International trade and risk aversion elasticities

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Abstract

This paper analyses, for the first time, risk-taking behaviour (under no-hedging possibilities) using two-moment model for a firm linked to both domestic and foreign markets simultaneously – in the first case, the firm is simultaneously serving both domestic and foreign markets; while in the second case, it is serving the domestic market, using the imported intermediate products as inputs. Uncertainties in the spot exchange rates impart production decisions of the firm in either case. In sum, the firm’s elasticity of risk aversion with respect to the standard deviation (or the mean) of the firm’s end-period random profit determines the direction of the impact of exchange rate volatilities on trade. This simple framework can be helpful to answer the seemingly non-intuitive empirical results.

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**Keywords:** Two-moment model; exports; imported intermediate inputs; exchange rate risk; elasticities of risk aversion.
1. Introduction

One of the most contentious issues in international economics is the effect of the uncertainties associated with the exchange rate fluctuations on the international firms, and therefore, on the entry and exit into export market (extensive margin) as well as on the volume of international trade (intensive margin). There is relatively little evidence on the responses of exports, due to the exchange rate volatilities, at the level of firms or individual producers. Exceptions include Cheung and Sengupta (2013); Berman et al. (2012); Fitzgerald and Haller (2012); Arize et al. (2008); Greenaway et al. (2007); Cheung (2005); Bernard and Jensen (2004a, b); Bugamelli and Infante (2003); and Forbes (2002). However, most of these empirical studies fail to explain how and why should the higher volatilities in foreign exchange rate lead to a reduction in international trade.

Among these, Cheung and Sengupta (2013) examined the impact of exchange rate changes on the volume of exports of the Indian manufacturing firms, i.e. at the intensive margin. They have found negative and significant effects on firms’ export shares of exchange rate appreciation. However, Cheung and Sengupta (2013) have also noted, for their sample of Indian firms from 2000 to 2010, that the exchange rate fluctuations have differential firm-specific effects on the export shares, with an asymmetric response towards the exchange rate movement.

There are, unfortunately, paucity of the theoretical contributions linking exchange rate risk and trade using portfolio theory, without any hedging possibilities. Among these, most notable have been Broll and Eckwert (1999) and Broll et al. (2006) in this context. The main message from both of these papers was that the impact on the export shares of an international firm owing to an increased exchange rate volatility should be contingent upon the degree of relative risk aversion of the firm.

However, in all of these theoretical contributions, the exporting firm under consideration cannot simultaneously serve both domestic and foreign markets. Given this, we have applied the mean-standard deviation approach (Eichner and Wagener 2011; 2014; Broll et al. 2015; and so on) in order to examine an international firm that serves simultaneously both domestic and export (foreign) markets. Risk preferences only contribute to alter the allocation of production between these two activities, keeping the total production unchanged. Therefore, we do not impose any
specific a priori assumption about the firm, for the sake of simplicity and ease of interpretation. This is one of the major contributions of our modelling approach, identifying the ‘gap’ the existing related theoretical literature that applied portfolio theory to explain a risk-averse firm’s export behaviour owing to exchange rate risks. Therefore, Sections 2 and 2.1 of this paper examine the optimal production and export decisions of a risk-averse firm facing exchange rate uncertainty under mean-variance preferences.

Meyer (1987) demonstrated that if the random variables under some choice set differ only in terms of the scale (standard deviation) and location (mean) parameters of the distribution, then an expected utility ranking of these random variables can be based on the means and standard deviations of the alternatives’ risky outcomes, if uncertainty represented by a stochastic variable and the decision maker’s decision variable interact in a linear way. The decision problem of a risk-averse firm can also be characterised by such a linear interaction between random influence of the (spot) foreign exchange rate and the production decision using mean – standard deviation modelling approach. This approach allows us to model the firm being linked to both domestic and the world market simultaneously, in contrast to the existing theoretical contributions.

Davis (1989) pointed out that increase in risk does have an unambiguous substitution effect, wherein the decision maker reacts by switching to less risky alternative, and an ambiguous income effect. Therefore, using the concept of risk aversion elasticity we show how changes in the mean or the standard deviation affect the firm’s decisions on domestic production and trade.

As demonstrated in Goldberg et al. (2010), for a large developing country like India, expansion in the sales of domestic products by the Indian manufacturing firms during the liberalised regime, even at the extensive margin, can be attributed largely to the increased imports of intermediate inputs from abroad by the domestic firms. Therefore, an immediate subsequent question that should arise in our mind is how far the volatilities in foreign exchange rate at the import market would affect domestic production? This paper, for the first time, also devotes itself to the analysis of the optimal import decision for an international firm that imports intermediate input from abroad for domestic production and facing exchange rate risks in the import market, using portfolio theory. Therefore, Sections 3 and 3.1 investigate optimal import
decision for such a risk-averse firm facing exchange rate uncertainty under mean-variance preferences.

We aim at systematic analyses of economic response in the mean-variance framework. All comparative static effects are described by the marginal rate of substitution between risk and return, i.e. the willingness to pay for a reduction in risk.

2. Firm serving both domestic and export markets

Consider a firm that serves both the domestic market and a foreign country market under exchange rate uncertainty. There is one period with two dates. At \( t = 0 \) (i.e. at the beginning of the period), the firm produces a single good, according to a cost function, \( C(q) \), with \( C'(q) > 0 \), and \( C''(q) > 0 \). We suppress the riskless interest rate by compounding all operating profits to their future values at the end of the period (i.e. \( t = 1 \)).

The spot exchange rate is expressed in units of the home currency per unit of foreign currency. The foreign spot exchange rate, \( \delta \) is random, being distributed according to an objective cumulative distribution function over support \([\underline{\delta}, \overline{\delta}]\). \( p_x \) is the per-unit price of exportable \((x)\) in units of foreign currency; while \( p_y \) is the per-unit price of the product \((y)\) sold in domestic market, in units of domestic currency. Both prices are given parametrically.

With total output \( q = x + y \), the random operating profit of the firm is given by

\[
\Pi = \delta p_x x + p_y y - C(q)
\]

The domestic firm’s preferences are given by a two-parameter utility function

\[
V = V(\mu, \sigma)
\]  \hspace{1cm} (1)

Where \( \mu = p_x p_x x + p_y y - C(x + y) \) and \( \sigma = \sigma_y p_y x \) denote, respectively, the expected value and the standard deviation of random profit.
To have a non-trivial decision problem, we assume that $p_\sigma < p_y < \bar{p}$. We require the following properties to be satisfied for all $\mu, \sigma$: $V_\mu(\mu, \sigma) > 0$, $V_\sigma(\mu, \sigma) < 0$.

The marginal rate of substitution (MRS) between risk and return is defined by

$$S = \frac{V_\mu(\mu, \sigma)}{V_\sigma(\mu, \sigma)}$$

$S$ is the two-parameter equivalent to Arrow–Pratt measure of absolute risk aversion. Indifference curves in $(\sigma, \mu)$-space are upward-sloping, with their slopes measuring risk aversion.

The domestic firm solves the following problem

$$\max_{x \in \mathbb{R}^+} V(\mu(x, y), \sigma(x, y))$$

When we consider interior solutions of this decision problem, the optimum is then determined by

$$V_\mu(\mu^*, \sigma^*) (\mu^* p_x - C'(x^* + y^*)) + V_\sigma(\mu^*, \sigma^*) \mu^* p_x = 0 \quad (3)$$

$$V_\mu(\mu^*, \sigma^*) \left(p_y - C'(x^* + y^*)\right) = 0 \quad (4)$$

The second-order condition is satisfied due to the quasi-concavity of $V(\mu, \sigma)$. In the optimum we obtain for total production $C(x^* + y^*) = p_y$. Hence, we establish our first result.

**Proposition 1.** The firm optimally chooses its total output level, $q^* = x^* + y^*$, at which the marginal cost of production $C'(x^* + y^*)$, is equated to the domestic market price, $p_y$. However, the allocation of production depends on the firm’s risk preferences.

### 2.1 Comparative static effects for exporting firm

The purpose of this section is to demonstrate the comparative static properties of the model in relative terms, i.e., the comparative statics depend on how sensitively the firm’s risk aversion responds to changes in expected final profit and risk.
We are interested in how optimal risk taking responds to changes in the world market price. Our first result deals with the comparative statics for changes in the distribution of the (foreign) spot exchange rate risk. Before analysing a change in exchange rate risk and its impact upon trade and domestic sales, let us introduce the concept of ‘risk aversion elasticity’.

**Definition 1.** The elasticity of the marginal rate of substitution between risk and return with respect to the standard deviation of the firm’s end of period profit is

\[ \epsilon(\mu, \sigma) = \frac{\partial \delta(\mu, \sigma)}{\partial \sigma} \frac{\sigma}{\delta(\mu, \sigma)} \quad \sigma > 0 \]

The elasticity indicates the percentage change in risk aversion over the percentage change in final profit standard deviation, keeping the mean of the end-period profit \( \mu \) constant.

By using the marginal rate of substitution, \( \delta(\mu, \sigma) \), the first-order condition of the firm’s international trade problem becomes

\[ (\mu E - p_x) \sigma = S'(\mu E, \sigma) \]

The left hand side (5) is merely the expected spread between foreign and domestic market prices. Thus, when the expected spread is non positive, the firm will never export some of its production.

**Proposition 2.** A risk-averse firm may not reduce its optimal export \( x^* \) upon an increase in risk, \( \sigma_x \), and may even export more (optimally) when the risk aversion elasticity is less than \(-1\), i.e., \( \epsilon(\mu^*, \sigma^*) < -1 \).

**Proof.** Implicit differentiation of (5) with respect to \( \sigma_x \) leads to

\[ \text{sgn} \left( \frac{\partial x}{\partial p} \right) = \text{sgn} \left( S' + \sigma_x \frac{\partial x}{\partial p} \frac{\partial \sigma_x}{\partial \sigma} \right) = \text{sgn} \left( 1 + \epsilon(\mu^*, \sigma^*) \right) \]

The result shows that a risk averse firm may optimally export more when risk increases. This happens if and only if the elasticity of risk aversion is less than \(-1\).
An increase in price risk leads to a direct and indirect effect. The substitution effect (direct effect) is unequivocally negative, i.e., a higher price risk implies lower exports, and higher domestic sales. The income effect can be negative or positive, because it encompasses the possibilities that the exportable is priced at higher (lower) domestic currency per unit of foreign currency, and such income effect can even dominate the substitution effect, if and only if the elasticity of risk aversion is less than $-1$, i.e., $\left(\frac{\partial x^*}{\partial \sigma} \right) = - \left(\frac{\partial y^*}{\partial \sigma} \right) > 0$. That is, the firm then reacts to an increase in exchange rate risks by exporting more.

**Definition 2.** The elasticity of the marginal rate of substitution between risk and return with respect to the mean of final operating profit is defined as

$$
\epsilon_{\mu}(\mu, \sigma) = \frac{\frac{\partial \sigma_{\mu}}{\partial \mu}}{\sigma_{\mu}}
$$

The elasticity $\epsilon_{\mu}(\mu, \sigma)$ indicates the percentage change in risk aversion over the percentage change in expected final profit, keeping the standard deviation of the firm’s end-period profit ($\sigma$) constant.

Now we examine the relationship between the firm’s export and domestic sales with respect to a change in the expected foreign exchange rate, i.e., $\mu$. From the first order condition (5) applying the implicit function theorem we get

$$
\text{sgn} \left( \frac{\partial x^*}{\partial \mu} \right) = \text{sgn} \left( 1 - \sigma^* \frac{\partial \sigma^*}{\partial \mu} \frac{\partial \mu}{\partial \sigma} \right)
$$

By extension the last term on the right hand side we obtain

$$
\text{sgn} \left( \frac{\partial x^*}{\partial \mu} \right) = \text{sgn} \left( 1 - \epsilon_{\mu}(\mu^*, \sigma^*) \frac{R^*}{\mu} \right)
$$

Where $R^* = S^*(\mu^*, \sigma^*) \sigma^*/\mu$. We arrive at our next proposition.

**Proposition 3.** An increase in the expected foreign exchange rate at a given risk does not necessarily cause a risk-averse firm to decrease export. If and only if the elasticity of the
marginal rate of substitution between risk and return with respect to $\mu$ is less than 1, i.e., $\epsilon_\mu < 1$, exports of the firm will increase.

**Proof.** In the optimum risk aversion elasticity $\epsilon_\mu$ is less than 1, if and only if $R^* \in (0, 1)$. We are going to show this below.

With definition of the marginal rate of substitution and the first order condition we obtain

$$R^* = \frac{\sigma_\mu p_x \bar{x} \cdot \left(\mu_\mu p_x - C'(x^* + y^*)\right)}{\sigma_\mu p_x \bar{x} + p_y y^* - C'(x^* + y^*)}$$

$$= \left[\left(\mu_\mu p_x - C'(x^* + y^*)\bar{x}\right)\left(p_\mu p_x \bar{x}^* + p_y y^* - C'(x^* + y^*)\right)\right] < 1$$

Since $C'(x^* + y^*) - C'(x^* + y^*) \bar{x} \leq 0$. ■

Hence, the impact on firm’s optimal export behaviour depends once again on the interaction between the income and the substitution effect.

To sum up, our results can be generalised by stating that the firms with sufficient financial resources and therefore, with greater risk-taking capacity (i.e. with $\epsilon_\mu < 1$ and $\epsilon_x < -1$) may not necessarily reduce exports at the intensive margin owing to the exchange rate risks. This explains the asymmetries in the responses of different firms on their export activities to the fluctuations in foreign exchange rate.

### 3. Import of intermediate input and domestic production

Let us now consider a firm that is importing intermediate inputs from abroad for production in the domestic market. The firm is facing uncertainties in the foreign spot exchange rate of the import market, which is modelled again by a positive random variable $\tilde{\epsilon}$, having distributed according to an objective cumulative distribution function over support $[\underline{\epsilon}, \overline{\epsilon}]$. The spot exchange rate is again measured in terms of the home currency per unit of foreign currency. $p_x$ now denotes the per-unit price of imported inputs ($x$) in units of foreign currency.
\[ \dot{x} = p_x f(x) - \dot{p}_x x \]  
(9)

That is, domestic production \( y = f(x) \), where \( f(x) \) is concave production function with \( f'(x) > 0, f''(x) < 0 \). \( x > 0 \) is the amount of intermediate product imported. The firm’s preferences are again given by the two-parameter utility function \( V = V(\mu, \sigma) \), where

\[ \mu = p_x f(x) - \mu_x \dot{p}_x x \]  
(10)

and

\[ \sigma = \alpha_x \dot{p}_x x \]  
(11)

The importing domestic firm solves the following problem (w.r.t. (10) and (11))

\[ \max_{x \in \Omega} V(\mu(x), \sigma(x)) \]

The first-order condition (FOC) becomes

\[ V'_{\mu}(\mu^*, \sigma^*)(p_x f'(x^*) - \mu_x \dot{p}_x) + V'_{\sigma}(\mu^*, \sigma^*) \sigma_x \dot{p}_x = 0 \]

As before, by using the marginal rate of substitution, \( S(\mu, \sigma) \), the FOC becomes,

\[ \left( p_x f'(x^*) - \mu_x \dot{p}_x \right) / \sigma_x \dot{p}_x = S'(\mu^*, \sigma^*) \]  
(12)

Since \( S'(\mu^*, \sigma^*) > 0 \), the LHS states that the value of marginal product is greater than the expected marginal cost of imports at the optimum, which essentially implies that the firm likes to be compensated for exchange rate risks at the import market.

### 3.1 Comparative static effects for importing firm

As in Section 2.1, here also we are going to trace out comparative static responses for changes in the distribution of the (foreign) spot exchange rate risk, using the definitions of the risk aversion elasticities (already discussed in Section 2.1).
For simplicity, let us take \( p_x = p_y = 1 \). Then differentiating the FOC w.r.t. \( \sigma_x \) yields

\[
\frac{\partial x^*}{\partial \sigma_x} = -\sigma_x \Phi \frac{f'(x^*)}{x^*} \tag{13}
\]

By definition, \( f''(x^*) < 0 \). Hence, with the negative sign before, only sign of \( \Phi \) determines the intensive margin of firm’s import, at the optimum, in response to the higher volatilities in exchange rate at the import market (i.e. sign of \( \frac{\partial x^*}{\partial \sigma_x} \)).

\[
\text{sgn} \left( \frac{\partial x^*}{\partial \sigma_x} \right) = \text{sgn} \Phi
\]

\[
= \text{sgn} \left[ \left( f'(x^*) \frac{\partial x^*}{\partial \sigma_x} - \frac{\partial x^*}{\partial \sigma_x} \frac{\partial^2 x^*}{\partial x^2} \right) \right]
\]

\[
= \text{sgn} \left[ \left( f'(x^*) - \frac{\partial x^*}{\partial \sigma_x} \right) \frac{\partial x^*}{\partial \sigma_x} \frac{\partial^2 x^*}{\partial x^2} \right]
\]

Or,

\[
\text{sgn} \left( \frac{\partial x^*}{\partial \sigma_x} \right) = -\text{sgn} \left[ \left( f'(x^*) - \frac{\partial x^*}{\partial \sigma_x} \right) \frac{\partial x^*}{\partial \sigma_x} \frac{\partial^2 x^*}{\partial x^2} \right]
\]

\[
= -S^* \text{sgn}[1 + \epsilon_s \mu^*] \tag{14}
\]

Since from the FOC we have, \( (f'(x^*) - \mu_x)/\sigma_x = S^* \) (\( \because p_x = p_y = 1 \)).

Where \( \epsilon_s \) is the elasticity of the marginal rate of substitution between risk and return with respect to the standard deviation of the firm’s end-period \( t = 1 \) profit, defined in Section 2.1.

Hence, \( \frac{\partial x^*}{\partial \sigma_x} > 0 \) when \( \epsilon_s < -1 \). This yields the following proposition.

**Proposition 4:** A risk-averse firm may not reduce its optimal imports of intermediate products upon an increase in risk, \( \sigma_x \), and may even import more (optimally) when the risk aversion elasticity is less than \(-1\), i.e. \( \epsilon_s (\mu^*, \sigma^*) < -1 \).
Similarly differentiating the FOC w.r.t. $\mu$, we have,

$$
\left( \frac{\partial x^*}{\partial \mu} \right) = -\sigma \frac{\psi}{\mu_{\mu}}
$$

(15)

Since $f^*(.) < 0$, with –ve sign before, the sign of $\left( \frac{\partial x^*}{\partial \mu} \right)$ (i.e. the impact on the intensive margin of firm’s import, at the optimum, in response to the higher expected foreign exchange rate at a given risk) depends on the sign of $\psi$. Hence,

$$
\text{sgn} \left( \frac{\partial x^*}{\partial \mu} \right) = \text{sgn} \psi
$$

$$
= \text{sgn} \left( - \frac{1}{\sigma} \cdot \frac{\partial S^*}{\partial \mu} \times \frac{\partial \mu}{\partial \mu} \right) = \text{sgn} \left( - \frac{1}{\sigma} + \left( \frac{S^*}{\mu^*} \right) x^* e_\mu (\mu^*, \sigma^*) \right)
$$

$$
= \frac{1}{\sigma} \text{sgn} \left( -1 + \left( \frac{x^*}{\mu^*} \right) \sigma^* e_\mu (\mu^*, \sigma^*) \right).
$$

Since we also have $x^* = \frac{\sigma^*}{\mu}$

$$
= \frac{1}{\sigma} \text{sgn} \left( R^* e_\mu (\mu^*, \sigma^*) - 1 \right)
$$

(16)

Where $e_\mu$ is the elasticity of the marginal rate of substitution between risk and return with respect to the mean of the firm’s end-period ($t = 1$) profit, defined above.

Now, $R^* = \left[ S^*(\mu^*, \sigma^*) \left( \sigma^*/\mu \right) \right]$ can be shown as lying between 0 and 1. This is because,

$$
R^* = \left( \frac{f(x^*) - \mu^*}{\sigma^*} \right) \left( \frac{\sigma^* x^*}{f(x^*) - \mu^* x^*} \right)
$$

$$
= \frac{x^* f(x^*) - \mu^* x^*}{f(x^*) - \mu^* x^*} < 1 \text{, since } f'(x^*) < \frac{f(x^*)}{x^*}$$
This yields $\left( \frac{\sigma^*}{\mu^*} \right) > 0$ if and only if $\varepsilon_{\sigma}^*(\mu^*, \sigma^*) > 1$. Therefore, we have the following proposition.

Proposition 5: An increase in the expected foreign exchange rate at a given risk does not necessarily cause a risk-averse firm to decrease import of intermediate products. If the elasticity of the marginal rate of substitution between risk and return with respect to $\mu$ is greater than 1, imports of intermediate inputs by the firm will increase.

4. Summary

The theoretical analyses carried out in this paper examine the optimal production decisions of the international firm linked to both domestic and foreign markets simultaneously. In the first case, the firm under consideration performs domestic production and simultaneously serves the export market, while we have also considered another scenario when the firm only serves domestic market, but using imported inputs from abroad. In both cases, the firm faces international price risk owing to the uncertainties in the foreign (spot) exchange rate. Using a mean – standard deviation model, we have shown that the impact of an increase in the exchange rate volatilities (expectation and/or variances) on the risk-averse firm’s production decision is ambiguous. Whether or not should the firm produce more for domestic market depends on the firm’s elasticity of risk aversion with respect to the standard deviation (or the mean) of the firm’s end-period random profit. This application of the robust elasticity concept explains asymmetric responses of the firms (owing to the impact of exchange rate volatilities), in terms of their trade shares at the intensive margin, as the outcome of the unbalanced risk aversion elasticities over time. Therefore, the policymakers should not focus exclusively on exchange rate policies for export-oriented industrialisation, neglecting the other catalytic factors such as labour market reform that can promote greater participation of the workforce and benefit exporting industries (Mukherjee 2016).
References


