# ENDOGENOUS MODE OF COMPETITION IN GENERAL EQUILIBRIUM *,†

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Abstract

This paper endogenises the degree of intra-sectoral competition in a multi-sectoral model of oligopoly in general equilibrium. Firms choose capacity followed by prices. If the benefits of capacity investment in a given sector are below a threshold level, the sector exhibits Bertrand behaviour, otherwise it exhibits Cournot behaviour. By endogenising the threshold parameter in general equilibrium, we show how exogenous shocks alter the mix of sectors between "more" and "less" competitive, or Bertrand and Cournot. The model also has implications for the effects of trade liberalisation on the relative wages of skilled and unskilled workers.

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# 1 Introduction

One of the oldest themes in international economics is that larger or more open economies are likely to be more competitive. However, formalising this notion has so far proved elusive. In this paper we provide a framework which suggests how exogenous shocks such as growth or trade liberalisation can lead to changes in the degree of competitive behaviour throughout the economy. The paper embeds a model of firm behaviour along the lines of Kreps and Scheinkman (1983) in a framework of general oligopolistic equilibrium presented in Neary (2002). In the model of Kreps and Scheinkman, as simplified, and reduced to an equilibrium in pure strategies by Maggi (1996), firms compete in a Bertrand manner if the cost savings from prior investment in capacity are below a threshold level.[1] By contrast, if the cost savings exceed the threshold, then the outcome is "as if" the firms were playing a one-shot Cournot game (even though second-stage competition is always a price game). All previous applications of this approach have considered only a single sector in partial equilibrium. Moreover, they have assumed that the crucial threshold parameter is exogenous. Our contribution is to note that it is endogenous in general equilibrium. We assume that it consists of a "real" component, which varies across sectors, and a "nominal" one, linked to economy-wide factor prices. Shocks to the equilibrium, such as trade liberalisation, affect factor prices and therefore change the mix of sectors between "more" and "less" competitive, or Bertrand and Cournot. The model thus suggests a mechanism whereby exogenous changes can affect the degree of competition in an economy. It also throws light on the impact of trade liberalisation on the relative wages of skilled and unskilled workers.

It is helpful to begin by considering the model in the absence of oligopolistic interaction. Section 2 examines the case of a closed economy where each of a continuum of sectors has only a single firm. We show how the decision to invest in capacity is determined and in Section 3 illustrate the determination of equilibrium and the effects of shocks to the initial equilibrium. Section 4 extends this model to allow for more than one firm in each sector and shows how the mix between "Bertrand" and "Cournot" sectors is determined. Section 4 also shows how the duopoly case can be interpreted as a two-country world, and Section 5 explores how opening up to trade affects the degree of competition and the distribution of income.

# 2 Monopoly in General Equilibrium

## 2.1 Technology

We consider a continuum of sectors indexed by $z$, which varies along the unit interval: $z \in [0, 1]$. As already discussed in the introduction, we assume in this section that there is a single firm in each sector. Each

[1] The Kreps-Scheinkman model has been further explored by Davidson and Deneckere (1986), Friedman (1988) and Madden (1998), and has been applied to trade issues by Venables (1990) and Ben-Zvi and Helpman (1992) as well as by Maggi.

faces a choice between investing in capacity or not. Installing capacity requires skilled labour: the skilled labour requirement for a unit of capacity is the same across all sectors, equal to $\delta$, and the skilled wage is $r$. Production requires unskilled labour, which is paid a wage $w$. For all units up to capacity the unskilled labour requirement is $\gamma(z)$; while each unit of production above capacity requires $\theta(z)$ additional unskilled workers. Hence, letting $q(z)$ and $k(z)$ denote the levels of actual output and capacity output in sector $z$, respectively, total costs can be written as:

$$C\left[q\left(z\right)\right]=r\delta k\left(z\right)+\begin{cases} w\gamma(z)q\left(z\right) & \text{if} \quad q\left(z\right)\leq k\left(z\right) \\ w\gamma(z)q\left(z\right)+w\theta\left(z\right)\left[q\left(z\right)-k\left(z\right)\right] & \text{if} \quad q\left(z\right)>k\left(z\right) \end{cases} \tag{1}$$

As already noted, all sectors face the same factor prices $w$ and $r$. However, they differ in their technologies. We assume that sectors vary between "low-tech" and "hi-tech" or "traditional" and "modern". Sectors with low values of $z$ are more low-tech, in that they have relatively high unskilled labour requirements $\gamma(z)$ and relatively low penalties for producing above capacity $\theta(z)$. Sectors with high values of $z$ are more hi-tech and have the opposite configuration. We assume that the sectors can be ordered such that $\gamma(z)$ falls (or at least does not rise) and $\theta(z)$ rises (or at least does not fall) monotonically with $z$:

$$\gamma'(z)\leq 0 \qquad \text{and} \qquad \theta'(z)\geq 0 \tag{2}$$

In simulations it is convenient to specialise to the case where both $\gamma(z)$ and $\theta(z)$ are linear in $z$.[2]

## 2.2 Capacity Choice

When will a firm invest in capacity? If it does not invest its marginal production cost is $w\{\gamma(z)+\theta\left(z\right)\}$; while if it does invest its full marginal cost is $w\gamma(z)+r\delta$. The latter includes the cost of a unit of capacity $r\delta$, equal to the skilled wage times the number of skilled workers needed to produce the unit. The benefit of an extra unit of capacity is the saving of $w\theta\left(z\right)$ on the additional unit of output produced. Hence the marginal sector $\tilde{z}$ which is indifferent between investing in capacity and not is the solution to the equation:

$$r\delta=w\theta\left(\tilde{z}\right) \tag{3}$$

[2] Thus, $\gamma\left(z\right)=\gamma_0-\gamma_1 z$, where $\gamma_0>0$, $\gamma_1\geq 0$ and $\gamma\left(z\right)\geq 0$ for all $z$; and $\theta\left(z\right)=\theta_0+\theta_1 z$, where $\theta_0\geq 0$ and $\theta_1\geq 0$. In the case where both $\gamma'$ and $\theta'$ are zero (corresponding to $\gamma_1=\theta_1=0$ when the functions are linear), all sectors are identical. This is called the "featureless economy" in Neary (2003) and will not be considered further.

Because we have assumed that $\theta(z)$ is monotonic in $z$, the equation must have a unique solution. However, the value of $\tilde{z}$ need not lie strictly between zero and one. If it does, then some sectors (those for which $z > \tilde{z}$) invest in capacity while others (those for which $z \leq \tilde{z}$) do not.[3] If, instead, the value of $\tilde{z}$ which satisfies (3) lies outside these admissible bounds, then the effective value of $\tilde{z}$ must take on one or other boundary value. At one extreme, if the relative cost of skilled labour is so high or extra capacity is so unproductive that $r\delta$ exceeds $w\theta(z)$ for all $z \in [0,1]$, then no sectors invest in capacity and $\tilde{z}$ equals one. At the other extreme, if capacity is relatively cheap so $r\delta$ is less than $w\theta(z)$ for all $z \in [0,1]$, then all sectors invest in capacity and $\tilde{z}$ equals zero. In most of the paper we concentrate on the case where $\tilde{z}$ lies strictly between zero and one, since this gives the richest set of outcomes. Other cases will be mentioned in passing.

Note the comparative statics implications of (3). Both an increase in the skill premium $r/w$ and a fall in the productivity of skilled workers (i.e., a rise in $\delta$) are associated with a rise in $\tilde{z}$. Each of these shocks makes investing in capacity less attractive, and so fewer sectors choose to do so in equilibrium. In this case we can say that the extensive margin of capacity investment rises.

### 2.3 Preferences

Consumer preferences take a continuum quadratic form as in Neary (2002), extended to allow for differentiated products.[4] There are $\bar{L}$ identical consumers, each with additively separable preferences over their consumption of all goods:

$$U[\{x(z)\}] = \int_0^1 u\{x(z)\}dz \tag{4}$$

and a quadratic sub-utility function defined over the output of each sector:

$$u\{x(z)\} = a\left[x_1(z) + x_2(z)\right] - \frac{b}{2}\left[x_1(z)^2 + x_1(z)^2 + 2ex_1(z)x_2(z)\right] \tag{5}$$

Here $x_i(z)$ is the individual's demand for variety $i$ in sector $z$; and $e$ is an inverse measure of product differentiation, ranging from zero (the case of unrelated goods) to one (the case of identical products). To ensure an equilibrium in pure strategies exists we assume that $e$ is strictly less than one. In the monopoly case, only one good is produced in each sector and so $u\{x(z)\}$ is simply $ax(z) - \frac{b}{2}x(z)^2$.

[3] When the firm is indifferent (i.e., equation (3) is satisfied with equality), we assume that it does not invest in capacity.

[4] This treatment of differentiated product demand within each sector follows Dixit (1981), Vives (1985) and Ottaviano, Tabuchi and Thisse (2002).

Each individual maximises utility subject to the budget constraint:

$$\int_0^1 \left[p_1(z)x_1(z) + p_2(z)x_2(z)\right] dz \leq I \tag{6}$$

where $I$ is the individual's income. This yields inverse demand functions for each good as follows:

$$p_i(z) = \frac{a}{\lambda} - \frac{b}{\lambda}[x_i(z) + ex_j(z)] \qquad i, j = 1, 2; \quad i \neq j \tag{7}$$

Here $\lambda$ is the individual's marginal utility of income, which depends on income and on the distribution of prices. (Details are given in the Appendix.) Aggregating over all $\bar{L}$ individuals and imposing market clearing (so the total quantity sold by firm $i$ in sector $z$, $q_i(z)$, equals $\bar{L}x_i(z)$, for all $i, z$) yields the market inverse demand functions:

$$p_i(z) = \widetilde{a} - \widetilde{b}[q_i(z) + eq_j(z)] \qquad i, j = 1, 2; \quad i \neq j \tag{8}$$

where $\widetilde{a} \equiv a/\lambda$ and $\widetilde{b} \equiv b/\lambda\bar{L}$. Because of $\lambda$, the demand functions are highly non-linear. However, $\lambda$ depends only on economy-wide variables and not directly on variables in sector $z$. Hence, firms in sector $z$ take $\widetilde{a}$ and $\widetilde{b}$ as given in their decision-making, so from the perspective of individual sectors the demand functions are linear.

## 2.4 Factor markets

Assuming that wage flexibility brings about full employment, equilibrium in the market for unskilled workers is determined as follows:

$$\bar{L} = \int_0^{\tilde{z}} \left[\gamma(z) + \theta(z)\right] q^M[w\{\gamma(z) + \theta(z)\}]dz + \int_{\tilde{z}}^1 \gamma(z)\, q^M[w\gamma(z) + r\delta]dz \tag{9}$$

As already noted, $\bar{L}$ is the number of individuals, each of which is assumed to supply one unit of unskilled labour and $s$ units of skilled labour. This equals the aggregate demand for unskilled labour, which in turn equals the sum over all sectors of their output, times their unskilled labour requirement per unit output. The level of output in monopoly $q^M$ depends on the firm's full marginal cost and is given in the Appendix, equation (21). Note that the unskilled labour requirement drops discontinuously at $\tilde{z}$, the threshold sector where firms switch to investing in capacity.

Equilibrium in the market for skilled workers is determined in a similar manner:

$$s\bar{L} = \int_{\tilde{z}}^1 \delta q^M[w\gamma(z) + r\delta]dz \tag{10}$$

The endowment of skilled workers is $s$ times the endowment of unskilled workers, while the demand for skilled workers from each sector which invests in capacity equals $\delta$ times its output.

### 2.5 National Income

To complete the model we need to specify how profits are disbursed. It is convenient to assume that they are redistributed costlessly in equal shares to each of the $\bar{L}$ individuals. National income therefore equals the sum of factor payments and profits. In per capita terms, this is:

$$I = w + rs + \Pi/\bar{L} \tag{11}$$

where $\Pi$ is the sum of profits of all firms in the economy.

## 3 Equilibrium and Responses to Shocks

### 3.1 Determination of Equilibrium

The full model consists of the two labour-market equilibrium conditions, (9) and (10), with the level of output in each sector given by (21); the equation for the threshold sector or extensive margin $\tilde{z}$, (3); and the definitions of income and the marginal utility of income, (11) and (24). However, we have one degree of freedom in solving for nominal variables: as in Neary (2002), all real variables are homogeneous of degree zero in the nominal variables $w$, $r$ and $\lambda^{-1}$. Hence we can choose an arbitrary numeraire without affecting the model's properties, and it is convenient to choose the marginal utility of income itself as numeraire, setting $\lambda$ equal to one.

The model can be further simplified by using (3) to eliminate $r$. The model can thus be reduced to two equations in $w$ and $\tilde{z}$, which in turn can be illustrated in a single diagram as in Fig. 1.[5] The properties of these equations are derived formally in the Appendix. Here we give an intuitive account.

Consider first the equilibrium condition in the market for skilled labour, equation (10) with $r\delta$ replaced by $w\theta(\tilde{z})$. The demand for skilled labour is decreasing in the unskilled wage $w$ for two distinct reasons. On the one hand, a rise in $w$ raises costs directly, since skilled and unskilled labour are complements in production in each sector that invest in capacity.[6] On the other hand, a rise in $w$ also raises costs indirectly,

[5] It might seem that it would be simpler to eliminate $\tilde{z}$ rather than $r$. However, the slopes of the loci in the figures are not so clear-cut in that case. Where this approach becomes very useful is in the boundary case where all sectors invest in capacity so $\tilde{z}$ is always zero. The model then reduces to two excess factor demand equations which exhibit gross substitutability: $L = L^D(w,r)$ and $S = S^D(w,r)$. The comparative statics properties of this system are easily derived, and (provided $\tilde{z}$ remains equal to zero) they are very similar to those of the more complex case considered in the text.

[6] The technology could be described as "putty-clay". (See Solow (1962) and Bliss (1968).) There is a discrete choice between two techniques ex ante, while after capacity is installed the skilled-to-unskilled labour ratio equals $\delta/\gamma(z)$. Even the latter is

by raising the skilled wage $r$ needed to maintain the initial value of the threshold sector $\tilde{z}$. The demand for skilled labour is also decreasing in $\tilde{z}$ itself for similar reasons. On the one hand, an increase in $\tilde{z}$ reduces the demand for skilled labour at the extensive margin, as the marginal sector ceases to invest in capacity. On the other hand, it raises the equilibrium skilled wage $r$, at a given unskilled wage $w$, so inducing all capacity-using sectors (those for which $z > \tilde{z}$) to invest in less capacity at the intensive margin. The locus is thus downward-sloping, as shown in Fig. 1, with points above corresponding to states of excess supply of skilled labour.

Consider next the equilibrium condition in the market for unskilled labour, equation (9). A rise in the unskilled wage induces all sectors to reduce their labour demand at the intensive margin, both directly and indirectly (by raising the skilled wage). So, not surprisingly, unskilled labour demand is decreasing in the unskilled wage. However, the effect of an increase in $\tilde{z}$ on unskilled labour demand is ambiguous: see equation (29) in the Appendix. On the one hand, the increase in the extensive margin itself means that the marginal sector ceases to invest in capacity, increasing its demand for unskilled labour by $\theta(\tilde{z})$ times its output. On the other hand, the increase in $\tilde{z}$ raises the skilled wage, so reducing the demand for unskilled labour at the intensive margin in all the capacity-using sectors. When $\tilde{z}$ is high, the latter effect is insignificant, and so the locus must be upward-sloping. We can characterise this as the case where the economy is *skill-scarce*: relatively few sectors use skilled labour, so the effect at the extensive margin dominates, raising the demand for unskilled labour. By contrast, for low values of $\tilde{z}$, the economy is *skill-abundant*. Now the intensive-margin effect of an increase in $\tilde{z}$ lowers the demand for unskilled labour in sectors which invest in capacity. When this effect dominates the extensive-margin effect (which it may, though need not), the demand for unskilled labour is reduced by an increase in $\tilde{z}$ and so the locus is downward-sloping. Combining these two conflicting effects, the unskilled-labour-market equilibrium locus is likely to have a U-shape as illustrated in Fig. 1.

The equilibrium of the economy is therefore as illustrated in Fig. 1. The two loci may intersect more than once, but plausible stability conditions ensure that the out-of-equilibrium dynamics must be as shown by the arrows. Hence the local configuration in the neighbourhood of a stable equilibrium must be as shown by point $A_0$.[7] Alternatively, the two loci may not intersect at all for values of $\tilde{z}$ in the $[0, 1]$ interval. This corresponds to an economy in which either all sectors invest in capacity, so $\tilde{z}$ equals zero, or no sectors invest in capacity, so $\tilde{z}$ equals one. The latter case cannot be an equilibrium, since the demand for skilled labour

not fixed in the engineering sense, since firms could in principle produce above or below capacity. However, in equilibrium it is never profit-maximising to do so.

[7] At points above the unskilled labour-market equilibrium locus, excess supply tends to reduce the unskilled wage; conversely for points below. As for the skilled labour-market equilibrium locus, at points to the right of it there is excess supply of skilled workers, as we have seen. This puts downward pressure on the skilled wage, which, at a given unskilled wage, encourages more sectors to invest in capacity and so reduces the extensive margin $\tilde{z}$. The converse applies for points to the left of the skilled labour-market equilibrium locus.

would fall to zero.[8] The former is the case where skilled labour is extremely abundant, and as noted in footnote 5 its properties are less interesting and are easily derived. In most of the remainder of the text we concentrate on the case of a unique interior equilibrium, as shown in Fig. 1.

## 3.2 Comparative Statics

We can now consider the effects of shocks to an initial equilibrium. Consider first an increase in the size of the economy as measured by the labour force $\bar{L}$. Inspection of the labour-market equilibrium conditions, (9) and (10), and the equation for output in (21) (see the Appendix), shows that this type of growth is completely neutral. Endowments of both factors, and demands for all goods, rise in line with growth, and so no change in the initial factor prices and threshold sectors is required. This occurs despite the fact that demands are not homothetic.[9] It arises because, with investment in capacity chosen endogenously, there are no exogenous fixed costs in the economy. By contrast, in models with exogenous fixed costs, such as Krugman (1979), growth per se leads to an increase in productivity and firm size (and/or firm numbers if entry is possible). For medium- and long-run analysis it seems more plausible to assume that fixed costs arise from prior investments rather than from fundamental properties of the technology.

A change in relative factor endowments is not neutral of course. Consider the effects of an increase in $s$, the endowment of skilled labour. Inspecting the equilibrium loci, (9) and (10), and recalling that $r\delta = w\theta(\widetilde{z})$, this shock leaves the unskilled-labour-market equilibrium locus unaffected while it shifts downwards the skilled-labour-market equilibrium locus. As Fig. 2 shows, the effect is to lower the extensive margin $\widetilde{z}$. As a result, the skill premium $r/w$ definitely falls, although the unskilled wage itself may rise or fall. In the skill-scarce economy, where the initial equilibrium is along the upward-sloping portion of the unskilled-labour-market equilibrium locus, skilled and unskilled labour are complements in general equilibrium and the unskilled wage falls. Fig. 2 illustrates the skill-abundant case, where skilled and unskilled labour are substitutes in general equilibrium (despite being complements in production *within* each capacity-using sector) and the unskilled wage rises.

[8] As $\tilde{z}$ approaches one, the unskilled-labour-market equilibrium locus is upward-sloping, as we have seen. Hence the value of $w$ on this locus at $\tilde{z} = 1$ must be positive. However, the skilled-labour market equilibrium locus asymptotes to a value for $w$ of minus infinity, as the sectors which demand skilled workers disappear. Hence if $s$ is strictly positive there cannot be an equilibrium at $\tilde{z} = 1$.

[9] As shown in Neary (2002), continuum-quadratic preferences as in (4) and (5) are a member of the Gorman polar form or quasi-homothetic family of preferences. The implied Engel curves are straight lines, but do not pass through the origin.

# 4 Oligopoly

## 4.1 Firm Behaviour

We turn next to consider the oligopoly case. Note first that the analysis of capacity choice in Section 2 continues to apply. The decision by a firm in sector $z$ on whether or not to invest in capacity depends solely on whether $z$ is greater or less than the threshold $\tilde{z}$. Hence, from (3), it depends solely on factor prices and technology (summarised by $\delta$ and $\theta(z)$). In particular, it is independent of whether there are one or many firms in the sector. However, the amount of investment undertaken and the nature of the resulting equilibrium depend crucially on the number of firms and on the details of their behaviour. Here we assume that there are two firms in each sector and that they engage in a two-stage game, first choosing their levels of capacity and then, having observed each others' choices of capacity, choosing their prices.

The determination of equilibrium in a single sector is as described in Maggi (1996). First, note that, with no uncertainty and no threat of entry, firms will never choose to hold excess capacity in equilibrium; and provided it is profitable to invest in capacity (i.e., provided $z > \tilde{z}$), they will never produce less than their installed capacity.[10] Hence the equilibrium level of investment is:

$$k(z) = \begin{cases} 0 & \text{if} \quad z \leq \tilde{z} \\ q(z) & \text{if} \quad z > \tilde{z} \end{cases} \tag{12}$$

What is the optimal choice of capacity by each firm? Investment in capacity lowers the cost of producing the marginal unit of output. In addition, it serves as a commitment device, committing the firm to incurring a penalty if production exceeds capacity. (Crucial for this property is the assumption that capacity choices are observable before decisions on prices are taken.) This commitment is stronger the greater the cost penalty $\theta(z)$. Hence, as Maggi shows, firms which invest in capacity but have relatively low values of $\theta(z)$ choose a price equal to the equilibrium price which would be set in a one-stage Bertrand game with unit costs equal to $w\{\gamma(z) + \theta(z)\}$. This unit cost is higher than the true unit cost actually incurred by firms, which (since they must pay the unit cost of investing in capacity $r\delta$ but do not in practice incur the surcharge of $\theta(z)$ for producing above it) equals $w\gamma(z) + r\delta$. Hence investment in capacity sustains higher prices.[11]

The highest price which investment can sustain is the price that would obtain in a one-stage Cournot

[10] Following Maggi, we assume that firms never find it profitable to ration consumers. Boccard and Wauthy (2000) consider the extra complications which arise in this case.

[11] This result is analogous to the outcome of a one-stage Bertrand game with homogeneous products and heterogeneous costs. Here the firms actually incur a marginal cost of $w\gamma(z) + r\delta$ but they charge prices which reflect the marginal cost they would incur if they deviated from the low-output equilibrium, $w\{\gamma(z) + \theta(z)\}$. Similarly, in the one-shot Bertrand game, all firms produce zero except the lowest-cost firm, which charges a price equal not to its own marginal cost but to the marginal cost of the second-lowest-cost firm, since that is the price which would prevail if the lowest-cost firm deviated and produced less than equilibrium output.

equilibrium with unit costs equal to $w\gamma(z) + r\delta$. Hence, firms in sectors with sufficiently high values of $\theta(z)$ can credibly commit to charging this price. Denoting the threshold sector by $z^C$, equilibrium prices are therefore given by:

$$p(z) = \begin{cases} p^B[w\{\gamma(z) + \theta(z)\}] & \text{if} \quad z \leq z^C \\ p^C[w\gamma(z) + r\delta] & \text{if} \quad z \geq z^C \end{cases} \tag{13}$$

where $p^B(c)$ and $p^C(c)$ denote respectively the equilibrium prices in a symmetric one-stage Bertrand and Cournot game, conditional on marginal costs equal to $c$. (Explicit expressions for these are given in the Appendix.) Using similar notation to denote the corresponding equilibrium outputs, we can write them as follows:

$$q(z) = \begin{cases} q^B[w\{\gamma(z) + \theta(z)\}] & \text{if} \quad z \leq z^C \\ q^C[w\gamma(z) + r\delta] & \text{if} \quad z \geq z^C \end{cases} \tag{14}$$

As for the threshold sector $z^C$, it is determined by the condition that the Bertrand and Cournot equilibrium prices coincide:

$$p^B[w\{\gamma(z^C) + \theta\left(z^C\right)\}] = p^C[w\gamma(z^C) + r\delta] \tag{15}$$

For the model to make sense, the threshold $z^C$ at which sectors that invest in capacity switch to behaving "as if" a one-stage Cournot game was being played should be greater than the threshold $\widetilde{z}$ at which it becomes profitable to invest in capacity. A convenient sufficient condition for this to be the case is given in the following:

**Proposition 1** *A sufficient condition for $z^C$ to exceed $\widetilde{z}$ is that $H(z) \equiv \theta'(z) + [e^2/(2+e)]\gamma'(z)$ is positive for all $z \in [\widetilde{z}, z^C]$.*

(The proofs of this and subsequent results are in the Appendix.) Recall that, from (2), $\theta'(z)$ is non-negative and $\gamma'(z)$ is non-positive; moreover, the coefficient of $\gamma'(z)$ lies between zero (when $e = 0$) and 1/3 (when $e = 1$). Hence the sufficient condition would fail only if $\theta(z)$, the cost premium for producing above capacity, increases in $z$ less than a third as quickly as $\gamma(z)$, the marginal cost when producing at or below capacity, falls in $z$. This seems plausible, and henceforward it is convenient to assume that it holds. Provided it holds at $z = z^C$, we can also sign the responses of $z^C$ to changes in factor prices:

**Proposition 2** *The threshold Cournot sector $z^C$ is decreasing in the return to unskilled workers $w$ and increasing in the return to skilled workers $r$, if and only if $H\left(z^C\right)$ is positive.*

An immediate corollary to this result follows from the fact that, recalling (3), the return to skilled workers $r$ is increasing in both $w$ and $\widetilde{z}$. Hence, when $r$ is determined endogenously in general equilibrium, the threshold Cournot sector $z^C$ is increasing in the extensive margin $\widetilde{z}$.

## 4.2 Intersectoral Differences in Prices and Factor Demands

For given factor prices and threshold sectors, Fig. 3 illustrates how the equilibrium output price varies across sectors. (To avoid distracting non-linearities, Figs. 3 and 4 use the special linear functional forms for the technology distributions specified in footnote 2, with the added simplification that $\gamma_1$ is zero, so $\gamma(z)$ is independent of $z$.) Here and subsequently (except where otherwise specified) we assume that the equilibrium is an interior one, so both $\widetilde{z}$ and $z^C$ lie between zero and one; and that it is a well-behaved one, so $z^C$ exceeds $\widetilde{z}$.

Sectors with $z$ below $\widetilde{z}$ in Fig. 3 are denoted "pure Bertrand" sectors: here firms do not invest in capacity, and the equilibrium price is that in a standard one-stage game with marginal costs equal to $w\{\gamma(z)+\theta(z)\}$. The expression for the equilibrium price is the same in sectors with $z$ between $\widetilde{z}$ and $z^C$, but now this price is above the "pure Bertrand" level, since the full marginal cost in this range is $w\gamma(z)+\delta r$, which is less than $w\{\gamma(z)+\theta(z)\}$. Investment in capacity sustains a higher price than the one-stage Bertrand equilibrium price. Hence we call these "quasi-Bertrand" sectors. Finally, sectors with $z$ above $z^C$ have the highest sustainable price, the Cournot price corresponding to the full marginal cost $w\gamma(z)+\delta r$.

The full employment conditions are similar to those in the monopoly case, equations (9) and (10), with two added complications. First, factor demands differ between the "quasi-Bertrand" sectors (with $\widetilde{z}<z<z^C$) where the equilibrium price is lower than the Cournot price, and the "Cournot" sectors (with $z>z^C$) where it equals the Cournot price; and, second, demands from all sectors are higher because there are now two firms in each sector rather than one. Thus, the equilibrium condition in the market for unskilled labour is:

$$\begin{aligned}\bar{L} \quad = \quad & 2\int_0^{\tilde{z}}\left[\gamma(z)+\theta(z)\right]q^B\left[w\{\gamma(z)+\theta(z)\}\right]dz \\ & +2\int_{\tilde{z}}^{z^C}\gamma(z)\,q^B[w\{\gamma(z)+\theta(z)\}]dz \\ & \quad +2\int_{z^C}^{1}\gamma(z)\,q^C[w\gamma(z)+r\delta]dz \end{aligned} \tag{16}$$

Similarly the demand for skilled labour comes from both firms in each of the "quasi-Bertrand" and "Cournot" sectors:

$$s\bar{L}=2\int_{\tilde{z}}^{z^C}\delta q^B[w\{\gamma(z)+\theta(z)\}]dz+2\int_{z^C}^{1}\delta q^C[w\gamma(z)+r\delta]dz \tag{17}$$

These aggregate factor demands are most easily understood by considering how they vary across sectors, as illustrated in Fig. 4, where $s(z)$ and $l(z)$ denote the skilled and unskilled labour demand in sector $z$

respectively. Sectors with $z$ below $\tilde{z}$ do not invest in capacity and so demand unskilled labour only. Their demand for unskilled labour may either rise or fall with $z$: Fig. 4 illustrates the case where it falls with $z$.[12] At $z = \widetilde{z}$, there is a discrete drop in the demand for unskilled labour and a corresponding jump in demand for skilled labour, as sectors begin to invest in capacity. As $z$ increases further, demand for both factors falls, not because actual costs incurred rise, but because the penalty for producing beyond capacity rises and so higher prices can be sustained. Finally, beyond $z^C$, factor demands do not change further as $\gamma(z)$ is assumed to be independent of $z$ in the figure.

### 4.3 Equilibrium and Comparative Statics

Despite the added complexities of the duopoly case, the analysis of the monopoly case continues to apply in qualitative terms. In particular, the equilibrium conditions can be reduced to two equations in the unskilled wage $w$ and the extensive margin $\widetilde{z}$, and the qualitative properties of Fig. 1 are unchanged.[13] The skilled-labour-market equilibrium locus continues to be unambiguously downward-sloping, as a higher extensive margin leads to excess supply of skilled labour, requiring a fall in the unskilled wage to restore equilibrium. In addition, the unskilled-labour-market equilibrium locus continues to have a U-shape in general: a higher extensive margin may lead to excess supply of unskilled labour when skilled labour is abundant, but must lead to excess demand when skilled labour is scarce.

Comparative statics in the neighbourhood of an equilibrium also exhibit the same qualitative properties as before. Thus an increase in the endowment of skilled labour always reduces the extensive margin but has an ambiguous effect on the unskilled wage: Fig. 2 illustrates the skill-abundant case where the unskilled wage rises. The additional feature is of course that the threshold sector for Cournot behaviour, $z^C$, is endogenous in general equilibrium. Hence, given the sufficient condition in Proposition 2, we can conclude that the fall in the extensive margin $\tilde{z}$ is associated with a fall in $z^C$ in all cases. In this sense, the effect of an increased endowment of skilled labour is to make the economy less competitive, as more sectors exhibit Cournot behaviour.

[12] The responsiveness of $l(z)$ to an increase in $z$ in pure Bertrand sectors equals $\eta'(z)q^B\{c(z)\}[1-\varepsilon(z)]$. Here $\eta(z) \equiv \theta(z)+\gamma(z)$ and so $\eta'(z)$ may be either positive or negative in general, though it is positive under the special assumptions made in Fig. 4; $c(z)$ equals $w\eta(z)$ in this range; and $\varepsilon(z) \equiv -[c(z)/q^B\{c(z)\}][\partial q^B\{c(z)\}/\partial c(z)]$ is the elasticity of output with respect to marginal cost, which can be greater or less than one. Hence the case illustrated in Fig. 4 corresponds to a relatively high cost elasticity of output.

[13] Technically, this arises because, as a marginal sector switches from quasi-Bertrand to Cournot behaviour, its output, and hence its factor demands, do not change. Hence the labour-market equilibrium conditions, (16) and (17), are independent of the new variable $z^C$, and so the Jacobian of the coefficient matrix is block-triangular: the equilibrium values of $\tilde{z}$, $w$ and $r$ are determined by (3), (16) and (17), just as they were determined by (3), (9) and (10) in the monopoly case, while equation (15) alone determines the equilibrium value of $z^C$.

### 4.4 Autarky versus Free Trade

So far, we have interpreted the model as applying to a single economy only. However, the duopoly case can also be interpreted as an integrated world economy, formed by the elimination of trade barriers between two separate economies, each of which is identical to the monopoly case of Sections 2 and 3. To see this, note that the skilled-labour-market equilibrium condition in one country after trade can be written as:

$$s\bar{L} = \int_{\tilde{z}}^{z^C} \delta q^B [w\{\gamma(z) + \theta(z)\}]dz + \int_{z^C}^{1} \delta q^C [w\gamma(z) + r\delta]dz \tag{18}$$

Compared to the corresponding condition for the closed-economy duopoly case, equation (17), the labour demand in (18) comes from only one firm in each sector rather than two. However, the goods demands facing each sector are now greater, as free trade has increased the market size. In the case of free trade between two identical countries, the goods demands are exactly twice the monopoly level: they are given as before by equation (20) in the Appendix, except that the demand parameter $\widetilde{b}$, an inverse measure of market size, now equals $b/2\lambda\bar{L}$ instead of $b/\lambda\bar{L}$ in the monopoly case. These two differences between closed-economy duopoly and symmetric free trade exactly offset one another, so the equilibria are identical.[14] Hence we have an exact equivalence between two comparisons: that between monopoly and duopoly in a single country, and that between autarky (with monopoly in each sector) and symmetric free trade.

## 5 Comparing Monopoly and Oligopoly

Next we wish to compare the equilibria with monopoly and duopoly, which as we have seen also amounts to comparing autarky and symmetric free trade. This is a non-local comparison, so we proceed in a series of steps.[15]

Note first the effects of moving from autarky to free trade in those sectors which do not invest in capacity in autarky (i.e., those with $z < \tilde{z}^A$). On the one hand, they clearly increase their demand for unskilled labour at initial wages as the number of firms rises. On the other hand, since there is no demand for skilled labour from these sectors, there is no change in the demand for skilled labour at initial wages. This asymmetric change in factor demands by itself tends to raise the unskilled wage as the economy moves from autarky to free trade. A higher unskilled wage in turn encourages more sectors to invest in capacity, so both the extensive margin and the skill premium fall. In a skill-scarce economy, these sectors dominate: $\tilde{z}^A$ is close to one and so the primary impact of moving to free trade is to raise the skill premium. This result is

[14] Necessary conditions for this property are that demands are linear, there are no exogenous fixed costs, and the number of countries in symmetric free trade is the same as the number of firms in closed-economy oligopoly.

[15] Henceforward we use superscripts "$A$" and "$F$" to denote autarky (monopoly) and symmetric free trade (duopoly) respectively.

straightforward, but for completeness we state if formally:

**Proposition 3** *In a skill-scarce economy, where $\tilde{z}^A$ is close to one, a move from autarky to free trade leads to an increase in the unskilled wage, and a decrease in both the extensive margin and the skill premium.*

(The proof is immediate.)

Next, we show that the effect highlighted in Proposition 3 also dominates in the special case where variable costs are the same in all sectors, provided only that $\tilde{z}^A$ is strictly positive:

**Proposition 4** *When $\gamma(z)$ is independent of $z$, and $\tilde{z}^A$ is strictly positive, a move from autarky to free trade leads to an increase in the unskilled wage, and a decrease in both the extensive margin and the skill premium.*

The proof (in the Appendix) is complex but can be explained intuitively with the help of Fig. 5. With increased competition, the demand for both factors rises in the move from monopoly to duopoly. Hence the unskilled-labour-market equilibrium locus shifts upwards and the skilled-labour-market equilibrium locus shifts rightwards as shown. To prove the proposition we need to show that the vertical shift in the $L$ locus is greater than that in the $S$ locus. Let $w^S$ denote the new unskilled wage rate which is just sufficient to re-establish equilibrium on the market for skilled labour without any change in the extensive margin $\tilde{z}$. Hence point $A^S$, with an unskilled wage equal to $w^S$ and a threshold sector equal to the monopoly level $\tilde{z}^A$, lies on the duopoly skilled-labour-market equilibrium locus, and we need to show that it lies below the duopoly unskilled-labour-market equilibrium locus. Because $\gamma(z)$ is independent of $z$, the increase in $w$ to $w^S$ exactly offsets the effect of extra competition in raising demand for unskilled labour from the capacity-using sectors. But the remaining sectors (those with $z < \tilde{z}$) are unskilled-labour-intensive, both on average and at the margin, precisely because they do not invest in capacity. Hence this increase in $w$ is not sufficient to choke off their extra demand for unskilled labour, and the unskilled wage must rise by more, to the level denoted $w^F$ in the figure, to clear both markets. As a result, the new equilibrium is at $A^F$, where the threshold sector $\tilde{z}$ and the skill premium are lower in free trade relative to autarky.

Consider next the case where variable costs are strictly decreasing in $z$. Now the factor intensity differences across sectors work in the opposite direction to Proposition 4: the increased market size tends to encourage disproportionally the expansion of sectors which invest in capacity in autarky, increasing the relative demand for skilled labour, which in turn tends to raise the skill premium. This effect is most likely to dominate in a skill-abundant economy, where relatively few sectors use only unskilled labour in autarky; and in an economy where firms face little competition, so their factor demands expand to take full advantage of the larger market. Carrying both these effects to their extremes, we can state a formal set of conditions which ensure an increase in the skill premium:

**Proposition 5** *When $\gamma(z)$ is strictly decreasing in $z$, $e$ equals zero, and exogenous parameters are such that $\tilde{z}^A$ is zero, then a move from autarky to free trade increases both the extensive margin and the skill premium.*

Formally, the proof proceeds in a similar way to that of Proposition 4, except that now there is excess *supply* of unskilled labour at the autarky threshold sector ($\tilde{z}^A = 0$) and at the wage $w^S$ just sufficient to choke off excess demand for skilled labour.

The results of Propositions 4 and 5 are summarised in Fig. 6. Here we specialise to the case of a linear distribution of technology parameters, so $\gamma(z) = \gamma_0 - \gamma_1 z$. The vertical axis plots $\gamma_1$, which is a measure of intersectoral differences in factor intensity (specifically, in the unskilled labour requirement per unit output). We have seen from Proposition 4 that when sectors are identical in this respect (so $\gamma_1$ is zero) free trade leads capacity–using sectors to raise their demands for both factors in equal proportions. Provided there are some sectors in autarky which do not use capacity, their increased relative demand for unskilled labour therefore dominates the move from autarky to free trade and induces a fall in the skill premium. Hence Proposition 4 tells us that, at points along the horizontal axis in Fig. 6, free trade leads to a lower skill premium.

As for the horizontal axis in Fig. 6, it plots $e$, which is a measure of the intensity of competition within sectors. When this is low, the move from autarky to free trade has only a market size effect so encouraging a large increase in output by all firms. When all sectors use skilled labour in autarky and $\gamma_1$ is strictly positive, this effect encourages a larger relative increase in the demand for skilled labour. Hence, Proposition 5 tells us that at points along the vertical axis free trade leads to a higher skill premium.

The locus in the interior of Fig. 6, based on simulations for the linear case, shows combinations of $\gamma_1$ and $e$ which yield no change in the skill premium and the extensive margin between autarky and free trade. Thus a rise in the skill premium and the extensive margin is encouraged by greater differences in factor intensity between sectors, and by less intense competition within sectors.

# 6 Conclusion

This paper has presented a new model which integrates some key features of industrial organisation and general-equilibrium trade theory, and highlights a new mechanism whereby relative wages and the nature of competition within sectors are affected by exogenous shocks. The model extends to general equilibrium the work of Maggi (1996), which predicts that firms will exhibit Bertrand or Cournot behaviour depending on the costs of investing in capacity, where capacity serves as a commitment device to sustain higher prices. Maggi looked at normative questions only. In particular, he showed that the Kreps-Scheinkman approach resolves the apparent conflict between Brander and Spencer (1985) and Eaton and Grossman (1986), who proved respectively that the optimal export subsidy is positive in Cournot competition and negative in Bertrand competition.[16] Here we have focused instead on positive questions, including the effects of exogenous shocks such as changes in relative factor endowments and trade liberalisation on the distribution of income and on the margin between more and less competitive sectors.

Of course, trade liberalisation leads directly to more competition when the number of firms in each sector rises. However, just how much more competition is induced depends on whether firms are able to sustain prices above the Bertrand level. Our model shows that this in turn depends both on technology and on factor prices, with the latter determined endogenously in general equilibrium. When sectors differ in their requirements of unskilled labour, and when goods are more differentiated within sectors so inter-firm competition is less intense, trade between similar economies raises the relative return to skilled labour, making it more difficult to sustain higher prices through investment in capacity and so leading to greater competition throughout the economy.

The model also exhibits other novel features. It shows that the effects of exogenous shocks to factor endowments and technology differ greatly depending on wether the economy is skill-abundant or not. And, although preferences are non-homothetic and fixed costs play an important role, the fact that the fixed costs are endogenous implies that the economy exhibits constant returns to scale in the aggregate, in striking contrast to models with exogenous fixed costs. More work is needed to explore the robustness of these and other properties of the model to alternative specifications of the workings of factor markets and the ways in which technology and factor prices interact to affect the nature of competition between firms.

[16] An obvious extension of the present paper would be to consider optimal trade and industrial policy. This would qualify Maggi's results by adding general-equilibrium effects similar to those considered by Dixit and Grossman (1986).

# 7 Appendix

## 7.1 Bertrand, Cournot and Monopoly Equilibria

Given the market inverse demand functions (8), and letting $c$ denote the marginal production cost, routine calculations show that the prices and outputs in Bertrand and Cournot equilibria are as follows:

$$p^B(c) = \frac{\widetilde{a}(1-e)+c}{2-e} \qquad \text{and} \qquad p^C(c) = \frac{\widetilde{a}+(1+e)c}{2+e} \tag{19}$$

$$q^B(c) = \frac{\widetilde{a}-c}{\widetilde{b}(1+e)(2-e)} \qquad \text{and} \qquad q^C(c) = \frac{\widetilde{a}-c}{\widetilde{b}(2+e)} \tag{20}$$

As for the price and output in monopoly, they equal the values in both Bertrand and Cournot equilibrium when $e$ equals zero:

$$p^M(c) = \frac{\widetilde{a}+c}{2} \qquad \text{and} \qquad q^M(c) = \frac{\widetilde{a}-c}{2\widetilde{b}} \tag{21}$$

## 7.2 Derivation of the Marginal Utility of Income

An individual's marginal utility of income with continuum quadratic preferences and homogeneous products in each sector is derived in Neary (2002). With differentiated products, the steps are similar. First invert the individual inverse demand functions (7) to get the direct demand functions:

$$x_i(z) = \alpha - \lambda\beta[p_i(z) - ep_j(z)] \qquad i,j = 1,2; \quad i \neq j \tag{22}$$

where $\alpha$ and $\beta$ are related to the utility parameters $a$, $b$ and $e$ as follows

$$\alpha \equiv \frac{a}{b(1+e)} \qquad \text{and} \qquad \beta \equiv \frac{1}{b(1-e^2)} \tag{23}$$

Now multiply the direct demand functions for goods $i$ and $j$ in sector $z$ by $p_i(z)$ and $p_j(z)$ respectively, add to get the individual's expenditure on both goods in that sector, and integrate over all sectors to get her total expenditure. Substituting for $I$ and solving gives an explicit expression for $\lambda$:

$$\lambda = \frac{\alpha\mu_p^1 - I}{\beta(\mu_p^2 - e\nu_p)} \tag{24}$$

where $\mu_p^1$ and $\mu_p^2$ denote respectively the first and second moments of the distribution of prices and $\nu_p$ denotes the (uncentred) covariance of prices across sectors:

$$\mu_p^1 \equiv \int_0^1 \left[p_i(z) + p_j(z)\right] dz \qquad \mu_p^2 \equiv \int_0^1 \left[p_i(z)^2 + p_j(z)^2\right] dz$$
$$\nu_p \equiv 2\int_0^1 p_i(z)p_j(z)dz \tag{25}$$

When prices are the same in all sectors, $\nu_p$ reduces to $\mu_p^2$. Note, however, that we cannot take the limit of (24) as $e$ approaches 1, since the inverse demand functions cannot be inverted in this case. The value of $\lambda$ when $e$ equals 1 can instead be calculated directly by integrating over the inverse demand functions times the corresponding prices as in Neary (2002).

## 7.3 Properties of the Equilibrium Loci

Consider first the monopoly or autarky equilibrium of Sections 2 and 3. Write the equilibrium condition for the skilled-labour-market, equation (10), as $s\bar{L} = S^M(w, r, \widetilde{z})$, with $r$ determined by (3). The derivative of the demand function with respect to the unskilled wage is:

$$\begin{aligned}\frac{dS^M}{dw} &= \frac{\partial S^M}{\partial w} + \frac{\partial S^M}{\partial r}\frac{dr}{dw} \\ &= \delta\int_{\widetilde{z}}^1 \{\gamma(z) + \theta(\widetilde{z})\}\frac{\partial q^M}{\partial c}dz < 0\end{aligned} \tag{26}$$

while the derivative with respect to the extensive margin is:

$$\frac{dS^M}{d\widetilde{z}} = \frac{\partial S^M}{\partial \widetilde{z}} + \frac{\partial S^M}{\partial r}\frac{dr}{d\widetilde{z}} = -\delta q^M[w\{\gamma(\widetilde{z}) + \theta(\widetilde{z})\}] + \delta\int_{\widetilde{z}}^1 w\theta'(\widetilde{z})\frac{\partial q^M}{\partial c}dz < 0 \tag{27}$$

Similarly, the equilibrium condition for the unskilled-labour-market, equation (9), can be written as $\bar{L} = L^M(w, r, \widetilde{z})$, and the derivative of the demand function with respect to the unskilled wage is as follows:

$$\begin{aligned}\frac{dL^M}{dw} &= \frac{\partial L^M}{\partial w} + \frac{\partial L^M}{\partial r}\frac{dr}{dw} \\ = \int_0^{\tilde{z}} \{\gamma(z) + \theta(z)\}^2\frac{\partial q^M}{\partial c}dz &+ \int_{\widetilde{z}}^1 \gamma(z)\{\gamma(z) + \theta(\widetilde{z})\}\frac{\partial q^M}{\partial c}dz < 0\end{aligned} \tag{28}$$

However, the derivative with respect to the extensive margin is ambiguous in sign, as noted in the text:

$$\frac{dL^M}{d\widetilde{z}} = \frac{\partial L^M}{\partial \widetilde{z}} + \frac{\partial L^M}{\partial r}\frac{dr}{d\widetilde{z}} = \theta(\tilde{z})q^M[w\{\gamma(\widetilde{z}) + \theta(\widetilde{z})\}] + \int_{\widetilde{z}}^1 \gamma(z)w\theta'(\widetilde{z})\frac{\partial q^M}{\partial c}dz \gtrless 0 \tag{29}$$

Consider next the duopoly or free-trade case of Sections 4 and 5. As before, write the equilibrium condition for the skilled-labour market, equation (17), as $s\bar{L} = S^D(w, r, \widetilde{z})$, with $r$ determined by (3). The derivatives of the demand function are as follows:

$$\begin{aligned}\frac{dS^D}{dw} &= \frac{\partial S^D}{\partial w} + \frac{\partial S^D}{\partial r}\frac{dr}{dw}\\ &= \delta\int_{\tilde{z}}^{z^C}\{\gamma(z)+\theta(z)\}\frac{\partial q^B}{\partial c}dz + \delta\int_{z^C}^{1}\{\gamma(z)+\theta(\widetilde{z})\}\frac{\partial q^C}{\partial c}dz < 0\end{aligned} \tag{30}$$

$$\frac{dS^D}{d\widetilde{z}} = \frac{\partial S^D}{\partial \widetilde{z}} + \frac{\partial S^D}{\partial r}\frac{dr}{d\widetilde{z}} = -\delta q^B[w\{\gamma(\widetilde{z})+\theta(\widetilde{z})\}] + \delta\int_{z^C}^{1} w\theta'(\widetilde{z})\frac{\partial q^C}{\partial c}dz < 0 \tag{31}$$

Similarly, the equilibrium condition for the unskilled-labour market, equation (16), can be written as $\bar{L} = L^D(w, r, \widetilde{z})$, and the derivatives of the demand function are as follows:

$$\begin{aligned}\frac{dL^D}{dw} &= \frac{\partial L^D}{\partial w} + \frac{\partial L^D}{\partial r}\frac{dr}{dw}\\ = \int_0^{\tilde{z}}\{\gamma(z)+\theta(z)\}^2\frac{\partial q^B}{\partial c}dz &+ \int_{\tilde{z}}^{z^C}\gamma(z)\{\gamma(z)+\theta(z)\}\frac{\partial q^B}{\partial c}dz\\ &+ \int_{z^C}^{1}\gamma(z)\{\gamma(z)+\theta(\widetilde{z})\}\frac{\partial q^C}{\partial c}dz < 0\end{aligned} \tag{32}$$

$$\frac{dL^D}{d\widetilde{z}} = \frac{\partial L^D}{\partial \widetilde{z}} + \frac{\partial L^D}{\partial r}\frac{dr}{d\widetilde{z}} = \theta(\tilde{z})q^B[w\{\gamma(\widetilde{z})+\theta(\widetilde{z})\}] + \int_{z^C}^{1}\gamma(z)\,w\theta'(\widetilde{z})\frac{\partial q^C}{\partial c}dz \gtreqless 0 \tag{33}$$

## 7.4 Proofs of Propositions 1 and 2

To prove the propositions, define the following function:

$$\Delta(z) \equiv p^C[w\gamma(z) + r\delta] - p^B[w\{\gamma(z)+\theta(z)\}] \tag{34}$$

This equals the difference between the Cournot and Bertrand prices evaluated at an arbitrary $z$. From equation (15) which defines $z^C$, we know that $\Delta(z^C) = 0$. As for $\tilde{z}$, we know from (3) that the arguments of $p^C$ and $p^B$ in (34) are equal at $z = \tilde{z}$. We also know from (19) that the Cournot equilibrium price exceeds the Bertrand equilibrium price for all $c$, provided $e$ is strictly positive:

$$p^C(c) - p^B(c) = \frac{e^2(\widetilde{a}-c)}{4-e^2} \tag{35}$$

Hence it follows that $\Delta(\tilde{z}) > 0$.

Given that $\Delta(z^C) = 0$ and $\Delta(\tilde{z}) > 0$, a sufficient condition for $z^C$ to exceed $\tilde{z}$ is that $\Delta$ is decreasing in

$z$ at every point in the range $[z^C, \tilde{z}]$. Differentiating $\Delta$ with respect to $z$ gives:

$$\Delta_z(z) = \frac{dp^C}{dc} w\gamma'(z) - \frac{dp^B}{dc} w[\gamma'(z) + \theta'(z)] = -\frac{w}{2-e} H(z) \tag{36}$$

Hence a sufficient condition for $z^C$ to exceed $\tilde{z}$ is that $H(z)$ is positive at every point in the range $[z^C, \tilde{z}]$. This proves Proposition 1.

To prove Proposition 2, totally differentiate (34). From this we can conclude that, when $H(z^C)$ is positive and so from (36) $\Delta_z(z^C)$ is negative, $z^C$ is decreasing in $w$ if and only if $\Delta$ is decreasing in $w$. This is easily shown to be the case:

$$\Delta_w(z) = \frac{dp^C}{dc}\gamma(z) - \frac{dp^B}{dc}[\gamma(z) + \theta(z)] = -\frac{e^2\gamma(z) + (2+e)\theta(z)}{4-e^2} < 0 \tag{37}$$

By similar reasoning, $\Delta$ is increasing in $r$:

$$\Delta_r(z) = \frac{dp^C}{dc}\delta = \frac{(1+e)\delta}{2+e} > 0 \tag{38}$$

Hence $z^C$ is decreasing in $w$ and increasing in $r$ if and only if $H(z^C)$ is positive.

## 7.5 Proof of Proposition 4

Denote the equilibrium wage and extensive margin in the autarky equilibrium by $w^A$ and $\tilde{z}^A$ respectively. In the autarky equilibrium where we assume that $\gamma(z)$ is independent of $z$, so $\gamma(z) = \gamma_0$ for all $z$, the skilled-labour-market equilibrium condition (10) reduces to:

$$s\bar{L} = \int_{\tilde{z}^A}^{1} \delta q^M[w^A\{\gamma_0 + \theta\left(\tilde{z}^A\right)\}]dz = S^M\left(w^A, \tilde{z}^A\right) \tag{39}$$

Now, consider the move to free trade. Hold the extensive margin fixed at the autarky equilibrium level $\tilde{z}^A$, and assume that the unskilled wage needed to restore equilibrium on the skilled labour market is given by $w^S$. (See Fig. 5.) Hence the skilled-labour-market equilibrium condition (17) reduces to:

$$s\bar{L} = \int_{\tilde{z}^A}^{z^C} 2\delta q^B[w^S\{\gamma_0 + \theta\left(z\right)\}]dz + \int_{z^C}^{1} 2\delta q^C[w^S\{\gamma_0 + \theta\left(\tilde{z}^A\right)\}]dz = S^D\left(w^S, \tilde{z}^A\right) \tag{40}$$

Since the increase in the number of firms raises the demand for skilled labour in every sector, we have that $w^S > w^A$.

Consider next the market for unskilled workers. In the free trade case, the equilibrium condition for this market is given from (9) by the following:

$$\bar{L} = \int_0^{\tilde{z}^A} \{\gamma_0 + \theta(z)\} q^M [w^A\{\gamma_0 + \theta(z)\}] dz + \gamma_0 \int_{\tilde{z}^A}^1 q^M [w^A\{\gamma_0 + \theta(\tilde{z}^A)\}] dz \tag{41}$$

Using the equilibrium condition in the market for skilled workers from equation (39), the second integral can be replaced by $s\gamma_0\bar{L}/\delta$ to give:

$$\left(1 - \frac{s\gamma_0}{\delta}\right)\bar{L} = \int_0^{\tilde{z}^A} \{\gamma_0 + \theta(z)\} q^M [w^A\{\gamma_0 + \theta(z)\}] dz = L^{M1}\left(w^A, \tilde{z}^A\right) \tag{42}$$

where $L^{M1}(.)$ denotes the autarky demand for unskilled workers from those sectors which do *not* invest in capacity. Similarly, in free trade, the demand for unskilled workers from the non-capacity-using sectors only, evaluated at $w^S$ and $\tilde{z}^A$, is:

$$2\int_0^{\tilde{z}^A} \{\gamma_0 + \theta(z)\} q^B [w^S\{\gamma_0 + \theta(z)\}] dz = L^{D1}\left(w^S, \tilde{z}^A\right) \tag{43}$$

where $L^{D1}(.)$ denotes the free trade demand for unskilled workers from those sectors which do not invest in capacity. To prove the proposition, we need to show that $L^{D1}\left(w^S, \tilde{z}^A\right) - L^{M1}\left(w^A, \tilde{z}^A\right)$ is positive, so there is excess demand for unskilled labour at $w^S$ and $\tilde{z}^A$. When this is the case, the move from autarky to free trade will lead to an unskilled wage greater than $w^S$ and a threshold sector $\tilde{z}$ lower than $\tilde{z}^A$, as in Fig. 5.

Using the equations above, we have:

$$\begin{aligned} & L^{D1}\left(w^S, \tilde{z}^A\right) - L^{M1}\left(w^A, \tilde{z}^A\right) \\ = & \int_0^{\tilde{z}^A} \{\gamma_0 + \theta(z)\} \left\langle 2q^B\left[w^S\{\gamma_0 + \theta(z)\}\right] - q^M[w^A\{\gamma_0 + \theta(z)\}]\right\rangle dz \end{aligned} \tag{44}$$

A sufficient condition for the last expression to be positive is that for all $z \in \left[0, \tilde{z}^A\right]$:

$$2q^B[w^S\{\gamma_0 + \theta(z)\}] > q^M[w^A\{\gamma_0 + \theta(z)\}] \tag{45}$$

Substituting from the expressions for output in (20) and (21), this is equivalent to:

$$\Omega(z) \equiv \frac{\tilde{a} - w^A\{\gamma_0 + \theta(z)\}}{\tilde{a} - w^S\{\gamma_0 + \theta(z)\}} < \frac{4}{\eta} \tag{46}$$

where $\eta \equiv (2-e)(1+e)$.

The expression $\Omega(z)$ is increasing in $z$:

$$\begin{aligned}\frac{\partial\Omega(z)}{\partial z} &= \frac{-w^A\theta'(z)\left[\tilde{a}-w^S\{\gamma_0+\theta(z)\}\right]+w^S\theta'(z)\left[\tilde{a}-w^A\{\gamma_0+\theta(z)\}\right]}{\left[\tilde{a}-w^S\{\gamma_0+\theta(z)\}\right]^2} \\ &= \theta'(z)\frac{\tilde{a}\left(w^S-w^A\right)}{\left[\tilde{a}-w^S\{\gamma_0+\theta(z)\}\right]^2}>0 \end{aligned} \tag{47}$$

since $\theta'(z)>0$ and $w^S>w^A$.

Define a new function which gives the demand for skilled labour in the hypothetical situation where the equilibrium in all capacity-using sectors is as if firms behave in a Bertrand manner facing the true production cost $w(\gamma_0+r\delta)=w\{\gamma_0+\theta(\tilde{z})\}$:

$$S^{D'}(w,\tilde{z})\equiv 2\int_{\tilde{z}}^{1} q^B[w\{\gamma_0+\theta(\tilde{z})\}]dz$$

Now define a new unskilled wage $w'$ as the solution to $s\bar{L}=S^{D'}\left(w',\tilde{z}^A\right)$. $S^{D'}(w,\tilde{z})$ overestimates the demand for skilled workers relative to $S^D(w,\tilde{z})$ for all values of $w$ and $\tilde{z}$, since it assumes a a lower production cost in those sectors for which $\tilde{z}<z<z^C$ and a Bertrand rather than a Cournot outcome in those sectors for which $z>z^C$. Hence it must be the case that $w'>w_F^S$.

We have:

$$S^M\left(w^A,\tilde{z}^A\right)=S^{D'}\left(w',\tilde{z}^A\right) \tag{48}$$

since both are equal to $s\bar{L}$ by construction. Substituting once again from the expressions for output in (20) and (21), this implies:

$$\frac{\tilde{a}-w^A\{\gamma_0+\theta\left(\tilde{z}^A\right)\}}{\tilde{a}-w'\{\gamma_0+\theta\left(\tilde{z}^A\right)\}}=\frac{4}{\eta} \tag{49}$$

Since $w'>w^S$, we have

$$\Omega\left(\tilde{z}^A\right)=\frac{\tilde{a}-w^A\{\gamma_0+\theta\left(\tilde{z}^A\right)\}}{\tilde{a}-w^S\{\gamma_0+\theta\left(\tilde{z}^A\right)\}}<\frac{4}{\eta}$$

This, together with (47), means that (46) holds for all $z\in\left[0,\tilde{z}^A\right]$, which proves the proposition.

## 7.6 Proof of Proposition 5

The proof makes use of the following lemma:

*Lemma*: Given $Q(z)$ such that $Q'>0$, $\gamma(z)\geq 0$ such that $\gamma'<0$, and $\int_a^b Q(z)\,dz=0$, then $\int_a^b \gamma(z)\,Q(z)\,dz<0$.

*Proof*: Since $Q(z)$ is monotonically increasing in $z$ and $\int_a^b Q(z)\,dz=0$, there must exist a $z^Q\in]a,b[$ such that $Q(z) < 0$ for $z < z^Q$, $Q(z) = 0$ for $z = z^Q$, and $Q(z) > 0$ for $z > z^Q$. We can therefore split the integral of $Q$ at $z^Q$ as follows: $A \equiv -\int_a^{z^Q} Q(z)\,dz = \int_{z^Q}^b Q(z)\,dz > 0$. Next, define $G(z) \equiv \gamma(z)/\gamma(z^Q)$, so that $G(z) > 1$ for $z < z^Q$, $G(z) = 1$ for $z = z^Q$, and $G(z) < 1$ for $z > z^Q$. It follows that $\int_a^{z^Q} G(z)Q(z)\,dz < -A$ and $\int_{z^Q}^b G(z)Q(z)\,dz < A$. Hence:

$$\begin{aligned}\int_a^b \gamma(z)\,Q(z)\,dz &= \gamma\left(z^Q\right)\left[\int_a^{z^Q} G(z)\,Q(z)\,dz + \int_{z^Q}^b G(z)\,Q(z)\,dz\right] \\ &< \gamma\left(z^Q\right)[-A+A] = 0 \end{aligned} \tag{50}$$

*Q.E.D.*

The proof of Proposition 5 now proceeds in the same way as that of Proposition 4, with two key differences. First, we wish to show that there is excess supply of (not excess demand for) unskilled labour at $w^S$ and $\tilde{z}^A$. Second, the additional assumption that $e = 0$ implies from (20) and (21) that $q^B(c) = q^C(c) = 2q^M(c)$ (recalling that $\tilde{b}$ in free trade is twice the autarky level); and that $z^C = \tilde{z}$. With these assumptions equation (40) which defines $w^S$ becomes:

$$s\bar{L} = 2\int_{\tilde{z}^A}^1 \delta q^M[w^S\{\gamma(z) + \theta\left(\tilde{z}^A\right)\}]dz = S^D\left(w^S, \tilde{z}^A\right) \tag{51}$$

Equating this to the skilled-labour-market equilibrium locus in autarky, equation (10), yields $\int_{\tilde{z}^A}^1 Q(z)\,dz = 0$, where:

$$Q(z) \equiv 2\delta q^M[w^S\{\gamma(z) + \theta\left(\tilde{z}^A\right)\}] - \delta q^M[w^A\left\{\gamma(z) + \theta\left(\tilde{z}^A\right)\right\}] \tag{52}$$

It is easily checked that $Q'$ is positive:

$$Q' = \frac{\delta\lambda\bar{L}}{2b}\left(w^A - w^S\right)\gamma' > 0 \tag{53}$$

so $Q(z)$ satisfies the restrictions of the Lemma.

Next, consider the unskilled labour market. The equilibrium locus in autarky is given by (9). In free trade, evaluated at $w^S$ and $\tilde{z}^A$, the demand for unskilled labour is:

$$L^D\left(w^S, \tilde{z}^A\right) = 2\int_{\tilde{z}^A}^1 \gamma(z)\,q^M[w^S\{\gamma(z) + \theta\left(\tilde{z}^A\right)\}]dz \tag{54}$$

Hence the excess demand for unskilled labour at $w^S$ and $\tilde{z}^A$ equals:

$$L^D\left(w^S, \tilde{z}^A\right) - \bar{L} = \int_{\tilde{z}^A}^1 \gamma(z)\,Q(z)\,dz \tag{55}$$

Invoking the Lemma shows that this is negative, which proves the proposition.

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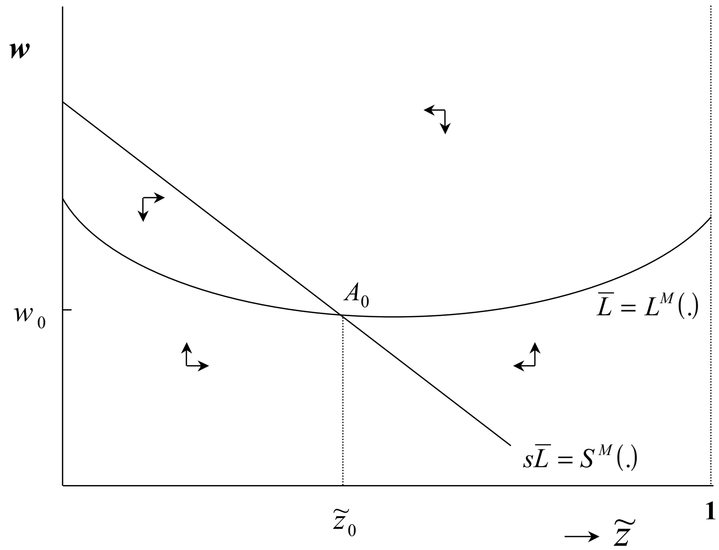


**Fig. 1: Simultaneous Determination of the Unskilled Wage and the Extensive Margin**

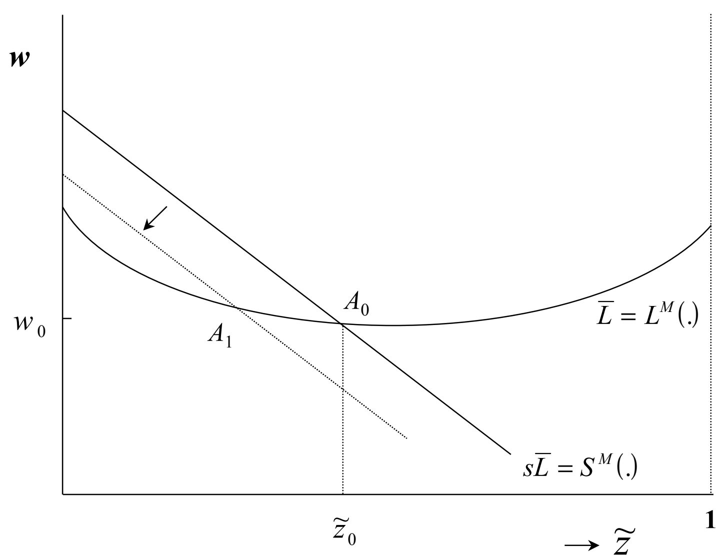


**Fig. 2: Effects of an Increase in the Endowment of Skilled Labour**

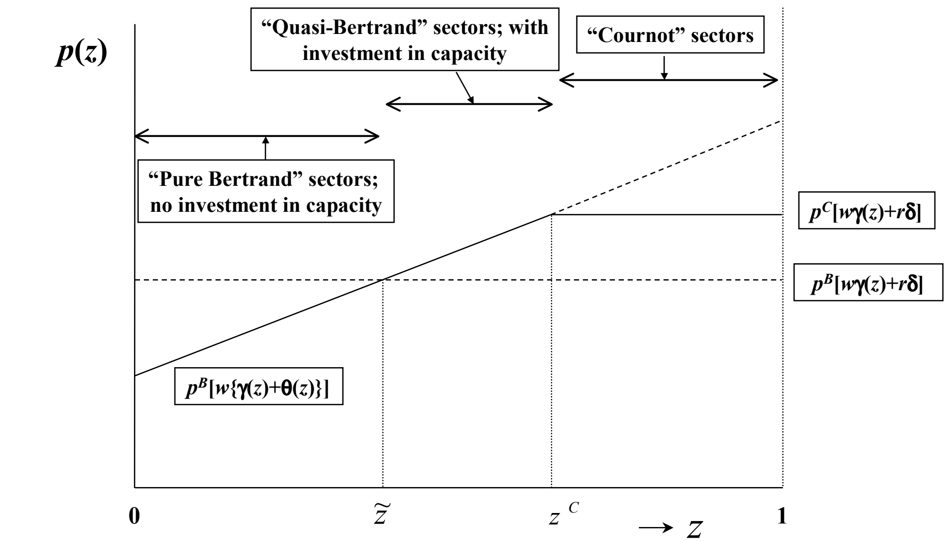


**Fig. 3: Distribution of Prices across Sectors at Given Factor Prices; $\gamma_1 = 0$, $\theta_1 > 0$.**

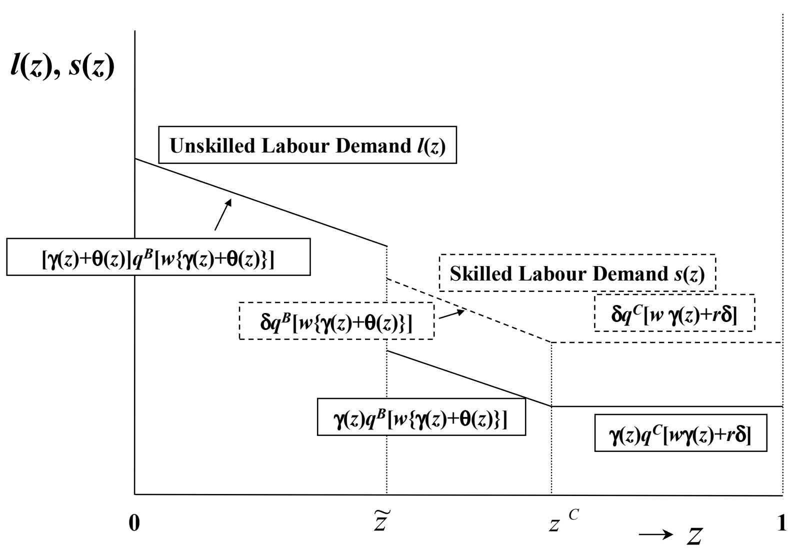


**Fig. 4: Demand for Skilled and Unskilled Labour across Sectors at Given Factor Prices; $\gamma_1 = 0$, $\theta_1 > 0$.**

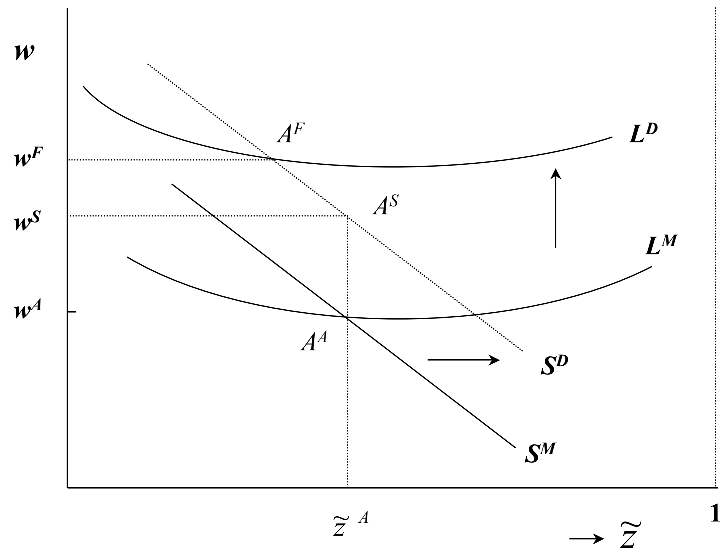


**Fig. 5: Autarky and Free Trade Equilibria with Uniform Variable Costs**

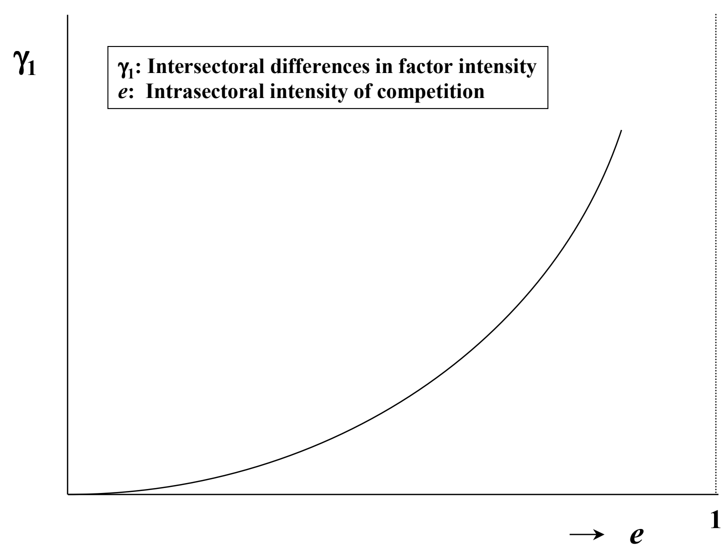


**Fig. 6: Autarky versus Free Trade**

**For parameter values above the curve, free trade leads to a higher skill premium and threshold sector than autarky; conversely for values below. See text for further details.**