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Competitive General Equilibrium with Finite Change and Theory of Policy Making

by

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Abstract

We construct a generalized model of finite change whereby exogenous shocks such as international trade or technological change, not only contract, but totally shut down production in some sectors. In such cases even in a competitive structure and in absolute contrast to the conventional wisdom, price based strategies to protect those vanishing sectors will not be equivalent to quantity based strategies. We also consider factor trade and a similar asymmetry between price based and quantity based interventionist policies.

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Outline

I. Introduction

It is widely understood that price and quantity based policy instruments yield symmetric results in a competitive framework. Quantity implications of price based policies and price implications of quantity based policies are symmetric, a particular example being the tariff-quota equivalence result. However, comparative static results in the standard competitive general equilibrium models are about local changes when outputs can contract and expand but the sectors keep producing the commodity. It is rather unusual, though fairly realistic to conceive of cases when exogenous shocks can eliminate some of the production. International price shocks or technological innovation can eliminate sectors or lead to the emergence of new ones.

In trade theory, such a regime change has come to be known as *finite change*. Jones and Findlay (2000) and Jones (2008) provide examples where a finite change can alter standard comparative static results. Jones and Findlay (2000) analyzed the effects of a finite change in techniques and showed, for illustration, that labor-saving technical progress can result in lowered real wages, even if it is concentrated in the more labor-intensive of two commodities initially produced. Jones (2008) showed that large shocks to equilibrium may change a country's production pattern with trade, and such shocks are provide a non-monotonic response. In an early paper, Jones and Marjit (1985) had explained a particular kind of finite change that converts a fairly arbitrary multi-good multi-factor autarkic model to either a specific-factors model (SFM) or a combination of the Heckscher-Ohlin-Samuelson (HOS) model and SFM. [1] In this paper, we provide a generalized version of finite change models when goods outnumber factors of production. While this fits well with continued interests in higher dimensional issues in trade theory,[2] we focus on the fact that certain exogenous shocks (such as, though certainly not limited to, exposure to international trade) allow only a subset of commodities to be produced under competitive conditions and domestic production of certain goods will vanish.

Typically, if policymakers are interested in pursuing quantity or price based instruments to support some of the sectors that keep producing in the changed environment, the effect will be symmetric. But once policymakers worry about supporting the sectors which are currently non-functioning, the price and quantity based instruments can offer entirely different results. One example is the tariff and quota non-equivalence, as shown in Choi and Marjit (1996). It is not a stretch to imagine several other policies that will yield a similar outcome Therefore,

[1] Recent applications of this can be found in Marjit and Mandal (2012).

comparative static results change under finite alterations of the general equilibrium structure. Our general set up embodies many special cases and results that are available with small scale general equilibrium structure.

The rest of this paper is organized as follows. Section II develops the basic analytical framework. Section III discusses policy experiments in terms of comparative statistics. The final section gathers our concluding remarks.

II. Model

Let us consider a standard neo-classical general equilibrium production model with *n* goods and m factors of production. Goods are indexed by $i = 1,...,n$ and factors by $j = 1,...,m$. Competitive equilibrium conditions ensure

$$\sum_{j=1}^{m} W_j a_{ji} = P_i \qquad (i = 1,2,...,n) \tag{1}$$

where W_j and P_i represent factor and commodity prices, respectively. We chose the n^{th} good as the numéraire.

Full employment conditions are given by

$$\sum_{i=1}^{n} a_{ji} X_i = V_j \qquad (j = 1,2,...,m) \tag{2}$$

where X_i and V_i are output and stock of factors of production, respectively. The market clearing conditions are

$$D(P_i, r) = X_i \quad (i = 1,2,...,n) \tag{3}$$

Note that collection of equations (1), (2), and (3) define a closed economic equilibrium: we have $[n + m + (n-1)]$ independent equations to solve for, $(n-1)$ relative commodity prices, m factor prices, and n output levels.[3] This is a standard existence result with usual curvature restrictions. The autarkic values are (P_i^0, W_j^0, X_i^0).

Now let us consider the possibility that this country opens up for trade and acts as a small open economy facing exogenously given commodity prices $(P_i * ..., P_{n-1} *)$. Note that this makes the set of equations denoted by (3) irrelevant as in trade there is no reason why local demand will

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[2] See Ethier (1982), Jones and Marjit (1985), Marjit and Beladi (1999), Feenstra (2003), and Jones (2014).

[3] By Walras Law one of the market clearing equations will be redundant and we shall define (*n*-1) relative prices.

match local supply. Therefore, given the set of prices in (1), we need to determine m factor prices for $n > m$. We now write down the following proposition.

<u>Proposition I</u>. *With trade the economy will produce at most m distinct goods.*

Proof. Let us consider an arbitrary set of m goods. Appealing to the global univalence property of the standard neo-classical trade model, à la McKenzie (1967) and Ethier (1982), a set of m prices will uniquely determine $\{W_1..., \mathrm{W}_m\}$. Now consider the complementary set of commodities indexed by $(m+1,...,n)$ and let us pick any k^{th} commodity and look at the competitive condition.

$$\sum_{j=1}^{m} W_j a_{jk} \geq P_k * \qquad (4)$$

Note that $P_k *$ is exogenous to the system and W_j has been predetermined from the commodity set $\{1...,m\}$ with constant returns to scale. a_{jk}'s are determined by $W_j, j = 1...,m$. Hence, there are two possibilities

$$\sum W_j a_{jk} \geq P_k *$$

or

$$\sum W_j a_{jk} < P_k *$$

In the former case, with strict inequality, the k^{th} good will not be produced. Thus, we ignore the equality part because the goods are distinct and the probability that the LHS and RHS in (4) will match is zero, given that LHS and RHS in (4) both are exogenous. In the latter case the k^{th} good won't be produced because at least one W_j can earn more in this sector. This will imply that at least one good in the set $\{1...,m\}$ must vanish. Since k is arbitrary, the outcome holds for any good. Therefore, more than m number of goods will not be produced. [QED]

Without any loss of generality, let us suppose that the first m goods are produced and $\{m+1...,n\}$ are imported from abroad. Suppose the policy maker is interested in supporting the production of one of the m goods. It can be done either through a price support or a quantity based policy instrument. If t is the extent of support per unit then, for the i^{th} good, the following holds in equilibrium.

$$\sum_{j=1}^{m} W_j a_{ji} = P_j * + t \qquad (5)$$

The rest of the system remains the same. Suppose the new equilibrium is denoted by $\{W_j^t\}$ and $\{X_i^t\}$, $i = 1...,n$, $j = 1...,m$. We assume such change to be local in the sense that factor price changes are consistent with the initial separation of commodity sets between $\{1...,m\}$ and $\{m+1...,n\}$ so that goods that were not produced earlier are still not produced. Also, with global univalence $\{W_j^t\}$, $\{X_i^t\}$ are unique given (P_i*,t). The initial equilibrium, for $t=0$, is denoted by $\{W_j*\}$, $\{X_i*\}$. It is well known in the literature that one needs to put more structure in the model to guarantee that for $t>0$, $X_r^t > X_r*$. Following Jones and Scheinkman (1977), one could argue the following must be true. For $t>0$, $\exists\ s$ such that $X_s^t > X_s*$, but r may be different from s. To prove our point we can assume $s=r$ or follow Jones and Marjit (1985) to provide a condition that this is necessarily so. But s or r, it will serve our purpose. Let us define,

$$\Delta X_r* = X_r^t - X_r* \tag{6}$$

Note that since X_r^t is unique for $t>0$ and X_r* is unique for $t=0$, ΔX_r* is unique too.

Proposition II. *If the policy maker purchases ΔX_r* additional amount from the r^{th} sector, the new local price of the r^{th} good will be $(P_r*+\Delta P_r)$ where $\Delta P_r = t$.*

Proof. Since ΔX_r* is uniquely determined by t and global univalence holds, the new price $P_r^t = (P_r*+t)$. [QED]

So $(t,\Delta X_r*)$ constitute equivalent symmetric policy combination. This is similar to tariff-quota equivalence result in the literature. With this backdrop we get into the next section.

III. Comparative Statics with Finite Change

a) Commodity Trade

We now try to repeat the same experiment with set of goods $(m+1...,n)$ which are not being produced as

$$\sum_{j=1}^{m} W_j a_{ji} > P_i* \text{ for } i = m+1...,n \tag{7}$$

Let us suppose the policymaker gives a price support to the s^{th} good, $s \in (m+1,...,n)$ such that,

$$\sum_{j=1}^{m} W_j a_{is} = P_S*+t \tag{8}$$

Note $\{W_j, a_{js}\}$ all are defined by the price-cost equality condition by the commodity set $\{1...,m\}$. So are X_i* for $i=1...,m$.

Lemma 1. Given $t>0$, X_s^t is indeterminate.

Consider the full employment conditions

$$\sum_{i=1}^{m} a_{ji} X_i^{\,t} + a_{js} X_s^{\,t} = \overline{V}_j \tag{9}$$

This set of equations m have $(m+1)$ variables to determine i.e. X_i^t, $i = 1..., m$ and X_s^t. In the price system m equations determine W_i, $i = 1..., m$ and (8) just balances the wedge. For a series of given values of X_s^t one can determine $\{X_i^t\}$. Hence, X_i^t will be functionally dependent on X_s^t. One such value of $X_s^t = 0$, we know $X_i^t(X_s^t = 0) = X_i*$. But many values of X_s^t are possible and hence X_s^t is non-unique given $t = \sum W_j a_{js} - P_s*$. [QED]

This shows that, for example, if t is a tariff applied to protect the s^{th} sector, it can not determine a unique output level X_s^t and, therefore, there is no unique equivalent quantity support $(X_s^t - X_s*) = X_s^t$. Since the system is solved with $X_i^t(X_s^t)$, many X_s^t are consistent with $t > 0$. Moreover, the price support system may be far more discretionary than the quantity support system. When $t > 0$, firms can produce a large amount of X_s^t. But a quota of a specific amount can contain such overproduction.

b) Factor Trade

Our argument can be easily extended in case of factor trade as well.

Consider the case where in autarky there are n goods and n factors of production and one of the factors, call it capital K, is allowed to emigrate as the world rate of return r* > r , the local autarkic rate of return. This immediately converts the model to one where good outnumber locally fully employed factors of production. But here we consider policies to control factor trade. One reason might be that if K earns r*, given the set of commodity prices some factor must earn less than its autarkic return and those factors might lobby for capital control.

It is clear with price control the government imposes a tax t on r* with r*(1-t) =r.

In fact following the previous argument, it is obvious that there is an equilibrium where K will not move out at all. On the other hand explicit quota on export of K will be less restrictive.

IV. Concluding Remarks

A vast majority of studies in the area of international trade theory typically relies on results from comparative static exercises hinged on infinitesimal changes. Such analyses fails to take into cognizance the realistic possibility that the direction of change in key variables can be sensitive to sufficiently large shocks to the original equilibrium, commonly known as finite changes, due to an alteration in the pattern of production. This paper offers a generalized model of such finite changes when goods outnumber factors of production. We demonstrate how, under competitive conditions, some exogenous shocks (e.g. exposure to international trade) can lead to only a subset of commodities being produced while eliminating the domestic production of some of the other goods.

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