^{nj}, and hence $\Gamma^{nj}(a)$ is decreasing in a. In other words, the more efficient the parent, the more likely the subsidiary is to survive. The intuition for this result is straightforward. The subsidiary differs from indigenous firms in that it is a branch of a multi-plant firm with pre-determined home costs prior to entry. A lower cost of the parent means a greater chance of a lower cost subsidiary and thus higher subsidiary profits. A lower parent cost also means a greater opportunity cost of utilising the subsidiary, however, since the profits from exporting from the parent are also decreasing in a. But since the difference between the operating profit and opportunity cost ($\pi_I^{nj} - \pi_X^{nj}$) is decreasing in a[13], a higher cost parent will be more likely to find the opportunity cost of utilising the local plant exceeds its operating profit, and therefore to revert to exporting. It is also apparent from (17) that $\Gamma^{nj}(a)$ is increasing in τ^{nj}, as the higher the transport cost, the lower the profit from exporting. Furthermore, Lemma 1 and equation (17) indicate that $\Gamma^{nj}(a)$ is monotonically increasing in a_D^j, and equation (15) implies that the domestic survival cost ceiling in a country is lower the lower the average cost of indigenous entrants ($\partial a_D^j / \partial \bar{a}^j > 0$). Thus $\Gamma^{nj}(a)$ is increasing in $\bar{a}^j$. A fall in the cost of the representative indigenous firm leads to lower survival probabilities of foreign firms. As indigenous firms become more productive, foreign entrants find it more difficult to compete and more likely to close. We summarize these results in the following proposition:

[13] Inspection of (10a) and (11) reveals that for given ϕ, $\frac{\partial\left[\pi_I^{nj}(a) - \pi_X^{nj}(a)\right]}{\partial a} < 0$.

Proposition 2 *The survival probability of a foreign-owned firm is decreasing in the pre-entry cost of it parent firm, but increasing in the transport costs between the FDI host and home country and the average cost of indigenous firms.*

3.2 *Market Size Effects*

Now we consider the effects of market size on firm survival. The host country market size L^j has both direct and indirect effects on Γ^{nj}. Holding a_D^j constant, a rise in L^j has a positive effect on the sales, profits and hence survival of foreign-owned firms. However, a_D^j will also respond to L^j, implying that the total effect of market size on survival of foreign firms could be ambiguous. To focus on the market size effect, we now consider the special case where countries are symmetric in all parameters except their sizes. This enables us to rewrite the equilibrium conditions as:

$$E(\pi^n) = \frac{L^n}{4\gamma} V(a_D^n) + E(\pi_{XI}^n) - f_E = 0 \tag{18}$$

where $V(a_D^n) = \int_{-\phi_L}^{a_D^n - \bar{a}} [a_D^n - \bar{a} - \phi]^2 d\Omega(\phi)$. Equation (18) implies that

$$\frac{\partial a_D^n}{\partial L^n} = \frac{-V(a_D^n)}{L^n \int_{-\phi}^{a_D^n - \bar{a}} \Omega(\phi) d\phi} < 0 \tag{19}$$

The domestic survival cost ceiling a_D^n is lower when the market size L^n is larger[14]. Recall that the survival probability of indigenous firms Γ_D^j is increasing in a_D^j, therefore a larger market size will lead to a lower survival probability of indigenous firms. This result corresponds to one of the key outcomes from Melitz and Ottaviano (2005) that a larger market size leads to tougher competition, in terms of a larger number of operating firms and therefore a lower price cost markup . This eventually results in a lower pre-entry probability of firm survival. However, there is no FDI[15] in Melitz and Ottaviano (2005) so we next investigate whether this result extends to the subsidiaries of multi-plant multinationals, which unlike indigenous firms are heterogeneous prior to entry. Interestingly, inspection of

[14] This result is based on the presumption that firms' expected profit of serving the foreign market *j* only depend on the size of foreign country, and is independent of its own market size i.e. $\partial E(\pi_{IX}^{nj}) / \partial L^j \neq 0$, $\partial E(\pi_{IX}^{nj}) / \partial L^n = 0$. Melitz and Ottaviano (2005) obtained similar results in a two country model. The derivation of this result is available from the authors.

[15] Or subsidiaries enter on exactly the same terms as indigenous firms.

(17) reveals that the sign of $\partial\Gamma^{nj}(a)\big/\partial L^j$ is ambiguous, depending on the cost of the parent:

$$\frac{\partial\Gamma^{nj}(a)}{\partial L^j} \geq (<) 0 \;\Leftrightarrow a \leq (>) \tilde{a}^j \equiv a_D^j - \tau + \frac{\gamma f_C}{L^j \Delta^j} - \Delta^j \qquad (20)$$

where $\Delta^j = V(a_D^j)\Big/\int_{-\phi_L}^{a_D^j - \bar{a}} \Omega\,(\phi)d\phi$. This means that an increase in the size of the host market will increase (reduce) the probability of a subsidiary's survival depending on whether its parent cost is lower (greater) than a threshold $\tilde{a}^j$ [16]. Hence:

> **Proposition 3** *A larger market size leads to* ***(a)*** *a lower survival probability for all indigenous firms; and* ***(b)*** *a higher (lower) survival probability of a foreign-owned firm when its parent's home cost is sufficiently low (high).*

Most interestingly, while the effect of market size is homogenous for the survival of indigenous firms, foreign firms are affected differentially depending on the pre-determined home costs of their parents. The intuition is as follows. The total effect of market size on a foreign entrant's survival can be separated into the two opposing effects, and parent heterogeneity plays an important role in both. Firstly, a larger L^j has a direct positive effect on the survival of the subsidiary, by increasing the number of customers and therefore profits, which will increase its probability of survival. Call this *sales effect*. This positive sales effect is *increasing* in the parent's cost, as low cost subsidiaries can expand their sales by more in a larger market as they sell at lower prices. Secondly, a larger market size will lead to more intense market competition as reflected in a lower domestic survival cost ceiling and thus lower price mark-ups, profitability and probabilities of survival. Hence market size tends to have an indirect negative effect on firm survival. Call this the *competition effect*. Since price mark ups are decreasing in firms' marginal costs, as shown in (7), high cost subsidiaries tend to have lower price mark-ups, and thus will be most "sensitive" to the falling price mark up. Therefore the magnitude of the negative competition effect on firm survival is *smaller* the greater the foreign entrant's pre-entry home costs. When the foreign firm's home cost is sufficiently low (high) the sales (competition) effect dominates, and the net effect of market size on the subsidiary's survival probability will be positive (negative). Thus, in contrast to its clear cut negative

[16] This proof is also available from the authors.

effects on the survival of all indigenous firms, the host country's market size may indeed *raise* the survival probability of some foreign subsidiaries, but only those whose parent firms are sufficiently productive.

3.3 A numerical example

We now give a simple numerical illustration of the important role of parent heterogeneity in the overall host market size effect on subsidiary survival. We consider the special case in which there is one FDI source country *n* and two FDI host countries (*j*=1,2). Further details of the specifications of the parameters are listed in Appendix 2. In Figures 2-1 and 2-2, we plot the survival probability of subsidiaries as a function of their parent firm productivities. Figure 2-1 illustrates Proposition 2 that the subsidiary survival probability increases with its parent's productivity, and is higher in a country with less competitive domestic firms (i.e. higher average domestic cost $\bar{a}^j$). Figure 2-2 reveals that the market size effect is ambiguous, however, depending on pre-entry firm heterogeneity. The survival probability schedules will pivot anti-clockwise for a larger market size, leading to an intersection between these two schedules that determines the productivity threshold $\frac{1}{\tilde{a}} = 0.195$. Foreign-owed firms with parent productivity above this threshold will have a higher survival probability in the larger country (1) than the smaller country (2), while those with productivity below this threshold will have lower chances of survival in the larger country.

3.4 Investment Liberalization

An alternative interpretation of $\Gamma^{nj}(a^n)$ is as the fraction of subsidiaries in *j* with parents in *n* having cost a^n that survive. We can derive the overall survival rate of subsidiaries from this source (Λ_I^{nj}) by aggregating over a^n, obtaining

$$\Lambda_I^{nj} = \frac{\int_{a_L^k}^{a_I^{nj}} \Gamma^{nj}(a) dG^n(a^n)}{G^n(a_I^{nj})} \tag{21}$$

Clearly Λ_I^{nj} depends on a_I^{nj}, the cost cut off of FDI entry from country *n* to country *j*. An increase in the FDI cost cut off from country *n* to *j* will lead more high cost firms from country *n* to undertake FDI in *j*. But entry of more high cost foreign firms will reduce the overall survival rates of foreign subsidiaries, since survival probabilities of individual foreign firms are decreasing in their home costs (Proposition 1). Thus[17]

Lemma 2: $\dfrac{\partial \Lambda_I^{nj}}{\partial a_I^{nj}} < 0$

Inspection of (12) and (13) reveals that a_I^{nj} is decreasing in $f_I^{\,j}$ [18]. Therefore $\partial \Lambda_I^{nj} \big/ \partial f_I^{\,j} > 0$.

Let $\Lambda_I^j = \sum_{n \neq j} \Lambda_I^{nj}$ denote the overall survival rate of foreign firms located in country *j* from all source countries. We then have the following:

> **Proposition 4**. *Unilateral foreign investment liberalization - in terms of a reduction in the foreign investment entry costs - will lead to a lower average productivity as well as a lower overall survival rate of foreign firms.*

A reduction in the entry barrier for foreign investors will make it “easier” for foreign firms to set up their subsidiaries in the host country, leading to an increase in the number of foreign entrants. But this also means weaker selection of foreign investors based on their productivities. As a result, the average productivity as well as the overall survival rates of foreign entrants will be lower. By contrast, for indigenous firms, a falling domestic entry barrier (f_E^n) will lead to an *increase* in their average productivity, as a result of increased competition[19]. An important implication of this result is that, whilst domestic investment liberalization tends to improve the quality of domestic investments by introducing more competition and forcing less productive producers to exit, deregulation of foreign investment may lead to a deterioration of the *quality* of inward FDI – in terms of both average productivity and the overall failure rate of foreign investment, even though the *quantity* of FDI inflow will rise at the extensive margin. This implication is particularly interesting in that the surge of inward FDI as result of dramatic foreign investment liberalization in some emerging markets, such as China, seems to be accompanied by a rising proportion of foreign investors that incur an unexpected loss on their investment[20].

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[17] A proof of Lemma 2 is available from the authors.

[18] Equations [12]-[13] shows that a_I^{nj} is increasing in a_D^j, which is increasing in $f_I^{\,j}$.

[19] As shown in (11), a_D^n is increasing in f_E^n.

[20] Business surveys conducted by independent consultant companies revealed that 35%-45% foreign companies are running a net deficit . (see Infatuation’s End , Economist 1999)

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4. Conclusion

Recent firm level empirical studies provide robust evidence that, other things equal, foreign-owned firms are more likely to close down plants than domestic-owned firms. This paper builds a general equilibrium model that incorporate heterogeneous firms and productivity uncertainty faced by multinationals in the FDI host country to investigate the survival of foreign firms. We show that, while a bigger market always induces stronger selection and thus lower survival probability for domestic firms, it may however lead to greater (lower) survival probability for foreign owned firms when the home productivities of their parent firms are sufficiently high (low).These results provide a new dimension – namely the interaction between market size and firm heterogeneity- for further empirical investigations on the survival or failure of multinational’s foreign subsidiaries. Furthermore, our model yields predictions on the effect of investment liberalization on the overall survival rate of foreign firms: a fall in the entry barrier to foreign investors will reduce the average productivity of foreign entrants, leading to a *lower* overall survival rate of foreign firms.

Our model very parsimoniously captures the role of market size and firm heterogeneity in the survival of foreign firms. To achieve this parsimony we concentrate on horizontal FDI, omitting vertical FDI and fragmentation, which constitutes a large proportion of FDI flows to emerging economies such as China. Future research could be directed towards extending the more complex organizational forms of multinational firms as described in Grossman, Helpman and Szeidl (2003), to include *local* production uncertainty and firm heterogeneity. We hope that our model sets up a useful framework towards that end.

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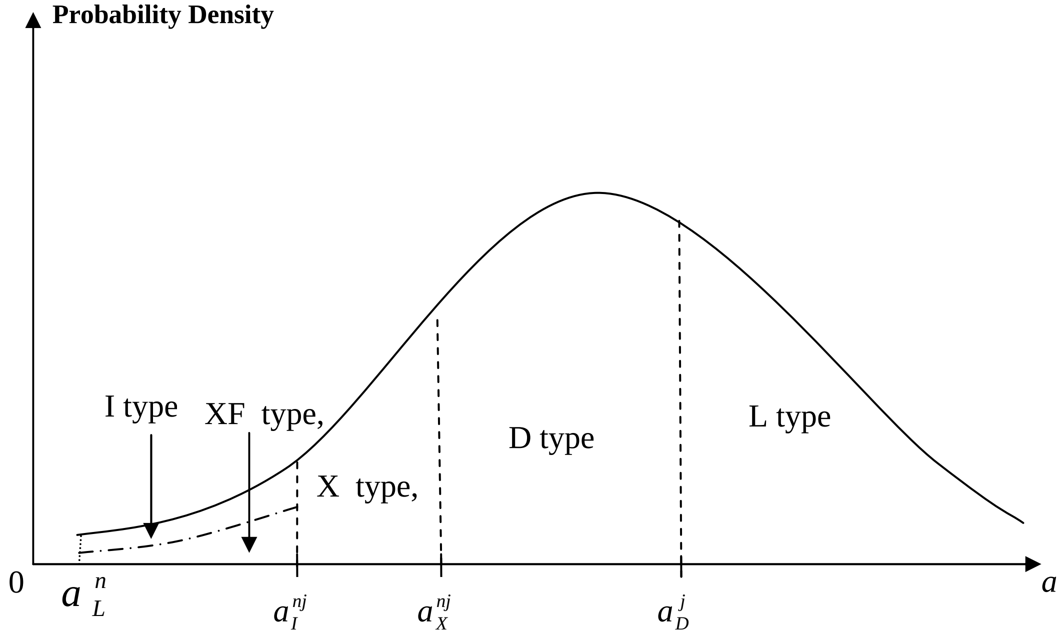
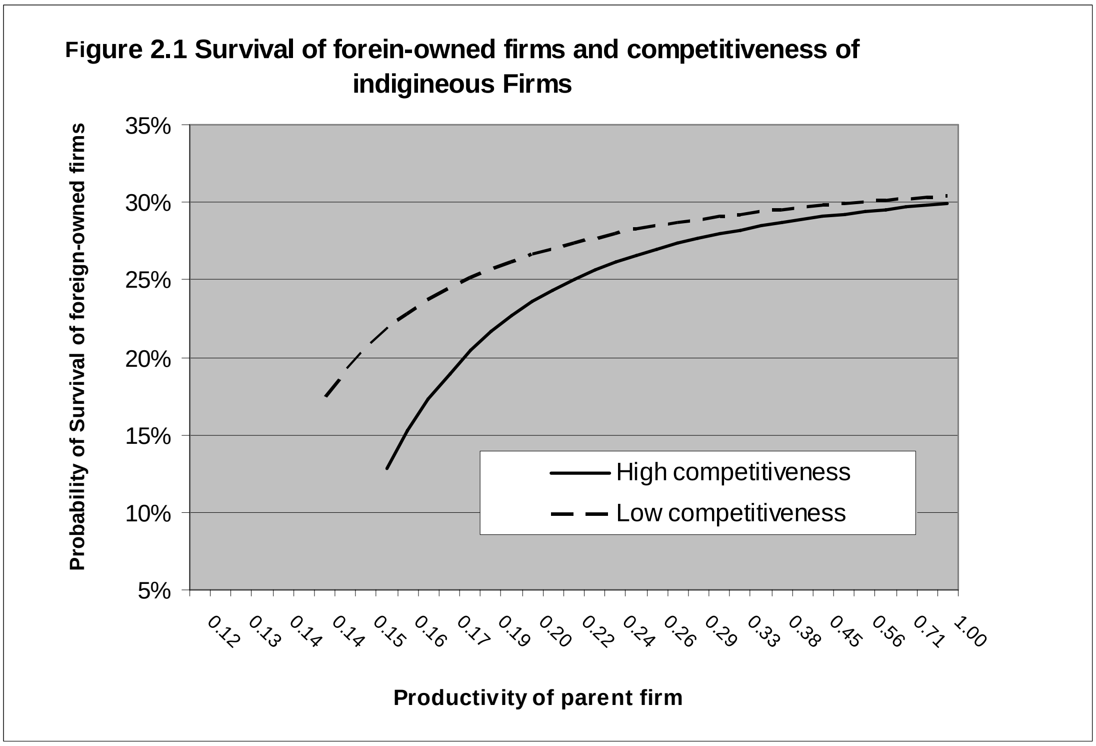
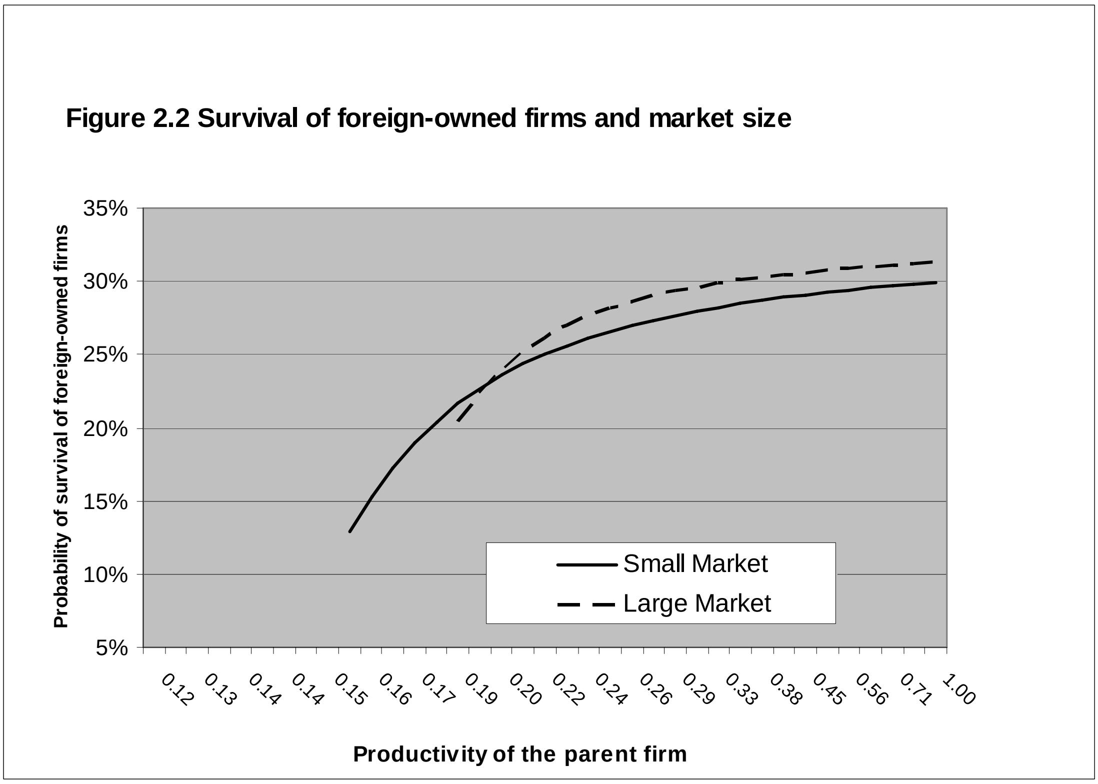


Figure 1. Productivity hierarchy under FDI uncertainty

This figure summarized the productivity distributions of firms from country *n* by their means of serving the domestic market and country *j*. L type: exiters who are lowest productivity firms that exit the industry ;D type :non-exporters who are low productivity firms that do not serve country *j* ;X type :exporters , who are high productivity firms that serve country *j* by exporting without previous experience of investment ,XF type : FDI losers , who are highest productivity firms that serve country *j* by exporting , but previously failed in their investment ,I type : FDI winners , who are highest productivity firms that serve country *j* via their local foreign subsidiary



Note: This figure shows the results of our simulation as described in appendix 2. Other things equal, foreign-owned firms will be more likely to survive in a country where indigenous firms are less "competitive" (higher $\bar{a}^{j}$).



Note: This figure shows the results of our simulation as described in appendix 2. Country 1 has a larger market size with $L^1 = 2000$, country 2 has a smaller market size with $L^2 = 1000$. Survival probability of a foreign firm is greater(smaller) in the larger markets if the pre-determined productivity of its parent firm is high(low).

Appendix 1

Here we provide further details on the free entry conditions. $E(\pi_D^n)$ and $E(\pi_{IX}^n)$ can be written as :

$$E(\pi_D^n) = \frac{L^n}{4\gamma}\int_{a_L^n}^{a_D^n}[a_D^n - a]^2 dG^n(a) = \frac{L^n}{4\gamma}\int_{-\phi_L^n}^{a_D^n - \bar{a}^n}[a_D^n - \bar{a}^n - \phi]^2 d\Omega^n(\phi)$$

$$E(\pi_{IX}^n) = \sum_{j\neq n} E\left[\pi_{IX}^{nj}\right]$$

$$= \sum_{j\neq n}\left\{\begin{array}{l} \int_{a_I^{nj}}^{a_X^{nj}} \pi_X^{nj}(a)dG^n(a) + \int_{a_L^n}^{a_I^{nj}}\left(\int_{-\phi_L^j}^{\hat{\phi}^{nj}(a)} \pi_I^{nj}(a,\phi)d\Omega^j(\phi) - f_I^j\right)dG^n(a) \\ + \int_{a_L^n}^{a_I^{nj}}\int_{\hat{\phi}^{nj}(a)}^{\infty} \pi_X^{nj}(a)d\Omega^j(\phi)\,dG^n(a) \end{array}\right\}$$

$$= \sum_{j\neq n}\left\{\begin{array}{l} \int_{a_I^{nj}}^{a_X^{nj}} \frac{L^j}{4\gamma}[a_D^j - a - \tau^{nj}]^2 dG^n(a) \\ + \int_{a_L^n}^{a_I^{nj}}\left[\int_{-\phi_L^j}^{\hat{\phi}^{nj}(a)} [\frac{L^j}{4\gamma}[a_D^j - a - \phi]^2 - f_c^{nj}]d\Omega^j(\phi) + \int_{\hat{\phi}^{nj}(a)}^{\infty}\frac{L^j}{4\gamma}[a_D^j - a - \tau^{nj}]^2 d\Omega^j(\phi) - f_I^j\right]dG^n(a) \end{array}\right\}$$

These conditions imply that the expected domestic profit is increasing in the domestic survival cost ceiling of the home country a_D^n , whilst the total expected profit from serving the foreign markets will depend on all the survival cost ceilings of all foreign countries a_D^j, $j \neq n$.

Appendix 2

Let n denote the FDI home country and $j=1,2$ denote the FDI host country. The specifications of the parameters for the numerical example shown in figure 2 are as follows:. $\bar{a}^n = 0.5$, $\bar{a}^j = 2$, $\tau^{nj} = 1$, $\gamma = 3$, $f_C = 200$, $f_E = 500$, L_1=1000, L_2=2000. The distribution of the cost draw is uniform: $\Omega^1(\phi) = \Omega^2(\phi) = \Omega^H(\phi) = \dfrac{\phi - \phi_L}{\phi_M - \phi_L}$, where $\phi_L = 1$, $\phi_M = 5$. The following equilibrium variables are derived given different market size:

	L_1=1000	L_2=2000
$1/a_D^j$	0.13	0.15

$1/a_X^{nj}$	0.15	0.17
$1/\tilde{a}^{n}$	0.195	

Appendix 3

Proof of *Proposition 3*.

(i) Let $V(a_D^n) = \int_{-\phi_L}^{a_D^n - \bar{a}} [a_D^n - \bar{a} - \phi]^2 \, d\Omega(\phi)$, we can rewrite condition [18] as:

[A1] $$V(a_D^n) = \frac{4\gamma\left[f_e \; - E(\pi_{XI}^n)\right]}{L^n}$$

Hence

[A2] $$\frac{\partial a_D^n}{\partial L^n} = \frac{-4\gamma\left[f_e \; - E(\pi_{XI}^n)\right]}{\frac{\partial V(a_D^n)}{\partial a_D^n}\left(L^n\right)^2}$$

Using Leibniz integral rule , we derive

[A3] $$\frac{\partial V(a_D^n)}{\partial a_D^n} = 2\int_{-\phi_L}^{a_D^n - \bar{a}} \Omega(\phi) d\phi$$

Substituting [A2] and [A1] back to [A3], we have

[A4] $$\frac{\partial a_D^n}{\partial L^n} = \frac{-V(a_D^n)}{2L^n \int_{-\phi_L}^{a_D^n - \bar{a}} \Omega(\phi) d\phi} < 0$$

(ii) $$\frac{\partial \phi^{nj}(a)}{L^j} = \frac{\partial a_D^j}{\partial L^j} - \frac{\left(a_D^j - a - \tau\right)\frac{\partial a_D^j}{\partial L^j} - \frac{2\gamma f_c}{\left(L^j\right)^2}}{\sqrt{\left(a_D^j - a - \tau\right)^2 + \frac{4\gamma f_c}{L^j}}}$$

Let $\Delta^j = \frac{-L^j \partial a_D^j}{\partial L^j} = \frac{V(a_D^j)}{\int_{-\phi^n}^{a_D^j - \bar{a}} \Omega(\phi) d\phi}$, we have

$$\frac{\partial \phi^{nj}(a)}{L^j} \geq 0 \Leftrightarrow \sqrt{\left(a_D^j - a - \tau\right)^2 + \frac{4\gamma f_c}{L^j}} - \left(a_D^j - a - \tau\right) < \frac{2 f_C \gamma}{L^j \Delta^j}$$

$$\Leftrightarrow a_D^j - a - \tau \geq \Delta^j - \frac{\gamma f_C}{L^j \Delta^j} \Leftrightarrow a \leq a_D^j - \tau + \frac{\gamma f_C}{\Delta^j L^j} - \Delta^j$$

Since $\Gamma^{nj}(a) = \Omega\left(\hat{\phi}^{nj}\right)$ is increasing in $\hat{\phi}^{nj}(a)$, we have the result in (23) :

$$\frac{\partial \Gamma^{nj}(a)}{\partial L^j} \geq 0 \Leftrightarrow \quad a \leq a_D^j - \tau + \frac{\gamma f_C}{\Delta^j L^j} - \Delta^j$$

Proof of *Lemma 2*

$$\frac{\partial \Lambda_I^{nj}}{\partial a_I^{nj}} = \frac{\Gamma^{nj}(a_I^{nj}) g^n(a_I^{nj}) G^n(a_I^{nj}) - g^n(a_I^{nj}) \Lambda_I^{nj}(a_I^{nj}) G^n(a_I^{nj})}{\left(G^n(a_I^{nj})\right)^2} = \frac{g(a_I^{knj})\left[\Gamma^{nj}(a_I^{nj}) - \Lambda_I^{nj}(a_I^{nj})\right]}{G^n(a_I^{nj})}$$

where $g^n(a) = \frac{dG^n(a)}{da}$.

Since

$$\Lambda_I^{nj}(a_I^{nj}) = \frac{\Gamma^{nj}(a_I^{nj}) G^n(a_I^{nj}) - \int_{a_L^n}^{a_I^{nj}} G^n(a) d\Gamma^{nj}(a_I^{nj})}{G^n(a_I^{nj})} = \Gamma^{nj}(a_I^{nj}) - \frac{\int_{a_L^n}^{a_I^n} G^n(a) d\Gamma^{nj}(a_I^{nj})}{G^n(a_I^{nj})}$$

,

We have

$$\frac{\partial \Lambda_I^{nj}}{\partial a_I^{nj}} = \frac{g(a_I^{nj}) \int_{a_L^n}^{a_I^{nj}} G^n(a) d\Gamma^{nj}(a_I^{nj})}{\left(G^n(a_I^{nj})\right)^2}$$

Recall that $\frac{d\Gamma^{nj}}{da_I^{nj}} < 0$, therefore $\frac{\partial \Lambda_I^{nj}}{\partial a_I^{nj}} < 0$