Research Paper 2018/02

The attitude of multinationals towards risk

By

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This version: February 2018

Abstract

This paper extends the decision problem of a multinational regarding how much to invest abroad optimally under uncertainties stemmed from the exchange rate movements, with the presence of a correlated background risk, in a two moment decision model. This framework is based upon the utility from the expected value and the standard deviation of the uncertain random total profit of the multinational firm. This modelling approach allows us to explore not only how much a risk averse investor optimally invests abroad when facing uncertainties regarding the exchange rate movements; but also to discover how does (and under what conditions) any perturbation in the background risk (which is linearly related to the endogenous exchange rate risks) affect the optimal foreign investment decision for a risk averse investor. All comparative static effects are described in terms of the relative sensitivity of the investor towards risk. This simplest possible analytical framework is useful for explicit empirical estimation of risk aversion elasticities in the literature of multinational firm and FDI decision.

Keywords: Multinational firm; Exchange rate risk; Two moment decision model; Background risk; Risk aversion elasticity.

JEL Classifications: D81; F23; G11.

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1. Introduction

Multinational firms (MNFs hereafter), with preferences towards cross-border investments, are one of the key drivers of the globalisation process. These firms are generally relatively larger, more productive, and having dominance in the host countries in terms of higher market power and bargaining strengths. The higher fixed costs (often sunk) of locating overseas generally comprise uncertainties associated with exchange rate movements (Broll and Wong, 2006; Schmidt and Broll, 2009; Russ, 2007; 2012; and so on). However, other than the exchange rate risks, the investor also faces uncertainties in non-portfolio wealth (such as housing, bequests, and so on) – can be viewed as an aggregated ‘background risk’ (see, for example, Eichner and Wagener, 2009; Franke, Schlesinger, and Stapleton, 2011; Wong, 2012; 2017; Broll and Wong, 2013; and so on), measured in domestic currencies. This second source of uncertainties is also correlated with the price risk brought about by the random exchange rate volatilities. In general, background risk is correlated with market risk brought about by the random exchange rate volatilities.

The unique contribution of this paper is to extend the optimal foreign investment decision problem of a MNF under uncertainties stemmed from the exchange rate movements, with and without the presence of the above-mentioned background risk, in a two moment decision model (see Broll and Mukherjee, 2017; and the references therein). Although this modelling technique sometimes is misinterpreted as the special case of the standard von Neumann–Morgenstern expected utility framework, the two moment decision making modelling approach is completely different and a novel-yet-simplest approach. The reason is: when the random variables under some choice set differ only in terms of the scale (standard deviation) and location (mean) parameters of the distribution, then an expected utility ranking of these random variables can be based on the means and standard deviations of the alternatives’ risky outcomes, if uncertainty represented by a stochastic variable and the decision maker’s decision variable interact in a linear way (Meyer, 1987).

The two moment modelling framework is based upon the utility from the expected value and the standard deviation of the uncertain random total profit of the MNF, without any hedging opportunities. This flexible framework allows us to explore not only how much a risk averse
MNF invests abroad optimally when facing uncertainties regarding the exchange rate movements; but also to discover how does (and under what conditions) any perturbation in the background risk (which is linearly related to the endogenous exchange rate risks) affect the optimal foreign investment decision for a risk averse MNF. The major advantage of the mean–standard deviation model lies in its simplicity and ease of interpretation. Its effects can be illustrated in terms of risk and returns, and such models remain two-dimensional even with multidimensional risks or choice variables. This approach enables us to directly model the multinational’s decision problem without any specific assumptions on the higher-order and cross derivatives of the utility function.

The purpose of this paper is to contribute to the comparative static results of the \((\mu, \sigma)\) – preference model for an international firm exposed to multiple risks. Several astounding results are shown. For example, increasing the mean (variance), keeping the variance (mean) constant, does not necessarily imply that the MNF is encouraged to invest more in the foreign country under exchange rate risk.

Comparative static effects are described in relative terms: in terms of the relative sensitivity of the MNF towards risk. The paper is organised as follows. Section 2 discusses the modelling framework in details. Section 3 evaluates the optimal foreign investment decision owing to the changes in exchange rate risk distribution only, while section 4 analyses the decision problem not only owing to the changes in the distribution of background risk, but also following the changes in the correlation between the exchange rate risk and the background risk. Section 5 illustrates the significance of the results of the comparative static exercises (i.e. in sections 3 and 4) in terms of a parametric example, and finally section 6 concludes.

2. The model

Our investigation rests on the following set of assumptions: We consider an MNF that invests \(K\) (in domestic currency) abroad. The foreign direct investment is positive under the Inada-conditions. The expression for the firm’s revenue from investing abroad in foreign currency is given by \(R(K)\), with \(R'(K) > 0, R''(K) < 0\). Therefore, the revenue in domestic currency
becomes $\tilde{e}R(K)$, where the exchange rate ($\tilde{e}$), defined in terms of domestic currency per unit of foreign currency, is uncertain. Assuming rental return of capital as $r$ in domestic currency, the rental cost of capital in domestic currency is $rK$. There is an aggregated background risk component, $\tilde{Z}$. Such background risk may be viewed as a weighted average of all random components affecting the investor’s profit within the source (domestic) country, such as uncertainties in non-portfolio wealth (such as housing, bequests, and so on).¹ We measure $\tilde{Z}$ in domestic country’s currency. It results in the uncertain total profit of the multinational firm, which can be written as

$$\tilde{\pi} = \tilde{e}R(K) - rK + \tilde{Z}.$$  

(1)

For analytical simplicity, we assume that the background risk is additive. Expectation of background risk is: $E(\tilde{Z}) = \mu_\tilde{Z}$. Exchange rate movements and background risks are correlated. We are assuming that the multinational firm has no access to any hedging opportunities, i.e. the firm is investing to a low-income developing country.

The preference function of the firm is $U = V(\mu, \sigma)$,² with $V_\mu(\mu, \sigma) > 0$, $V_\sigma(\mu, \sigma) < 0$, and $V(\mu, \sigma)$ is strictly quasi-concave in $(\sigma, \mu)$-space.

Expected profit is given by:

$$\mu = E(\tilde{\pi}) = \mu_\tilde{e}R(K) - rK + \mu_\tilde{Z}.$$ 

Profit risk is defined as follows:

$$\sigma = \sqrt{\sigma_\tilde{e}^2R(K)^2 + \sigma_\tilde{Z}^2 + 2R(K)\text{cov}(\tilde{e}, \tilde{Z})}$$

where $\sigma_\tilde{e}$, $\sigma_\tilde{Z}$ and $\text{cov}(\tilde{e}, \tilde{Z})$ are respectively the standard deviation of exchange rate, standard deviation of background risk, and the correlation between both sources of risk. The degree of

¹ For analytical simplicity, we are not going to focus on the political risks or other sources of uncertainties in the host (i.e. foreign) country in this paper. Future research should be extended along this line.

² The profit function is convex in the random spot exchange rate. This implies that the multinational is risk averse. It is reasonable to assume risk averse behaviour. Even cases like the multinational firm is publicly listed, corporate taxes, costs of financial distress, capital market imperfections and so on (see Froot et al., 1993) all will imply a concave objective function of the MNF. We use risk aversion as an approximation for these imperfections (see, also, Broll and Wong, 2006).
the linear dependence between \( (\tilde{e}, \tilde{Z}) \) is measured by the Pearson correlation coefficient \( \rho = \frac{\text{cov}(\tilde{e}, \tilde{Z})}{\sigma_{e} \sigma_{Z}} \). Therefore, while expected profits exhibit a ‘trend shock’, profit risk does not. Notwithstanding, the standard deviation of profits depends upon both the random (spot) foreign exchange rate, \( \tilde{e} \), and the random background risk component \( \tilde{Z} \); since optimum profits depend upon the realisation of \( \tilde{e} \) and \( \tilde{Z} \). Hence both arguments in the preference function are affected by volatilities in exchange rate movements and the background risks.

The marginal rate of substitution (MRS) between risk and return is defined by

\[
S = -\frac{V_{\sigma}(\mu, \sigma)}{V_{\mu}(\mu, \sigma)}
\]

\( S \) is the two-parameter equivalent to Arrow–Pratt measure of absolute risk aversion. The multinational firm solves the following problem

\[
\max_{(K \geq 0)} V(\mu, \sigma).
\]  

(2)

Before proceeding to the comparative static exercises, let us introduce fewer concepts that will be used in the analyses.

**Definition 1.** The elasticity of the marginal rate of substitution between risk and return with respect to the standard deviation of the multinational firm’s end of period profit is

\[
\epsilon_{\sigma}(\mu, \sigma) = \frac{\partial S(\mu, \sigma)}{\partial \sigma} \frac{\sigma}{S(\mu, \sigma)}, \quad \text{with } \sigma > 0.
\]

The elasticity indicates the percentage change in risk aversion over the percentage change in final profit standard deviation, keeping the mean of the end-period profit \( \mu \) constant.

**Definition 2.** The elasticity of the marginal rate of substitution between risk and return with respect to the mean of final profit is defined as

\[
\epsilon_{\mu}(\mu, \sigma) = \frac{\partial S(\mu, \sigma)}{\partial \mu} \frac{\mu}{S(\mu, \sigma)}
\]
The elasticity $\epsilon_\mu(\mu, \sigma)$ indicates the percentage change in risk aversion over the percentage change in expected final profit, keeping the standard deviation of the firm’s end-period profit $\sigma$ constant.

3. Optimum investment without background risk

To start with, we are assuming background risks, $\tilde{Z}$, is zero, i.e.

$$\pi = \tilde{e}R(K) - rK.$$  

The key comparative static exercise we are going to explore is how much capital investment does a risk averse multinational allocate between domestic and foreign markets when facing uncertainties regarding the exchange rate movements.

When we consider interior solutions of this decision problem, the optimum is then determined by

$$V_\mu(\mu^*, \sigma^*)(\mu_eR'(K^*) - r) + V_\sigma(\mu^*, \sigma^*)\sigma_eR'(K^*) = 0. \quad (3)$$

$V_\mu(\mu^*, \sigma^*) > 0, V_\sigma(\mu^*, \sigma^*) < 0$ (in other words, the MNF exhibits nonsatiation and risk aversion), where the asterisk denotes the optimum. Hence, we are now going to demonstrate the comparative static properties of the model in relative terms, i.e., the comparative statics depend on how sensitively the firm’s risk aversion responds to changes in expected final profit and risk.

We are interested in how optimal investment allocation decision of the multinational firm responds to changes in the world market. Our first result deals with the comparative statics for changes in the distribution of the (foreign) spot exchange rate.

By using the marginal rate of substitution, $S(\mu, \sigma)$, the first-order condition of the multinational firm’s investment decision problem becomes

$$(\mu_eR'(K^*) - r)/\sigma_eR'(K^*) = S(\mu^*, \sigma^*). \quad (4)$$

The left-hand side of this marginal condition (i.e. Equation (4)) describes the slope of the opportunity line; the right-hand side denotes the slope of the indifference curve. Also, it is
easy to infer from Equation (4) that when two firms are investing abroad, but with different
degrees of risk aversion, for example, $S_2 > S_1$ (i.e. firm 2 is more risk averse than firm 1), we
would obtain $K_2^* < K_1^*$.

Now let us first trace out the change in optimum investment abroad owing to the increase in
the exchange rate risk, i.e. of $\sigma_e$.

**Proposition 1.** Higher exchange rate volatility leads to a decrease in optimum foreign
investment if and only if $\epsilon_{\sigma} (\mu^*, \sigma^*) > -1$.

**Proof.** Implicit differentiation of (4) gives

$$\text{sgn}(\partial K^*/\partial \sigma_e) = - \text{sgn}(\mu^*, \sigma^*) R'(K^*) [1 + \epsilon_{\sigma} (\mu^*, \sigma^*)]$$

(5)

Therefore, $(\partial K^*/\partial \sigma_e) < 0$, if and only if $\epsilon_{\sigma} (\mu^*, \sigma^*) > -1$. (Q.E.D.)

Now we are going to examine the relationship between the firm’s optimum investment
allocation decision with respect to a change in the expected foreign exchange rate, i.e., $\mu_e$.

**Proposition 2.** An increase in the expected exchange rate will lead to an increase in optimum
foreign investment if and only if the risk aversion elasticity with respect to expected exchange
rate, $\epsilon_{\mu}$, is less than the unity ($\epsilon_{\mu} < 1$).

**Proof.** From the first order condition (4) applying the implicit function theorem we get

$$\text{sgn} \left( \frac{\partial K^*}{\partial \mu_e} \right) = \text{sgn} \left( 1 - \alpha^* \epsilon_{\mu} (\mu^*, \sigma^*) \right),$$

(6)

where $\alpha^* = \sigma^* S(\mu^*, \sigma^*) / \mu^*$.

With the definitions of the MRS and the F.O.C. we obtain

$$\frac{\mu_e R'(K^*) - r}{R'(K^*)} < \frac{\mu_e R(K^*) - r K^*}{R(K^*)}$$

This is because, by the property of $R(.)$ mentioned earlier we have, $R(K^*)/K^* > R'(K^*)$, from
which the above inequality is easy to obtain. Hence, $\alpha^* \in (0,1)$. 

7
Therefore, from Equation (6), we can easily conclude \((\partial K^* / \partial \mu_e) > 0\) if and only if \(\epsilon_\mu < 1\).

(Q.E.D.)

What are the significance of propositions 1 and 2? Any change in the distribution of the spot exchange rate in the world market leads to an unambiguously negative substitution effect (lower investment due to higher risk) and an income effect (or wealth effect) that could be either positive or negative. Thus, the total effect on \(K^*\) depends on the relative magnitudes of the income and substitution effects.\(^3\)

4. Optimum investment with background risk

From (1) we have the following total profit function with foreign background risk:

\[
\tilde{\pi} = \delta R(K) - rK + \tilde{Z}.
\]

The equivalent F.O.C. is:

\[
\mu_e R'(K^*) - r - \left( \frac{\partial \sigma}{\partial K^*} \right) S(\mu^*, \sigma^*) = 0 \quad (7)
\]

Point to be noted that in this scenario with no hedging possibilities, engaging in FDI is risky. Higher investment increases standard deviation of final profit at the margin, since we assume positive correlation between both sources of risk. Therefore, we have

\[
\left( \frac{\partial \sigma}{\partial K^*} \right) = \left[ \sigma_e^2 + \text{cov}(\tilde{e}, \tilde{Z}) \right] \frac{R'(K^*)}{\sigma} > 0.
\]

Now let us trace out the impact on the decision optimally invest abroad owing to the changes in the distribution of background risk.

Proposition 3. The firm will decrease optimum foreign investment, \(K^*\), upon an increase in background risk \(\sigma_Z\), if and only if \(S_\sigma(\mu^*, \sigma^*) > 0\) holds.

\(^3\) See Broll and Mukherjee (2017) for details.
Proof. From Equation (7), implicit differentiation with respect to (w.r.t. hereafter) $\sigma_Z$ yields
\[
\text{sgn} \left( \frac{\partial K^*}{\partial \sigma_Z} \right) = -\text{sgn} \left[ \left( \frac{\partial \sigma / \partial K^*}{\sigma_Z / \sigma(K^*)} \right) S_\sigma(\mu^*, \sigma^*) \right]
\]
The above expression is less than 0, if and only if $S_\sigma(\mu^*, \sigma^*) > 0$.

In other words, higher degree of background risks will prevent the risk averse investor from investing abroad if and only if.

Moving on to the scenario of an increase in the expected background risk, let us start with defining $S_\mu$. The term $S_\mu$ determines whether the decision maker becomes more or less risk averse in terms of the degree of absolute risk aversion. With $S_\mu < 0$, the MNF shows decreasing absolute risk aversion (DARA).

**Proposition 4.** Foreign investment will increase optimally upon an increase in the expected background risk, if and only if $S_\mu < 0$ holds.

Proof. Implicit differentiation of Equation (7) w.r.t. $\mu_Z$ entails
\[
\text{sgn} \left( \frac{\partial K^*}{\partial \mu_Z} \right) = -\text{sgn} \left( (d\sigma / dK^*) S_\mu(\mu^*, \sigma^*) \right)
\]
The above expression is positive, if and only if $S_\mu(\mu^*, \sigma^*) < 0$, i.e. the MNF’s preferences follow DARA. (Q.E.D.)

Therefore, our comparative static results do not depend both on exchange rate volatility and the correlated background risk simultaneously.

Now let us examine the consequences of an increase in $\text{cov}(\tilde{e}, \tilde{Z})$ on the optimum investment abroad.

**Proposition 5.** The optimum investment abroad will decrease upon an increase in $\text{cov}(\tilde{e}, \tilde{Z})$ if and only if $S_\sigma > 0$ holds true.

Proof. Implicit differentiation of Equation (7) w.r.t. $\text{cov}(\tilde{e}, \tilde{Z})$ yields
\[
\text{sgn} \left( \frac{\partial K^*}{\partial \text{cov}(\tilde{e}, \tilde{Z})} \right) = -\text{sgn} \left[ S_\sigma(\mu^*, \sigma^*) \left( R(K^*) \frac{\partial \sigma}{\partial K^*} \right) \right]
\]
This expression is negative if and only if $S_{\sigma}(\mu^*, \sigma^*) > 0$. (Q.E.D.)

Therefore, with the higher background risk, the investor would optimally behave in more risk averse fashion under exchange rate volatility.

5. A Parametric Example

Let us exemplify our propositions and their significance by a parametric example. We apply the following specific utility function, likewise Saha (1997)

$$U = \mu^a - \sigma^b$$ (8)

The first-order condition of the MNF’s decision problem suggests slope of the opportunity line must be equal to the MRS.

Monotonicity and quasi-concavity of the utility function implies $a > 1, b > 1$.

Our optimization exercise becomes

$$\text{Max } U(\mu, \sigma) = \mu^a - \sigma^b$$

with $\mu = \mu_e R(K) - rK + \mu_Z$

and

$$\sigma = [\sigma_e^2 R(K)^2 + \sigma_Z^2 + 2R(K)\text{cov}(e, Z)]^{1/2}.$$

The first order condition would become:

$$a\mu^{a-1}[\mu_e R'(K^*) - r] - b[\sigma_e^2 R(K^*)^2 + \sigma_Z^2 + 2R(K^*)\text{cov}(\hat{e}, \hat{Z})]^{b-2}\left[\sigma_e^2 + \text{cov}(\hat{e}, \hat{Z})\right]R'(K^*) = 0$$ (9)

According to the definition of risk aversion elasticities, $\varepsilon_\sigma = b - 1, \varepsilon_\mu = 1 - a$. Hence, from the F.O.C. we can derive the following results as corollaries to the propositions 1-5.

**Corollary 1:** Under the preferences given by (8), we have
(a) When $\mu_Z$ increases, the multinational will be induced to invest more if and only if $a > 1$.

Proof.

$$\frac{\partial K^*}{\partial \mu_Z} = a(a - 1)(\mu_e R'(K^*) - r)^{a-1} > 0, \text{ if and only if } a > 1.$$ 

(b) With higher background risk, the multinational will be induced to invest less abroad, if and only if $b > 2$.

Proof.

$$\frac{\partial K^*}{\partial \sigma_Z} = -b(b - 2)\sigma_Z^{b-4}\{\sigma_e^2 + \text{cov}(e, Z)\}R'(K^*) < 0, \text{ if and only if, } b > 2.$$ 

(c) An increase in $\mu_e$ induces the investor into more risk taking behaviour if and only if, $a > 0$.

Proof.

$$\frac{\partial K^*}{\partial \mu_e} = a\left[\mu_e R'(K^*) + \mu_Z - rK^*\right]^{a-1}R'(K^*) > 0, \text{ if and only if, } a > 0.$$ 

(d) The optimum foreign investment will decrease upon an increase in $\text{cov}(\tilde{e}, \tilde{Z})$ if and only if $b > 2$.

Proof.

$$\frac{\partial K^*}{\partial \text{cov}(\tilde{e}, \tilde{Z})} = -b\sigma_e^{-3}R(K^*)[2R(K^*)(b - 2)\{\sigma_e^2 + \text{cov}(\tilde{e}, \tilde{Z})\} + \sigma] < 0, \text{ if and only if, } b > 2.$$ 

(e) With higher $\sigma_e$, the investor will behave in more risk averse fashion if and only if $b > 2$.

Proof.

$$\frac{\partial K^*}{\partial \sigma_e} = -b\sigma_e^{-3}\{(b - 2)R(K^*)^2\{\sigma_e^2 + \text{cov}(\tilde{e}, \tilde{Z})\} + \sigma\} < 0, \text{ if and only if } b > 2.$$
6. Concluding remarks

The aim of this paper has been to explore the decision of a risk averse multinational on how much to be optimally invested abroad under exchange rate movement in the world market, with the presence of a correlated background risk. In the first case, when the multinational only faces changes in the distribution of the foreign exchange rate, the risk averse investor’s optimum investment decision are contingent upon the relative sensitivity of the risk aversion, i.e. the willingness to invest abroad for changes in the distribution of the spot exchange rate. In the second case, the presence of domestic market risks, clubbed as a collective “background risk” affects the decision making through additional riskiness and correlation with the revenue risk. When the risk averse investor confronts with higher background risk or the correlation between the background risk and the revenue risk rises, the investor behaves in more risk averse way; while with higher expected background risk, the investor may optimally invest more abroad if and only if the risk preference structure exhibits DARA (in the context of two moment decision model).

An attractive feature of the conditions we derive for the decision problem of the multinational using the two moment approach is their simplicity in interpretation: with minimal assumption on preference structure like monotonicity and quasi-concavity, the sufficiency conditions based on the investor’s relative sensitivity towards risks are more intuitive and appealing as empirically testable predictions; in contrast to the alternative (such as expected utility) approaches, which would depend on higher-order derivatives of utility functions and their composites.

References


