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Trade, firm selection, and innovation: the competition channel

By

Giammario Impullitti and Omar Licandro

GEP

Trade, firm selection, and innovation: the competition channel

Giammario Impullitti* and Omar Licandro†

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Abstract

We study the welfare gains originating from pro-competitive effects of trade liberalization in an economy with heterogeneous firms, variable markups and endogenous growth. Variable markups arise from oligopoly trade in similar goods, and cost-reducing innovation is the engine of sustained productivity growth. Trade liberalization stiffens product market competition by reducing markups, generating tougher firm selection and increasing the aggregate productivity *level*. Market share reallocations triggered by selection increase firms' incentives to innovate, thereby leading to a higher aggregate productivity *growth* rate. Endogenous productivity growth boosts the selection gains from trade, leading to substantial welfare improvements. A calibrated version of the model shows that growth doubles the gains from trade obtainable in models with static firm-level productivity.

JEL Classification: F1, F43, F6, O4.

Keywords: Endogenous Growth, Heterogeneous Firms, Oligopoly, Variable Markups, Dynamic Gains from Trade.

* *University of Nottingham.* Email: giammario.impullitti@nottingham.ac.uk.
† *University of Nottingham and IAE-CSIC Barcelona. Email:* Omar.Licandro@nottingham.ac.uk.

1 Introduction

If the gains from trade are small, is it worth it facing the potentially disruptive distributional consequences of globalization?[1] Assessing the size and identifying the sources of gains from trade is a long standing challenge for economists. In the last decades, a new line of research introducing firm heterogeneity in trade models has highlighted the possibility of a new source of welfare gains. Trade-induced reallocations of market shares from low to highly productive firms within the same industries increase sectorial efficiency, leading to improvements in aggregate productivity and to potentially large welfare gains. However large these productivity gains are, selection carries also welfare losses that can possibly outweigh the gains: firms' exit has a negative effect on welfare by reducing the variety of goods available in the economy.

Theoretical and quantitative analyses assessing the contribution of the 'selection margin' to welfare gains from trade have mainly featured market structures with perfect or monopolistic competition, focused on static models, or on dynamic economies without long-run productivity growth. The goal of this paper is to fill these gaps by providing a theory of trade with heterogeneous firms under oligopolistic competition and innovation-driven productivity growth. On the one hand, since it involves 'cross-hauling' of identical goods, oligopoly trade is potentially wasteful, therefore representing a more difficult environment to obtain old a new gains from trade. On the other hand, trade reduces markups thereby generating pro-competitive effects of trade which are absent in environments where firms market power is constant. Moreover, innovation-driven growth can possibly magnify the gains from selection, as market share reallocations can affect not only the productivity level but also its growth rate. We show that the new gains due to selection can be substantial and that their dynamic component due to the interaction between selection, innovation and productivity growth magnifies the gains obtainable in static models with firm heterogeneity.

Our model economy features a continuum of imperfectly substitutable varieties, or product lines, brought to the market by firms with different productivities. Differently from the standard Melitz (2003) model, each variety is produced by a small number of identical firms operating in an oligopolistic market. So individual firms are "large in the small but small in the large":

[1] Recent theoretical and empirical work has shown that trade can have adverse effects on employment and on wage and income inequality. See e.g. Cosar, Guner, and Tybout (2014), Felbermayr, Impullitti, and Prat (2014), Helpman, Itskhoki, Muendler, and Redding (2014), Autor, Dorn, Hanson, and Song (2014), and Acemoglu, Autor, Dorn, Hanson, and Pirce (2015).

relevant actors in their own market, interacting strategically with their direct competitors, but infinitesimal in the economy as a whole (Neary, 2010). Upon paying a sunk cost, a small number of firms enters each product line by drawing their productivity from a distribution of existing technologies. If entry is successful, firms compete Cournot with their rivals in their product line, and also invest in innovation to improve their productivity over time. Firms' innovation activity generates endogenous growth through within-variety knowledge spillovers typical of quality ladders models (e.g. Grossman and Helpman, 1991). The open economy features two symmetric countries engaging in costly trade. Since firms in each product line produce perfectly substitutable goods, two-ways trade takes place because of strategic interaction between firms, as in Brander and Krugman (1983).

We first present a simple and analytically tractable version of the model in which the number of oligopolistic firms per product line is fixed, and all operating firms export. Trade liberalization produces *static* and *dynamic* gains from selection. First, as in the standard model of oligopoly trade, a reduction in trade costs increases product market competition reducing markups. Reductions in markups force the least productive firms out of the market, reallocating resources toward surviving firms, increasing their average size and the average productivity *level* of the economy. Second, the increase in the size of surviving firms produced by selection stimulates cost-reducing investment in innovation thereby leading to faster productivity *growth*. Finally, selection involves potential losses since firms' exit reduces the number of varieties available to consumers. We show analytically that these potential losses from selection are outweighed by the gains.

We perform a quantitative evaluation of our theory in a more general version of the model where we endogenize the number of oligopolistic firms competing in each product line through free entry. We also introduce fixed export costs, which leads to an equilibrium where only the most productive firms export and they charge a different markup compared to domestic firms. This markup difference proves to be crucial in generating trade-induced selection effects in this economy with free entry. Without markup dispersion the marginal and the entering firm would have the same profit opportunities, the marginal firm would be indifferent to profit changes and, as a consequence, trade liberalization would not trigger firm exit. We calibrate this version of the model to match salient firm-level and aggregate statistics of the US economy, and solve it numerically.

Moving from a prohibitive level of variable trade costs, autarky, to a 10% import penetration ratio reduces the markup of non-exporting firms forcing the less productive of them out of the market. Exporters reduce their markup on domestic sales due to fiercer foreign competition and increase the markup on foreign sales: pricing-to-market allows them to avoid passing the whole reduction in trade costs on foreign consumers. As profitability of export increases, more firms enter the export market, this is the well known new exporter margin of trade. The average markup in the economy drops by 31% generating a large pro-competitive effect of trade on prices. Trade-induced increase in competition and selection increases the market size of more productive firms (exporters), thereby increasing their incentives to innovate ultimately leading to a 56% increase in the aggregate growth rate of productivity. These effects compound into large welfare gains from trade: long-rum consumption increases by 50%, and about half of this change is accounted for by the dynamic gains from trade. Hence dynamic welfare gains double the gains obtainable in an oligopoly trade model with firm heterogeneity and static firm-level productivity. Moreover, we compare the benchmark economy with a version of it where the selection margins do not respond to trade liberalization. We find that the overall welfare gains from trade are about 8 times and the dynamic gains are about 5 times higher in the benchmark model compared to its version where selection is not operative. This suggests that the interaction between selection and innovation/driven growth is fundamental in generating large welfare gains from trade liberalization.

Literature review. The paper is related to several strands of literature. A novel set of empirical regularities about trade, competition, and innovation has recently emerged from a large number of studies using firm-level data. First, trade-induced selection reallocates resources from less to more productive firms triggering increases in aggregate productivity.[2] A second line of research has highlighted the joint selection and innovation effect of trade, showing that trade liberalization cleans the market of inefficient firms forcing surviving firms to innovate more (e.g. Bustos, 2010, Aw, Roberts and Xu, 2010, Lleiva and Trefler, 2010, and Bloom, Draca and Van Reenen, 2016). A third piece of evidence shows that trade liberalization has pro-competitive effects by reducing prices and markups. Feenstra and Weinstein (2010) find a substantial reduction in average markups in the US between 1992 and 2005 associated to a large increase in import shares. De Loecker, Goldberg, Khandelwal and Pavcnik (2016),

[2] See for example, Pavcnik (2002), Topalova (2011), and Bernard, Jensen and Schott (2012) for a survey.

once controlling for changes in input tariffs, find that India's reduction of tariffs on final goods in the period 1989-2003 reduced markups.[3] Finally, Aghion, Blundell, Griffith, Howitt, and Prantl (2009) find that incumbent productivity growth and patenting in UK firms is positively correlated with foreign firm entry in technologically advanced industries. Griffith, Harrison, and Simpson (2010) find that the EU Single Market Programme (SMP) was associated with increased product market competition, as measured by a reduction in average profitability, and with a subsequent increase in innovation intensity and productivity growth. Our paper presents a rich and tractable model providing a coherent interpretation of these empirical regularities. Trade liberalization increases product market competition, triggers firm selection, increases the incentives to innovate leading to higher aggregate productivity growth.

Our model generates trade under oligopoly as in Brander (1981) and Brander and Krugman (1983) pioneering work.[4] Although two-ways trade in similar goods is inherently wasteful, trade can potentially be beneficial because of the pro-competitive welfare gains brought about by lower markups. Brander and Krugman show that introducing free entry, oligopoly trade yields welfare gains. We extend their oligopoly trade model to an economy with firm heterogeneity and productivity growth. Similarly to Brander and Krugman, in our simple model without free entry the wasteful nature of oligopoly trade can potentially offset the pro-competitive effects leading to losses from trade, but the new channels of firms selection and endogenous growth introduce additional sources of gains from trade. In line with Brander and Krugman, in our general model we introduce free entry which implies that trade liberalization is always welfare enhancing even in the absence of heterogeneity.

Our paper is also related to the endogenous growth literature in several ways. In surveying recent work on globalization and growth, Grossman and Helpman (2015) discuss the fundamental channels linking trade and growth: international knowledge spillovers, market size and competition, relative prices, and technology diffusion. In modern environments with firm heterogeneity, the link between trade openness and productivity growth is still shaped by these classical channels. Baldwin and Robert-Nicoud (2008), introduce firm heterogeneity in an endogenous growth model of expanding product varieties (Romer, 1990) and find that the

[3]Further evidence from firm-level studies on the pro-competitive effects of trade can be found in Levinson (1993) for Turkey, Harrison (1994) for the Ivory Coast, Roberts (1997) for Colombia, Krishna and Mitra (1998) for India, Bugamelli, Fabiani, and Sette (2015) for Italy.

[4]See Neary (2003) and Eckel and Neary (2010) for recent applications, and Neary (2010) for a review of the literature on oligopoly and trade.

selection effect of trade on growth depends on the form of international knowledge spillovers. In a more general setup, with firm and worker heterogeneity, Grossman and Helpman (2014) show that under an arbitrary (positive) pattern of international knowledge spillovers, open economies innovate and grow more than closed ones. In old and new endogenous growth models, lower trade barriers tend to increase the size of the market, thereby increasing the incentive to innovate. This market size effect can be offset by an adverse competition effect: the successful innovators must share the market with foreign competitors. In Grossman and Helpman's baseline economy these two effects exactly offset each other. Finally, recent papers have explored the role of trade in accelerating knowledge diffusion. Sampson (2016) studies welfare gains in a trade model where productivity growth is driven by knowledge diffusion at the entry stage. Trade-induced selection accelerates knowledge diffusion and growth, thereby tripling the gains from trade relative to heterogeneous firms' economies with static productivity at the firm level. Along similar lines Perla, Tonetti, and Waugh (2015), set up a model where growth is driven by knowledge diffusion between incumbent firms, and show that trade accelerates technology diffusion and growth.

Our economy does not feature international knowledge spillovers, nor growth driven by technology diffusion. The fundamental channel linking trade and growth is that of product market competition. In a key departure from the literature discussed above, trade-induced increases in competition stimulate innovation leading to faster growth. Similarly to Grossman and Helpman (2014), the market size effect of trade is offset by the competition of foreign firms, but in our oligopolistic economy, fiercer foreign competition reduces markups thereby triggering selection, reallocation and faster productivity growth. Peretto (2003), is the first paper to analyse the effects of trade on growth in an endogenous growth model with variable markups. Bertrand competition among oligopolistic producers can generate, under some conditions, pro-competitive effects of trade triggering faster innovation and growth. In line with Peretto (2003), our economy features markups responding endogenously to trade costs, but in a key departure we introduce firm heterogeneity, which allows us to analyse the effects of trade on growth through firm selection.[5] We depart from Sampson (2016) by generating growth

[5]Besides considering homogeneous firms, Peretto's economy differs from ours in other dimensions. First, different countries produce imperfectly substitutable goods, and countries trade because of love for variety. Since there is no two-way trade in identical goods ("reciprocal dumping" in the words of Brander and Krugman, 1983), trade cannot be inherently wasteful. Secondly, the growth effects of trade hinges on the presence of 'international' knowledge spillovers. Licandro and Navas-Ruiz (2011), work out a version of Peretto (2003) with Cournot competition and show that the growth effect of trade can be obtained independently on international

through innovation instead of technology diffusion, and complement his results showing that dynamic gains from trade through innovation-driven growth can be substantial. This suggests that a unifying framework with growth sustained by innovation and knowledge diffusion could potentially generate very large gains from trade-induced selection.

A stream of papers has recently analyzed the welfare effects of selection in static models, or in models without long-run productivity growth. Arkolakis, Costinot, and Rodriguez-Clare (2012) show that welfare gains in a wide class of old and new trade models depend only on the change in the trade share and on the Armington elasticity of trade to changes in trade costs. Provided that a given change in trade costs generates the same change in trade shares across models, the gains from trade will be the same. Melitz and Redding (2015) show that Arkolakis et al. results hold only under some parameter restrictions, and primarily on assuming that firm productivity follows an unbounded Pareto distribution. Deviations from these restrictions, such as assuming a bounded Pareto distribution of productivity (Melitz and Redding, 2015), or a log-normal distribution (Head, Mayer, and Thoenig, 2014), allow heterogeneous firms models to generate substantial welfare gains from trade-induced selection. All these papers zero in on a class of models with either perfect or monopolistic competition. Focusing on oligopoly we stand outside this class of models and contribute to the debate by showing that with an oligopolistic market structure large gains from selection can be obtained even calibrating the model with a standard unbounded Pareto.

Finally, Atkeson and Burstein (2010) set up a dynamic model with constant markups, process and product innovation, with trade having only transitional effects on growth. They show that selection has positive welfare gains from trade through process innovation that are offset by negative effects through product innovation (entry). Our results complement their findings by showing that the presence of endogenous markups and endogenous growth leads to sizable gains from trade-induced selection.

Section 2 describes the simple model with an exogenous number of competitors and studies its autarkic equilibrium. Section 3 analyses the equilibrium with two symmetric countries incurring in iceberg trade costs, and characterizes the welfare gains of trade liberalization. The simple model is extended in Section 4, allowing for an endogenous number of oligopolistic firms in each product line through free entry and for selection into the export market. Section 5

knowledge spillovers.

presents the calibration of the generalized model and a numerical simulation of the effects of trade liberalization on innovation, growth and welfare. Section 6 contains a discussion of the role of endogenous markups in shaping the contribution of selection to the aggregate welfare gains from trade. Section 7 concludes.

2 Basic model: closed economy

This section presents a simple version of the model economy designed to illustrate the basic properties of the suggested theory. The general version of the model used to tackle the data is developed in Section 4. In both cases, this paper only explores the properties of the steady state equilibrium.

2.1 Economic environment

Preferences. The economy is populated by a continuum of identical consumers of measure one. Time is continuous and denoted by t, with initial time $t = 0$. Preferences of the representative consumer are described by

$$U = \int_0^\infty (\ln X_t + \beta \ln Y_t)\ \mathrm{e}^{-\rho t}\ \mathrm{d}t, \tag{1}$$

with $\beta > 0$ and discount factor $\rho > 0$. There are two types of goods, a homogeneous good Y and a differentiated good X. Consumers are endowed with a unit flow of labor, which among other uses can be transformed one-to-one into the homogeneous good. In this sense, Y can also be interpreted as leisure. The labor endowment, or equivalently the homogeneous good, is taken as the numeraire.

The differentiated good is composed of a continuum of varieties of endogenous mass M_t, $M_t \in [0, 1]$, according to

$$X_t = \left(\int_0^{M_t} x_{jt}^{\alpha}\ \mathrm{d}j \right)^{\frac{1}{\alpha}}, \tag{2}$$

where x_{jt} represents consumption of variety j, and $1/(1-\alpha)$ is the elasticity of substitution across varieties, $\alpha \in (0, 1)$.

Technology and market structure. Each variety $j \in [0, M_t]$ is produced by n identical firms, manufacturing perfectly substitutable goods. Firms use labor to cover both variable production costs and a fixed production cost λ, $\lambda > 0$. Productivity is assumed to differ across varieties, but firms producing the same variety are equally productive. Let us omit index j and identify varieties with their productivity, which we denote by $\tilde{z}_t$. A firm with productivity $\tilde{z}_t$ has the following production technology

$$l_t = \tilde{z}_t^{-\eta} q_t + \lambda, \tag{3}$$

where l represents labor and q production. Variable costs are assumed to be decreasing in the firm's state of technology, with $\eta > 0$. Irrespective of their productivity, varieties exogenously exit at rate $\delta > 0$; in which case, the variety becomes obsolete and all firms exit simultaneously.

We may think of this technological structure as a streamlined representation of a real economy in the following way: first, the set of firms is divided in small groups producing the closest possible goods in terms of their substitutability; substitutability has to be almost perfect. We call the goods they produce a 'variety' or product line. Second, we assume that the degree of substitutability across varieties is constant. Finally, to keep the model tractable, we assume homogeneity in productivity within varieties, but heterogeneity across varieties. This simplified technological structure does not capture only heterogeneity across the 'few' observable sectors in the data, but also the productivity difference across the 'many' firms producing imperfectly substitutable goods.

Incumbent firms can increase their productivity $\tilde{z}_t$ through the following process innovation technology

$$\dot{\tilde{z}}_t = A k_t h_t, \tag{4}$$

where h_t represents labor allocated to innovation and $A > 0$ is an efficiency parameter. The externality k_t is defined as

$$k_t = D_t \ \tilde{z}_t^c. \tag{5}$$

It encompasses an increasing difficulty component and a knowledge spillover component. The knowledge spillover component comes from the average productivity of direct competitors – those producing the same variety – which is denoted by $\tilde{z}^c$: the more productive direct competitors are the more effective a firm's current innovation is in enhancing productivity. This specification of innovation technology is commonly used in the endogenous growth literature to generate a constant growth rate in steady state.[6] Since we are assuming that all firms producing the same variety have the same productivity, in a symmetric equilibrium $\tilde{z}^c$ is equal to $\tilde{z}$.

[6] Aghion and Howitt (1992) and Grossman and Helpman (1991), for example, adopt a similar knowledge

The term D_t represents the degree of difficulty in innovation, under the assumption that innovating is harder for firms producing highly productive varieties. The degree of difficulty in innovation is measured as the distance between the average productivity of the overall economy

$$\tilde{Z}_t = \frac{1}{M_t} \int_0^{M_t} \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j,$$

$\hat{\eta} \equiv \eta\alpha/(1-\alpha)$, and the productivity of direct competitors, $D_t = \tilde{Z}_t/(\tilde{z}_t^c)^{\hat{\eta}}$. Since, in the symmetric Cournot equilibrium that we will characterize, $\tilde{z}^c = \tilde{z}$, the definition of D implies that more productive firms find it harder to innovate. As we show later, this assumption on the innovation technology makes productivity growth rates equal across varieties, yielding an equilibrium stationary distribution of productivity. A similar assumption is commonly used in R&D-driven growth models with homogeneous firms to eliminate counterfactual scale effects, and stationarize models with growing population (e.g. Jones, 1995, Kortum, 1997, and Segerstrom, 1998).[7] In recent models of endogenous growth with heterogeneous firms, such as Klette and Kortum (2004), increasing innovation difficulty is introduced to stationarize the distribution of productivity and to match a robust stylized fact in the data: more productive/larger firms invest more in R&D, but the growth rate of productivity does not scale with size.[8] As we show later, in our stationary equilibrium more productive firms innovate more but the steady state growth rate is the same for all firms.

Finally, let us define the entry technology. There is a mass of unit measure of potential varieties of which $M_t \in [0, 1]$ are operative, and each of them is produced by n firms. In this benchmark model a new variety can be introduced at zero cost. At any time t, there are n firms associated to any potential new variety waiting outside the economy, and they jointly draw a productivity from an initial productivity distribution. In the quantitative analysis we will remove the assumptions that new varieties are created at zero cost and that the number of oligopolistic firms per varieties n is constant.

spillovers structure to obtain sustained growth in Schumpeterian growth models where only entrants innovate. While Peretto (1996) uses this innovation technology to obtain positive long-run growth with innovation performed by incumbents.

[7] Empirical evidence supports decreasing returns to innovation. The evidence surveyed in Kortum (1993) suggests point estimates for the patent/R&D elasticity that range between 0.1 and 0.6. More recently, Blundell et al. (2002) find a long-run elasticity of 0.5.

[8] See Klette and Kortum (2004), fact 1, and Griliches (2000) for the supporting empirical evidence.

2.2 Equilibrium

Next, we characterize the steady-state equilibrium properties of the model.

Households. The representative household maximizes utility subject to its instantaneous budget constraint. The corresponding first order conditions are

$$Y = \beta E \tag{6}$$

$$\frac{\dot{E}}{E} = r - \rho \tag{7}$$

$$p_{jt} = \frac{E}{X_t^{\alpha}}\, x_{jt}^{\alpha-1}, \tag{8}$$

where r is the interest rate and p_{jt} is the price of good j. For variables that are constant in steady state, such as r, Y, E and M, index t is omitted to simplify notation. Total household expenditure on the composite good X is

$$E = \int_0^M p_{jt} x_{jt}\, \mathrm{d}j.$$

Because of log preferences, total spending in the homogeneous good is β times total spending in the differentiated good. Equation (7) is the standard Euler equation implying $r = \rho$ at the stationary equilibrium, and (8) is the inverse demand function for variety j.

Firms' problem: Cournot equilibrium. The n identical firms competing in the production of each variety j play a dynamic Cournot game. They behave non-cooperatively and maximize the expected present value of their net cash flow, denoted by V_{ijs} for firm i producing variety j at time s. This differential game is solved focusing on Nash Equilibrium in open loop strategies. Let $a_{ijt} = (q_{ijt}, h_{ijt})$, $t \geq s$, be a strategy for firm i producing j at time t. Let us denote by a_{ij} firm i's strategy path for quantities and innovation. At time s a vector of strategies $(a_{1j}, ..., a_{ij}, ..., a_{nj})$ is an equilibrium in market j if

$$V_{ijs}(a_{1j}, ..., a_{ij}, ..., a_{nj}) \geq V_{ijs}(a_{1j}, ..., a'_{ij}, ..., a_{nj}) \geq 0,$$

for all firms $\{1, 2, ..., n\}$, where in $(a_{1j}, ..., a'_{ij}, ..., a_{nj})$ only firm i deviates from the equilibrium path. The first inequality states that firm i maximizes its value taking the strategy paths of the others as givens, and the second requires firm i's value to be positive.[9]

[9]We choose the open loop equilibrium because it allows for closed form solution. The drawback of focusing on the open loop equilibrium is that it does not generally have the property of subgame perfection, as firms

The characterization of the open loop Nash equilibrium proceeds as follows: at time s a firm producing a particular variety solves (we suppress indexes i and j to simplify notation)

$$V_s = \max_{\{q_t, h_t, \tilde{z}_t\}_{t=s}^{\infty}} \int_s^{\infty} \Big((p_t - \tilde{z}_t^{-\eta}) q_t - h_t - \lambda\Big) \mathrm{e}^{-(\rho+\delta)(t-s)}\, \mathrm{d}t, \qquad s.t. \tag{9}$$

$$p_t = \frac{E}{X_t^{\alpha}}\, x_t^{\alpha-1}$$
$$x_t = \hat{x}_t + q_t$$
$$\dot{\tilde{z}}_t = A k_t h_t$$
$$\tilde{z}_s > 0,$$

where the firm cash flow is discounted with the steady-state interest rate ρ and the exogenous firm death rate δ. The first constraint is the indirect demand function for each variety; the second is the quantity constraint that splits the total size of the market for a variety x_t between this firm and its direct $(n-1)$ competitors in the product line, $\hat{x}_t$. The third constraint is the innovation technology. In a Cournot game a firm takes as given the path of its competitors' production $\hat{x}_t$, the path of the externality k_t, as well as the path of the aggregates E and X_t, and the exit shock δ. The current value Hamiltonian for this problem is:

$$\begin{aligned}\mathcal{H}_t &= \big[(p_t - \tilde{z}_t^{-\eta}) q_t - h_t - \lambda\big] + v_t A k_t h_t \\ &= \left[\left(\frac{E_t}{X_t^{\alpha}}\, (\hat{x}_t + q_t)^{\alpha-1} - \tilde{z}_t^{-\eta}\right) q_t - h_t - \lambda\right] + v_t A k_t h_t,\end{aligned}$$

where v_t is the costate variable. The first order conditions for the problem above, under symmetry, are

$$\frac{\partial \mathcal{H}_t}{\partial q} = 0: \quad \tilde{z}_t^{-\eta} = \theta \underbrace{\frac{E}{X_t^{\alpha}} x_t^{\alpha-1}}_{p_t} \tag{10}$$

$$\frac{\partial \mathcal{H}_t}{\partial h} = 0: \quad v_t A k_t = 1 \tag{11}$$

$$\frac{\partial \mathcal{H}_t}{\partial\, \tilde{z}} = \eta \tilde{z}^{-\eta-1} q_t = -\dot{v}_t + (\rho+\delta)\, v_t. \tag{12}$$

choose their optimal time-paths strategies at the initial time and stick to them forever. In closed loop equilibria, instead, firms do not pre-commit to any path and their strategies at any time depend on the whole past history. The Nash equilibrium in this case is strongly time-consistent and therefore sub-game perfect. Unfortunately, closed loop or feedback equilibria generally do not allow for a closed form solution and, often, they do not allow for a solution at all. The literature on differential games has uncovered classes of games in which the open loop equilibrium degenerates into a closed loop and therefore is subgame perfect (e.g. Reingaum, 1982, Fershtman, 1987, and Cellini and Lambertini, 2005). A sufficient condition for the open loop Nash equilibrium to be subgame perfect is that the state variables of other firms do not appear in the first order conditions for each firm.

From (10), firms charge a markup over marginal costs, with $\theta \equiv (n-1+\alpha)/n$ being the inverse of the markup. This is the well-known result in Cournot-type equilibria that the markup depends on the perceived demand elasticity, which is a function of both the demand elasticity and the number of competitors.[10]

Firms producing the same variety are assumed to face the same initial conditions, resulting in a symmetric equilibrium with $x_t = nq_t$. As shown in Appendix A, substituting (10) into (2), we obtain the demand for variable inputs

$$\tilde{z}_t^{-\eta} q_t = \theta e \, \frac{z}{\bar{z}}, \tag{13}$$

where $e \equiv E/nM$ is expenditure per firm. Variable $z = \tilde{z}_{jt}^{\hat{\eta}} \, e^{-\hat{\eta} g t}$ is the measure of firm detrended productivity, where g is the endogenous growth rate of average productivity which will be computed below, and $\bar{z} = (1/M) \int_0^M z_j dj$ is the average detrended productivity. Notice that the amount of resources used by a firm in (13) is the product of average expenditures per firm, the inverse of the markup and the relative productivity of the variety the firm produces. When the environment becomes more competitive, θ increases, prices decline, produced quantities increase and firms demand more inputs. Moreover, (13) shows that more productive firms produce more.

The optimal growth rate of productivity is

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A \theta e - \rho - \delta, \tag{14}$$

the same for all $\tilde{z}$. This is obtained using (5), (11), (12), (13) and the definition of D_t. Equilibrium innovation for firm z can be derived using (4), (5) and (14),

$$h = (\eta \theta e - \hat{\rho}) \, \frac{z}{\bar{z}}, \tag{15}$$

where $\hat{\rho} = (\rho + \delta)/A$. Labor resources allocated to innovation h are directly proportional to the firm's relative productivity $z/\bar{z}$. Equation (15) shows that more productive firms innovate more: since more productive firms are larger and innovation is cost-reducing, the benefits of a reduction in the unit production cost are increasing in the units of goods produced and sold. By the same token, a reduction in the markup, increases market efficiency and incumbent firms'

[10] It can be easily shown that condition (10) is the solution of the corresponding static Cournot game with given productivities. Consistently with Cellini and Lambertini (2005), the solution to the open loop equilibrium coincides with the closed loop when $\hat{\eta} = 1$, since in this case the externality k in the FOC (11) does not depend on the productivity of direct competitors.

market size, thereby stimulating innovation. This is consistent with the empirical evidence showing that more productive firms spend more on innovation without featuring higher growth rates of productivity (e.g. Lentz and Mortensen, 2008, and Akcigit and Kerr, 2011). The specific form of the externality k in (5) allows for the growth rate to be equal across firms in steady state, offsetting the positive effect that the relative productivity has on innovation. Intuitively, as firms become more productive they spend more in innovation and they grow more in the way to the steady state, but as further innovation becomes harder and harder for them and at the steady state they will end up growing at the common rate g. Since there is no innovation in the homogeneous good sector, (1) and (3) imply that the growth rate of output is

$$g_{gdp} = \frac{\eta}{1+\beta}\, g, \tag{16}$$

where $1/(1+\beta)$ represents the share of the composite good in total consumption expenditure.

In a stationary equilibrium, the productivity of all firms grow at the same rate. As a consequence, their demand for variable inputs, as described by (13), is constant along the balance growth path. More importantly, even when firms do R&D and their productivity endogenously grow, in a stationary equilibrium they all stay in their initial position in the productivity distribution, and the model remains highly tractable. Therefore the productivity distribution becomes a traveling wave, the lower bound increasing over time at the steady state growth rate.

Exit and entry. At any time, outside the market there is a mass of potential varieties $1-M$, each produced by n firms, drawing a productivity z from a time-invariant initial productivity distribution $\Gamma(z)$, which is assumed to be continuous in $(z_{\min}, \infty)$, with $0 \leq z_{\min} < \infty$. This productivity distribution is defined on detrended productivity z. Incumbent firms are involved in innovation activities making their productivity grow at the endogenous rate g. This makes the distribution of incumbent firms move permanently to the right at rate g. By defining the entry distribution as a function of detrended productivity z, we allow the productivity of entrants to grow on average at the same rate as that of incumbent firms.[11] This entry structure allows us to support a stationary productivity distribution, but it does not represent an engine

[11]Note that this entry structure allows us to support a stationary productivity distribution, but the entry process does represent an engine of growth. Growth is driven by firm-level innovation, in the absence of which (15) and (14) yield zero long-run growth.

of growth. Growth in our economy is driven by firm-level innovation, in the absence of which (15) and (14) yield zero long-run growth. In models of technology diffusion, such as Sampson (2014), instead, the entry process is the driver of long-run productivity growth.

Profits are a linear function of the relative productivity $z/\bar{z}$,

$$\pi(z/\bar{z}) = (1-\theta)\, e\ z/\bar{z} - \underbrace{(\eta\theta e - \hat{\rho})\, z/\bar{z}}_{h} - \lambda. \tag{17}$$

Notice that both net revenues from production and innovation costs depend on a firm's distance from average productivity $z/\bar{z}$. In the following, we assume η to be small enough such that $1-(1+\eta)\theta > 0$, a sufficient condition for the cash flow to depend positively on z. Let us denote by z^* the stationary cutoff productivity below which varieties exit. At a stationary state, the cutoff productivity makes firm's profits equal to zero, implying

$$e = \frac{\frac{\lambda}{z^*/\bar{z}} - \hat{\rho}}{1-(1+\eta)\theta}. \tag{EC}$$

We refer to it as the exit condition.[12] We can now write average productivity $\bar{z}$ as a function of z^*

$$\bar{z}(z^*) = \frac{1}{1-\Gamma(z^*)} \int_{z^*}^{\infty} z f(z)\ \mathrm{d}z. \tag{18}$$

Let us denote by $\mu(z)$ the stationary equilibrium density distribution defined on the z domain. The endogenous exit process related to the cutoff productivity z^* implies $\mu(z) = 0$ for all $z < z^*$. Since the equilibrium productivity growth rates are the same irrespective of z, and firm death shock δ is independent of firm productivity, in a stationary environment surviving firms remain always at their initial position in the distribution Γ. Consequently, the stationary equilibrium distribution is $\mu(z) = f(z)/(1-\Gamma(z^*))$, for $z \geq z^*$, where f is the density associated to the entry distribution Γ.

Since varieties exit at the rate δ, stationarity of the mass of product lines M requires

$$(1-M)(1-\Gamma(z^*)) = \delta M. \tag{19}$$

[12] Notice that problem (9) does not explicitly include positive cash flow as a restriction. By doing so and then imposing the exit condition (EC), we implicitly forbid firms with $z < z^*$ to innovate and potentially grow at some growth rate smaller than g. If they were allowed to do so, they will optimally invest in innovation up to the point in which the cash flow would be zero. In such a case, firms with initial productivity smaller than the cutoff value will be growing at a rate smaller than g, moving to the left of the distribution and eventually exiting. Such an extension would make the problem unnecessarily cumbersome without affecting the main results.

This condition states that the exit flow, δM, equals the entry flow defined by the number of entrants, $1 - M$, times the probability of surviving, $1 - \Gamma(z^*)$. Consequently, the mass of operative varieties is a function of the productivity cutoff z^*,

$$M(z^*) = \frac{1 - \Gamma(z^*)}{1 + \delta - \Gamma(z^*)}. \tag{20}$$

It is easy to see that M is decreasing in z^*, going from $1/(1+\delta)$ to zero.

Market clearing. The labor market clearing condition can be written as

$$n \int_0^M (l_j + h_j) \, \mathrm{d}j + Y = n \int_0^M \left(\tilde{z}_j^{-\eta} q_j + h_j + \lambda \right) \, \mathrm{d}j + \beta E = 1.$$

The labor endowment is allocated to production and innovation activities in the composite sector, as well as to production in the homogeneous sector. The first equality is obtained substituting l from (3) and Y from (6). Let us change the integration domain from varieties $j \in [0, M]$ to productivities $z \in [z^*, \infty)$ and use (4), (13) and (15) to rewrite the labor market clearing condition as

$$\int_{z^*}^{\infty} \left[(1+\eta) \, \theta e \, z/\bar{z} - \hat{\rho} z/\bar{z} + \lambda \right] \, \mu(z) \, \mathrm{d}z + \beta e = \frac{1}{nM}.$$

Since $\int_{z^*}^{\infty} \mu(z) \, \mathrm{d}z = \int_{z^*}^{\infty} z/\bar{z} \, \mu(z) \, dz = 1$, after integrating over all varieties we obtain

$$e = \frac{\frac{1}{nM(z^*)} + \hat{\rho} - \lambda}{\beta + (1+\eta)\theta}, \tag{MC}$$

a positive relationship between e and z^*.

The market clearing condition is characterized by labor allocation between the homogeneous and the composite good sectors, and between production and innovation. Note that θe is the amount of labor allocated by the average firm to cover variable production costs. While E is total consumer expenditure on the differentiated good, price distortions imply that θE units of labor are required as variable costs to produce it, where $E = nMe$. Recall that we have taken labor as the numeraire. Because of log preferences, when E is spent in the consumption good, βE units of labor are allocated to the homogeneous good. Since the marginal return on innovation depends on firm's production, $\eta(\theta E)$ is allocated to innovation (minus the user cost of innovation, as measured by $\hat{\rho}$). Finally, firms also assign labor to cover the fixed cost λ.

Equilibrium existence. Here we prove the equilibrium existence and uniqueness, and show some of its key properties.

Assumption 1. The entry distribution is such that

$$z^*/\bar{z}(z^*) \text{ is increasing in } z^*, \tag{a}$$

and the following parameter restrictions hold:

$$\bar{z}_e/z_{\min} > \hat{\rho} \tag{b}$$

$$(1+\eta)\theta > \Psi \tag{c}$$

where

$$\Psi = \frac{\frac{(1+\delta)}{n} + \hat{\rho}(1+\beta) - \lambda\left(1 + \beta\frac{\bar{z}_e}{z_{\min}}\right)}{\frac{(1+\delta)}{n} + \lambda\left(\frac{\bar{z}_e}{z_{\min}} - 1\right)},$$

and $\bar{z}_e$ is the average productivity at entry.

Assumption (a) makes the (EC) curve decreasing in z^*. As discussed in Melitz (2003), many common distributions satisfy condition (a).[13] Indeed, if the productivity distribution is Pareto, (EC) is horizontal.[14] As stated in Proposition 1 below, under assumptions (b) and (c) the (EC) curve cuts the (MC) curve from above, which is sufficient for existence and unicity of equilibrium. Figure 1 provides a graphical representation.

Proposition 1 *Under Assumption 1, there exits a unique interior solution* (e, z^*) *of (MC) and (EC), with* M *determined by (20).*

Proof. See Appendix B. ■

Proposition 2 below states the main effects of an exogenous increase in product market competition as measured by θ.[15]

Proposition 2 *An increase in* θ *raises the productivity cutoff* z^**, reduces the number of operative varieties* M*, has an ambiguous effect on the labor resources allocated to the homogeneous sector* e *and increases the growth rate* g*.*

[13] More precisely, condition (a) is satisfied by the Lognormal, Exponential, Gamma, Weibul, or truncations on $(0, +\infty)$ of the Normal, Logistic, Extreme value, or Laplace distributions. See Melitz (2003).

[14] Consistently with evidence on US firm size distribution (e.g. Axtell, 2001, and Luttmer, 2007), in the quantitative analysis we will assume that firms' size/productivity is distributed Pareto.

[15] Notice that a reduction in the markup rate $1/\theta$, as $\theta \equiv (n-1+\alpha)/n$, can potentially be produced by either an increase in the substitutability parameter α, or by an increase in the number of firms n.

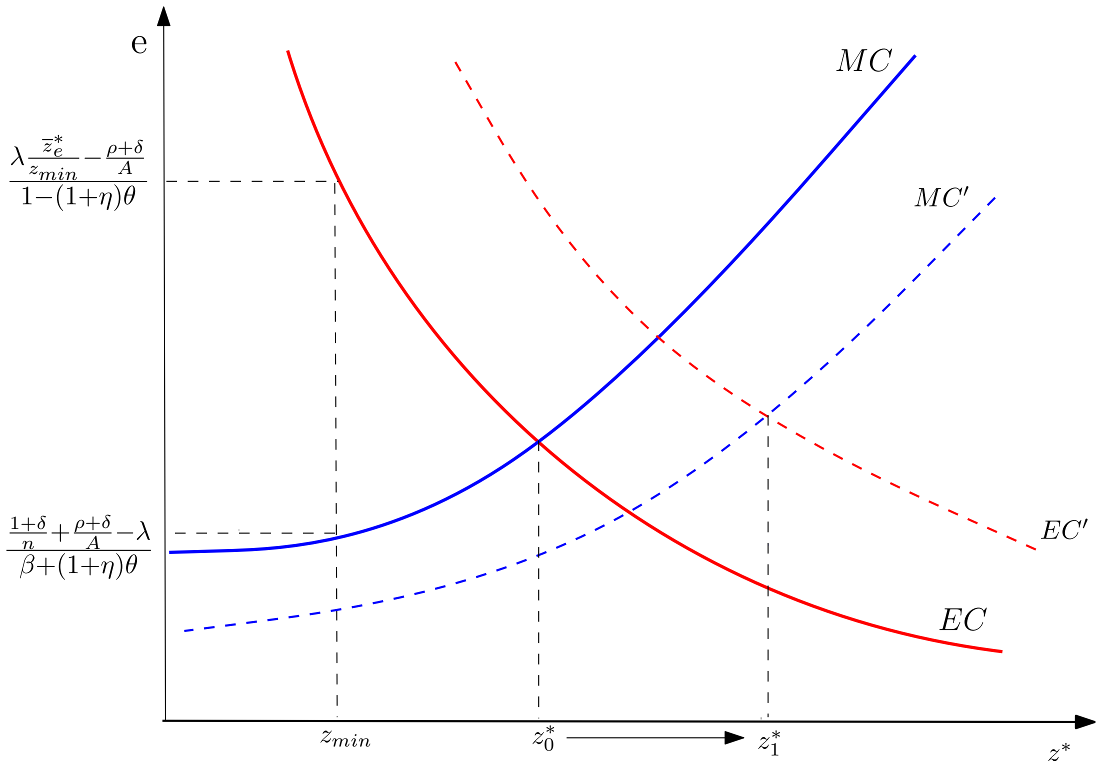

Figure 1: Closed economy equilibrium.

Proof. See Appendix B. ■

Two mechanisms contribute to the rise in growth, a *direct* growth effect and an *selection* effect. In a Cournot equilibrium, a reduction in the markup raises produced quantities; this can be easily seen from (13) which shows that the quantity produced is positively related to θ. The increase in quantities is feasible since the homogeneous good becomes relatively more expensive (i.e., the relative efficiency of the differentiated sector increases), and consumers' demand moves away from it towards the differentiated sector. Since the benefits of cost-reducing innovation are increasing in the quantity produced, but the innovation cost does not scale with the quantity sold, the higher static efficiency associated with lower markups affects positively innovation and growth. This mechanism does not depend on firm heterogeneity: it is easy to check that assuming away the dependence of M on z^*, setting $M = 1$, the equilibrium growth is still increasing in θ.[16] This direct effect of competition on growth can in fact be found in representative firm models of growth with endogenous market structure (see e.g. Peretto, 1996 and 2003, and Licandro and Navas-Ruiz, 2011). Notice that in this simple exercise the

[16]Shutting down firm heterogeneity the only equilibrium variable is expenditure e and is pinned down by the market clearing condition (MC). It is easy to see from (MC) that $|de/d\theta| < 1$, hence g is increasing in θ.

direct growth of competition effect depends on the presence of the homogeneous good.[17] As we show below, in open economy the increase in competition is generated by a reduction in trade costs and it leads to an increase in quantities produced by firms even in the absence of the homogeneous good. The increase in quantities is feasible because resources are freed by the reduction in trade costs.

The selection effect is specifically related to the heterogeneous firms structure of the model. A reduction in the markup raises the productivity threshold above which firms can profitably produce, the cutoff z^*, thus forcing the least productive firms to exit the market. Market shares are reallocated from exiting to surviving firms, thereby increasing their market size and their incentive to innovate. Therefore this selection mechanism leads to higher aggregate productivity level and higher innovation and productivity growth. This effect is independent on the presence of the homogenous good, since it mainly plays out through a reallocation of market shares between firms in the differentiated good sector.

3 Basic model: open economy

Consider a world economy populated by two symmetric countries with the same technologies, preferences, and endowments as described in the previous section. We assume that both countries produce exactly the same varieties and that trade costs are of the iceberg type: τ units of goods must be shipped abroad for each unit sold at destination, $\tau \geq 1$. They represent transportation costs and trade barriers created by policy. Cournot competition between firms in each product line, generates two-ways trade in identical commodities as in Brander and Krugman (1983). In each product line domestic and foreign firms compete strategically for market shares, so in open economy the number of firms per product is $2n$, twice as many firms as in autarky. To keep the model tractable, we assume that there are no fixed cost to enter the export market, and all operating firms sell both to the domestic and foreign markets.

[17]Since, the effect on g of a change in θ is determined by its effect on θe, without the homogeneous good ($\beta = 0$), the (MC) becomes

$$\theta e = \frac{\frac{1}{n} + \hat{\rho} - \lambda}{(1 + \eta)}$$

which is not affected by changes in θ.

3.1 Equilibrium

Since the two countries are perfectly symmetric, we can focus on one of them. Let q_t and $\tau\breve{q}_t$ be the quantities produced by a firm for the domestic and the foreign market, respectively, and let $q_{x,t} = q_t + \tau\breve{q}_t$ be total firm's output. Total consumption in the domestic market is $x_t = n\left(q_t + \breve{q}_t\right)$, with $x_t \leq nq_{x,t}$ the difference being equal to the trade cost. Because of the trade costs, firms face different marginal costs and have different sales in the domestic and foreign markets. Under Cournot competition countries export and import goods that are perfectly substitutable to domestic production, even in the presence of positive variable trade costs.[18] Firms solve a problem similar to the one in the closed economy. In Appendix C, we show that the equilibrium price in this open economy p_x is

$$p_x = \frac{\tilde{z}^{-\eta}}{\theta_d} = \frac{\tau\tilde{z}^{-\eta}}{\theta_f}, \tag{21}$$

where $\theta_d = \left(2n + \alpha - 1\right)/n\left(1 + \tau\right)$ and $\theta_f = \tau\theta_d$, are the inverse of the markups charged in the domestic and foreign markets, respectively.

Notice that a reduction in trade costs τ raises θ_d since the domestic market becomes more competitive, due to the penetration of foreign firms, but reduces θ_f, since a lower fraction of the goods shipped abroad is lost in transportation. Hence, our economy features pricing-to-market: as the trade cost declines, firm reduces their domestic markup and increase their export markup. This happens because a reduction in trade costs implies that exporters enjoy a cost reduction in their shipments to the foreign market while their direct local competitors in that market do not experience any cost change. Hence, exporters can optimally charge a higher markup and not pass the whole cost reduction due to lower trade costs onto foreign consumers.[19]

In fact, θ_f is increasing in τ becoming equal to one when $\tau = 1/\theta$, the closed economy markup defined in the previous section: $\bar{\tau} = 1/\theta = n/\left(n + \alpha - 1\right)$ operates as a *prohibitive iceberg cost*, since no firm exports if the variable trade cost is larger than it.[20] Moreover at

[18] This is a standard result in the literature of trade under oligopoly since the pioneering contribution of Brander and Krugman (1983). Intuitively, in imperfectly competitive markets firms equal marginal revenues (not prices) to marginal costs. In the presence of variable export costs, marginal costs of exporting are higher than those of selling domestically. Hence, setting marginal revenues equal to marginal costs leads exporters to sell a lower quantity in the foreign market, compared to domestic sales. This leads to "cross-hauling", i.e., intra-industry trade of highly similar goods.

[19] Pricing to market in Atkeson and Burnstein (2008) is generated through the same mechanism.

[20] Notice that when the trade cost is at the prohibitive level, the inverse of the exporter's markup in the foreign market θ_f becomes equal to one. For any $\tau > n/(n + \alpha - 1)$, θ_f is larger than one, which implies that the markup rate is negative. Firms will never export under such conditions.

$\bar{\tau}$, $\theta_d = \theta$ since domestic firms face no foreign competition in the domestic market. Then, θ_d increases with the reduction of trade costs until reaching its free trade value $(2n + \alpha - 1)/2n$, which has the same structure of the autarky markup but a double number of firms. When the trade cost decreases, foreign firms increase their participation in the domestic market, reducing the markup of domestic firms. The pro-competitive effect of trade operates through this mechanism.

The firm's optimal production choice leads to the following expression for variable production costs,

$$\tilde{z}_t^{-\eta} q_{x,t} = \theta_x e \, z/\bar{z}, \tag{22}$$

where $\tilde{z}$ and $\bar{z}$ are defined as in autarky, and

$$\theta_x = \mathcal{A}\theta_d \tag{23}$$

where

$$\mathcal{A} = \frac{(1 - n - \alpha)(1 + \tau^2) + 2n\tau}{(1 - \alpha)(1 + \tau)},$$

and θ_x is the inverse of the average markup charged by a firm across markets. In line with Brander and Krugman (1983), $\mathcal{A}$ measures the cost of importing goods that could be otherwise produced locally. In fact, it can be shown that

$$\mathcal{A} = \frac{q_t + \tau\breve{q}_t}{q_t + \breve{q}_t} \geq 1.$$

Notice that θ_x is decreasing in variable trade costs τ, with θ_x reaching its maximum value $(2n - 1 + \alpha)/2n$ when $\tau = 1$. The autarky value $\theta = (n - 1 + \alpha)/n$ is reached when $\tau = \bar{\tau}$. The average markups is increasing in τ; intuitively, an increase in τ increases the domestic markups $1/\theta_d$ and decreases the export markup $1/\theta_f$, but since $1/\theta_d > 1/\theta_f$, the former effect prevails and the average markup of exporting firms increases with trade costs. This is the source of the pro-competitive effect of trade that we will explore more in detail later.

The dynamic conditions of the firm problem yield the following equilibrium growth rate of productivity

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A\theta_x e - \rho - \delta, \tag{24}$$

which has the same functional form as in the closed economy, differing only in the structure of the markup θ_x. Consequently, opening to trade only affects the equilibrium growth rate through changes in the markup θ_x. The productivity cutoff is determined by the exit condition

$$\pi(z^*/\bar{z}) = (1-\theta_x)\, e\, z^*/\bar{z} - \underbrace{(\eta\theta_x e - \hat{\rho})\, z^*/\bar{z}}_{h} - \lambda = 0,$$

which yields

$$e = \frac{\frac{\lambda}{z^*/\bar{z}(z^*)} - \hat{\rho}}{1-(1+\eta)\,\theta_x}. \tag{EC^T}$$

Since firms fully compensate losses in local market shares by increased shares in the foreign market, profits are only affected by the change in the markup. Consequently, the exit condition has the same functional form as in (EC) except for θ_x.

The market clearing condition, proceeding as in the closed economy, becomes

$$e = \frac{\frac{1}{nM(z^*)} + \hat{\rho} - \lambda}{\beta + (1+\eta)\,\theta_x}, \tag{MC^T}$$

which is equal in all aspects to (MC) except for the markup, with θ_x instead of θ. Equations (EC^T) and (MC^T) yield the equilibrium (e, z^*) in open economy, with $M(z^*)$ determined by (20). The equilibrium growth rate is characterized by (24).

Proposition 3 *Under Assumption 1 and for $\tau \in [1, \bar{\tau}]$, there exists a unique interior solution (e, z^*) of (MC^T)-(EC^T).*

At the prohibitive trade cost $\bar{\tau}$, markups under trade and autarky are equal, $\theta_x = \theta$. Thus, for $\tau \geq \bar{\tau}$ firms do not have incentives to export, and trade does not take place. For $\tau < \bar{\tau}$ the proof of existence and unicity is similar to that in the closed economy, and we omit it for brevity.

3.2 Trade liberalization

A reduction in trade barriers makes markets more competitive by reducing the markup $1/\theta_x$. Hence the selection effect of trade liberalization is similar to that triggered by an exogenous decline in the markup in closed economy shown in Proposition 2. The economy with costly trade is characterized by a level of product market competition higher than in autarky, with $\theta_x > \theta$, due to the participation of foreign firms in the domestic market. A larger number of firms in the domestic market raises product market competition, thus lowering the markup rate. From the definition of θ and θ_x we obtain

$$\theta_x - \theta = \frac{\tau\,(1-\alpha)^2 - n\,(\tau-1)^2\,(n+\alpha-1)}{n\,(1+\tau)^2\,(1-\alpha)}, \tag{25}$$

which is positive for any non-prohibitive level of trade costs ($\tau \leq \bar{\tau}$). Differentiating the expression above, it is easy to see that the distance between θ_x and θ is decreasing in τ, since θ_x is decreasing in τ (see Appendix E). Hence we have two results: first, when a country goes from autarky to costly trade, it experiences an increase in product market competition. Secondly, incremental trade liberalization increases product market competition as well. When trade is completely free, $\tau = 1$, product market competition reaches its maximum level, $\theta_{\max} \equiv (2n - 1 + \alpha)/2n$. Notice that $\theta_{\max}$ has the same functional form as the inverse of the markup in autarky, θ, but with twice the number of firms. Once established that trade reduces markups, it is easy to see that trade liberalization has effects on selection and innovation similar to those produced by an exogenous change in the markup in the closed economy.[21] Hence, we can conclude that trade liberalization makes markets more competitive by lowering markups $1/\theta_x$, thereby triggering a selection and reallocation process which leads to higher aggregate productivity level and growth rate.

Next we decompose the growth effect and the welfare effects of trade into their different channels. Following a procedure similar to that in Melitz and Redding (2015), we compare the effects of trade on growth and welfare in our economy with those obtainable in a counterfactual economy where the selection margin of trade liberalization is not operative. More precisely, this economy has the same initial equilibrium of our benchmark economy, but changes in trade costs do not affect the cutoffs z^* and z_x^*. This allows us to isolate the direct gains of trade, and compute the contribution of selection comparing these gains with those obtained in the benchmark model.

Growth effect decomposition. Trade openness only affects equilibrium through competition, represented here by the inverse of the markup θ_x. Before doing the decomposition, let us introduce the following notation representing the right-hand-side of (MC^T)

$$mc(z^*, \theta_x) = \frac{\frac{1}{nM(z^*)} + \hat{\rho} - \lambda}{\beta + (1+\eta)\theta_x}.$$

[21] The effects are similar but not exactly identical. The increase in competition generated by moving from autarky to costly trade is less that the one produced by doubling the number of firms because of the presence of trade costs. Trade costs imply that foreign firms cannot compete as aggressively in the domestic market as domestic firms. Moreover, notice that moving from autarky to trade does not change firms' market shares, since the number of competitors doubles with the size of the market. Conversely, in the closed economy, an increase in the number of firms reduces firms market shares.

It can be easily shown that the signs of the partial derivatives of function $mc(.)$ are $mc_1 > 0$ and $mc_2 < 0$. Recall that equilibrium e and z^* are simultaneously determined by the exit and labor market clearing conditions (EC^T) and (MC^T). At equilibrium, expenditure per-firm e and the cutoff productivity z^* depend on variable trade costs only through θ_x. The equilibrium effects of a change in θ_x on e and z^* are denoted by $de/d\theta_x$ and $dz^*/d\theta_x$, respectively. The equilibrium growth rate is given by (24) above. The total derivative of g w.r.t. to θ_x is then

$$\frac{dg}{d\theta_x} = \eta A \left(e + \theta_x \frac{de}{d\theta_x} \right) = \underbrace{\eta A \big(e + \theta_x \, mc_2 \big)}_{\texttt{direct effect}} + \underbrace{\eta A \theta_x \, mc_1 \, \frac{dz^*}{d\theta_x}}_{\texttt{selection effect}}.$$

The direct effect is defined as the change of g that follows a change in θ_x, under the conditions that z^* remains unchanged and consequently (EC) is not binding. In other words, it only takes into account that e changes because the (MC) condition moves. Using the derivative of $mc(z^*, \theta_x)$ w.r.t. θ_x, it is easy to show that

$$\texttt{Direct effect} = \frac{\beta e}{\beta + (1+\eta)\theta_x} > 0.$$

The selection effect, indeed, is the change of g that is not explained by the direct effect, i.e., the change induced by an increase in the cutoff productivity z^* only. The selection effect is positive since as shown above $mc_1 > 0$, and from Proposition 2, $dz^*/d\theta_x > 0$.

Welfare gains decomposition. We now move to the analysis of the welfare gains from trade. As pointed out by Atkeson and Burstein (2010), as long as love-for-variety matters, the positive welfare effect of improved selection may be offset by the reduction in the mass of varieties triggered by the same process. Since in our model the mass of potential entrants $(1 - M)$ is bounded above by one, selection always leads to a reduction in the equilibrium mass of varieties, as can be seen in (20). Hence, selection has a starker negative welfare effect through the loss of varieties, compared to the standard Melitz model and its dynamic versions in Atkeson and Burstein. In the Melitz model in fact, the mass of entrants can respond endogenously to changes in trade costs, potentially taming or even offsetting the loss of varieties due to selection.

In our model, aggregate steady state welfare can be written as

$$\rho U = \underbrace{\frac{1-\alpha}{\alpha} \ln \left(\bar{z} M \right)}_{\texttt{Productivity}} + \underbrace{\ln(\theta_d E)}_{\texttt{Labor}} + \underbrace{\beta \ln(\beta E)}_{\texttt{Homogeneous good}} + \underbrace{\frac{\eta g}{\rho}}_{\texttt{Growth}}. \tag{26}$$

We have shown above that $\theta_d < \theta_x$, implying that aggregate consumption $\theta_d E$ is smaller than total firms' production $\theta_x E$, because some resources are wasted in trade costs. Steady-state welfare in (26) has four terms. The first three correspond to static gains: the first two are associated to consumption of the differentiated good and the third to homogeneous good consumption. The last term corresponds to dynamic gains. Since, as shown above, both the direct and selection effects of trade on growth are positive, a reduction in trade costs generates positive dynamic welfare gains from both channels.

Are static gains from trade positive too? The direct static effect operates through $\theta_d\, e$, with e depending on θ_x.[22] From the analyses above concerning the direct growth effect of trade liberalization, we know that $\theta_x\, e$ increases with θ_x. Indeed, $\theta_d = \theta_x/\mathcal{A}$ and $\mathcal{A}$ is hump-shaped, being equal to one in the extreme cases of free trade, $\tau = 1$, and prohibitive trade costs, $\tau = n/(n + \alpha - 1)$, and for $\tau \in (1, n/(n + \alpha - 1))$ it is larger than one.[23] Consequently, even if $\theta_x\, e$ is increasing with θ_x, it may be that trade liberalization reduces $\theta_d\, e$ at large values of τ. Brander and Krugman (1983) show that the welfare gains from trade in an economy with oligopoly trade without free entry are ambiguous. Intuitively, trade has positive effect on welfare through reduction in markups which increases consumer surplus. Trade liberalization has two opposite effects on producer surplus: on the one hand, foreign competition reduces profits on domestic sales and, on the other hand, it increases profits on export sales. When trade costs are high, there is little export and the negative effect of trade liberalization on domestic profits dominates leading to lower producer surplus and potentially negative overall effects on welfare. Viceversa, when trade costs are low, trade liberalization increases producer surplus yielding positive welfare gains. Hence, the welfare effects of trade liberalization have an inverted U-shape relationship with trade costs. Introducing free entry, the negative effect of trade on producer surplus disappears, and trade always increases welfare through the pro-competitive effect. Similarly, in our model $\mathcal{A}$ is hump-shaped, implying that the direct gains $\theta_d e = \theta_x e/\mathcal{A}$ can have an inverted U-shape relationship with trade costs. In the next section we introduce free entry and show that, in line with Brander and Krugman, the direct welfare

[22] The direct effect on total expenditure E and total expenditure per firm e, are the same when z^* is constant, since the mass of varieties M is constant.

[23] To show that, notice that the sign of the partial derivative $\partial\mathcal{A}/\partial\tau$ is equal to the sign of

$$(1 - n - \alpha)(1 + \tau)^2 + 2(2n + \alpha - 1),$$

which has a zero at $1 + \tau = \sqrt{2(2n + \alpha - 1)/\,(n + \alpha - 1)}$, for τ in the interval $(1, n/(n + \alpha - 1))$. $\mathcal{A}$ is increasing before that maximum and decreasing after.

effects of trade are only pinned down by the pro-competitive effects on consumer surplus, which are always positive.

The selection static welfare gains operate through $\bar{z}$, M and e, which also depends on z^*. They can be decomposed into two sources, as can be seen by differentiating (26), after substituting $E = nMe$, with respect to z^*:

$$\texttt{Static selection gains} = \underbrace{\frac{1-\alpha}{\alpha}\left(\frac{1}{\bar{z}}\frac{\partial \bar{z}}{\partial z^*} + \frac{1}{M}\frac{\partial M}{\partial z^*}\right)}_{\texttt{Productivity/LFV}} + \underbrace{(1+\beta)\left(\frac{1}{e}\frac{\partial e}{\partial M} + \frac{1}{M}\right)\frac{\partial M}{\partial z^*}}_{\texttt{Fixed cost}} e, \quad (27)$$

where $\partial e/\partial M$ is obtained by differentiating (MC^T), $\partial M/\partial z^*$ by differentiating (19) and $\partial \bar{z}/\partial z^*$ by differentiating the definition of $\bar{z}$. Starting with the first component, we see a static effect bringing welfare gains from selection-induced increases in the average productivity level of the economy. This component also includes welfare losses due to love for variety (LFV); they are brought about by selection-induced reductions in the mass of available varieties M. The second static component represents the change of labor allocated to the production of the differentiated good (excluding the fixed cost). Selection forces some firms out of the market, thereby reducing the amount of resources needed to cover fixed production costs. These resources are allocated to the remaining more productive firms. In other words, the reduction in the mass of varieties following selection reduces labor allocated to cover total fixed operating costs, thus freeing resources for production. The following proposition states conditions under which the static selection effects of trade liberalization are always positive.

Proposition 4 *Selection produces static welfare gains through the fixed cost channel. Under Pareto, for sufficiently large values of the exogenous death rate δ, the productivity/LFV trade-off channel results in positive welfare gains as well.*

Proof. See appendix E. ■

In the next section, we bring the model to the data in an attempt to quantify the static and dynamic gains from selection. Before performing our quantitative analysis we generalize the model along two relevant dimensions: first, we allow vertical entry, that is we assume that in order to enter the market firms pay a fixed cost $\phi > 0$. This implies that the number of firms per product n is endogenously determined by a free entry condition. Second, we remove the assumption that all firms export.

4 General model

We remove the simplifying assumption that all firms export by introducing a fixed export cost λ_x. We also remove the assumption that the number of firms per variety is exogenously given by adding a free entry condition. Firms' entry works as follows: firms pay a sunk entry cost, giving them the right to produce a particular variety, before the productivity of this variety is reveled. The constitution of a new variety implies that n firms enter altogether and if they are not productive enough to cover the fixed production costs, they will all exit. Consequently, the number of firms per variety n becomes endogenous, but it is the same for all varieties as in the model of Sections 2 and 3, since firms' productivity is revealed only after entry. Markets for non-exporters behave as in the simple autarky model of Section 2, and markets for exporters behave as under the costly trade economy discussed in Section 3. The only difference between these markets is in the markup, non-exporting firms will charge the closed economy markup $1/\theta$, while exporters will charge $1/\theta_d$ on domestic sales, $1/\theta_f$ on export, and earn an average markups on their total sales of $1/\theta_x$. All these markup expressions are the same as in the basic model presented above. With these differences in mind, we proceed as in Section 3 in the characterization of the equilibrium. Below we only show the key equations, the procedure to derive them and the rest of the equilibrium can be found in the appendix.

Non-exporters and exporters demand for variable inputs are structurally similar to (13) and (22),

$$\tilde{z}_t^{-\eta} q_t = \theta e \left(\frac{\bar{p}}{p(z)}\right)^{\frac{\alpha}{1-\alpha}} = \theta^{\frac{1}{1-\alpha}} e \, \frac{z}{\bar{p}^{\frac{\alpha}{\alpha-1}}} \tag{28}$$

$$\tilde{z}_t^{-\eta} q_{x,t} = \theta_x e \left(\frac{\bar{p}}{p_x(z)}\right)^{\frac{\alpha}{1-\alpha}} = \theta_x^{\frac{1}{1-\alpha}} e \, \frac{z}{\bar{p}^{\frac{\alpha}{\alpha-1}}}, \tag{29}$$

where $q_{x,t}$ is now the production of exporting firms including domestic and foreign sales. Detrended prizes derive from (10) and the definition of stationary productivity $z \equiv \tilde{z}_t^{\hat{\eta}} \ e^{-\hat{\eta}gt}$, yielding $p(z)^{\frac{\alpha}{\alpha-1}} = \theta^{\frac{\alpha}{1-\alpha}} z$ and $p_x(z)^{\frac{\alpha}{\alpha-1}} = \theta_x^{\frac{\alpha}{1-\alpha}} z$. The average detrended price is

$$\bar{p}^{\frac{\alpha}{\alpha-1}} = \left(\theta^{\frac{\alpha}{1-\alpha}} \int_{z^*}^{z_x^*} z\mu(z)\mathrm{d}z + \theta_x{}^{\frac{\alpha}{1-\alpha}} \int_{z_x^*}^{\infty} z\mu(z)\mathrm{d}z\right).$$

In the particular case where $\theta = \theta_x$, the ratios $(\bar{p}/p_x(z))^{\frac{\alpha}{1-\alpha}}$ and $(\bar{p}/p(z))^{\frac{\alpha}{1-\alpha}}$ become both equal to $z/\bar{z}$ as in (13) and (22) in the basic formulation of the previous sections. As in the simple model, more productive firms are larger, that is variable labor demand is positively related to productivity. Differently from the simple model where all firms exports, here the non-exporting firms do not obtain the size premium related to export status.

In order to keep the model stationary, we assume that the degree of innovation difficulty in the externality (5) follows

$$D_t = \frac{\theta_x e}{(\tilde{z}_t^c)^{-\eta}\, y_t^c} = \left\{ \begin{array}{ll} \dfrac{\theta_x}{\theta^{\frac{1}{1-\alpha}}} \dfrac{\bar{p}^{\frac{\alpha}{\alpha-1}}}{z} & \text{if } z^* \leq z \leq z_x^* \\ \dfrac{1}{\theta_x^{\frac{\alpha}{1-\alpha}}} \dfrac{\bar{p}^{\frac{\alpha}{\alpha-1}}}{z} & \text{if } z^* > z_x^* \end{array} \right\} \tag{30}$$

where production y is q for non exporters and q_x for exporters, and the second equality is obtained using (28) and (29). Both productivity $\tilde{z}$ and production y refer to direct competitors. This assumption is equivalent to the corresponding assumption in Section 2. By the definitions of average and detrended productivity, in the simple model the degree of difficulty D, defined as $\tilde{Z}/(\tilde{z}^c)^{\hat{\eta}}$, becomes equal to $\bar{z}/z^c$, where z^c is the detrended productivity of direct competitors. Using $\tilde{z}_t^{-\eta} q_t = \theta e\, z/\bar{z}$ from (13), we can obtain $D_t = \theta e/(\tilde{z}_t^c)^{-\eta} q_t^c$, which has the same structure as in (30) except for a different scale factor, θ instead of θ_x, and does not account for quantity differences between exporters and non-exporters. It is possible to show that when all firms export, $\theta = \theta_x$ and $D_t = \bar{z}/z$, as in the simple model.

As it was the case for the simple model of Section 2, the difficulty index in (30) makes innovation harder for more productive/larger firms and, as we show below, allows us to obtain symmetric growth across varieties and a stationary distribution of productivity in steady state. Since in our model more productive firms are also larger we can interpret D both in terms of productivities (as we did in Section 2) or in terms of the firm size, as it is more appropriate for (30). Hence, we can think of D as measuring the distance between the labor size of the average firm, θe, and the labor resources employed by firms producing the variety $\tilde{z}$, $\tilde{z}_t^{c-\eta}\, y_t^c$.[24] Larger (more productive) firms face higher innovation difficulty than the average firm.

The firm problem for non-exporters is similar to that in the closed economy is Section 2.1, and the equilibrium for exporters is derived similarly to the open economy of section 3.1. Using the new definition of the externality D_t, it can be easily shown that the growth rate of productivity is the same for exporters and non-exporters and reads

$$g \equiv \frac{\dot{\tilde{z}}}{\tilde{z}} = \eta A \theta_x e - \rho - \delta. \tag{31}$$

[24]Recall that in symmetric Cournot equilibrium $\tilde{z}^c = \tilde{z}$, hence all firms producing the same variety have the same productivity.

The steady state innovation for exporters is

$$h_x(z) = \left(\frac{\bar{p}}{p_x(z)}\right)^{\frac{\alpha}{1-\alpha}} (\eta\theta_x e - \hat{\rho}) \tag{32}$$

where $\hat{\rho} = (\rho + \delta)/A$, and

$$h(z) = \left(\frac{\bar{p}}{p(z)}\right)^{\frac{\alpha}{1-\alpha}} (\eta\theta_x e - \hat{\rho}) \frac{\theta}{\theta_x} \tag{33}$$

for non-exporters. Using the definition of $p_x(z)$ and $p(z)$ we can immediately see that, as in the simple model, more productive firms innovate more. Moreover, since $\theta < \theta_x$, $p_x(z) < p(z)$ which implies that exporting firms invest in innovation more than non-exporters. Intuitively, due to different markups, exporters operate in more competitive markets allowing them to obtain larger equilibrium size, which in turn, leads to more innovation.

The productivity cutoff for non-exporters, z^*, is obtained setting $\pi(z^*) = 0$, which leads to the following cutoff condition,

$$e = \frac{\lambda\left(\frac{p(z^*)}{\bar{p}}\right)^{\frac{\alpha}{1-\alpha}} - \hat{\rho}\frac{\theta}{\theta_x}}{1-(1+\eta)\theta}. \tag{EC'}$$

Since innovation affects simultaneously the productivity of both domestic production and exports, even if production costs are linear in output, domestic and export profits are not separable. The cutoff condition for exporting firms is determined by $\pi_x(z_x^*) = \hat{\pi}_x(z_x^*)$, where $\pi_x(z)$ is the total profit of a firms operating in an traded product line z who sells to both markets and $\hat{\pi}_x(z)$ is the profit of a firm in the same line z that decides to deviate by selling only domestically. The export condition implies that no firm should have incentives to deviate by not exporting and saving the fixed export costs at equilibrium prices. Hence the marginal exporter is the firm that is perfectly indifferent between the exporting and selling domestically. As shown in the appendix this condition yields,

$$e = \frac{\lambda_x\left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{1-\alpha}}}{(1-(1+\eta)\theta_x) - (1-(1+\eta)\theta_d)\frac{\tau - \frac{\theta_x}{\theta_d}}{\tau-1}}. \tag{XC}$$

Notice though, that since the average markup of exporters $1/\theta_x$ is lower than that of non exporters, it is possible that for some parameters combinations exporters total profits are zero or negative for productivity levels above z^*. Hence, (XC) pins down the export cutoff provided that total profits of the marginal exporter are non-negative. In our numerical solution that follows, we use condition (XC) to pin down the export cutoff and check that the marginal exporter makes always non-negative profits.

Firms entering the economy pay a fixed entry cost $\phi > 0$ before they observe the productivity of the variety they will produce. In this entry process, n firms draw a technology z for producing a variety and they enter altogether. Free entry implies that the expected value of the firm must be equal to the entry cost, $(1-\Gamma(z^*))\,\bar{\pi}/(\rho+\delta) = \phi$, where expected profits are given by

$$\bar{\pi} = \int_{z^*}^{z_x^*} \Big((p(\tilde{z}) - \tilde{z}^{-\eta})q - h\,(z) - \lambda\Big)\mu(z)dz + \int_{z_x^*}^{\infty} \Big((p(\tilde{z}) - \tilde{z}^{-\eta})q_x - h_x\,(z) - \lambda - \lambda_x\Big)\mu(z)dz\,.$$

Using (28) and (29) the free entry condition can be written as

$$\big(1-(1+\eta)\,\bar{\theta}\big)\,e + \hat{\rho}\frac{\bar{\theta}}{\theta_x} - \lambda - \frac{1-\Gamma(z_x^*)}{1-\Gamma(z^*)}\lambda_x = \frac{\rho+\delta}{1-\Gamma(z^*)}\phi, \tag{FE}$$

where

$$\bar{\theta} = \theta\int_{z^*}^{z_x^*}\left(\frac{\bar{p}}{p(z)}\right)^{\frac{\alpha}{1-\alpha}}\mu(z)\mathrm{d}z + \theta_x\int_{z_x^*}^{\infty}\left(\frac{\bar{p}}{p_x(z)}\right)^{\frac{\alpha}{1-\alpha}}\mu(z)\mathrm{d}z.$$

As in the simple model of Section 2, firms/products exit the market at the exogenous destruction flow δ, and the stationarity condition for the mass of firms M is the same as in (20). Finally, the labor market clearing condition

$$\int_{z^*}^{z_x^*}\left(\tilde{z}^{-\eta}q + h(z) + \lambda\right)\mu(z)\mathrm{d}z + \int_{z_x^*}^{\infty}\left(\tilde{z}^{-\eta}q_x + h_x(z) + \lambda + \lambda_x\right)\mu(z)\mathrm{d}z + \beta e + \frac{1-M(z^*)}{M(z^*)}\phi = \frac{1}{nM(z^*)}$$

equals the labor resources used by domestic and exporting firms plus those devoted to entry, $(1-M(z^*))\phi$, to the labor endowment of the economy. From (19) and the definition of $\bar{\theta}$ above, the market clearing condition can be written as

$$\big(\beta + (1+\eta)\,\bar{\theta}\big)\,e + \left(\lambda + \frac{1-\Gamma(z_x^*)}{1-\Gamma(z^*)}\lambda_x + \frac{\delta}{1-\Gamma(z^*)}\phi\right) - \hat{\rho}\frac{\bar{\theta}}{\theta_x} = \frac{1+\delta/(1-\Gamma(z^*))}{n}. \tag{MC'}$$

A stationary equilibrium for this economy is a vector $\{z^*,\ z_x^*,\ e,\ n\}$ solving the system (EC')-(XC)-(FE)-(MC'), with $M(z^*)$ determined by (20).

5 Quantitative analysis

The purpose of this section is twofold. First, we explore numerically the equilibrium properties of the generalized model. Secondly, we compute the growth and welfare gains from trade, decomposing them in their static and dynamic part, and highlighting the role of firm heterogeneity in shaping these gains.

5.1 Calibration

We target the US economy, for which micro data are widely available. Consistent with the evidence on firm size and productivity distribution, we assume that the entry distribution is Pareto with shape parameter κ, and scale parameter $z_{\min}$ (see e.g. Axtell, 2001, and Luttmer, 2007).[25] We have to calibrate 11 parameters $(\alpha, \tau, \delta, \rho, \beta, A, \lambda, \lambda_x, \phi, \kappa, z_{\min})$. The discount factor ρ is equal to the interest rate in steady state, following the business cycle literature we set $\rho = 0.02$ which corresponds to an annual discount factor of about 98%. Using Census 2004 data, we set $\delta = 0.09$ to match the average enterprise annual death rate in manufacturing observed in period 1998-2004.[26] Rauch (1999) classifies goods into homogeneous and differentiated, and finds that differentiated goods represent between 64.6 and 67.1 percent of total US manufactures, depending on the chosen aggregation scheme. We set $\beta = 0.5$ to get the share of differentiated goods $1/(1+\beta)$ equal to 2/3. We normalize the minimum value of the productivity distribution $z_{\min}$ to 1, without loss of generality.[27]

Parameters $(\alpha, \tau, A, \lambda, \lambda_x, \phi, \kappa)$ are jointly calibrated in order to match seven steady state moments predicted by the model to the corresponding firm-level and aggregate statistics. We target a productivity growth rate of 1.2% as reported by Corrado, Hulten and Sichel (2009),[28] and a profit share of GDP of 11% as found in US data by Akcigit and Kerr (2015). Bernard, Redding, and Schott (2007) using the 2002 Census data for US manufacturing firms report the following statistics: first, exporters account for 18 percent of manufacturing firms; second, exporting and importing firms have a size (log employment) premium of 75%. We target these

[25] In our economy very finely defined product lines have standard production technologies to which a few active firms have access. An example of a product line could be smart phones. In this line a few top-end powerful firms share the global market and operate with similar productivities. To get a sense of the empirical mapping, the US NAICS industry classification finest sectoral disaggregation is at the 6-digit level. Our smart phone example belongs to sector 334220, "Radio and Television Broadcasting and Wireless Communications Equipment Manufacturing", the sector includes a large range of products from Airborne radios to cellular phones (smart phones are not specified) to televisions (about 30 different and quite broadly defined types of products). A product line in our model cannot be NAICS 334220, since we have a small number of firms (two or three in the calibration) competing tightly in the production of highly substitutable goods: Iphone 6 competes with Galaxy 5s, but not with Sony Home TV X. Hence, if we think about our product lines as sectors, there would not be a clear empirical counterpart for them, not even at the 6-digit level.

[26] For each year the death rates are computed as follows: taking year 2000 as an example, the death rate is the ratio of firms dead between March 2000 and March 2001 to the total number of firms in March 2000. Data can be downloaded at http://www.sba.gov/advo/research/data.html#ne, file data_uspdf.xls.

[27] The role of parameter η is to guarantee that $1-(1+\eta)\theta > 0$, so that the profit function is always increasing in productivity. We set it to an arbitrary small value such that this restriction is always satisfied.

[28] Since the model does not include tangible capital, investment in tangible capital has to be subtracted from total income in the data to compute labor productivity. After this adjustment, Corrado, Hulten and Sichel (2009) report an average growth of labor productivity of about 1.2% a year in the period 1973-2003.

Table 1: Calibration and model fit

Parameters taken from external sources				
Parameters	Value	Interpretation	Source	
ρ	0.02	Interest rate	standard	
δ	0.09	Firms death rate	Census, 2004	
β	0.5	Share of differentiated goods	Rauch, 1999	
Calibrated parameters and model fit				
Parameters	Value	Moment	Data	Model
Varieties substitutability, α	0.32	Productivity growth %	1.2	1.2
Pareto shape , κ	1.14	Markup avg. %	20	20
Innovation technology shifter, A	50	Share of exporters	18	18
Fixed cost, domestic, λ	0.01	Import penetration ratio	10	9.7
Fized cost, export, λ_x	0.0022	Exporters log employment premium %	75	70
Variable trade costs, τ	1.08	Std. firm productivity %	75	80
Sunk entry cost, ϕ	0.1	Profits/GDP %	11	13

trade statistics, together with an import penetration ratio (import share of output) of about 10 percent, obtained from the World Bank Development Indicators (2007), and a standard deviation of the log of firm productivity of 75% reported in Bernard, Eaton, Jensen and Kortum (2003). Finally, the average markup is set to 20 percent, an intermediate value in the range of estimates reported in Basu (1996) and close to the median markup found in recent work by De Loecker and Warzynski (2012). Table 1 shows that the calibrated parameters deliver a fairly good model fit.

5.2 Trade liberalization

We use the calibrated economy to simulate the steady state equilibrium response to changes in trade costs τ. More precisely, we analyze the response of product market competition, selection, innovation, growth and welfare when the iceberg trade cost moves from one, the theoretical lower bound representing the absence of iceberg trade costs, to the prohibitive trade cost, $\bar{\tau} \equiv n/(n+\alpha-1)$. Since we are doing steady state comparison, the welfare gains that we compute must be interpreted as gains coming from comparing the welfare of two global economies similar in all features except for the variable trade cost. Figure 2 shows the results.

Figure 2 shows that trade liberalization has a pro-competitive effect on both exporters and non-exporters. The pro-competitive effect results, first, from an increase in import competition induced by the reduction in trade costs, and second, from the increase in the number of domestic

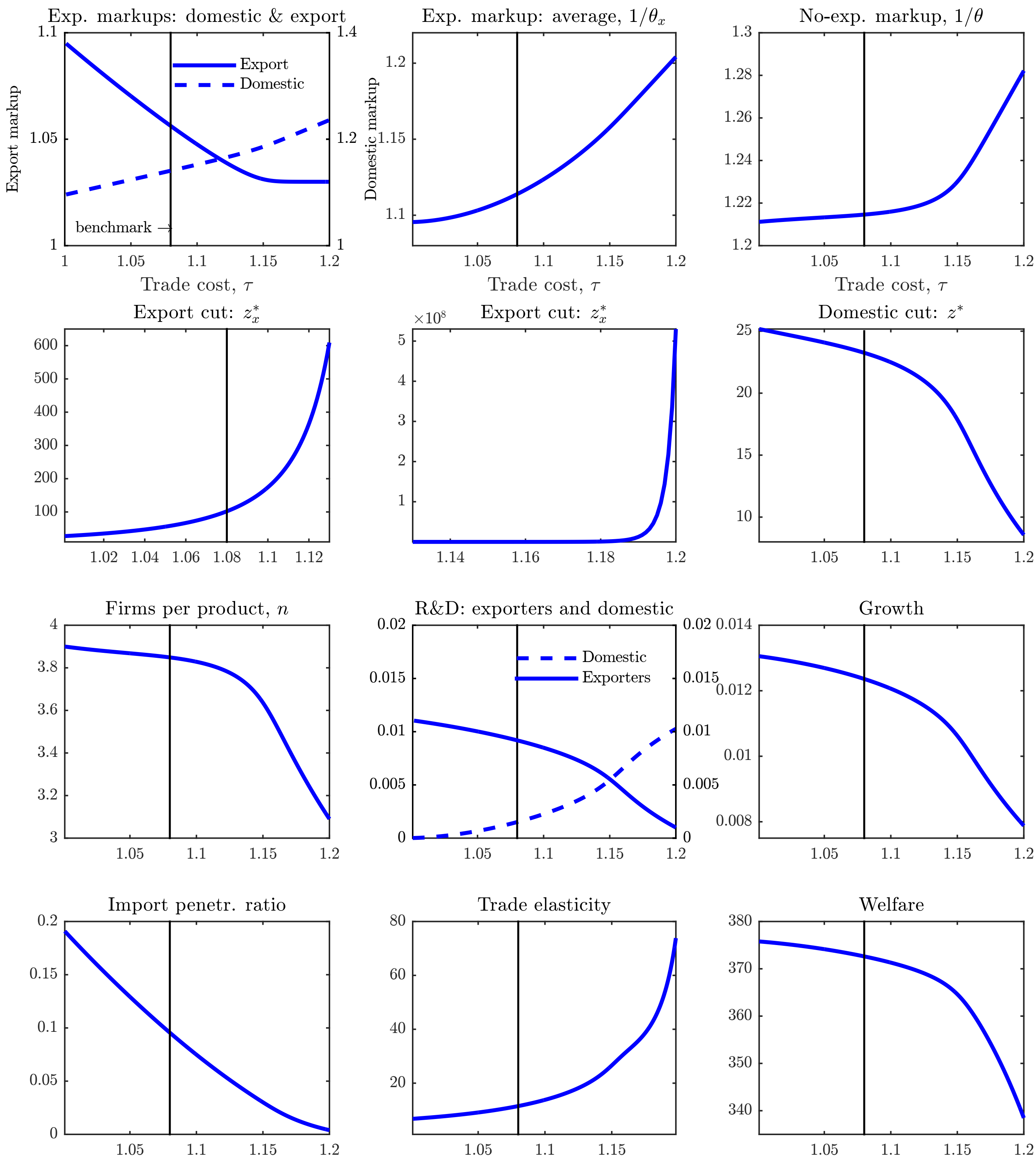

Figure 2: Trade liberalization

firms n that follows. Exporters experience a markup reduction on their domestic sales, $1/\theta_d$, due to increasing foreign competition. While they face an increase in the markups on their export sales, $1/\theta_f$, due to the drop in the variable export cost.[29] Notice also that although exporting firms experience an increase in their export markup this is not strong enough to offset the reduction in their domestic markups, therefore the average markup of exporters decline with trade liberalization. In the simple model we have proved that the average markup of exporting firms $1/\theta_x$ falls with a reduction in trade costs. In the general model, the average export markup is identical to that in the simple model (eq. 23) and the numerical simulations in Figure 2 show that the two models qualitatively predict similar pro-competitive effects on exporters. In addition to that, in the numerical simulation of the general model we find that n increases, as shown in panel seven, which makes the previous effect even stronger.

The export condition (XC) pins down the export cutoff by equalizing the profits of exporting and that of deviating and selling only domestically. Hence, a drop in the domestic markup of exporting firms and an increase in their export markup encourage more firms to start exporting, implying that the export cutoff z_x^* decreases when τ declines, and goes to infinite when the trade cost approaches its prohibitive level $\bar{\tau}$. In Figure 2, in order to provide a clear image of this effect, we first show in panel four the export cutoff from free trade $\tau = 1$ to $\tau = 1.13$, then in panel five we show the cutoff from $\tau = 1.13$ to the prohibitive level. A pro-competitive effect is also experienced by non-exporting firms, although the markup for non-traded varieties $1/\theta$ is not directly affected by changes in trade costs. The drop in the markup of non-exporting firms is generated by the increases in the number of firms per variety n. Similarly to Melitz and Ottaviano (2008), the endogenous market structure of our model implies that trade liberalization has a positive effect on firms' production that outweigths the direct competition effect on prices and markups and allows surviving firms to be bigger, sell more, and earn higher profits on average. Hence, expected profits are larger in a more open economy, and this leads a larger number of oligopolistic firms to enter the market. The reduction in markups for non-exporting firms triggers selection, forcing the less productive domestic firms out of the market, as can be seen from the increase in the domestic cutoff z^*. As in the simple model where all firms are exporting, trade generates a pro-competitive effect that forces the least productive firms to exit the market. In the general model, where only a fraction of firms

[29] Notice that in the first panel of Figure 2 corresponding to the markups of traded varieties, the left scale is the one for the export markup and the right scale for the domestic markup. Both markups are equal at $\tau = 1$.

exports, a reduction in trade costs induces the least productive non-exporting firms to exit the market, but the most productive non-exporters are encouraged enter the export market. This is in line with the recent empirical evidence on the extensive margin of international trade.[30]

In the baseline model, more productive firms innovate more. Similarly, in this generalized framework, exporters are more productive than non-exporters, hence, as suggested by equilibrium condition (32) and (33), exporters allocate more resources to innovation activities. Figure 2, shows that average innovation of exporters is substantially larger than that of non-exporters. Moreover, our simulations show that trade liberalization increases innovation for exporters but not for non-exporters. As innovation in our economy is proportional to firms size, this also implies that trade increases market shares of exporters and decreases those of non exporters. What are the economic mechanisms behind these results? Stronger product market competition forces the least productive firms out of the market, reallocating their market shares to more productive firms. Hence surviving firms, both exporters and non-exporters, can potentially increase their size through reallocation of market shares from exiting firms and from the homogeneous good. Since only exporters increase their innovation when the trade cost declines, Figure 2 suggests that only exporters become bigger after liberalization. This happens because the average markup for exporters $1/\theta_x$ drops more than that of non-exporting firms $1/\theta$, hence oligopolistic inefficiency declines more for the former. Since the increase in efficiency is then higher for exporters, market demand shifts towards these firms and away from domestic producers and from the homogeneous good, where efficiency has improved less and not at all respectively. Finally, as exporters innovate more than non-exporters and trade increases the share of exporting firms, aggregate innovation increases with trade liberalization, thereby leading to a higher productivity growth rate.

In order to give a sense of the volume of trade produced by different levels of the trade cost, in Figure 2 we also report the import penetration ratio. Moving from $\bar{\tau} = 1.2$ to no variable trade costs, produces a substantial change in the volume of trade, with the import penetration ratio being close to 20% at $\tau = 1$. Notice also that the absolute elasticity of trade to the trade cost varies massively for different levels of τ; it is extremely high at autarky and substantially lower close to free trade. Similarly to what shown by Melitz and Redding (2015)

[30]Bernard et al. (2006) for example, using firm-level US data find that y high-productivity non-exporters are more likely to start exporting in response to falling trade costs.

for the standard Melitz model, variable trade elasticity will prove important in shaping the selection gains from trade, as we will discuss later. For the moment it suffices to notice that that trade liberalization has a positive effect on welfare. As in the simple model, welfare effects are both static and dynamic, and they originate directly through intensive margin adjustments, and indirectly through trade-induced selection margins. In the next section we break down the various sources of gains from trade and explore the economic forces governing them. Before moving to the channels of welfare gains from trade we discuss the empirical evidence in support of the predictions of our model.

Trade, markups and innovation: empirical evidence. Here we briefly discuss the empirical evidence on the pro-competitive and innovation effects of trade.

Trade and markups. There is a large literature documenting pro-competitive effects of trade. Harrison (1994) finds robust negative effects on firm-level profit margins of a large trade reform in Cote d'Ivoire implemented in 1995. She also shows that accounting for the effects of trade on product market competition leads to larger positive effect of trade on the growth rate of firm productivity. Levinsohn (1993), using firm-level Turkish data, finds evidence of pro-competitive effects associated to a trade liberalization reform in 1984. More recently, De Loecker, Goldberg, Khandelwal and Pavcnik (2016), find that Indian firms do not entirely pass the cost reduction due to input tariff reduction in the period 1989-2003 to consumers. This happens because they cash in the lower costs by increasing markups. This mechanism is similar to what we observe in our model, although we do not have trade in intermediate goods. In our economy, trade reduces the cost of reaching foreign markets, and firms use some of the cost reduction to increase their export markup. Moreover, when focusing on changes in final goods tariffs De Loecker et al. (2016) find that trade liberalization has a negative effect on total markups of exporting firms, similarly to what our model suggests.

Trade, selection and innovation. There is a growing empirical literature documenting the effects of trade on selection and innovation. Bloom et al. (2016), using a new data set of firms across twelve European countries find that import competition from China, led to faster technological change along several dimensions, including patenting, IT intensity and TFP. Bustos (2010) studies the impact of a large regional trade agreement, MERCOSUR, on a measure of firm-level innovation which includes R&D, spending for technology transfer, and capital goods that embody new technologies. She finds that increase in revenues generated by tariff reductions lead exporters to innovate more. In line with this results, in our model trade increase innovation only among exporters.

Trade, markups, and innovation/growth. Griffith, Harrison, and Simpson (2010), using European sector-level data, find that the EU Single Market Programme (SMP), a large program deregulating the product market which also includes reductions in trade barriers, is associated with increased product market competition, as measured by a reduction in average profitability, and with a subsequent increase in innovation intensity and productivity growth. Aghion, Blundell, Griffith, Howitt, and Prantl (2009) using firm-level UK data, find that incumbent productivity growth and patenting in UK firms is positively correlated with foreign firms' entry in technologically advanced industries. This technologically advanced industries are those where foreign and domestic firms are more neck-and-neck, that is they have similar levels of technology and market shares.

5.3 Growth and welfare gains from trade

In this section we quantify the growth and welfare gains from trade and decompose them into their different sources. First, we decompose welfare gains into their static and dynamic part, to highlight the role of productivity growth. Second, we explore the role of firm heterogeneity decomposing growth and welfare gains from trade into their direct and selection components.

Static and dynamic gains. Welfare gains of moving from autarky to trade are quantified with a consumption compensating variation measure. For any balanced growth equilibrium consumption path $\{X_t, Y\}$, X_t growing at the rate ηg and Y constant, let us denote stationary consumption by $\Omega = \{X, Y\}$, $X = X_t \,\mathrm{e}^{-\eta g}$. Consequently, Ω_A corresponds to stationary consumption in autarky and Ω_τ to stationary consumption in a trade equilibrium with $\tau \in (1, \bar{\tau})$. Welfare can be then denoted by $U(\Omega)$, representing the corresponding value of the discounted utility flow, resulting from substituting the equilibrium path in the welfare function (1):

$$\rho U(\Omega) = \rho \int_0^\infty (\ln X_t + \beta \ln Y)\ \mathrm{e}^{-\rho t}\ \mathrm{d}t$$
$$= \underbrace{\ln X + \beta \ln Y}_{\texttt{Static}} + \underbrace{\frac{\eta g}{\rho}}_{\texttt{Dynamic}} .$$

Let us define the compensating variation measure ω, $\omega \in \Re^{+}$, such that $U_A(\omega\,\Omega_A) = U_\tau(\Omega_\tau)$, for any $\tau \in (1, \bar{\tau})$. It can be easily shown that

$$\frac{1+\beta}{\rho}\log(\omega) = U_\tau(\Omega_\tau) - U_A(\Omega_A).$$

It measures the percentage gains in terms of lifetime consumption of comparing the trade economy with the autarkic economy. The contribution of growth to these gains can be expressed as

$$\log(\omega_g) = \frac{\eta}{(1+\beta)\,\rho}\,(g_\tau - g_A)\,. \tag{34}$$

The static gains can be obtained as a residual subtracting ω_g from the total gains ω. In table (2) we report the quantitative effects of moving from autarky to the benchmark economy, where the import penetration ratio is 9.7%.

Table 2: Gains from trade

	Trade	Autarky	% Change
Avg. markup (%)	20	29	−31
Avg. productivity	4.02	3.02	33
Growth (%)	1.25	0.8	56.3
	Total	Dynamic	Static
Welfare gains	51.1	25.1	26

The average markup drops by about 31%. Moving from autarky to the benchmark economy trade level increases productivity by 33%. The annual growth in autarky is 0.79% while under trade it goes up to 1.245%, a 56% increase. The last row shows the welfare gains in consumption equivalent terms and its decomposition into static and dynamic gains according to (34). Moving from autarky to a 9.7% import penetration ratio increases welfare by about 50%, half of this increase is attributable to trade-induced productivity growth.

Figure 3 reports the decomposition of the gains of moving from autarky to all levels of trade costs. The first panel reports welfare gains, as measured by the compensating consumption measure ω. Not surprisingly, welfare gains are increasing in the magnitude of liberalization, with the dynamic gains rising more steeply than static gains. Although the main scope of the paper is to explore the structure of the gains from trade assessing the role of productivity dynamics as an additional source, it is worth noticing that the absolute size of the total gains from trade is quite large. Perla et al. (2016) show that in their dynamic economy moving from autarky to a 23% total trade share leads to a 24% increases in welfare in terms of compesanting variation. This entire increase is due to static gains, while the potential gains coming from trade-induced growth are offset by the decline of product variety due to selection, as in Atkeson and Burstein (2010), and reallocation of labor away from production. The size of our static gains is similar to theirs but in our model trade-induced growth generates large dynamic gains that are not completely offset by the loss of variety induced by selection.

The second panel of 3 reports the ratio of the overall welfare gains to the static gains for different levels of the trade cost, thus measuring the proportional increase in gains due to productivity growth. Starting from the benchmark calibration of the trade cost, in line with Table 2, the total gains are about *two times* higher than those obtainable in an economy which admits only static gains. Starting from higher initial trade cost, the ratio declines but total gains are still substantially higher then static gains, about 40% higher close to autarky; starting from trade costs lower than the benchmark total gains are more than twice as large as static gains. In line with our results, but in a model where productivity growth is driven by knowledge diffusion at the entry stage and not by innovation, Sampson (2016) finds that 'dynamic' selection leads to welfare gains at least *three times* the gains from trade relative to heterogeneous firms' economies with static steady states. Finally, the figure reports the ratio between growth under costly trade and growth under autarky, showing that growth is about 55% higher at the benchmark trade cost compared to autarky, and always robustly higher at all other levels of the trade cost. Similar results are obtained in Perla et al. (2016), while the growth rate increases by about 25% in Sampson (2016) when the import penetration ratio goes from zero to 10%.

Direct and selection effects. We now decompose the gains from trade into *direct gains*, which do not depend on trade-induced selection, and *selection gains* hinging on the reallocation of market shares from low to high productive firms. Following Melitz and Redding (2015), and in line with our decomposition in the simple model of Section 3.2, we construct a counterfactual economy where the growth rate, welfare, the import penetration ratio, the share of exporting firms, and the average productivity of exporters and non-exporters are the same as in our benchmark economy; essentially, the two economies have the same initial equilibrium. We then compute the gains from trade in this economy, shutting down the extensive margins: we do not let the cutoffs z^* and z_x^* respond to changes in trade costs. This allows us to isolate the

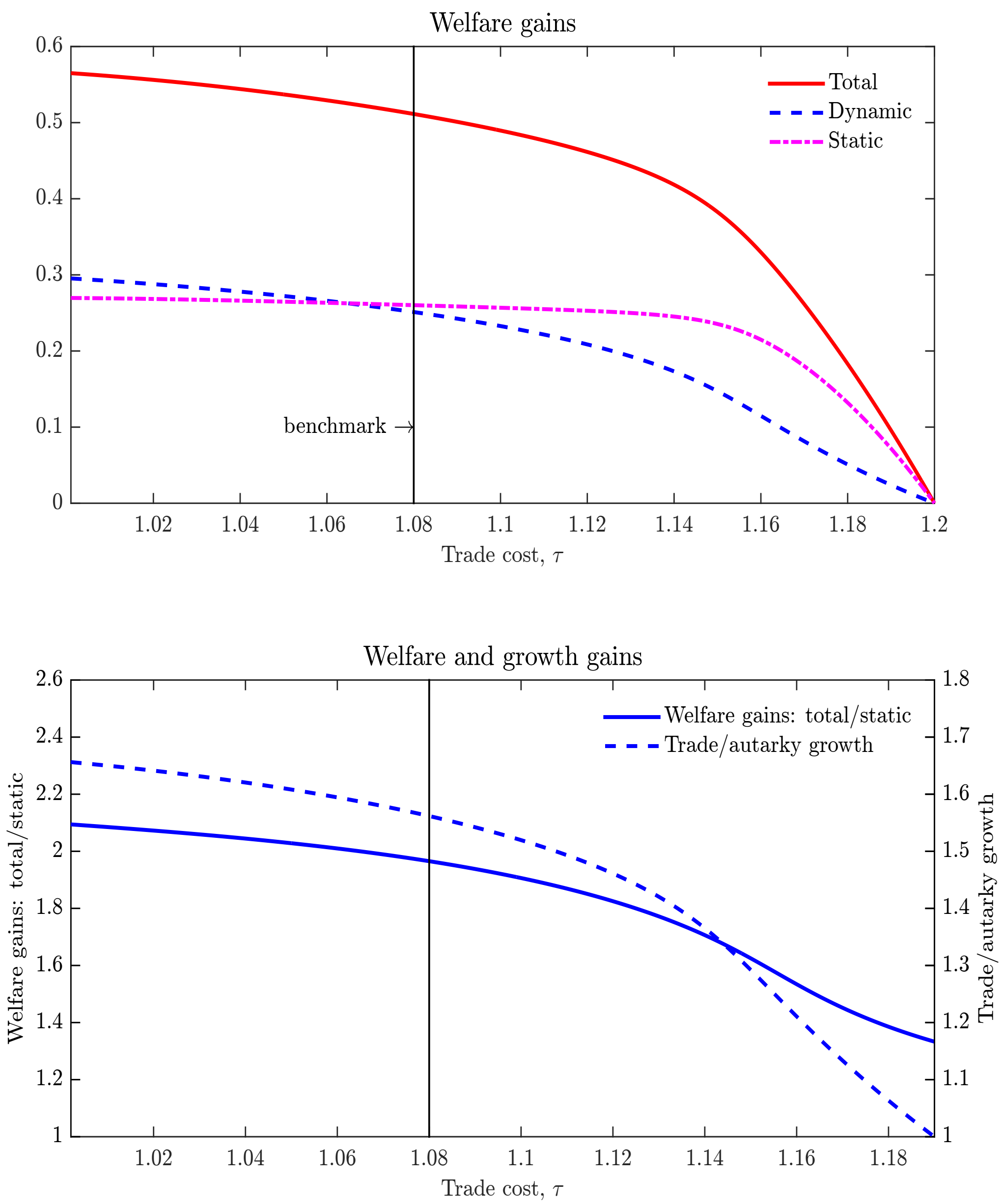

Figure 3: Static and dynamic gains from trade

direct gains of trade, and compute the contribution of selection comparing these gains with those obtained in the benchmark model.

The first panel in Figure 4, shows the total direct welfare gains of moving from each value of the trade cost τ, starting from the free trade level, to autarky, and its static and dynamic components. As discussed in Section 3.2, and in line with Brander and Krugman (1983) free entry implies that the direct gains from trade are pinned down by the pro-competitive effect of trade on consumer surplus, which is always positive. Although the gains are positive, we can see that they are substantially lower compared to those in Figure 3, where selection is operative. Moreover, in the second panel we can see that dynamic gains account for a bigger share of the total gains compared to the benchmark case. Shutting down the selection margins also reduces the growth effect of trade, with a mere 6% difference between the growth rate at the benchmark trade cost and that in autarky. In order to get a clear image of the importance of selection, in the third panel we compare the welfare gains obtained in the benchmark model with the direct gains. The figure shows that the presence of selection in the benchmark model generates total welfare gains of moving from the benchmark trade cost to autarky that are about 8 times larger than the direct gains. Moreover, the dynamic gains in the benchmark model are about 5 time larger than those generated shutting down selection. The figure also shows that these results are robust to wider liberalization scenarios, and that role of selection is bigger the closer we are to autarky.

Why are the gains from trade produced by selection several orders of magnitude higher that those generated by a model where the extensive margins of exit, entry and exporting are shut down? Arkolakis et al (2012) show that in a large class of models satisfying three macro-level restrictions the gains from trade are pinned down by two sufficient statistics, the domestic trade share and the trade elasticity, and they do not depend on the different micro-speficications of the model. These restrictions are: 1) balanced trade; 2) aggregate profits are a constant share of aggregate revenues; 3) CES demand system with a constant elasticity of trade with respect to variable trade costs. They show that the standard Melitz (2003) model with unbounded Pareto distribution of productivity satisfies these restrictions. As a consequence, the welfare gains from a given increase in the domestic trade share in this model are identical to those obtained in a version of the model without firm heterogeneity, such as Krugman (1980). Melitz and Redding (2015) find that small variations from these restrictions imply that the trade elasticity is not

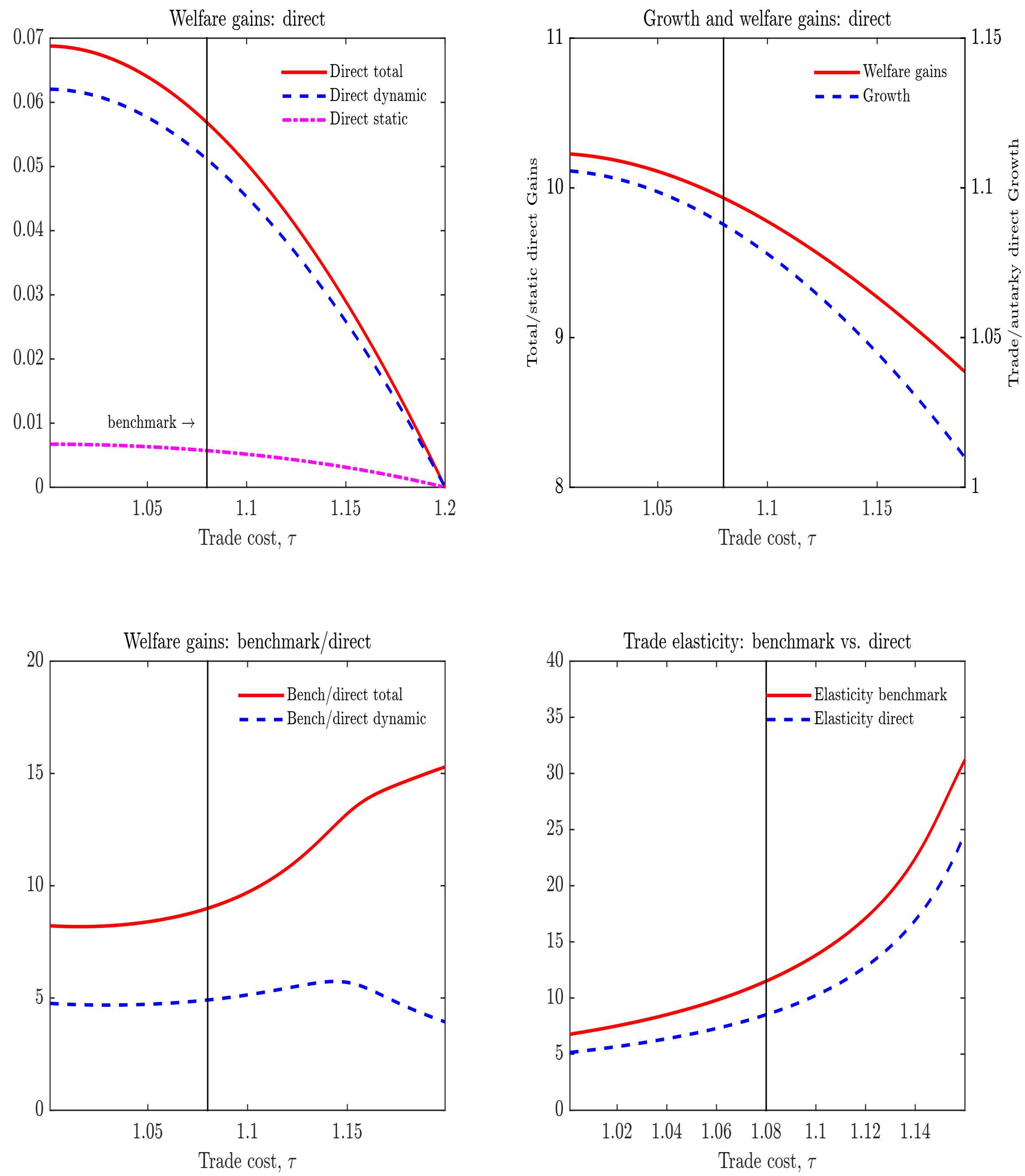

Figure 4: Direct and selection growth and welfare gains

constant and therefore it is not a sufficient statistics for welfare. They show that replacing the unbounded Pareto with a bounded Pareto, the CES demand system does not yield constant trade elasticity and that the welfare gains generated by a model with firm heterogeneity are arbitrarily larger than those obtainable with homogeneous firms.

The class of models considered in Arkolakis et al. (2012) features either perfect or monopolistic competition, hence our oligopolistic economy is outside that class and violates both restrictions 2) and 3). Figure 2 shows that the elasticity of trade to trade cost varies massively at different levels of openness. Moreover, in the last panel of Figure 4, we see that trade elasticity also varies across models. Precisely, the elasticity in the benchmark model is on average 30% higher than that in the model where the selection margins have been shut down, and this difference is increasing the closer we are to autarky. This suggests that, similarly to Melitz and Redding (2015), in our dynamic oligopolistic economy variable trade elasticity generates different gains in models with and without extensive margins. Finally, notice that differently from the monopolistic competitive environment of the standard Melitz model considered in Melitz and Redding (2015), with our oligopolistic market structure we obtain variable trade elasticity even with unbounded Pareto. Head, Mayer and Thoenig (2015) show that the new gains from trade due to selection can be substantially larger if instead of using an unbounded Pareto the model is calibrated using a log-normal distribution of firm productivity. The reason is that the log-normal delivers variable trade elasticity. Our results show that with an oligopolistic market structure, variable trade elasticity and large gains from selection can be obtained even calibrating the model with the typical unbounded Pareto.

We can draw two conclusions from these experiments. First, economies with oligopoly trade feature variable trade elasticity and generate new gains from trade-induced selection not obtainable in models with homogeneous firms. Second, innovation-driven productivity growth increases the gains from trade originating from firm selection substantially. Hence, evaluating gains from trade in models with static steady-state productivity, such as Melitz (2003), may lead to underestimating the size of these gains. Second, by abstracting from firm heterogeneity, the previous generation of trade and growth models, such as Grossman and Helpman (1991), and Peretto (2003), are likely to largely underestimate the welfare gains from trade.

Robustness. Next, we explore the robustness of these results to a wide range of parameters' specifications different from our benchmark calibration. Precisely, we compute the total gains, the ratio of total to static gains, the relative growth rate, and the direct gains of moving from the benchmark trade cost (9.7% import penetration ratio) to autarky, moving one parameter at the time away from its calibrated value.[31] Table (3), show the results.

Table (3) shows that total gains are substantially above static gains, suggesting that the sizable contribution of dynamic gains found in the benchmark calibration is sufficiently robust. Not surprisingly, the level of total gains and the ratio of total over static gains falls below two for higher levels of the discount factor ($\rho = 0.05$) and low levels of the innovation efficiency parameter A. High discount rate implies that consumers care less about future growth and, as a consequence, the impact of growth on their lifetime consumption is lower. Low efficiency of the innovation technology implies lower returns innovation, lower equilibrium growth, and larger share of labor needed to keep the economy growing. Although trade increases growth more under low innovation efficiency, due to its lower level growth is a smaller component of welfare and comes at a higher cost, since it diverts more resources away from production. It follows that both the total gains and their dynamic component are lower the lower innovation efficiency is.

Table 3: Robustness

	Bench	$\underline{\rho} = .01$	$\overline{\rho} = .05$	$\underline{A} = 30$	$\overline{A} = 120$	$\underline{\alpha} = .1$	$\overline{\alpha} = .5$
Total welfare gains	0.51	0.84	0.49	0.39	0.95	0.29	0.5
Total/static welfare gains	1.97	5.04	1.19	1.55	3.71	1.05	14
Trade/autarky growth	1.56	1.62	1.52	1.68	1.53	1.46	1.68
Total/direct gains	8.9	7.63	19.6	11.14	7.3	5.9	9.09
		$\underline{\beta} = .25$	$\overline{\beta} = .75$	$\underline{\phi} = .05$	$\overline{\phi} = .2$		
Total welfare gains	0.51	0.75	0.35	0.45	0.62		
Total/static welfare gains	1.97	2.32	2.18	8.7	1.69		
Trade/autarky growth	1.56	1.55	1.51	1.68	1.10		
Total/direct gains	8.9	12.93	6.7	22.5	8.8		

Furthermore, lower demand elasticity (lower α), decreases overall gains and their dynamic component. Higher substitutability across varieties implies weaker intensive margins of reallocations, hence weaker dynamic and static gains. A higher share of the differentiated good in the economy (lower β) increases total gains and the share of total over static gains. Intuitively, more important is the differentiated sector in consumer utility, stronger are the selection effects

[31] Since parameters changes affect the value of the prohibitive trade cost $\bar{\tau}$ which defines the autarky state of the economy, in computing the welfare gains we take this into account and use the appropriate prohibitive cost $\bar{\tau}$ for each departure from the benchmark calibration.

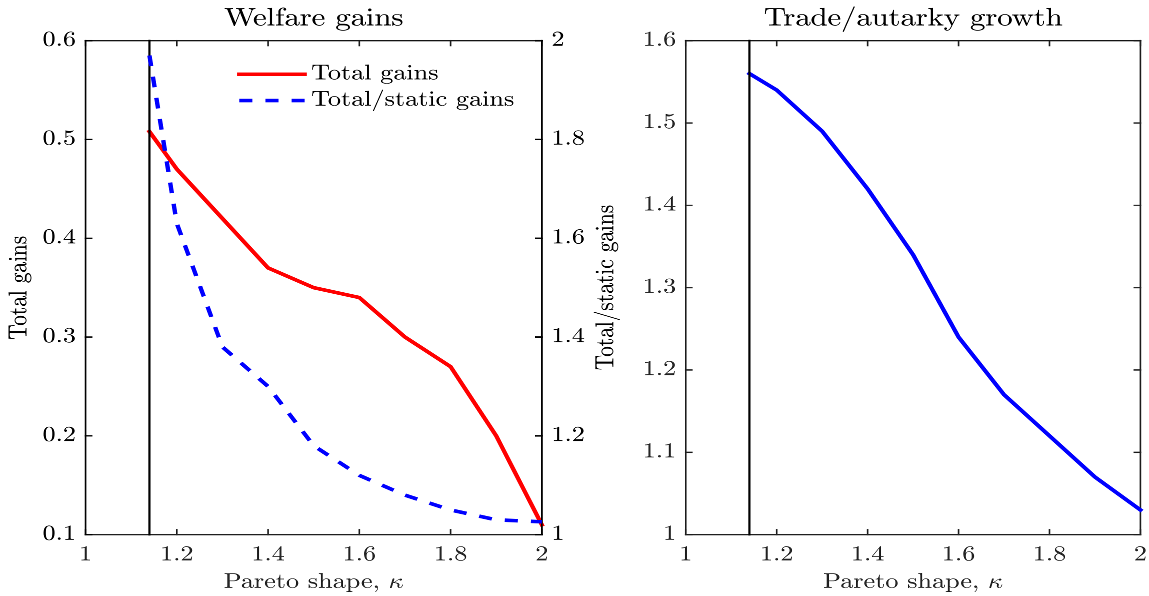

Figure 5: Firm heterogeneity and welfare gains

of trade, and consequently the welfare gains. Notice also that in all these different scenarios the total gains from trade obtainable with our heterogeneous firms economy are substantially higher than the direct gains, obtained in a version of our economy where the selection margins have been shut down. This suggests that firm selection has an important role in generating additional welfare gains from trade.

Finally, since our initial productivity distribution is an untruncated Pareto, the degree of heterogeneity is summarized only by the shape parameter κ. The homogeneous firm model corresponds to the limit case in which the $\kappa \to \infty$. Following Melitz and Redding (2015), we can study the role of firm heterogeneity showing how welfare gains change as we move from the benchmark economy toward an economy with less firm heterogeneity. This yields a continuous comparative statics with respect to the degree of firm heterogeneity κ. In Figure (5) we perform a wider robustness on the role of firm heterogeneity in shaping the gains from trade and its composition. As for the robustness analysis above, the welfare gains are computed comparing autarky with the benchmark level of the trade cost.

The results show that less heterogeneity, higher values of κ, are associated with lower total gains and lower ratio of total to static gains. Intuitively, the lower the degree of firm heterogeneity, the lower is the scope of the selection margin in affecting trade induced reallocations and welfare gains. The total welfare gains decrease substantially as the degree of firm heterogeneity declines. Interestingly, a lower degree of heterogeneity among firms affects the dynamic gains more than the static gains. In fact, as the Pareto shape increases the ratio between the total and static welfare gains from trade tends to one. This is because the growth effect of trade declines substantially at lower degrees of firm heterogeneity. As shown in the second panel, growth is about 55% under trade than in autarky in the benchmark economy ($\kappa = 1.14$) and only 3% higher when the Pareto shape increases to $\kappa = 2$. This suggests that heterogeneity is key in delivering larger growth and welfare effects of trade. Consequently, quantitatively assessing welfare and growth gains from trade in endogenous growth frameworks with representative firms, such as the first generation trade and growth models of Grossman and Helpman (1991) and Peretto (2003), may lead to miss substantial parts of these gains.

6 Discussion

In this section, we dig deeper into the role of endogenous markups in driving the quantitative results shown above. We show that in the general model, the presence of markup dispersion is necessary for our oligopolistic economy with free entry to generate trade-induced selection. We develop our argument in two stages. We start showing that trade does not generate selection in the simple model of Section 3 if we allow the number of firm per product line to be endogenously determined by the free entry condition, but we keep the assumption that all firms export. This result echoes the neutrality result in Atkeson and Burstein (2010). We then move to the general model where the number of firms is endogenous, only the most productive firms export, and markups for exporters and non-exporters are different, and show that the selection effect of trade reappears.

The absence in Atkeson and Burstein (2010) of selection welfare gains from trade is fundamentally due to the role played by the free entry condition, which by shrinking the mass of operative varieties generates welfare losses that compensate the gains due to selection. We show below that the free entry condition plays a similar and even more extreme role in our simple model. This can be easily seen by adding the free entry condition to the simple model of Section 3.[32] Notice that, when all varieties are traded, the free entry condition becomes

$$(1-(1+\eta)\theta_x)e+\hat{\rho}-\lambda=\frac{\rho+\delta}{1-\Gamma(z^*)}\phi$$

where the left-hand-side represents the average profit at entry. Combining this with the exit condition (EC^T) and rearranging terms, we obtain

$$\frac{\bar{z}(z^*)}{z^*}\lambda=\lambda+\frac{\rho+\delta}{1-\Gamma(z^*)}\phi, \tag{35}$$

which determines z^* independently of θ_x and, consequently, independently of the iceberg trade cost τ. Hence, changes in trade costs do not affect selection and cannot have any welfare effect through the selection channel. This result can be easily understood in terms of arbitrage. Incumbent firms face two alternatives: operating their current technology or exiting and entering again by paying the fixed entry cost and drawing a new productivity level. This trade-off is represented in (35), which combines the entry and exit conditions. Since both exiting and entering firms have the same markup, arbitrage makes the marginal firm indifferent to markup changes and, consequently, to changes in trade costs. Hence, in our simple model, free entry implies that the selection effect of trade vanishes.

While in the simple model the markup is the same for all firms, the general model features exporting firms charging lower markups than non-exporters, as shown in (28) and (29). Hence, entering and exiting firms face different profit opportunities. As can be seen by comparing the free entry condition (FE) and the exit condition (EC'), the marginal firm is not indifferent anymore to changes in profits induced by trade liberalization, and the entry/exit arbitrage does not imply that selection is independent of the markup. Hence, free entry does no shut down the selection effect of trade which, as shown in Table 2, yields positive welfare gains.

7 Conclusion

In this paper, we have built a rich model of oligopoly trade featuring heterogeneous firms, endogenous markups, and innovation-driven endogenous growth, to identify and quantify the sources of gains from trade originating from firm selection. We have shown that under an oligopolistic market structure, trade increases product market competition by reducing markups,

[32] Free entry is introduced to endogenize the number of firms. An equivalent conclusion would be reached if the number of firms remains constant, but the the mass of varieties is determined by the free entry condition, as in Atkeson and Burstein.

thereby triggering firm selection and innovation-driven productivity growth. Markup differentials between exporters and non-exporters is key in generating trade-induced selection. Moreover, trade leads to substantial welfare gains, about half of which are accounted for by the effect of selection on productivity growth. Dynamic gains due to the growth effects of selection double the welfare gains from selection obtainable in a static version of our economy.

In the current debate on the new welfare gains from trade due to firms selection, it has been shown that in a large class of models with monopolistically competitive economies and intra-industry trade firm heterogeneity does not generate new gains from trade if firms draw their productivity from an unbounded Pareto distribution (Arkolakis et al. 2010, Melitz and Redding, 2015). We have shown that under an oligopolistic market structure trade-induced selection has first order effects on welfare even with unbounded Pareto productivity distribution.

Since introducing free entry is a notable challenge in general equilibrium models of oligopoly trade, we have considered an simple entry strategy that only generates markup differences between exporters and non-exporters. This is a limitation of the model and a challenge for future research would be to generalize the model to a full distribution of markups across heterogeneous firms. In an ongoing follow up project, Impullitti and Licandro (2016), we are generalizing a static version of the model along this and other dimensions.

References

[1] Acemoglu, D., D. Autor, D. Dorn, G. Hanson, and B. Price (2015), "Import Competition and the Great U.S. Employment Sag of the 2000s," mimeo.

[2] Aghion, P. and P. Howitt (1992) "A Model of Growth through Creative Destruction," *Econometrica*, 60(2), 323–351.

[3] Aghion, P., R. Blundell, R. Griffith, P. Howitt and S. Prantl (2009) 'The effects of entry on incumbent innovation and productivity', *Review of Economics and Statistics*, Vol. 91, No. 1, pp. 20-32.

[4] Akcigit, U. and W.R. Kerr (2011) "Growth through Heterogeneous Innovations," NBER 16443.

[5] Anderson, J. and E. van Wincoop (2002) "Trade Costs," *Journal of Economic Literature*, 42, 691–751.

[6] Arkolakis, C., A. Costinot and A. Rodriguez-Clare (2012) "New Trade Models, Same Old Gains?" *American Economic Review*, 102(1), 94–130.

[7] Atkeson, A. and A. Burstein (2008). "Pricing to Market, Trade Costs, and International Relative Prices," with Andrew Atkeson, *American Economic Review*, December.

[8] Atkeson, A. and A. Burstein (2010) "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, 118(3), 433–484.

[9] Autor, D., D. Dorn, G. Hanson, and J. Song (2014) "Trade Adjustment: Worker Level Evidence," *Quarterly Journal of Economics*, 129(4), 1799-1860.

[10] Aw, B., M. Roberts and D. Xu (2011) "R&D Investment, Exporting, and Productivity Dynamics," *American Economic Review*, 101, 1312-1344.

[11] Axtell, R.L. (2001) "Zipf distribution of U.S. firm sizes," *Science*, 293, 1818–1820.

[12] Baldwin, R. and F. Robert-Nicoud (2008) "Trade and Growth with Heterogeneous Firms," *Journal of International Economics*, 74(1), 21–34.

[13] Basu, S. (1996) "Procyclical Productivity: Increasing Returns or Cyclical Utilization?" *Quarterly Journal of Economics*, 111, 709–751.

[14] Benedetti, C. (2009) "Trade and Growth: Selection versus Process and Product Innovation," mimeo.

[15] Bernard, A.B. and J.B. Jensen (1999) "Exceptional Exporter Performance: Cause, Effect or Both?" *Journal of International Economics*, 47, 1–25.

[16] Bernard, A.B., J.B. Jensen, J. Eaton and S. Kortum (2003). "Plants and Productivity in International Trade," *American Economic Review*, 93(4), 1268–1290.

[17] Bernard, A.B., J.B. Jensen and P. Schott (2006) "Trade Costs, Firms and Productivity", *Journal of Monetary Economics*, Volume 53, Issue 5, July: 917-937.

[18] Bernard, A.B., J.B. Jensen and P. Schott (2012). "The Empirics of Firm Heterogeneity and International Trade," *Annual Review of Economics*, 4, 283-313.

[19] Bernard, A.B., S. Redding and P. Schott (2007) "Comparative Advantage and Heterogeneous Firms," *Review of Economic Studies*, 74, 31–66.

[20] Bloom, N., M. Draca and J. Van Reenen (2016) "Trade Induced Technical Change: The Impact of Chinese Imports on Innovation, Diffusion and Productivity," *Review of Economic Studies*, forthcoming.

[21] Blundell, R., R. Griffith, and F. Windmeijer (2002), "Individual effects and dynamics in count data models", Journal of Econometrics, 180, 113-131.

[22] Brander, J. (1981). "Intra-industry Trade in Identical Commodities", *Journal of International Economics* 11 (February), 1-14.

[23] Brander, J. and P. Krugman (1983) "A Reciprocal Dumping Model of International Trade," *Journal of International Economics*, 15, 313–321.

[24] Bugamelli, M., S. Fabiani, E. Sette (2015). "The Age of the Dragon: The Effect of Imports from China on Firm-Level Prices," J*ournal of Money, Credit and Banking*, vol. 47(6),1091-1118, 09.

[25] Cellini, R. and L. Lambertini (2005) "R&D Incentives and Market Structure: A Dynamic Analysis," *Journal of Optimization Theory and Applications*, 126, 85–96.

[26] Corrado, C., C. Hulten and D. Sichel (2009) "Intangible Capital and U.S. Economic Growth," *Review of Income and Wealth*, 55(3), 661–685.

[27] Coşar, K., N. Guner, and J. Tybout (2014) "Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy", *NBER Working paper* 16326.

[28] De Loecker, J., and F. Warzynski (2016). "Markups and Firm-level Export Status", *American Economic Review*, Vol. 102, No. 6., 2437-2471.

[29] De Loecker, J., P. Goldberg, A. Khandelwal, and N. Pavcnik, (2016). "Prices, Markups and Trade Reform," *Econometrica*, forthcoming.

[30] Djankov, S., La Porta, R., Lopez-de-Silanes, F., and A. Shleifer, (2002), "The Regulation of Entry", *Quarterly Journal of Economics,* 117(1): 1-37

[31] Edmond, C., V. Midrigan and D. Xu (2012) "Competition, Markups, and the Gains from International Trade," *American Economic Review*, forthcoming.

[32] Epifani, P., and G. Gancia (2011) "Trade, Markup Heterogeneity and Misallocations," *Journal of International Economics*, 83, 1–13.

[33] Feenstra, R., and D.. Weinstein (2010) "Globalization, Markups, and the U.S. Price Level," 2010. NBER Working Paper 15749.

[34] Felbermayr, G., G. Impullitti, and J. Prat (2014). "Firm Dynamics and Residual Inequality in Open Economies", *CEPR Discussion Papers* 9855.

[35] Fershtman, C. (1987) "Identification of Classes of Differential Games for Which the Open-Loop is a degenerated Feedback Nash Equilibrium," *Journal of Optimization Theory and Applications*, 55, 217–31.

[36] Gabler, A. and O. Licandro (2009) "Firm Dynamics Support the Importance of the Embodied Question." IAE Working Paper 782.09.

[37] Griliches, Z. (2000) *R&D, Education, and Productivity: A Retrospective*. Cambridge, Harvard University Press.

[38] Griffith, R., R. Harrison and H. Simpson (2010) "Product market reforms and innovation in the E," *Scandinavian Journal of Economics*, 112 (2), June 2010, 389-415 .

[39] Grossman, G. and E. Helpman (1991) "Quality Ladders in the Theory of Growth," *Review of Economic Studies*, 58, 43–61.

[40] Grossman, G. and E. Helpman (2014) "Growth, Trade, and Inequality", mimeo.

[41] Grossman, G. and E. Helpman (2015) "Globalization and Growth", *American Economic Review Papers and Proceedings*, forthcoming.

[42] Harrison, A. (1994), "Productivity, Imperfect Competition and Trade Reform: Theory and Evidence," *Journal of International Economics* 36:53-73.

[43] Head, K., T. Mayer and M. Thoenig, (2014), "Welfare and Trade Without Pareto", *American Economic Review Papers and Proceedings*, 104(5): 310-316.

[44] Helpman, E., O. Itskhoki, A. Muendler, and S. Redding, 2014, "Inequality and Unemployment in a Global Economy", *NBER Working Paper* No. 17991.

[45] Hsieh, C. and P. Klenow (2009) "Misallocation and Manufacturing TFP in China and India," *Quartely Journal of Economics*, 124, 1403–1448.

[46] Impullitti, G., and O. Licandro (2016). "Endogenous Market Structure, Innovation, and the New and Old Gains from Trade", mimeo.

[47] Jones, C. (1995) "R&D-Based Models of Economic Growth," *Journal of Political Economy*, 103, 759–784.

[48] Klette, T.J. and S. Kortum (2004) "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*, 112, 986–1018.

[49] Kortum, S. (1997) "Research, Patenting, and Technological Change," Econometrica, 65(6), 1389–1419.

[50] Kortum, S. (1993) "Equilibrium R&D Ratio and the Patent-R&D Ratio: US Evidence," *American Economic Review, Papers and Proceedings*, 83, 450-457.

[51] Krishna, P. and D. Mitra, "Trade Liberalization, Market Discipline, and Productivity Growth: New Evidence from India," *Journal of Development Economics* 46 (1998):447-52.

[52] Lentz, R. and D. Mortensen (2008) "An Empirical Model of Growth through Product Innovation," *Econometrica*, 76(6), 1317–1373.

[53] Levinsohn, J. (1993), "Testing the Imports-as-Market-Discipline Hypothesis," Journal of International Economics 35:1-12.

[54] LLeiva, A., and D. Trefler (2010) "Improved Access to Foreign Markets Raises Plant-Level Productivity ... for Some Plants." *Quarterly Journal of Economics*, 125(3) (August 2010):1051-1099.

[55] Licandro, O. and A. Navas-Ruiz (2011) "Trade Liberalization, Competition and Growth," *The B.E. Journal of Macroeconomics*, 11(1).

[56] Luttmer, E.G. (2007) "Selection, Growth, and the Size Distribution of Firms." *Quarterly Journal of Economics*, 122(3), 1103–1144.

[57] Melitz, M. (2003) "The Impact of Trade on Aggregate Industry Productivity and Intra-Industry Reallocations," *Econometrica*, 71, 1695–1725.

[58] Melitz, M., and S. Redding (2015). "New Trade Models, New Welfare Implications." *American Economic Review* 105 (3): 1105-46.

[59] National Science Foundation (2011). *Science and Engineering Indicators 2011.*

[60] Neary, P. (2003) "Globalisation and market structure," *Journal of the European Economic Association*, 1(2-3), 245–271.

[61] Neary, P. (2009) "International trade in general oligopolistic equilibrium," mimeo.

[62] Neary, P. (2010) "Two and a Half Theories of Trade," *The World Economy*, 33(1), 1–19.

[63] Peretto, P. (1996) "Sunk Costs, Market Structure, and Growth," *International Economic Review*, 37(4), 895–923.

[64] Peretto, P. (2003) “Endogenous Market Structure, and the Growth and Welfare Effects of Economic Integration,” *Journal of International Economics*, 60, 177–201.

[65] Perla, J., C. Tonetti, and M. Waugh (2015). “Equilibrium Technology Diffusion, Trade, and Growth,” mimeo.

[66] Poschke, M. (2009) “Employment Protection, Firm Selection, and Growth,” *Journal of Monetary Economics*, 56(8), 1074–1085.

[67] Rauch, James E., (1999). ”Networks Versus Markets in International Trade,” *Journal of International Economics,* Vol. 48 (June): 7-35.

[68] Restuccia, D., and R. Rogerson (2008). ”Policy Distortions and Aggregate Productivity with Heterogeneous Establishments,” *Review of Economic Dynamics*, Volume 11 (4), pp. 707-20.

[69] Roberts, M. (1997). “The Decision to Export in Colombia,” American Economic Review 87:545-565.

[70] Romer, P. (1990) “Endogenous Technological Change,” *Journal of Political Economy*, 98(5), S71-S102.

[71] Sampson, T. (2014). “Dynamic Selection: an Idea Flows Theory of Entry, Trade, and Growth”, mimeo LSE.

[72] Segerstrom, P. (1998) “Endogenous Growth Without Scale Effects,” *American Economic Review,* 88(5), 1290-1310.

A Derivation of equation (13), the stationary growth rate, and the welfare function

Equation (13). Rearranging (10), we obtain $x_t = \tilde{z}_t^{\eta \frac{1}{1-\alpha}} (\theta E/X_t^{\alpha})^{\frac{1}{1-\alpha}}$. Substituting it into (2) yields

$$X_t^{\alpha} = \left(\int_0^M \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j \right)^{1-\alpha} (\theta E)^{\alpha} ,$$

where $\hat{\eta} \equiv \eta\alpha/(1-\alpha)$. Using this into the expression for x above, we find

$$x_t^{\alpha} = (\theta E)^{\alpha} \, \tilde{z}_t^{\hat{\eta}} \left(\int_0^M \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j \right)^{-\alpha} .$$

Substituting these expressions for x and X into (10), considering that in a symmetric equilibrium $x = nq$, using the definition of stationary productivity $z \equiv \tilde{z}_t^{\hat{\eta}} \, e^{-\hat{\eta} g t}$, and the definition of $e = E/(nM)$, we obtain (13).

Steady state growth. The stationary growth rate (14) is obtained differentiating (11) with respect to time, which yields $-\dot{v}/v = \dot{k}/k = \dot{\tilde{z}}/\tilde{z}$, where the second equality is obtained using $k_t(\tilde{z}) = (\bar{z}/z)\tilde{z}_t$ in which by definition $\bar{z}$ and z are stationary. Plugging it, (13), and (11) into (12), we obtain (14).

B Equilibrium existence and comparative statics

Proof of Proposition 1. Since M is decreasing in z^*, the (MC) locus is increasing starting at

$$\frac{\frac{(1+\delta)}{n} + \hat{\rho} - \lambda}{\beta + (1+\eta)\theta},$$

when $z^* = z_{min}$, and going to infinity when z^* goes to infinity. Under Assumption 1(a), the (EC) locus is decreasing, starting at

$$\frac{\lambda \frac{\bar{z}_e}{z_{\min}} - \hat{\rho}}{1 - (1+\eta)\theta},$$

for $z^* = z_{min}$, and going to $\big(\lambda - (\rho+\delta)/A\big)/\big(1-(1+\eta)\theta\big)$ when z^* goes to ∞. Assumption 1.b implies $\Psi < 1$ and substituting this into 1.c leads to $1+\eta < 1/\theta$, which guarantees that profits (17) are increasing in productivity z. Since $\Psi < 1$ it is easy to show that 1.c is a sufficient condition for the intercept of the (EC) curve be larger than the (MC) curve at $z^* = z_{min}$, which implies single-crossing of the two equilibrium conditions.

Proof of Proposition 2. Figure 1 shows the effect of an increase in the degree of competition (reduction in the markup $1/\theta$) on the equilibrium values of z^* and e. An increase in θ shifts both the (EC) and the (MC) curves to the right, thereby increasing the equilibrium productivity cutoff z^*. Depending on the relative strengths of the shift of the two curves e can increase or decrease, but the average growth rate g always increases. From (14), the effect on g of a change in θ is determined by its effect on θe. Multiplying the market clearing condition (MC) by θ, we obtain θe as a function of θ and $M(z^*)$, and since in equilibrium $M(z^*)$ is decreasing in θ, we can conclude that θe is increasing in θ.

C Firm problem in open economy

Each domestic firm solves the following problem

$$
\begin{aligned}
&V_s = \max \int_s^\infty \left(\left(p_{d,t} - \frac{1}{\tilde{z}_t^\eta} \right) q_{d,t} + \left(p_{f,t} - \frac{\tau}{\tilde{z}_t^\eta} \right) q_{f,t} - h_t - \lambda \right) e^{-(\rho+\delta)(t-s)} \mathrm{d}t \\
&s.t. \\
&p_{d,t} = \frac{E_{d,t}}{X_{d,t}^\alpha}\, x_{d,t}^{\alpha-1} \qquad \text{and} \qquad p_{f,t} = \frac{E_{f,t}}{X_{f,t}^\alpha}\, x_{f,t}^{\alpha-1} \\
&x_{d,t} = \hat{x}_{d,t} + q_{d,t} \qquad \text{and} \qquad x_{f,t} = \hat{x}_{f,t} + q_{f,t} \\
&\dot{\tilde{z}}_t = A k_t h_t,
\end{aligned}
$$

where $q_{j,t}, p_{j,t}$, $E_{j,t}$, $X_{j,t}$ and $\hat{x}_{j,t}$ are the quantity, price, expenditure, total composite good and direct competitors sales, respectively, for $j \in \{d, f\}$ representing domestic and foreign markets, respectively. Writing down the current-value Hamiltonian and solving it yields the following first order conditions

$$
\left((\alpha - 1) \frac{q_{d,t}}{x_t} + 1 \right) p_{d,t} = \frac{1}{\tilde{z}_t^\eta} \tag{36}
$$

$$
\left((\alpha - 1) \frac{q_{f,t}}{x_t} + 1 \right) p_{f,t} = \frac{\tau}{\tilde{z}_t^\eta} \tag{37}
$$

$$
1 = v_t A k_t, \tag{38}
$$

$$
\frac{\eta \tilde{z}_t^{-\eta-1}}{v_t} (q_{d,t} + \tau q_{f,t}) = \frac{-\dot{v}_t}{v_t} + \rho + \delta, \tag{39}
$$

where v_t is the costate variable. Since the two countries are symmetric, $x_{d,t} = x_{f,t} = x_t$, $E_{d,t} = E_{f,t} = E_t$, $X_{d,t} = X_{f,t} = X_t$, $p_{d,t} = p_{f,t} = p_t$. In the following, we denote $q_{d,t} = q_t$ and

$q_{f,t} = \breve{q}_t$. From (36) and (37) and using $q_t/x_t + \ \breve{q}_t/x_t = 1/n$ yields

$$\left((\alpha - 1) \frac{q_t}{x_t} + 1 \right) = \frac{2n - 1 + \alpha}{n(1+\tau)} \equiv \theta_d \tag{40}$$

$$\left((\alpha - 1) \frac{\breve{q}_t}{x_t} + 1 \right) = \tau \frac{2n - 1 + \alpha}{n(1+\tau)} \equiv \theta_f = \tau\theta_d \tag{41}$$

which allows us to write the price of exported goods as

$$p_t = \frac{\tilde{z}_t^{-\eta}}{\theta_d} = \frac{\tau \tilde{z}_t^{-\eta}}{\theta_f}$$

where θ_d and θ_f are the markups charged in the domestic market and in the export market, respectively. Equations (40) and (41) allow us to rewrite (36) and (37) as follows

$$\theta_d \frac{E_t}{X_t^{\alpha}} \, x_t^{\alpha-1} = \frac{1}{\tilde{z}_t^{\eta}} \quad \text{and} \quad \tau\theta_d \frac{E_t}{X_t^{\alpha}} \, x_t^{\alpha-1} = \frac{\tau}{\tilde{z}_t^{\eta}}.$$

Multiplying the above equations by q_t and $\breve{q}_t$ and summing up we obtain

$$\frac{q_t + \tau\breve{q}_t}{\tilde{z}_t^{\eta}} = n \left(\theta_d \frac{q_t}{x_t} + \tau\theta_d \frac{\breve{q}_t}{x_t} \right) \frac{E_t}{n} \left(\frac{x_t}{X_t} \right)^{\alpha}.$$

Using $x_t = \left((1/\tilde{z}_t^{\eta}) \, (X_t^{\alpha}/\theta_d E_t) \right)^{\frac{1}{\alpha-1}}$, it is easy to prove that $(x_t/X_t)^{\alpha} = z/(M\bar{z})$. From (40) and using $q_t/x_t + \ \breve{q}_t/x_t = 1/n$, we obtain

$$\frac{q_t + \tau\breve{q}_t}{\tilde{z}_t^{\eta}} = \theta_x e_t \frac{z}{\bar{z}} \tag{42}$$

where

$$\theta_x = \left(\theta_d \frac{q_t}{x_t} + \tau\theta_d \frac{\breve{q}_t}{x_t} \right)$$

is the inverse of the average markup in the open economy. In the main text, we rearranged it to express the average markup as a proportion of the domestic markup.

In order to derive the steady-state growth rate, from the FOCs we have $v_t = (Ak_t)^{-1} = (AD_t\tilde{z}_t)^{-1}$, where we used the externality definition (5). Substituting this into (12) and using the definition $D_t = \tilde{Z}_t/(\tilde{z}_t^c)^{\hat{\eta}} = \bar{z}/z = D$ we obtain the growth rate (24). Finally, from the innovation technology we have

$$h = \frac{g}{AD} = (\eta\theta_x e - \hat{\rho}) \frac{z}{\bar{z}}.$$

D Exit in open economy

The productivity cutoff is determined solving the following equation

$$\pi_t(\tilde{z}^*) = \left(p_t - \frac{1}{\tilde{z}_t^{*\eta}}\right) q_t + \left(p_t - \frac{\tau}{\tilde{z}_t^{*\eta}}\right) \breve{q}_t - h_t - \lambda = 0$$

Using $p_t = 1/\theta_d \tilde{z}_t^{\eta}$ and $h_t = \eta\theta_x e_t z_t - \hat{\rho}$ obtained from (38) and (39) yields

$$\frac{1}{\theta_d}\frac{q_t + \breve{q}_t}{\tilde{z}_t^{*\eta}} - \left(\frac{q_t + \tau\breve{q}_t}{\tilde{z}_t^{*\eta}}\right)(1+\eta) + \hat{\rho} - \lambda = 0.$$

With the same procedure used to derive (42) we obtain

$$\frac{q_t + \breve{q}_t}{\tilde{z}_t^{*\eta}} = \theta_d e_t\, z_t/\bar{z}_t$$

which, together with (42), yields

$$\left(1 - (1+\eta)\,\theta_x\right) e_t z_t^*/\bar{z}_t + \hat{\rho} - \lambda = 0.$$

This expression is similar to (EC) except for the markup $1/\theta_x$ in the place of $1/\theta$.

E Welfare and the pro-competitive effect

Pro-competitive effect. Differentiating θ_x with respect to τ

$$\frac{\partial\theta_x}{\partial\tau} = -\frac{2(\tau-1)\left(2n-1+\alpha\right)^2}{n\left(1+\tau\right)^3\left(1-\alpha\right)} \leq 0,$$

thus trade liberalization reduces the markup. Moreover, differentiating it with respect to n we find

$$\frac{\partial^2\theta_x}{\partial\tau\partial n} = -\frac{2(\tau-1)\left(2n-1+\alpha\right)}{n^2\left(1+\tau\right)^3} < 0,$$

which implies that the competition effect of incremental trade liberalization is decreasing in the number of firms n.

Welfare. In the open economy, optimal quantities consumed in the domestic and foreign markets are

$$q_t = \left(1 - \frac{1}{\tilde{z}_t^{\eta} p_t}\right)\frac{x_t}{1-\alpha}$$

$$\breve{q}_t = \left(1 - \frac{\tau}{\tilde{z}_t^{\eta} p_t}\right)\frac{x_t}{1-\alpha},.$$

respectively. Since total consumption $x_t = n(q_t + q_{x,t})$, multiplying both sides of both equations by n, adding up right and left hand side terms and substituting p_t by the inverse demand function, we get

$$x_t^{1-\alpha} = \theta_d \frac{E}{X_t^{\alpha}} \tilde{z}_t^{\eta},$$

where $\theta_d = \frac{2n+\alpha-1}{n(1+\tau)}$. For $\tau \leq \bar{\tau} = n/(n+\alpha-1)$, it can be easily proved that $\theta_d < \theta_x$.

Substituting x_t in the definition of composite consumption good X_t and using the definition of average productivity $\bar{z}$, we get

$$X_t = (\bar{z}M)^{\frac{1-\alpha}{\alpha}} \theta_d E \ e^{\eta g t}.$$

The solution for X_t can now be substituted into the discounted utility (1). Using (6) yields the steady state welfare function

$$\begin{aligned}\rho U &= \rho \int_0^{\infty} (\ln X_t + \beta \ln Y_t) \ \mathrm{e}^{-\rho t} \ \mathrm{d}t \\ &= \frac{1-\alpha}{\alpha} \ln(\bar{z}M) + \ln(\theta_d n M e) + \beta \ln(\beta n M e) + \frac{\eta g}{\rho}.\end{aligned}$$

Next, we sign the different components of the static welfare effects of selection in (27). Differentiating the definition of $\bar{z}$, we get

$$\frac{\partial \bar{z}}{\partial z^*} = (\bar{z} - z^*) \frac{f(z^*)}{1 - \Gamma(z^*)} > 0.$$

Differentiating (19), we get

$$\frac{\partial M}{\partial z^*} = -\delta \frac{f(z^*)}{(1 + \delta - \Gamma(z^*))^2} < 0.$$

Differentiating (MC^T), we get

$$\frac{\partial e}{\partial M} = -\frac{1}{(\beta + (1+\eta)\theta_x) n M^2} < 0.$$

Given that $\partial M / \partial z^* < 0$, the sign of the fixed cost component in (27) is the opposite of the sign of

$$\frac{1}{e}\frac{\partial e}{\partial M} + \frac{1}{M} = \frac{1}{M} - \frac{1}{(\beta + (1+\eta)\theta_x) n M^2 e} = \frac{(\hat{\rho} - \lambda) n}{(\hat{\rho} - \lambda) n M + 1},$$

which is strictly negative since $\hat{\rho} < \lambda$ by Assumption 1 (b). The last term results from using (MC^T) to substitute for e.

Finally, the productivity/LFV trade off in (27) can be written as

$$\frac{1}{\bar{z}}\frac{\partial \bar{z}}{\partial z^*}+\frac{1}{M}\frac{\partial M}{\partial z^*}=\frac{(M-z^*/\bar{z})f(z^*)}{(1-\Gamma(z^*))(1+\delta-\Gamma(z^*))},$$

which is positive iff $M > z^*/\bar{z}$. Let us assume the entry distribution is Pareto with tail parameter κ, $\kappa > 1$. In this case, the (EC^T) condition becomes

$$e=\frac{\frac{\lambda\kappa}{\kappa-1}-\hat{\rho}}{1-(1+\eta)\,\theta_x}.$$

Combining it with the (MC^T) condition

$$\frac{1}{nM}=\frac{\Big(\beta+(1+\eta)\theta_x\Big)\Big(\frac{\lambda\kappa}{\kappa-1}-\hat{\rho}\Big)}{1-(1+\eta)\,\theta_x}-\hat{\rho}+\lambda=\frac{\Big(\beta+(1+\eta)\theta_x\Big)\Big(\frac{\lambda\kappa}{\kappa-1}-\hat{\rho}\Big)+\Big(1-(1+\eta)\,\theta_x\Big)\Big(\lambda-\hat{\rho}\Big)}{(1+\eta)\theta_x}$$

$$=\frac{\Big((1+\beta)\kappa+(1+\eta)\theta_x-1\Big)\lambda-(\kappa-1)(1+\beta)\hat{\rho}}{(\kappa-1)\Big(1-(1+\eta)\theta_x\Big)}.$$

Then, $M < (\kappa-1)/\kappa$ iff

$$\hat{\rho}>\frac{(1+\beta)\kappa\lambda-\Big(1-(1+\eta)\theta_x\Big)(\kappa+\lambda)}{(\kappa-1)(1+\beta)}\equiv\bar{\rho}.$$

Notice that this condition always hold if λ is large enough. Notice also that this condition is equivalent to $\delta > A\bar{\rho}-\rho\equiv\bar{\delta}$, which could be negative is A is small enough.

F Derivations for the generalized model

Variable costs. We want to derive the variable costs for non-exporters $\tilde{z}^{-\eta}q$ and exporters $\tilde{z}^{-\eta}q_x$. The first order condition for non-exporters will be again (10), simply stating that their price, given by the inverse market demand, is equal to a markup θ over marginal costs $\tilde{z}^{-\eta}$

$$\tilde{z}_t^{-\eta}=\theta\frac{E}{X_t^{\alpha}}x_t^{\alpha-1} \tag{43}$$

The exporter will solve the same problem as in the open economy version for the benchmark model (Section 3), and face a price equal to a markup θ_x over marginal costs $\tilde{z}^{-\eta}$, that is

$$\tilde{z}_t^{-\eta}=\theta_x\frac{E}{X_t^{\alpha}}x_{xt}^{\alpha-1} \tag{44}$$

Rearranging we obtain $x_t^{\alpha}=\tilde{z}_t^{\hat{\eta}}\left(\theta E/X_t^{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$ and $x_{xt}^{\alpha}=\tilde{z}_t^{\hat{\eta}}\left(\theta E/X_t^{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$, where $\hat{\eta}\equiv\eta\alpha/(1-\alpha)$. Substituting these into (2) we obtain

$$X_t^{\alpha}=\left(\theta^{\frac{\alpha}{1-\alpha}}\int_0^{M_d}\tilde{z}_{jt}^{\hat{\eta}}\,\mathrm{d}j+\theta_x^{\frac{\alpha}{1-\alpha}}\int_0^{M_x}\tilde{z}_{jt}^{\hat{\eta}}\,\mathrm{d}j\right)^{1-\alpha}E^{\alpha}=M^{1-\alpha}\bar{p}^{-\alpha}E^{\alpha} \tag{45}$$

where with a slight abuse of notation we temporarily define the mass of exporting firm M_x and the mass of non-exporters M_d. Substituting back into the expressions for x_t^α and x_{xt}^α yields

$$x_t^\alpha = \frac{\theta^{\frac{\alpha}{1-\alpha}} E^\alpha \tilde{z}_t^{\hat{\eta}}}{\left(\theta^{\frac{\alpha}{1-\alpha}} \int_0^{M_d} \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}} \int_0^{M_x} \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j\right)^\alpha} \text{ and } x_{x,t}^\alpha = \frac{\theta_x^{\frac{\alpha}{1-\alpha}} E^\alpha \tilde{z}_t^{\hat{\eta}}}{\left(\theta^{\frac{\alpha}{1-\alpha}} \int_0^{M_d} \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j + \theta_x^{\frac{\alpha}{1-\alpha}} \int_0^{M_x} \tilde{z}_{jt}^{\hat{\eta}} \, \mathrm{d}j\right)^\alpha}.$$

Using $z \equiv \tilde{z}_t^{\hat{\eta}} \, e^{-gt}$ we can write

$$\left(\frac{x_t}{X_t}\right)^\alpha = \frac{\theta^{\frac{\alpha}{1-\alpha}} z}{M \bar{p}^{\frac{\alpha}{\alpha-1}}} \text{ and } \left(\frac{x_{x,t}}{X_t}\right)^\alpha = \frac{\theta_x^{\frac{\alpha}{1-\alpha}} z}{M \bar{p}^{\frac{\alpha}{\alpha-1}}}.$$

Plugging these into (43) and (44) and using the symmetric equilibrium condition $x = nq$ we obtain (28) and (29).

In order to derive the steady-state growth rate, from the FOCs we have $v_t = (Ak_t)^{-1} = (AD_t \tilde{z}_t)^{-1}$, where we used the externality definition (5). Substituting this into (12) and using the definition $D_t = \theta_x e \tilde{z}_t^{c\eta} / y_t^c$, we obtain the growth rate (31). Finally, from the innovation technology we have $h = g/AD$, and substituting D_t we obtain the steady state innovation rate for non exporting and exporting firms (33) and (32).

Cutoff conditions. The cutoff survival productivity for non-exporters, z^*, is given by the exit condition

$$\pi(z^*) = \left(\big(1 - (1+\eta)\theta\big) e + \hat{\rho} \frac{\theta}{\theta_x}\right) \left(\frac{\bar{p}}{p(z^*)}\right)^{\frac{\alpha}{1-\alpha}} - \lambda = 0.$$

which rearranged gives (EC').

The derivation of the export cutoff is a bit more cumbersome. Exporters profit is

$$\pi_x(z) = p(z)\,(q + \breve{q}) - \left(\frac{q + \tau \breve{q}}{\tilde{z}^\eta}\right) - h_x(z) - \lambda - \lambda_x.$$

The total quantity sold by an exporter is $q_x = q + \breve{q}$, and the revenues can be obtained using a similar procedure as in the baseline model described in Appendix C and D, $p_x(z)^{\frac{\alpha}{\alpha-1}} = \theta_x^{\frac{\alpha}{1-\alpha}} z$, and the expression for x_x/X derived above. Total exporters variable cost (29) and total quantity sold are,

$$\tilde{z}^{-\eta} q_x = \frac{q + \tau \breve{q}}{\tilde{z}^\eta} = \theta_x e \left(\frac{\bar{p}}{p_x(z)}\right)^{\frac{\alpha}{1-\alpha}} \tag{46}$$

$$\frac{q + \breve{q}}{\tilde{z}^\eta} = \theta_D e \left(\frac{\bar{p}}{p_x(z_x)}\right)^{\frac{\alpha}{1-\alpha}}. \tag{47}$$

For a given $\tau \in (1, \bar{\tau})$, where $\bar{\tau} = 1/\alpha$ is the prohibitive trade costs, a firm producing variety z will face price $p_x(z)$ in both the domestic and foreign market. If the firm decides to export, profits are

$$\pi_x(z) = (1-(1+\eta)\theta_x)\, e \left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{\alpha-1}} + \hat{\rho}\left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{\alpha-1}} - (\lambda + \lambda_x),$$

where $\hat{\rho} = \frac{\rho+\delta}{A}$. The firm will export only if $\pi_x(z) > 0$, which happens for all $z > z_x^*$, where z_x^* solves $\pi_x(z_x^*) = 0$. However, this condition is not enough for a firm be an exporter. Let us analyze the situation where a firm producing a traded variety would like to deviate by only serving the domestic market, even when the foreign firm exports towards the domestic market. In this case, profits will become

$$\hat{\pi}_x(z) = \big(p_x(z) - \tilde{z}^{-\eta}\big)q - h - \lambda = \frac{1-\theta_d}{\theta_d}\ \tilde{z}^{-\eta}q - h - \lambda.$$

Using (46) and (47) we obtain the variable cost in the case the firm decides to produce only for the domestic market,

$$\tilde{z}^{-\eta}q = \tilde{z}^{-\eta}(q - \breve{q}) - \tilde{z}^{-\eta}\breve{q} = \theta_d \frac{\tau - \frac{\theta_x}{\theta_d}}{\tau - 1}\, e \left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{\alpha-1}}.$$

From the first order condition for innovation we obtain

$$h = \left(\eta\theta_d \frac{\tau - \frac{\theta_x}{\theta}}{\tau - 1}\, e - \hat{\rho}\right)\left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{\alpha-1}}$$

implying that

$$\hat{\pi}_x(z) = \left((1-(1+\eta)\theta_d)\ \frac{\tau - \frac{\theta_x}{\theta_d}}{\tau - 1}\right) e \left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{\alpha-1}} + \hat{\rho}\left(\frac{p_x(z_x^*)}{\bar{p}}\right)^{\frac{\alpha}{\alpha-1}} - \lambda.$$

A firm will decides to export only if $\pi_x(z) \geq \hat{\pi}_x(z)$, which requires

$$\left((1-(1+\eta)\theta_x) - (1-(1+\eta)\theta_d)\ \frac{\tau - \frac{\theta_x}{\theta_d}}{\tau - 1}\right) e \left(\frac{p_x(z)}{\bar{p}}\right)^{\frac{\alpha}{\alpha-1}} \geq \lambda_x.$$

It is easy to see that for $\tau \in (1, 1/\alpha)$,

$$\left((1-(1+\eta)\theta_x) - (1-(1+\eta)\theta_d)\ \frac{\tau - \frac{\theta_x}{\theta_d}}{\tau - 1}\right) \equiv \Theta,$$

which, for $\tau \in (1, 1/\alpha)$, is decreasing in τ. Hence the condition $\pi_x(z_x^*) = \hat{\pi}_x(z_x^*)$, which is (XC) in the text, defines the export cutoff.

Growth and Welfare. In order to derive the steady-state growth rate, from the FOCs we have $v_t = (Ak_t)^{-1} = (AD_t\tilde{z}_t)^{-1}$, where we used the externality definition (5). Substituting this into (12) and using the definition $D_t = \theta_x e \tilde{z}_t^{c\eta}/y_t^c$, we obtain the growth rate (31). Finally, from the innovation technology we have $h = g/AD$, and substituting D_t we obtain the steady state innovation rate for non exporting and exporting firms (33) and (32). Using (45) into (1) we obtain:

$$\begin{aligned}\rho U &= \rho \int_0^\infty (\ln X_t + \beta \ln Y_t)\, \mathrm{e}^{-\rho t}\, \mathrm{d}t \\ &= \ln\left(\frac{nM^{\frac{1}{\alpha}}e}{\bar{p}}\right) + \beta \ln(\beta n M e) + \frac{\eta g}{\rho},\end{aligned}$$

which is the steady state equilibrium welfare.

G