Shadow Pricing Market Access: A Trade Benefit Function Approach

Nancy H. Chau* Rolf Färe†

This version: August 2007

Abstract: Appropriate assessment of the social value of market access is at the core of a broad range of inquiries in trade research. A selection include: the appraisal of industry-level production and consumption distortions due to selective trade liberalization and partial tax reform; the construction of national-level quantity indicators of market access consistent with welfare change, and the use of international trade re-balancing as sanctions to discourage trade agreement violations, or as compensation in trade dispute settlement. In order to obtain shadow prices, we propose a new approach integrating the Luenberger benefit function and the directional output distance function. This yields a trade benefit function which represents trade preferences à la Meade in the context of a canonical general equilibrium model of trade. We first show that our approach is in keeping with well-established and commonly used measurement techniques of trade welfare, for the standard trade expenditure function is in fact dual to the trade benefit function. We then show that this dual relation allows for a direct retrieval of the shadow values of net imports from the trade benefit function. The usefulness and operationality of our approach is then demonstrated in a series of applications and simulations.

JEL Classification: F 13.
Keywords: Trade Benefit Function, Shadow Prices of Net Imports

*Department of Applied Economics and Management, Cornell University, Ithaca NY 14853. Email: hyc3@cornell.edu
†Department of Economics, Oregon State University, Corvallis, OR 97331. Email: Rolf.Fare@orst.edu

1 Introduction

The importance that countries attach to the value of net imports of traded goods, often referred to simply as market access, surfaces in a wide variety of contexts. A prime example involves multilateral trade agreement, such as the GATT/WTO, which provides a negotiation ground wherein the rights of market access can be exchanged. This widely noted concern over market access has been referred to as a trade agreement negotiation rule-of-thumb (Krugman 1997), and also more fundamentally as principles embodied in the articles of the GATT/WTO, such as GATT Article XXVIII and Article 22.4 of the Dispute Settlement Understanding.[1] The issue of market access takes center stage in the practice of each of these principles, where for example a trade agreement violation or withdrawal of concessions by any one party is to be met by an equivalent and compensatory market access re-balancing, leaving total trade unchanged for all parties concerned (Anderson 2002, WTO 2005).

This focus on securing market access is related to at least three important and distinctive lines of research inquiries, each with their own collection of implications that have not yet been fully understood as part of a coherent framework in which the welfare implications of market access can be ascertained. First, the use of market access as chips in trade agreement negotiation ignores the intrinsic value of unilateral concessions in market access. Some of the oldest arguments of gains from trade hold that a country's own welfare gains from opening up should not depend on whether other countries have decided to adopt trade-related policies against their own self-interests. But opening up need not imply trading more. Indeed, the world value of imports and exports is generally not well-suited to convey information about trade restrictiveness, national welfare, or economic efficiency, since important country-level differences in preference and endowments (Belassa 1985, Leamer 1989), policy distortions (Chau, Färe and Grosskopf 2003) and composition effects (Hummels and Klenow 2005) can simultaneously confound observed trade outcomes.

By contrast, the vital role of rule-based reciprocal exchanges in market access of equivalent (world) value has been shown in a body of recent studies. The key insights of this important

[1] To wit, GATT Article XXVIII legitimizes compensation in the form of the withdrawal of "substantially equivalent concessions", when a trade partner withdraws a previous concession. Article 22.4 of the Dispute Settlement Understanding of the WTO allows an injured party in a legitimate trade dispute to "suspend previous concessions equivalent to the nullification or impairment of benefits".

line of inquiry is two-fold. First, a rule-based negotiation requiring reciprocal exchanges in market access of equivalent value (Bagwell and Staiger 1999, 2002) – the reciprocity principle – can guide self-interested policy-makers and negotiators to an efficient international trade agreement, for doing so effectively rules out the ability on the part of countries to manipulate the terms of trade. Second, given a trade agreement, the requirement that any trade agreement violation is to be matched by a compensatory displacement of trade of equal value (Anderson 2002, World Trade Report 2005) – the nullification or impairment rule – can likewise cancel out any terms of trade gains that a violation ex post can offer (Bown 2002). Taken together, the central theme then is that the world value of market access does in fact convey important information about aggregate welfare of open economies through the terms of trade, and putting checks on its value can be expected to alter the behavior of self-interested governments.

The third line of inquiry deals with the lack of participation in trade dispute settlements (Dam 1970, Bown 2002, 2000, Busch and Hauser 2000). Two sets of issues arise here which again call into question the value of securing market access. First, and evidently, trade agreement violations do occur despite the nullification or impairment rule, and governments have elected to raise illegal trade barriers to protect both export and import competing sectors (Bown 2002, WTO 2007). Second, when trade violations do occur, non-participation in trade disputes is nonetheless widespread. Put another way, there are countries that may appear to prefer non-compliance knowing that any market access gains may be nullified. Meanwhile, there are others who do not avail themselves to a re-balancing in market access that they are eligible to make. In the end, one may be left with the question, is more market access better, or worse?

In this paper, we approach these questions by introducing a set of new tools based on which the shadow value of net imports can be evaluated. We do so by proposing an alternative representation of the trade preferences of an open economy, which we call the trade benefit function. The trade benefit function is rooted in the Meade trade utility function in the context of a general equilibrium model of international trade (Meade 1952, Chipman 1970, Woodland 1980). It is a directional distance function in units of net imports, which integrates the Luenberger benefit function in the consumer context (Luenberger 1992, 1995), and the directional output distance function in the producer context (Chambers, Chung and Färe 1996, Färe and Primont 2005).[2]

This paper contains three main sets of results. First, the theory of the trade benefit function is worked out and its properties explored. In particular, the trade benefit function is shown to be dual to the familiar trade expenditure function, and as such it allows for direct retrieval of the shadow prices of net imports as its import derivatives. As a matter of interpretation, these shadow prices give the vector of sector-specific *trade tax ridden price equivalents*, which yields the same vector of net imports as any given set of trade-related policies. Thus, the shadow prices of net imports summarize the general equilibrium trade impacts of any combination of tax reforms and trade policies in an open economy, and re-express them in the form of a net import equivalent vector of sector-specific trade tax ridden prices.

Interestingly, our findings also allow for a re-interpretation of the Le-Chatelier's principle in the context of trade, noted in Meade (1952), and shown formally in Woodland (1980). Interpreted in the context of trade utility function, consumer preferences and technologies, the principle holds that the trade indifference curve is *less convex* than the corresponding consumer indifference curve and the production frontier, since efficiency gains can typically be achieved via a reallocation of consumption and production, given the same level of net imports. Interpreted instead in the context of shadow prices and the trade benefit function, we find the same principle to imply that the shadow prices of net imports are in general unobservable and require estimation, for they are not equal to observed consumer prices, observed producer prices, or world prices. The only exception is that of pure trade taxes, so that consumer price and producer price coincide in every sector (e.g. free trade, with zero trade tax everywhere).[3]

The second set of results applies the shadow value of market access developed here to the three distinctive research questions alluded to in the outset, with an aim to resolve some of

[2]The roots of the directional distance function approach to technology / preference representation can be traced back to the concept of disposable surplus of Allais (1943) and the benefit function of Luenberger (1992, 1995). The full implication in terms of duality theory; input, output, and profit efficiency measurements; empirical implementation strategies; with the inclusion of the production and consumption both of goods and bads, have been explored recently in a series of studies (Färe and Primont 2006, Chambers, Chung and Färe 1996, Färe and Grosskopf 2000).

[3]The issue of multiple and asymmetric consumer and producer price distortions has been of longstanding interest (Lipsey and Lancaster 1956, Foster and Sonnenschein 1970, Kawamata 1974). Its practical relevance has also been shown in a series of recent studies (Keen and Ligthart 2002, Michael et. al. 1991), where the removal of tariff barriers and a corresponding matching increase in consumption tax is shown to be a revenue neutral way of liberalizing trade, and improving welfare.

the apparent contradictions implied by the literature with regards to the relationship between changes in welfare and changes in market access. We approach the first question – the welfare implication of changes in market access – by proposing a pair of Laspeyres and Paasche quantity indicators of net imports using shadow prices. We show that the duality between trade benefit function and the trade expenditure function straightforwardly implies that the quantity indicators of net imports, evaluated at the relevant shadow prices, are indeed systematically related to changes in trade welfare. The gains from trade proposition is accordingly presented as a special case, with the additional advantage here being that the results are stated independently of whether there is a departure from free trade (Deardorff 1980, Kemp 1964, Ohyama 1972, Chipman 1979, Woodland 1982, Diewert and Woodland 2004).

These indicators are compared with their counterparts evaluated using either ex ante or ex post world prices, and also to a pair of Laspeyres and Paasche terms of trade indices (Krueger and Sonnenschein 1967, Woodland 1982). It is shown that a constant level of market access, evaluated at world prices, is synonymous with a constant terms of trade, when expressed in the form of Laspeyres and Paasche terms of trade indices. This corresponds exactly with the Bagwell-Staiger (1999) insight originally formulated for a two goods, two countries world, where the corresponding terms of trade index simplifies to a price ratio. In our context, however, this synonymity in fact reinforces the usefulness of shadow pricing market access. In particular, by capturing only changes in the terms of trade, the use of world prices to evaluate changes in market access will in general give an incomplete assessment of the corresponding welfare change, since it leaves out other possible trade / allocative inefficiencies, which are embodied instead in the shadow prices of net imports.

The second question is concerned with the welfare implication of *market access equivalence*, in the context of dispute settlements in international trade agreements. In particular, we seek a set of conditions under which a country may be deterred from violating *any* given trade agreement (accommodating either pure production subsidies, consumption taxes, quantitative restrictions, or combinations thereof), if non-compliance will be countered by a re-balancing in market access, in accordance with the nullification or impairment rule. We show that this twin-purpose – nullification and non-compliance deterrence – can be fulfilled, if and only if market access is evaluated at the shadow price of imports of the violating country before the violation.

Put differently, even in the absence of political considerations, one reason why the nullification or impairment rule can fail to deter trade agreement violations is simply that market access is valued at the wrong set of prices.

We then turn to the issue of lack of participation in trade dispute settlement, given the nullification or impairment rule. It is shown that while the key to violation deterrence requires evaluating market access in terms of the violator's shadow prices, the incentive to participate in trade dispute settlement depends instead on the compliant countries' shadow prices. We then restrict analysis to the case of efficient trade agreements in a two-country world economy, wherein shadow prices of trade partners coincide. Ironically, since the nullification or impairment rule guarantees that a violating country will be worse off in the retaliation phase compared to the trade agreement phase, the same rule means that the compliant country can not be better off retaliating compared to the trade agreement phase either, precisely because the shadow prices of the two countries are the same. In other words, the nullification or impairment rule should not be construed as a means of fully compensating the welfare losses of a complainant country in the context of a trade dispute. To further add to this list of reasons why there may be a lack of participation in trade disputes, it is shown that there is a class of violations that will never be met with retaliation in a two-country world given the nullification or impairment rule and an efficient trade agreement. These include export subsidies, production subsidies / consumption taxes in export sectors, which guarantee an improvement in the terms of trade in the *compliant* country thanks to the violation itself.

The paper concludes with an exploration of two ways in which the trade benefit function can be operationalized. One approach uses an approximation formula, and the other invokes the duality between the trade benefit function and the trade expenditure function. In a series of simulations, we employ a pair of popularly used functional forms for the consumer expenditure function and the revenue function. Our findings reiterate the implications of the theory section, and show that with the exception of pure trade taxes in every sector, the shadow price of imports are country-specific even when observed ad valorem tax and subsidy rates are the exactly same across countries. In each case, the shadow prices of imports are shown to depend on the joint interactions between the size of trade-related policies, the composition of such policies, and the technology, endowment and preferences of the country.

2 The Open Economy

We begin with an open economy in which production possibilities are given by the output set $Y(v) \subseteq \Re^M_+$. $v \in \Re^N_+$ denotes a vector of N input endowments. $Y(v)$ is a compact, convex set and $Y(0) = \{0\}$. Inputs and outputs are freely disposable, with $y \leq y' \Rightarrow y \in Y(v)$ whenever $y' \in Y(v)$ and $v \leq v' \Rightarrow Y(v) \subseteq Y(v')$.[4]

Preferences of the aggregate household are characterized by a utility function of the M goods ($x \geq 0$), $u(x)$, where $u(x)$ is assumed to satisfy standard concavity and monotonicity properties, with $u(x') \geq u(x)$ whenever $x' \geq x$.

2.1 Trade Preferences and Trade Expenditure

Let the set of feasible net import vectors $\mathcal{M}(v)$ be

$$\mathcal{M}(v) \equiv \{m | m = x - y, \ \ x \geq 0, \ \ y \in Y(v)\},$$

where $m_i > (<) 0$ whenever there is strictly positive net import (export) of good i. For any $m \in \mathcal{M}(v)$, the Meade direct trade utility function gives the maximal utility that can be achieved given the output set and a vector of net imports m (Meade 1952, Chipman 1970, Woodland 1980):

$$U(m, v) \ \ \equiv \ \ \max_{x,y} \{u(x), m = x - y, x \geq 0, y \in Y(v)\}. \tag{1}$$

Given our assumptions on $Y(v)$ and $u(x)$, $U(m, v)$ is a continuous real valued function, quasi-concave jointly in (m, v), and non-decreasing in m and v respectively, with $U(m', v) \geq U(m, v)$ if $m' \geq m$, and $U(m, v') \geq U(m, v)$ if $v' \geq v$.

The trade expenditure function, in turn, gives the smallest import expenditure required to achieve direct trade utility μ, and evaluated at given price vector $\pi > 0$ (Woodland 1980):

$$\begin{aligned} S(\pi, \mu, v) &\equiv \inf_m \{\pi \cdot m, \ \ U(m, v) \geq \mu\} \\ &= E(\pi, \mu) - G(\pi, v) \\ &\equiv \inf_x \{\pi \cdot x, \ \ u(x) \geq \mu)\} - \sup_y \{\pi \cdot y, \ \ y \in Y(v))\}, \end{aligned} \tag{2}$$

[4] See Färe and Primont (1995) for a discussion of these properties in the production context, and Woodland (1980) in the context of an open economy.

where $E(\pi, \mu)$ and $G(\pi, v)$ are respectively the expenditure and the revenue functions. From (2), the trade expenditure function exhibits properties that follow directly from $E(\pi, \mu)$ and $-G(\pi, v)$, including homogeneity of degree one and concavity in π. The vector of compensated import demand is simply: $\nabla_\pi S(\pi, \mu, v) \equiv m(\pi, \mu, v) = \nabla_\pi E(\pi, \mu) - \nabla_\pi G(\pi, v) = x(\pi, \mu) - y(\pi, v)$, where $x(\pi, \mu)$ and $y(\pi, v)$ are respectively the expenditure minimizing compensated consumption vector and the revenue maximizing output vector.

3 The Trade Benefit Function

The trade benefit function is an alternative primal representation of trade preferences. Let $g \geq 0$ be an M dimensional directional vector denoting a reference basket of net imports.[5] The trade benefit function is defined as[6]

$$\vec{D}^m(m, \mu, v; g) = \max\{\beta | U(m - \beta g, v) \geq \mu\} \tag{3}$$

if there exists some scalar β such that $m - \beta g \in \mathcal{M}(v)$, and $-\infty$ otherwise. $\vec{D}^m(m, \mu, v; g)$ is thus a directional distance function in units of net imports, and measures the market access adjustments required to maintain a given level of welfare in terms of net import contractions / expansions in the direction g. Alternatively, $\vec{D}^m$ can also be understood as a measure of the willingness to trade one net import vector for another, as $\vec{D}^m$ gives the units of the reference basket of net imports g that an economy should be maximally willing to forgo starting from m, in order to avoid adopting any other vector of net imports along the trade indifference curve $\{m' | U(m', v) = \mu\}$. Figure 1 illustrates.

Since the Meade trade utility function integrates the production and consumption sides of an open economy, the associated trade benefit function can also be shown to exhibit a number general properties that are directly analogous to the properties of the Luenberger (1992) benefit function which deals with consumption inefficiencies, and the directional output distance

[5]Thus, the only requirement on g is that at least one element of the reference basket is strictly positive, but otherwise it accommodates the choice of any combination of (possibly differing) units of the M goods. As examples $g = (1, 0, 0, ..., 0)$ takes good 1 as the numeraire, while $g = (1/M, ..., 1/M)$ corresponds to a reference baskets with equal units of every good. In the sequel, the independence of the trade benefit function with respect to proportional changes in g will also be demonstrated.

[6]Since elements of m can take on both positive, zero and negative values, standard radial distance function measures (e.g. Shephard (1970)) cannot be employed in the context of trade. The indicator approach adopted here based on differences rather than ratios overcomes this difficulty.

function of Chambers, Chung and Färe (1996)[7] which deals with production inefficiencies. Each of these properties will be useful in the sequel and their proofs are relegated to the Appendix:

Trade preference representation: $\vec{D}^m(m, \mu, v; g) \geq 0$ if and only if $U(m, v) \geq \mu$. Thus $\vec{D}^m(m, \mu, v; g)$ completely characterizes trade preferences as given by the trade utility function.

Monotonicity: Given $m' \geq m$, $v' \geq v$ and $\mu' \geq \mu$, $\vec{D}^m(m', \mu, v; g) \geq \vec{D}^m(m, \mu, v; g)$, $\vec{D}^m(m, \mu, v'; g) \geq \vec{D}^m(m, \mu, v; g)$, and $\vec{D}^m(m, \mu', v; g) \leq \vec{D}^m(m, \mu, v; g)$. Equivalently, the M imports are goods, not bads,[8] while an increase in any one or more of the N inputs cannot decrease trade utility.

g-homogeneity of degree -1 and unit independence: For any strictly positive scalar γ, $\vec{D}^m(m, \mu, v; \gamma g) = \gamma^{-1}\vec{D}^m(m, \mu, v; g)$, and $\nabla_g \vec{D}^m(m, \mu, v; g) \cdot g = -\vec{D}^m(m, \mu, v; g)$, where

$$\frac{\partial \vec{D}^m(m, \mu, v; g)}{\partial g_j} = -\vec{D}^m(m, \mu, v; g)\frac{\partial \vec{D}^m(m, \mu, v; g)}{\partial m_j}$$

which shows that the import contraction $\vec{D}^m g$ required to reach μ is invariant to proportional increases in the directional vector g.

Translation property: $\vec{D}^m(m - \alpha g, \mu, v; g) = \vec{D}^m(m, \mu, v; g) - \alpha$, $\alpha \in \Re$, and thus the net import vector $m - \vec{D}^m(m, \mu, v; g)g$ at trade utility μ is invariant to any translation of the starting import vector in the direction g.

Concavity: $\vec{D}^m(m, \mu, v; g)$ is concave jointly in (m, v). Thus, the Hessian matrices $\nabla_{mm}\vec{D}^m(m, \mu, v; g)$ and $\nabla_{vv}\vec{D}^m(m, \mu, v; g)$ are both negative semidefinite.

3.1 Duality and Normalized Shadow Price of Net Import

One important property of a distance function approach relates to shadow pricing. In the context of trade, we demonstrate in what follows that the trade benefit function can also be used to generate shadow prices of net imports, as distinct from but nevertheless related

[7] Also see Färe and Primont (2006) for a synthesis of alternative directional distance function concepts.
[8] See Chau, Färe and Grosskopf (2007) for an analysis when bads are introduced.

to consumer and producer prices. To this end, note that by definition of the trade benefit function, the trade expenditure function can be expressed as an unconstrained infimum:[9]

$$S(\pi, \mu, v) \;=\; \inf_m\{\pi \cdot m,\;\; U(m, v) \geq \mu\} = \inf_m\{\pi \cdot (m - \vec{D}^m(m, \mu, v; g)g)\}. \tag{4}$$

Since $S(\pi, \mu, v)$ is the minimal trade expenditure at given prices, (4) implies the following relationship between the trade expenditure function and the trade benefit function:

$$S(\pi, \mu, v) \leq \pi \cdot (m - \vec{D}^m(m, \mu, v; g)g) \;\;\Leftrightarrow\;\; \vec{D}^m(m, \mu, v; g) \leq \frac{\pi \cdot m - S(\pi, \mu, v)}{\pi \cdot g}. \tag{5}$$

In fact, (5) above can be strengthened, and the trade benefit function can be directly retrieved from the expenditure and the revenue functions via the following minimization problem:

$$\vec{D}^m(m, \mu, v; g) \;=\; \inf_\pi \left\{\frac{\pi \cdot m - S(\pi, \mu, v)}{\pi \cdot g}\right\}. \tag{6}$$

The formal proof is relegated to Appendix A. The intuition of (6) follows from quasi-concavity of the trade utility function, and is illustrated in Figure 2 for a two-good case. As shown, given any observed net import vector $\bar{m}$, the smallest difference between observed trade expenditure and minimal trade expenditure $(\pi \cdot \bar{m} - S(\pi, \mu, v))/\pi \cdot g$ is indeed just $\vec{D}^m(\bar{m}, \mu, v; g)$, by choice of a relative price ratio equal to the marginal rate of substitution of the trade indifference curve evaluated at $\bar{m} - \vec{D}^m(\bar{m}, \mu, v; g)g$.

Now, to see the relationship between shadow price of net imports and the trade benefit function, suppose that $\hat{m}$ solves the unconstrained minimization problem (4), and let $\tilde{\pi}$ denote the normalized price vector, $\tilde{\pi} = \pi/\pi \cdot g$, expressed in units of the reference import basket g. Since $S(\pi, \mu, v)$ is homogeneous of degree one in π, it follows by definition of the uncompensated net import demand $\nabla_\pi S(\pi, \mu, v) = m(\pi, \mu, v)$ and the first order condition from (4) that

$$m(\tilde{\pi}, \mu, v) \;=\; \hat{m} - \vec{D}^m(\hat{m}, \mu, v; g)g \tag{7}$$

$$\tilde{\pi} \;=\; \nabla_m \vec{D}^m(\hat{m}, \mu, v; g). \tag{8}$$

Equations (7) and (8) show that $\nabla_m \vec{D}^m(\hat{m}, \mu, v; g)$ gives the vector of normalized prices which

[9] Since the objective function (3) takes both imports m and input endowments v as given, one may also derive from the trade benefit function the shadow price of inputs evaluated at the shadow price of imports. This issue is of independent interest, and will be the subject of a companion paper.

exactly induces the expenditure minimizing import vector $\hat{m} - \vec{D}^m(\hat{m}, \mu, v; g)g$.[10] One solution to the unconstrained problem in (4) that is of particular interest is $\hat{m} = m(\pi, \mu, v)$ itself. We have thus $\vec{D}^m(m(\tilde{\pi}, \mu, v), \mu, v; g) = 0$ or equivalently $U(m(\tilde{\pi}, \mu, v), v) = \mu$ from (7). Using (8),

$$\tilde{\pi} = \nabla_m \vec{D}^m(m(\tilde{\pi}, \mu, v), \mu, v; g). \tag{9}$$

Put another way, given an observed net import vector m, import derivatives of the trade benefit function evaluated at m along its trade indifference curve $\mu = U(m, v)$, $\nabla_m \vec{D}^m(m, \mu, v; g)$, gives the vector of normalized shadow prices which exactly induces the net import vector m: $m = m(\nabla_m \vec{D}^m(m, \mu, v; g), \mu, v)$. Interestingly, this is exactly how trade taxes operate, since the same set of trade tax distorted prices applies both to producers and consumers: $m(\pi, \mu, v) = x(\pi, \mu) - y(\pi, v)$. Thus, the normalized shadow prices of net imports $\nabla_m \vec{D}^m(m, \mu, v; g)$ can indeed be interpreted as the vector of normalized trade tax ridden prices that induces production and consumption responses to yield the net import vector m, regardless of whether m is the result of tax reforms, trade policies, and / or quantitative restrictions to begin with.

To briefly summarize, (4) and (6) jointly establish the duality between the trade expenditure function $S(\pi, \mu, v)$ and the trade benefit function $\vec{D}^m(m, \mu, v; g)$: the trade benefit function can be directly retrieved from the trade expenditure function via the minimization problem (6), and vice versa using (4). The normalized shadow price of a given vector of net imports can then be obtained as import derivatives of the trade benefit function as in (9). From (8), it can be further shown that $\nabla_m \vec{D}^m(m, \mu, v; g) \cdot g = 1$, since $\tilde{\pi} = \pi / \pi \cdot g$. It follows that the normalized shadow value of the reference import basket g is unity, and is therefore independent of the choice of g.[11]

[10] One may also directly note that since $U(m(\tilde{\pi}, \mu, v), \mu, v) = \mu$, routine differentiation using (5) gives

$$\frac{\partial \vec{D}^m(\hat{m}, \mu, v; g)/\partial m_i}{\partial \vec{D}^m(\hat{m}, \mu, v; g)/\partial m_j} = \frac{\partial U(\hat{m} - \vec{D}^m(\hat{m}, \mu, v; g)g, v)/\partial m_i}{\partial U(\hat{m} - \vec{D}^m(\hat{m}, \mu, v; g)g, v))/\partial m_j}$$

which indicates in familiar fashion that the relative shadow price of any two net imports i and j is equal to the marginal rate of substitution between the same two net imports associated with the trade utility function $U(m, v)$.

[11] Additionally, it follows from g-homogeneity and the translation property of the trade benefit function that the shadow value of g is invariant to proportional changes in the directional vector, $\gamma \nabla_m \vec{D}^m(m, \mu, v; \gamma g) \cdot g = \nabla_m \vec{D}^m(m, \mu, v; g) \cdot g$, and any translation of the starting important vector in the direction g, from m to $m - \alpha g$, $\nabla_m \vec{D}^m(m - \alpha g, \mu, v; g) = \nabla_m \vec{D}^m(m, \mu, v; g)$.

3.2 The Shadow Price of Net Imports with Multiple Distortions

In order to relate the trade benefit function with consumption and production inefficiencies, and accordingly to draw the link between the shadow price of imports with consumer and producer prices, define the Luenberger benefit function and the directional output distance function as:[12]

$$\vec{D}^x(x,\mu;g) = sup\{\beta : u(x-\beta g) \geq \mu\}, \ \ \vec{D}^y(y,v;g) = sup\{\beta : y+\beta g \in Y(v)\}. \tag{10}$$

Thus, $\vec{D}^x(x,\mu;g)$ projects consumption x to the indifference curve at μ, and $\vec{D}^y(y,v;g)$ projects production y to the frontier of $Y(v)$, both along direction g. An analogous set of duality relations apply as in (4) and (6) (Luenberger 1992, 1995, Chambers, Chung and Färe 1996)[13]

$$E(q,\mu) = \inf_x\{q\cdot(x-\vec{D}^x(x,\mu;g)g)\}, \ \ G(p,v) = \sup_y\{p\cdot(y+\vec{D}^y(y,v;g)g)\}, \tag{11}$$

$$\vec{D}^x(x,\mu;g) = \inf_q\left\{\frac{q\cdot x - E(q,\mu)}{q\cdot g}\right\}, \ \ \vec{D}^y(y,v;g) = \inf_p\left\{\frac{G(p,v)-p\cdot y}{p\cdot g}\right\}. \tag{12}$$

As before, the gradient vector of the Luenberger benefit function with respect to x give the normalized vector of consumer prices, while the gradient vector of the directional output distance function with respect to y gives the normalized vector of producer prices, respectively $\nabla_x\vec{D}^x(x(\tilde{q},\mu),\mu;g) = q/q\cdot g = \tilde{q}$ and $\nabla_y\vec{D}^y(y(\tilde{p},v),v;g) = p/p\cdot g = \tilde{p}$.

The relationship between the trade benefit function, the Luenberger benefit function, and the directional output distance function can now be investigated. For any given consumption and utility pair (x,μ), along with an output and input pair (y,v), such that $x-\vec{D}^x(x,\mu;g)g \geq 0$ and $y+\vec{D}^y(y,v;g)g \in Y(v)$, the associated net import vector is by definition feasible: $x - \vec{D}^x(x,\mu;g)g - y - \vec{D}^y(y,v;g)g \in \mathcal{M}(v)$. Thus, by the translation property,

$$\begin{aligned}\vec{D}^m(x-y,\mu,v;g) &= \vec{D}^x(x,\mu;g) + \vec{D}^y(y,v;g) \\ &\quad + \vec{D}^m(x-\vec{D}^x(x,\mu;g)g - y - \vec{D}^y(y,v;g)g,\mu,v;g).\end{aligned} \tag{13}$$

From the definition of the direct trade utility function, since $y+\vec{D}^y(y,v;g)g \in Y(v)$,

$$U(x-\vec{D}^x(x,\mu;g)g - y - \vec{D}^y(y,v;g)g,v) \geq u(x-\vec{D}^x(x,\mu;g)g) = \mu \tag{14}$$

[12] The latter is a special case of the shortage function. See for example Luenberger (1995).

[13] Diewert and Woodland (2004) recently use a similarly defined set of shadow prices of outputs and consumption goods evaluated at autarky $m = 0$. These are then applied to examine the gains from trade proposition in the presence of technological change, and the introduction of new goods.

We have thus $\vec{D}^m(x - \vec{D}^x(x, \mu; g)g - y - \vec{D}^y(y, v; g)g, \mu, v; g) \geq 0$, or

Proposition 1 *For any (x, μ) and (y, v), such that $x - \vec{D}^x(x, \mu; g)g \geq 0$ and $y + \vec{D}^y(y, v; g)g \in Y(v)$, $\vec{D}^m(x - y, \mu, v; g) \geq \vec{D}^x(x, \mu; g) + \vec{D}^y(y, v; g)$.*

Thus, consumption and production inefficiencies as captured by $\vec{D}^x(x, \mu; g) > 0$ and $\vec{D}^y(y, v; g) > 0$ imply trade inefficiencies $\vec{D}^m(x - y, \mu, v; g) > 0$. The converse, however, need not be true. In fact, Proposition 1 suggests that the trade indifference curve is *less convex* than the corresponding consumption indifference curve, and the production possibility frontier. This observation, first made in Meade (1952), and proven formally in Woodland (1980), is presented in Proposition 1 in terms of the trade benefit function. This difference in curvature arises as there are almost always additional welfare gains that can be achieved by simultaneously adjusting both consumption and production, keeping import constant. Importantly, this insight essentially implies that the shadow price of imports (which should now account for trade inefficiencies through resource and consumption mis-allocations) are generally different from observed consumer and producer prices, except of course when there is no allocative inefficiencies, in the sense of $p/(p \cdot g) = q/(q \cdot g)$

From (13) and (14) above, the potential for trade efficiency gains through resource and consumption reallocation are exhausted whenever $\vec{D}^m(x - y, \mu, v; g) = \vec{D}^x(x, \mu; g) + \vec{D}^y(y, v; g)$, or equivalently

$$\vec{D}^m(x - \vec{D}^x(x, \mu; g)g - y - \vec{D}^y(y, v; g)g, \mu, v; g) = 0. \tag{15}$$

This allows us to prove the following, the details of which are provided in Appendix A:

Proposition 2 *The following are equivalent statements:*

1. *$\vec{D}^m(x - y, \mu, v; g) = \vec{D}^x(x, \mu; g) + \vec{D}^y(y, v; g)$, and*

2. *The vector normalized shadow prices of imports coincides with the normalized shadow prices of consumption, and of output:*

$$\nabla_m \vec{D}^m(x - y, \mu, v; g) = \nabla_x \vec{D}^x(x, \mu; g) = -\nabla_y \vec{D}^y(y, v; g)$$

evaluated respectively at $x - y - \vec{D}^m(x - y, \mu, v; g)g$, $x - \vec{D}^x(x, \mu; g)g$ and $y + \vec{D}^y(y, v; g)g$.

As an instructive special case, let $q > 0$ be the vector of consumer prices, and $p > 0$ the vector producer prices, not necessarily equal to q. Also let $p^* > 0$ be the vector of world prices. Given any reference level of utility μ and input vector v, the expenditure minimizing consumption and revenue maximizing output vectors are $x(q, \mu)$ and $y(p, v)$. Of course, by virtue of expenditure minimization given μ and revenue maximization given v:

$$\vec{D}^x(x(q, \mu), \mu; g) = 0, \quad \vec{D}^y(y(p, v), v; g) = 0,$$

where the normalized shadow price vector of consumption and production are simply $q/q \cdot g$ and $p/p \cdot g$. Proposition 1 essentially finds that in general

$$\vec{D}^m(x(q, \mu) - y(p, v), \mu, v; g) \geq 0 \Leftrightarrow U(x(q, \mu) - y(p, v), v) \geq \mu.$$

As such, welfare gains beyond μ may be achieved by adjusting consumption and production, while keeping import constant at $m = x(q, \mu) - y(p, v)$. Also from Proposition 2, such gains are exhausted if and only if $\vec{D}^m(x(q, \mu) - y(p, v), \mu, v; g) = 0$, or equivalently, when the normalized shadow price of imports, consumption and output all coincide:[14]

$$\begin{aligned} \nabla_m \vec{D}^m(x(q, \mu) - y(p, v), \mu, v; g) &= \nabla_x \vec{D}^x(x(q, \mu), \mu; g) = \frac{q}{q \cdot g} \\ &= -\nabla_y \vec{D}^y(y(p, v), v; g) = \frac{p}{p \cdot g}. \end{aligned}$$

Conversely, the above also implies that if consumer and producer prices differ, the shadow price of imports cannot be directly observed, for it is in general not equal to $q/(q \cdot g)$, $p/(p \cdot g)$, or the corresponding normalized world price. To formally establish the link between the shadow price of net imports, and producer and consumer prices when $q \neq p$, we have a preliminary result, proven in Appendix A:

Proposition 3 *The Hessian matrix of the trade benefit function evaluated at the import vector $(m(\pi, \mu, v))$ is given by*

$$\begin{aligned} \nabla_{mm} \vec{D}^m(m(\pi, \mu, v), \mu, v; g) &= \left(\nabla_{xx} \vec{D}^x(x(\pi, \mu), \mu; g)^{-1} + \nabla_{yy} \vec{D}^y(y(\pi, v), v; g)^{-1} \right)^{-1}. \\ &= A(\pi, g) \left(\nabla_{\pi\pi} E(\pi, \mu) - \nabla_{\pi\pi} G(\pi, v) \right)^{-1} \\ &= A(\pi, g) \nabla_{\pi\pi} S(\pi, \mu, v)^{-1} \end{aligned}$$

[14]For an economy in autarky, for example, $\nabla_m \vec{D}^m(0, \mu^a, v; g)$ gives the normalized shadow price of imports evaluated at the utility at autarky μ^a, which in turn coincides with normalized consumer and producer prices in the familiar way, with $x(\nabla_m \vec{D}^m(0, \mu^a, v; g), \mu^a) = y(\nabla_m \vec{D}^m(0, \mu^a, v; g), v)$.

where $A(\pi, g)$ *is the* $M \times M$ *matrix* $\frac{1}{\pi \cdot g}(1 - \pi g^T)$.

Proposition 3 provides an alternative relationship between the trade benefit function and the trade expenditure function, now expressed in terms of their respective substitution matrices. In addition, it also provides a direct link between the Hessian matrix of the trade benefit function in terms of the Hessian matrices of the Luenberger benefit function, and the directional output distance function, when the normalized shadow prices of imports, consumption and production coincide.

Consider therefore an import vector $x(q, \mu) - y(p, v)$, and the associated trade utility $\bar{\mu} = U(x(q, v) - y(p, v), v)$. Linearly approximate $\nabla_m \vec{D}^m(x(q, \mu) - y(p, v), \bar{\mu}, v; g)$ around a baseline price vector π,[15]

$$\begin{aligned} & \nabla_m \vec{D}^m(x(q, \mu) - y(p, v), \bar{\mu}, v; g) \\ \approx \; & \frac{\pi}{\pi \cdot g} + \nabla_{mm} \vec{D}^m(m(\pi, \mu, v), \mu, v; g)(x(q, \mu) - x(\pi, \mu)) \\ & + \nabla_{mm} \vec{D}^m(m(\pi, \mu, v), \mu, v; g)(y(\pi, v) - y(p, v)) \\ \approx \; & \frac{\pi}{\pi \cdot g} + \nabla_{mm} \vec{D}^m(m(\pi, \mu, v), \mu, v; g) \nabla_{xx} \vec{D}^x(x(\pi, \mu), \mu; g)^{-1} (\frac{q}{q \cdot g} - \frac{\pi}{\pi \cdot g}) \\ & + \nabla_{mm} \vec{D}^m(m(\pi, \mu, v), \mu, v; g) \nabla_{yy} \vec{D}^y(y(\pi, v), v; g)^{-1} (\frac{p}{p \cdot g} - \frac{\pi}{\pi \cdot g}) \end{aligned}$$

Proposition 3 can now be used to simplify the above expression to yield

Proposition 4 *The linear approximation of the normalized shadow price of imports can be expressed as a linear combination of normalized consumer and producer prices, with*

$$\nabla_m \vec{D}^m(x(q, \mu) - y(p, v), \bar{\mu}, v; g) \approx D \frac{q}{q \cdot g} + (1 - D) \frac{p}{p \cdot g} \qquad (16)$$

where D *denotes the* $M \times M$ *matrix* $\nabla_{mm} \vec{D}^m(m(\pi, \mu, v), \mu, v; g) \nabla_{xx} \vec{D}^x(x(\pi, \mu), \mu; g)^{-1}$, *and* $\bar{\mu} = U(x(q, \mu) - y(p, v), v)$

The shadow price of net imports can thus be directly seen as a sector-specific summary of the general equilibrium collective impact of all trade-related policies on net imports. A

[15]The first approximation formula follows since $x(\pi, \mu)$ and $y(\pi, v)$ are homogeneous of degree zero π, and

$$\frac{\nabla_m U(m(\pi, \mu, v), v)}{\nabla_m U(m(\pi, \mu, v), v) \cdot g} = \frac{\pi}{\pi \cdot g}.$$

The second follows since $\nabla_x \vec{D}^x(x(q, \mu), \mu; g) = q/(q \cdot g)$ and $\nabla_y \vec{D}^y(y(p, v), v; g) = -p/(p \cdot g)$.

few special cases may be considered. Suppose first that the country in question practices free trade, so that $p = q = p^*$. It follows that the normalized shadow price of imports correspond exactly to a similarly normalized set of world prices: $p^*/(p^* \cdot g)$. For trade taxes, where $q = p$, but not equal to p^*, the normalized vector of the shadow price of imports coincides with $q/(q \cdot g) = p/(p \cdot g)$, or equivalently the normalized vector of trade tax ridden prices. In each of these two cases, the normalized shadow price of imports can in principle be observed, corresponding respectively to $p^*/(p^* \cdot g)$, and $q/(q \cdot g) = p/(p \cdot g)$

Suppose now that consumption taxes and production subsidies differ, so that $p_i \neq q_i$ for at least one good i. Equation (16) reveals that the shadow prices of imports are no longer equal to observed prices. This will be the case when consumption tax is the only form of trade-related policies while no production subsidies are offered, for example.[16] Since the matrix D in (16) above is in general country-specific, depending on the nature of preferences, endowments, and technology, Proposition 4 additionally implies that two countries can have completely different sets of internal shadow prices of net imports, even when faced with the same prevailing set of world prices, and the same vectors of producer subsidies and consumption taxes.

A series of simulations of the optimization problem (6) will be performed in Section 5 to demonstrate these points numerically. In the next section, we first turn to three applications of the shadow prices of net import developed here.

4 Applications

By definition, the trade benefit function evaluates the market access adjustments required to achieve a given level of welfare. In what follows, we go in the opposite direction and apply the trade benefit function to examine how trade welfare $U(m, v)$ responds to a given change in market access. We do so in three related applications, involving the different stages of a trade dispute.

Consider therefore two countries (home and foreign), $j = h, f$, and three import vectors

[16]The relevance of this type of policy configuration which gives rise to asymmetric consumer and producer prices, and its importance as an IMF policy recommendation to countries undergoing trade liberalization, are discussed in Keen and Ligthart (2002). In particular, it is shown that trade liberalization, coupled with a point by point increase in consumption taxes can be a tax revenue neutral way of liberalizing trade, and improving welfare.

for each country observed over three periods, $m_j^t = x_j^t - y_j^t$, $t = 0, 1, 2$. Let the corresponding welfare of the aggregate household be $\mu_j^t = U_j(m_j^t, v_j)$. Also let p^{*t}, q_j^t and p_j^t respectively be the rest of the observables including the world price, consumer price, and producer price vectors, with $x_j^t = \nabla_q E_j(q^t, \mu_j^t)$ and $y_j^t = \nabla_p G_j(p_j^t, v_j)$. With balanced trade in all three periods, it must also be the case that $p^{*t} \cdot m_j^t = 0, \quad t = 0, 1, 2$. For each commodity $i = 1, ..., N$, market clearance requires $m_{h_i}^t + m_{f_i}^t = 0$.

We envisage the three time periods as corresponding to: (i) a base period trade agreement in which the two countries agree upon a set of trade related policies and associated market access given by (m_h^0, m_f^0); (ii) a period 1 violation of the trade agreement by the home country, with $m_j^1 \neq m_j^0$, and (iii) a compensatory set of policies in period 2 and the corresponding volume of trade m_j^2. Other than market clearance in all three periods, we wish to keep our setup in as general a set of terms as possible, and accordingly we will not put any further restrictions on the policy choices (production or consumption taxes, trade taxes, or quantitative restrictions) in the base period trade agreement phase, the violation in the first period, or the retaliatory response in the second, except for the requirement of balanced trade.

The basic setup embodies a whole host of possible sources of welfare change. A typical example of a trade agreement violation is of course an improvement in home welfare at the expense of the foreign country, due for example to production subsidies in an import competing sector or by import tariffs. This results in $\mu_f^0 > \mu_f^1$, and $\mu_h^0 < \mu_h^1$. In terms of the trade benefit function, $\vec{D}_f^m(m_f^1, \mu_f^0, v_f; g) < 0$, and $\vec{D}_h^m(m_h^1, \mu_h^0, v_h; g) > 0$. But a violation can of course also include production subsidies in the export sector, in which case a violation can in fact lead to $\vec{D}_f^m(m_f^1, \mu_f^0, v_f; g) > 0$, and $\vec{D}_h^m(m_h^1, \mu_h^0, v_h; g) < 0$.

In the compensation period, which may for example differ from period 1 in market access terms because of a set of countervailing foreign import tariffs, the trade benefit function can be used as a gauge of the effectiveness of the retaliation in nullifying welfare gains / losses of the two countries. Thus, say if $\vec{D}_f^m(m_f^2, \mu_f^t, v_f; g) \leq 0$, and $t = 0, 1$, foreign welfare in the retaliatory period falls short compared both to the base period and to the period during which violation takes place.

4.1 Import Quantity Indicators and Trade Welfare

We begin by defining a set of import quantity indicators, which can be applied to ascertain welfare change from base to period $t = 1, 2$. For each country $j = h, f$, define the Laspeyres ($Q^t_{L_j}$) and Paasche ($Q^t_{P_j}$) import quantity indicators in units of the reference import basket g, respectively at base period and current period world prices, as:

$$Q^t_{L_j} \equiv \tilde{p}^{*0} \cdot (m^t_j - m^0_j), \;\; Q^t_P \equiv \tilde{p}^{*t} \cdot (m^t_j - m^0_j) \tag{17}$$

where $\tilde{p}^{*t}$ is the normalized price vector $p^{*t}/(p^{*t} \cdot g)$. When $Q^t_{L_j}$ or $Q^t_{P_j}$ takes on a value greater (less) than zero, the world value of net imports rises (declines) from base to period t.

Also define the corresponding import quantity indicators $SQ^t_{L_j}$ and $SQ^t_{P_j}$ as:

$$SQ^t_{L_j} \equiv \tilde{\pi}^0_j \cdot (m^t_j - m^0_j), \;\; SQ^t_{P_j} \equiv \tilde{\pi}^t_j \cdot (m^t_j - m^0_j) \tag{18}$$

where $\tilde{\pi}^t_j = \nabla_m \vec{D}^m_j(m^t_j, \mu^t_j, v_j; g)$, $t = 0, 1, 2$. Here, net imports are valued respectively at normalized past and current shadow prices of net imports. From (6)

$$\begin{aligned} \vec{D}^m_j(m^t_j, \mu^0_j, v_j; g) &= \inf_\pi \left\{ \frac{\pi m^t_j - S_j(\pi, \mu^0_j, v_j)}{\pi \cdot g} \right\} \\ &\leq \tilde{\pi}^0_j m^t_j - S_j(\tilde{\pi}^0_j, \mu^0_j, v_j) \\ &= \tilde{\pi}^0_j \cdot (m^t_j - m^0_j) = SQ^t_{L_j} \end{aligned}$$

the second to last equality follows since the normalized shadow price vector by definition induces the trade expenditure minimization import m^0_j, with $S_j(\tilde{\pi}^0_j, \mu^0_j, v_j) = \tilde{\pi}^0_j \cdot m^0_j$. Analogously,

$$\vec{D}^m_j(m^0_j, \mu^t_j, v_j; g) \;\; \leq \;\; \tilde{\pi}^t_j \cdot (m^0_j - m^t_j) = -SQ^t_{P_j}.$$

We have

Proposition 5 *Trade utility deteriorates going from base to period t ($\vec{D}^m_j(m^t_j, \mu^0_j, v_j; g) \leq 0$), if $SQ^t_{L_j} \leq 0$, and improves ($\vec{D}^m_j(m^0_j, \mu^t_j, v_j; g) \leq 0$), if $SQ^t_{P_j} \geq 0$.*
If, in addition, there is free trade in the base period, ($\tilde{\pi}^0_j = \tilde{p}^{*0}$), trade utility deteriorates if $Q^t_{L_j} \leq 0$. If there is free trade in period t, ($\tilde{\pi}^t_j = \tilde{p}^{*t}), t = 1, 2$, trade utility improves if $Q^t_{P_j} \geq 0$.

Whenever $SQ^t_{L_j} \leq 0$, there is a reduction in net imports valued at base period shadow prices. Proposition 5 shows such a change in market access implies a deterioration in welfare

$\mu_j^t \leq \mu_j^0$. Intuitively, since m_j^0 minimizes trade expenditure required to achieve μ_j^0 at the shadow price vector $\tilde{\pi}^0$, and m_j^t is affordable in base period shadow prices ($\tilde{\pi}_j^0 \cdot (m_j^0 - m_j^t) \geq 0$), it must be the case that trade welfare has deteriorated $\mu_j^0 = U_j(m_j^0, v_j) > U_j(m_j^t, v_j) = \mu_j^t$. Similarly, when the net import quantity indicator in current shadow prices $SQ_{P_j}^t$ takes on a value great than zero, m_j^0 is affordable and yet not chosen as the trade expenditure minimizing vector of imports at μ_j^t in the current period. Therefore, there must have been a welfare improvement from base to period t. The second part of the proposition shows free trade as a special case, when external world prices accurately reflects internal shadow valuation of net imports.

We note in addition that the familiar gains from trade proposition (Chipman 1962, Dixit and Norman 1980, Deardorff 1980) with or without policy distortions (Ohyama 1972) readily follows from Proposition 5. Specifically, let $m_j^1 = 0$, $j = h, f$, which will be the case, for example, if the home violation involves a complete ban on trade. It follows that some trade in the base period is better than no trade at all subsequent to home violation as long as

$$SQ_{L_j}^t = \tilde{\pi}_j^0 \cdot (m_j^1 - m_j^0) \leq 0 \Leftrightarrow \tilde{\pi}_j^0 \cdot m_j^0 \geq 0$$

for country $j = h, f$. With balanced trade ($\tilde{p}^{*0} \cdot m_j^0 = 0$), this simply requires that country j engages in natural trade during the base period (Deardorff 1980):

$$(\tilde{\pi}_j^0 - \tilde{p}^{*0}) \cdot m_j^0 \geq 0. \tag{19}$$

Thus, if all trade related policies are in the form of import tariffs / export subsidies so that producer and consumer prices coincide, $p_j^0 = q_j^0$, and accordingly $\tilde{\pi}_j^0 = \tilde{p}_j^0 = \tilde{q}_j^0$, (19) is but the familiar requirement that some trade in the base period is better than no trade in period 1 if tariff revenue is non-negative in the base period.

To further develop intuitions, we show next that the import quantity indicators in (18) can be decomposed into a terms of trade change component and a trade inefficiency component. In particular, following Krueger and Sonnenschein (1967), Kemp (1964) and Woodland (1982), we define the Paasche export price index P_{Pe}^t as the period t value of period t exports divided by the base period value of period t exports. The Paasche import price index P_{Pm}^t is similarly defined as the period t value of period t imports divided by the base period value of period t imports. Thus, let I_j be the set of imports of country j, so that $m_{j_i}^t > 0$ whenever $i \in I_j$ and

$m_{j_i}^t \leq 0$ otherwise, we have

$$P_{Pe_j}^t = \frac{\sum_{i \notin I_j} p_i^{*t} m_{j_i}^t}{\sum_{i \notin I_j} p_i^{*0} m_{j_i}^t}, \quad P_{Pm_j}^t = \frac{\sum_{i \in I_j} p_i^{*t} m_{j_i}^t}{\sum_{i \in I_j} p_i^{*0} m_{j_i}^t}.$$

The ratio $P_{P_j}^t \equiv P_{Pe_j}^t / P_{Pm_j}^t$ is the Paasche terms of trade index of country j, and $P_{P_j}^t > 1$ represents a terms of trade improvement in period t relative to the base. The Laspeyres export and import price indices can be similarly defined as:

$$P_{Le_j}^t = \frac{\sum_{i \notin I_j} p_i^{*t} m_{j_i}^0}{\sum_{i \notin I_j} p_i^{*0} m_{j_i}^0}, \quad P_{Lm_j}^t = \frac{\sum_{i \in I_j} p_i^{*t} m_{j_i}^0}{\sum_{i \in I_j} p_i^{*0} m_{j_i}^0}.$$

The ratio $P_{L_j}^t \equiv P_{Le_j}^t / P_{Lm_j}^t$ gives the Laspeyres terms of trade index of country j. From (17), and with balanced trade in all periods ($\tilde{p}^{*t} \cdot m_j^t = 0,\ t = 0, 1, 2$), $Q_{L_j}^t = \tilde{p}^{*0} \cdot (m_j^t - m_j^0) = (\tilde{p}^{*0} - \tilde{p}^{*t}) \cdot m_j^t$. Thus, $Q_{L_j}^t$ directly reflects price deviations $(\tilde{p}^{*0} - \tilde{p}^{*t})$ weighted by net import m_j^t. Dividing and multiplying through by $\sum_{i \notin I_j} p_i^{*0}(-m_{j_i}^t)$ and noting that $\sum_{i \notin I_j} p_i^{*t}(-m_{j_i}^t) = \sum_{i \in I_j} p_i^{*t} m_{j_i}^t$ with balanced trade, it follows that the quantity indicators of net imports and the terms of trade indices are in fact closely related by definition:

$$\begin{aligned} Q_{L_j}^t &= (P_{P_j}^t - 1) \sum_{i \notin I_j} p_i^{*0}(-m_{j_i}^t), \\ Q_{P_j}^t &= (P_{L_j}^t - 1) \sum_{i \in I_j} p_i^{*t} m_{j_i}^0, \end{aligned} \tag{20}$$

where the second expression follows analogously, since $Q_{P_j}^t = \tilde{p}^{*t} \cdot (m_j^t - m_j^0) = (\tilde{p}^{*0} - \tilde{p}^{*t}) \cdot m_j^0$ with balanced trade. From (20), an increase in net imports valued at past world price $Q_{L_h}^t > 0$ is synonymous with an increase in the price of exports over imports. This is expressed here as a terms of trade improvement, weighted by current import volumes $P_{P_j}^t > 1$. Similarly, an increase in net imports valued at current world prices is synonymous with a terms of trade improvement, weighted by past import volumes. When normalized shadow prices are used to evaluate net imports, however

$$\begin{aligned} SQ_{L_j}^t &= Q_{L_j}^t + (\tilde{\pi}_j^0 - \tilde{p}^{*0}) \cdot (m_j^t - m_j^0), \\ &= (P_{P_j}^t - 1) \sum_{i \notin I_j} p_i^{*0}(-m_{j_i}^t) + (\tilde{\pi}_j^0 - \tilde{p}^{*0}) \cdot (m_j^t - m_j^0), \\ SQ_{P_j}^t &= Q_{P_j}^t + (\tilde{\pi}_j^t - \tilde{p}^{*t}) \cdot (m_j^t - m_j^0) \\ &= (P_{L_j}^t - 1) \sum_{i \in I_j} p_i^{*t} m_{j_i}^0 + (\tilde{\pi}_j^t - \tilde{p}^{*t}) \cdot (m_j^t - m_j^0). \end{aligned} \tag{21}$$

As long as external world prices coincide with internal shadow prices of imports $((\tilde{\pi}_j^0 = \tilde{p}^{*0}),\ \ (\tilde{\pi}_j^t = \tilde{p}^{*t}))$, Proposition 5 and (20) - (21) together reiterate the findings in Krueger and Sonnenschein (1967), Kemp (1964) and Woodland (1982), wherein an improvement in a similarly measured Laspeyres terms of trade index $(P_{L_j}^t)$ implies a welfare improvement.

In addition, (20) - (21) also show that since the net import quantity indicators $Q_{L_j}^t$ and $Q_{P_j}^t$ are synonymous *only* with changes in terms of trade, they are not well-suited to capture welfare change whenever internal shadow prices differ from world prices $(\tilde{p}^{*0} \neq \tilde{\pi}_j^0)$. In familiar fashion, (21) shows that net import changes evaluated at shadow prices are made up of two parts: a terms of trade component, and another that captures changes in trade inefficiencies given the same terms of trade, $(\tilde{\pi}_j^t - \tilde{p}^{*t}) \cdot (m_j^t - m_j^0)$.

In particular, given no change in the terms of trade, $P_{P_j}^t = 1$, Proposition 5 and (21) above shows that welfare cannot improve (i) if there exists no trade inefficiency in the base period, $\tilde{\pi}_j^0 = \tilde{p}^{*0}$. Additionally, welfare will decline if (ii) existing trade inefficiencies are *reinforced* going from base to period 1, or $(\tilde{\pi}_j^0 - \tilde{p}^{*0}) \cdot (m_j^t - m_j^0) < 0$, so that imports are increased in those sectors where the base period internal shadow prices of net imports are already lower than the prevailing world price, and decreased when the opposite in fact the case.

Finally, it also follows from (17) as a matter of accounting identity that

$$Q_{L_h}^t + Q_{L_f}^t = 0, \ \ s_{P_h}^0 P_{P_h}^t + s_{P_f}^0 P_{P_f}^t = 1$$

where the $s_{P_h}^0$ is simply the home country's period t exports $\sum_{i \notin I_h} p_i^{*0}(-m_{h_i}^t)$ as a share of world exports $\sum_{j=h,f} \sum_{i \notin I_j} p_i^{*0}(-m_{j_i}^t)$, evaluated at base period world prices. Similarly

$$Q_{P_h}^t + Q_{P_f}^t = 0, \ \ s_{L_h}^t P_{L_h}^t + s_{L_f}^t P_{L_f}^t = 1$$

where the $s_{L_h}^t = \sum_{i \notin I_h} p_i^{*t}(-m_{h_i}^0)/(\sum_{j=h,f} \sum_{i \notin I_j} p_i^{*t}(-m_{j_i}^0))$. Of course, these literally reflect the familiar notion that market access rivalry, evaluated at world prices, is a zero sum enterprise, and in such a way that any improvement in one country's terms of trade comes at the other's expense, regardless of any trade-related policies that the two countries may have in place. Meanwhile, these also imply another set of identities. From (18), and market clearance:

$$\begin{aligned} SQ_{L_h}^t + SQ_{L_f}^t &= (\tilde{\pi}_h^0 - \tilde{\pi}_f^0) \cdot (m_h^t - m_h^0), \\ SQ_{P_h}^t + SQ_{P_f}^t &= (\tilde{\pi}_h^t - \tilde{\pi}_f^t) \cdot (m_h^t - m_h^0) \end{aligned} \tag{22}$$

which sum up respectively to zeros only when there is efficient allocation of market access across the two countries $\tilde{\pi}_h^t = \tilde{\pi}_f^t$, $t = 0, 1, 2$. More generally, the former is negative whenever any base period world trade inefficiencies, if they exist, are reinforced through an increase in home country imports in sectors where the base period home country shadow price is lower than foreign, while the latter positive when current world trade inefficiencies was worse that what it currently is. This applies when an increase in home import occurs where current home shadow prices are higher than foreign.[17]

4.2 Market Access and the Enforcement of Trade Agreements

We now turn specifically to base to period 2 welfare changes, between the status quo trade agreement, and the retaliatory period 2. As discussed in the introduction, we consider retaliations subsequent to non-compliance in terms of a re-balancing in market access. In accordance with GATT/WTO parlance, we work with market access re-balancing that serves the purpose of nullifying trade impairments.

While the basic idea is to keep the value of trade of the foreign country unchanged, the question remains as to what prices should be used to value exports and imports. In addition, as has been shown in (17) - (18), some candidates include both world prices and shadow prices of imports, in both ex ante (base period) and ex post (period 2) terms.

Let $p^n > 0$ be a positive M dimensional price vector used in the evaluation of market access adjustments, and also let $\tilde{p}^n$ be the normalized price vector $p^n / p^n \cdot g$. A nullification of the change in market access due to a home country violation at prices $\tilde{p}^n$ will be taken to require

$$\tilde{p}^n \cdot (m_h^2 - m_h^0) = 0, \tag{23}$$

or equivalently, that

$$\sum_{i \in I_h} \tilde{p}_i^n (m_{h_i}^2 - m_{h_i}^0) = -\sum_{i \notin I_h} \tilde{p}_i^n (m_{h_i}^2 - m_{h_i}^0) \tag{24}$$

where as before, I_h is the set of imported goods for the home country. Nullification thus requires that any import displacement of the home country be countered by an equivalent displacement

[17] In a multiple country framework $j = 1, ..., J$, similar identities hold: $\sum_{i=1}^J Q_{L_j}^t = 0$, $\sum_{i=1}^J s_{P_j}^0 P_{P_j}^t = 1$. Meanwhile, $\sum_{i=1}^J SQ_{L_j}^t = \sum_{j=2}^J \sum_i (\tilde{\pi}_{1_i}^0 - \tilde{\pi}_{j_i}^0)(m_{j_i}^t - m_{j_i}^0)$.

in home country exports, where imports and exports are evaluated at $\tilde{p}^n$. We note that (24) above is the multiple commodities analogue of the principle of reciprocity adopted in Bagwell and Staiger (1999, 2002), as well as the Dispute Settlement Understanding reciprocity adopted in Bown (2002), with the only difference being that p^n is defined as world prices in these studies.

In order for the nullification of changes in market access to be effective in deterring non-compliance incentives on the part of the home country, it must also be the case that home country trade welfare in the retaliation phase is no higher than μ_h^0, or $\vec{D}_h^m(m_h^2, \mu_h^0, v_h; g) \leq 0$. The proof of following is relegated to Appendix A.

Proposition 6 *A vector of normalized prices* $\tilde{p}^n > 0$ *ensures a constant value of market access at* $\tilde{p}^n \cdot (m_h^2 - m_h^0) = 0$*, and a period 2 level of home trade welfare no higher than* μ_h^0*,* $\vec{D}_h^m(m_h^2, \mu_h^0, v_h; g) \leq 0$*, if and only if*

$$\tilde{p}^n = \nabla_m \vec{D}_h^m(m_h^0, \mu_h^0, v_h; g) = \tilde{\pi}_h^0.$$

Equivalently, a constant market access in the sense of $SQ_{L_h}^2 = \tilde{\pi}_h^0 \cdot (m_h^2 - m_h^0) = 0$ guarantees that the violating home country's trade welfare is no higher than μ_h^0 in the retaliation phase. Intuitively, Proposition 6 seeks a price vector p^n with which to value market access adjustments, so as to guide the home country to *prefer* compliance. But this coincides with the definition of the normalized shadow value of imports, $\tilde{\pi}_h^0$, which by definition in (9), is the unique normalized price vector that induces the home to import m_h^0, at the base period level of trade welfare μ_h^0.

Two additional observations follow directly from Proposition 6 for the class of Pareto efficient trade agreement, where the normalized shadow price of net imports in the two countries coincide $\tilde{\pi}_h^0 = \tilde{\pi}_f^0$. Such an efficient trade agreement need not require free trade (Mayer 1980), but of course includes free trade as a special case.[18]

Note that while Proposition 6 is stated in terms of the shadow price and the change in net imports in the violating (home) country, both can be relaxed in the context of an efficient trade agreement with two countries, since

$$SQ_{L_h}^2 = \tilde{\pi}_h^0 \cdot (m_h^2 - m_h^0) = 0 = \tilde{\pi}_f^0 \cdot (m_f^0 - m_f^2) = -SQ_{L_f}^2$$

[18] For example, in the two-goods case where the home country is an importer of good 1 and the foreign country an importer of 2, any pair of ad valorem import tariff in the two countries satisfying $(1 + t_{h_1}^0)(1 + t_{f_2}^0) = 1$ constitutes a Pareto efficient trade agreement. It can be easily verified that the normalized shadow prices of the two countries are the same in the base period (Mayer 1980).

from (18). Accordingly, the nullification of *foreign country* market access changes at *foreign shadow prices* will nonetheless deter non-compliance incentives by the home country.[19] Finally, consider now a free trade agreement, in which the shadow prices of imports of the two countries coincide with world prices: $\tilde{\pi}_h^0 = \tilde{\pi}_f^0 = \tilde{p}^{*0}$.[20] We have,

$$SQ^2_{L_h} = Q^2_{L_h} = 0 = -Q^2_{L_f} = -SQ^2_{L_f}.$$

Thus, a guarantee that any changes in market access evaluated at free trade prices will be nullified in the retaliation phase is indeed sufficient to deter incentives to violate a free trade agreement by any of the two countries.

4.3 Market Access and Dispute Settlement Participation

In this application, we consider changes in market access going from period 1 to period 2, and the foreign welfare implications of launching a trade dispute, in exchange for a re-balancing in market access of the type specified in Proposition 6. Even in the presence of rule-based dispute settlement mechanisms, the reasons why countries may still deviate from trade agreements are diverse. In addition to political considerations, one possible reason is simply that the foreseeable punishment is not as high as the benefits from doing so, as shown in Proposition 6. But another equally valid reason may well be that compliant members of a trade agreement choose not to participate in a trade dispute, and as such retaliation will not be forthcoming despite the violation.

Starting therefore with Proposition 6 and (21), effective deterrence against non-compliance based on the nullification or impairment rule requires that the foreign country has trade instruments at her disposal to implement:

$$SQ^2_{L_h} = (P^2_{P_h} - 1) \sum_{i \notin I_h} p_i^{*0}(-m^2_{h_i}) + (\tilde{\pi}^0_h - \tilde{p}^{*0}) \cdot (m^2_h - m^0_h) = 0. \tag{25}$$

[19] Note however that in a world economy with multiple countries, including the trade agreement violating home and multiple other compliant foreign countries, the nullification of foreign changes in market access will deliver sufficient incentives to deter non-compliance by the home country if all foreign countries participate in trade dispute settlement.

[20] When instruments of redistribution via compensating transfers are available, and if the two countries in question are sufficiently similar in size, it has been shown that free trade constitutes one possible trade agreement between the two countries, that is Pareto superior to outright trade war (Johnson 1953-1954, Mayer 1981, Kennan and Riezman 1988, Bagwell and Staiger 1999, 2002).

Equally important, and from Proposition 5, a necessary condition for the complainant country to be better off retaliating, or, $U_f(m_f^2, v_f) \geq U_f(m_f^1, v_f)$, is

$$\tilde{\pi}_f^1 \cdot (m_f^1 - m_f^2) \leq 0, \tag{26}$$

which requires information on the shadow price of net imports of the complainant. To see whether effective deterrence through nullification (25) is consistent with participation (26), one needs a full-fledged model in which specifics are given about technologies, preferences, existing trade policy distortions, and the nature of trade agreement violation that gives rise to the period 1 shadow price and import pair, $(\tilde{\pi}_f^1,\ m_f^1)$ (Bown 2002). Without making these further assumptions, three observations may nevertheless be made.

Starting from the simplest case of a free trade agreement, so that $\tilde{\pi}_h^0 = \tilde{p}^{*0}$, effective nullification of market access in equation (25) above essentially requires that $P_{P_h}^2 = 1$. Thus, the foreign country is required to have at her disposal trade-related policy instruments to overturn any home country actions that may have impacted the terms of trade in the interim period 1. In other words, having market power is a pre-condition for effective nullification of changes in market access. Consistent with the original Bagwell and Staiger (1999, 2002) line of reasoning, countries with little market power have been found, both in theory and empirically (Dam 1970, Horn, Mavroidis and Nordström 1999, Bown 2002, 2000, Busch and Hauser 2000), to be less likely to participate in trade disputes.

Consider now more generally any efficient trade agreements. It follows from (22) and Proposition 5 that $SQ_{L_h}^2 + SQ_{L_f}^2 = 0$, or

$$SQ_{L_h}^2 = 0 = -SQ_{L_f}^2 \quad \Leftrightarrow \quad \vec{D}_f^m(m_f^2, \mu_f^0, v_f; g) \leq 0. \tag{27}$$

Thus, deterrence of violation (25) does not imply welfare compensation. Indeed, what (27) shows is that the nullification or impairment rule cannot fully compensate the welfare loss of the foreign country starting from an efficient trade agreement.

Finally, and again starting with an efficient trade agreement,[21] a class of trade agreement violation that in fact *discourages* the compliant foreign country from launching a trade dispute

[21] For inefficient trade agreements, there exist by definition trade agreement violations that can lead to Pareto improvements in welfare of all participating countries. Clearly, compliant countries will have no incentive to dispute this type of violation.

in exchange for market access nullification can readily be found. In particular, suppose that the violator's actions give rise to an *increase* in foreign net import volume so that $SQ^1_{P_f} \geq 0$, Proposition 5 and market clearing gives

$$\vec{D}^m_f(m^1_f, \mu^0_f, v_f; g) \geq \vec{D}^m_f(m^0_f, \mu^0_f, v_f; g) = 0 \geq \vec{D}^m_f(m^2_f, \mu^0_f, v_f; g)$$

where the last inequality follows from (27). In other words, the promise of a nullification of any change in market access can in fact discourage the report of a trade agreement violation, whenever the home country violation in fact raises the welfare of the foreign country beyond the trade agreement level. This occurs, for example, when the home country subsidizes of her exports, and by doing so improves the terms of trade of the foreign country.

5 Evaluation and Simulation

A key set of inputs in each of the aforementioned applications is the shadow prices of net imports. In this section, we explore ways to operationalize the trade benefit function, in order to obtain these shadow prices. The analysis in section 3 naturally suggests two possible approaches. The first is the linear approximation formula (16) shown in Proposition 4, which requires as inputs the vectors of consumer and producer prices, along with the Hessian matrices of the expenditure and revenue functions, made up of own- and cross-price slopes of demand and supply from Proposition 3.

A second approach directly makes use of the duality of the trade expenditure function and the trade benefit function implied by the minimization problem in (6). We report the results of a series of simulations based on this approach in Tables 1 - 3, Figures 3A and B. A constant elasticity of substitution expenditure function and a constant elasticity of substitution revenue function are adopted (Appendix B). Table 1 summarizes the four types of trade regimes examined here: (I) pure trade taxes, (II) consumption taxes only, (III) production subsidies only, and (IV) a combination of production subsidies and consumption taxes. Table 2 lists two sets of parametric assumptions, reflecting two small open economies A and B, each with 20 sectors, and each facing world prices normalized to unity in all sectors. The two economies are otherwise identical, but their production shares are ranked in decreasing and increasing order of importance respectively in A and B. Thus, sector 1 is a net exporting sector in economy A, and a net importing sector in economy B with free trade. Tables 3A and B list the results of the simulations. The column entries under "normalized shadow price" in the tables give the normalized sector-specific trade tax ridden price equivalents that yields the same net import vector as the four trade regimes listed in Table 1. Finally, the reference basket of net imports g puts equal weight on all 20 sectors, $g = (1/20, ..., 1/20)$.

These results illustrate a number of general points, and reiterate the findings of the theory sections. First, whenever producer and consumer prices coincide (the case of pure trade taxes), the vector of normalized shadow prices always coincide with the similarly normalized vector of internal consumer and producer prices. This is shown in Figure 3A and Table 1A for the case of export subsidies in sectors 1 - 10 of economy A, and in Figure 3B and Table 1B for the case of import tariffs also in sectors 1 - 10 of economy B. But since the subsidies and tax rates differ across sectors, it can be readily seen in Figure 3A for example that the schedule of export subsidies (regime AI) (i) raises the normalized shadow prices in *most* of the subsidized sectors, (ii) simultaneously raises the normalized consumer and producer prices above normalized free trade prices, but (iii) leaves the normalized shadow prices of the rest of the sectors (including some with the smallest subsidies) below free trade level.

Second, in the case of a pure consumption tax (trade policy regimes AII and BII), and the case of a pure production subsidy (trade policy regimes AIII and BIII), the corresponding schedules of normalized shadow prices are, roughly speaking, flatter than the corresponding schedules of normalized consumer and producer prices (Figures 3AII, 3AIII and 3BII, 3BIII). This corresponds well with the interpretation of shadow prices as the net import equivalent trade tax ridden prices. In particular, since a trade tax by definition simultaneously impact both consumers and producers, a larger consumption tax is required to generate the same net import impact as an import tariff, for example.

As a third observation, the results show that whenever producer and consumer prices differ, the relative sectoral ranking of ad valorem subsidies or tax rates may no longer accurately reflect the general equilibrium trade distortion implied by the entire schedule of subsidies or taxes. The case of pure production subsidy (regime BIII) in Figure 3B and Table 3B is one such example. Note that even though production subsidy rates are monotonically decreasing going from sector 1 to sector 20, the corresponding normalized shadow price of net imports is nonlinear and inverted-U shaped as shown. Put differently, in order to generate the same net import vector implied by the monotonically decreasing schedule of production subsidies, a nonlinear trade tax schedule as shown in the corresponding diagram in Figure 3B and Table 3B will be required. Intuitively, since the production share of sector 1 is the smallest, the trade distortionary impact of production subsidies in the rest of the economy overshadows the direct trade effect of a high production subsidy in sector 1. Indeed, the normalized shadow price in sector 3, with only the third highest ad valorem production subsidy across all 20 sectors, exhibits the largest positive deviation from its normalized free trade price.

Finally, a comparison of the simulation results associated with the four trade regimes across the two economies illustrates another important observation. Even when countries face the same set of world prices, and adopt identical trade-related policy measures across the board, the shadow price of net imports can still differ widely. In the context of our simulation, even though economies A and B both employ exactly the same schedules of monotonically decreasing production subsidies and increasing consumption taxes respectively in regimes AIV and BIV, the corresponding diagrams in Figures 3A and B, and columns in Tables 3A and B, show that the implied schedules of normalized shadow prices in the two economies are far from similar. Sector 1 in the two economies, with the largest production share in A, and the smallest in B, can once again be taken as an illustrative case in point. The schedule of trade-related policies implies a net import equivalent trade tax in the form of an export tax in sector 1 of economy A, thus encouraging net imports in general equilibrium. In sharp contrast, the same schedule of policies discourages net imports in sector 1 in economy B, and implies instead an import tariff for the same sector in economy B in general equilibrium.

6 Conclusion

In this paper, we introduced the trade benefit function as an alternative primal representation of trade preferences. The trade benefit function is a directional distance function in the space of net imports, and measures the maximal contraction in net imports required to meet a given level of trade utility. It is shown that the trade benefit function is dual to the familiar trade expenditure function, and is therefore readily operational, and consistent with standard approaches of trade welfare measurement. The trade benefit function yields a set shadow prices of net imports, or, the vector of net import equivalent trade tax ridden prices in general equilibrium. These shadow prices summarize the general equilibrium trade impact of any set of trade-related policies, and re-express them in terms of their sector specific impact in the form of a normalized set of trade tax ridden prices. A series of simulation is conducted which show, among other things, that the shadow prices of net imports are in general country-specific (except for a well-defined set of special cases), even when the same set of trade-related policies are in place across countries.

The usefulness of the shadow prices of net imports is demonstrated in three applications. In doing so, the analysis also resolves a number of apparent contradictions implied by the literature with respect to the relationship between changes in trade welfare, and changes in market access in general equilibrium. We introduce a pair of quantity indicators of changes in market access, evaluated at shadow prices, which are indeed systematically related to changes in trade welfare of an open economy. In a second application, we find these indicators to be useful not just as indicators of welfare and market access changes, but also in understanding the design of dispute settlement mechanisms in the context of international trade agreements. In particular, if the objective is to discourage trade agreement violations, it suffices to put in place a dispute settlement mechanism that promises a nullification of any market access changes caused by a violation, evaluated at the violator's ex ante shadow prices. Importantly, we find that effective deterrence does not imply adequate compensation, nor does it imply sufficient built-in incentives for compliant countries to rectify the changes in market access caused by a violation. In the last application, incentives to participate in trade dispute settlements founded on the nullification or impairment rule is shown to depend instead on the shadow price of net imports of the compliant country.

Appendix A

Properties of the Trade Benefit Function:[22]

Representation: By definition, if $U(m,v) \geq \mu$, $\vec{D}^m(m,\mu,v;g) \geq 0$. Now if $\vec{D}^m(m,\mu,v;g) \geq 0$, and since $U(m,v)$ is increasing in m and $g \geq 0$, $U(m,v) \geq U(m - \vec{D}^m(m,\mu,v;g)g, v) \geq \mu$.

Monotonicity: Given $m' \geq m$, $U(m' - \vec{D}^m(m',\mu,v;g)g, v) = \mu = U(m - \vec{D}^m(m,\mu,v;g)g, v) \leq U(m' - \vec{D}^m(m,\mu,v;g)g, v)$ since $U(m,v)$ is increasing in m and $g \geq 0$. Thus, $\vec{D}^m(m',\mu,v;g) > \vec{D}^m(m,\mu,v;g)$. The rest of the monotonicity results can be shown analogously.

g-homogeneity of degree -1 and unit independence: $\vec{D}^m(m,\mu,v;\gamma g) = \max\{\beta | U(m - \beta\gamma g) \geq \mu\} = \max\{\alpha/\gamma | U(m - \alpha g) \geq \mu\} = \vec{D}^m(m,\mu,v;g)/\gamma$ where $\alpha = \beta\gamma$ and γ is a positive scalar. Since $\vec{D}^m(m,\mu,v;\gamma g) = \vec{D}^m(m,\mu,v;g)/\gamma$, differentiating both sides by γ, and evaluating at $\gamma = 1$ gives $\nabla_g \vec{D}^m(m,\mu,v;g) \cdot g = -\vec{D}^m(m,\mu,v;g)$. Finally, since $U(m - \vec{D}^m(m,\mu,v;g)g, v) = \mu$, differentiating with respect to g_j gives $\partial \vec{D}^m(m,\mu,v;g)/\partial g_j = -\vec{D}^m(m,\mu,v;g)\left(\partial \vec{D}^m(m,\mu,v;g)/\partial m_j\right)$.

Translation property: $\vec{D}^m(m - \alpha g, \mu, v; g) = \max\{\beta | U(m - \alpha g - \beta g) \geq \mu\} = \max\{\gamma - \alpha | U(m - \gamma g) \geq \mu\} = \vec{D}^m(m,\mu,v;g) - \alpha$.

Concavity: Let (m^1, v^1) and (m^2, v^2) be two import and input vectors and $\lambda \in [0,1]$. Also let $(m^\lambda, v^\lambda) = (\lambda m^1 + (1-\lambda)m^2, \lambda v^1 + (1-\lambda)v^2)$ By definition, and since $U(m,v)$ is quasi-concave in (m,v)

$$\begin{aligned} \mu &= \lambda U(m^1 - \vec{D}^m(m^1,\mu,v^1;g)g, v^1) + (1-\lambda)U(m^2 - \vec{D}^m(m^2,\mu,v^2;g)g, v^2) \\ &\leq U(m^\lambda - \lambda\vec{D}^m(m^1,\mu,v^1;g)g - (1-\lambda)\vec{D}^m(m^2,\mu,v^2;g)g), v^\lambda). \end{aligned}$$

Now, $\mu = U(m^\lambda - \vec{D}^m(m^\lambda,\mu,v^\lambda;g)g), v^\lambda)$ by definition and thus $\lambda\vec{D}^m(m^1,\mu,v^1;g)g + (1-\lambda)\vec{D}^m(m^2,\mu,v^2;g) \leq \vec{D}^m(m^\lambda,\mu,v^\lambda;g)$.

[22] With the exceptions of concavity and the second part of g-homogeneity below, the corresponding set of results for the directional output distance function can be found, for example, in Chambers, Chung and Färe (1996).

Duality:[23] Note that since $\vec{D}^m(m,\mu,v;g) \leq (\pi\cdot m - S(\pi,\mu,v))/(\pi\cdot g)$ from (4), $\vec{D}^m(m,\mu,v;g) \leq \inf_\pi(\pi\cdot m - S(\pi,\mu,v))/(\pi\cdot g)$. Suppose that the inequality is strict, such that $\vec{D}^m(m,\mu,v;g) < \hat{D} = \inf_\pi(\pi\cdot m - S(\pi,\mu,v))/(\pi\cdot g)$ From the translation property, $\vec{D}^m(m-\hat{D}g,\mu,v;g) < 0$ and thus, $U(m-\hat{D}g,v) < \mu$. By quasi-concavity of the trade utility function, it follows from standard arguments that there exist a price vector $\hat{\pi} > 0$ such that $\hat{\pi}\cdot(m-\hat{D}g) < S(\hat{\pi},\mu,v)$, or, $\hat{D} > (\hat{\pi}\cdot m - S(\hat{\pi},\mu,v))/(\hat{\pi}\cdot g) \geq \inf_\pi(\pi\cdot m - S(\pi,\mu,v))/(\pi\cdot g)$, a contradiction.

Proof of Proposition 2: Taking derivatives with respect to x and y, $\vec{D}^m(x-y,\mu,v;g) = \vec{D}^x(x,\mu;g) + \vec{D}^y(y,v;g)$ implies straightforwardly $\nabla_m\vec{D}^m(x-y,\mu,v;g) = \nabla_x\vec{D}^x(x,\mu;g) = -\nabla_y\vec{D}^y(y,v;g)$ evaluated respectively at $x-y-\vec{D}^m(x-y,\mu,v;g)g$, $x-\vec{D}^x(x,\mu;g)g$ and $y+\vec{D}^y(y,v;g)g$. Suppose instead $\nabla_m\vec{D}^m(x-y,\mu,v;g) = \nabla_x\vec{D}^x(x,\mu;g) = -\nabla_y\vec{D}^y(y,v;g)$. It follows from (4) - (6) that

$$
\begin{aligned}
S(\nabla_m\vec{D}^m(x-y,\mu,v;g),\mu,v) &= E(\nabla_m\vec{D}^m(x-y,\mu,v;g),\mu) - G(\nabla_m\vec{D}^m(x-y,\mu,v;g),v) \\
&= \nabla_m\vec{D}^m(x-y,\mu,v;g)\cdot(x-y-\vec{D}^m(x-y,\mu,v;g)g)
\end{aligned}
$$

since $S(\pi,\mu,v)$ is homogeneous of degree one in π. Similarly, from (11) - (12),

$$
\begin{aligned}
E(\nabla_x\vec{D}^x(x,\mu;g),\mu) &= \nabla_x\vec{D}^x(x,\mu;g)\cdot(x-\vec{D}^x(x,\mu;g)g) \\
G(-\nabla_y\vec{D}^y(y,v;g),v) &= -\nabla_y\vec{D}^y(y,v;g)\cdot(y+\vec{D}^y(y,v;g)g)
\end{aligned}
$$

Thus, if $\nabla_m\vec{D}^m(x-y,\mu,v;g) = \nabla_x\vec{D}^x(x,\mu;g) = -\nabla_y\vec{D}^y(y,v;g)$,

$$
\begin{aligned}
& S(\nabla_m\vec{D}^m(x-y,\mu,v;g),\mu,v) \\
= \; & E(\nabla_m\vec{D}^m(x-y,\mu,v;g),\mu) - G(\nabla_m\vec{D}^m(x-y,\mu,v;g),\mu,v) \\
= \; & E(\nabla_x\vec{D}^x(x,\mu;g),\mu) - G(-\nabla_y\vec{D}^y(y,v;g),v).
\end{aligned}
$$

Or

$$
\begin{aligned}
& \nabla_m\vec{D}^m(x-y,\mu,v;g)\cdot(x-y-\vec{D}^m(x-y,\mu,v;g)g) \\
= \; & \nabla_x\vec{D}^x(x,\mu;g)\cdot(x-\vec{D}^x(x,\mu;g)g) + \nabla_y\vec{D}^y(y,v;g)\cdot(y+\vec{D}^y(y,\mu;g)g)
\end{aligned}
$$

[23] An analogous proof of duality for the cases of directional output and input distance functions can be found in in Färe and Primont (2006).

and thus $\vec{D}^m(x-y,\mu,v;g) = \vec{D}^x(x,\mu;g) + \vec{D}^y(y,v;g)$ as indicated in Proposition 2.

Proof of Proposition 3: Given a price vector π, the discussion following Proposition 2 gives $\nabla_m \vec{D}^m(m(\pi,\mu,v),\mu,v;g) = \pi/(\pi\cdot g)$. In addition, $\nabla_x \vec{D}^x(x(\pi,\mu),\mu;g) = \pi/(\pi\cdot g)$, and $\nabla_y \vec{D}^y(y(\pi,v),v;g) = -\pi/(\pi\cdot g)$. Differentiating both sides of these three equations with respect to π gives

$$\begin{aligned} & \nabla_{mm}\vec{D}^m(m(\pi,\mu,v),\mu,v;g)\nabla_{\pi\pi}(E(\pi,\mu)-G(\pi,v)) \\ = \; & \nabla_{xx}\vec{D}^x(x(\pi,\mu),\mu;g)\nabla_{\pi\pi}E(\pi,\mu) \\ = \; & -\nabla_{yy}\vec{D}^y(y(\pi,v),v;g)\nabla_{\pi\pi}G(\pi,v) = A(\pi,g) \end{aligned} \tag{28}$$

It follows that

$$\begin{aligned} \nabla_{\pi\pi}E(\pi,\mu) &= \left(\nabla_{xx}\vec{D}^x(x(\pi,\mu),\mu;g)\right)^{-1} A(\pi,g), \\ \nabla_{\pi\pi}G(\pi,v) &= -\left(\nabla_{yy}\vec{D}^y(y(\pi,v),v;g)\right)^{-1} A(\pi,g). \end{aligned} \tag{29}$$

Proposition 4 follows upon substituting (29) into (28).

Proof of Proposition 6: First, suppose that $\bar{p}/(\bar{p}\cdot g) = \nabla_m \vec{D}^m(m_j^0,\mu_h^0,v_h;g) = \tilde{\pi}_h^0$. Proposition 5 shows that $SQ_{L_h}^2 = \tilde{\pi}_h^0 \cdot (m_h^0 - m_h^2) = 0$ is sufficient for trade welfare to never improve beyond $\mu_h^0 = U_h(m_h^0, v_h)$ since $\vec{D}^m(m_h^2,\mu_h^0,v;g) \le SQ_{L_h}^2 = 0$.

Suppose instead that there exists an alternative price vector $\hat{p} > 0$, with $\hat{p}/(\hat{p}\cdot g) \neq \tilde{\pi}_h^0$, but $\vec{D}^m(m_h^2,\mu_h^0,v;g) \le 0$ for any m_h^2 such that $\frac{\hat{p}}{\hat{p}\cdot g}\cdot(m_h^0 - m_h^2) = 0$. Since $\hat{p}/(\hat{p}\cdot g) \neq \tilde{\pi}_h^0$, it must be the case that $S_h(\hat{p}/(\hat{p}\cdot g),\mu_h^0,v) < \hat{p}\cdot m_h^0/(\hat{p}\cdot g)$. Implicitly define $\hat{\mu}$ by

$$S_h(\hat{p}/(\hat{p}\cdot g),\hat{\mu},v_h) = \frac{\hat{p}}{\hat{p}\cdot g}\cdot m_h^0$$

and choose the compensated import vector $m_h(\hat{p}/(\hat{p}\cdot g),\hat{\mu},v_h)$. Collecting terms,

$$\frac{\hat{p}}{\hat{p}\cdot g}\cdot m_h(\frac{\hat{p}}{\hat{p}\cdot g},\hat{\mu},v_h) = S_h(\frac{\hat{p}}{\hat{p}\cdot g},\hat{\mu},v_h) = \frac{\hat{p}}{\hat{p}\cdot g}\cdot m_h^0 > S_h(\frac{\hat{p}}{\hat{p}\cdot g},\mu_h^0,v_h)$$

or, $\hat{u} > \mu_h^0$ since the trade expenditure function is monotonically increasing in trade utility. We have thus

$$\frac{\hat{p}}{\hat{p}\cdot g}\cdot m_h(\frac{\hat{p}}{\hat{p}\cdot g},\hat{\mu},v_h) = \frac{\hat{p}}{\hat{p}\cdot g}\cdot m_h^0, \;\text{ but }\; \vec{D}^m(m_h(\frac{\hat{p}}{\hat{p}\cdot g},\mu_h^0,v_h;g) > 0,$$

a contradiction.

Appendix B

The expenditure and revenue functions employed in the simulation are:

$$E(q,u) = u\sum_{i=1}^{20}\left(s_{c_i}q_i^{1-\rho_c}\right)^{1/(1-\rho_c)}, \quad G(p,v) = \phi(v)\sum_{i=1}^{20}\left(s_{p_i}p_i^{1+\delta_p}\right)^{1/(1+\delta_p)}$$

where s_{c_i} and s_{p_i} are consumption and production share parameters with $\sum_{i=1}^{20} s_{c_i} = \sum_{i=1}^{20} s_{p_i} = 1$. The elasticities of substitution parameters are assumed to take the values of $\rho_c = 0.5$ and $\delta_p = 2$. Finally, we assume also that $\phi(v)$, along with free trade prices p^* are equal to unity for every i throughout. The equilibrium distorted level of net imports $\hat{m}$ and utility $\hat{\mu}$ is obtained as joint solution to $\hat{m} = \nabla_q E(q,\hat{\mu}) - \nabla_p G(p,v)$ and the balance of trade constraint $p^* \cdot (\nabla_q E(q,\hat{\mu}) - \nabla_p G(p,v)) = 0$.

Given a policy-induced vector of net imports $\hat{m}$ above, the corresponding vector of normalized shadow prices of net imports, $\bar{\pi}$, solves the optimization problem

$$\vec{D}^m(\hat{m},\bar{\mu},v) = \inf_{\pi}\left\{\frac{\pi\cdot\hat{m} - S(\pi,\bar{\mu},v)}{\pi\cdot g}\right\}$$

subject to $\hat{m} = \nabla_q E(q,\hat{\mu}) - \nabla_p G(p,v) = \nabla_\pi S(\bar{\pi},\bar{\mu},v)$. The equilibrium trade utility $U(\hat{m},v) = \bar{\mu}$ can once again be obtained from the balance of trade constraint, since $0 = p^*\cdot\hat{m} = p^*\cdot\nabla_\pi S(\bar{\pi},\bar{\mu},v)$. The results of the simulations reported in Tables 3 A and B are obtained using the Excel Solver, and are available upon request.

Reference

Anderson, K. 2002. "Peculiarities of Retaliation in WTO Dispute Settlement," *World Trade Review* 1: 123 - 134.

Allais, M. 1943. *Traitè D'Économie Pure,* Vol, 3 (Paris: Imprimerie Nationale).

Bagwell, K. and R. Staiger. 1999. "An Economic Theory of the GATT", *American Economic Review* 89: 215-248.

Bagwell, K. and R. W. Staiger. 2002. *The Economics of the World Trading System*, Cambridge MA: MIT Press.

Balassa, B. 1985. ”Exports, Policy Choices and Economic Growth in Developing Countries After the 1973 Oil Shock,” *Journal of Development Economics* 18(1): 23-35.

Baldwin, R. 1989. ”Measuring Nontariff Trade Policies,” NBER Working Paper no. 2978. (May): 34-39, Cambridge, Massachusetts.

Bown, C. P. 2000. “Participation in WTO Dispute Settlement: Complainants, Interested Parties and Free Riders,” *The World Bank Economic Review* 19(2): 287-310.

——. 2002. “The Economics of Trade Disputes, the GATT’s Article XXIII, and the WTO’s Dispute Settlement Understanding.” *Economics and Politics* 14(3):283-323.

Butler, M. and H. Hauser. 2000. ‘’The WTO Dispute Settlement Mechanism: A First Assessment from an Economic Perspective,” *Journal of Law, Economics and Organization* 16(2): 503-533.

Chau, N. H., R. Färe and S. Grosskopf. 2003. “Trade Restrictiveness and Efficiency”, *International Economic Review* 44 (3): 1079-1095.

Chambers, R. G., Y. Chung, and R. Färe. 1996. “Benefit and Distance Functions”. *Journal of Economic Theory* 70 (2): 407-19.

Chambers, R. G. and R. Färe (1998b), “Translation Homotheticity,” *Economic Theory* 11(3): 629-41.

Chipman, J. 1979. “The Theory and Applications of Trade Utility Functions”, in J. R. Green, and J.A. Scheinkman (eds.) *General Equilibrium, Growth and Trade: Essays in Honor of Lionel McKenzie*. New York: Academic Press. pp.277-296.

Dam, K. 1970. *The GATT: Law and International Organization*. Chicago IL: University of Chicago.

Deardorff, A. 1980. “The General Validity of the Law of Comparative Advantage,” *Journal of Political Economy* 88(5): 941-957.

Diewert, E. W., and A. D. Woodland. 2004. “The Gains from Trade and Policy Reform Revisited,” *Review of International Economics* 12(4): 591-608.

Dixit, A. 1997. "Economists as Advisors to Politicians and to Society" *Economics and Politics* 9: 225- 230.

Dixit, A. K. and V. Norman. 1980. *Theory of International Trade: A Dual General Equilibrium Approach*. Welwyn: Cambridge University Press.

Färe, R. E. and S. Grosskopf. 2000. "Theory and Application of Directional Distance Functions," *Journal of Productivity Analysis* 13: 93-103.

Färe, R. E. and S. Grosskopf. 1996. *Intertemporal Production Frontiers: With Dynamic DEA*, Boston: Kluwer Academic Publishers.

Färe, R. E. and D. Primont. 1995. *Multi-Output Production and Duality: Theory and Applications*, Boston: Kluwer Academic Publishers.

—— 2006. "Directional Duality Theory", forthcoming in *Economic Theory* 29: 239-247.

Foster, E. and H. Sonnenschein. 1970. "Resource Allocation and the Public Sector," *Econometrica* 38: 281-297.

Horn, H., P. Mavroidis, H. Nordströ. 1999. "Is the Use of the WTO Dispute Settlement System Biased?" CEPR Discussion Paper No. 2340.

Hummels, D., and P. J. Klenow. 2005."The Variety and Quality of a Nation's Exports," *American Economic Review* 95: 704-723.

Johnson, H. G. 1953-1954. "Optimum Tariffs and Retaliation," *Review of Economic Studies* 21: 142-153.

Kawamata, K. 1974. "Price Distortion and Potential Welfare," *Econometrica* 42: 435-460.

Keen, M. and J. Ligthart. 2002. "Coordinating Tariff Reduction and Domestic Tax Reform," *Journal of International Economics* 56: 289-507.

Kemp, M. C. 1964. *The Pure Theory of International Trade*. New Jersey: Prentice-Hall.

Kennan, J. and R. Riezman. 1988. "Do Big Countries Win Tariff Wars?" *International Economic Review* 29(1): 81-85.

Krueger, A. O. and H. Sonnenschein. 1967. “The Terms of Trade, the Gains from Trade and Price Divergence,” *International Economic Review* 8: 121 - 127.

Krugman, P. 1997. “What Should Trade Negotiators Negotiate About?” *Journal of Economic Literature* XXXV (March): 113-120.

Leamer, E. 1988. ”Measures of Openness” in R. Baldwin (ed.), *Trade Policy Issues and Empirical Analysis*, Chicago: University of Chicago Press. pp. 147-200.

Luenberger, D. G. 1992. “Benefit Functions and Duality,” *Journal of Mathematical Economics* 21: 461-481.

——. 1995. *Microeconomic Theory*, McGraw-Hill, New York, New York, 1995.

Mayer, W. 1981. “Theoretical Considerations on Negotiated Tariff Adjustments,” *Oxford Economic Papers* 33(1): 135-153.

Meade, J. E. 1951. *A Geometry of International Trade*, London: Gerogen Allen.

Michael, S., P. Hatzipanayotou, and S. M. Miller. 1991. “Integrated Reforms of Tariffs and Consumption Taxes,” *Journal of Public Economics* 52: 417-428.

Ohyama, M. 1972. “Trade and Welfare in General Equilibrium,” *Keio Economic Studies* 9: 37-33.

Shephard, R. W. 1970. *Theory of Cost and Production Functions*, Princeton: Princeton University Press.

Woodland, A. D. 1982. *International Trade and Resource Allocation*. New York: North-Holland Publishing Company.

——. 1980. “Direct and Indirect Trade Utility Functions”, *Review of Economic Studies* 47(5): 907-926

World Trade Organization. 2005. *World Trade Report 2005.* Geneva. WTO Publications.

World Trade Organization. 2007. *WTO Dispute Settlement.* Geneva. WTO Publications.

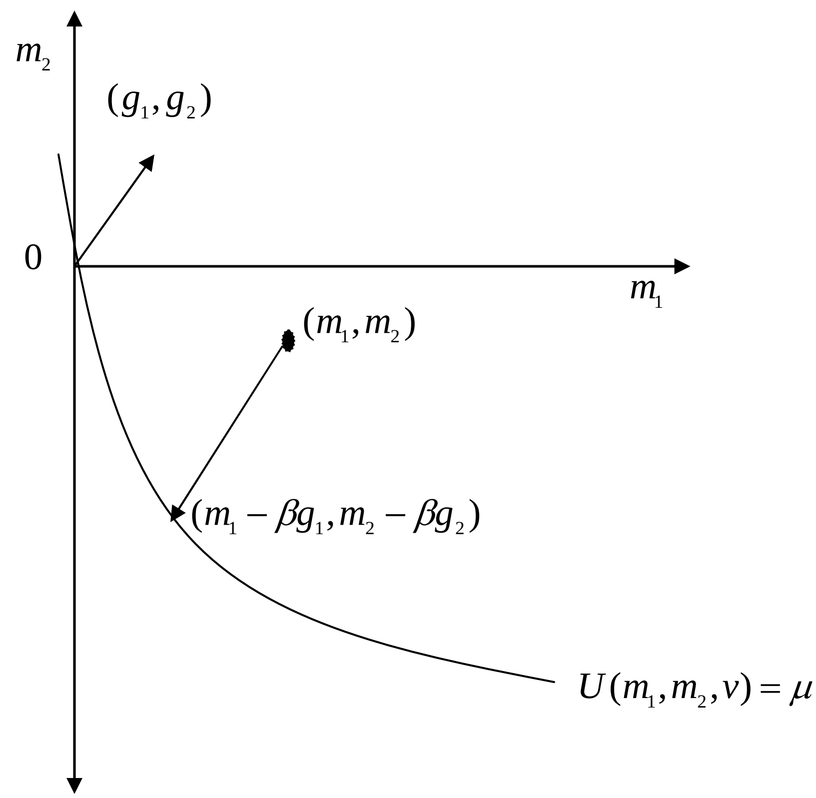


Figure 1. Trade Benefit Function

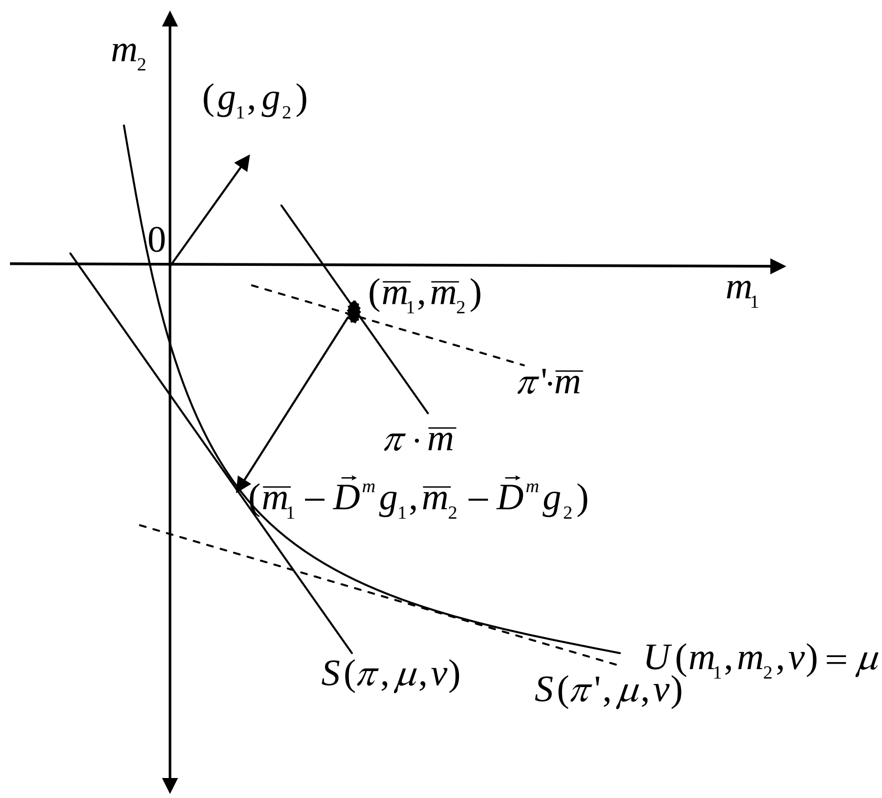
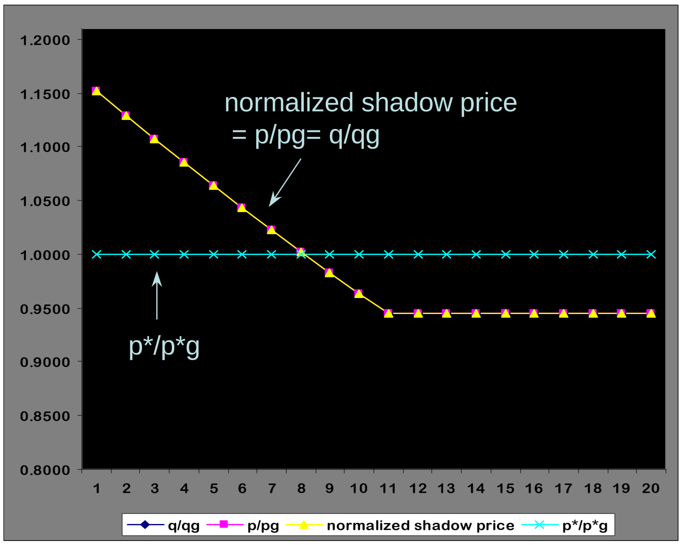


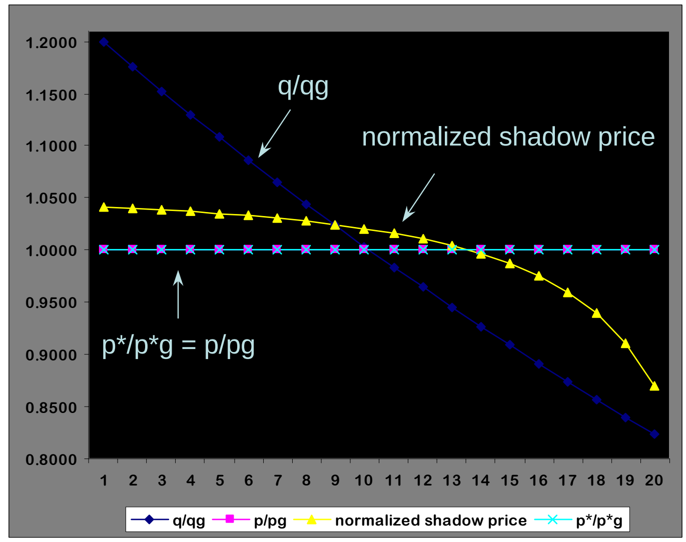
Figure 2. Duality

Figure 3A: Simulated Shadow Prices of Imports with Decreasing Production Shares

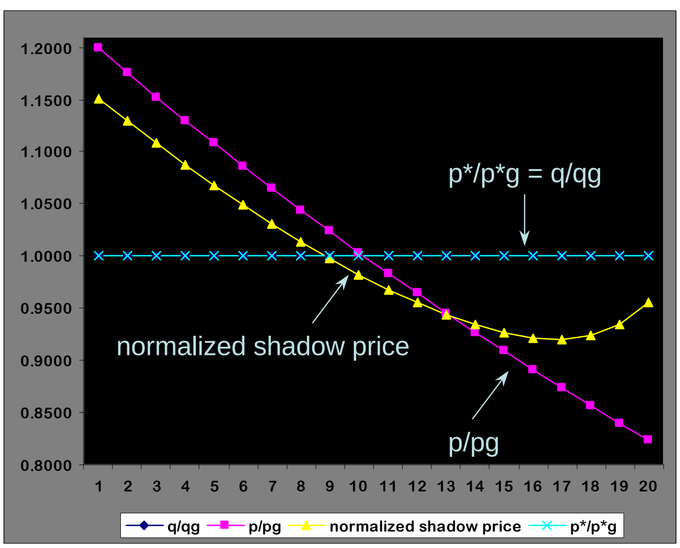
AI. Trade Tax



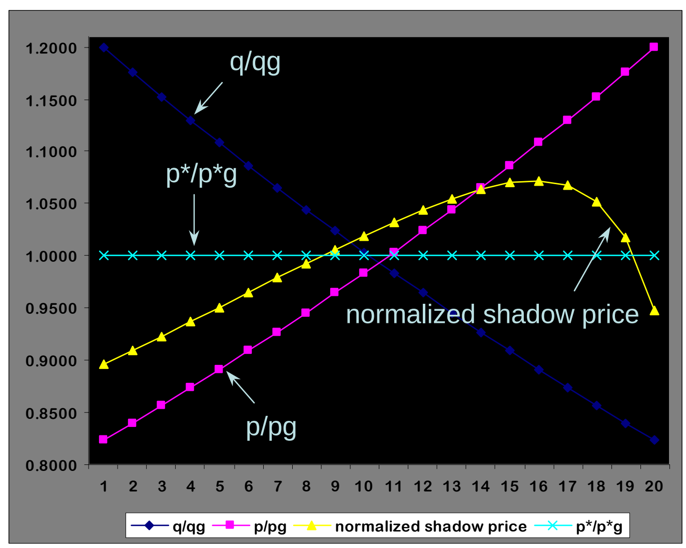
AII. Consumption Tax



AIII. Production Subsidy



AIV. Consumption Tax and Production Subsidy



High production shares ← Low production shares

High production shares ← Low production shares

Figure 3B: Simulated Shadow Prices of Imports with Increasing Production Shares

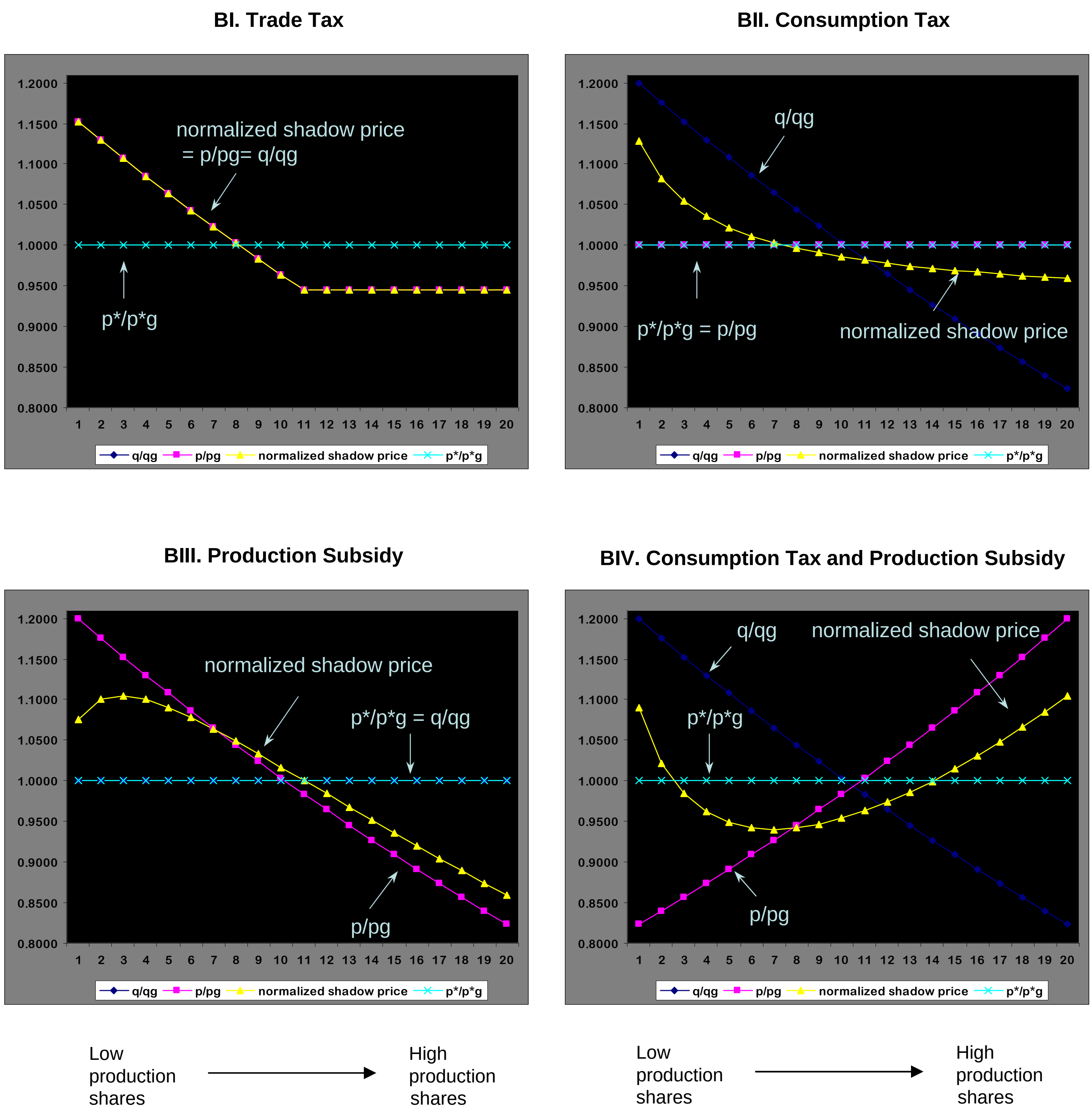


Table 1. Simulation Input: Trade Policy Regimes

Sector	I. Pure Trade Taxes		II. Consumption Tax		III. Production Subsidy		IV. Consumption Tax and Production Subsidy	
	Production Subsidy (%)	Consumption Tax (%)	Production Subsidy (%)	Consumption Tax (%)	Production Subsidy (%)	Consumption Tax (%)	Production Subsidy (%)	Consumption Tax (%)
1	22%	22%	0%	46%	46%	0%	0%	46%
2	20%	20%	0%	43%	43%	0%	2%	43%
3	17%	17%	0%	40%	40%	0%	4%	40%
4	15%	15%	0%	37%	37%	0%	6%	37%
5	13%	13%	0%	35%	35%	0%	8%	35%
6	10%	10%	0%	32%	32%	0%	10%	32%
7	8%	8%	0%	29%	29%	0%	13%	29%
8	6%	6%	0%	27%	27%	0%	15%	27%
9	4%	4%	0%	24%	24%	0%	17%	24%
10	2%	2%	0%	22%	22%	0%	20%	22%
11	0%	0%	0%	20%	20%	0%	22%	20%
12	0%	0%	0%	17%	17%	0%	24%	17%
13	0%	0%	0%	15%	15%	0%	27%	15%
14	0%	0%	0%	13%	13%	0%	29%	13%
15	0%	0%	0%	10%	10%	0%	32%	10%
16	0%	0%	0%	8%	8%	0%	35%	8%
17	0%	0%	0%	6%	6%	0%	37%	6%
18	0%	0%	0%	4%	4%	0%	40%	4%
19	0%	0%	0%	2%	2%	0%	43%	2%
20	0%	0%	0%	0%	0%	0%	46%	0%

Table 2. Simulation Input: Technologies and Preferences

Sector	A. Decreasing Production Shares		B. Increasing Production Shares	
	s_p	s_c	s_p	s_c
1	0.095	0.05	0.005	0.05
2	0.090	0.05	0.010	0.05
3	0.086	0.05	0.014	0.05
4	0.081	0.05	0.019	0.05
5	0.076	0.05	0.024	0.05
6	0.071	0.05	0.029	0.05
7	0.067	0.05	0.033	0.05
8	0.062	0.05	0.038	0.05
9	0.057	0.05	0.043	0.05
10	0.052	0.05	0.048	0.05
11	0.048	0.05	0.052	0.05
12	0.043	0.05	0.057	0.05
13	0.038	0.05	0.062	0.05
14	0.033	0.05	0.067	0.05
15	0.029	0.05	0.071	0.05
16	0.024	0.05	0.076	0.05
17	0.019	0.05	0.081	0.05
18	0.014	0.05	0.086	0.05
19	0.010	0.05	0.090	0.05
20	0.005	0.05	0.095	0.05

Table 3A. Simulation Results with Decreasing Production Shares

Sector	AI. Pure Trade Taxes			AII. Consumption Tax			AIII. Production Subsidy			AIV. Consumption Tax and Production Subsidy		
	Normalized Producer Price (p/pq)	Normalized Consumer Price (q/qg)	Normalized Shadow Price (π/πg)	Normalized Producer Price (p/pq)	Normalized Consumer Price (q/qg)	Normalized Shadow Price (π/πg)	Normalized Producer Price (p/pq)	Normalized Consumer Price (q/qg)	Normalized Shadow Price (π/πg)	Normalized Producer Price (p/pq)	Normalized Consumer Price (q/qg)	Normalized Shadow Price (π/πg)
1	1.15	1.15	1.15	1.00	1.20	1.04	1.20	1.00	1.15	0.82	1.20	0.90
2	1.13	1.13	1.13	1.00	1.18	1.04	1.18	1.00	1.13	0.84	1.18	0.91
3	1.11	1.11	1.11	1.00	1.15	1.04	1.15	1.00	1.11	0.86	1.15	0.92
4	1.09	1.09	1.09	1.00	1.13	1.04	1.13	1.00	1.09	0.87	1.13	0.94
5	1.06	1.06	1.06	1.00	1.11	1.04	1.11	1.00	1.07	0.89	1.11	0.95
6	1.04	1.04	1.04	1.00	1.09	1.03	1.09	1.00	1.05	0.91	1.09	0.96
7	1.02	1.02	1.02	1.00	1.06	1.03	1.06	1.00	1.03	0.93	1.06	0.98
8	1.00	1.00	1.00	1.00	1.04	1.03	1.04	1.00	1.01	0.95	1.04	0.99
9	0.98	0.98	0.98	1.00	1.02	1.02	1.02	1.00	1.00	0.96	1.02	1.01
10	0.96	0.96	0.96	1.00	1.00	1.02	1.00	1.00	0.98	0.98	1.00	1.02
11	0.94	0.94	0.94	1.00	0.98	1.02	0.98	1.00	0.97	1.00	0.98	1.03
12	0.94	0.94	0.94	1.00	0.96	1.01	0.96	1.00	0.96	1.02	0.96	1.04
13	0.94	0.94	0.94	1.00	0.95	1.00	0.95	1.00	0.94	1.04	0.95	1.05
14	0.94	0.94	0.94	1.00	0.93	1.00	0.93	1.00	0.93	1.06	0.93	1.06
15	0.94	0.94	0.94	1.00	0.91	0.99	0.91	1.00	0.93	1.09	0.91	1.07
16	0.94	0.94	0.94	1.00	0.89	0.98	0.89	1.00	0.92	1.11	0.89	1.07
17	0.94	0.94	0.94	1.00	0.87	0.96	0.87	1.00	0.92	1.13	0.87	1.07
18	0.94	0.94	0.94	1.00	0.86	0.94	0.86	1.00	0.92	1.15	0.86	1.05
19	0.94	0.94	0.94	1.00	0.84	0.91	0.84	1.00	0.93	1.18	0.84	1.02
20	0.94	0.94	0.94	1.00	0.82	0.87	0.82	1.00	0.96	1.20	0.82	0.95

Table 3B. Simulation Results with Increasing Production Shares

Sector	BI. Pure Trade Taxes			BII. Consumption Tax			BIII. Production Subsidy			BIV. Consumption Tax and Production Subsidy		
	Normalized Producer Price (p/pq)	Normalized Consumer Price (q/qg)	Normalized Shadow Price (π/πg)	Normalized Producer Price (p/pq)	Normalized Consumer Price (q/qg)	Normalized Shadow Price (π/πg)	Normalized Producer Price (p/pq)	Normalized Consumer Price (q/qg)	Normalized Shadow Price (π/πg)	Normalized Producer Price (p/pq)	Normalized Consumer Price (q/qg)	Normalized Shadow Price (π/πg)
1	1.15	1.15	1.15	1.00	1.20	1.13	1.20	1.00	1.08	0.82	1.20	1.09
2	1.13	1.13	1.13	1.00	1.18	1.08	1.18	1.00	1.10	0.84	1.18	1.02
3	1.11	1.11	1.11	1.00	1.15	1.05	1.15	1.00	1.10	0.86	1.15	0.98
4	1.09	1.09	1.09	1.00	1.13	1.04	1.13	1.00	1.10	0.87	1.13	0.96
5	1.06	1.06	1.06	1.00	1.11	1.02	1.11	1.00	1.09	0.89	1.11	0.95
6	1.04	1.04	1.04	1.00	1.09	1.01	1.09	1.00	1.08	0.91	1.09	0.94
7	1.02	1.02	1.02	1.00	1.06	1.00	1.06	1.00	1.06	0.93	1.06	0.94
8	1.00	1.00	1.00	1.00	1.04	1.00	1.04	1.00	1.05	0.95	1.04	0.94
9	0.98	0.98	0.98	1.00	1.02	0.99	1.02	1.00	1.03	0.96	1.02	0.95
10	0.96	0.96	0.96	1.00	1.00	0.99	1.00	1.00	1.02	0.98	1.00	0.95
11	0.94	0.94	0.94	1.00	0.98	0.98	0.98	1.00	1.00	1.00	0.98	0.96
12	0.94	0.94	0.94	1.00	0.96	0.98	0.96	1.00	0.98	1.02	0.96	0.97
13	0.94	0.94	0.94	1.00	0.95	0.97	0.95	1.00	0.97	1.04	0.95	0.99
14	0.94	0.94	0.94	1.00	0.93	0.97	0.93	1.00	0.95	1.06	0.93	1.00
15	0.94	0.94	0.94	1.00	0.91	0.97	0.91	1.00	0.94	1.09	0.91	1.01
16	0.94	0.94	0.94	1.00	0.89	0.97	0.89	1.00	0.92	1.11	0.89	1.03
17	0.94	0.94	0.94	1.00	0.87	0.96	0.87	1.00	0.90	1.13	0.87	1.05
18	0.94	0.94	0.94	1.00	0.86	0.96	0.86	1.00	0.89	1.15	0.86	1.07
19	0.94	0.94	0.94	1.00	0.84	0.96	0.84	1.00	0.87	1.18	0.84	1.08
20	0.94	0.94	0.94	1.00	0.82	0.96	0.82	1.00	0.86	1.20	0.82	1.10