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Predicting the Pattern of International Trade in the Neoclassical Model: A Synthesis

by

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Abstract

I propose a framework that takes a set of conceivable outcomes as the primitive and a prediction is defined by identifying a subset on the set of conceivable outcomes. This notion of predictability serves as an organizing principle for characterizing pattern of trade predictions in single economy and integrated equilibrium formulations of the neoclassical trade model. I identify allocative efficiency as the unifying subset selection criterion for the different formulations of the neoclassical trade model, ranging from Ricardo's (1817) original comparative advantage formulation to the multi-cone Heckscher-Ohlin specification with multiple countries, goods and factors.

JEL classification: B10, F11

Keywords: Predictability, pattern of trade, neoclassical model

Outline

Non-Technical Summary

This paper suggests a unifying framework for pattern of trade predictability in the neoclassical trade theory. My framework is based on the Popperian key characteristic of a scientific theory: the imposition of a restriction on possible outcomes. As a result, there are two parts to a theoretical prediction: the identification of a set of conceivable outcomes and a restriction criterion. Applying this apparatus to the neoclassical trade model reveals a remarkable coherence between the various formulations of the model and allows for an intuitive interpretation of the predictions. In addition, it highlights the intellectual continuity between Ricardo's (1817) original formulation of comparative advantage and the modern general equilibrium formulations.

1. Introduction

This paper examines pattern of trade predictions in neoclassical trade theory. I propose a simple notion of predictability and use it as an organizing principle for characterizing pattern of trade predictions in various formulations of the neoclassical trade model. In this framework a set of conceivable outcomes is taken as the primitive and a theory makes a prediction by identifying a subset on the set of conceivable outcomes. This notion of predictability is rooted in Popper's (1953) statement that "Every good scientific theory is a prohibition: it forbids certain things to happen".

The paper makes the following contributions. First, building on Ruffin's (2002) reinterpretation of Ricardo's (1817) "four magic numbers" as labour embodied in trade rather than labour unit coefficients, I show that Ricardo implicitly used this intuitive notion of predictability in what is arguably the first formal model in the history of economic thought. I discuss Ricardo's comparative advantage formulation in a new graphical framework which illustrates the intellectual continuity between Ricardo's first prediction and the higher dimensional formulations which were developed over one and a half centuries later.

Second, I provide a model taxonomy which is organized around the different specifications of the set of conceivable outcomes. I distinguish between single economy predictions (class (i) models) and integrated equilibrium predictions (class (ii) models). In class (i) models, the terms of trade defines the set of conceivable trading patterns and autarky prices impose a single restriction on the pattern of an economy's multilateral trade. In this specification national allocative efficiency is shown to be the subset selection criterion for predicting the pattern of commodity and factor content of trade. From this perspective single economy predictions are invariant to dimensionality in goods and in factor content space. This questions the popular perception that 2-dimensional formulations provide strong predictions, whereas the n-dimensional extensions provide only weak restrictions.

In integrated equilibrium predictions, the set of conceivable outcomes is the set of goods or industries in which countries could specialize in equilibrium. In this framework differences in international factor prices is a prerequisite for the ability to predict in which industries countries will specialize. Free trade factor prices are shown to impose restrictions on predictive specialization based on global efficiency in productive allocation, independent of preferences. I use a continuum of goods framework to highlight the duality between the chain of comparative advantage goods predictions and the multi-cone factor content predictions. A key result is that the pattern of specialization is determined by factor price information from all trading partners in the world economy.

2. Defining predictability

Let us motivate the definition of predictability with a situation outside of economics. A month prior to the 2006 Football World Cup tournament in Germany, a school teacher poses the following question to his students: Who do you predict will win the world cup? Assume the teacher gets the following three answers. Answer A: Brazil will win. Answer B: A European team will win. Answer C: Wales will win. Which of these answers are valid predictions? Clearly, Answer A is a valid prediction. However, Answer B is a valid prediction, too. Although Answer B does not identify a single country as a winner, it provides a prediction by reducing the set of conceivable winners to a European team.[1] On the other hand, Answer C is not a prediction. Since Wales did not qualify for the tournament, this country is not a conceivable winner.

The example illustrates that there are two parts to a prediction: the determination of a set of conceivable outcomes and the identification of a subset. Formally:

Definition: *Given a set Ω of outcomes that are either directly observed or estimated, a theory T is said to make a prediction on the set of conceivable outcomes through the specification of a subset Ω_P of Ω. Ω_P is called the prediction set and $\Omega_A = \Omega/\Omega_P$ is called the alternative.*

The advantage of this notion of predictability is that it leaves room for the specification of an alternative which is often ignored in empirical tests that aim to link theoretical formulations to data.[2] For example, if Ω_A is identified by an alternative

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[1] In fact, historically Answer B turned out to be the best prediction since, with the exception of Brazil in 1958, a European team has always won when the tournament was played in Europe.

[2] This definition of predictability has some similarity with an area theory in experimental economics, where an area theory predicts a subset of all possible outcomes. However, the notion of an area theory appears to be more restrictive since it assumes a size measure on the set of possible outcomes. Selten (1991) investigates properties of a measure of predictive success, assuming there exist an appropriate size measure. Thanks to Chris Starmer for providing me the reference to the Selten paper.

theory T_A, then the theories T and T_A can be distinguished by whether the observed/estimated outcomes fall either in Ω_P or Ω_A. If there is no alternative theory that restricts Ω, which is more common, one can postulate 'chance' as the alternative hypothesis.

Let us apply this framework to the well-known question of how the imposition of an excise tax affects the volume of sales in a well-defined market. Prior to any economic theorizing, there are four conceivable outcomes: the tax will increase sales, it will decrease sales, it will keep sales unchanged or the relationship is ambiguous. Denoting sales by x and the excise tax by t, the set of conceivable outcomes is given by $\Omega=\{\partial x/\partial t>0,\ \partial x/\partial t<0,\ \partial x/\partial t=0,\ \text{ambiguous}\}$. Given the standard ceteris paribus assumptions, partial equilibrium theory predicts that the sales volume will decline, i.e. $\Omega_P = \{\partial x^*/\partial t<0\}$.

This example illustrates that the comparative statics logic can be viewed as a special case of this notion of predictability. Assume we are interested in how changes in a variable α affect a variable x, where the focus is on the direction of the effect, rather than the magnitude. We construct then a theory T which is characterized by $f(x,\alpha)=0$ or a fixed point equation $x=g(x,\alpha)$, where x is the equilibrium variable and α is a parameter of the model. In comparative statics we consider the functional relationship $x^*(\alpha)$ where x^* is the solution to the fixed point equation. Given that T predicts that x^* is increasing in α, the theory's comparative statics prediction can be written as follows: $\Omega=\{\partial x/\partial\alpha>0,\ \partial x/\partial\alpha<0,\ \partial x/\partial\alpha=0,\ \text{ambiguous}\}$and $\Omega_P = \{\partial x^*/\partial\alpha>0\}$.[3]

Although the comparative statics framework is extremely powerful when the variables of interest are univariate, its applications are limited in higher dimensional settings, which are particularly important in international trade theory.[4]

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[3] If an alternative theory T_A were to predict that $\partial x/\partial\alpha<0$, then the two theories could be distinguished from each other.

[4] See Milgrom and Roberts (1994) for developing an ordinal approach to comparing equilibria to remedy some of the shortcomings of the comparative statics framework.

3. David Ricardo's four magic numbers[5]

The genesis of the theory of comparative advantage is found in the following passage from chapter VII of Ricardo's (1817) *Principles of Political Economy and Taxation*:

"The quantity of wine which she [Portugal] shall give in exchange for the cloth of England, is not determined by the respective quantities of labour devoted to the production of each, as it would be, if both commodities were manufactured in England, or both in Portugal.

England may be so circumstanced, that to produce the cloth may require the labour of **100** men if she attempted to make the wine, it might require the labour of **120** men for the same time. England would therefore find it her interest to import wine, and to purchase it by the exportation of cloth.

To produce the wine in Portugal, might require only the labour of **80** men for one year, and to produce the cloth in the same country, might require the labour of **90** men for the same time, It would therefore be advantageous for her to export wine in exchange for cloth." (Ricardo, 1817, p.82)

Following the lead of John Stuart Mill, Ricardo's four magic numbers have been interpreted as the labour units necessary to produce one unit of cloth and wine in England and Portugal, i.e. $a_c^{Eng}=100$, $a_w^{Eng}=120$, $a_c^{Por}=90$, $a_w^{Por}=80$. [6] Given this interpretation, England is predicted to export cloth and import wine because the relative cost of cloth is lower in England than in Portugal: i.e. $a_c^{Eng}/a_w^{Eng}<a_c^{Por}/a_w^{Por}$. However, a disturbing fact of this interpretation is that "the principle (which) is of the very heart and soul of our field" (Ethier, 1984, p. 132) had an illogical beginning. Ricardo draws a conclusion about England's pattern of trade based on the first two numbers; however, a pattern of trade prediction based on relative labour cost comparisons requires information on all four numbers.

In a series of insightful papers, Ruffin (2002) and Maneschi (2004) have rescued Ricardo from the accusation of 'illogical conclusion' by suggesting that Ricardo's numbers pertain to the labour units embodied in actual trade rather than the

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[5] This section builds on an earlier working paper (Bernhofen (2007a)).

[6] Maneschi (2004) provides a brief history of the input coefficient interpretation of Ricardo's four numbers. The term "magic" has been coined by Paul Samuelson.

country's per unit labour coefficients.[7] Building on Ruffin and Maneschi, I argue that Ricardo made implicit use of the framework discussed in section 2 and illustrate the logic underlying his prediction in a new diagram. This diagram reveals the amazing generality of Ricardo's pattern of trade prediction. In fact, the discussion in section 4 shows that the nature of Ricardo's pattern of trade prediction carries over to the modern higher dimensional formulations of the neoclassical trade model.[8]

Ricardo's development of comparative advantage is tightly linked to his labour theory of value. In Ricardo's formulation, the value of a commodity is measured by the quantity of labour embodied in it.[9] The logic inherent in Ricardo's labour value formulation is captured in Figure 1. The horizontal axis pertains to the labour content of cloth; it is positive if cloth is imported and negative if it is exported. The vertical axis pertains to the labour content of wine; it is positive if wine is imported and negative if it is exported.

The 45^0 line in Figure 1 depicts the rule governing domestic exchange: the labour of 100 workers embodied in domestic cloth production must always be exchanged for the labour of 100 workers embodied in domestic wine production. Ricardo postulated that in international trade the labour exchange rate will be different.

"*The same rule which regulates the relative value of commodities in one country does not regulate the relative value of the commodities exchanged between two or more countr*ies….*The labour of 100 Englishmen cannot be given for that of 80 Englishmen, but the produce of the labour of 100 Englishmen may be given for the produce of the labour of 80 Portugese, 60 Russians, or 120 East Indians* (Ricardo, 1817, p.81ff).

The first step in Ricardo's logic is that he postulated a given terms of trade, or international exchange ratio, between cloth and wine. Since Ricardo's trade theory

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[7] Ruffin also brough to light the neglected paper by Sraffa (1930) which provides the same interpreation.

[8] Neither Maneschi nor Ruffin discuss how Ricardo's logic extends to higher dimensional predictions in commodity and factor content space.

[9] By contrast, the familiar textbook transformation curve formulation of the law of comparative advantage is based on Gottfried Haberler's (1930) opportunity cost formulation of the law where the value of good X is measured in terms of forgone units of good Y. A straightjacket of the opportunity cost formulation is that the underlying logic is not extendable to higher dimensions.

was rooted in his labour theory of value, he gave this international exchange ratio in English labour units. If England is able to exchange T_c^{Eng} units of cloth for T_w^{Eng} units of wine, this is equivalent to trading $a_c^{Eng}\ T_c^{Eng}$ English workers embodied in cloth for $a_w^{Eng}T_w^{Eng}$ English workers embodied in wine. Ricardo's first two numbers pertain then to the English labour content of international exchange, i.e. $100 = a_c^{Eng}\ T_c^{Eng}$ and $120 = a_w^{Eng}T_w^{Eng}$. These two numbers predict England's pattern of trade by assuming that England will only be willing to engage in trade which yields gains. If England imports cloth and exports wine, it gains 100 workers at the expense of 120 workers, which results in a net loss of 20 English workers. If it imports wine and exports cloth, it gains 120 workers at the expense of 100 workers, which results in a net gain of 20 workers. *"England would therefore find it her interest to import wine, and to purchase it by the exportation of cloth*".

Formally, England's conceivable trading possibilities can be written as $\Omega^{Eng}=\{T_1^{Eng},\ T_2^{Eng}\}$, where T_1^{Eng} and T_2^{Eng} are England's labour content vectors, defined as $T_1^{Eng}\ =(-a_c^{Eng}T_c^{Eng},\ a_w^{Eng}T_w^{Eng}) = (-100,\ 120)$ and $T_2^{Eng} = (a_c^{Eng}\ T_c^{Eng},\ -a_w^{Eng}\ T_w^{Eng}) = (100,-120)$. England's prediction set is then $\Omega_P^{Eng}=\{T_1^{Eng}\}$ These vectors are depicted in Figure 1. The 45^0 degree line can be interpreted as the autarky reference line which splits the set of conceivable outcomes according to the gains from trade criteria.

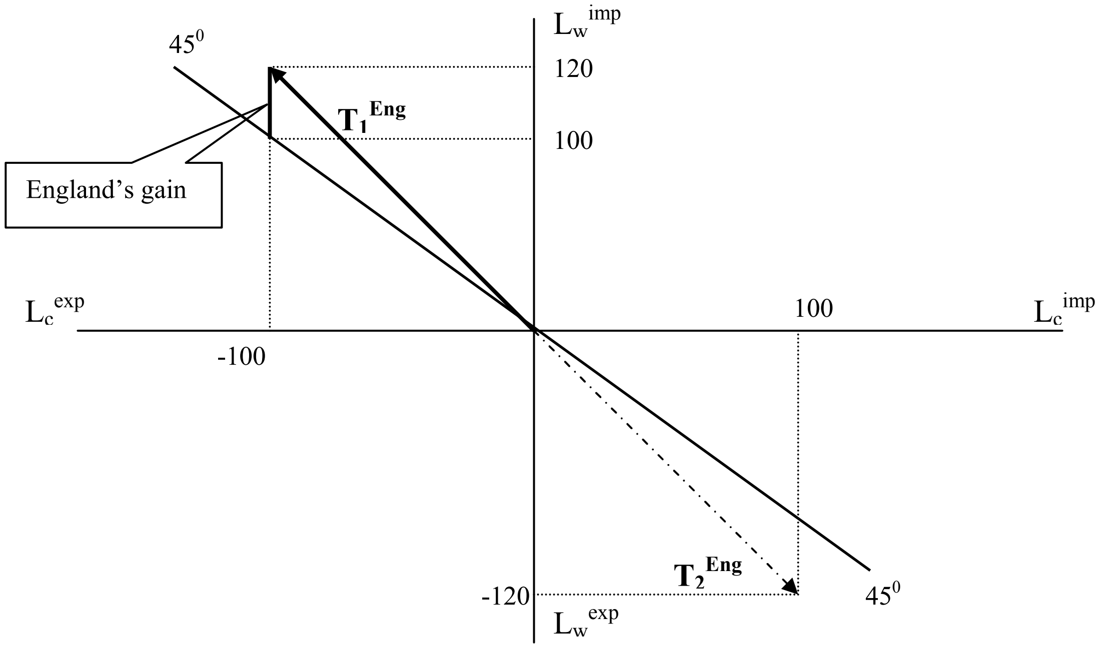

Figure 1: England's labour content of trade

Ricardo's second set of numbers pertain to the Portugese labour content of trade: $90=a_c^{Por}T_c^{Por}$ and $80=a_w^{Por}T_w^{Por}$.[10] Portugal's conceivable trading possibilites are then given by $\Omega^{Por}=\{\mathbf{T_1^{Por}},\mathbf{T_2^{Por}}\}$ where $\mathbf{T_1^{Por}}=(-a_c^{Por}T_c^{Por}, a_w^{Por}T_w^{Por}) = (-90, 80)$ and $\mathbf{T_2^{Por}} =(a_c^{Por}T_c^{Por}, -a_w^{Por}T_w^{Por}) = (90,-80)$. Figure 2 illustrate that the gains from criteria restricts the set of conceivable trading possibilities predicting that Portugal will export wine for cloth since this yields a gain of 10 Portugese workers relative to no trade. Portugal's prediction set is $\Omega_P^{Por}=\{\mathbf{T_2^{Por}}\}$

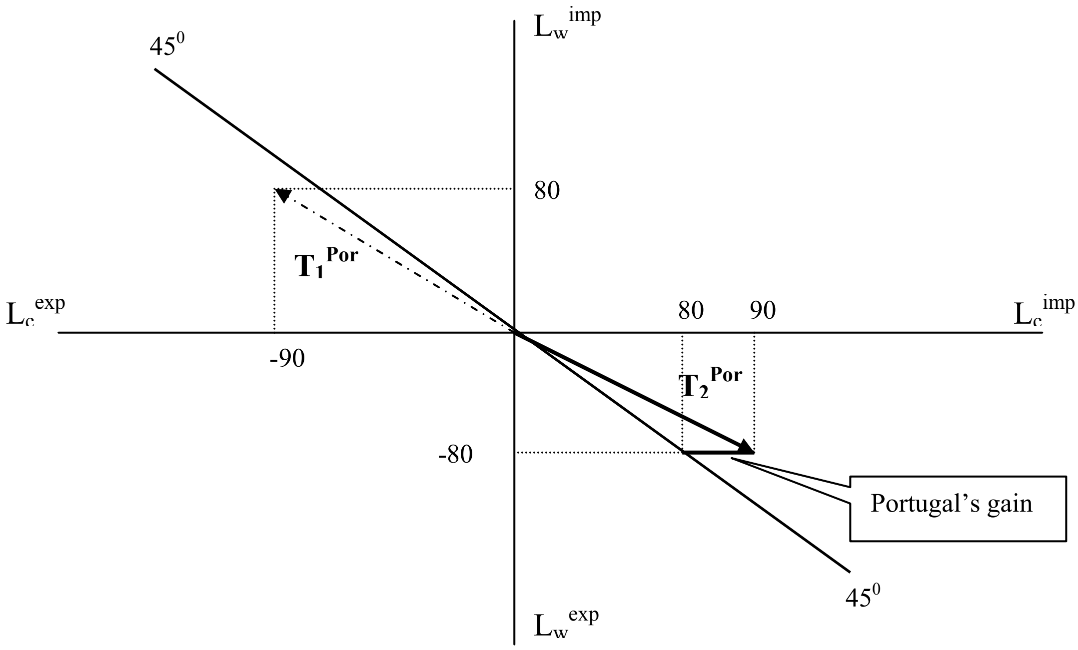

Figure 2: Portugal's labour content of trade

Two things should be noted. Although the domain of Ricardo's prediction pertains to the pattern of commodity trade, the logic is inherently tied to the labour content of trade. Hence, the idea of trade in factor services, or the factor content of trade, is not a 20th century invention, but has its genesis in Ricardo. Second,
by taking the terms of trade as given, Ricardo linked its pattern of trade prediction to the gains from trade without requiring specific information about its trading partner.

[10] The fact that England and Portugal face the same international commodity exchange ratio implies that $T_c^{Eng}/T_w^{Eng}=T_c^{Por}/T_w^{Por}$.

The next section will show that this underlying logic is inherent to higher dimensional formulations of comparative advantage.

4. A taxonomy of pattern of trade predictions

4.1 Single economy formulations

Consider the case of a single economy that faces an exogenous set of world prices. Building on Deardorff (1980, 1982) and Neary and Schweinberger (1986), we apply our predictability framework to commodity and factor content predictions and show that the nature of the underlying prediction is invariant to dimensionality in goods and factor content space. In addition, the analysis reveals that Ricardo's formulation is a special case of either formulation.

4.1.1 Commodity trade predictions

We start out with the 2-good formulation of comparative advantage for a single economy that considers trading with the rest of the world. In this formulation, the world prices p_1^w, p_2^w are exogenously given and determine the terms of trade. The familiar relative price (or opportunity cost) formulation is then:

$$\text{if } \frac{p_1^a}{p_2^a} < (>) \frac{p_1^w}{p_2^w} \text{ then } T_1<(>)0 \text{ and } T_2>(<)0, \quad (1)$$

where p_1^a and p_2^a denote the economy's autarky prices and T_1 and T_2 the corresponding net import quantities.[11] A shortcoming of the price comparison formulation is that it is not extendable to higher dimensions (see Ethier, 1984).
However, the price comparison formulation (1) can be rewritten in terms of a restriction on the set of conceivable outcomes. The set of conceivable outcomes can then be defined as those trading possibilities that satisfy the balanced trade condition, i.e. $\Omega=\{T\in R^2 | p^w{}_1T_1+p^w{}_2T_2=0\}$, and the prediction set is given by:

$$\Omega_P=\{T\in R^2 | p^w{}_1T_1+p^w{}_2T_2=0 \text{ and } p_1^aT_1+p_2^aT_2 >0\}. \quad (2)$$

[11] If T_i >(<)0, good i is imported (exported).

It is easily verified that (1) are (2) are equivalent. However an advantage of the formulation in (2) is that it is invariant to dimensionality:

$$\Omega_P=\{T\in R^n|\ p^w{}_1T_1+\ldots+p^w{}_nT_n=0 \text{ and } p^aT_1+\ldots+p_2{}^aT_n>0\}. \qquad (3)$$

Note that (3) is the n-dimensional comparative advantage formulation developed by Deardorff (1980). The underlying nature of the prediction is illustrated in Figure 3. The balanced trade condition defines a hyperplane in R^n, which is cut into half by the restriction $p^aT>0$. In the case of two goods, the hyperplane is a line with only two conceivable directions for trade, which is illustrated by the vectors T^1 and T^2. In this special case, the restriction predicts a unique trading configuration T^1 where good 1 is exported and good 2 is imported. In higher dimensions, the set of conceivable permissible trading outcomes are also cut into half, however, this does not identify which goods are exported or imported.

In Ricardo's one factor formulation, a country's relative autarky prices are given by the labour input coefficients: $p_c{}^a=a_c$, $p_w{}^a=a_w$.[12] Ricardo's numbers pertain then to the formulation in (2). The restriction on conceivable trading possibilities in the 2-commodity (cloth-wine) world is then $T_ca_c+T_wa_w>0$, postulating that there must be labour savings from international trade.

[12] Because the labour coefficients determine only relative prices, we would have to include a factor of proportionality k. However, without loss of generality, we assume that prices are normalized such that k=1.

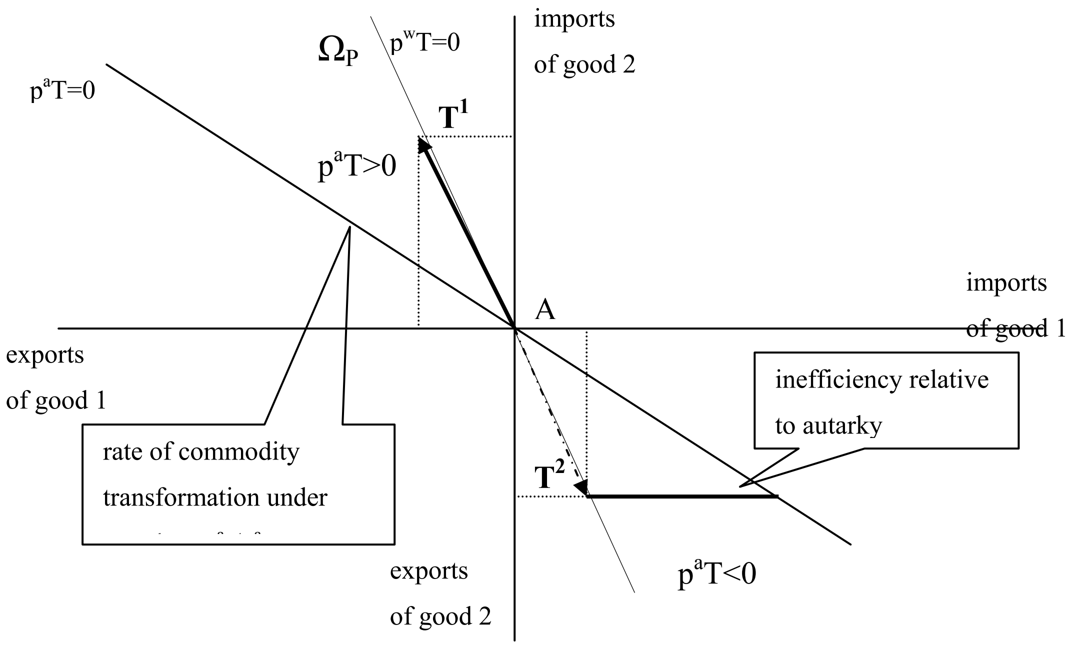

Figure 3: Commodity pattern of trade prediction

The selection criterion is intimately related to the gains from trade resulting from a more efficient allocation of resources. In particular, the trading vector T^2 in Figure 3 is excluded since it is associated with an international transformation of good 2 (i.e. the exportable) into good 1 (i.e. the importable) that is inefficient relative to the autarky transformation, i.e. along the line $p^aT=0$.

Finally, from a testing perspective, the *n* and 2-good formulations are completely equivalent with regard to the specification of the alternative hypothesis. As there exist no alternative theory that imposes restrictions on the set of conceivable outcomes, we can postulate "chance" as the alternative. Under the assumption of "chance", each element of the set of conceivable outcomes is equally likely. Therefore, we can define the null and the alternative hypothesis:

$$H_0: \ \Pr(T \in \Omega_P)=1; \quad H_1: \Pr(T \in \Omega_P)=0.5, \tag{4}$$

where Pr(.) denotes the probability measure. The key point here is that the probability statement in the alternative hypothesis is independent of dimensionality.[13]

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[13] Using autarky price data from 19th century Japan, Bernhofen and Brown (2004) were able to reject the alternative hypothesis at a 99% significance level.

4.1.2 Factor content prediction

Alternatively, we can investigate predictions pertaining to the factor content of trade. Technologies are such that *n* goods are produced from *l* factors under standard CRS production functions. A key point in factor content analysis is the definition of the factor content of trade in a world with unequal technologies.[14] In the context of our framework, we calculate the economy's factor content using the domestic technology matrix A. We can then define then the set of conceivable outcomes as:

$$\Omega=\{F \in R^l | F=AT \text{ and } p^w T=0\}. \quad (5)$$

The prediction or selection criterion identifies again the trading configurations that are efficient for the economy.

Let us now split the net import vector T into its individual components: T=M-X, where M is the *n*-good import vector and X is the *n*-good export vector.[15] Given a particular trading vector T, the economy is giving up actual factor services AX embodied in its exports in exchange for the factor services embodied in its imports. AM are the domestic resource gains embodied in imports. Interpreting the autarky factor price vector w^a as the shadow prices at which the economy evaluates factor services embodied in trade, the economy would be willing to engage in the trading opportunity T only if the 'gain from factor imports' exceeds the loss from factor exports, i.e. $w^a(AM)>w^a(AX)$. The corresponding prediction can be stated as follows:

$$\Omega_P=\{AT \in R^l | \; w^a(AT)>0\}. \quad (6)$$

Figure 4 illustrates factor content of trade triangles in the two-factor case. Given two conceivable factor content of trade vectors AT^1 and AT^2, trade in factor

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[14] Deardorff (1982) considers three different variations of the factor content of trade, but assumes identical technologies. Neary and Schweinberger (1986) define the factor content of trade based on domestic techniques of production.

[15] The entries in X and M are now all non-negative. Since a particular good is either imported or exported, X will have entries of "0"for goods that are imported and M will have entries of "0" for goods that are exported.

services can be thought of an augmentation of the country's endowment vector $V=(V_1,V_2)$. The factor content of trade is decomposed in the factor content of exports AX^i and the factor content of imports AM^i (i=1,2). The factor content vector AT^1 leads to a welfare gain since $w^a(V+AT^1)>w^aV$. By comparison, the factor content vector AT^2 leads to a welfare loss since $w^a(V+AT^2)<w^aV$.

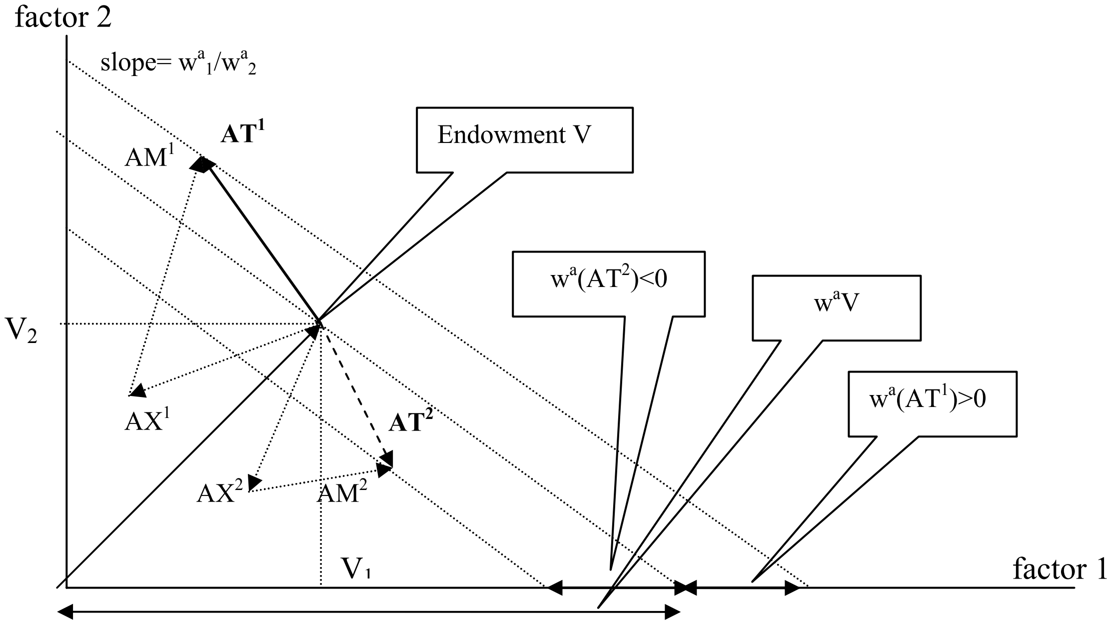

Figure 4: Factor content triangles and welfare

Alternatively, the factor content prediction is illustrated in Figure 5, which can be viewed as the factor content dual to Figure 3. The factor content of trade vector AT^2 is excluded from the set of conceivable outcomes as it leads to an inefficient international factor transformation relative to the situation of no trade.

Ricardo's prediction is then a special formulation of (6): $w^a(T_ca_c+T_wa_w)>0$. In the case of a single factor, the magnitude of the autarky price w^a does not matter for the sign of the left-hand side, so w^a can be normalized to 1 and we obtain Ricardo's prediction.

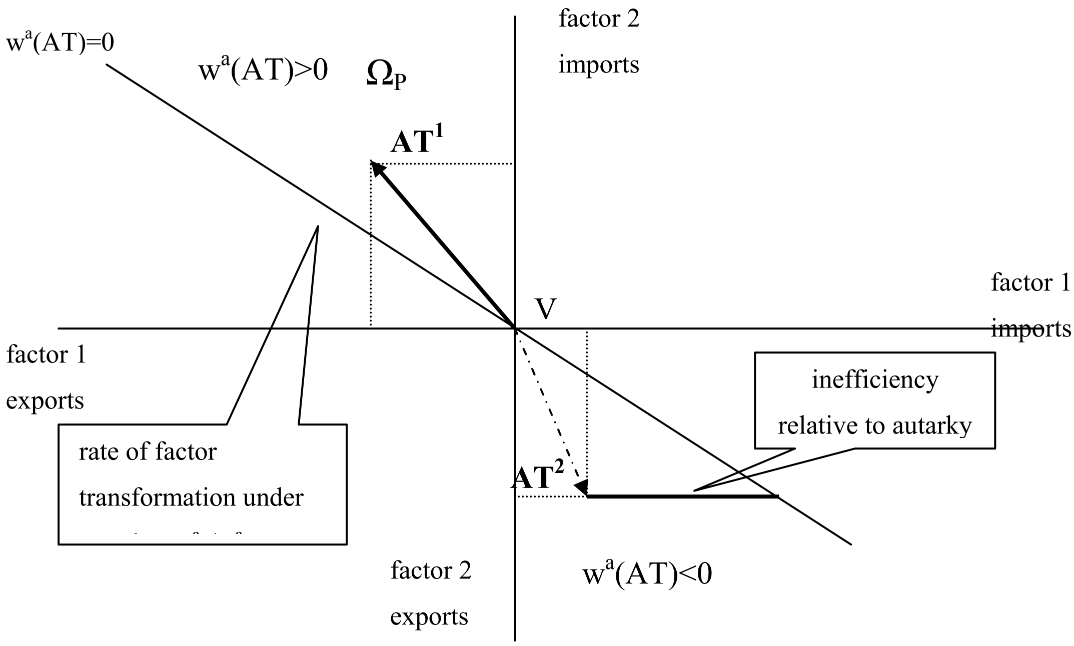

Figure 5: Factor content prediction

4.2 Integrated equilibrium formulations

A drawback of the specifications in section 4.1 is that they rely on autarky price data, which are not usually observable.[16] In this section we characterize restrictions on the pattern of international trade based on factor prices that are observable in a trading regime. The analysis is motivated by an emerging empirical literature claiming evidence in favour of the neoclassical trade model by testing restrictions on bilateral trade flows.[17] The theoretical foundation of these studies is based on Helpman (1984), who has shown that in an integrated equilibrium without international factor price equalization, the factor content F^{ij} of any bilateral trade flow from country *i* to country *j*, is restricted (or predicted) by the corresponding factor price difference (w^j-w^i) between these two countries: $(w^j-w^i)F^{ij} \geq 0$.

In what follows, I will show two things. First, I show that in an integrated equilibrium the predictive domain of the theory is a country's export vector, or factor content of exports, rather than its bilateral export vector. Second, a country's

[16] An exception is Bernhofen and Brown (2004).

[17] See for instance the recent papers by Choi and Krishna (2004), Lai and Zhu (2007) and the earlier work by Brecher and Choudhri (1993).

equilibrium exports (or factor content of exports) is restricted by the factor prices of all trading partners.[18]

Our analytical framework is based on the continuum of goods formulations pioneered by Dornbusch, Fischer and Samuleson, DFS (1977, 1980). In this set-up, the set of conceivable outcomes Ω is the set of industries in the world economy, characterized by the unit interval Ω = [0,1]. Free trade factor prices impose restrictions on Ω = [0,1] which predict in which industries an economy will specialize. Since the emphasis is on the production side of the economy, one does not need to make any specific assumptions about the demand side of the economy, except that preferences are such that an equilibrium exists. To develop the intuition, we first characterize predictions in the Ricardian specification (DFS, 1977) and then move on to the Heckscher-Ohlin specifications (DFS, 1980).

4.2.1 Ricardian continuum of good formulation

In the Ricardian formulation, Ω = [0,1] is a continuum of industries which are exogenously ranked according to their relative labour productivity $A(z)=a^2(z)/a^1(z)$, where A(z) is decreasing in z so that country 1 (home) has a productivity advantage in low-indexed industries and country 2 (foreign) has a productivity advantage in high-indexed industries. In this specification, home and foreign free trade factor prices w^1 and w^2 determine the dividing, or marginal, good *m*, defined as $A(m)=w^1/w^2$. The prediction set for the home economy is $\Omega_1= [0,m(w^1/w^2)]$, which can be characterized by

$$\Omega_1=\{z\in[0,1] |\ a^2(z)w^2-a^1(z)w^1 \geq 0\}. \qquad (7)$$

The prediction in (7) can be interpreted as saying that free trade factor prices impose a restriction on Ω = [0,1] that guarantee that the country 1 specializes in those industries in which it is most efficient relative to country 2, i.e. the left-side of the interval.[19]

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[18] In a companion paper (Bernhofen (2007b)), I have derived these additional restrictions using Helpman's analytical apparatus and applied them to the data domain of Choi and Krishna. However, the focus of this paper is just to characterize these restrictions and identify the links to other specifications.

[19] Alternatively, the prediction set for the foreign economy is $\Omega_2= [m(w^1/w^2),1]$. If one incorporates uniform iceberg transportation costs, the efficiency criterion is modified such that there are two border

The exact location of the border good *m* will depend on w^1/w^2 which embodies all the relevant information about preferences, endowments etc. In sum, the pattern of specialization is characterized by a single restriction.[20]

4.2.2 Multi-cone Heckscher-Ohlin formulation: 2 countries

Consider now a Heckscher-Ohlin specification with 2 factors (capital and labour), 2 countries (country 1 and 2) and identical CRS technologies. The set of conceivable outcomes is again a continuum of industries in the unit interval $\Omega = [0,1]$, where each industry z in Ω is characterized by its capital-labour ratio $\beta(z) = a_K(z)/a_L(z)$. Industries are ranked in order of decreasing capital intensity, i.e. $\beta(z)$ is decreasing in z.[21]

Assume that country 1 is relatively capital abundant, i.e. $K^1/L^1 > K^2/L^2$.

If the free trade equilibrium is characterized by factor price equalization, i.e. $w^1 = w^2$ and $r^1 = r^2$, then the model does not provide any prediction on sectorial specialization. The reason for this is if factor prices are identical, it is equally efficient to produce the goods in either country. Consequently, there is no global efficiency criterion that imposes a restriction on where the goods are produced.[22] Lack of international factor price equalization is central to predictability.

Assume now that factor endowments are sufficiently dissimilar so that factor prices are different in equilibrium. Because country 1 is assumed to be relatively capital abundant, country 1 must have a higher wage-rental ratio in equilibrium: $w^1/r^1 > w^2/r^2$. Then there exists again a border good m_1, such that home specializes in $\Omega_1 = [0, m_1]$ and foreign specializes in $\Omega_2 = [m_1, 1]$. Ω_1 is characterized by the following restriction

$$\Omega_1 = \{z \in [0,1] \mid a_K(z)(r^2 - r^1) + a_L(z)(w^2 - w^1) \geq 0\}. \qquad (8)$$

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goods, m_1 and m_2, with country 1 specializing in $[0,m_1]$, country 2 specializing in $[m_2,1]$ and both countries producing the non-traded goods $[m_1, m_2]$.

[20] Here we focus only on the two-country specification since it has been a challenge to extend DFS (1977) to multiple countries in a tractable way. See Matsuyama (2007) for an excellent survey on the Ricardian trade literature and the extensions to multiple countries.

[21] Without loss of generality, and for ease of exposition, we assume fixed coefficient technologies, i.e. $\beta(z)$ does not depend on factor prices.

[22] Under the assumption of identical homothetic preferences, we obtain the Heckscher-Ohlin-Vanek prediction, which has been the workhorse equation for testing the neoclassical trade model.

The underlying logic in (8) is the same as in (7). Factor prices in (8) impose a single restriction which partitions the industry set Ω such that the relatively capital abundant country 1 will specialize in the most capital-intensive goods $[0,m_1]$ and the relatively labour abundant country will specializes in the most labour-intensive goods $[m_1,1]$. Factor price differences determine the equilibrium location of production based on global cost minimization.

Figure 6 illustrates this prediction with the help of the well-known Lerner-Pearce Diagram. The country's equilibrium factor prices determine each country's unit value iso-cost line, defined by $w_iL+r_iK=1$ (i=1,2). Cost minimization implies that the production equilibria are characterized by the tangency between the unit-value iso-cost lines and the isoquants that generate $1 worth of revenue. Since it is equally efficient to produce the border good in either country, i.e. $a_K(m_1)(r^2-r^1) + a_L(m_1)(w^2-w^1)=0$, its position is determined by the intersection of the country's iso-cost curves. The restriction in (8) implies then that country 1 will specialize and export goods z that satisfy $a_K(z)/a_L(z) \geq a_K(m_1)/a_L(m_1)=(r^2-r^1)/(w^1-w^2)$.

Alternatively, we can characterize the factor content dual to (8) by considering the factor content of production or exports. The set of conceivable outcomes in factor content space is $\Omega^{FC}=\{(K,L)|\ 0\leq K/L<\infty\}$. Free trade factor prices generate then a partitioning on the set of conceivable outcomes, i.e. $\Omega^{FC}= \Omega_1^{FC} \cup \Omega_2^{FC}$, where

$$\Omega_1^{FC}= \{(K,L)|\ a_K(m_1)/a_L(m_1) \leq K/L < \infty\}. \qquad (9)$$

The factor content prediction is illustrated in Figure 6, which shows that Ω_1^{FC} and Ω_2^{FC} define country-specific cones. This specification predicts that any factor content of export vector F^i of country i must lie in Ω_i^{FC} and is implicitly restricted by home and foreign factor prices.

Alternatively, one can view the restriction on each country's pattern of specialization as a solution to a global social planner's problem who, for given countries' factor prices, decides on the productive allocation of resources in the most cost-efficient way. Graphically, globally efficient specialization can be characterized by the social planner's iso-cost curve. Figure 6 identifies the social planner's iso-cost curve as the bold segments of the countries' iso-cost curves, with the kink occurring at the marginal good m_1. Global efficiency requires then that the more capital abundant country must specialize in the most capital-intensive goods.

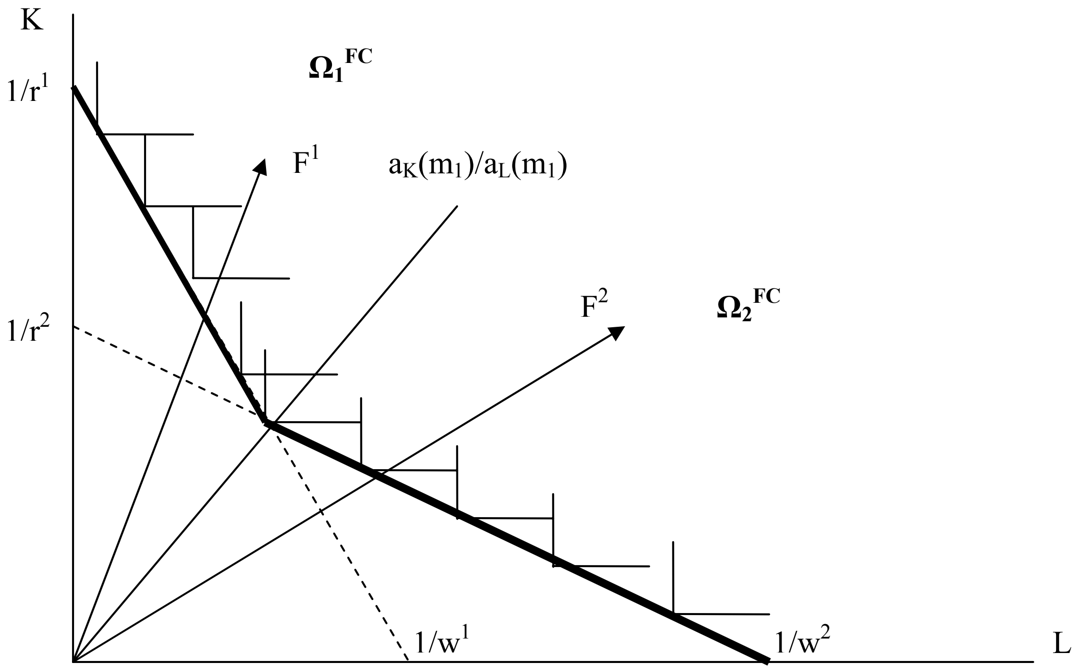

Figure 6: Two-cone Heckscher-Ohlin specification based on a single restriction

4.2.3 Multi-cone Heckscher-Ohlin formulation: n countries.

Consider now the case of *n* countries where we continue to assume that all countries have the same CRS technologies which enable them to produce any of the goods in [0,1]. Ranking the countries in decreasing relative capital abundance, we obtain: $K^1/L^1 > K^2/L^2 > K^3/L^3 > \ldots > K^n/L^n$. Assuming again that factor endowments are sufficiently dissimilar, the equilibrium factor price ratios will reflect the endowment ranking: $w^1/r^1 > w^2/r^2 > w^3/r^3 > \ldots > w^n/r^n$. The integrated equilibrium will then be characterized by n-1 border goods $m_1, m_2, \ldots, m_{n-1}$ which define the ranges of specialization for the individual countries: $\Omega_1=[0,m_1]$, $\Omega_2=[m_1, m_2], \ldots$ $\Omega_i=[m_{i-1},m_i] \ldots$ $\Omega_n=[m_{n-1},1]$. Ω_i is then characterized by the following global cost efficiency criterion:

$$\Omega_i=\{z \in [0,1] |\ a_K(z)(r^k - r^i) + a_L(z)(w^k - w^i) \geq 0;\ k \neq i\}. \quad (10)$$

The key characteristic of the specification in (10) is that the prediction set Ω_i of country i is determined by *n-1* restrictions involving the free trade factor prices of all trading partners. Consequently, any border good m_i can be viewed as an implicit function of all factor prices: $m_i=m_i(w_1,r_1,w_2,r_2,\ldots,w_n,r_n)$. The intuition for this is that since the factor price ratios embody information on countries' relative factor scarcities, efficient multilateral specialization requires information on the factor scarcities of all trading partners.

We can again characterize the factor content dual to (10). The factor content space is characterized by *n* country-specific cones $\Omega_1^{FC}, \Omega_2^{FC},\ldots, \Omega_{n-1}^{FC}$, where Ω_i^{FC} is given by[23]:

$$\Omega_i^{FC}= \{(K,L)|\ a_K(m_i)/a_L(m_i)\leq K/L\leq a_K(m_{i-1})/a_L(m_{i-1})\ ] \qquad (11)$$

Figure 7 illustrates the multi-cone Heckscher-Ohlin specification in the case of 3 countries. The border goods m_1 and m_2 identify the social planner's kinked iso-cost curve, which consists now of three segments. The three segments correspond to the 3 country-specific cones of diversification Ω_1^{FC}, Ω_2^{FC} and Ω_3^{FC}.

A few comments are in order regarding the nature of the prediction. First, since a factor content set Ω_i^{FC} characterizes the production side of the economy, the prediction pertains to the factor content of production or exports, independent of where the exports are shipped. Given any factor content of exports F^i originating in country i, the theory predicts that $F^i \in \Omega_i^{FC}$, as seen in Figure 7.

Second, the number of trading partners matters.[24] In particular, the theory predicts that the size of the cones becomes smaller, the more trading partners there are. Ω_1^{FC} is smaller in Figure 7 than in Figure 6 since the additional trading partners enables country 1, which is most capital abundant, to specialize in a smaller set of the most capital-intensive goods.

[23] We need to define $a_K(m_0)/a_L(m_0)=\infty$ and $a_K(m_n)/a_L(m_n)=0$.

[24] Assuming, of course, that countries' factor endowments are sufficiently dissimilar.

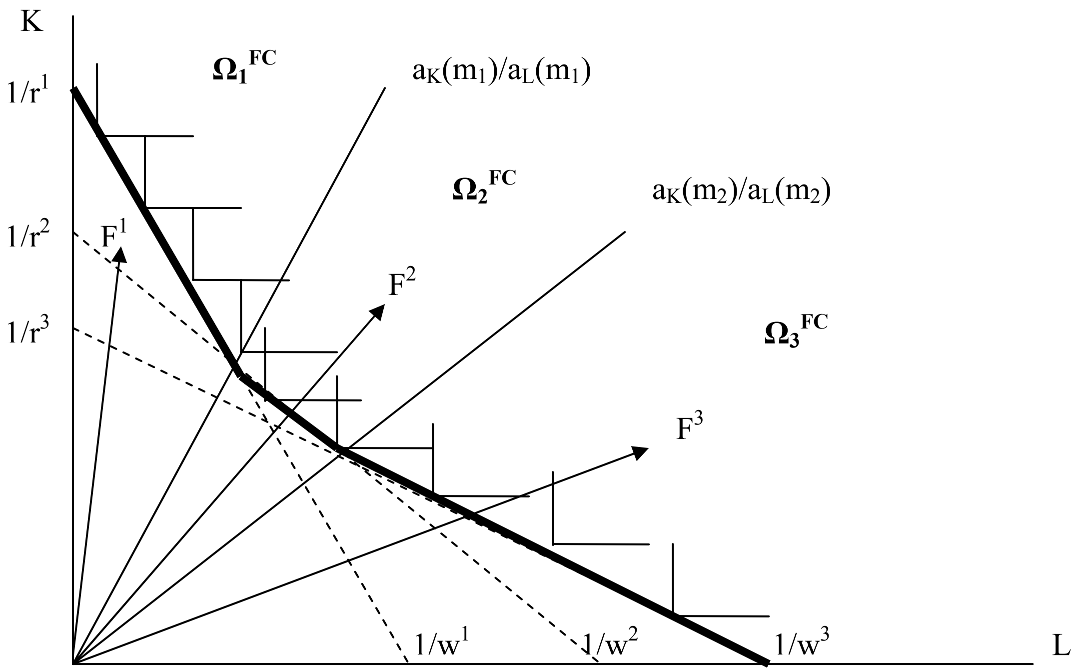

Figure 7: n-cone Heckscher-Ohlin specification based on *n*-1 restrictions (here: *n=3*):

5. Concluding remarks

Using a single analytical framework, I have characterized a whole class of pattern of trade predictions, from Ricardo (1817) to the multi-cone specification. One of the key messages of this paper is that the pattern of trade predictions can be linked to efficiency. For small open economy predictions, efficiency is directly related to the gains from trade since an economy will only be willing to engage in trading activities that are more efficient than what it can do under autarky. As a result, autarky goods and factor prices impose restrictions on observable trading patterns. For integrated equilibrium predictions, the pattern of international specialization is governed by global efficiency. Lack of factor price equalization is central and free trade factor prices of all trading partners restrict patterns of specialization in goods and factor content space.

The message that pattern of trade predictions are directly related to efficiency gains in models without factor price equalization provides an important justification for testing these models. For instance, if empirical tests confirm these predictions, they provide implicit evidence for efficiency gains resulting from international specialization.

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