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*Openness, Managerial Incentives and Heterogeneous Firms*

by

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## Abstract

Motivated by new evidence that managerial incentives play an important role in determining firm productivity, this paper incorporates the principal-agent mechanism into the new heterogeneous firm trade framework to examine the link between openness and endogenous firm productivity. We show that firm heterogeneity plays a crucial role in the effects of openness on firms' optimal incentive contracts via the trade-induced "carrot and stick" effect. This mechanism increases the marginal value of managerial effort, which motivates the firm owners (principals) to offer a higher power contract to the managers (agents) to reduce managerial slacks. The intra-firm managerial incentive mechanism stressed in this paper could be viewed as complementary to the inter-firm reallocation effect in the Melitz (2003) model in explaining the observed link between openness and aggregate productivity.

**JEL Classification:** F12, F13

**Keywords:** openness, managerial incentives, heterogeneous firms, productivity.

## Outline

1. *Introduction*
2. *The Closed Economy*
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4. *Why Firm Heterogeneity Matters? The Carrot and Stick Effects*
5. *Concluding Remarks*
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## Non-Technical Summary

Why might openness enhance productivity? Motivated by new empirical evidence that managerial incentives play an important role in determining firm productivity, this paper examines the link between openness and endogenous firm productivity by incorporating the principal-agent mechanism into the new heterogeneous firm trade framework. We show that firm heterogeneity plays a crucial role in the effects of openness on firms' optimal incentive contracts and productivity via the trade-induced "carrot and stick" effect. In the equilibrium where firms are substantially heterogeneous and self-select into the export market, increasing trade openness may reward the low-cost global exporters whilst penalising the high-cost purely domestic firms. This will increase the value of managerial effort devoted to cost reduction, which motivates the firm owners (principals) to offer a higher power contract to the managers (agents) to reduce managerial slacks. As a result, managerial incentives and efforts increase, leading to improvements in firm and industrial productivity. However, this mechanism may not be at work when trade liberalization is driven by falling fixed cost of exporting, or when firms are quasi-homogenous and all export in equilibrium. In the later case, increasing openness may even lead to weaker managerial incentives and losses in firm productivity.

Recent micro level trade studies have found robust empirical evidence that increasing exposure to trade via declining trade costs leads to significant productivity gains at both firm and industry level. We argue that, the interactions between the intra-firm managerial incentive mechanism and the degree of firm heterogeneity stressed in this paper, could be viewed as complementary to the inter-firm reallocation effect in the Melitz (2003) model in explaining the observed link between openness and aggregate productivity.

# 1. Introduction

The relationship between openness and productivity might be one of the most fundamental questions in international economic studies. It is often argued that openness could raise productivity, yet there has been little consensus on the mechanisms through which such positive effects would occur. Recent micro-level empirical trade studies shed some new light on this issue by providing robust evidence on the link between firm heterogeneity and trade. Firstly, a new empirical regularity emerging from the literature is that firms exhibit astounding differences in their productivity performances even within very narrowly defined industries<sup>1</sup>, and such heterogeneity is found to be an important dimension to international trade. Specifically, exporting firms are more productive, larger and more skill intensive than non-exporters and this is mainly due to the self-selection effect into the export market. (See for example, the pioneering work by Bernard and Jensen 1999, Clerides et al. 1998, and Bernard 2007 and Greenaway and Kneller 2007 for survey). Secondly, the literature also offers strong evidence on productivity gains stemming from firm level responses to increasing exposure to trade driven by falling trade barriers (see for example, Pavnick 2002, Bernard, Jensen and Schott. 2006). In particular, after analysing the US plants' responses to increasing exposure to trade via falling trade costs, Bernard, Jensen and Schott (2006) concludes:

“ We find that greater exposure to international trade via declining trade costs promotes productivity gains at three levels: across industries within manufacturing, across plants within industries, and *within* plants.” (p 918 , emphasis added)

“...we provide the first comprehensive evidence of a relationship between trade liberalization and productivity growth *within* plants in a developed economy...” (p 934 , emphasis added)

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<sup>1</sup> See for example, Foster, Haltiwanger and Syverson (2005) shows the dramatic differences in their labour productivity in the same five digit industry.

Whilst the across plant reallocation effects of trade is well explored in the burgeoning heterogeneous firm trade models pioneered by Melitz (2003)<sup>2</sup>, there is little consensus on why increasing exposure to trade could lead to *within* firm productivity gains, which is perhaps an equally important channel as the reallocation effect via which trade boosts aggregate productivity. One seemingly straightforward explanation for the trade-induced firm productivity gains is that more fierce competition from foreign rivals reduces managerial slack. But most of the theoretical IO literature on competition and X-(in)efficiency derives ambiguous results ( see for example, Holmstrom 1982, Nalebuff and Stiglitz 1983, Hart 1983, Scharfstein 1988, Schmidt 1997, Raith 2006), and none of these models explicitly examines the effects of openness and international trade<sup>3</sup>. Further, as the current wave of globalization is increasingly driven by multilateral reduction in trade costs, the “innovation upon import competition” argument may not be able to fully capture the effects of openness when an economy is opened up to both import competition and export opportunities. One explanation for why opening up to export market may boost firm productivity is the learning-by-exporting hypothesis, but the empirical evidence on this is weak and highly ambiguous. (see for example, Clerides et. al 1998 and Greenaway and Kneller 2007 for a review).

In this paper we attempt to bring trade theory closer to the new firm level evidence on the openness-productivity nexus by incorporating the principal-agent mechanism into the new heterogeneous firm trade framework pioneered by Melitz (2003). Motivated by recent evidence that managerial incentives could be an important determinant of firm productivity (Bandiera, Barankay, Rasul 2007)<sup>4</sup>, we build a monopolistic competitive industry model where firm owners (principals) provide incentive contracts to the managers (agents) to reduce managerial slacks. Firm productivity is a stochastic outcome that increases in managerial efforts, which in turn respond to the power of the optimal incentive scheme. Firm productivity is thus determined by the optimal contractual choice of the managerial incentives, which is endogenous to the characteristics of the industry such as entry costs,

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<sup>2</sup> In Melitz (2003) where firms differ substantially in their productivities, opening up to trade leads to aggregate productivity gains by reallocating market shares towards more productive firms that export and expand, away from least productive firms that remain purely domestic and shrink or exit. Bernard, Redding and Schott (2006) incorporates heterogeneous firms into Heckcher-Ohlin framework and shows opening up to trade leads to both across industry and within industry reallocation of resources.

<sup>4</sup> For example, Bandiera, Barankay and Rasul (2007) show from their firm level field experiment that that stronger managerial incentives raises worker productivity. In another empirical study, Bloom and Reenen (2007) shows that management practice is highly associated with firm productivity.

product differentiation etc. and, most importantly, the degree of the openness of the economy.

One distinguishing feature of our model is that firm heterogeneity plays a crucial role in the directions of the effects of increasing openness on managerial incentives and firm productivity. To stress its importance we investigate two possible types of equilibria in the open economy. In the first scenario, firms differ substantially in their marginal costs and trade barriers are high, so that in equilibrium only a fraction of firms self-select into the export market: only lowest cost firms will export to the global markets whilst the highest cost firms will remain purely domestic. We call such equilibrium “heterogeneous firm trade equilibrium”. (HFT) In this case, increasing openness- triggered by falling variable trade costs - will unambiguously increase the power of the incentive contract, leading to stronger managerial incentives and compensation, and thus reduces managerial slacks. The main reason for this is that the “carrot and stick” effect is at work: when trade costs fall, the lowest-cost exporting firms will be rewarded with the “carrot” of increasing profits and outputs due to their further expansions into the global markets, whilst the high cost domestic firms will be penalized by the “stick” of decreasing domestic profits and outputs as a result of the increasing foreign competition. Consequently, the marginal value of cost-reducing managerial effort, determined by the profit differential between the low and high cost status, will increase<sup>5</sup>. As the managerial effort becomes more “valuable” to the firm, the owner is motivated to induce the managers to exert greater effort by providing a more powerful incentive contract and raise managerial compensation. In other words, when trade costs fall, firms have greater incentives to motivate their managers to work harder, so as to increase the probability of being rewarded with the “carrot” and reduce the risk of being penalized by the “stick”. This could ultimately reduce managerial slacks and boosts firm level and industrial productivity. Finally, both consumers and managers are better off, since the improvement of firms’ internal efficiencies leads to falling prices and boosts managerial pay. However, in contrast to Melitz (2003), falling fixed costs of exporting may not always lead to stronger managerial incentives and boost productivity. This is because the “carrot and stick” effect becomes ambiguous: the profits of low-cost exporters may not

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<sup>5</sup> This is a common feature in the Schumpeterian literature and endogenous growth models that the incentive of cost reduction depends not on the absolute level of profits, but the profit differential between high and low cost status. Changes in the economic environment may dampen the incentive to reduce cost as long as the profit differential falls, even if the absolute size of profits increases. See for example , Schmidt (1997).

always increase their profits, although the high-cost purely domestic firms will always suffer from a profit loss.

To examine the importance of heterogeneity in driving these results, we investigate a second type of equilibrium in which trade costs are low and the degree of firm heterogeneity is small so that all firms export in equilibrium, we call this “quasi-homogeneous firms trade equilibrium”(QHF). In contrast to the HFT equilibrium, increasing openness will never enhance managerial incentives and firm productivity in this case. The main reason for this is that when all firms export in equilibrium, falling trade costs affect the profits of all firms in the same direction: all firms increase their profits in the export markets but lose profits and market shares in their domestic markets. The net result is unchanged firm level total profits and output (when variable trade costs fall) or a decreasing profit differential between low and high cost firms (when the fixed cost of exporting falls). The “carrot and stick effects” is not at work at all. Consequently, when the economy is more opened up , the value of managerial efforts to the firm does not increase, and thus firms has no incentives to further reduce managerial slack and firms’ internal efficiencies will not improve.

The contrasts between the HFT equilibrium and the QHF equilibrium reveal that , firm heterogeneity is not only vital to the *inter*-firm reallocation effects via which openness boosts productivity as stressed in Melitz (2003), but also could be the key dimension to explain the *intra*-firm productivity gains induced by trade liberalization. Whilst increasing openness of the economy may reallocate the market shares away from low productivity firms towards high productivity firms, our analysis show that firms may not only react *passively* by adjusting their output margins, but also foresee such new threats and opportunities and thus adapt their internal contract structure *proactively* to cope with the changing global market environment. Hence, by incorporating the principal-agent problem into the heterogeneous firm trade framework, we may attribute the trade-induced intra-firm productivity gains at least partly to the optimal incentive contract mechanism, which could be viewed as complementary to the inter-firm reallocation mechanism in the original Melitz (2003) model in explaining the link between openness and aggregate productivity.

Our model adds to the small but fast growing theoretical literature on the link between openness on and endogenous firm productivity. The model closest to ours is Trindade



(2005), whose approach mainly focuses on the leisure-income trade-off decisions by firm owners. Motivated by the responses of firm owners facing import competition in Portugal, he reveals a new mechanism via which import competition boosts firm productivity. In his model, import competition increases total varieties of goods available to the consumers, which reduces the real price of consumption basket and increases the value of income. Since firm owners are also consumers who “love variety”, they are motivated to work harder (seeking for better technology) and raise income, leading to higher productivity. In a related recent study, Davidson, Matusz and Shevchenko (2008) builds a model with perfectly competitive product market and imperfect labour market with heterogeneous workers, they explore the impact of openness on firm productivity via the mechanism of changing skill mix within firms that differ in their technology choices. Our model focuses on a quite different mechanism from the above two papers, stressing the importance of export market selection based on the recent heterogeneous firm trade framework, and explains the trade-induced productivity gains focuses on the role of principle-agent mechanism within firms that is perhaps more prevalent for firms in developed countries such as those in the US (see Bernard et al.2006).

Our model can also be viewed as complementary to Horn, Lang, Lundgren (1994) and Marin and Verdier (2007), which we found to be the few theoretical contributions focusing the link between openness and managerial incentives. Horn, Lang, Lundgren (1994) adopt a very different industry structure with a fixed number of oligopolistic competitive firms whose owners hire managers to organise production. Moving from autarky to free trade increases output of all firms that export, and also raises labour costs, leading to stronger managerial incentives to save labour costs and affect firms X-efficiency. Marin and Verdier (2007) argue that international trade affects industry productivity by affecting the distribution of the modes of corporation organization – in terms of the internal power allocation between the headquarter and middle manager.

The remainder of the paper is organized as follows. In the next section we introduce the key features of the model. In section 3 we proceed to the open economy model by investigating the HFT and QHF equilibrium separately. This is followed by section 4 , in which we compare and contrast the results from the two equilibriums , and discuss the important role of firm heterogeneity and the “carrot and stick” effects. Some concluding remarks are made in section 5. Most of the details of proofs are relegated to the Appendix.

## 2. The Closed Economy

The model builds upon the well-known workhorse Dixit-Stiglitz-Krugman model of trade in the presence of horizontal product differentiation and monopolistic competition (Dixit and Stiglitz, 1977; Krugman, 1980). We incorporate *across* firm heterogeneity (Melitz 2003) and the *within* firm incentive contract between the owner and the manager as the key mechanism that endogenises firm productivity. We start with considering a closed economy in which a single factor (labour) is used to produce output in two sectors. Sector H produces a homogenous good and sector D differentiated products.

### 2.1 Demand and production

The preferences of a representative consumer are Cobb-Douglas across the outputs of the two sectors, with  $E$  being the total expenditure and  $\beta$  the fraction of expenditure on differentiated products. Production in sector H exhibits constant return to scales and we choose the homogenous good as the numeraire. Selecting units so that one unit of labour is required to produce one unit of the homogenous good, implies the wage rates are also unity. Full employment is maintained through adjustment in the size of the H sector, so labour supply to sector D is perfectly elastic. Preferences for sector D products are assumed to be the well-known C.E.S (Constant Elasticity of Substitution) form over a continuum of varieties indexed by  $i$  :

$$[1] \quad U = \left( \int_{i \in \Theta} q_i^\rho di \right)^{\frac{1}{\rho}}, \quad 0 < \rho < 1$$

Where the set  $\Theta$  represents the varieties available to consumers. Elasticity of substitution between varieties is assumed to be constant given by  $\sigma = \frac{1}{1-\rho} > 1$ . Consumer behavior in sector D can be modeled as if they were consuming an aggregate product  $Q \equiv U$  with aggregate price

$$[2] \quad P = \left( \int_{i \in \Theta} p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}$$

This yields a constant elasticity of demand function for each variety:

$$[3] \quad q_i = Q \left( \frac{p_i}{P} \right)^{-\sigma} = A \cdot p_i^{-\sigma}$$

where  $A \equiv \frac{\beta E}{P^{1-\sigma}}$  is the demand shifter for each individual variety  $i$  that is produced by a unique firm. Technology of production is characterised by the following cost function:

$$[4] \quad C_i = a_i q_i + f_E$$

Where  $C_i$  is total cost, and  $a_i$  and  $f_E$  are constant marginal cost and fixed entry cost in labour units, respectively.

It is well-known that under the Dixit-Stiglitz monopolistic competition, the pricing rule by a profit maximising firm will be a constant mark-up over its marginal cost :

$$[5] \quad p_i = \frac{a_i}{\rho}$$

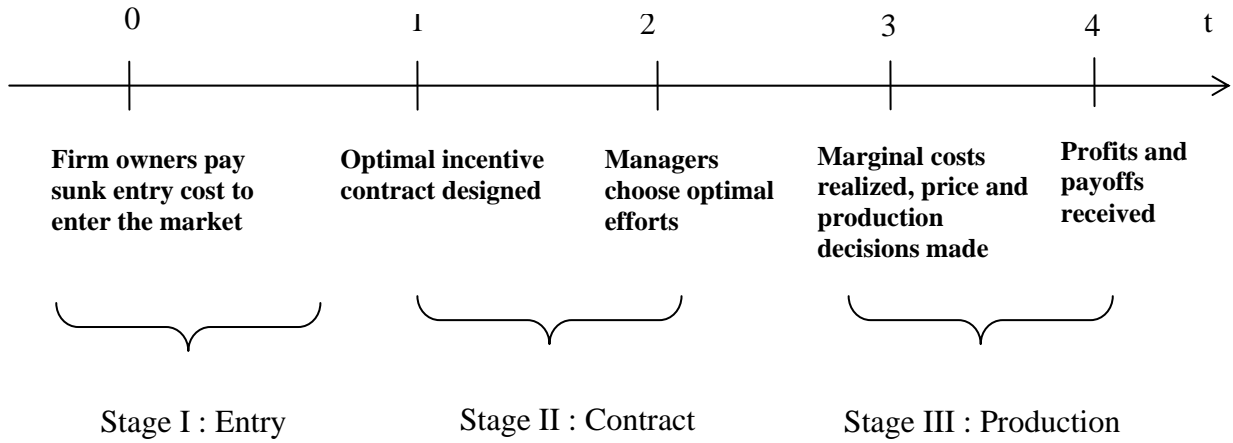
Operating firm profit and revenues are then given by

$$[6] \quad \pi_i(a_i) = p_i q_i - C_i = B \cdot a_i^{1-\sigma} - f_E \quad , \quad r_i(a_i) = p_i q_i = \sigma \cdot B \cdot a_i^{1-\sigma}$$

where

$$[7] \quad B \equiv \rho^{\sigma-1} (1-\rho) A = \rho^{\sigma-1} (1-\rho) \beta E \cdot P^{\sigma-1}$$

**Figure 1. Time Sequencing of the Firm Behavior**



$B$  is a transformed demand shifter that is considered as exogenous by individual firms. It can also be viewed as an inverse measure of the competitiveness of the product market: a lower  $B$  corresponds to a lower aggregate price ( $P$ ), leading to a lower firm level profit, indicating a market with greater competition.

## 2.2 The optimal incentive contract

### *The agency problem*

We assume the separation of ownership and control within the firm, leading to the existence of a standard principal-agent problem between the owner and the manager. The basic set up of the sequencing of firm behaviour is illustrated in figure 1. There is an unbounded mass of prospective entrants to sector  $D$ . During stage I, at date 0 firm owners decide whether to pay an irreversible sunk fixed cost  $f_E$  to enter the market. If the owner decided to enter, she will set up a plant and then hire a manager to organise the production and reduce costs at stage II. In this stage, the owner designs a managerial incentive contract that optimises her net expected payoff, and the manager will respond by choosing her optimal effort that maximises her expected utility, depending on the power of the incentive provided by the contract. Finally at stage III the production costs, which depend on the managerial efforts, will be realised, then firms will make decisions on their prices and

outputs to maximise profits. Payments to the owners and the managers are received, all markets clear.

Next we show the details of the model stage by stage. Firstly, assume each firm in the market is owned by a principal (owner or shareholders), who pays the entry cost  $f_E$  (e.g. research and development of the variety) to start a firm and then hires an agent (the manager) on a competitive market for identical agents. The main task of the manager is to exert effort and improve the firm's productive efficiency by reducing the marginal cost of production  $a_i$ . For example, she can be the head of a division monitoring the workers, controlling the quality of inputs, experimenting with new production methods etc. We assume that the outcome of the manager's effort denoted as  $e_i$ , is uncertain because the marginal cost is affected by an independent and identically-distributed (i.i.d.) random influence  $\alpha_i$  :

**Assumption 1**  $a_i = [1 - \gamma(e_i)]\alpha_i$ , where  $\gamma(0) = 0$ ,  $\lim_{e_i \rightarrow \infty} \gamma(e_i) = 1$ ,  $\gamma'(e_i) \geq 0$ ,  $\gamma''(e_i) \leq 0$  and  $\alpha_i$  is a i.i.d. random variable with cumulative distribution  $F(\alpha)$  and positive support  $[\alpha_L, \alpha_M]$ .

Let  $\bar{\alpha} > 0$  denote the positive mean of  $\alpha$ . This assumption implies the firm's expected marginal cost is  $\bar{a} = \bar{\alpha} - \gamma(e_i)\bar{\alpha}$ . Hence, if the manager exerts zero effort ( $e_i=0$ ), the firm's marginal costs will be purely random with mean  $\bar{\alpha}$ , and  $\gamma(e_i)$  denotes the proportional expected cost reduction resulting from increasing managerial effort. Also note that  $\gamma''(e) \leq 0$ , which indicates decreasing returns to efforts, i.e. the marginal (expected) reduction in costs decreases with increasing efforts. The c.d.f. of cost  $a_i$  is therefore  $G(a;e) = F[a(1-\gamma(e))^{-1}]$  with support  $[a_L, a_M]$ , where  $a_L = \alpha_L[1-\gamma(e)]$  and  $a_M = \alpha_M[1-\gamma(e)]$ <sup>6</sup>.

The firm owner(principal) is assumed to be risk neutral. Her payoff function is firm profit net of managerial compensation  $W_i$  :

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<sup>6</sup> Note that by this assumption, an increase in  $e$  will change the upper and lower boundaries of  $G(a;e)$ .

$$[8] \quad \Pi_i = \pi_i - W_i$$

For simplicity the manager is also assumed to be risk neutral. Her utility function is

$$[9] \quad U_M = W_i - D(e_i)$$

where  $D(e_i)$  is the manager's disutility of effort and  $D'(e) > 0, D''(e) > 0$ , with  $D(0) = 0$ . We assume that, due to information asymmetry the manager's effort is not observable to the owner. As a result, due to the steep disutility function of effort, the manager may get slack and not exert her maximum effort level. Hence, there is a standard moral hazard problem between the owner and the manager. The owner therefore will design an incentive scheme to reduce managerial slack and induce the manager to exert greater effort. Since managers are identical, they will choose the same effort level simultaneously, which will in turn determine the distribution as well as the average level of firm productivity.

#### *Optimal effort and the incentive contract*

The realized marginal cost  $a$  is assumed to be fully observable by the owner and therefore contractible. We assume that the owners offer their managers the following compensation scheme:<sup>7</sup>

$$[10] \quad W_i = s + b(1 - (a_i/\bar{\alpha}))$$

where  $s$  is fixed salary,  $b$  is the "piece rate" and  $1 - (a_i/\bar{\alpha})$  the (proportional) cost reduction observable by the principal. The firm owner's problem is to design an optimal incentive scheme i.e. choose appropriate  $s$  and  $b$  in order to maximize her net expected payoff, which equals the expected operating profit net of expected manager

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<sup>7</sup>Such linear compensation scheme is very common in business practice. It is also common in the theoretical IO literature on competition and X-(in) efficiency to assume a linear incentive contract (see for example, Raith 2006). In international economics studies, Grossman and Helpman (2004) also assume a linear contract between the firm and the agent to study firms' outsourcing decisions. Also see Holmstrom and Milgrom (1987) that shows the circumstances under which it is optimal for the principal to offer a linear contract. For a full discussion on the conditions under which the linear incentive scheme would be optimal, see Bolton and Dewatripont (2003).

compensation:  $E(\Pi) = E(\pi) - E(W)$ . Using [6] the firm's expected operating profit can be written as:

$$[11] \quad E(\pi) = \int_{a_L}^{a_M} \pi(a) dG(a; e) = BV(e) - f_E$$

where  $V(e) = \int_{a_L(e)}^{a_M(e)} a^{1-\sigma} dG(a; e) = \Delta(e)\Omega$  ,  $\Delta(e) = [1 - \gamma(e)]^{1-\sigma}$  and

$\Omega = \int_{\alpha_L}^{\alpha_M} \alpha^{1-\sigma} dF(\alpha)$ . Note that  $V'(e) > 0$  , i.e firms' expected operating profit increases with managerial effort. Hence, the owner's problem can be written as (we suppress the firm index  $i$  hereafter):

$$[12a] \quad \max_{s,b,e} \{E(\Pi)\} = \max_{s,b,e} \{E(\pi) - E(W)\} = \max_{s,b,e} \{BV(e) - [s + b\gamma(e)] - f_E\}$$

Subject to :

$$[12b] \quad e \in \arg \max_{\tilde{e}} \{s + b \cdot \gamma(\tilde{e}) - D(\tilde{e})\} \quad (\text{Incentive Compatible Constraint})$$

$$[12c] \quad E(U_M) = s + b\gamma(e) - D(e) \geq U_r \quad (\text{Participation Constraint})$$

The incentive compatible constraint gives the optimal effort exerted by the manager for given compensation scheme  $\{b, s\}$ . [12b] can be replaced with the following first order

condition<sup>8</sup>:  $b = \frac{D'(e)}{\gamma'(e)}$ . It can be shown that  $e$  is increasing in  $b$ <sup>9</sup>, indicating that the

manager will exert greater effort, the larger the reward of cost reduction offered by the owner. Further, the participation constraint [12c] gives the minimum compensation acceptable by the manager: she will accept any contract with pair  $\{b, s\}$  that yields an

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<sup>8</sup> Assumption 1 ensures that the second order condition is always satisfied, which guarantees an internal solution.

<sup>9</sup> This can be derived straightforwardly from the assumption  $D''(e) > 0, \gamma''(e) < 0$ .

expected utility of at least her reserve utility  $U_r$ .<sup>10</sup> In order to maximise her payoff the owner will choose a salary scheme that drives the expected manager utility down to zero, implying that the owner will choose  $s = D(e) - b\gamma(e) + U_r$  so that  $E(W) = s + b\gamma(e) = D(e) + U_r$ .<sup>11</sup> Substitute this to [12a], we can rewrite the owner's problem as:

$$[13] \quad \max_e \{BV(e) - D(e) - U_r - f_E\}$$

The owner's problem therefore is equivalent to choosing an optimal managerial effort ( $e$ ) that maximises her expected net payoff. A higher effort will increase the owner's expected operating profit, but also increases the compensation paid to the manager. Due to this trade-off, the optimal  $e$  therefore depend on the shape of the profit function as well as the manager's disutility function. Specifically, we can solve [13] by its first order condition:

$$[14] \quad B = \frac{D'(e^*)}{V'(e^*)} \quad (\text{OI :Optimal Incentive condition})$$

Where  $e^*$  denotes the optimal effort as a solution for problem [13].<sup>12</sup> We can therefore obtain the optimal piece rate ( $b$ ) and salary ( $s$ ) set by the owner in the incentive contract:

$$[15] \quad b = \frac{D'(e^*)}{\gamma'(e^*)}$$

$$[16] \quad s = D(e^*) - \frac{D'(e^*)}{\gamma'(e^*)} \gamma(e^*)$$

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<sup>10</sup> For simplicity we assume  $U_r$  is exogenous, in appendix we discuss the possibility of endogenous reserve utility and why the qualitative result of our model will largely remain unaffected.

<sup>11</sup> This implies that the manager generate zero "rent" or "surplus" utility on average, a simplification that is common in existing trade models with endogenous manager/owner efforts, see for example Grossman and Helpman (2004), Horn et al. (1994) and Trindade (2005).

<sup>12</sup> Note that the second order condition  $\frac{D'(e^*)}{V'(e^*)} V''(e^*) - D''(e^*) < 0$  is always satisfied since  $\gamma'(e) < 0$ . It

also implies  $\left(\frac{D'(e^*)}{V'(e^*)}\right)' > 0$



It can be further shown that  $\frac{\partial e^*}{\partial B} > 0, \frac{\partial b}{\partial e^*} > 0, \frac{\partial s}{\partial e^*} < 0$  .<sup>13</sup> The following lemma summarises the above results:

**Lemma 1** *For given market condition  $B$ , the owner will design an optimal incentive contract with piece rate ( $b$ ) and salary ( $s$ ) given in (15)-(16), where the manager choose optimal effort  $e^*$  specified in (14).  $e^*$  is increasing in  $B$  and  $b$ , but decreasing in  $s$ .*

We close the model by assuming free entry of firms until the owner's expected pay off is zero. From [13], this condition can be written as:

$$[17] \quad E(\Pi) \equiv BV(e^*) - D(e^*) - U_r - f_E = 0 \quad (\text{ZEP: Zero expected payoff condition})$$

### 2.3 Existence and uniqueness of the equilibrium

The equilibrium can be determined by combining the ZEP condition ( equation [17] ) and the OI condition ( equation [14] ) :

$$[18] \quad \left\{ \begin{array}{l} B = \frac{D'(e^*)}{V'(e^*)} \quad (\text{OI}) \\ BV(e^*) - D(e^*) = f_E + U_r \quad (\text{ZEP}) \end{array} \right.$$

The equilibrium  $e^*$  and  $B$  are thus determined by the two different relationships between these two key endogenous variables. Firstly, as can be seen in figure 2a, the (OI) condition defines an increasing relationship between  $e^*$  and  $B$ , whereas the (ZEP) condition defines a U shape relationship. The intuition for the U shape ZEP curve can be described as below. Along the ZEP curve, each pair of  $e^*$  and  $B$  yields same (zero) net expected payoff for prospective owners. When  $e^*$  is low and  $B$  is high, i.e. to the left of point  $E$ , an increase in  $e^*$  tends to increase the owner's expected net payoff (profit net of compensation), this is

<sup>13</sup> Use footnote 12 and assumption  $\gamma''(e) \leq 0$ .

because the marginal increase in profit from cost reduction is high (due to high  $B$ ) and the increase in compensation is low (due to low  $e^*$  and therefore low marginal disutility of the manager). Hence, when  $e^*$  increases  $B$  has to decline so as to keep the net payoff equal zero, yielding a downward slope. However when the pair  $\{e^*, B\}$  passes point  $E$  so that  $e^*$  is high, an increase in  $e^*$  tends to decrease owner's payoff because the increase in manager compensation will be high as a result of her steep disutility function, whilst the increase in profit from cost reduction is low due to decreasing return of effort. This requires an increase in  $B$  in order to keep the owner's net payoff unchanged, implying an upward slope. The intersection of the (OI) and (ZEP) schedules is always at the ZEP curve's bottom point  $E$ , which defines a unique pair of  $(e^*, B)$  at the equilibrium.<sup>14</sup> A more formal proof of the existence and uniqueness of the equilibrium can be shown as follows. We can substitute [14] to [17] to obtain:

$$[19] \quad J(e^*) \equiv \frac{D'(e^*)}{V'(e^*)}V(e^*) - D(e^*) = f_E + U_r$$

It can be shown that  $J(e^*)$  is monotonically increasing in  $e^*$  <sup>15</sup>, which ensures a unique and positive solution. The following proposition summarises the above results:

**PROPOSITION 1** *Under free entry, there exists unique industry equilibrium in managerial efforts  $e^*$  as defined in [19], and incentive contractual choices, in which each firm chooses an optimal piece rate and salary as defined in [15]-[16].*

Since the equilibrium distribution over firms' marginal costs  $G(a; e^*) = F[a(1 - \gamma(e^*))^{-1}]$  is endogenous to  $e^*$ , the distribution of firm level performances such as productivity, profit, size, etc. are also dependent on  $e^*$ , which is in turn determined by our model parameters such as the entry costs, the elasticity of substitution and the shape of manager's disutility function etc. However, it is worth noting that as shown in (19) the optimal effort  $e^*$  is independent of aggregate revenue  $E$ , therefore the equilibrium distribution of firm marginal

<sup>14</sup> See appendix A for a formal proof of why the ZEP curve is U shape.

<sup>15</sup>  $J'(e^*) = \left(\frac{D'(e^*)}{V'(e^*)}\right)'V(e^*) + \left(\frac{D'(e^*)}{V'(e^*)}\right)V'(e^*) - D'(e^*) = \left(\frac{D'(e^*)}{V'(e^*)}\right)'V(e^*)$ . From footnote 12,  $\left(\frac{D'(e^*)}{V'(e^*)}\right)' > 0$ . so  $J'(e^*) > 0$ .

cost  $G(a; e^*)$  is also independent of the size of the market<sup>16</sup>. Thus, changes in market size will have no impact on firm level behaviors such as cost, price, output etc. The only effect of market size is to increase (proportionally) the number of firms.

## 2.4 Analysis of the equilibrium

Next we show how other key aggregate variables of the closed economy are determined in equilibrium and then conduct some comparative statics analysis. Firstly we can derive the number of firms  $N$  as a function of  $e^*$ <sup>17</sup>

$$[20] \quad N(e^*) = (1 - \rho)\beta E \cdot \left( \frac{V'(e^*)}{D'(e^*)} \right) V^{-1}(e^*)$$

Since  $\left( \frac{D'(e^*)}{V'(e^*)} \right)' > 0$  (see footnote 11),  $N'(e^*) < 0$ , so the greater the optimal managerial effort and incentive, the smaller the number of firms producing in the market. Intuitively, greater managerial efforts leads to lower industry cost ( $\bar{a}(e^*)$ ), which intensifies market competition and therefore deters the entry of new firms. Reasoning analogously, other endogenous model variables can be derived as follows<sup>18</sup>:

$$[21a] \quad \bar{a}(e^*) = \int_{a_L(e^*)}^{a_M(e^*)} a dG(a; e^*) = (1 - \gamma(e^*)) \int_{\alpha_L}^{\alpha_M} \alpha dF(\alpha), \quad (\text{Average industry cost})$$

$$[21b] \quad b(e^*) = \frac{D'(e^*)}{\gamma'(e^*)}, \quad (\text{Managerial incentive})$$

<sup>16</sup>. This result is common to models with constant elasticity of substitution between varieties. In Melitz(2003) where the elasticity of substitution is exogenous, market size also has no impact on the productivity cut off that determines the aggregate industry productivity. However, in a more general case where the elasticity of substitution is endogenous to market size, the firm level variables may indeed respond to market size.

<sup>17</sup> Rewrite the aggregate price as a function of  $N$  and  $e$ :

$$P = \left( \int_i p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = N^{\frac{1}{1-\sigma}} \left( \int_{a_L}^{a_M} a^{1-\sigma} dG(a; e) \right)^{\frac{1}{1-\sigma}} \rho^{-1} = N^{\frac{1}{1-\sigma}} V_d(e)^{\frac{1}{1-\sigma}} \rho^{-1}$$

Substitute this expression to [7] and we obtain  $B \equiv \beta E \cdot P^{\sigma-1} \cdot \rho^{\sigma-1} (1 - \rho) = (1 - \rho)\beta E \cdot V_d(e)^{-1} N^{-1}$ . Using the expression of equilibrium  $B$  in equation [14], we obtain the equilibrium number of firms as a function of the optimal effort:

<sup>18</sup> In this paper we adopt the presumption that the aggregate revenue  $E$  is exogenous. in appendix we discuss the case where  $E$  is endogenous and why our main results concerning managerial incentives and productivity remain unaffected.

$$[21c] \quad \bar{W}(e^*) \equiv E(W) = D(e^*) + U_r, \quad (\text{Average managerial compensation})$$

$$[21d] \quad P(e^*) = N^{\frac{1}{1-\sigma}} V(e^*)^{\frac{1}{1-\sigma}} \rho^{-1} = \left( \frac{D'(e^*)}{V'(e^*) \rho^{\sigma-1} (1-\rho) E} \right)^{\frac{1}{\sigma-1}} \quad (\text{Aggregate price})$$

$$[21e] \quad U(e^*) = \beta E \cdot P(e^*)^{-1} = \left( \frac{D'(e^*)}{V'(e^*) \rho^{\sigma-1} (1-\rho)} \right)^{\frac{1}{1-\sigma}} E^{\frac{\sigma}{\sigma-1}} \quad (\text{Welfare})$$

Clearly, these key variables of the economy all depend on  $e^*$ . It can further be shown that they are also monotonic functions of  $e^*$ :  $\bar{a}'(e^*) < 0$ ,  $b'(e^*) > 0$ ,  $\bar{W}'(e^*) > 0$ ,  $P'(e^*) > 0$ ,  $U'(e^*) < 0$ . Next we conduct comparative static analysis on how various shocks to the economy affect the equilibrium  $e^*$ , leading to changes in the above economy-wide variables.

### *Comparative Statics*

First consider the effect of the entry cost  $f_E$ . From [18], it is obvious that an increase in  $f_E$  has no effects on the (OI) curve but shifts upwards the (ZEP) curve. Consequently, both the equilibrium  $e^*$  and  $B$  will increase when  $f_E$  rises. Secondly, an increase in the elasticity of substitution  $\sigma$  also increases  $e^*$  and  $B$ . This can be shown in figure 2b: the ZEP curve shifts upward when  $\sigma$  increases, whilst the OI curve will shift to the right<sup>19</sup>, the new equilibrium  $E'$  therefore must locate at the top right of the original equilibrium.

**PROPOSITION 2** *The power of the contract ( piece rate ), managerial efforts and average managerial compensation are higher, the higher the fixed entry cost ( $f_E$ ), or varieties are more substitutable ( $\sigma$  higher).*

PROOF: see Appendix B

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<sup>19</sup> [14] can be written as  $B = \frac{D'(e^*)[1-\gamma(e^*)]^\sigma}{(\sigma-1)\gamma(e^*)}$ . Since  $\gamma(e^*) \in [0,1]$ , increasing  $\sigma$  reduces  $B$  for

given  $e^*$ , meaning the OI curve will shift to the right for greater  $\sigma$ . On the other hand, for the ZEP condition in [17], since increasing  $\sigma$  increases  $V(e^*) = [1-\gamma(e^*)]^{1-\sigma} \Omega$ , the ZEP curve will shift upwards for greater  $\sigma$ .

The intuition behind proposition 2 is that higher entry costs deter the entry of new firms, therefore each firm has greater market share and output, leading to a greater value of cost reduction. Thus, firm owners provide higher-power contracts (a higher piece rate  $b$ ) to managers to induce them greater exert efforts. Managerial compensation also has to increase to compensate for managers' increasing disutility. Furthermore, an increase in the elasticity of substitution also reduces the number of firms as each firm charges lower prices, this again increases the output and market share of existing firms, leading to greater managerial incentives and compensation. Based on proposition 2, we can further derive the effects of entry costs and the degree of product substitutability on welfare.

**COROLLARY 1.** *Industrial cost, the number of varieties and total consumer welfare are higher, the lower fixed entry cost ( $f_E$ ), or the higher the degree of product differentiation ( $\sigma$  lower).*

When entry costs fall or products are more substitutable, managerial efforts decreases, leading to higher average industry costs and lower productivity. On one hand, the increasing industry costs tend to push up average price and therefore be welfare reducing (the “efficiency effect”). But, on the other hand, the number of firms and total varieties will actually increase due to falling entry barriers or greater product differentiation. Such “variety effect” is certainly welfare enhancing, since consumers “love variety”. Perhaps somewhat surprisingly, it turns out that the variety effect dominates the efficiency effect, leading to a decrease in the aggregate price and an increase in welfare.

The above results from the comparative statics, especially those on industrial productivity, are consistent with Raith (2003) but somewhat differ from those in the original Melitz model. Raith (2003) sets up a model of oligopolistic industry to study the relation between competition and managerial incentives. Despite the very different setup of his model to ours, both models yield the conclusion that lower degree of entry costs and product substitution will reduces managerial incentives and industrial productivity, but increase welfare. On the other hand, in Melitz(2003) lower entry costs will increase the threshold productivity required for firm survival ,and therefore raises industrial productivity. In our model, however, firm level productivity fall as firms provide weaker incentives to managers, who respond by exerting less effort that lowers firm and industry productivity. However, in both

models the welfare effects are positive for falling entry costs due to the dominate role of the variety effect.

### 3. The Open Economy

Now we examine the case of open economy, where the world is composed of  $m+1$  identical countries. We adopt the standard simplifying assumption that all countries produce the homogeneous good, which is always costlessly traded. Assuming that equilibrium in the constant returns-to-scale, homogeneous sector ties down the equilibrium wage, the wage will then be equalised across countries, which is normalized to one. In the absence of international trade costs in the differentiated good sector, opening up to trade will allow countries to replicate the outcomes of an integrated world economy. Firms will sell in all countries and therefore behave as if they were operating in an integrated world market. Trade has the same impact on countries in an open economy as would an increase in market size on a closed economy. As was previously described changes in market size has no impact on equilibrium firm behaviors. Most importantly, the optimal incentive contract within firms remain unchanged. As a result, the firm level performances such as marginal cost, prices and output are also the same as those in the closed economy. The difference, however, is that firms now divide their outputs into domestic and foreign sales. Further, consumers in each economy are better off, since they have access to a larger number of varieties. Hence, the existence of *across firm* heterogeneity and *within firm* incentive contract do not make substantial differences relative to the original Krugman model.

But what if trade is costly? Recent empirical studies on firm level trade revealed that exporting incurs not only variable trade costs such as transport costs, but also fixed costs that is invariant to the export volume (see for example, Robert and Tybout 1997). To capture such stylized facts we follow the heterogeneous firm trade literature pioneered by Melitz (2003) to assume that there exists fixed cost of exporting  $f_x > 0$  in addition to the

traditional melting iceberg trade cost  $\tau > 120$ . The assumption of a positive fixed export cost is crucial for firms' self-selection into the export markets when they are heterogeneous in terms of productivity<sup>21</sup>. In the following analysis we first consider the equilibrium in which the degree of firm heterogeneity is high and trade costs are significant so that only a fraction of firms export. We then compare and contrast the results to the second type of equilibrium where trade costs are low and firms are homogeneous so that all firms export. Such comparisons help to reveal the crucial role of firm heterogeneity in the directions of the effects of increasing openness on intra-firm incentive contract and industrial productivity.

### 3.1 Export selection, Firm heterogeneity and Managerial Incentives

In this section we examine the effects of trade on managerial incentives where firms are substantially heterogeneous and trade barriers is high so that exporters self-select into the global markets. We call this "heterogeneous firm trade equilibrium" (HFT).

#### Equilibrium

For a prospective exporter with marginal cost  $a$ , its potential export profit is given by (note the symmetric country assumption):

$$[22] \quad \pi_x(a) = m \left[ B(\tau a)^{1-\sigma} - f_x \right], \quad \forall a \in [a_L, a_M]$$

To ensure that firms select into the export market, the following assumption is imposed throughout this section:

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<sup>20</sup> For every one unit of good to arrive the export destination,  $\tau$  units of goods have to be shipped.

<sup>21</sup> There is robust empirical evidence that only more productive firms become exporters because they can overcome the high fixed costs of breaking into the foreign markets, see Greenaway and Kneller (2007) and Tybout (2003) for a survey.

**Assumption 2** Trade costs  $(f_X, \tau)$  are high relative to the fixed entry cost  $(f_E)$  so that the following inequality holds<sup>22</sup>

$$[23] \quad f_X \tau^{\sigma-1} > B(a_M)^{1-\sigma}$$

This assumption guarantees the partitioning of firms by export status and productivity levels: only a fraction of firms with the lowest costs can earn a positive export profit, whereas export profits for the highest-cost firms are always negative:  $\pi_X(a_M) = B(a_M \tau)^{1-\sigma} - f_X < 0$ . Now define  $a_X$  the export cost cut off such that  $\pi_X(a_X) = 0$ . From [22] we can obtain:

$$[24] \quad a_X = B^{\frac{1}{\sigma-1}} \phi^{-1}$$

Where  $\phi \equiv f_X^{\frac{1}{\sigma-1}} \tau$  is a measure of the trade costs. Firms with marginal cost  $a \in [a_L, a_X]$  will export to all foreign markets, whilst those with  $a \in (a_X, a_M]$  will remain purely domestic. The firm owner's operating profit and revenue is then given by

$$[25] \quad \pi(a) = \begin{cases} \pi_d(a) = Ba^{1-\sigma} - f_E & , \text{ non-exporters with } a \in (a_X, a_M] \\ \pi_d(a) + \pi_X(a) = Ba^{1-\sigma}(1 + m\tau^{1-\sigma}) - mf_X - f_E & , \\ & \text{exporters with } a \in [a_L, a_X] \end{cases}$$

$$r(a) = \begin{cases} r_d(a) = \sigma \cdot B(a\tau)^{1-\sigma} & , \text{ non-exporters with } a \in (a_X, a_M] \\ r_d(a) + r_X(a) = \sigma \cdot Ba^{1-\sigma}(1 + m\tau^{1-\sigma}) & , \\ & \text{exporters with } a \in [a_L, a_X] \end{cases}$$

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<sup>22</sup> Note that both  $B$  and  $a_M$  are endogenous variables, but as will be shown later,  $B$  is increasing in  $f_E$  and  $a_M$  is decreasing in  $f_E$ . Thus when  $f_E$  is sufficiently low, the right hand side of the inequality (20) will be sufficiently small to ensure that the inequality holds.



Thus, using [23], an entrant's expected operating profit in the domestic and foreign markets are, respectively:

$$[26] \quad E(\pi_d) = \int_{a_L}^{a_M} \pi_d(a) dG(a; e) - f_E = BV(e) - f_E$$

$$[27] \quad E(\pi_X) = m \int_{a_L}^{a_X} \pi_X(a) dG(a; e) = Y(B, e, \tau, f_X)$$

Where

[28]

$$Y(B, e, \tau, f_X) \equiv m \int_{a_L}^{(B)^{\frac{1}{\sigma-1}} \phi^{-1}} [B(\tau a)^{1-\sigma} - f_X] dG(a; e) = m \int_{a_L}^{(B)^{\frac{1}{\sigma-1}} \phi^{-1} \Gamma^{-1}(e)} [B(\tau \alpha)^{1-\sigma} \Gamma^{-1}(e) - f_X] dF(\alpha)$$

$$, \Gamma(e) \equiv 1 - \gamma(e)$$

Hence, the firm's total expected operating profit is

$$[29] \quad E(\pi) = E(\pi_d) + E(\pi_X) = BV(e) + Y(B, e, \tau, f_X) - f_E$$

The compensation scheme to the managers will take the form of  $W = s + b[1 - (a/\bar{\alpha})]$  as in the closed economy. Reasoning analogous to the case of the closed economy, the owner's problem is then to design a contract that maximises her expected net payoff  $E(\pi) - E(W)$ , subject to the incentive compatible constraint and the participation constraint as described in [12b]-[12c]. So the owner's total expected net payoff can be written as 23

$$[30] \quad \Pi(e, B, \tau) = E(\pi) - E(W) = BV(e) + Y(B, e, \tau, f_X) - D(e) - f_E - U_r$$

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<sup>23</sup> Again, reasoning analogous to the case of the closed economy, we can obtain  $b = D'(e) / \gamma'(e)$  and  $E(W) = D(e) + U_r$ .

Hence, the owner's problem is

$$[31] \quad e^* = \arg \max_e (BV(e) + Y(B, e, \tau, f_X) - D(e) - f_E - U_r)$$

This yields the following first order condition:

$$[32] \quad BV'(e^*) + Y_{e^*}(B, e^*, \tau, f_X) = D'(e^*) \quad \text{(Optimal Incentive)}$$

Furthermore, to ensure that  $e^*$  is the internal solution to the owner's optimisation problem [31], we impose the following assumption so that the second order condition is satisfied:

$$\textbf{Assumption 3} \quad \left( \frac{Y_e(B, e, \tau)}{V'(e)} \right)' < \left( \frac{D'(e)}{V'(e)} \right)'$$

Analogous to the case of the closed economy, [32] gives the condition under which the firm owner will choose the optimal managerial effort  $e^*$  to maximise her net payoff in an open economy with trade costs  $\tau, f_X$ . It can further be shown that

**Lemma 2 :** *Equation (32) defines  $B$  as an implicit function of  $e^*$ , and  $B$  is monotonically increasing in  $e^*$ .*

*Proof:* see appendix C

Finally, as in the closed economy equilibrium, the free entry condition leads to the zero total expected profit of prospective firm owners:

$$[33] \quad \Pi(B, e^*, \tau, f_X) \equiv BV(e^*) + Y(B, e^*, \tau, f_X) - D(e^*) - f_E - U_r = 0$$

Thus, [32] and [33] gives two different relationships between  $e^*$  and  $B$ :

$$\left\{ \begin{array}{l} BV'(e^*) + Y_{e^*}(B, e^*, \tau) = D'(e^*) \end{array} \right. \quad \text{[OI]}$$

$$BV(e^*) + Y(B, e^*, \tau, f_X) - D(e^*) = f_E + U_r \quad [\text{ZEP}]$$

Again, analogous to the case of the closed economy, the OI and ZEP condition defines one upward sloping curve and one U shape relationship between  $B$  and  $e^*$ . The unique intersection of these two schedules determines the two endogenous variables  $\{e^*, B\}$ .

**PROPOSITION 3** Equation (32) and (33) determines a unique equilibrium pair  $\{e^*, B\}$ ,

which in turn determines the unique contractual choice (optimal piece rate)  $b = \frac{D'(e^*)}{\gamma'(e^*)}$ .

*Proof:* See appendix D.

It is noteworthy that the equilibrium  $B$  is increasing in entry cost  $f_E$ . This is because increasing  $f_E$  shifts the ZEP curve upwards whereas left OI curve unchanged, leading to a higher  $B$  and  $e^*$ .

The probability of exporting is then given by

$$p_{ex} = G(a_X; e^*) = F(\alpha_X) \quad , \quad \alpha_X = B[\phi(1 - \gamma(e^*))]^{-1}$$

As was the case of the closed economy,  $e^*$  plays an important role in determining the aggregate variables of the system. The aggregate price now can be written as

$$[34] \quad P = \left\{ \int_{i \in \Omega_d} [p(i)]^{1-\sigma} di + \int_{i \in \Omega_x} [p_X(i)]^{1-\sigma} di \right\}^{\frac{1}{1-\sigma}} = \rho^{-1} \left[ NV(e) + m \cdot p_{ex} \cdot N \int_{a_L}^{a_X} (\pi)^{1-\sigma} dG(a, e) \right]^{\frac{1}{1-\sigma}}$$

Substitute [34] to [7] and using [33] we obtain the equilibrium number of firms

$$[35] \quad N = \frac{\beta E}{\sigma \left[ BV(e^*) + p_{ex} m \int_{a_L}^{a_X} B(\pi)^{1-\sigma} dG(a, e^*) \right]} = \frac{\beta E}{\sigma \{ p_{ex} [D(e^*) + f_E + U_r + m f_X p_{ex}] + (1 - p_{ex}) BV(e^*) \}}$$

The aggregate price and welfare are

$$[36] \quad P = \left[ \frac{B\sigma}{\beta E} \right]^{\frac{1}{\sigma-1}} \rho^{-1}$$

$$[37] \quad U = \beta E P^{-1} = (\beta E)^{\frac{\sigma}{\sigma-1}} [B\sigma]^{\frac{1}{1-\sigma}} \rho$$

It is noteworthy that welfare is decreasing in  $B$ , consumers will be better off in a more competitive market. Next we investigate the impacts of openness and trade liberalisation on the economy, focusing on the managerial incentives, productivity and welfare effects.

### Effects of trade openness and trade liberalisation

#### *Autarky to Trade*

We first consider the impact of a transition from the closed economy to the open economy. Let  $e_T^*$  and  $e_A^*$  denote the equilibrium optimal efforts in the open economy and autarky, respectively. Substitute [32] to [33] and rearrange we can rewrite [33] as:

$$[38] \quad J(e_T^*) + H(B, e_T^*, \tau, f_X) = f_E + U_r$$

Where  $H(B, e_T^*, \tau, f_X) \equiv Y(B, e_T^*, \tau, f_X) - \frac{Y_{e_T^*}(B, e_T^*, \tau, f_X) \cdot V(e_T^*)}{V'(e_T^*)}$ . Recall that

$J(e_A^*) = f_E + U_r$  (as shown in [19]), we obtain :

$$[39] \quad J(e_T^*) - J(e_A^*) = H(B, e_T^*)$$

It can be shown that  $H(B, e_T^*) < 0$  (see appendix E). Recall that  $J(e^*)$  is monotonically increasing in  $e^*$ , therefore [39] implies that  $e_T^* > e_A^*$ : the optimal effort is greater in the open economy than that in the closed economy. Since the contractual choice (piece rate)

$b(e^*) = \frac{D'(e^*)}{\gamma'(e^*)}$  is increasing in  $e^*$  and the average industry marginal cost

$\bar{a}(e^*) = [1 - \gamma(e^*)] \int_{\alpha_L}^{\alpha_M} \alpha dF(\alpha)$  decreasing in  $e^*$ , the open economy equilibrium exhibit a

greater power of the incentive contract , managerial efforts , and productivity than autarky. The above result can also be shown graphically in Figure 3a. Comparing equation [14] and [32] it is straightforward that the OI schedule is lower in the open economy than that in the closed economy<sup>24</sup> . Reasoning analogously, the ZEP schedule is also lower in the open economy than that in the closed economy. Thus, both (OI) and (ZEP) schedules will shift downwards when the economy moves from autarky to open economy. Intuitively, for a given level of  $e^*$  , firm owner’s net payoff is greater in the open economy than in the closed economy , since she can earn extra prospective profits from the export markets. Thus if  $B$  remains unchanged or increases, the OI or ZEP condition will fail to hold. As a result, for any given  $e^*$  ,  $B$  has to be lower in the open economy on both schedules. Hence , in the new open economy equilibrium ,  $B$  must be lower than the old autarky equilibrium , indicating an increasing competitiveness of the market , reflected in the lower demand shifter for each individual firm in the domestic market. Since by definition (equation [7]) the aggregate price  $P$  increases in  $B$  , moving from autarky to the open economy also reduces aggregate price of the differentiated good sector and is welfare improving. The following proposition summarises:

**PROPOSITION 4** *Under the (HFT) equilibrium, the power of incentive contract ( piece rate), managerial effort , managerial compensation , and industry productivity will be higher when the economy moves from autarky to costly trade. Aggregate price decreases and welfare increases.*

It is noteworthy that the welfare gains from openness is mainly due to the increasing firm productivity (falling marginal costs) – as a result of rising managerial efforts – rather than changes in the number of total varieties. Indeed , the impact of changes in the number of total varieties is very similar to that shown in Melitz (2003) : whether  $N_v = N_T(1 + mp_{ex}) > N_A$  is ambiguous , since openness allows consumers to access foreign varieties but may also lead to loss of domestic varieties due to more intensified competition in the domestic market. It is the “efficiency effect” that always dominates the “variety effect “and leads to improving welfare.

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<sup>24</sup> Note that [32] can be rewritten as  $B = \frac{D'(e^*) - Y_{e^*}(B, e^*, \tau, f_X)}{V'(e^*)}$ , so for given  $e^*$  ,  $B$  is lower on [32] than [14].

### *Falling $\tau$*

The above analysis shows that a transition of the regime from autarky to an open economy increases managerial incentives and firm productivity, and is welfare enhancing. However, very few of the economies of the world today are under autarky, so it is perhaps more important to ask how does trade liberalisation – in terms of incremental decrease in trade costs – affect firm level managerial incentives and the productivity of the economy. Now we analyse the effects of decreasing variable trade costs  $\tau$  on  $e^*$  and other key endogenous variables. As shown in figure 3b, analogous to the effects of moving from autarky to trade, falling  $\tau$  shifts both OI and FE curves downwards<sup>25</sup>. Intuitively, for a given level of  $e^*$ , when trade costs fall, firm owner's net payoff increases as a result of increasing expected profits from the export market. Thus, if  $B$  remains unchanged or increases, the OI or ZEP condition will not hold. Consequently for any given  $e^*$ ,  $B$  has to be lower for lower trade costs on both schedules. Since both schedules shifts downwards, and the equilibrium point is at the bottom of the U shape ZEP schedule,  $B$  must be lower at the new equilibrium (lower trade costs), indicating an increasing competitiveness of the markets. Since by definition (equation [7]) the aggregate price  $P$  increases in  $B$ , falling trade costs also reduces aggregate price of the differentiated good sector and is welfare improving. More importantly, it is yet to be shown whether the new equilibrium  $e^*$  locates at the right or left of the old equilibrium. In appendix F, we prove that  $e^*$  is decreasing in  $\tau$ , i.e. the lower the trade costs the higher equilibrium managerial efforts. Hence, for lower  $\tau$ , the equilibrium pair of  $\{e^*, B\}$  will move to the right bottom, meaning a lower  $B$  but higher  $e^*$ . We summarise the above results in the following proposition.

**PROPOSITION 5** *In the (HFT) equilibrium, the power of the incentive contract (piece rate), managerial effort and compensation, and industry productivity increase when the trade cost  $\tau$  decreases. Aggregate price falls and welfare improves.*

Proposition 5 immediately implies a negative link between the power of the incentive contract and variable trade costs. Although there is little direct empirical evidence on this so far, our result seems to be consistent with the finding in Lemieux et al. (2006) that incentive

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<sup>25</sup> This can be shown by inspection of [32] and [33], noting that  $Y(e^*, B; \tau, f_X)$  is decreasing in  $\tau, f_X$ .

pay has become more important for various occupations in the past two decades while barriers to international trade has been falling significantly due to both technological improvement and substantial reductions in tariffs. Furthermore, proposition 5 may also suggest that the positive correlation between competition and efficiency could be driven by a third factor, namely the magnitude of international trade costs, rather than a direct causality between the two. As was previously described from figure 3b, falling trade costs simultaneously reduces  $B$  and increases  $e^*$ . Recall that  $B$  is the demand shifter that represents the degree of competition (the lower  $B$ , the smaller the share of each firm in their domestic markets, reflecting a greater “toughness” of the market competition). Thus, falling trade costs *simultaneously* leads to increasing firm productivity - due to greater managerial effort- and increasing product market competition – due to the entry of foreign competitors, without a direct causality running from the latter to the former.

Finally, it is also note worthy that in our model falling  $\tau$  increases both allocative efficiency and productive efficiency: allocative efficiency increases since the share of output reallocate from less productive non-exporters to high productive exporters<sup>26</sup>, whereas productive efficiency increases (or X-efficiency reduces) since managers work harder and less slack, which is achieved with higher agency costs.

### *Falling $f_x$*

Reasoning analogous to the effects of falling  $\tau$  on the OI and ZEP schedule, falling  $f_x$  shifts both schedules downwards, leading to decrease in  $B$ . However, different from the effects of falling  $\tau$ , we show in the appendix that decrease in  $f_x$  has ambiguous effect on  $e^*$ . We defer the discussion of the intuition of this result to section 3.4, but it is noteworthy that despite its ambiguous effects on managerial incentives and firm productivity, falling fixed costs of exporting still unambiguously increases welfare by allowing consumers to access more foreign varieties.

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<sup>26</sup> The within industry reallocation effect is well known in Melitz (2003). In appendix H we show that such effect is also present in our model: when trade costs fall, exporting firms that are more productive will expand their shares in total output, whereas non-exporters that are less productive will lose.

## How important is firm heterogeneity? The case of quasi-homogeneous firms

The preceding analysis assumes that in equilibrium firms are substantially heterogeneous and self-select into the export market. But one may wonder how important is the role of firm heterogeneity in driving the above results? Would the positive effects of trade liberalization on managerial incentives still hold if firms are homogeneous? This question is non-trivial, because one contribution of the burgeoning heterogeneous firm trade literature is to show how firm heterogeneity adds an important new dimension to understand international trade that could not be otherwise explained in a homogeneous firm framework (see for example , Helpman , Melitz and Yeaple (2004) and Chaney (2005)).

To answer this question, in this section we investigate a benchmark case where firms are quasi-homogeneous: the degree of firm heterogeneity is small and trade barrier is low so that firm heterogeneity does not matter for their export status i.e. all firms export in the equilibrium. (an extreme example is that the range of  $\alpha$  is compressed into one single level so that firms are identical in their marginal costs, in which case all firms export for low trade costs.). We call this quasi-homogeneous firm trade (QHF) equilibrium.

The following assumption defines the QHF equilibrium that ensures  $\pi_x(a) > 0$  for  $\forall a \in [a_L, a_M]$  i.e. all firms earn positive export profits and sell in the foreign markets in the open economy:

**Assumption 4** Trade costs ( $\tau$  and  $f_x$ ) are low relative to the fixed entry cost  $f_E$  so that

$$B(a_M)^{1-\sigma} > f_x \tau^{\sigma-1} .27$$

Since all firms export, the total operating profit and revenue for each firm is :

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<sup>27</sup> Again , as was the case of assumption 2 , it will be shown below that  $B$  is increasing in  $f_E$  and  $a_M$  decreasing in  $f_E$  , so such equilibrium will exist if  $f_E$  is high relative to  $f_x, \tau$  . Assumption 4 is also equivalent to assume that  $\alpha_M$  , the upper bound of the stochastic cost element is low , so that firm heterogeneity is small and all firms earn positive export profits.



$$[40] \quad \pi(a) = Ba^{1-\sigma} + m \left[ B(\tau a)^{1-\sigma} - f_x \right], \quad r(a) = \sigma \cdot B \cdot (1 + m\tau^{1-\sigma}) a_i^{1-\sigma}$$

This yields the following pre-entry expected operating profit:

$$[41] \quad E(\pi) = \int_{a_L}^{a_M} \pi(a) dG(a; e) - f_E = B(1 + m\tau^{1-\tau})V(e) - (mf_x + f_E)$$

Analogous to the case of closed economy, the firm owner will design a contract that optimises her expected payoff  $E(\pi) - E(W)$ , leading to the optimal managerial effort  $e^*$  given by F.O.C. 28

$$[42] \quad B(1 + m\tau^{1-\tau})V'(e^*) = D'(e^*) \quad (\text{OI})$$

Again, analogous to the case of closed economy, in equilibrium free entry yields zero net expected payoff:

$$[43] \quad B(1 + m\tau^{1-\tau})V(e^*) - (mf_x + f_E) - D(e^*) = 0 \quad (\text{ZEP})$$

Combining the (OI) and (ZEP) equations we obtain the following equilibrium condition:

$$[44] \quad J(e^*) \equiv \frac{D'(e^*)}{V_d'(e^*)} V_d(e^*) - D(e^*) = f_E + mf_x$$

This equilibrium condition is very similar to that in closed economy as shown in [19], the only difference is that the fixed costs is bid up by  $mf_x$ . Intuitively, since all firms sell in all foreign markets, the equilibrium will look very similar to that in a integrated world market with extra fixed costs of selling in each foreign country. The existence and uniqueness of the equilibrium is therefore straightforward:  $J(e^*)$  is monotonically increasing in  $e^*$ , which ensures that [44] has a unique solution Further, as in the case of closed economy, equilibrium aggregate variables can all be expressed as monotonic functions of  $e^*$  ( see appendix G for more details).

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<sup>28</sup> The second order condition is always satisfied given assumption 2, also see footnote 11.

It is noteworthy that the fixed cost of exporting  $f_x$  is now playing the same role as the industry entry cost  $f_E$ , since *all* firms immediately start exporting after their entry to the industry.

Next we consider the effects of falling trade costs  $\tau$  and  $f_x$ . Firstly, [44] shows that  $e^*$  is independent of  $\tau$ , therefore changes in  $\tau$  have no impacts on the equilibrium  $e^*$ . Since the piece rate  $b(e^*) = \frac{D'(e^*)}{\gamma'(e^*)}$ , average managerial compensation  $E(W) = U_r + D(e^*)$  and average industrial cost  $\bar{a}(e^*) = [1 - \gamma(e^*)] \int_{\alpha_L}^{\alpha_M} \alpha dF(\alpha)$  all depends on  $e^*$ , falling transport cost  $\tau$  has no impact on the power of the incentive contract, managerial efforts and compensation, and industrial productivity. However, as can be seen in figure 4a: since both the OI and ZEP curves shift downwards when  $\tau$  decreases, the new equilibrium point at  $E'$  is lower than the old equilibrium  $E$ , meaning a fall in equilibrium  $B$ . Thus, the equilibrium aggregate price falls and welfare increases when  $\tau$  decreases, although  $e^*$  remains unchanged. Secondly, as was shown in figure 4b, falling  $f_x$  will shift the ZEP curve downwards, leading to a new equilibrium point at the lower left of the old equilibrium, indicating a lower  $B$  and  $e^*$ . Therefore, falling  $f_x$  has the same effect on aggregate price and welfare as falling  $\tau$ , but leads to a unambiguously *lower* power of the incentive contract, managerial efforts, and industrial productivity.

These results regarding the effects of falling trade costs on managerial incentives run contrast to those shown in the (HFT) equilibrium. Most interestingly, falling fixed costs of exporting unambiguously decreases managerial incentives and efforts, leading to lower firm level and industrial productivities. The reason could be explained as follows. When all firms export, the total fixed cost bared by each firm is the sum of the fixed cost of industry entry  $f_E$  and exporting  $f_x$ . A fall in  $f_x$  thus has the same effect as that of a decreasing  $f_E$  would have in the closed economy. As was shown in proposition 2, falling  $f_E$  reduces manager incentives, since it induces new domestic entrants that reduces firms' market shares. In the case of open economy where all firms export, falling  $f_x$  will not only attract the entry of new domestic firms since it leads to an increase in the expected profit of exporting, but also encourages the entry of new foreign firms. Such massive entry of both

foreign and domestic firms reduces firms' total output  $r(a) = \sigma B(1 + m\tau^{1-\sigma})a^{1-\sigma}$  and market share due to falling  $B$ . As a result, firm owners have weaker incentives to encourage managers to exert efforts to reduce costs. Thus, firm level marginal cost increases and the economy incurs a productivity loss. By contrast, however, decrease in  $\tau$  does not decrease firm's total output: they lost their domestic market share since foreign exporters become more competitive, but this is compensated by their gains from the export markets due to falling transport costs. Hence, the firm level total output was not affected by  $\tau$ , so the managerial incentives provided by firm owners are also independent to  $\tau$  <sup>29</sup>.

## 4. Why firm heterogeneity matters? The Carrot and Stick Effects

Comparing the above results from the (QHF) equilibrium and those from (HFT) equilibrium highlighted the important role of firm heterogeneity and export market selection in the effects of openness on managerial incentives. In this section we reveal why firm heterogeneity is so important, and through what *mechanism* it works to create the positive *causal* link from openness to firm productivity.

The key channel through which openness may raise managerial is the trade-induced “carrot and stick effect”. This effects occurs, when the lowest-cost-exporting firms will be rewarded with the “carrot” of increasing total output and profits from trade liberalization, whilst the high-cost-non-exporting firms penalised by the “stick” of shrinking output and domestic profit due to increasing competition. Thus, the firm owners have stronger incentives to provide a high power contract to the managers to encourage them to exert greater efforts. Because this will increase (decrease) the firm's pre-entry probability of drawing a low (high) cost and therefore become an exporter (non-exporter), so that the firm is more likely to seize the “carrot” and avoid the “stick” in a more open economy. An

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<sup>29</sup> Despite their different effects on managerial incentives and industrial productivity, falling  $\tau$  and  $f_X$  both lead to welfare gains. Falling  $f_X$  raises welfare because it leads to increasing number of total varieties that dominates the negative productivity effects, whilst a lower  $\tau$  decreases the c.i.f. export prices delivered to customers, which reduces the aggregate price and makes them better off.

alternative interpretation for this mechanism is that trade liberalization increases the profit differential between the high cost and low cost status drawn from the cost distribution, which raises the “value of cost reduction”.<sup>30</sup> Consequently, managerial effort devoted to reduce cost reduction becomes more “valuable” to the firms, which in turn motivates the owners to design a higher power incentive contract to induce managers to work harder.

To see this mechanism more clearly, first consider the effects of falling  $\tau$  on firms profits  $\pi(a)$  and outputs  $r(a)$ . If the firm draws a high cost (above the export cost cut off)  $a \in [a_X, a_M]$  after entry, it becomes a non-exporter so that  $\pi(a) = \pi_d(a) = Ba^{1-\sigma} - f_e$  and  $r(a) = r_d(a) = \sigma Ba^{1-\sigma}$ . Apparently, a lower  $\tau$  leads to lower profits and outputs ( $\partial\pi(a)/\partial\tau > 0, \partial r(a)/\partial\tau > 0$ ), since  $B$  decreases with falling  $\tau$  (as shown in figure 3b and proved in the appendix). Intuitively, falling trade costs leads to increasing product market competition that reduces firms’ domestic profit and output. On the contrary, if a firm draws a low cost (below the export cost cutoff)  $a \in [a_L, a_X]$  and becomes an exporter, its total profit and output is

$$[45] \quad \begin{aligned} \pi(a) &= \pi_d + \pi_x = Ba^{1-\sigma} (1 + m\tau^{1-\sigma}) - (f_e + mf_x) , \\ r(a) &= r_d + r_x = \sigma Ba^{1-\sigma} (1 + m\tau^{1-\sigma}) \end{aligned}$$

As was shown the appendix H,  $\partial\pi(a)/\partial\tau < 0$  and  $\partial r(a)/\partial\tau < 0$ : when  $\tau$  falls, the firms’ loss of the domestic profit ( $\partial\pi_d/\partial\tau > 0$ ) and output will be *more than compensated* by the increase in its export profit ( $\partial\pi_x/\partial\tau < 0$ ) and output, leading to a net increase in  $\pi(a)$  and  $r(a)$  for  $a \in [a_L, a_X]$ . Hence, falling  $\tau$  unambiguously magnifies the profit differential between the low cost and high cost status. As a result, firm owners would like to reward the managers by more if they achieve a “good” cost draw and export, but also penalize them by more if the cost draw turns out to be “bad” and the firm remains non-exporting. This means a higher optimal contractual piece rate  $b$  in equilibrium, leading to a greater managerial effort and average firm productivity.

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<sup>30</sup> It is common in the Schumpeterian literature that the incentives to innovate depends on the differences, rather than the absolute size of pre- and post innovation rents. See for example Trindade (2005).

Secondly, such “carrot and stick” effect, may not be necessarily at work when  $f_x$  falls. As was shown in the appendix, the effects of falling  $f_x$  on the profits of exporting firms ( $\pi_x(a)$  and  $r_x(a)$ ,  $a \in [a_L, a_X]$ ) is ambiguous<sup>31</sup>, which may even lead to a decrease in the profit differential between exporter and non exporters. This well explains the ambiguous results regarding the effects of falling  $f_x$  on the managerial incentives and firm productivity as described in section 3.1.

Finally, in the (QHF) equilibrium, such “carrot and stick” effect is totally shut down, since firms do not select into the export market so that trade liberalization affect firm profits and outputs in the same directions. More precisely, when all firms export,  $\frac{\partial \pi(a)}{\partial \tau} = 0$ ,  $\frac{\partial \pi(a)}{\partial f_x} > 0$ , indicating that the profit differential between high cost and low firms remains unchanged when  $\tau$  falls, but decreases when  $f_x$  falls<sup>32</sup>. Hence, the value of cost reduction and the marginal value of managerial effort are non-increasing under falling trade costs, leading to unchanged or even a lower optimal contractual piece rate and therefore lower managerial effort and firm productivity.<sup>33</sup>

To summarise, in Table 1 we list the impacts of falling trade costs on managerial incentives under different scenarios via the carrot and stick mechanism:

<sup>31</sup> This result is also pointed out in the original Melitz (2003) model. However in his model falling  $f_x$  unambiguously increases aggregate productivity despite its ambiguous effects on market share reallocation.

<sup>32</sup> Let  $\bar{a}$  and  $\underline{a}$  represent any cost level within the boundaries of cost distribution  $\bar{a} > \underline{a}$ ,

and  $\Delta\pi = [\pi(\underline{a}) - \pi(\bar{a})]$  denote the profit differential. Using [45] we obtain  $\frac{\partial(\Delta\pi)}{\partial \tau} = 0$ , and

$$\frac{\partial(\Delta\pi)}{\partial f_x} = \frac{\partial B}{\partial \tau} (\underline{a}^{1-\sigma} - \bar{a}^{1-\sigma}) (1 + m\tau^{1-\sigma}) > 0$$

<sup>33</sup> It is also worth noting that there is possibly a second mechanism at work through which firms provide stronger managerial incentives to reduce costs when trade costs fall: saving labour costs. As was shown previously, the aggregate price falls when trade costs decrease, which means an increase in the real wage. Since labour becomes more expensive, firms have stronger incentives to save labour costs by raising labour productivity. This “labour cost saving” effect is also discussed in Hornet a. (1994) in an oligopoly industry model.

**Table 1 : Falling Trade costs and the carrot and stick effects**

Degree of Firm Heterogeneity	Types of trade costs	Changes in Managerial Incentives	Changes in profit differential
HFT equilibrium (export selection)	$\tau$	+	Increase
	$f_x$	+/-	Undetermined
QHF equilibrium (all firms export)	$\tau$	0	unaffected
	$f_x$	-	Decrease

It is noteworthy that the result that trade liberalization might reward the high productivity firms and punishes the least productive firms within the same industry is not completely earth breaking. This is actually the central mechanism revealed by Melitz (2003) that leads to inter firm reallocations and thus aggregate productivity gains. What is new in our model, however, is that such “reward and penalty“ effect stemming from the importance of firm heterogeneity is not only important to the *inter-firm* reallocation effect as stressed in Melitz (2003) , but also the key dimension to explain the *intra-firm* productivity gains/losses induced by trade liberalization. Whilst increasing openness of the economy may reallocate the market shares away from low productivity firms towards high productivity firms, our analysis show that firm may not only react *passively* by adjusting their output margins, but also might foresee the threats and opportunities brought by increasing openness and therefore adapt their internal contract structure *proactively* to cope with the changing global market environment. Hence, by incorporating the principal-agent problem into the heterogeneous firm trade framework, we can attribute the trade-induced intra-firm productivity gains at least partly to the optimal incentive contract mechanism, which could be viewed as complementary to the inter-firm reallocation mechanism in the original Melitz (2003) model in explaining the link between trade barriers and aggregate productivity.

## 5. Concluding Remarks

Recent empirical trade studies often find that increasing openness boosts firm productivity ( see Tybout 2003 and Greenaway and Kneller 2007 for a survey). One theoretical puzzle

arising from this result is that, if productivity improvement is profit-increasing, why firms did not implement them earlier but waited until the economy becomes more opened up? Perhaps one plausible approach to solve this puzzle is to break into the “black box” of cost-minimising firms and investigate how openness could affect the organizational structure within the firm. Recent developments in trade theory have made several successful attempts in that direction, but far from reaching a consensus yet.

Motivated by recent empirical evidence that firm level productivity is highly associated with (good) management practice and (strong) managerial incentives (Bandiera , Barankay and Rasul 2007, Bloom and Rennan 2007), we have presented a model that incorporates the principal-agent mechanism into the heterogeneous firm trade framework based on Melitz (2003), in which firms could self-select into the export market whilst their productivities depends on the intra-firm optimal managerial incentive contract. One new mechanism we reveal in our model via which “openness boosts productivity” is the “carrot and stick effect”: when the economy is more opened up, the low-cost-exporting firms could be rewarded with increasing profit due to their expansion in the global market, whereas the high-cost-purely-domestic firms would be penalized by decreasing profit resulting from fiercer import competition. If such “carrot and stick” effect is at work, cost-reducing managerial effort will generate a greater value to the firm owners , who would therefore like to provide the managers with a contract with stronger managerial incentives to motivate them working harder, and thus improve management practice and boosts productivity.

Our paper further shows that there are, however, conditions under which such openness-induced “carrot and stick” effect is at work. Specifically, this mechanism is present only when firms are substantially heterogeneous and openness is triggered by falling variable trade costs. When firms are quasi-homogeneous – defined as low productivity spread and trade costs – so that all firms export in equilibrium, openness affects profits of all firms in the same direction, therefore will never increase the profit differential between low-cost and high-cost firms, and thus do not raise managerial incentives. We have also shown that falling fixed costs of exporting could have ambiguous results on profits of low-cost-exporting firms in the heterogeneous firm equilibrium, which therefore may not necessarily boosts managerial incentives and productivity.

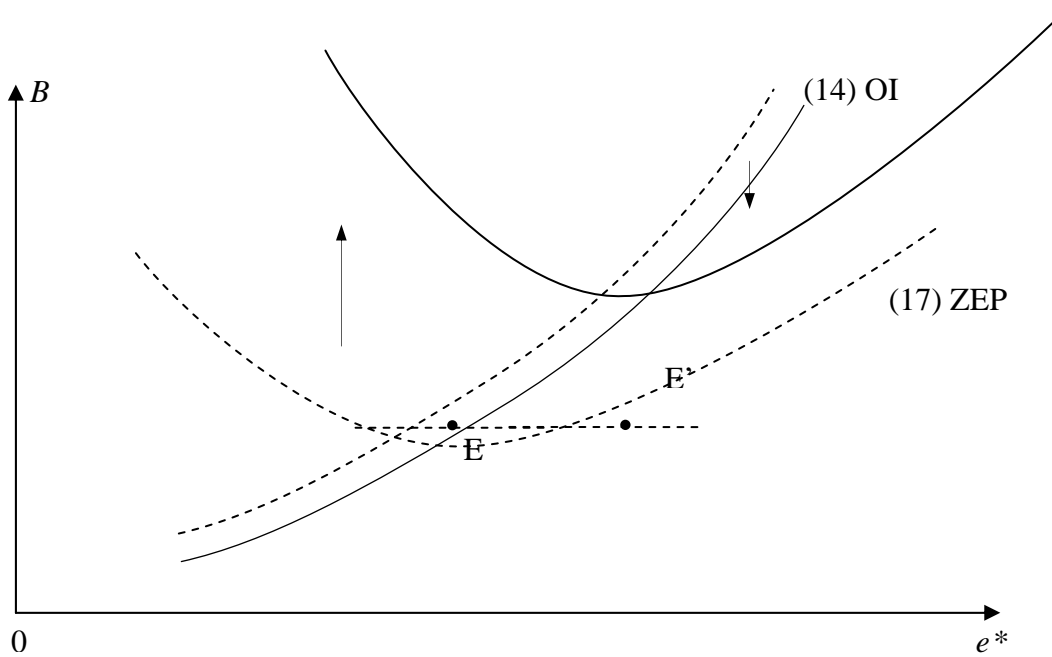
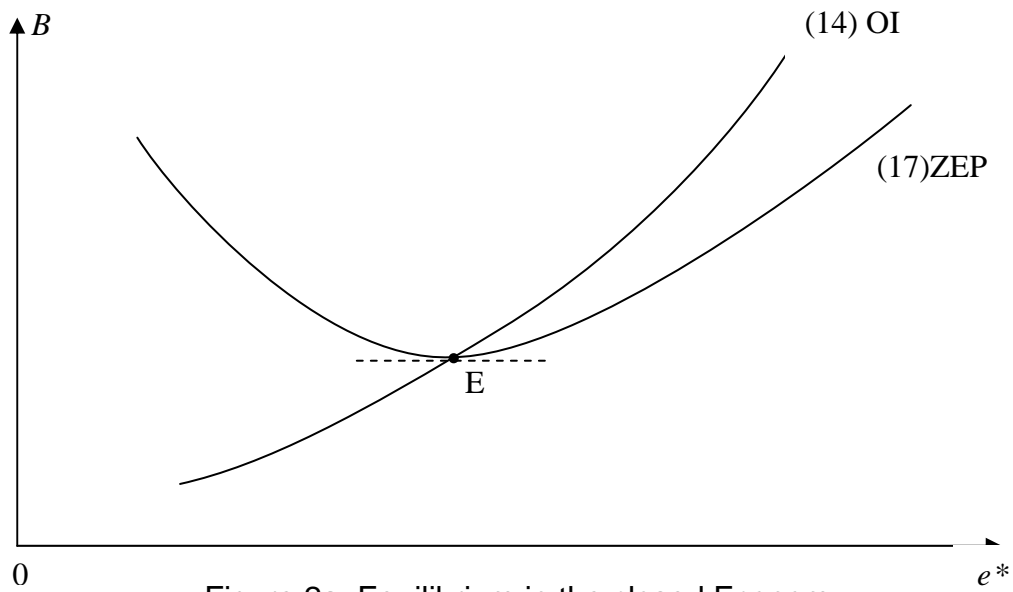
Our model has revealed the crucial role of firm heterogeneity in the openness-induced productivity gains via the managerial incentives mechanism. It is often argued that globalisation brings both threat and opportunity to the economy, and one specific mechanism is that only the best firms can benefit from the opportunity, whilst the worst firms will unambiguously lose. Such reallocation effects, which heavily relies on the assumption of firm heterogeneity, is the main building block in the new heterogeneous firm trade framework pioneered by Melitz (2003), Bernard et al. (2003) , Yeaple (2005) and Melitz and Ottaviano (2007). By incorporating the managerial incentive mechanism into the Melitz (2003) model, we are able to show that the *second order effect* of the reallocation effect is that firms may also react proactively to such new opportunity and threats by adjusting their internal incentive schemes, which could provide a useful mechanism to understand the “openness boosts productivity” argument.

While our model stresses the importance of firm heterogeneity and managerial incentive mechanism, there are certainly other plausible mechanisms explaining the positive openness-productivity link. As pointed out by Bernard et al. (2006), international technology transfer, incentives to invest in research and development and changes in product mix of firms may also play a role. However, our model yields some distinguishing testable hypotheses that may differentiate us from other explanations. In particular, we show that that the power of the incentive contract (piece rate) and the average managerial compensation is greater when variable trade costs fall, which is generally consistent with recent evidence that the incidences of incentive pay in the U.S. has increases substantially in the last two decades during which period the variable trade costs have declined sharply. Furthermore, our model allows us to derive how the effects of openness on managerial incentives and firm productivity vary with the degree of firm heterogeneity and export selection across industries; these may be interesting questions for further empirical investigations using more detailed firm level data.

Our model very parsimoniously captures the significantly complex role of managerial incentive structure in shaping firm behaviours. To achieve this parsimony we abstracted from many features of managerial characteristics in the real world. For example, we omitted the heterogeneity of managers’ abilities and their attitude towards risks, we completely ignored the dynamics of managerial behaviours, and we assumed away the financial constraint problems that are one of the main obstacles to firms’ overseas



expansions. Future research could be directed towards extending this line of research to a comprehensive framework that incorporates these features and explains the complex interactions between international trade and managerial behaviours. We hope our model has brought trade theory closer to the evidence , and serves as an important first step towards that end.



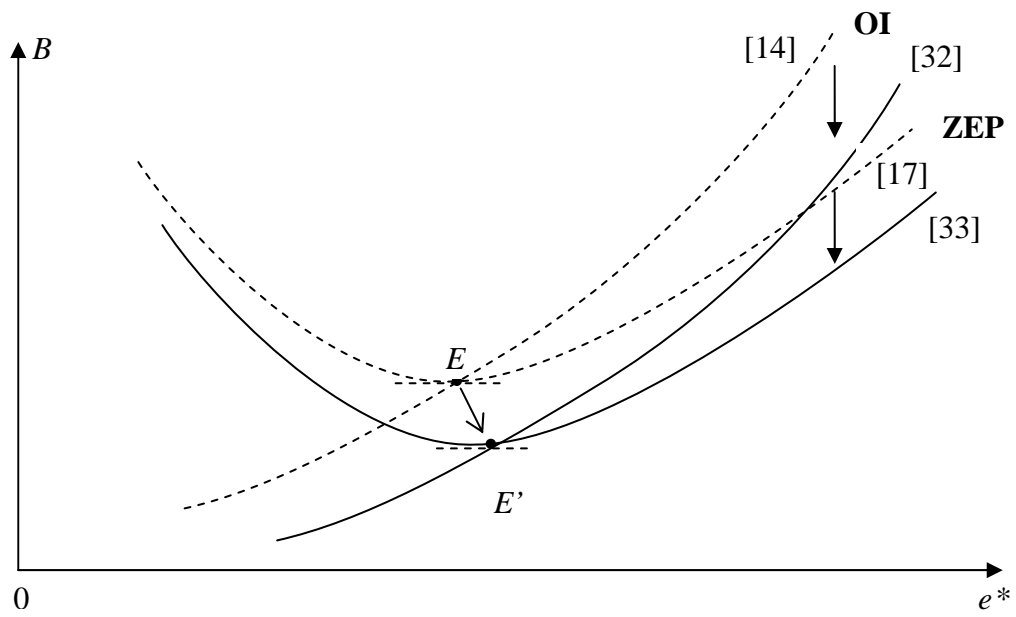


Figure 3a. Effects of moving from autarky to open economy

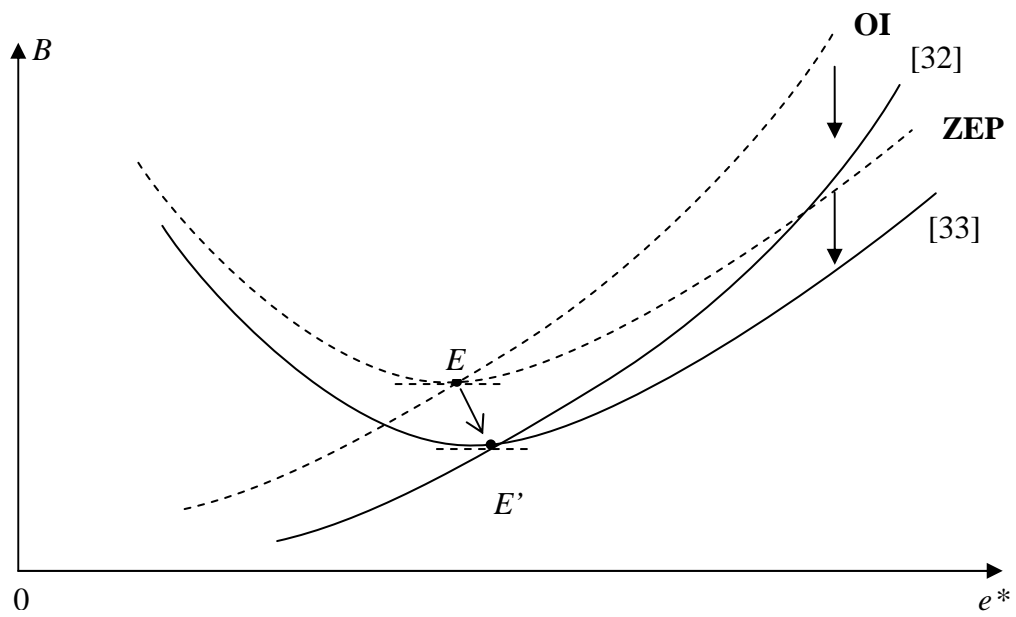


Figure 3b. Effects of falling Trade costs

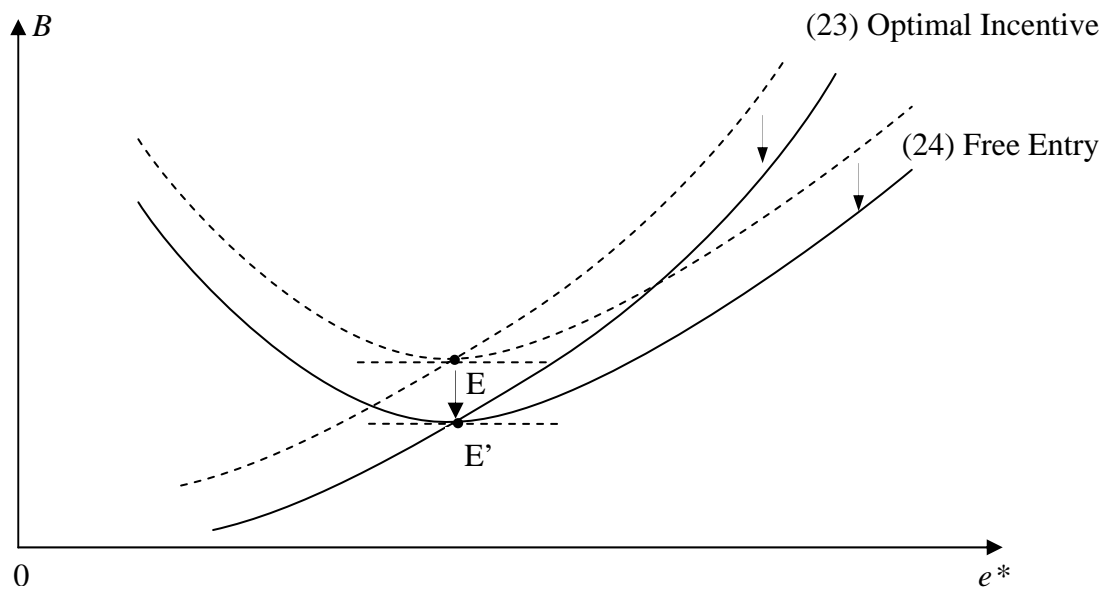


Figure 4a Effects of falling  $\tau$  in quasi-homogenous firms case

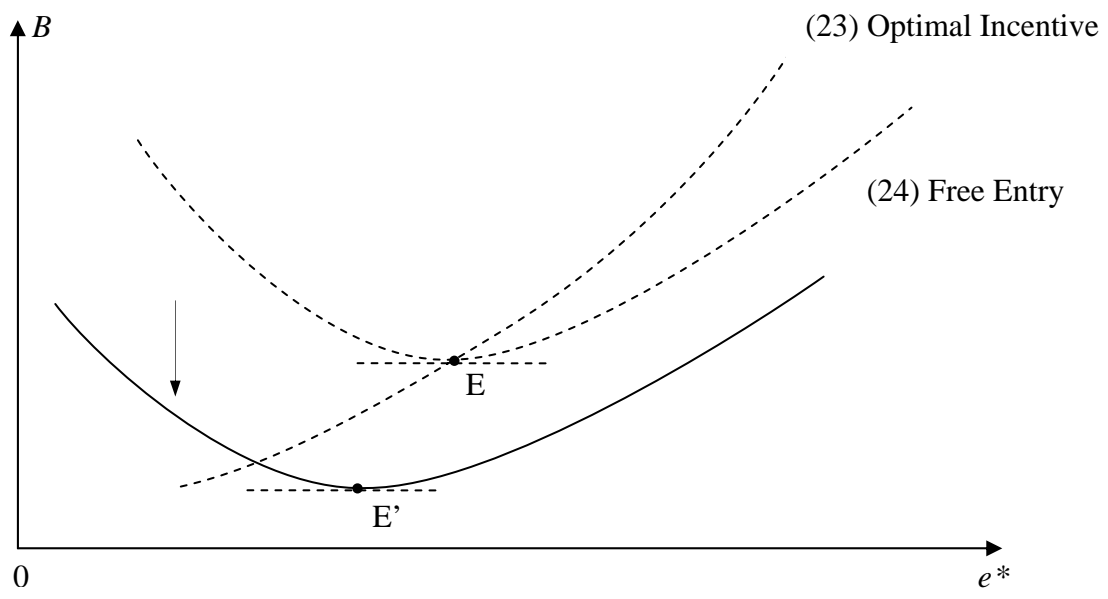


Figure 4b. Effects of falling  $f_x$  in quasi-homogenous firms case

## References

- Bandiera, O., Barankay I. and Rasul, I. 2007. "Incentives for Managers and Inequality Among Workers: Evidence From a Firm-Level Experiment," *The Quarterly Journal of Economics*, Vol. 122 , No, 2, pp. 729-773
- Bernard, A.B.and Jensen, J.B., 1999. "Exceptional exporter performance: cause, effect, or both?" *Journal of International Economics* Vol. 47, No. 1, pp. 1–26.
- Bernard, A. B., Redding, S. J. and Schott, P. K., 2007, "Comparative Advantage and Heterogeneous Firms" . *Review of Economic Studies*, Vol. 74, No. 1, pp. 31-66
- Bernard, A.B., Jensen, J.B., Schott, P.K. 2006 , "Trade Costs, Firms and Productivity" *Journal of Monetary Economics* , Vol. 53, Issue 5, pp. 917-937.
- Bernard, A., Jensen. B., Redding, S., Schott, P., 2007,"Firms in International Trade" *The Journal of Economic Perspectives* , Vol. 21, No. 3, pp. 105-130
- Bloom, N. and Van Reenen, J. , 2007, "Measuring and Explaining Management Practices Across Firms and Countries," *The Quarterly Journal of Economics*, Vol. 122, No. 4, pp. 1351-1408
- Bolton, P. and Dewatripont , M., 2005 , Contract theory, MIT Press: Cambridge and London.
- Clerides, S., Lach, S., Tybout, J., 1998. "Is 'learning-by-exporting' important?", *Quarterly Journal of Economics*, CXIII , pp. 903–948.
- Davidson, C., Matusz, S., and Shevchenko, A., 2008 , "Globalization and Firm level Adjustment with Imperfect Labor Markets" , *Journal of International Economics*, forthcoming

Dixit, A. K., and Stiglitz, J. E., 1977. "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, Vol. 67, No. 3, pp. 297-308

Greenaway D. and Kneller, R., 2007. "Firm heterogeneity, exporting and foreign direct investment," *Economic Journal*, Vol. 117, pp. F134-F161

Hart, O., 1983 , "The Market as an Incentive Mechanism." *Bell Journal of Economics*, Vol. 14, No. 2, pp. 366– 82

Holmstrom , B. and Milgrom , P.,1987, "Aggregation and Linearity in the Provision of Intertemporal Incentives", *Econometrica*, Vol.55, pp.303-328

Horn, H., Lang H. and Lundgren S., 1995 , "Managerial effort incentives, X-inefficiency and international trade", *European Economic Review* , Vol. 39, pp. 117-138

Krugman, P., 1980. "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, American Economic Association, Vol. 70 , No. 5, pp. 950-59

Marin, D. and Verdier, T., 2007. "Competing in Organizations: Firm Heterogeneity and International Trade," *CEPR Discussion Papers 6342*, C.E.P.R. Discussion Papers.

Melitz, M.J., 2003. "The impact of trade on intra-industry reallocations and aggregate industry productivity", *Econometrica* , Vol.71, pp. 1695–1725.

Melitz, M. and Ottaviano, G. , 2008, Market Size, Trade, and Productivity" *Review of Economic Studies*, Vol. 75, Issue 1, pp. 295-316

Nalebuff, B. J and Stiglitz, J. E., 1983, "Information, Competition, and Markets," *American Economic Review*, , Vol. 73, No.2, pp. 278-83

Pavcnick , N., 2002 , "Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Plants," *The Review of Economic Studies* , Vol. 69, pp. 245-76.

Raith, M., "Competition, Risk and Managerial Incentives", *American Economic Review* Vol. 93 , pp. 1425-1436

Scharfstein, D., 1988. "Product-Market Competition and Managerial Slack," *RAND Journal of Economics*, The RAND Corporation, Vol. 19, pp. 147-155

Schmidt, K. M.,1997, "Managerial Incentives and Product Market Competition." *Review of Economic Studies*, Vol. 64 No. 2, pp. 191–213.

Trindade, V., "Openness and Productivity: a Model of Trade and Firm-Owners' Effort", mimeo

Tybout , J., "Plant- and Firm-Level Evidence on 'New' Trade Theories," in E. Kwan Choi and James Harrigan, eds., *Handbook of International Economics*, Oxford: Basil-Blackwell, 2003.

Yeaple, S. R. , 2005. "A Simple Model of Firm Heterogeneity, International Trade, and Wages," *Journal of International Economics*, Vol. 65, pp. 1-20



# Appendix

## A Proof of U shape ZEP curve

The ZEP condition in [18] can be rewritten as

$$[A.1] \quad B(e^*) = \frac{f_E + U_r}{V(e^*)} + \frac{D(e^*)}{V(e^*)}$$

$$\Rightarrow B'(e^*) = \frac{-V'(e^*)[f_E + U_r + D(e^*)] + D'(e^*)V(e^*)}{V^2(e^*)}$$

$$\Rightarrow B'(e^*) < (>) 0 \quad \text{if and only if} \quad J(e^*) \equiv \frac{D'(e^*)}{V'(e^*)}V(e^*) - D(e^*) < (>) f_E + U_r$$

As shown in footnote 13,  $J'(e^*) > 0$ , therefore  $B'(e^*) < (>) 0$  when  $e^* < (>) e_E^*$ , where  $e_E^*$  satisfies  $J(e_E^*) = 0$ . In words, the ZEP curve is decreasing in  $e^*$  before point  $E$  and then increasing in  $e^*$  after crossing  $E$ . Furthermore, it can be shown that  $B''(e^*) > 0$ , since  $[V^{-1}(e^*)]'' > 0$  and  $\left(\frac{D(e^*)}{V(e^*)}\right)'' > 0$  due to  $V''(e^*) < 0$ .

## B Proof of proposition 2

Using  $V(e) = [1 - \gamma(e)]^{1-\sigma} \Omega$ , we can rewrite equation [19] as :

$$[B.1] \quad J(e^*; \sigma) = \frac{D'(e^*)[1 - \gamma(e^*)]}{\gamma'(e^*)(\sigma - 1)} - D(e^*) = f_E + U_r$$

Since  $J_\sigma(\sigma, e^*) < 0$  and  $J_{e^*}(\sigma, e^*) > 0$  (from footnote 13), we have  $\frac{\partial e^*}{\partial \sigma} > 0$  where

$$J(e^*, \sigma) = f_E + U_r.$$

## C Proof of lemma 2

Equation (32) can be rewritten as

$$[C.1]) \quad B = \frac{D'(e^*) - Y_{e^*}(B, e^*, \tau)}{V'(e^*)}$$

By assumption 3, the right hand side of [C.1] is increasing in  $e^*$  :  $\frac{\partial \left( \frac{D'(e^*) - Y_{e^*}(B, e^*, \tau)}{V'(e^*)} \right)}{\partial e^*} > 0$  but decreasing in  $B$ . Now suppose  $B$  is decreasing in  $e^*$ . If so, then when  $e^*$  increases the right hand side of [C.1] increases, and the left hand side decreases, thus equation [C.1] can not be hold, leading to contradiction. Therefore  $B$  must be increasing in  $e^*$ .

### D Proof of proposition 3

Under lemma 2, the left hand side of (33) is a function of  $e^*$ . So we can rewrite the ZEPcondition as :

$$[D.1] \quad Q(e^*) \equiv B(e^*)V(e^*) + Y(B(e^*), e^*, \tau, f_X) - D(e^*) = f_E + U_r$$

Taking differentiation of the left hand side with respect to  $e^*$  yields:

[D.2]

$$\frac{dQ(e^*)}{de^*} = B'(e^*)V(e^*) + Y_B(B(e^*), e^*, \tau, f_X)B'(e^*) + Y_{e^*}(B(e^*), e^*, \tau, f_X) + B(e^*)V'(e^*) - D'(e^*)$$

Using equation (32) we obtain:

$$[D.3] \quad \frac{dQ(e^*)}{de^*} = B'(e^*)[V(e^*) + Y_B(B(e^*), e^*, \tau, f_X)] > 0$$

Thus, since  $Q(e^*)$  is monotonically increasing  $e^*$  and  $Q(0) = 0$ , there exists a unique and strictly positive solution  $e^*$  to  $Q(e^*) = f_E + U_r$ . Further, from lemma 2, there also exists a unique equilibrium  $B$  determined by equilibrium  $e^*$  according to equation (32).

**E proof  $H(B, e^*) < 0$**

Rewrite [32] as  $B = \frac{D'(e^*) - Y_{e^*}(B, e^*, \tau, f_X)}{V'(e^*)}$  and substitute it into (33), the left hand side of (33) can

be written as

$$\frac{D'(e^*)}{V'(e^*)}V(e^*) - D(e^*) + Y_{e^*}(B, e^*, \tau, f_X) - \frac{Y_{e^*}(B, e^*, \tau, f_X)}{V'(e^*)}V(e^*) = J(e^*) + H(e^*, B, \tau, f_X)$$

$$\text{Where } H(e^*, B, \tau, f_X) \equiv Y(B, e^*, \tau, f_X) - \frac{Y_{e^*}(B, e^*, \tau, f_X)}{V'(e^*)}V(e^*)$$

For expositional convenience next we use  $Y(e^*)$  and  $Y'(e^*)$  to represent  $Y(B, e^*, \tau, f_X)$  and  $Y_{e^*}(B, e^*, \tau, f_X)$ , and  $\Gamma(e^*)$  represent  $1 - \gamma(e^*)$ . We need to prove:

$$[E.1] \quad H(e^*, B) < 0 \Leftrightarrow \frac{V(e^*)}{V'(e^*)} > \frac{Y(e^*)}{Y'(e^*)}$$

Using  $a = \Gamma(e)\alpha$ , we can rewrite  $Y(e^*)$  as  $\int_{\alpha_L}^{\alpha_X(e^*)} \{B[\Gamma(e^*)\alpha]^{1-\sigma} - f_X\} dF(\alpha)$ , where

$\alpha_X = B^{\frac{1}{\sigma-1}} \phi^{-1} \Gamma^{-1}(e^*)$ . Using Leibiniz rule, we obtain:

$$Y'(e^*) = 0 - 0 + (1 - \sigma) \Gamma'(e^*)^{-\sigma} \Gamma'(e^*) \int_{\alpha_L}^{\alpha_X} B[\Gamma(e^*)\alpha]^{1-\sigma} dF(\alpha) = (\sigma - 1) \{\gamma'(e^*)/[1 - \gamma(e^*)]\} \int_{\alpha_L}^{\alpha_X} B[\Gamma(e^*)\alpha]^{1-\sigma} dF(\alpha) > 0$$

Therefore the right hand side of inequality is

$$\begin{aligned} \frac{Y(e^*)}{Y'(e^*)} &= \frac{\int_{\alpha_L}^{\alpha_X} \{B[\Gamma(e^*)\alpha]^{1-\sigma} - f_X\} dF(\alpha)}{(\sigma - 1) \{\gamma'(e^*)/[1 - \gamma(e^*)]\} \int_{\alpha_L}^{\alpha_X} B[\Gamma(e^*)\alpha]^{1-\sigma} dF(\alpha)} > 0 \\ &= (\sigma - 1)^{-1} \{[1 - \gamma(e^*)]/\gamma'(e^*)\} \left[ 1 - \frac{f_X p_{ex}(e^*)}{\int_{\alpha_L}^{\alpha_X} B[\Gamma(e)\alpha]^{1-\sigma} dF(\alpha)} \right] \end{aligned}$$

where  $p_{ex}(e^*) = F(\alpha_x)$ . Since  $\frac{V(e^*)}{V'(e^*)} = \frac{\Delta(e^*)}{\Delta'(e^*)} = (\sigma - 1)^{-1} \{[1 - \gamma(e^*)]/\gamma'(e^*)\} > 0$ , and

$$\text{obviously } \left[ 1 - \frac{f_x p_{ex}(e^*)}{\int_{\alpha_L}^{\alpha_x} B[\pi(e)\alpha]^{1-\sigma} dF(\alpha)} \right] \in [0,1], \quad \text{we obtain } \frac{V(e^*)}{V'(e^*)} > \frac{Y(e^*)}{Y'(e^*)},$$

therefore  $H(e, B^*) < 0$ .

## F. Proof of proposition 5

Take differentiation of [32] and [33] with respect to  $\tau$  we obtain :

$$\frac{\partial B}{\partial \tau} \left[ V'(e^*) + \frac{\partial Y(e^*, B, \tau)}{\partial B} \right] + \frac{\partial e^*}{\partial \tau} \left[ BV'(e^*) + \frac{\partial Y(e^*, B, \tau)}{\partial e^*} - D'(e^*) \right] = \frac{-\partial Y(e^*, B, \tau)}{\partial \tau} \quad [\text{F.1}]$$

$$\frac{\partial B}{\partial \tau} \left[ V'(e^*) + \frac{\partial^2 Y(e^*, B, \tau)}{\partial e^* \partial B} \right] + \frac{\partial e^*}{\partial \tau} \left[ BV'(e^*) + \frac{\partial^2 Y(e^*, B, \tau)}{\partial (e^*)^2} - D'(e^*) \right] = \frac{-\partial^2 Y(e^*, B, \tau)}{\partial e^* \partial \tau} \quad [\text{F.2}]$$

Hence ,

$$(F.1) \Rightarrow \frac{\partial B}{\partial \tau} = \left( \frac{-\partial Y}{\partial \tau} \right) / \left[ V'(e^*) + \frac{\partial Y}{\partial B} \right] \quad [\text{F.3}] \quad (\text{using [32]})$$

Furthermore ,

$$[\text{F.2}] \Rightarrow \frac{\partial e^*}{\partial \tau} = \frac{\left\{ \frac{-\partial^2 Y}{\partial e^* \partial \tau} - \frac{\partial B}{\partial \tau} \left[ V'(e^*) + \frac{\partial^2 Y}{\partial e^* \partial B} \right] \right\}}{BV''(e^*) + \frac{\partial^2 Y}{\partial (e^*)^2} - D''(e^*)} \quad [\text{F.4}] \quad (\text{using [E.3]})$$

From [F.3] it is straightforward that  $\frac{\partial B}{\partial \tau} > 0$ , since  $\frac{\partial Y}{\partial \tau} < 0$ . Next we prove  $\frac{\partial e^*}{\partial \tau} < 0$ .

Since by assumption 3, the denominator of the right hand side of [F.4]  $BV''(e^*) + \frac{\partial^2 Y}{\partial (e^*)^2} - D''(e^*) < 0$ .

So we only need to prove the numerator

$$\begin{aligned} & \frac{-\partial^2 Y}{\partial e^* \partial \tau} - \frac{\partial B}{\partial \tau} \left[ V'(e^*) + \frac{\partial^2 Y}{\partial e^* \partial B} \right] > 0 \\ \Leftrightarrow & \frac{V(e^*) + \frac{\partial Y}{\partial B}}{\frac{\partial Y}{\partial \tau}} < \frac{V'(e^*) + \frac{\partial^2 Y}{\partial e^* \partial B}}{\frac{\partial^2 Y}{\partial e^* \partial \tau}} \quad \text{[F.5] (using F.1)} \end{aligned}$$

Since the right hand side of [F.5] can be written as :

$$\begin{aligned} & \frac{V'(e^*) + \frac{\partial^2 Y}{\partial e^* \partial B}}{\frac{\partial^2 Y}{\partial e^* \partial \tau}} \\ &= \frac{(1-\sigma)[\Gamma'(e^*)/\Gamma(e^*)]V(e) + (1-\sigma)[\Gamma'(e^*)/\Gamma(e^*)](\partial Z/\partial B)}{(1-\sigma)[\Gamma'(e^*)/\Gamma(e^*)](\partial Z/\partial \tau)} \\ &= \frac{V(e^*) + \frac{\partial Y}{\partial B} + f_x \frac{\partial p_{ex}}{\partial B}}{\frac{\partial Y}{\partial \tau} + f_x \frac{\partial p_{ex}}{\partial \tau}} \end{aligned}$$

where  $Z \equiv \int_{\alpha_L}^{\alpha_X} B[\pi(e)\alpha]^{1-\sigma} dF(\alpha) = Y + p_{ex}f_x$ , and  $p_{ex} = G(\alpha_X) = F(\alpha_X)$

Hence, equilibrium [F.5] is equivalent to :

$$\frac{V(e^*) + \frac{\partial Y}{\partial B}}{\frac{\partial Y}{\partial \tau}} < \frac{V(e^*) + \frac{\partial Y}{\partial B} + f_x \frac{\partial p_{ex}}{\partial B}}{\frac{\partial Y}{\partial \tau} + f_x \frac{\partial p_{ex}}{\partial \tau}} \quad \text{[F.6]}$$

$$\Leftrightarrow \left[ V(e^*) + \frac{\partial Y}{\partial B} \right] \frac{\partial p_{ex}}{\partial \tau} < \frac{\partial p_{ex}}{\partial B} \frac{\partial Y}{\partial \tau} \quad \text{[F.7]}$$

$$\text{Since } \frac{\frac{\partial p_{ex}}{\partial B}}{\frac{\partial p_{ex}}{\partial \tau}} = \frac{\frac{\partial \alpha_X}{\partial B}}{\frac{\partial \alpha_X}{\partial \tau}} = \frac{\tau}{B(1-\sigma)} = \frac{\frac{\partial Y}{\partial B}}{\frac{\partial Y}{\partial \tau}} \text{ and } \frac{\partial Y}{\partial \tau} < 0$$

, [F.7] always holds. Hence, inequality (F.5) holds, which proves  $\frac{\partial e^*}{\partial \tau} < 0$ .

## G Proof of proposition 6

$$[G.1] B(e^*) = \frac{D'(e^*)}{(1 + m\tau^{1-\sigma})V_d'(e^*)},$$

$$[G.2] b^* = \frac{D'(e^*)}{\gamma'(e^*)},$$

$$[G.3] P(e^*) = \left( \frac{B(e^*)}{\rho^{\sigma-1}(1-\rho)\beta E} \right)^{\frac{1}{\sigma-1}} = \left( \frac{D'(e^*)}{(1 + m\tau^{1-\sigma})V_d'(e^*)\rho^{\sigma-1}(1-\rho)\beta E} \right)^{\frac{1}{\sigma-1}},$$

$$[G.4] \bar{a}(e^*) = [1 - \gamma(e^*)] \int_{\alpha_L}^{\alpha_M} \alpha dF(\alpha)$$

$$[G.5] U = \beta EP(e^*)^{-1} = \left( \frac{D'(e^*)}{(1 + m\tau^{1-\sigma})V_d'(e^*)\rho^{\sigma-1}(1-\rho)} \right)^{\frac{1}{1-\sigma}} \beta E^{\frac{\sigma}{\sigma-1}}$$

$$[G.6] N(e^*) = (1 - \rho)\beta E \cdot \left( \frac{V_d'(e^*)}{D'(e^*)} \right) V_d^{-1}(e^*)$$

## H Carrot and Stick Effects

In this appendix we prove a low cost exporting firm's profit and total output increases when trade cost  $\tau$  falls.

Profit of an exporter is  $\pi(a) = B(1 + m\tau^{1-\sigma})a^{1-\sigma} - (mf_X + f_E)$ , therefore

$$\begin{aligned} \frac{\partial \pi(a)}{\partial \tau} &= a^{1-\sigma} \left[ \frac{\partial B}{\partial \tau} (1 + m\tau^{1-\sigma}) - (\sigma - 1)mB\tau^{-\sigma} \right] < 0 \\ \Leftrightarrow \frac{\partial B}{\partial \tau} &< \frac{(\sigma - 1)Bm}{(\tau^\sigma + m\tau)} \end{aligned} \quad [H.1]$$

Note that

$$\frac{-\partial Y(e^*, B, \tau)}{\partial \tau} = (\sigma - 1)m\tau^{-\sigma} B\Gamma^{1-\sigma}(e^*) \int_{\alpha_L}^{\alpha_X} \alpha^{1-\sigma} dF(\alpha) \quad [H.2]$$

and

$$\frac{Y(e^*, B, \tau)}{\partial B} = m\tau^{1-\sigma} \Gamma^{1-\sigma}(e^*) \int_{\alpha_L}^{\alpha_X} \alpha^{1-\sigma} dF(\alpha) \quad [H.3]$$

$$V(e^*) = \Gamma^{1-\sigma}(e^*) \int_{\alpha_L}^{\alpha_M} \alpha^{1-\sigma} dF(\alpha) \quad [\text{H.4}]$$

Using [F.3], [H2] -[H4] we obtain

$$\begin{aligned} \frac{\partial B}{\partial \tau} &= \left( (\sigma-1)\tau^{-\sigma} mB \int_{\alpha_L}^{\alpha_X} \alpha^{1-\sigma} dF(\alpha) \right) / \left[ \int_{\alpha_L}^{\alpha_M} \alpha^{1-\sigma} dF(\alpha) + m\tau^{1-\sigma} \int_{\alpha_L}^{\alpha_X} \alpha^{1-\sigma} dF(\alpha) \right] \\ &< \left( (\sigma-1)\tau^{-\sigma} mB \int_{\alpha_L}^{\alpha_X} \alpha^{1-\sigma} dF(\alpha) \right) / \left[ \int_{\alpha_L}^{\alpha_X} \alpha^{1-\sigma} dF(\alpha) + m\tau^{1-\sigma} \int_{\alpha_L}^{\alpha_X} \alpha^{1-\sigma} dF(\alpha) \right] \text{ (since } \alpha_X < \alpha_M \text{)} \\ &= \frac{(\sigma-1)Bm}{(\tau^\sigma + m\tau)}. \end{aligned}$$

Thus,  $\frac{\partial \pi(a)}{\partial \tau} < 0$ . Further, since  $r(a) = \sigma B(1 + m\tau^{1-\sigma})a^{1-\sigma}$ , so exporting firms also increase their total

output when trade costs fall. Finally, note that if  $\alpha_X = \alpha_M$  i.e. all firms export, then

$$\begin{aligned} \frac{\partial B}{\partial \tau} &= \left( (\sigma-1)\tau^{-\sigma} mB \int_{\alpha_L}^{\alpha_M} \alpha^{1-\sigma} dF(\alpha) \right) / \left[ \int_{\alpha_L}^{\alpha_M} \alpha^{1-\sigma} dF(\alpha) + m\tau^{1-\sigma} \int_{\alpha_L}^{\alpha_M} \alpha^{1-\sigma} dF(\alpha) \right] \\ &= \frac{(\sigma-1)Bm}{(\tau^\sigma + m\tau)} \end{aligned}$$

Hence  $\frac{\partial \pi(a)}{\partial \tau} = 0$  i.e. firm level operating profit (and thus total sales) does not respond to trade costs when

all firms export.

In equilibrium  $\frac{\partial E(\pi)}{\partial e^*} = E(W) = D'(e^*)$ . When firms are heterogeneous, as was proved in appendix

$E, e^*$  decreases in  $\tau$ . Since  $D'(e^*)$  increases in  $e^*$ ,  $D'(e^*)$  is higher for lower  $\tau$ , therefore  $\frac{\partial E(\pi)}{\partial e^*}$

must also be greater when  $\tau$  is lower. Reasoning analogously it can be shown that when firms are quasi-

homogeneous,  $\frac{\partial E(\pi)}{\partial e^*}$  is unchanged when  $\tau$  falls, but decreases when  $f_x$  falls.

(q.e.d.)