research paper series
Theory and Methods

Research Paper 2010/16
Offshoring and Growth with Two Factors
By
Andreas Hoefele

The Centre acknowledges financial support from The Leverhulme Trust
under Programme Grant F/00 114/AM
The Authors
Andreas Hoefele is an internal GEP research fellow.

Acknowledgements
I would like to thank Ian Wooton for the insightful discussions. Further I would like to thank Marta Aloï, Daniel Bernhofen, Alex Dickson, Joseph Francois, Gabriela Grotkowska, Pascalis Raimondos-Moller and Alejandro Riaño for comments. Further, I would like to thank the participants of the Strathclyde Seminar and the GEP workshop in Nottingham for their remarks.
Offshoring and Growth with Two Factors
by
Andreas Hoefele

Abstract

Offshoring has received a lot of attention lately. However, the debate is largely confined to the static effects. This paper shows that there are important implications of offshoring on the growth rate in an economy. In this paper I show that offshoring a larger share of unskilled workers tasks, has an ambiguous effect on the growth rate of the economy. Recent work on offshoring emphasizes the cost-savings effect due to offshoring. However, I highlight the factor markets as an important channel by showing that the condition under which offshoring has a positive or negative impact on the growth rate in the economy is linked to the factor markets.

Keywords: Endogenous Growth; Offshoring; Tasks Trade

JEL Classification: O41; F43; F12

Outline

1. Introduction
2. The Model
3. Offshoring and Growth
4. Discussion
5. Conclusion
Non-Technical Summary

In this work I show that offshoring has an ambiguous effect on the growth rate of a small open economy which has the possibility to offshore parts of the production chain. Producers can reduce production costs by offshoring. However, in the framework, offshoring increases the wage of skilled and unskilled labour as production becomes more efficient due to offshoring. This increase in the wages increases the costs of research which only skilled labour is able to do. I argue that offshoring has a positive impact on growth if the increase in production efficiency allows producers to increase their profits enough to offset the increase in research costs. Furthermore, I connect this to a condition which emphasizes the role of labour markets.
1 Introduction

In this paper I argue that offshoring may not increase the growth rate of a small open economy. In fact I provide a condition under which an increase in the extent of offshoring has a negative or positive effect on the growth rate of the economy. In particular I highlight the role played by factor markets, namely the markets for skilled and unskilled workers. In the model, offshoring reduces the costs of production by having access to low cost (unskilled) labour. The cost savings aspect of offshoring creates rents which are responsible for the increase in factor rewards. As a result the costs of doing research increase as well. The thought experiment of an increase in the fraction of tasks offshored results in a higher steady state growth if the cost savings in production outweigh the cost increase in research. I show that this is linked to a condition which arises from the labour market.

I introduce offshoring in a model of endogenous growth with two factor endowments a la Grossman and Helpman (1991a). Offshoring operates through two channels in the model. Firstly, due to the cost-savings of offshoring, the costs of unskilled tasks necessary to produce one unit of output fall. Owing to this effect, unskilled labour is substituted for skilled labour in production. Secondly, the wages of both factors increases due to rents occurring when a larger fraction of tasks is offshored. This increases the costs of research, as I assume that only skilled labour is able to do research. Therefore, only if the production becomes sufficiently unskilled labour intensive, and skilled labour is able to reallocate in research, has offshoring a positive effect on growth. This corresponds to the condition presented in the paper that the labour-supply effect must dominate the cost-savings effect for offshoring to have a positive effect on the growth rate. Only in this case is the economy able to increase the output sufficiently to compensate for the increase in research costs and reallocate skilled labour to research.

Offshoring is modelled following the idea of task trade developed in Grossman and Rossi-Hansberg (2008). In an earlier contribution, Jones and Kierzkowski (1990) regard offshoring as the spatial fragmentation of the production process, which is organized in production blocks linked by services.\(^1\) Grossman and Rossi-Hansberg (2008) follow the idea of a fragmented production process, however, propose that fragments - or tasks - are heterogeneous with respect to the costs of being provided from abroad. More specifically, a task is subject not only to transportation

\(^1\)See Francois (1990b) and Francois (1990a) for a more formal theory of service links and fragmentation.
costs, but there might also be some additional cost of performing it at a distance. The often cited example is the job of a janitor which is hard to perform at a distance, whereas basic stages of accounting might be easy to offshore, given modern communication technology.

Most of the theoretical works have studied the static effects of offshoring for the economy.\textsuperscript{2} Glass and Saggi (2001) and Rodriguez-Clare (2007) analyze the effect of offshoring on the relative wage of a North and a South in a dynamic setting. Both works show that an increase in the extent of offshoring causes an increase in research employment. Naghavi and Ottaviano (2009) argue that offshoring might lead to a slower growth rate as communication links between production and research are weakened. My work is most closely related to the first two papers. I differ from both works in conceding the assumption of one factor in favour of allowing for heterogeneity in factors. With this assumption I model The idea is that not all workers have the necessary skills to successfully develop new ideas for research.

The paper is organized as follows. In section two I develop the basic framework and derive the equilibrium conditions and the growth rate. In section three I analyze the effect of an increase in the extent of offshoring on the growth rate. In section four I discuss further aspects of the model and conclude in section five.

\section{The Model}

The structure of the economy in the paper is depicted in figure 1. There are two final good sectors \(X\) and \(Y\). Both final sectors use an intermediate input and either skilled or unskilled labour. I assume that both sectors use the same intermediates in order to be able focus on the role of offshoring on growth. Both final goods are traded. The intermediate sector uses skilled and unskilled labour to produce the input. Offshoring takes place in the intermediate sector and only unskilled tasks are offshored. Skilled labour is the only input used in research, where it invents new varieties of the intermediate input. Growth is modelled as an increase in the number of intermediate varieties\textsuperscript{3}.


\textsuperscript{3}Grossman and Helpman (1991c) show that this mechanism is similar to one with quality ladders except for the welfare analysis. I do not analyze welfare in this work. Therefore, the results hold in a quality ladder model as well.
2.1 Households

I assume an infinitely living representative household which consumes two final goods, \( X \) and \( Y \), at each period \( t \). The intertemporal utility function of the household is the formal dynamics problem of the household is

\[
U = \int_0^\infty e^{-\rho t} \ln u(c_{xt}, c_{yt}) \, dt,
\]

where the instantaneous utility function \( u(c_{xt}, c_{yt}) \) is non-decreasing, quasi-concave and homogeneous of degree one in consumption. The household faces both a static- and a dynamic-optimization problem. Firstly, the household maximizes instantaneous utility in each period by optimizing expenditures \( E_t \) on the two final goods. Secondly, a household maximizes lifetime utility over time subject to the intertemporal budget constraint

\[
\int_0^\infty e^{-R(t)} E_t \, dt \leq \int_0^\infty I_t \, dt + W_0
\]

where \( I_t \) is the income of a household in period \( t \) and \( \rho \) is the subjective discount rate. \( R(t) = \int_0^t r(s) \, ds \) is the cumulative discount rate depending on the interest rate \( r_t \). Wealth in the initial period is denoted by \( W_0 \). From the maximization follows the optimal path of expenditures, which is

\[
\frac{\dot{E}}{E} = r - \rho,
\]

where the dot indicates a change over time.

2.2 Production

I assume that the economy is endowed with skilled workers \( H \) and unskilled workers \( L \). The composition of skills in the economy does not change over time in the
sense that technological progress increases the effective endowments of both skilled and unskilled labour, leaving the effective relative endowment unchanged. Due to homothetic preferences, I do not have to distinguish between the two types of workers but can consider a representative household. The endowed factors are perfectly mobile within the economy, but not internationally. Each of the final goods uses one of the factors and a continuum of intermediates in its production process. I assume that the $X$ sector uses skilled workers and the $Y$ sector uses unskilled workers. The intermediates are assumed to be capital inputs that are used in both sectors. Let $Z_i$ denote the aggregate index of intermediates used in sector $i = X,Y$. I assume that both final goods are traded, whereas the capital goods are for domestic use only. The final goods are produced using the production functions

$$
X = B Z_X^\beta H_X^{1-\beta},
$$

$$
Y = B Z_Y^\beta L_Y^{1-\beta},
$$

(3)

where $\beta$ is the input share of the intermediate input and $B$ is a constant. Both sectors are perfectly competitive. By assuming that both sectors use the intermediates at the same intensity, I balance the effect of offshoring on final good production. If, instead I would assume different intensities, one final-good sector might benefit more from offshoring relative to the other final-good sector which would imply one final-good sector expanding relative to the other.\footnote{See for example in \cite{footnote} for a static analysis of offshoring on an economy where sector benefit differently.} As this kind reallocation is not of interest to the argument I choose to neglect it. From minimizing cost in each final good sector I obtain

$$
p_X = \frac{1}{w_H^\beta Z},
p_Y = \frac{1}{w_L^\beta Z},
$$

(4)

which implies that the relative factor price in the economy is fixed.

Each capital input, indexed by $\omega$, is manufactured by a different producer. The aggregate output index takes the functional form $Z_t = (\int_{\omega \in \Omega} z_{\omega t} \, d\omega)^{\frac{1}{\sigma}}$, where $z_{\omega t}$ is the output of each individual producer. The intermediate inputs are substitutes
with \(0 < \sigma < 1\) and an elasticity of substitution between any two varieties of \(\varepsilon = 1/1 - \sigma > 1\). The number of potential varieties is infinite. I assume, however, that varieties have to be invented before they can be used in the production of a final good. I denote the set of existing varieties by \(\Omega_t\). As I will show, \(\Omega_t\) grows over time which implies productivity gains in the economy. For simplicity I skip the time subscript of the number of varieties. Let \(p_z\) be the price set by a particular intermediate producer. The implied aggregate price index of \(Z\) is

\[
P_Z = \left( \int_{\omega \in \Omega} p_z^{1-\varepsilon} d\omega \right)^{\frac{1}{1-\varepsilon}}. \tag{6}
\]

Each producer of a capital input maximizes profits, facing a downwards sloping demand curve,

\[
z_\omega = Z \left( \frac{p_z}{P_Z} \right)^{-\varepsilon}. \tag{7}
\]

The capital input is produced by using skilled and unskilled workers. I assume that unskilled workers have to perform a continuum of tasks in order to provide one unit of a labour input. To simplify the analysis I assume that a task needs one unit of labour input. The development of trade in tasks follows Grossman and Rossi-Hansberg (2006). I normalize the mass of tasks to be from zero to one. It is assumed that all tasks are offshorable. However, each task has a specific trade cost \(\tau_j > 1\), where \(j\) indexes the task. I further assume that tasks are ordered such that trade costs are non-decreasing in \(j\), which orders the task according to their offshorability. A firm offshores a task as long as it is cheaper to import the task than produce it at home. This implies that

\[
w_L \geq w_L^* \tau_j, \tag{8}
\]

where the asterisk denotes the rest of the world. Each of the intermediate producers offshore up to the point where there are no more cost savings possible. Let \(J\) denote the marginal tasks for which a firm is indifferent between offshoring or domestic production and equation (8) hold with equality. The marginal task is a function of the wages of unskilled workers at home and in the rest of the world, \(J \equiv J(w_L, w_L^*)\). For simplicity, however, I skip the arguments and denote the marginal task by \(J\).

The wage of unskilled workers in the rest of the world is assumed to be lower than in the domestic economy, \(w_L^* < w_L\) for a marginal task to exist. Further, I assume that the wage of unskilled workers in the rest of the world grows at the same rate as the domestic wage. I make these assumptions in order to ensure that the marginal task exists over time and no corner solution arises.

5
The production function of a capital input is assumed to be \( z(\omega) = \Lambda \psi^{\alpha} \omega^{1-\alpha} \), where \( \Lambda \) is a parameter and \( \psi \) is the aggregate index of tasks performed. Note that due to the assumptions made on the tasks, I can think of them as a Leontief technology; each task has to be performed exactly once to produce one unit of the labour input. Cost minimization yields a unit cost function of \( \tilde{c}(w_L, w_L^*) \equiv \Theta(J)^\alpha c(w_L) \), where \( c(w_L) = w_L^\alpha \omega^{1-\alpha} \). The second expression in the unit costs is

\[
\Theta(J) \equiv 1 - J + \int_0^J \frac{\tau(j) dj}{\tau(J)}. \tag{9}
\]

The intuition for \( \Theta(J) \) is that it is a cost savings parameter. If the economy is able to offshore a fraction of the tasks, domestic labour is replaced by lower cost labour from the rest of the world, which reduces the costs of production. This is similar to an increase of the productivity of domestic labour.

The behaviour of each intermediate producer is characterized by a mark-up over marginal costs. The pricing rule is

\[
p(w_L, w_L^*) = \frac{\tilde{c}}{\sigma} \quad \forall \omega \in \Omega_t, \tag{10}
\]

where I have dropped the arguments on the right hand side. The mark-up is set over effective marginal costs. The price of a single variety falls in the effective marginal costs. The per-period profits of an intermediate producer is

\[
\pi_\omega(w_L, w_L^*) = (1 - \sigma) p_z z(\omega) \quad \forall \omega \in \Omega_t. \tag{11}
\]

Due to symmetrical producers, all capital input producers set a price equal to equation (10). Therefore, I can rewrite the aggregate price index in (6) as

\[
P_Z = n^{\frac{1+\sigma}{\sigma}} p_z, \tag{12}
\]

where \( n \) denotes the number of intermediate producers in period \( t \).

### 2.3 Research

Before entering the production stage in the intermediate sector, a potential producer of a capital input must invest in research and development of a blueprint for a new capital input. With the invention of a blueprint, the innovator receives a patent. I assume that inventing around the patent is prohibitively costly and thus an incumbent intermediate producer faces no (direct) competition for her variety

\footnote{It holds that \( n_t = \Omega_t \) in each period. Otherwise some firms would invent blueprints which are not used.}
because the latter is protected by the patent. I assume that patents are non-contractible.\footnote{This assumption is made analogous to Grossman and Helpman (1989).} Therefore, an intermediate producer that invests in research and development also becomes the producer of the capital input. I assume that research uses human capital as its sole input.

A potential entrant makes an investment if the cost of the investment is not larger than the present discounted profits it earns from its investment. Let $v$ be the present discounted value of an investment and $aw_H/K$ the investment cost. The investment cost is composed of the input requirements of human capital $a$, the wage the producer has to pay to employ one unit of human capital in research and the capital stock $K$. The capital stock $K$ represents the existing experience in the economy in research. With each new variety the capital stock increases. Therefore, successful research has a positive externality on the investment costs. I make the assumption that the capital stock equals the number of already invented varieties, $K = n$.\footnote{See Grossman and Helpman (1991b) for further discussion.} Therefore, investment costs decline over time, which permits more entry into the intermediate market. I assume free entry into the intermediate market. Accordingly, all the blueprints are marketed. If that was not the case, some R&D investment would be wasted. The free entry condition is

$$\int_0^\infty e^{-R(t)\pi(t)}dt = \frac{aw_H}{n}. \quad (13)$$

The discounted profits of successful innovation must equal the costs of developing a variety. If the costs are lower than the intertemporal profits then profitable opportunities exist in innovation. Differentiating equation (13) with respect to the initial period yields

$$r = \frac{n\pi}{aw_H} + \frac{\dot{w_H}}{w_H} - \frac{n}{n} \quad (14)$$

which is a no-arbitrage condition. The intuition is similar to the one of the free-entry condition. Potential investors are able to issue a bond on the financial market to finance research. The issuer of the bond has to pay interest $r$ per period. The return from inventing a blueprint is the pure profits in the period of invention and the evolution of the future profits. An investor would issue a bond as long as rent payment of the bond is not more than the return of investment, with equality in equilibrium.
2.4 Equilibrium Conditions

An equilibrium in the economy is characterized by a steady state, where all variables grow at a constant rate. I define \( g \equiv \dot{n}/n \) to be the growth rate of new varieties.\(^8\)

For the economy to be in equilibrium, the no-arbitrage condition in (14) must be satisfied and the factor markets have to clear. Each final good sector indirectly uses both factors. For example, the \( X \) sector uses skilled workers directly in its production and unskilled workers indirectly in the form of the capital input. Let \( a_{ki} \) denote the unit-input coefficients of input \( k = H, L \) used in sector \( i = X, Y, Z \). Further, let \( a_{ZX} \) and \( a_{ZY} \) be the unit input coefficients of the capital good in the respective final good sector. The input coefficients are derived from the unit-cost functions of the final good sectors in equation (4) and using Shephard’s lemma. The detailed derivation of the unit input coefficients is found in the appendix. The demand for skilled workers from the research sector is its input requirements \( a/n \) multiplied by the number of new entrants \( \dot{n} \). Therefore, I write the factor market clearing conditions as

\[
\begin{align*}
H &= a_{Hz}(a_{ZX}X + a_{ZY}Y) + a_{HX}X + aq \\
L &= (1 - J)a_{Lz}(a_{ZX}X + a_{ZY}Y) + a_{LY}Y.
\end{align*}
\]  

(15)

As has been assumed, offshoring affects the labour market clearing of unskilled labour only. An increase in marginal task \( J \) reduces the demand for unskilled labour from the intermediate sector. The input coefficients are affected by the introduction of new varieties. Rewriting the factor prices in their productivity-adjusted form enables me to solve for the equilibrium growth rate. Let the productivity adjusted wage be \( \bar{w}_k \equiv w_k A^\beta \), where \( A \equiv n^{1/\epsilon} \). I therefore rewrite the pricing equations in (10) as

\[
\begin{align*}
px &= c_X(\bar{w}_H, \bar{w}_L) \\
py &= c_Y(\bar{w}_L, \bar{w}_H).
\end{align*}
\]  

(16)

I define the coefficients as \( b_{HX} = a_{HX} + a_{Hz}a_{ZX} \), \( b_{HY} = a_{Hz}a_{ZY} \), \( b_{LX} = a_{Lz}a_{ZX} \) and \( b_{LY} = a_{LY} + a_{Lz}a_{ZY} \). Given those definitions, I can rewrite the factor market clearing as

\[
\begin{align*}
H &= b_{HX}\dot{X} + b_{HY}\dot{Y} + ag \\
L &= b_{LX}\dot{X} + b_{LY}\dot{Y} - Ja_{Lz}(a_{ZX}X + a_{ZY}Y)
\end{align*}
\]  

(17)

\(^8\)Solving for the equilibrium follows Grossman and Helpman (1991a)
where $\bar{X} = XA^\beta$ and $\bar{Y} = YA^\beta$ are the productivity-adjusted final outputs. Multiplying both equations in (17) with the respective effective wage $\bar{w}_k$, adding them together and using the appropriate definitions of the unit input coefficients yields

$$\bar{w}_L L + \bar{w}_H H = \chi_1 (p_X \bar{X} + p_Y \bar{Y}) + \bar{w}_H a g,$$

where $\chi_1 = \{1 - \beta + \beta \sigma (1 - \alpha) + \alpha \beta \sigma (1 - J) / \Theta \}$. This is a modified income-expenditure (in)equality.\(^9\)

I now turn to the evolution of the expenditures in the economy. Expenditures in this economy are not equal to output. Trade must be balanced and hence the import value of final good and tasks must equal export value. Therefore, the per period expenditures in the economy are the value of the production less the cost of the imported tasks, $E = p_X X + p_Y Y - \text{import value of tasks}$. In the Appendix I show that this results in

$$E = \chi_2 (p_X X + p_Y Y),$$

where $\chi_2 = 1 - \alpha \beta \int_0^J \tau (j) dj / [\Theta (J) \tau (J)]$, with $\chi_2 \in [0, 1]$.\(^{10}\) I will refer to $\chi_2$ as the wedge between the value of the output in the economy and domestic expenditures. The growth rate of the expenditures is

$$\frac{\dot{E}}{E} = \beta \frac{1}{1 - \epsilon} g,$$

which is derived in the appendix in more detail. Equation (20) implies that the growth of expenditures is proportional to the growth rate, showing the trade-off the economy faces. For example, if more resources are invested in research, current output reduces, but future consumption possibilities are enhanced.

Finally, I rewrite the no-arbitrage condition in (14) using (20) and the expression for the profits in (11). Therefore,

$$\beta \frac{1 - \sigma}{a \bar{w}_H} (p_x \bar{X} + p_y \bar{Y}) = g + \rho.$$

The no-arbitrage condition links current output to the growth rate, showing the trade-off the economy faces. For example, if more resources are invested in research, current output reduces, but future consumption possibilities are enhanced.

\(^9\)The left-hand side is the factor income whereas the right-hand side is the expenditures, less the profit income in the economy.

\(^{10}\)It is straight forward to show the upper bound. For the lower bound, note that $1 - \alpha \beta \int_0^J \tau (j) dj / [\Theta (J) \tau (J)] \geq 0$. I can rewrite the latter expression as $1 \geq \Theta (J) \geq \alpha \beta \int_0^J \tau (j) dj / \tau (J)$. The first inequality in the second expression is from the definition of $\Theta (J)$. This proofs the lower bound.
2.5 Equilibrium

I am now able to discuss a steady-state growth rate in the economy. In an equilibrium, the economy must satisfy the income-expenditure equality in (18) and the no-arbitrage condition in equation (21).\footnote{11I consider an economy that is diversified in the production of both final goods, which is the case if the economy grows at a moderate rate. However, if the growth rate is too high in the economy, all skilled labour is employed in research and the production of the capital input.}

I proceed with a graphical presentation of the equilibrium and the trade-off between current output and growth. The income-expenditure equality and the no-arbitrage condition are drawn in figure 2 with the effective per period output $Q$ on the vertical axis and the growth rate $g$ on the horizontal axis. The $RR$ line represents the income-expenditure equality. The negative slope of the income-expenditure equality reflects the trade off between growth and output. The factor endowments can be allocated either in production or research. The more skilled labour is allocated to research, the higher is the growth rate, however, the lower is the output in the economy. The $AA$ line represents the no-arbitrage condition. This line is upwards sloping because a higher current output implies higher profits and therefore a bigger incentive to invest in new blueprints, thereby increasing the growth rate. Both constraints are linear if the economy is diversified. In case of specialization, both constraints become non-linear. The equilibrium is found at the intersection of the income-expenditure equality and the no-arbitrage condition and is denoted by $G$.

For the analytical solution, substituting the income-expenditure equality into the no-arbitrage condition (14) yields

$$g = \frac{\eta}{a(1+\eta)} \left( \frac{w_L}{w_H} L + H \right) - \frac{1}{1+\eta} \rho,$$

(22)

where $\eta \equiv \frac{\beta}{\alpha} \frac{L}{H}$. By assumption, the growth rate is positive. The basic structure of the growth rate is that growth is increased if either or both of the endowments increase. Growth decreases with an increase in the discounting rate, as consumers become more impatient and invest less.

3 Offshoring and Growth

How is the growth rate affected by the extent of offshoring? If offshoring affects the allocation of factors of production, growth is affected as well. On the one hand, unskilled workers lose their jobs in the intermediate sector as offshoring enables
intermediate producers to reduce their costs by moving tasks overseas. On the other hand, due to the reduced costs, the intermediate sector might expand its output, which might absorb the job losses and the two effects might cancel each other out. In this section, I show that an increase in the extent of offshoring has an ambiguous effect on growth. To this end I assume that the economy is initially in a steady state when it experiences the shock and I compare it to the economy after the adjustment process. However, before I investigate the link of offshoring and growth, I develop some results that are helpful to build an intuitive understanding for the underlying mechanisms.

Throughout this section I consider an increase in the extent of offshoring. The extent of offshoring is measured by the marginal task $J$. If $J$ increases, a larger fraction of tasks is offshored. Two reasons for a shift in the marginal task exist. Firstly, the wage in the rest of the world falls. To fix ideas, by the condition for the marginal task in equation (8) I see that $J$ has to increase if the domestic wage remains constant.\(^{12}\) Secondly, the transportation costs of the task falls. For example, communication links to the rest of the world improve, which reduces the costs $\tau$. If the transport cost for each individual task falls, then, by equation (8), $J$ must increase. In terms of their effect on the marginal task, both reasons are equivalent. However, I will restrict the analysis in this section to a fall in trade costs.

\(^{12}\)Below I show that the domestic wage is not fixed, but $J$ must change nevertheless.
costs in order to be able to track down the effect of the change of cost savings parameter $\Omega(J)$.

In my discussion, I follow Grossman and Rossi-Hansberg (2008) and assume a uniform fall in the trade costs of all tasks. Formally, I assume that the trade costs fall by $\nu < 1$, where $(1 - \nu)\tau_j$ is the new level of trade costs of task $j$. Inspecting the definition of $\Theta$ in (9) reveals that it is only affected by a change in $J$ and not affected by change in the trade costs itself.\(^\text{14}\)

**Proposition 1.** Let the extent of offshoring, $J$, in the economy increase. Then the productivity adjusted wage $\hat{w}_i$ of each factor $i = L, H$ increases.

**Proof.** Totally differentiating the log of the pricing equation (27) in the Appendix for either final good sector, yields \( \hat{p}_i = \alpha\hat{\Theta} + \alpha\beta\hat{w}_k + (1 + \beta - \alpha(1 + \beta))\hat{w}_{-k} \), where the hat indicates a percentage change and $k \in H, L$. Note that the change in both wages must be equal, $\hat{w}_H = \hat{w}_L$, because the relative factor prices are determined by the relative final price which is unchanged. Taking this into account results in

\[
\hat{w}_k = -\alpha\beta\hat{\Theta} \quad k = H, L,
\]

This establishes a positive correlation between wages and offshoring. \qed

Domestic unskilled labour is mixed with cheaper foreign unskilled labour which raises the effective productivity of domestic labour. As with Hicks-neutral technological progress, the marginal product of labour increases and this increases the return for labour. This is the productivity effect in Grossman and Rossi-Hansberg (2008). Offshoring reduces the costs in the intermediate sector. These cost savings are passed on to the final good sectors which become relatively more intensive in their use of capital inputs.\(^\text{15}\) The substitution effect raises the marginal product of the respective factor. In combination with constant final good prices the factor prices must increase.

Will all factors in this set-up support offshoring? In a standard HOS model, the Stolper-Samuelson theorem indicates that not all factors of production gain from liberalizing final goods trade. In this model, none of the factors has an incentive to oppose offshoring because they all gain from higher wages, as shown in proposition 1.

\(^{13}\)A fall in the trade costs is similar to a cost reduction in service links in Jones and Kierzkowski (1990).

\(^{14}\)For a discussion of a proportional fall in trade costs, see Grossman and Rossi-Hansberg (2008)

\(^{15}\)The aggregate price index in the intermediate sector is $P_Z = \Theta^\alpha\hat{w}_L^{\alpha}\hat{w}_H^{1-\alpha}$. Rewriting the latter expression in percentage changes and using the extent of the wage changes given in the above proof yields $\hat{P}_Z = \alpha(1 - \beta)\hat{\Theta}$, which is a smaller one but positive if $J$ increases.
I now turn to the analysis of the growth rate in equation (22). The result I am interested in is a marginal change of the extent of offshoring on the rate of growth. This implies that I have to find the sign of
\[ \frac{\partial g}{\partial J} = \frac{\partial g}{\partial \eta} \frac{\partial \eta}{\partial J}. \]

(23)
The first derivative, \( \partial g/\partial \eta \), is always positive. Using the definition of \( \eta \), its derivative is
\[ \frac{\partial \eta}{\partial J} = -\beta 1 - \sigma \frac{\partial \chi_1}{\partial J}, \]

(24)
where a prim indicates the derivative with respect to \( J \). The sign of the derivative depends on the sign of \( \partial \chi_1/\partial J \), which is negative if \( -\epsilon_\Theta < \frac{J}{1-J} \), where \( \epsilon_\Theta < 0 \) is the elasticity of cost-savings \( \Theta \) with respect to the marginal tasks \( J \). The latter condition states that the additional cost savings of offshoring around the marginal costs might not be too large for all \( J \in [0,1] \). Note that this condition originates from the labour market clearing conditions. It is instructive to rewrite the condition as \( -\epsilon_\Theta < -\epsilon_{1-J} \), where \( \epsilon_{1-J} \equiv -J/(1-J) \) is the elasticity of the labour-supply effect. To summarize this paragraph:

**Corollary 1.** Let the extent of offshoring increase. If the labour-supply effect dominates the demand effect \( -\epsilon_\Theta < -\epsilon_{1-J} \), the growth rate in the economy increases. If the labour-supply effect is dominated by the demand effect or both effect cancel each other out, offshoring has a non-increasing effect on the growth rate in the economy.

In the following I will discuss the implication of an increase in the extent of offshoring on the two equilibrium conditions and subsequently link the discussion to the stated condition.

In figure 3 the income-expenditure equality \( RR \) and the no-arbitrage condition \( AA \) are depicted. The focus is on the linear part of the two equilibrium conditions. From proposition 1 follows that the return to both endowment increases. Accordingly, research becomes more expensive, which discourages firms to invest, which is revealed by inspecting the free entry condition in equation (13). In the figure, the no-arbitrage condition tilts upwards as a result. A tilt upward implies that domestic output, which provides the incentive to invest for entrepreneurs, must increase to be able to achieve a given growth rate.

To analyze the shift of the income-expenditure equality I start by considering a positive effect of offshoring on growth. The new income-expenditure equality is depicted by \( R'R' \). Offshoring shifts the income-expenditure equality out and increases
its slope. The shift is a result of the increase in the wages and the corresponding increase of the factor income. The change in the slope is a result of the increase in the relative costs of research. Although there are no savings in the model, postponing consumption can be seen as an equivalent. As investing in research becomes more costly, the incentive to postpone consumption for a consumer reduces. Thus, for a given output, consumers prefer to invest less.

In order to achieve a higher growth rate, the shift in the income-expenditure equality must be large enough for the equilibrium to be to the right of the dotted line, which represents that pre-change growth rate. Thus, with a sufficient shift of the resourced constraint, the increase in output is large enough to compensate for the increase in the costs of research.

\[ Q \]

\[ R \]

\[ R' \]

\[ R'' \]

\[ A \]

\[ A' \]

\[ R' \]

\[ R'' \]

\[ A \]

\[ A' \]

\[ A'' \]

\[ R \]

\[ R' \]

\[ R'' \]

In the case of a slowdown in growth due an increase in the extent of offshoring, the income-expenditure equality, which is labeled $R'' R''$, shifts out as well, whereas the new no-arbitrage condition is still given by $A' A'$. In this case the new equilibrium growth rate must be to the left of the dotted line. Therefore, the shift in the income-expenditure equality is not sufficiently large enough to compensate for the increase in the research costs.

So far I neglected the intuition of the shift size of the income-expenditure equality. With more offshoring, the price of the intermediate good reduces which leads to a substitution towards intermediates in both final good sectors. This effect, how-
ever, is of second order - it is already accounted for in the unit input requirements of the sectors. The important aspect of the labour market adjustments takes place in the intermediate production where unskilled labour tasks are substituted for skilled labour due to a fall in the costs of unskilled labour tasks. This shift towards a more unskilled intensive technology implies an increase in the demand for unskilled labour, which is the labour demand effect. Due to an increase in the extent of offshoring, the number of task that were produced domestically reduces which makes unskilled labour available for reallocation. This is the labour-supply effect. Only if the freed labour is not absorbed by the demand effect, then the growth rate increases. In other words, the shift of the income-expenditure equality is large if the labour supply effect dominates and sufficiently enough unskilled labour is freed to account for the demand effect and additionally reallocate unskilled labour to increase output. Furthermore, in equilibrium, there is sufficiently enough unskilled labour to substitute for skilled labour who can be reallocated towards research to increase growth.

4 Discussion

An important aspect of the model is that the condition of the effect of offshoring on growth provided depends on the marginal task $J$. A question that arises is whether countries that already offshore a large or a small fraction of their tasks are more likely to gain from more offshoring? The answer depends on the trade cost schedule. If, for example, the trade cost schedule is initially very steep, the first tasks are relatively costly to offshore, but the cost savings are initially large. Hence, a country starting to offshore might be less likely to benefit from offshoring in terms of a higher growth rate. If the trade cost schedule is convex, the initial stages are easy to offshore and the cost savings of early stages are relatively low. In this case, a country that offshores a small fraction of its tasks is more likely to experience an increase in the growth rate. Therefore, when assessing the impact of offshoring on growth, it is of importance not only to know the extent of offshoring in a country, but also the trade cost schedule.

Another interesting aspect of the model is to contrast the implication for growth of offshoring with immigration. In the existing literature, Rodriguez-Clare (2007) argues that research employment in a country increases if immigration takes place.\footnote{Again, the main focus of Rodriguez-Clare (2007) is on the wages effects of offshoring and immigration respectively.}
In this work, to contrast the implications of immigration with the ones of offshoring, I simply assume that immigration increases the unskilled labour endowment by a factor $\Lambda > 1$. Using the expression for the growth rate in equation (22), immigration unambiguously increases growth. The reason for the difference in the effects is that there are no cost savings associated with immigration. A worker is paid the local wage rate, which in the case of migration is the one of the offshoring economy. Thus in the case of migration research is not becoming more expensive which explains the ambiguity of migration.

5 Conclusion

In the paper I introduced trade in tasks into a model of endogenous growth. I then asked the question of how is the growth rate affected if a larger fraction of tasks is offshored. The implication of the model is that an increase in the extent of offshoring has an ambiguous effect on growth in an economy. In contrast, the existing literature has found an unambiguous increase in research employment. In the paper I show that the ambiguity of the effect depends on a condition which originates from the factor market clearing. The interpretation of the condition is that if the economy is able to increase the effective unskilled labour supply sufficiently, unskilled labour is able to substitute for skilled labour in the production and increase output beyond what is suggested by the cost-savings of offshoring. The substitution effect allows skilled workers to reallocate to research without diminishing the output in the economy.

Acknowledgments

I would like to thank Ian Wooton for many insightful discussions. Further I would like to thank Marta Aloï, Daniel Bernhofen, Alex Dickson, Joseph Francois, Gabriela Grotkowska, Pascalis Raimondos-Moller and Alejandro Riano for their comments. I also would like to thank the seminar participants at the University of Strathclyde and Tuebingen University, and the GEP workshop in Nottingham for their remarks.
A Appendix

A.1 Unit Input Coefficients

In this section, I derive the unit input coefficients. I start with the pricing equations

\[ p_X = A_Z^{\beta} p_Z^{\beta} w_H^{1-\beta} \]
\[ p_Y = A_Z^{\beta} p_Z^{\beta} w_L^{1-\beta} , \]

where \( A_Z \equiv n^{1-\beta} \). The pricing rule for the intermediate input is

\[ p_Z = \tilde{c}(w_L) \sigma, \]

where \( \tilde{c}(w_L) = \Theta(J)^{\alpha} c(w_L) \). Let the productivity adjusted factor prices be \( \bar{w}_i \equiv w_i A_Z^{\beta} \) for \( i = H, L \). I can therefore rewrite the pricing equations as

\[ p_X = \left( c(z(\bar{w}_L)) \right)^\beta \bar{w}_H^{1-\beta} \]
\[ p_Y = \left( c(z(\bar{w}_L)) \right)^\beta \bar{w}_L^{1-\beta} \]
\[ p_Z = \tilde{c}(w_L) A_Z^{\beta} \]

(27)

I define \( p_X \equiv \tilde{c}_X(\bar{w}_L) \) and \( p_Y \equiv \tilde{c}_Y(\bar{w}_L) \) to be the productivity adjusted cost functions of firms in the \( X \) and \( Y \) sector. The unit input coefficients are defined as the derivative of the cost function with respect to the input price. This results in

\[ a_{HX} = (1 - \beta) \frac{p_X}{\bar{w}_H} \]
\[ a_{LY} = (1 - \beta) \frac{p_Y}{\bar{w}_L} \]
\[ a_{ZX} = \beta \frac{p_X}{p_Z} \]
\[ a_{ZY} = \beta \frac{p_Y}{p_Z} \]

(28)

I can now write the a’s. \( b_{HX} = a_{HX} + a_{HZ} a_{ZX} \)

\[ b_{HX} = p_X \left( \frac{1-\beta}{w_H} + \frac{\beta \sigma a_{HZ}}{c} \right) \]
\[ b_{LY} = p_Y \left( \frac{1-\beta}{w_L} + \frac{\beta \sigma a_{HZ}}{c} \right) \]
\[ b_{LX} = p_X \frac{\beta \sigma a_{HZ}}{c} \]
\[ b_{HY} = p_Y \frac{\beta \sigma a_{HZ}}{c} \]

(29)

A.2 Derivation of Expenditures

In the main text I state that \( E = p_X X + p_Y Y - \) import value tasks. The import value of tasks is simply the aggregate demand for the tasks, which is \( \int_0^J w_L^a a_L a_Z (a_{ZX} X + a_{ZY} Y) \tau(j) dj \). Using the definition of the a’s and the marginal tasks I am able to derive \( E = \{1 - \alpha \beta[\int_0^J \tau(j) dj]/[\theta(J) \tau(J)]\} \{p_X X + p_Y Y\} \), where I define the first term in brackets to be \( \chi_2 \).
I can now determine the growth rate of the expenditures $\dot{E}/E$ by taking logs of the above expression and differentiating with respect to time. Prices and productivity adjusted outputs are constant and so is $\chi_2$ if the marginal tasks is constant in a steady state. Expenditures grow at the same rate as wages $\dot{E}/E = \beta g/\varepsilon - 1$.

A.3 Derivation of Equation (21)

In this appendix, I derive the no-arbitrage condition in equation (21) in more detail. I start by considering the basic no-arbitrage condition in equation (14). If this condition holds, the investment sector is in equilibrium as no firms have an incentive to enter or exit research. Consumer optimization yields the condition (20), which I substitute in the no-arbitrage condition. The evolution of expenditures is determined by the prevailing interest rate $r_t$ and the discount factor $\rho$. The correlation between change in income and the interest rate is positive. For instance, if the interest rate is high, consumers are willing to save more. Because savings must equal investments, income grows faster. I substitute the definition of the growth rate $g \equiv \dot{n}/n$, and the growth of the high skilled wage $\dot{w}_H/w_H = g\beta/\varepsilon - 1$. The growth of the skilled wage is derived from the definition of the effective wage, $\bar{w}_H = A^\beta w_h$, which is constant. As I argue in the text the growth rate of the expenditures is $\dot{E}/E = g\beta/(\varepsilon - 1)$. These substitutions yield the modified no-arbitrage condition

$$\frac{\beta}{\varepsilon - 1}g + \rho = \frac{n\pi}{aw_H} + \frac{\beta}{\varepsilon - 1}g - g.$$  \hspace{1cm} (30)

The profits of an intermediate producer is given in equation (11). I rewrite the profit function by using $nz = nz_x + nz_y$ which are the demands for the capital input from each of the final input sectors. The demands are $a_zx \bar{X}$ and $a_zy \bar{Y}$. Multiplying both demands with $p_z$ yields $\beta(p_x \bar{X} + p_y \bar{Y})$. Substituting this and making the appropriate cancellations yields the no-arbitrage condition in equation (21)

References


