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Partial International Emission Trading

by

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Abstract

In a model inspired by the EU Emissions Trading Scheme, non-cooperative countries allocate their emissions to internationally trading and non-trading sectors. Each country is better off with trading than without, and aggregate welfare is maximized with all sectors in the trading scheme. We simulate the effects of expanding the trading scheme in a two-country model with quadratic abatement costs. If only the original trading sector is asymmetric between countries, the welfare change is always positive and the same in both countries. If only the additional trading sector is asymmetric, one country might lose, but there is an aggregate welfare gain. If only the non-trading sector is asymmetric, both countries always gain.

JEL classification: H23; Q28; Q56

Keywords: International emission trading; EU Emission Trading Scheme

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Non-Technical Summary

When it comes to choosing instruments for environmental policy, economists are greatly in favour of tradable emission permits. With this instrument, emission reduction will be undertaken by those firms who can do it at the lowest cost.

Recently, emission trading has been applied more and more in environmental policy. The largest scheme so far is the European Union Emission Trading Scheme (EU ETS) for the carbon dioxide (CO_2) emissions of around 12,000 firms in energy-intensive industries in all 27 Member States. The scheme covers about 50% of the EU's total CO_2 emissions. The EU ETS started in 2005 and Phase 2 runs from 2008-2012, coinciding with the Kyoto commitment period. All EU Member States have committed to certain greenhouse gas emission reductions in this period under the Kyoto Protocol (1997). Currently, the European Commission's proposals for Phase 3, which will run from 2013-2020, are being discussed. The proposals include an expansion of the scheme to other industries and greenhouse gases.

Each Member State has to submit a National Allocation Plan (NAP) to the Commission for approval. This NAP details the total amount of permits the MS intends to distribute, and how it intends to allocate these among its firms. The Commission's main criteria for the Phase 2 NAPs were that they should be consistent with the Member State's Kyoto target and they should take into account the expected development of emissions and technical reduction potential.

In this paper, we analyze what would happen if Member States could determine by themselves (without Commission approval) how many permits to allocate to their firms in the EU ETS. Each country will still have to meet its national (Kyoto) target, therefore allocating more permits to the internationally trading sectors means that there are less emissions left for the non-trading sectors.

Countries will try to manipulate the international permit price with their permit allocation. A net seller of permits tries to drive up the permit price by reducing its allocation to its trading sectors, while a net buyer tries to reduce the permit price by allocating more permits to its trading sectors.

We find that each country is better off with any international trading scheme than without. This is the standard economic argument of gains from trade. Moreover, the welfare of all countries together would be maximized with an international trading scheme covering all sectors, because then there is nothing to manipulate anymore. However, this does not mean that each country will benefit from an expansion of the trading scheme, like the one that is proposed for Phase 3 of the EU ETS.

We look at the effects of expanding the trading scheme more closely in a two-country model. If only the original trading sector is asymmetric between countries, the welfare change is always positive and the same in both countries. If only the additional trading sector is asymmetric, one country might lose, but there is an aggregate welfare gain. If only the non-trading sector is asymmetric, both countries always gain.

1 Introduction

The concept of emissions trading is appealing in principle and has been shown to work in practice. If the market works well, all polluting firms set their marginal abatement cost equal to the permit price and the efficient allocation of emissions is achieved. After the success of the Sulfur Allowance Trading scheme in the US (Ellerman et al., 2000), the EU set up its Emissions Trading Scheme (EU ETS) for the CO_2 emissions of large industrial sources. The EU ETS came into operation in January 2005 and now includes around half of the EU's CO_2 emissions from 12,000 installations across all 27 Member States. It is currently the largest company-based emission trading scheme in the world.

However, the fact that the EU ETS is an international scheme introduces a new complication. While individual firms may take the permit price as given, an individual country might be large enough to manipulate the permit price with its allocation of permits to its internationally trading firms. As an example, let us take Phase 2 of the EU ETS which corresponds with the Kyoto compliance period (2008-2012). Although each Member State[1] has a national greenhouse gas emission ceiling under the Kyoto Protocol or the Burden Sharing Agreement, a country still has room to manoeuvre because it can decide how to allocate this ceiling between those sectors participating in the EU ETS (the trading sectors) and those outside the scheme (the non-trading sectors). A country that will sell permits within the EU ETS will want to drive up the permit price. It can do so by reducing the allocation of permits to its trading sectors, which means more will be left for its non-trading sectors. Conversely, a permit buying country will try to decrease the permit price by issuing more permits to its trading sectors. The strategic interaction between countries trying to manipulate the permit price with their permit allocations, and especially the welfare effects of this behaviour, is the subject of this paper.

It is easily seen that a country is always better of with any international emission trading scheme than without it. This is simply the standard economic argument that trade improves welfare. It is also intuitive that the countries' joint welfare is maximized when all polluting sectors are included in the international scheme. This would leave the individual countries no room to manipulate the permit price, and all polluters would have marginal

[1] Except for Cyprus and Malta.

abatement costs equal to the permit price. However, this does not imply that each country's welfare is maximized when all sectors are included in the international trading scheme. Nor can we be sure that each country would always gain from an expansion of the trading scheme to include more firms.

We will set up the simplest possible model to analyze the complicated issue of the welfare effects of expanding the international trading scheme. This is not only an intriguing theoretical issue, but it is also relevant to the EU ETS. The European Commission is planning to increase the coverage of the EU ETS from 2013, as we will discuss in more detail in Section 2.

While our model was inspired by the EU ETS, it applies to any international emissions trading scheme. We can expect to see more of these schemes in the future, as countries around the world are starting to take climate change policy more seriously. The success of the EU ETS may lead other countries to link their domestic trading scheme to it or to set up another international trading scheme. Any scheme is likely to be partial in its coverage. Small firms and households will probably be excluded, because the transaction costs of setting up and running the scheme for them would be prohibitive. Thus, a country that participates in an international emission trading scheme will always face the choice of how to allocate emissions between its internationally trading and its non-trading sectors, and the temptation to manipulate the permit price with this choice.

The seminal study on market power in markets of transferable property rights is provided by Hahn (1984). Assuming a single firm has market power, he demonstrates that the total expenditure on abatement will exceed the cost-minimizing solution unless the firm with market power receives an amount of permits equal to the number that it holds in equilibrium. Thus the distribution of permits matters not only in terms of equity, but also for efficiency.

Hahn (1984) also shows that, when a regular interior minimum exists, a transfer of permits from any of the price takers to the firm with market power will result in an increase in the equilibrium price. A direct corollary of this result is that the firm with market power itself uses more permits with an increase in its initial allocation. The inefficiency of the market increases as the number of permits allocated to the firm with market power raises above or falls below the amount of permits it holds in equilibrium.

Hahn's (1984) model was extended by Van Egteren and Weber (1996) to allow for the non-compliance of firms with their regulatory obligations. Their findings are supportive of Hahn's (1984) result that the initial distribution of permits influences the degree of market power in the permit market. Malik (2002) shows that the firm with market power may choose to hold more permits than it needs such that it effectively retires permits from the market. Moreover, this behaviour is not found to be a function of the possible non-compliance of firms. Armstrong (2008) considers a two-period model with market power in which the allocation of allowances is made dependent upon historic allowance acquisitions. It is shown that such a model can ensure either market efficiency (elimination of market power) or time efficiency (optimal behaviour is achieved across time periods as opposed to within time periods), and under specific conditions both can be achieved.

Helm's (2003) paper is closest to ours. Helm (2003) models the endogenous choice of emission allowances by non-cooperative countries for a global pollutant in regimes with and without permit trading. The major difference is that Helm (2003) assumes that each country sets its own national emission level and that the permit trading regime covers all sectors of the economy in all countries. In our paper, national emission ceilings are given and the international trading scheme does not cover all sectors. Then countries have to decide how to allocate their fixed national emission ceiling between trading and non-trading sectors.

Helm (2003) shows that environmentally less concerned countries tend to choose more allowances if these are tradable, while environmentally more concerned countries choose fewer allowances. The overall effect on emissions is ambiguous. In addition, it is found that as countries are affected differently by a trading scheme, there may be no unanimous agreement on trading even if it leads to a fall in pollution. Conversely, even if aggregate emissions are higher, a trading regime may be unanimously approved by countries due to the efficiency gains of the permit market. In our paper, by contrast, international emission trading always increases a country's welfare, because total emissions remain the same.

Maeda (2003) analyzes a permit market consisting of one large buyer, one large seller and many price-taking parties. He finds that the large seller has effective market power (the ability to move the permit price away from its competitive level) if and only if the volume of his excess permits exceeds the net shortage of permits in the market. The large buyer cannot have effective market power.

Babiker et al. (2004) demonstrate how an international emission trading scheme may lead to a direct welfare loss in some countries due to general equilibrium effects when there are market distortions. This occurs in countries exporting emission permits when efficiency costs associated with the pre-existing distortionary taxes are larger than the primary gains from emission trading. Similarly, Böhringer et al. (2008) show that there could be substantial efficiency losses to the imposition of emission taxes on sectors that are covered by the EU ETS.

The rest of this paper is organized as follows. Section 2 reviews the experience so far with the EU ETS, as well as the literature on this subject. Section 3 constructs a theoretical model of the EU ETS which is then developed in Section 4 in order to provide conditions which determine whether a country buys or sells permits. Section 5 discusses the welfare implications of the trading scheme, whilst Section 6 looks at the possible impact of expanding the trading scheme in the context of a simple example. Finally, Section 7 concludes.

2 The EU Emissions Trading Scheme (EU ETS)

The EU Emissions Trading Scheme ETS came into operation in January 2005 and now includes around half of the EU's CO_2 emissions from 12,000 installations across all 27 Member States.[2] It covers energy activities (combustion, mineral oil refineries, coke ovens), ferrous metals, the mineral industry (cement, glass, ceramics) and pulp and paper. All installations in these sectors above a certain minimum size are included in the EU ETS. Plants whose total emission level exceeds their total EU allowance (EUA) holdings either have to reduce their emissions or buy unused allowances from other firms. Otherwise firms will be fined 40 Euros for every excess tonne of CO_2 beyond the number of allowances they hold.

[2] Convery and Redmond (2007), Ellerman and Buchner (2007) and Kruger et al. (2007) discuss the workings of the EU ETS and its operation in Phase 1.

Phase 1 of the EU ETS ran from 2005 through 2007. Phase 2 coincides with the Kyoto compliance period (2008-2012). Phase 3 will run from 2013 to 2020.

For Phases 1 and 2, each Member State had to submit a National Allocation Plan (NAP) to the Commission for approval. A country's NAP specifies the total amount of EUAs it will issue to the plants covered by the EU ETS, and how this total allowance is distributed between the individual plants. It also details how the Member State plans to deal with new entrants and how many allowances it plans to auction. The maximum amount that can be auctioned is 5% in Phase 1 and 10% in Phase 2. The majority of allowances is grandfathered, i.e. distributed for free.

In Phase 1, the allowance price rose to around €30 per Mton CO2 before the 2005 compliance figures were announced in April and May 2006. These figures revealed a large overallocation of allowances, so that the price dropped to around €15. By the end of Phase 1, allowances had become virtually worthless. Banking of allowances into Phase 2 was practically ruled out.

The main criteria on which the Commission assessed the Phase 2 NAPs were that they should be consistent with the Member State's emission development, its reduction potential, and its commitment under the Kyoto Protocol or burden sharing agreement (BSA). Under the Kyoto Protocol of 1997, the EU as a whole (15 members) took on a greenhouse gas reduction target of 8% in 2008-2012 compared to 1990. The Member States later distributed this reduction among themselves in the BSA. The new EU members (except for Cyprus and Malta) have a national emission reduction target under the Kyoto Protocol.

In their Phase 2 NAPs, the Member States proposed to allocate allowances for a total of 2325 Mton CO_2. The Commission only allowed 2083 Mton, a reduction of 10%. It looked comparatively favourably upon the Western European Member States' NAPs, approving the amounts proposed by the UK and France and mandating cuts of 6% for Italy and Germany, 7.5% for Belgium and 8.5% for Greece. It also approved the amount proposed by Slovenia, but it was much harsher for the rest of the new Member States, with cuts of 27% for Poland, 37% for Bulgaria and 49% on average for the Baltic states. The new Member States typically had no problem reaching their generous Kyoto targets, but fell foul of the combined expected emissions/reduction potential criterion. Many Eastern European Member States have appealed against the Commission's decision. However, while the appeal is pending (which could take a few years), the Member States have to implement the Commission's decision.

In January 2008 the European Commission announced its proposals for Phase 3 (2013-2020) of the EU ETS (EC, 2008), as part of its overall plan to reduce EU greenhouse gas emissions to 20% by 2020. The Commission plans to extend the coverage of the EU ETS to additional sectors (aluminium and aviation) and greenhouse gases (nitrous oxide and PFCs). It proposes a formula for the Member States' allowance allocation in each year, so that Member States would no longer have to submit NAPs to the Commission. Finally, the Commission would like the share of auctioned allowances to rise to 100% by 2020, at least for those sectors that are not too exposed to international competition.

A number of studies have conducted computable general equilibrium analyses of EU trading schemes. The Commission asked Capros and Mantzos (2000) to use the PRIMES (EC, 1995) energy system model to analyze the impact of alternative emission trading schemes implemented in the EU-15 in 2005. The emission reduction commitments are assumed to apply for the year 2010. In the Reference case it is assumed that each Member State separately implements its burden sharing target at least cost. The total compliance cost is estimated at € 9bn, with marginal abatement costs differing substantially across Member States. The compliance cost rises to € 20.5bn Euros in the Alternative Reference case in which it is assumed the Kyoto commitment must apply to each individual sector, with no emission trading between sectors.

Capros and Mantzos (2000) then consider alternative EU-wide emission trading schemes in which different sets of sectors participate. If emission trading takes place only among energy supply sectors in the EU, total compliance costs are 7.2bn Euros. If the trading scheme is expanded to include energy intensive industries in the EU, total compliance costs fall by a further 0.3bn Euros. Coverage in this scenario is quite close to the actual EU ETS. If the emission trading takes place among all sectors in the EU, total costs fall to 6bn Euros, a 34% reduction relative to the Reference case.

In the two partial trading scenarios, Capros and Mantzos (2000) let each Member

State allocate the same emissions to its trading sector as it does in the Reference case. However, this is not optimal in any sense. Indeed, as Bernard et al. (2004) already note, the full benefits of EU-wide trading among all sectors can also be realized in a partial trading scheme by a suitable set of national allocations. All that is needed is for each Member State to allocate to its non-trading sectors the amount they would emit under full trading, and the rest of its allowed national emissions to its trading sectors. A less rosy view of Member States' behaviour would have each country choose the allocation that minimizes its own compliance cost, given the other countries' allocations. That is exactly the scenario that we will examine in this paper.

Bernard et al. (2004) assume, as we will, that each Member State allocates its emissions between trading and non-trading sectors so as to maximize its own welfare, given the other countries' allocations. Using the GEMINI-E3 model for the EU-15 in 2010-2020, they identify three major players: Germany operates as a potential seller while Italy and the Netherlands are assumed to collude as potential buyers. The combined power of Italy and the Netherlands is similar to that of Germany. However, the three countries' deviations from the competitive allocation are quite small, as are their welfare losses.

Viguier et al. (2006) also assess the strategic allocation of emission allowances in the EU ETS, using a two-level computable equilibrium model. Four groups of players are considered: Germany, the UK, Italy and the rest of the EU. These regions have to choose among four different rules to allocate emission allowances across economic sectors. The equilibrium solutions are found to the different possible strategy choices and from this the payoff matrices are obtained by running the GEMINI-E3 model. It is shown that the EU Member States characterised by high abatement costs could be tempted to give a generous initial allocation of allowances to their energy-intensive industries. However, the incentive to act strategically is relatively small as there would only be a limited impact on country payoffs.

De Muizon (2006) uses the GTAP-ECAT model to analyze the efficiency of the EU ETS. In particular, the cost implications are investigated of allocating the trading sectors a quantity of allowances for the 2008-2012 period such that their total emissions, as a share of national emissions, is the same as that granted by the 2005-2007 NAPs. Each

Member State is then assumed to respect its Kyoto commitment in 2010 by imposing an appropriate CO_2 tax on the non-trading sectors. Comparing the difference between the market allowance price and the shadow price of emissions in the non-trading sectors it is found that there would be inefficient burden sharing between the trading and non-trading sectors. For instance, while the equilibrium permit price is found to equal € 6.5, the tax level ranges from zero for each of the Eastern European countries to €430 for Denmark. In fact, an average tax of €113 per tonne of CO_2 is required by the EU-15. De Muizon (2006) investigates three possible solutions to mitigate this inefficient outcome: Allocating the optimal amount of allowances to the trading sectors, importing credits from Joint Implementation and Clean Development Mechanism projects, and expanding the sectoral coverage of the trading scheme.

Böhringer and Rosendahl (2008) focus specifically on the impact the strategic choice of emission allowances by Member States may have on the outcome of the EU ETS. They do this by running a number of simulations of the non-cooperative equilibrium based on a partial equilibrium multi-regional model for the EU-27 countries. This model is based on marginal abatement cost curves for trading and non-trading sectors in the EU-27 that are calibrated to empirical data. Crucially, they find that single countries can indeed have a significant impact by exploiting their market power. For instance, it will lead to substantial differentiation of marginal abatement costs across countries in the sectors outside the EU ETS, although the effects on the quota price and total abatement costs are relatively small. Overall costs are €314 million, only 3.8% higher than in the cost-effective outcome with costs of € 305m. In addition, in comparison to the cost-effective outcome, more abatement is found to take place in the initial EU Member States when countries have market power. Perhaps surprisingly, it should be noted that Böhringer and Rosendahl (2008) find that the EU could achieve its emission reduction commitments at a lower cost without a trading scheme (costs of € 5.3bn) than with the EU ETS based on the actual Phase 2 NAPs agreed upon (costs of € 8.2bn). This is mainly because abatement requirements are very tough for the non-trading sectors in Portugal and Spain.

3 The model

Let there be n countries, $i = 1, \cdots, n$. Country i has an exogenously given emission ceiling E_i. Polluters in each country are divided into a trading and a non-trading sector. The rules according to which polluters (or polluting activities) are divided into the two sectors are exogenous. Total benefits of emissions e_i in the trading sector of country i are $\pi_i(e_i)$, with marginal benefits positive ($\pi_i'(e_i) > 0$) and decreasing in emissions ($\pi_i''(e_i) \leq 0$). Total benefits of emissions ε_i in the non-trading sector of country i are $\phi_i(\varepsilon_i)$, again with $\phi_i' > 0$, $\phi_i'' \leq 0$. When there is an international permit trading scheme, the polluters in the trading sector can trade internationally with each other, but the polluters in the non-trading sector cannot.

Let us start with the autarky benchmark.[3] Each country i has to decide how to divide its ceiling E_i between the trading and the non-trading sector. It allocates $e_i \geq 0$ emissions to the trading sector and the rest $\varepsilon_i = E_i - e_i \geq 0$ to the non-trading sector. Each country i maximizes total benefits:

$$\max \ \pi_i(e_i) + \phi_i(E_i - e_i) \tag{1}$$

Let e_i^a be the emissions allocated to and taking place in the trading sector under autarky, with e_i^a given by the first order condition for e_i in (1):

$$\pi_i'(e_i^a) = \phi_i'(E_i - e_i^a) \tag{2}$$

We have assumed here that firms that do not trade permits internationally can still be regulated efficiently such that they all have the same marginal benefits. This may be because these sectors participate in a domestic trading scheme or they are subject to a carbon tax.

With international emission trading, country i's trading sector emissions e_i can differ from the amount of emissions $\bar{e}_i$ allocated to the sector by country i's emission authority.[4] Total allowed emissions, $\bar{e}_i$ for the trading sector plus ε_i for the non-trading sector, must

[3] Needless to say, autarky only refers to emission trading. There may very well be international trade in goods.

[4] It makes no difference to our analysis whether the national government auctions or grandfathers the permits to its firms, or (in the latter case) how it distributes the permits among its firms. This is because we assume the permit market is perfectly competitive for firms, and because we are only interested in the welfare of the country as a whole.

add up to the national ceiling E_i. After each country has set its $\bar{e}_i$ and distributed the permits among its trading sector, the polluters in the trading sectors trade the permits among each other. We assume that each individual polluter is too small to have market power. Thus each polluter takes the permit price P as given, so that e_i is determined by:

$$\pi_i'(e_i) = P \tag{3}$$

i.e. marginal benefits of emissions equal the permit price, under the restriction:

$$e \equiv \sum_{j=1}^{n} e_j = \sum_{j=1}^{n} \bar{e}_j \tag{4}$$

i.e. total emissions in all trading sectors equal the total amount of permits for the trading sectors. Equations (3) and (4) implicitly define P and e_i as a function of total trading sector emissions e, with:

$$P'(e) = \frac{1}{\sum_{j=1}^{n} \frac{1}{\pi_j''(e_j)}} \tag{5}$$

Totally differentiating (3) with respect to e and substituting (5) yields:

$$\pi_i''(e_i)\frac{de_i}{de} = P'(e) = \frac{1}{\sum_{j=1}^{n} \frac{1}{\pi_j''(e_j)}} < 0 \tag{6}$$

Thus we have:

$$0 < \frac{de_i}{d\bar{e}_i} = \frac{de_i}{de} = \frac{1/\pi_i''(e_i)}{\sum_{j=1}^{n} \frac{1}{\pi_j''(e_j)}} < 1 \tag{7}$$

Country i now chooses $\bar{e}_i$ to maximize:

$$\pi_i(e_i) + \phi_i(E_i - \bar{e}_i) + P(\bar{e}_i - e_i) \tag{8}$$

which is assumed to have a unique maximum. The first order condition is:

$$-\phi_i'(E_i - \bar{e}_i) + P + P'(\bar{e}_i - e_i) = 0 \tag{9}$$

4 Buyers and sellers

Define Marginal Revenue MR_i for country i as:

$$MR_i(\bar{e}_i, \bar{e}_{-i}) \equiv P(\bar{e}_i + \bar{e}_{-i}) + P'(\bar{e}_i + \bar{e}_{-i})\,[\bar{e}_i - e_i] \tag{10}$$

We will now see that a country's Marginal Revenue curve is between the international demand curve and the trading sector's Marginal Benefit curve, and that the three curves intersect only once (if at all).

Lemma 1 *For a given vector $\bar{e}_{-i}$:*

1. *If there is an $e_i^0 \in [0, E_i]$ with*

$$MR_i(e_i^0, \bar{e}_{-i}) = \pi_i'(e_i^0) \tag{11}$$

then this e_i^0 is unique.

2.

$$MR_i(\bar{e}_i, \bar{e}_{-i}) \gtrless P(\bar{e}_i + \bar{e}_{-i}) \qquad for\ \bar{e}_i \lessgtr e_i^0$$

3.

$$MR_i(\bar{e}_i, \bar{e}_{-i}) \lessgtr \pi_i'(\bar{e}_i) \qquad for\ \bar{e}_i \lessgtr e_i^0$$

Proof.

1. By (3) and (10), if $\bar{e}_i = e_i^0$, then given (11) it must be that $e_i = e_i^0$ and therefore:

$$MR_i(e_i^0, \bar{e}_{-i}) = P(e_i^0 + \bar{e}_{-i})$$

By (10):

$$MR_i(\bar{e}_i, \bar{e}_{-i}) - P(\bar{e}_i + \bar{e}_{-i}) = P'(\bar{e}_i + \bar{e}_{-i})\left[\bar{e}_i - e_i\right] \tag{12}$$

Since $P' < 0$ by (5), the LHS of (12) is zero if and only if $\bar{e}_i - e_i = 0$. By (7), $\bar{e}_i - e_i$ is increasing in $\bar{e}_i$. Thus e_i^0 is unique.

2. Since e_i^0 is unique by Lemma 1.1, then if $MR_i(\bar{e}_i, \bar{e}_{-i}) > (<) P(\bar{e}_i + \bar{e}_{-i})$ for one $\bar{e}_i < (>) e_i^0$, then it holds for all $\bar{e}_i < (>) e_i^0$. Let us look at e_i just above and below e_i^0. If:

$$\frac{\partial MR_i(e_i^0, \bar{e}_{-i})}{\partial \bar{e}_i} < P'(e_i^0 + \bar{e}_{-i}) \tag{13}$$

then $MR_i(\bar{e}_i, \bar{e}_{-i}) > (<) P(\bar{e}_i + \bar{e}_{-i})$ for $\bar{e}_i$ just below (above) e_i^0, and thereby for all $\bar{e}_i < (>) e_i^0$. From (10) and since $e_i = \bar{e}_i$ at e_i^0:

$$\frac{\partial MR_i(e_i^0, \bar{e}_{-i})}{\partial \bar{e}_i} = P'\left[2 - \frac{de_i}{d\bar{e}_i}\right] < P' \tag{14}$$

The inequality follows from (7). Thus (13) is satisfied.

3. Since e_i^0 is unique by Lemma 1.1, then if $MR_i(\bar{e}_i, \bar{e}_{-i}) < (>) \pi_i'(\bar{e}_i)$ for one $\bar{e}_i < (>) e_i^0$, then it holds for all $\bar{e}_i < (>) e_i^0$. Let us look at e_i just above and below e_i^0. If:

$$\frac{\partial MR_i(e_i^0, \bar{e}_{-i})}{\partial e_i} > \pi_i''(e_i^0) \tag{15}$$

then $MR_i(\bar{e}_i, \bar{e}_{-i}) < (>) \pi_i'(\bar{e}_i)$ for $\bar{e}_i$ just below (above) e_i^0, and thereby for all $\bar{e}_i < (>) e_i^0$. From (10) and since $e_i = \bar{e}_i$ at e_i^0:

$$\frac{\partial MR_i(e_i^0, \bar{e}_{-i})}{\partial e_i} = P'\left[2 - \frac{de_i}{d\bar{e}_i}\right] = P'\left[2 - \frac{P'}{\pi_i''(e_i^0)}\right] \tag{16}$$

The second equality follows from (6). Substituting (16) into (15):

$$P'\left[2 - \frac{P'}{\pi_i''(e_i^0)}\right] > \pi_i''(e_i^0)$$

The inequality holds because multiplying both sides by $-\pi_i''(e_i^0) > 0$ and rearranging yields:

$$[P']^2 - 2P'\pi_i''(e_i^0) + \left[\pi_i''(e_i^0)\right]^2 = \left[P' - \pi_i''(e_i^0)\right]^2 > 0$$

■

We can now determine which countries will be buyers and which will be sellers of permits:

Proposition 1 *For a given vector of $\bar{e}_{-i}$, define:*

$$\tilde{P}_i^a \equiv P(e_i^a + \bar{e}_{-i})$$

Then country i will buy (sell) permits when $\pi_i'(e_i^a) > (<) \tilde{P}_i^a$. A buyer (seller) country will allocate less (more) permits to the trading sector than under autarky, and emissions by the trading sector in a buyer (seller's) country will be higher (lower) than in autarky. That is: $\bar{e}_i^t < e_i^a < e_i^t$ for a buyer and $e_i^t < e_i^a < \bar{e}_i^t$ for a seller country.

Proof. From Lemma 1 and using (2) it can be seen that:

$$\phi_i'(E_i - e_i^a) = \pi_i'(e_i^a) \gtrless MR_i(e_i^a, \bar{e}_{-i}) \gtrless P_i^a \qquad for\ e_i^a \lessgtr e_i^0 \tag{17}$$

Hence $\pi_i'(e_i^a) > (<) P_i^a$ will hold if $e_i^a < (>) e_i^0$, in which case $MR_i(e_i^a, \bar{e}_{-i}) < (>) \phi_i'(E_i - e_i^a)$. In addition, we know from (9) know that $\phi_i'(E - \bar{e}_i^t) = MR_i(\bar{e}_i^t, \bar{e}_{-i})$. Therefore it must be that $\bar{e}_i^t < (>) e_i^a$. Futhermore, given that if $\bar{e}_i^t < (>) e_i^a$ then $\phi_i'(E - e_i^a) > (<) \phi_i'(E - \bar{e}_i^t)$, and using Lemma 1:

$$\pi_i'(e_i^a) = \phi_i'(E - e_i^a) \underset{<}{\overset{>}{}} \phi_i'(E - \bar{e}_i^t) = MR_i(\bar{e}_i^t, \bar{e}_{-i}) \underset{<}{\overset{>}{}} P(\bar{e}_i^t + \bar{e}_{-i}^t) \qquad for\ \bar{e}_i^t \underset{>}{\overset{<}{}} e_i^a$$

Therefore for $\bar{e}_i^t < (>) e_i^a$ then $\pi_i'(e_i^a) > (<) P(\bar{e}_i^t + \bar{e}_{-i}^t)$, and given the first order condition $\pi_i'(e_i^t) = P(\bar{e}_i^t + \bar{e}_{-i}^t)$ specified by (3), it directly follows that $\pi_i'(e_i^t) < (>) \pi_i'(e_i^a)$. Hence $e_i^t > (<) e_i^a$. ■

The intuition behind these results is as follows. Firstly, $\bar{e}_i^t < (>) e_i^a$ holds for a buyer (seller), because at e_i^a the marginal revenue of reducing (increasing) the allocation to the trading sector by one permit is less (more) than the marginal abatement costs associated with increasing (decreasing) emissions in the non-trading sector. Hence decreasing (increasing) $\bar{e}_i$ must lead to an increase in welfare. Secondly, $e_i^t > (<) e_i^a$ holds for a buyer (seller), because at e_i^a if country i increases (reduces) its trading sector emisssions, it is able to buy (sell) more permits, for which the marginal revenue is less (greater) than the associated marginal benefit from emissions in the trading sector. Hence increasing (decreasing) e_i must lead to an increase in welfare.

5 Welfare

Let us consider a country's welfare as a function of which sectors are included in the trading scheme. It is easily seen that:

1. Every country's welfare is higher with an international trading scheme (regardless of which sectors are included) than without any trading scheme. This is the standard economics result that trade improves welfare and is illustrated graphically below.

2. Aggregate welfare is highest when all sectors are included in the trading scheme. In this case, all sectors in all countries will have equal marginal abatement costs, and aggregate welfare is maximized. As long as there are non-trading sectors, however, a country will allocate its emissions between trading and non-trading sectors in a way that manipulates the permit price in its favour. This will increase this country's welfare at the expense of other countries and of aggregate welfare.

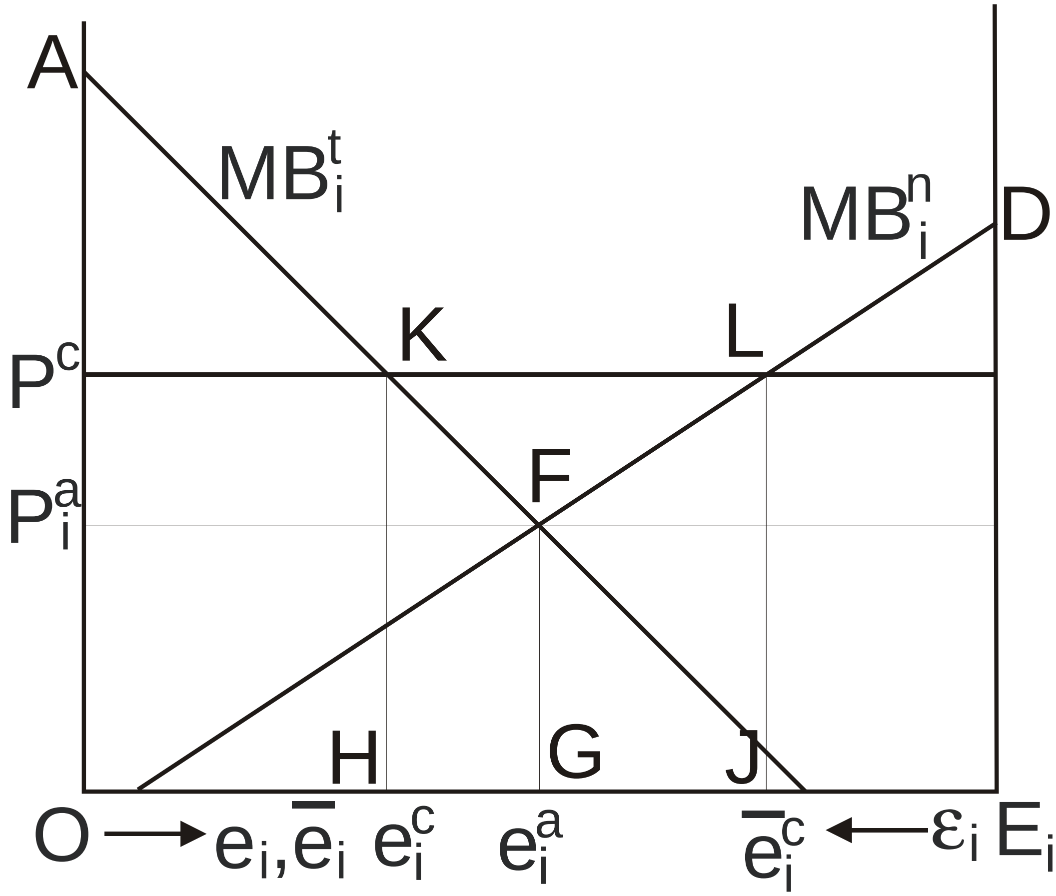

Figure 1: The switch from autarky to competitive international trade

Let us consider a country's change in welfare from autarky to the international trading scheme in two steps. First we look at the change from autarky to competitive international trade, illustrated in Figure 1. Then we consider the change in the international permit market from competitive to one with market power (Figure 2). Country i has a national emission ceiling of E_i. It has marginal benefits MB_i^t of emissions e_i in the trading sector, measured from left to right in Figure 1. It has marginal benefits MB_i^n of emissions ε_i in the non-trading sector, measured from right to left. In autarky, the country sets $MB_i^t = MB_i^n$, so that emissions in the trading sector are e_i^a and marginal benefits in both sectors are P_i^a. Total benefits are AFG for the trading sector and DFG for the non-trading sector.

Now consider a competitive international trading regime, with a permit price P^c that the country takes as given. Country i now has to divide its national emissions E_i between

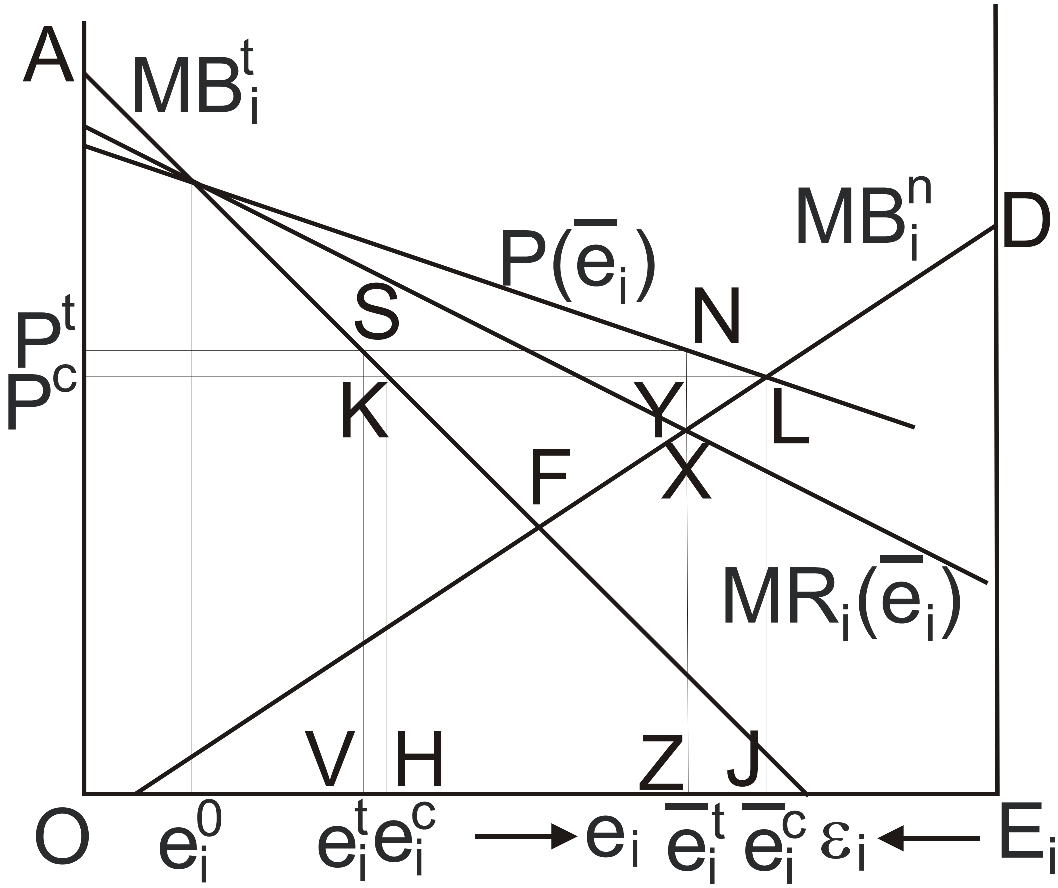

Figure 2: The switch from competitive trade to trade with market power

emissions for the non-trading sector ε_i and permits $\bar{e}_i$ allocated to the trading sector, the latter measured from left to right in Figure 1. The country sets $MB_i^n = P^c$, so that it allocates $\bar{e}_i^c$ permits to the trading sector. This sector will only emit e_i^c and sell the rest of its permits abroad. Total benefits from emissions are now AKH in the trading sector and DLJ in the non-trading sector. In addition, the trading sector receives the revenue of $HKLJ$ from selling HJ permits. Total welfare has increased by KLF compared to autarky.

Figure 2 illustrates the switch from competitive trade to trade with market power. As before, the international permit price will be P^c if the country issues $\bar{e}_i^c$ permits to its trading sector, but the permit price is now a decreasing function $P(\bar{e}_i)$ of $\bar{e}_i$. The country will be a net seller (buyer) of permits if it issues less (more) than e_i^0 permits to its trading sector. We know from Lemma 1 that marginal revenue MR_i lies above (below) $P(\bar{e}_i)$ for any $\bar{e}_i < (>) e_i^0$ because allocating more permits to the trading sector depresses the permit price, which is an extra benefit when the country is buying permits, but an extra cost when it is selling. The MR_i curve lies below (above) MB_i^t, so that the country will allocate less (more) permits to the trading sector when it can buy (sell) them abroad. The country now sets $MR_i = MB_i^n$, reducing its permit allocation to the trading sectors to $\bar{e}_i^t$ and driving the price up to P^t in Figure 2. The trading sector will now emit e_i^t. Total benefits from emissions are ASV in the trading sector and DXZ in the non-trading sector. In addition, the trading sector receives the revenue of $VSNZ$ from selling VZ permits. The gain in welfare compared to competitive trade is $KSNY - YLX$. This is positive, because by definition, allocating $\bar{e}_i^t$ to the trading sector when faced with the inverse demand function $P(\bar{e}_i)$ maximizes country i's welfare. The loss YLX is the efficiency loss from the domestic distortion of letting MB_i^n deviate from the permit price. The gain $KSNY$ is the gain from being able to sell the permits at a higher price.

6 Expanding the trading scheme

6.1 A two-country model

In this section we analyze the welfare effects of expanding the trading scheme. For simplicity, we assume there are just two countries i, where $i = 1, 2$, and benefit functions are quadratic. We divide the economy into three sectors j, where $j = 1, 2, 3$. Sector j of country i has a benefit function $\theta_{ji}(e_{ji})$ given by:

$$\theta_{ji}(e_{ji}) = b_0 e_{ji} - \frac{1}{2} b_{ji} e_{ji}^2 \tag{18}$$

where marginal benefits are:

$$\theta'_{ji}(e_{ji}) = b_0 - b_{ji} e_{ji} \tag{19}$$

Thus b_{ji} represents the slope of the marginal abatement cost curve in sector j of country i. We consider two scenarios S, $S = A, B$. In scenario A, sector 1 is in the trading scheme while sectors 2 and 3 are outside. In scenario B, sectors 1 and 2 are in the trading scheme, and sector 3 is outside.

Let us define:

$$\gamma_i^A \equiv b_{1i}, \qquad \gamma_i^B \equiv \frac{b_{1i}b_{2i}}{b_{1i}+b_{2i}}, \qquad \eta_i^A \equiv \frac{b_{2i}b_{3i}}{b_{2i}+b_{3i}}, \qquad \eta_i^B \equiv b_{3i} \tag{20}$$

Aggregate benefit functions for the trading and non-trading sectors in country i in scenario S, $S = A, B$, are respectively:

$$\begin{aligned} \pi_i(e_i^S) &= b_0 e_i^S - \frac{1}{2}\gamma_i^S \left(e_i^S\right)^2 \\ \phi_i(\varepsilon_i^S) &= b_0 \varepsilon_i^S - \frac{1}{2}\eta_i^S \left(\varepsilon_i^S\right)^2 \end{aligned}$$

The aggregate marginal benefit functions are:

$$\pi_i'(e_i^S) = b_0 - \gamma_i^S e_i^S \tag{21}$$

$$\phi_i'(\varepsilon_i^S) = b_0 - \eta_i^S \varepsilon_i^S \tag{22}$$

In autarky, marginal benefits are equal across sectors in the same country. From (19), the autarky price in country i with national emission ceiling E_i is given by:

$$P_i^a = b_0 - \frac{b_{1i}b_{2i}b_{3i}}{b_{1i}b_{2i}+b_{1i}b_{3i}+b_{2i}b_{3i}}E_i \tag{23}$$

Let us call the buyer country 1. This is the country with the higher autarky price, i.e. $P_1^a > P_2^a$, which implies from (23) that:

$$\Phi \equiv b_{11}b_{12}b_{21}b_{22}(b_{32}E_2 - b_{31}E_1) + b_{11}b_{12}b_{31}b_{32}(b_{22}E_2 - b_{21}E_1) + b_{21}b_{22}b_{31}b_{32}(b_{12}E_2 - b_{11}E_1) > 0 \tag{24}$$

or from (20):[5]

$$\Theta \equiv \frac{\gamma_1\gamma_2\left(\eta_2 E_2 - \eta_1 E_1\right) + \eta_1\eta_2\left(\gamma_2 E_2 - \gamma_1 E_1\right)}{\gamma_1\gamma_2\eta_1\eta_2} > 0 \tag{25}$$

In the joint welfare-maximizing outcome, marginal benefits in all sectors would be equal to each other at P^* given by:

$$P^* = b_0 - \frac{\gamma_1\gamma_2\eta_1\eta_2(E_1+E_2)}{\gamma_1\gamma_2\eta_1 + \gamma_1\gamma_2\eta_2 + \gamma_1\eta_1\eta_2 + \gamma_2\eta_1\eta_2} \tag{26}$$

With a partial emission trading scheme, the firms that participate in the scheme set their marginal benefits equal to the permit price:

$$P^S = b_0 - \gamma_i^S e_i^S \tag{27}$$

[5] In the following two equations we suppress the scenario superscript S, because the values of Θ and P^* do not depend on the scenario.

It follows from (27) for $i = 1, 2$ that:

$$P^S = b_0 - \frac{\gamma_1^S \gamma_2^S \left(e_1^S + e_2^S\right)}{\gamma_1^S + \gamma_2^S} = b_0 - \frac{\gamma_1^S \gamma_2^S \left(\bar{e}_1^S + \bar{e}_2^S\right)}{\gamma_1^S + \gamma_2^S} \tag{28}$$

and:

$$e_i^S = \frac{\gamma_j^S \left(\bar{e}_1^S + \bar{e}_2^S\right)}{\gamma_i^S + \gamma_j^S} \tag{29}$$

The welfare of country i in scenario S is given by:

$$W_i^S = b_0 e_i^S - \frac{1}{2}\gamma_i^S \left(e_i^S\right)^2 + b_0 \varepsilon_i^S - \frac{1}{2}\eta_i^S \left(\varepsilon_i^S\right)^2 + P^S(\bar{e}_i^S - e_i^S)$$

The first order condition for $\bar{e}_i^S$ is:

$$-b_0 + \eta_i^S \left(E_i - \bar{e}_i^S\right) + P^S + P^{S\prime}(\bar{e}_i^S - e_i^S) = 0 \tag{30}$$

Substituting (28) and (29) into (30) and suppressing the scenario superscript S for notational simplicity gives country i's first order condition with marginal revenues MR_i as defined in Section 4:

$$MB_i^n \equiv b_0 - \eta_i \left(E_i - \bar{e}_i\right) = b_0 - \frac{\gamma_i \gamma_j}{\gamma_i + \gamma_j}\left(\bar{e}_i + \frac{\gamma_i}{\gamma_i + \gamma_j}(\bar{e}_i + \bar{e}_j)\right) \equiv MR_i \tag{31}$$

Solving (31) for the two countries yields:

$$\bar{e}_i = \frac{1}{\Delta}\left(\left[(\gamma_i + 2\gamma_j)\gamma_i\gamma_j + \left(\gamma_i + \gamma_j\right)^2 \eta_j\right]\eta_i E_i - \gamma_i^2 \gamma_j \eta_j E_j\right) \tag{32}$$

$$e_i = \frac{\gamma_j}{\Delta}\left[\gamma_i\gamma_j(\eta_i E_i + \eta_j E_j) + (\gamma_i + \gamma_j)\eta_i\eta_j(E_i + E_j)\right] \tag{33}$$

with

$$\Delta \equiv 2\gamma_1^2\gamma_2^2 + \gamma_1\gamma_2(\gamma_1\eta_1 + \gamma_2\eta_2 + 2\left[\gamma_1\eta_2 + \gamma_2\eta_1\right]) + (\gamma_1 + \gamma_2)^2\eta_1\eta_2 \tag{34}$$

Substituting (33) into (28), we find the permit price in the international trading scheme as:

$$P = b_0 - \frac{\gamma_1\gamma_2}{\Delta}\left[\gamma_1\gamma_2(\eta_1 E_1 + \eta_2 E_2) + (\gamma_1 + \gamma_2)\eta_1\eta_2(E_1 + E_2)\right] \tag{35}$$

Combining (26) and (35), we see that:

$$P - P^* = \frac{\Theta\gamma_1^3\gamma_2^3\eta_1\eta_2\left(\eta_1 - \eta_2\right)}{\Delta\left(\gamma_1\gamma_2\eta_1 + \gamma_1\gamma_2\eta_2 + \gamma_1\eta_1\eta_2 + \gamma_2\eta_1\eta_2\right)} \tag{36}$$

Since $\Theta > 0$ by (25), we have:

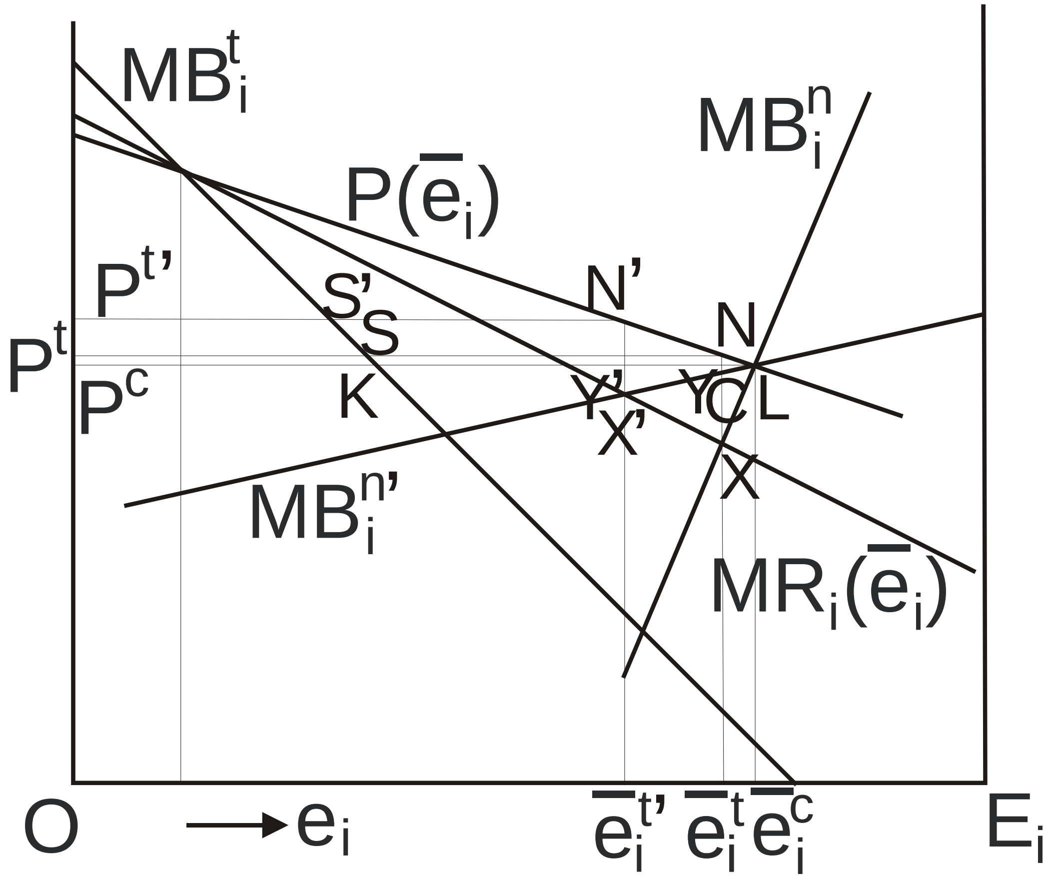

Figure 3: Effect of the slope of the MB_i^n curve

Proposition 2 *Comparing the equilibrium international permit price P in (35) to the joint welfare-maximizing price P^* in (26), we find:*

1. *When $\eta_1 = \eta_2$, $P = P^*$;*
2. *When $\eta_1 < (>)\eta_2$, $P < (>)P^*$, i.e. the country with the lowest η_i manages to manipulate the permit price in its favour.*

Figure 3 illustrates the effect of η_i which is the slope of the MB_i^n curve. If country i took the permit price P^c as given, it would issue $\bar{e}_i^c$ permits to it trading sector. When η_i is large so that country i's marginal benefits in the non-trading sector are given by the steep curve, it will reduce its permit allocation to the trading sector to $\bar{e}_i^t$, driving the permit price up to P^t. It would lose YLX relative to the competitive outcome because of the domestic distortion, but gain $KSNY$ from the higher permit price. Now suppose that

η_i is small, so that country i's marginal benefits in the non-trading sector are given by the much flatter curve $MB_i^{n\prime}$ instead. The country would still issue $\bar{e}_i^c$ permits to the trading sector if it took the permit price P^c as given. However, its loss from reducing permits to the trading sector to $\bar{e}_i^t$ is now only YLC. Since the domestic distortion of manipulating the permit price is much smaller now, the country will go further in driving up the permit price. In fact, it will reduce permits to the trading sector to $\bar{e}_i^{t\prime}$, driving the permit price up to $P^{t\prime}$. This will lead to a domestic distortion loss of $Y'LX'$ and a gain of $KS'N'Y'$ from the higher permit price.

This shows that the flatter a country's marginal benefits in the non-trading sector, the further it will manipulate the permit price. A flat marginal benefit curve means that there is only a small cost of letting the permit allocation deviate from the point where marginal benefits equal the permit price. In our two-country model with asymmetric non-trading sectors, the country with the lower η_i will succeed in pulling the equilibrium permit price away from P^* in its preferred direction (higher for a seller, lower for a buyer). If the non-trading sectors are symmetric, the two countries' efforts to pull the permit price in opposite directions cancel each other out and the permit price remains at P^*.

We wish to avoid corner solutions in $\bar{e}_i$. Country 1 that buys permits will allocate less permits to its trading sectors than in autarky, because it can now buy the permits cheaper from abroad. We want to make sure that $\bar{e}_1 > 0$ in (32). Since this condition is most likely to be violated in Scenario A, with just sector 1 in the international trading scheme, we find by substituting (20):

$$\left[(b_{11} + 2b_{12})\, b_{11} b_{12} + (b_{11} + b_{12})^2 \frac{b_{22} b_{32}}{b_{22} + b_{32}}\right] \frac{b_{21} b_{31}}{b_{21} + b_{31}} E_1 > b_{11}^2 b_{12} \frac{b_{22} b_{32}}{b_{22} + b_{32}} E_2 \tag{37}$$

Country 2 that sells permits will allocate more permits to its trading sectors than in autarky, because it can now sell the permits abroad at a higher price. The country would never want to allocate more than its national ceiling to the trading sectors, however. This would mean that its marginal benefits in the non-trading sectors (MB_2^n in (31)) would exceed b_0. However, it is clear from (31) that marginal revenues in the trading sectors MR_2 can never exceed b_0.

Finally, we would like to make sure that marginal benefits (and permit prices) in all sectors are always positive in all scenarios. It follows from (23), (24), (28), (30), (32) and

(33) that $\phi_2'\left(\varepsilon_2^S\right) < P_2^a < P^S < P_1^a < \phi_1'\left(\varepsilon_1^S\right)$ for $S = A, B$. Thus as long as $\phi_2'\left(\varepsilon_2^S\right) > 0$, all marginal benefits are positive. From (**??**) and (32), this implies:

$$b_0 > \frac{\gamma_1\gamma_2\eta_2\left[2\gamma_1\gamma_2 E_2 + \gamma_1\eta_1 E_2 + \gamma_2\eta_1 E_1 + 2\gamma_2\eta_1 E_2\right]}{2\gamma_1^2\gamma_2^2 + 2\gamma_1\gamma_2^2\eta_1 + \gamma_1^2\gamma_2\eta_1 + \gamma_1\gamma_2^2\eta_2 + 2\gamma_1^2\gamma_2\eta_2 + \gamma_1^2\eta_1\eta_2 + \gamma_2^2\eta_1\eta_2 + 2\gamma_1\gamma_2\eta_1\eta_2}$$

This inequality has to be checked for both scenarios $S,\ S = A, B$. It can be shown that $\phi_2'\left(\varepsilon_2^A\right) > \phi_2'\left(\varepsilon_2^B\right)$ if and only if:

$$2b_{11}b_{12}b_{21}b_{22} + 2b_{11}b_{12}b_{22}b_{31} + b_{11}b_{21}b_{22}b_{31} + 2b_{12}b_{21}b_{22}b_{31} > b_{12}b_{21}^2 b_{31}$$

In order to develop a deeper understanding of the structure of the model, whilst maintaining a tractable analysis, we will allow the b_{ji} to be asymmetric between countries for one sector j at a time. We are interested in country i's change in welfare ΔW_i when moving from scenario A to scenario B:

$$\Delta W_i \equiv W_i^B - W_i^A \tag{38}$$

6.2 Asymmetries in the original trading sector

In this subsection we consider asymmetries in sector 1 only, so that $b_{21} = b_{22} = b_2$ and $b_{31} = b_{32} = b_3$ in (18), which implies $\eta_1 = \eta_2 = \eta$ by (20) The expression for the welfare change is then given by (40) in Section 8.1 of the Appendix. We see that the welfare change is the same for both countries, and always positive.

To understand this result, note that in both scenarios A and B, the non-trading sectors are symmetric across countries. Then we know from Proposition 2 that in both scenarios the equilibrium permit price is equal to the permit price P^* in the joint-welfare maximizing scenario. Thus, the permit price does not change when expanding the international trading scheme. Furthermore, as can be seen from adding up the first order conditions (30) for the two countries, the equilibrium permit price is the average of the marginal benefits in the non-trading sectors in each country. Thus, letting P_i^j denote the marginal benefits in non-trading sector(s) j in country i, we have $P_1^{2,3} - P^* = P^* - P_2^{2,3}$ and $P_1^3 - P^* = P^* - P_2^3$. Then also $P_1^{2,3} - P_1^3 = P_2^3 - P_2^{2,3}$.

Finally we will see that $P_1^3 < P_1^{2,3}$ for buying country 1. The proof that $P_2^3 > P_2^{2,3}$ for selling country 2 is analogous. From (32) and (34) with $\eta_1 = \eta_2 = \eta$, we find:

$$\eta(E_1 - \bar{e}_1) = \frac{\eta\gamma_1\gamma_2\left(2\gamma_1\gamma_2 E_1 + \gamma_2\eta E_1 + 2\gamma_1\eta E_1 + \gamma_1\eta E_2\right)}{2\gamma_1^2\gamma_2^2 + 3\gamma_1\gamma_2\eta(\gamma_1 + \gamma_2) + (\gamma_1 + \gamma_2)^2\eta^2}$$

Substituting this and (20) into (22), we see that $P_1^3 < P_1^{2,3}$ if and only if:

$$\begin{aligned} & \frac{(2b_2b_{11} + b_3b_{11} + b_2b_3)\, b_{12}E_1 + b_{11}b_3(b_{11} + b_{12})(2E_1 + E_2)}{b_{11}b_{12}(b_2 + 2b_3) + b_2b_3(b_{11} + b_{12})} \\ > \; & \frac{(b_2b_3 + 2b_2b_{11} + 2b_3b_{11})\, b_{12}E_1 + b_{11}b_2b_3(2E_1 + E_2)}{b_{11}b_{12}(b_2 + b_3) + b_2b_3(b_{11} + b_{12})} \end{aligned}$$

This inequality reduces to (24).

Thus P_i^3 is closer to P^* than is $P_i^{2,3}$. Intuitively, both countries have a smaller non-trading sector in scenario B which makes it more difficult to manipulate the permit price. For country 1, for instance, raising marginal benefits in the non-trading sector by a certain amount releases less permits to the trading sector in scenario B. Moreover, issuing a certain amount of permits to the trading sector leads to a smaller reduction in the permit price in scenario B, since the trading sector is larger.

Figure 4 illustrates the welfare assessment.[6] Country i's emission ceiling is given by E_i. Emissions in the trading sectors are measured from left to right, starting at O. Emissions in the non-trading sectors are measured from right to left, starting at E_i. Country i's marginal benefits in sector(s) j are given by the curve MB_i^j. The countries' changes in welfare from the expansion of the trading scheme are best assessed against the benchmark of the joint welfare-maximizing outcome. In this outcome, marginal benefits in all sectors would be P^* and country 1's emissions in sectors 2 and 3 together would be N_1E_1. Instead, they are F_1E_1 in scenario A. The emission reduction of N_1F_1 means that benefits are $N_1S_1K_1F_1$ lower in sectors 2 and 3, however they also save the expenditure $N_1S_1G_1F_1$of buying N_1F_1 permits at a price of P^*. In scenario B, country 1's sector-3 emissions are L_1E_1, which is J_1L_1 less than the sector's emissions J_1E_1 in the welfare optimum. Country 1's welfare is $Z_1U_1T_1$ lower than in the welfare optimum: Forgone sector-3 benefits of $J_1Z_1U_1L_1$ minus $J_1Z_1T_1L_1$ saved on buying permits.

Country 1's welfare gain of including sector 2 into the trading scheme is then $S_1K_1G_1 - Z_1U_1T_1 > 0$. The difference is positive, because $G_1K_1 > T_1U_1$ since $P_1^{2,3} > P_1^3$, and because $S_1G_1 > Z_1T_1$ since $MB_1^{2,3}$ is the horizontal summation of MB_1^2 and MB_1^3 and thereby flatter than MB_1^3. In the same way it follows that country 2's welfare gain is

[6] Figures 4 to 6 can be derived graphically, but this is a rather tortuous process. Details are available from the corresponding author upon request.

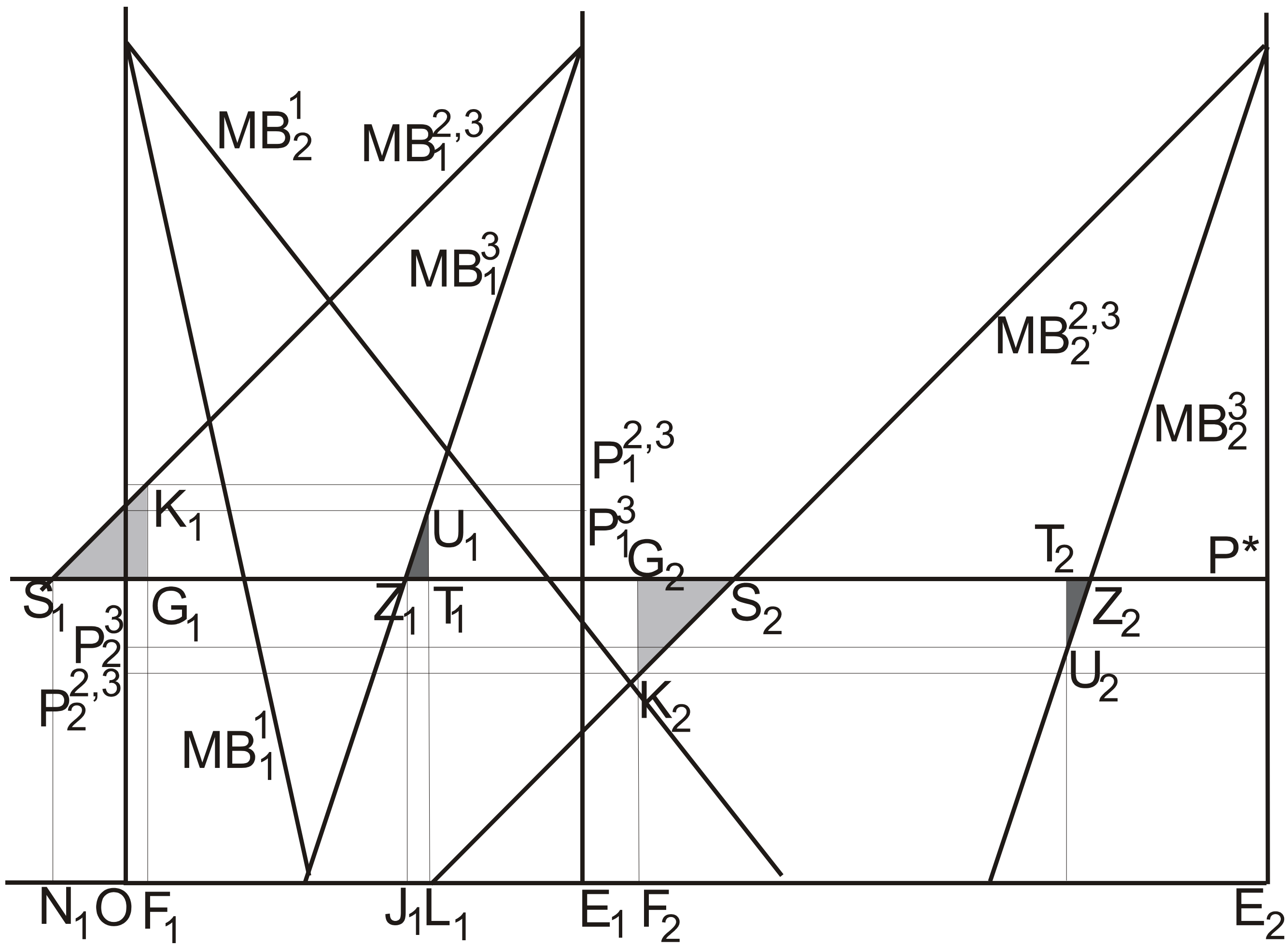

Figure 4: Asymmetries in sector 1 only

$K_2G_2S_2 - U_2T_2Z_2 > 0$. It is easily seen that $K_2G_2S_2$ and $U_2Z_2T_2$ are equal in size to $S_1K_1G_1$ and $Z_1U_1T_1$ respectively, because sectors 2 and 3 are symmetric across the two countries.

We conclude that both countries gain from the expansion of the international trading scheme, because the reduction in the size of the non-trading sector reduces the distortion in this sector. Marginal benefits as well as emissions in the non-trading sector are closer to the optimum in Scenario B.

6.3 Asymmetries in the additional trading sector

In this scenario, there are asymmetries in sector 2 only, so that $b_{11} = b_{12} = b_1$ and $b_{31} = b_{32} = b_3$ in (18). In this case, ΔW_i is given by (41) in Section 8.2 of the Appendix. The welfare effects of expanding the trading scheme differ between the two countries and can even be negative for one country. However, there is always an aggregate welfare gain, as shown by (44) and (42).

To investigate the possiblity that one country could be made worse off, numerical simulations are conducted for the case in which b_{21} is variable. The sign of the welfare change ΔW_i in (41) is the sign of V_i. First, let us set $b_1 = b_{22} = b_3 = 1$. Then V_i is, for country 1 and 2 respectively:

$$V_1 = 623b_{21}^4 + 506b_{21}^3 + 104b_{21}^2 - 6b_{21} - 3, \qquad V_2 = -81b_{21}^4 + 602b_{21}^3 + 540b_{21}^2 + 150b_{21} + 13$$

Three scenarios may arise: (a) if $b_{21} < \frac{1}{7}$, welfare will fall in country 1 but rise in country 2; (b) if $\frac{1}{7} < b_{21} < 8.266$, welfare will rise in both countries; (c) if $b_{21} > 8.266$, welfare will rise in country 1 but fall in country 2.

Figure 5 illustrates the case with $b_1 = b_{22} = b_3 = 1$ and $b_{21} = \frac{1}{16} < \frac{1}{7}$, so that country 1 loses from the expansion of the trading scheme. From Proposition 2, we know that in scenario A, the international permit price P^A is below the joint welfare-maximizing permit price P^*, because the buying country 1 has the flatter $MB^{2,3}$ curve. In scenario B, however, the equilibrium permit price equals P^*, because the non-trading sectors 3 are symmetric in the two countries.

A country's welfare in scenarios A and B is best assessed as compared to the benchmark of autarky. Country 1's autarky equilibrium is at K_1, with emissions OF_1 in sector 1 and F_1E_1 in sectors 2 and 3 together.

In scenario A, sector 1 emissions have increased by F_1e^1 to Oe^1. This has raised sector 1 benefits by $F_1K_1We^1$, but the extra permits had to be bought abroad at a price of P^A and an expense of $F_1I_1We^1$. The net increase in benefits in sector 1 is then I_1K_1W. Emissions in sectors 2 and 3 have increased by L_1F_1 to L_1E_1. This has raised benefits in these sectors by $L_1Y_1K_1F_1$, but if sector 1 had received these L_1F_1 permits instead, it would not have had to buy them abroad at price P^A and an expenditure of $L_1H_1I_1F_1$ would have been

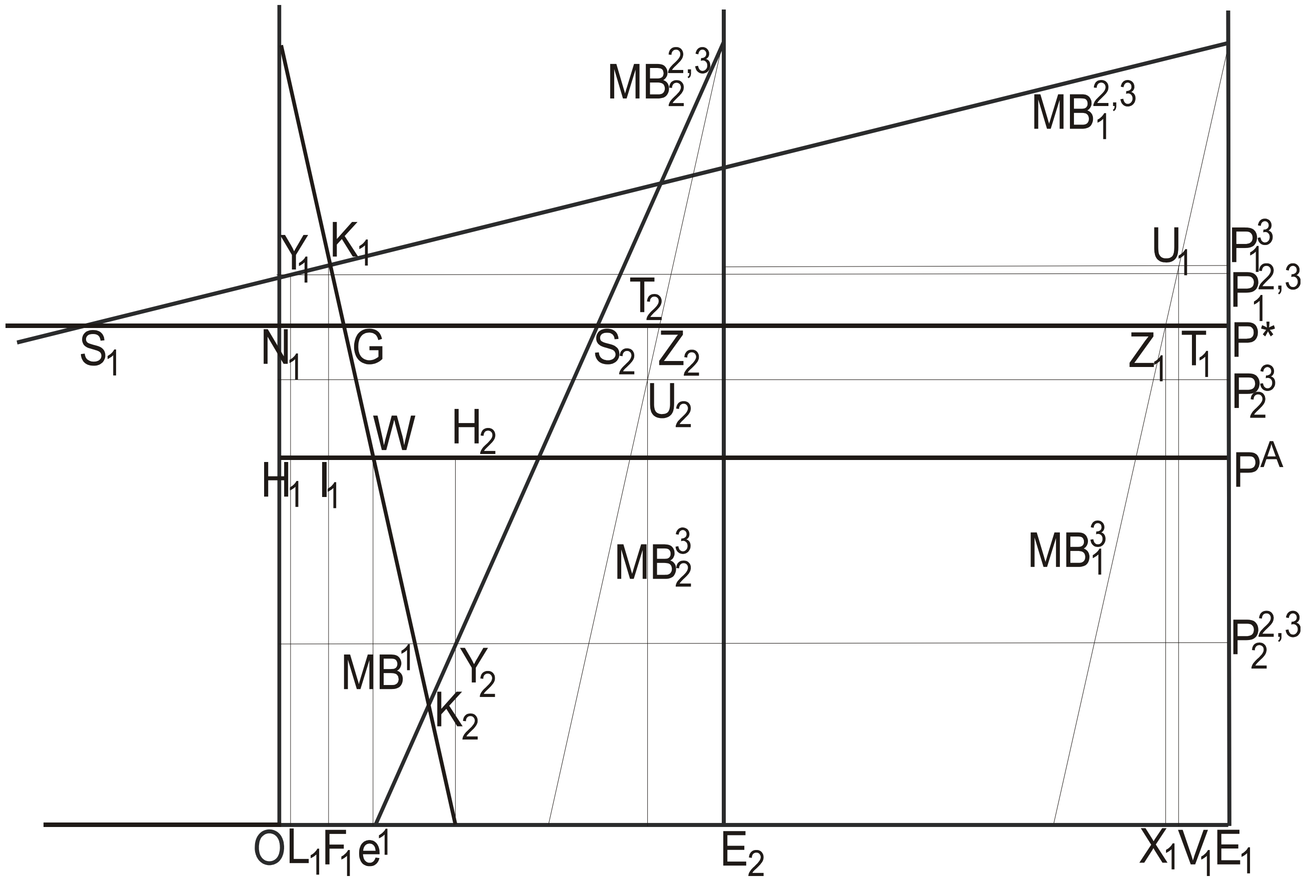

Figure 5: Asymmetries in sector 2 only

saved. The net increase in benefits in sectors 2 and 3 is then $H_1Y_1K_1I_1$. Country 1's total increase in benefits moving from autarky to scenario A is then $H_1Y_1K_1W$.

In scenario B, if all sectors faced the welfare optimum price of P^*, country 1 would have gained S_1K_1G relative to autarky. However, while the international permit price is P^* for sectors 1 and 2, sector 3 has marginal benefits above P^*. Its emissions are V_1E_1 while they would be X_1E_1 at marginal benefits of P^*. This reduces sector 3's benefits by $X_1Z_1U_1V_1$. However, if sector 3 had emissions of X_1E_1, sectors 1 and 2 would have had X_1V_1 permits less. They would then have had to buy these permits abroad at price P^*, which would have cost them $X_1Z_1T_1V_1$. Thus, the efficiency loss of sector 3's marginal benefits deviating from P^* is $Z_1U_1T_1$. Country 1's total increase in benefits moving from autarky to scenario B is then $S_1K_1G - U_1T_1Z_1$.

Country 1's change in welfare when moving from scenario A to B is then $S_1K_1G -$

$U_1T_1Z_1 - H_1Y_1K_1W = S_1Y_1N_1 - H_1N_1GW - U_1T_1Z_1$, which is clearly negative. Thus, country 1 loses from the expansion of the trading scheme.

In the same way, country 2's change in welfare when moving from scenario A to B is $GS_2K_2 - U_2T_2Z_2 - WH_2Y_2K_2 = WGS_2Y_2H_2 - U_2T_2Z_2$, which is clearly positive. Thus, country 2 gains from the expansion of the trading scheme. Moreover, comparing the welfare changes of both countries, it is clear that there is an overall welfare gain from the expansion of the trading scheme.

Intuitively, the reason why country 1 loses from the expansion of the trading scheme is that its extremely flat marginal benefit curve for the non-trading sectors gives it a large advantage in scenario A. We have already seen the effect of a flat MB^n curve in Figure 3. Note that country 1 will buy permits abroad and therefore wants to decrease the permit price by allocating more permits to its trading sector and less permits to its non-trading sector. It can achieve the latter quite painlessly in scenario A, because this results in just a small efficiency loss due to the flatness of the $MB_1^{2,3}$ curve. In scenario A, country 1 manages to depress the permit price to P^A. In scenario B, sector 2 has joined the international trading scheme. Now country 1 does not have the advantage of a flat MB^n curve anymore and the permit price equals the joint welfare-maximizing price P^*.

Of course, country 1's large gain in scenario A is at the expense of country 2. That is why country 2 gains substantially from the expansion of the trading scheme. Indeed, country 2 gains more than country 1 loses.

Having established that one country can lose from the expansion of the trading scheme when $b_1 = b_{22} = b_3 = 1$, let us now examine whether this can still occur when we vary b_1 and b_3. First, let b_3 approach infinity. This is equivalent to there being no non-trading sector in scenario B: The trading scheme is extended to include the whole economy. As we know, the sign of the welfare change ΔW_i in (41) is the sign of V_i. For $b_1 = b_{22} = 1$ and $b_3 \to \infty$ we find from (42) that $V_i > 0$ if and only if $X_i > 0$ with:

$$X_1 = 13b_{21}^2 + 6b_{21} - 3 \qquad\qquad X_2 = -7b_{21}^2 + 14b_{21} + 9$$

We find again that three scenarios may arise; (a) if $b_{21} < 0.302$, welfare will fall in country 1 but rise in country 2; (b) if $0.302 < b_{21} < 2.512$, welfare will rise in both countries; (c) if $b_{21} > 2.512$, welfare will rise in country 1 but fall in country 2. Compared to the case with $b_3 = 1$, a welfare decrease for either country occurs for a larger range of parameter values. Although joint welfare is maximized with all sectors in the trading scheme (as we argued in the Introduction), it is very well possible that one of the two countries loses in the move from partial to full international trading.

Finally, consider the case for which $b_{22} = b_3 = 1$ and b_1 becomes arbitrarily large. This implies that there is no original trading sector: Scenario A amounts to autarky. We have already seen in Section 5 that a move from autarky to international trade (however partial) always increases a country's welfare. This finding is supported by the numerical simulation: As b_1 tends towards infinity, both countries are made better off regardless of the value of b_{21}. Even for very large (but finite) values of b_1, country 1 still loses from the expansion of the trading scheme if b_{21} is very small. Country 2, however, always gains for sufficiently large values of b_1. The expression for ΔW_2 with $b_{11} = b_{12} = b_1$ and $b_{22} = b_{31} = b_{32} = b_3$ is given by (46) in Section 8.2 of the Appendix. We see that the welfare change can only be negative (for large enough b_{21}) if:

$$Y_2 \equiv 16b_1^4 + 12b_1^3 b_3 - 41b_1^2 b_3^2 - 52b_1 b_3^3 - 16b_3^4 < 0$$

The only non-negative solution for $Y_2 = 0$ is $b_1/b_3 = 1.815$. Thus if $b_1/b_3 > 1.815$, welfare cannot fall in country 2 as the trading scheme is expanded.

6.4 Asymmetries in the non-trading sector

If sectors 1 and 2 are symmetric ($b_{11} = b_{12} = b_1$ and $b_{21} = b_{22} = b_2$ in (18), and thus $\gamma_1 = \gamma_2 = \gamma$ by (20)), then ΔW_i is given by (47) in Section 8.3 of the Appendix. We see that both countries gain from an expansion of the trading scheme, but the country i with the smallest b_{3i} gains the least.

The reason for the latter result is that the country with the smallest b_{3i} experiences an adverse price change when the trading scheme is expanded. This can be shown as follows. We can rewrite (36) as:

$$P^S - P^* = \frac{\Omega^S \Theta \gamma_1 \gamma_2 \eta_1 \eta_2}{\gamma_1 \gamma_2 \eta_1 + \gamma_1 \gamma_2 \eta_2 + \gamma_1 \eta_1 \eta_2 + \gamma_2 \eta_1 \eta_2}$$

where Ω^S is the only part of the RHS that is changing with the scenario S:

$$\Omega^S \equiv \frac{\gamma_1^2\gamma_2^2\left(\eta_1-\eta_2\right)}{\Delta} \tag{39}$$

Substituting (20) and (34) with $\gamma_1 = \gamma_2 = \gamma$ into (39), we find:

$$\begin{aligned}
\Omega^A &= \frac{b_1^2b_2^2(b_{31}-b_{32})}{2b_1^2b_2^2+2b_1^2b_2(b_{31}+b_{32})+3b_1b_2^2(b_{31}+b_{32})+2(b_1+b_2)(b_1+2b_2)b_{31}b_{32}} \\
\Omega^B &= \frac{b_1^2b_2^2\left(b_{31}-b_{32}\right)}{2b_1^2b_2^2+3b_1^2b_2(b_{31}+b_{32})+3b_1b_2^2(b_{31}+b_{32})+4(b_1+b_2)^2b_{31}b_{32}}
\end{aligned}$$

We see that while the numerators are the same, the denominator is larger for Ω^B. Thus Ω^B is smaller than Ω^A in absolute terms and P^B is closer to P^* than is P^A. Considering the case where $b_{31} < b_{32}$, we know from Proposition 2 that buyer country 1 can manipulate the international permit price to its advantage in both scenarios, reducing it below P^*. However, its power to manipulate the permit price diminishes with the expansion of the international trading scheme: $P^A < P^B < P^*$. Intuitively, the smaller the non-trading sector, the more its marginal benefits rise when country 1 moves permits toward the trading sector. In addition, the larger the trading sector, the less the permit declines rises when country 1 issues more permits to its trading sectors.

Figure 6 illustrates the welfare effects for $b_1 = b_2 = b_{32} = 1$ and $b_{31} = \frac{1}{16}$. We are particularly interested in country 1, because we know that this country will have the smallest welfare gain as $b_{31} < b_{32}$. We see that the expansion of the trading scheme causes the permit price to rise from P^A to P^B. This is to the advantage of the seller country 2, but to the disadvantage of the buyer country 1.

In autarky, country 1 sets the marginal benefits for all its sectors equal to P_1^a. Emissions are OF_1 in sector 1, F_1X_1 in sector 2 and X_1E_1 in sector 3.[7] In scenario A, sector-1 emissions have increased from OF_1 to Oe^1. Analogous to our discussion of the welfare effects in subsection 6.3, this implies a net gain of I_1K_1W in sector 1. Sectors 2 and 3 have marginal benefits of $P_1^{2,3}$, which means their emissions have increased by L_1F_1, yielding a net welfare gain of $H_1Y_1K_1I_1$. Country 1's welfare gain when moving from autarky to scenario A is then $H_1Y_1K_1W$. Moving from autarky to scenario B, emissions in sectors 1 and 2 have increased from OX_1 to OJ, resulting in a net gain of U_1T_1G. Sector

[7] To improve legibility, Figure 6 has been stretched horizontally, so that E_1 no longer appears in it.

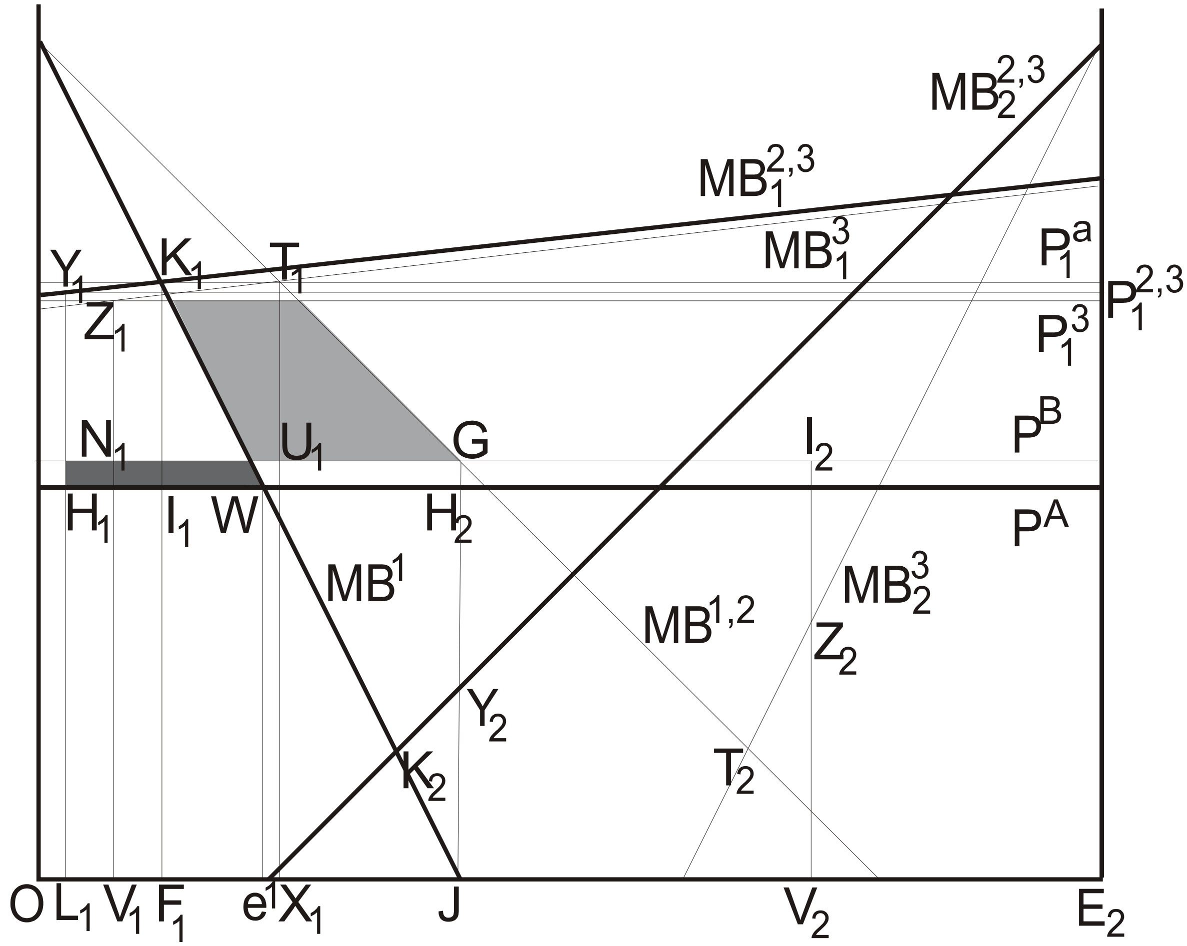

Figure 6: Asymmetries in sector 3 only

3's marginal benefits are now P_1^3, which means its emissions have increased by V_1X_1, giving a welfare gain of $N_1Z_1T_1U_1$. Country 1's welfare gain when moving from autarky to scenario B is then $N_1Z_1T_1G$.

Comparing country 1's welfare in scenarios A and B, we see that the changes in sector 3 are minuscule, because its marginal benefits decrease only slightly from $P_1^{2,3}$ to P_1^3. Therefore we will concentrate on sectors 1 and 2, setting marginal benefits in the non-trading sectors in both scenarios at P_1^3. When moving from scenario A to B, country 1 then gains the light-shaded area, but loses the dark-shaded area. The former area represents sector 2's gain from a decrease of its marginal benefits from $P_1^{2,3} \approx P_1^3$ to P^B. The latter area represents sector 1's loss from a rise in the international permit price from P^A to P^B. It is clear that the welfare gain exceeds the welfare loss: Country 1 gains from the expansion of the international trading scheme.

Finally, country 2's welfare gain when moving from scenario A to B can be determined in a similar way as $GI_2Z_2T_2 - WH_2Y_2K_2$, which is clearly positive and larger than country 1's gain.

7 Conclusion

We have constructed a model of a partial international emission trading scheme, such as the EU Emissions Trading Scheme (EU ETS), in which countries decide non-cooperatively how to allocate their nationally allowed emissions between their trading and their non-trading sectors. Countries that sell permits will want to reduce their permit allocation to the trading sector in a bid to drive up the permit price. Buying countries want to do the opposite.

It is easily seen that countries always gain when moving from autarky to any partial international emission trading scheme, and that aggregate welfare is maximized when all polluting sectors are included in the scheme. However, this does not mean that every country will always gain from an expansion of the scheme. Studying the welfare effects of an expansion of the trading scheme is policy-relevant, as the European Commission plans to expand the EU ETS in 2013.

We have simulated the effects of expanding the trading scheme in a two-country model. If the additional trading sectors and the non-trading sectors are symmetric between countries, the welfare change is the same in both countries and is always positive. If the original trading sectors and the non-trading sectors are symmetric, one country might lose, but there is an aggregate welfare gain. Moreover, the asymmetries in the additional trading sector have to be quite pronounced for one country to lose from the expansion. If the original and the additional trading sectors are symmetric, both countries always gain.

An obvious extension to the analysis would be to run simulations of the model in a computable general equilibrium modelling framework. This would allow for the quantification of the theoretical assertions regarding which countries will be permit buyers and which will be permit sellers. The EU ETS has now completed its first phase of operation, running from 2005-2007, and thus such data are now available for this period. However, the dynamics of the EU ETS will be completely different in the second Phase since it runs from 2008-2012 and therefore coincides with the Kyoto commitment period. Thus countries now face a national ceiling as assumed by this analysis. Moreover, CGE modelling would allow for an assessment of the welfare effects of expanding the EU ETS.

In order to keep our analysis manageable, we have assumed that several parameters were exogenously fixed. These include the national emission ceilings and the identity of the trading sectors. We could model both of these as the product of negotiations between the countries involved, before they decide non-cooperatively how to allocate their emissions between trading and non-trading sectors.

Most importantly, we have assumed that countries can freely choose how to allocate their emissions, whereas the National Allocation Plans for the EU ETS are subject to the Commission's approval. Any future international emission trading scheme is also likely to constrain the participating countries' allocations in order to maintain the integrity of the system. For the EU ETS, the Commission wants to ensure that Member States do not overallocate emissions to their trading sectors, for two main reasons. The first is that it would distort competition in the product market. The second is that it will make it more difficult for Member States to meet their Kyoto or burden sharing target. If a country were really committed to its Kyoto target, then it would already take this effect into account itself. However, the Commission may be concerned that Member States do not take their Kyoto target seriously enough. This consideration is difficult to incorporate in our current model.

We could model one aspect of the Commission's intervention by capping each country's trading sector allocation to its autarky allocation. This would constrain the sellers, but not the buyers in their allocation. However, the buyers will lose because the restriction will drive up the permit price. The sellers benefit from the higher permit price, but they cannot take full advantage of it because their allocation is restricted. Aggregate welfare will rise if the permit price rises toward the aggregate welfare-maximizing level.

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8 Appendix: Welfare change from expanding the trading scheme

8.1 Asymmetries in the original trading sector

When $b_{21} = b_{22} = b_2$ and $b_{31} = b_{32} = b_3$, then $\Delta W_i,\ i = 1, 2$, in (38) is given by:

$$\Delta W_1 = \Delta W_2 = \frac{b_{11}^2 b_{12}^2 \Phi^2 V}{2b_2 d^2} \tag{40}$$

where Φ is given by (24),

$$V \equiv b_{11}^2 b_{12}^2 (b_2 + b_3)(3b_2 + 4b_3) + 4b_{11}b_{12}(b_{11} + b_{12}) b_2 b_3 (b_2 + b_3) + (b_{11} + b_{12})^2 b_2^2 b_3^2$$

and

$$\begin{aligned} d \;\equiv\;& 2b_{11}^3 b_{12}^3 (b_2 + b_3)^2 (b_2 + 2b_3) + b_{11}^2 b_{12}^2 (b_{11} + b_{12}) b_2 b_3 (b_2 + b_3)(5b_2 + 8b_3) \\ & + b_{11}b_{12}(b_{11} + b_{12})^2 b_2^2 b_3^2 (4b_2 + 5b_3) + (b_{11} + b_{12})^3 b_2^3 b_3^3 \end{aligned}$$

8.2 Asymmetries in the additional trading sector

When $b_{11} = b_{12} = b_1$ and $b_{31} = b_{32} = b_3$, then $\Delta W_i,\ i = 1, 2$, in (38) is given by:

$$\Delta W_i = \frac{\Phi^2 V_i}{2d^2} \tag{41}$$

where Φ is given by (24),

$$\begin{aligned} V_i \;\equiv\;& 8b_1^4 b_{2i}^4 b_{2j}^3 + 16b_3^4 b_{2i}^3 b_{2j}^3 (2b_{2i} - b_{2j}) + 4b_1^4 b_3^3 (b_{2i} + b_{2j}) \left(b_{2i}^3 + 5b_{2i}^2 b_{2j} + b_{2i}b_{2j}^2 + b_{2j}^3\right) \\ & + 4b_1^4 b_3 b_{2i}^2 b_{2j}^2 \left(2b_{2j}^2 + 6b_{2i}b_{2j} + 5b_{2i}^2\right) + 2b_1^3 b_{2j}^3 \left(7b_3^3 b_{2i}^2 - 2b_1 b_{2j} b_3^3 + 2b_1 b_{2i}^3 b_{2j}\right) \\ & + 4b_1^4 b_3^2 b_{2i} b_{2j} \left(4b_{2i}^3 + 10b_{2i}^2 b_{2j} + 6b_{2i}b_{2j}^2 + b_{2j}^3\right) + 4b_1^3 b_3 b_{2i}^3 b_{2j}^3 (11b_{2i} + 3b_{2j}) \\ & + b_1^3 b_3^2 b_{2i}^2 b_{2j}^2 \left(9b_{2j}^2 + 86b_{2i}b_{2j} + 81b_{2i}^2\right) + 2b_1^3 b_3^3 b_{2i} b_{2j} \left(21b_{2i}^3 + 53b_{2i}^2 b_{2j} + 14b_{2i}b_{2j}^2 - 3b_{2j}^3\right) \\ & + b_1^3 b_3^4 (b_{2i} + b_{2j}) \left(5b_{2i}^3 + 27b_{2i}^2 b_{2j} + 3b_{2i}b_{2j}^2 - 3b_{2j}^3\right) + 4b_1^4 b_3^4 b_{2i} (b_{2i} + b_{2j})^2 \\ & + b_1^2 b_3^3 b_{2i}^2 b_{2j}^2 \left(111b_{2i}^2 + 102b_{2i}b_{2j} - 25b_{2j}^2\right) + 4b_1^2 b_3^4 b_{2i} b_{2j} \left(7b_{2i}^3 + 18b_{2i}^2 b_{2j} + 3b_{2i}b_{2j}^2 - 4b_{2j}^3\right) \\ & + 8b_1 b_3^3 b_{2i}^3 b_{2j}^3 (11b_{2i} - 3b_{2j}) + 4b_1 b_3^4 b_{2i}^2 b_{2j}^2 \left(13b_{2i}^2 + 10b_{2i}b_{2j} - 7b_{2j}^2\right) + 92b_1^2 b_3^2 b_{2i}^4 b_{2j}^3 \end{aligned} \tag{42}$$

and

$$\begin{aligned} d \;\equiv\;& 10b_1^3 b_3^3 b_{21}^2 b_{22}^2 + 4(b_1 + b_3)^2 (b_1 + 2b_3)^2 b_{21}^3 b_{22}^3 + b_1^3 b_3^3 (2b_1 + 3b_3)(b_{21} + b_{22})^3 + 2b_1^4 b_3^4 (b_{21} + b_{22})^2 \\ & + 2b_1 b_3 (b_1 + b_3) \left(5b_1^2 + 17b_1 b_3 + 14b_3^2\right) b_{21}^2 b_{22}^2 (b_{21} + b_{22}) + 2b_1^2 b_3^2 \left(2b_1^2 + b_1 b_3 + 4b_3^2\right) b_{21}^2 b_{22}^2 \\ & + b_1^2 b_3^2 (8b_1^2 + 23b_1 b_3 + 16b_3^2) b_{21} b_{22}^2 (b_{21} + b_{22}) + 2b_1^3 b_3^3 (3b_1 + 4b_3) b_{21} b_{22} (b_{21} + b_{22}) \end{aligned} \tag{43}$$

The aggregate welfare change is, from (41) to (43):

$$\Delta W_i + \Delta W_j = \frac{\Phi^2 V}{d^2} \tag{44}$$

where d is given by (43) and:

$$\begin{aligned} V \equiv\ & 2b_1^4 b_3^4 (b_{21}+b_{22})^3 + b_1^3 b_3^3 (2b_1+b_3)(b_{21}+b_{22})^4 + 12 b_1^3 b_3^4 b_{21} b_{22} (b_{21}+b_{22})^2 \\ & +2b_1^2 b_3^2 \left(5b_1^2 + 9b_1 b_3 + 3b_3^2\right) b_{21} b_{22} \left(b_{21}+b_{22}\right)^3 + 7 b_1^2 b_3^2 (b_1+b_3) b_{21}^2 b_{22}^2 \left(b_{21}+b_{22}\right)^2 \\ & +2\left(b_1+b_3\right)^2 \left(3b_1^2 + 8b_1 b_3 + 4b_3^2\right) b_{21}^3 b_{22}^3 \left(b_{21}+b_{22}\right) + 4 b_1^4 b_2^3 b_{21} b_{22} \left(2b_{21}^2 + 3b_{21} b_{22} + 2b_{22}^2\right) \\ & +12 b_1 b_3 (b_1+b_3)^3 b_{21}^2 b_{22}^2 \left(b_{21}+b_{22}\right)^2 + 2b_1^2 b_3^2 \left(b_1^2 + 10 b_1 b_3 + 12 b_3^2\right) b_{21}^2 b_{22}^2 \left(b_{21}+b_{22}\right) \\ & +2 b_1^3 b_3 (b_1+b_3) b_{21}^2 b_{22}^2 \left(b_{21}^2 + b_{22}^2\right) + 8 b_1 b_3^3 (2b_1+b_3) b_{21}^3 b_{22}^3 \end{aligned} \tag{45}$$

From (41) to (43), ΔW_2 with $b_{11} = b_{12} = b_1$ and $b_{22} = b_{31} = b_{32} = b_3$ is given by:

$$\Delta W_2 = \frac{\Phi^2 V_2}{2 b_3^3 d^2} \tag{46}$$

where:

$$\begin{aligned} V_2 \equiv\ & b_{21}^3 \left[16 b_1^4 + 12 b_1^3 b_3 - 41 b_1^2 b_3^2 - 52 b_1 b_3^3 - 16 b_3^4\right] \\ & +2 b_{21}^2 b_3 \left[32 b_1^4 + 86 b_1^3 b_3 + 103 b_1^2 b_3^2 + 64 b_1 b_3^3 + 16 b_3^4\right] \\ & +b_{21} b_1 b_3^2 \left[88 b_1^3 + 217 b_1^2 b_3 + 183 b_1 b_3^2 + 52 b_3^3\right] + 2 b_1^2 b_3^3 \left[24 b_1^2 + 37 b_1 b_3 + 14 b_3^2\right] + b_1^3 b_3^4 \left[5 b_3 + 8 b_1\right] \end{aligned}$$

and:

$$\begin{aligned} d \equiv\ & 4 b_1^4 \left(b_3^3 + 6 b_3^2 b_{21} + 11 b_3 b_{21}^2 + 6 b_{21}^3\right) + b_1^3 b_3 \left(3 b_3^3 + 40 b_3^2 b_{21} + 119 b_3 b_{21}^2 + 94 b_{21}^3\right) \\ & +2 b_1^2 b_3^2 b_{21} \left(8 b_3^2 + 51 b_3 b_{21} + 65 b_{21}^2\right) + 4 \left(7 b_1 b_3 + 19 b_1 b_{21} + 4 b_3 b_{21}\right) b_3^3 b_{21}^2 \end{aligned}$$

8.3 Asymmetries in the non-trading sector

When $b_{11} = b_{12} = b_1$ and $b_{21} = b_{22} = b_2$, then $\Delta W_i,\ i = 1, 2$, in (38) is given by:

$$\Delta W_i = \frac{\Phi^2 \left(4 b_1 b_2 b_{3i} V + Z\right)}{2 b_2 d^2} \tag{47}$$

where Φ is given by (24),

$$V \equiv b_1^2 b_2^2 \left(b_1 + 2b_2\right) + b_1 b_2 (b_1+b_2)(b_1+3b_2)\left(b_{31}+b_{32}\right) + (b_1+b_2)^2 (b_1+4b_2) b_{31} b_{32}$$

and

$$\begin{aligned}
Z \equiv\ & 16\left[(b_1+b_2)^4+b_1^4\right]b_{31}^2b_{32}^2+3b_1^2b_2^2(b_1+b_2)(4b_1+3b_2)\left(b_{31}+b_{32}\right)^2 \\
&+4b_1^2b_2^2(b_1+b_2)(7b_1+8b_2)b_{31}b_{32}+24b_1^3b_2^3(b_1+b_2)(b_{31}+b_{32}) \\
&+4b_1b_2(b_1+b_2)^2(7b_1+6b_2)b_{31}b_{32}(b_{31}+b_{32})+12b_1^4b_2^4-16b_1^4b_{31}^2b_{32}^2
\end{aligned}$$

and finally

$$\begin{aligned}
d \equiv\ & 2b_1^3b_2^3\left(5b_1+6b_2\right)\left(b_{31}+b_{32}\right)+4b_1^2b_2^2(b_1+b_2)(3b_1+4b_2)b_{31}b_{32}+8\left(b_1+2b_2\right)\left(b_1+b_2\right)^3b_{31}^2b_{32}^2 \\
&+2b_1b_2(b_1+b_2)^2(7b_1+12b_2)b_{31}b_{32}(b_{31}+b_{32})+3b_1^2b_2^2(b_1+b_2)(2b_1+3b_2)(b_{31}+b_{32})^2+4b_1^4b_2^4
\end{aligned}$$