# Trade negotiations when market access matters* 

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#### Abstract

This paper analyses trade negotiations in an oligopolistic setting that provides a rationale for trade negotiation rules in terms of market access concessions. Consistent with real-world negotiations, countries give up protection of their domestic market in exchange for market access abroad. The multi-country model sheds light on the impact of asymmetry between countries on trade negotiations. Asymmetric participation in trade liberalisation is also addressed and it is shown that the multilateral negotiations system can sustain a certain level of free-riding which suggests why multilateralism was successful in the past, but is having problems nowadays.


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## 1 Introduction

During the past 50 years, multilateral trade negotiations have achieved an extraordinary trade liberalisation. Since the end of World War II, average ad valorem tariffs on industrial goods have been reduced significantly from over 40 percent to less than 4 percent. This successful liberalisation led to an exceptional growth in world trade. Merchandise exports grew on average by $6 \%$ annually and total trade in 2000 was 22 -times the level of 1950 world trade.

It is generally acknowledged that the General Agreement on Tariffs and Trade (GATT) and later the World Trade Organisation (WTO) played a key role in achieving the historically low tariff levels through a series of eight trade negotiation rounds (the ninth, Doha Round, is currently in progress). ${ }^{1}$ Recently however multilateral trade negotiations have progressed rather slowly and with difficulties. The next to last Uruguay Round took seven and a half years, almost twice the original schedule. The Doha Development Round started in November 2001 and was set to be concluded in four years, but as of 2008, talks have stalled over a divide between the developed nations led by the European Union, the United States and Japan and the major developing countries led and represented mainly by India, Brazil, China and South Africa.

The GATT/WTO is a forum for governments to negotiate trade agreements according to a pre-agreed set of rules. These rules are lengthy and complex legal texts covering a wide range of issues, but two fundamental principles are considered as the foundation of the multilateral trading system: the principles of reciprocity and non-discrimination.

The principle of reciprocity requires countries to make reciprocal changes in their trade policy, in particular when negotiating trade liberalisation to exchange reciprocal concessions. Bagwell and Staiger (1999) interpret the principle of reciprocity as a requirement for mutual changes in trade policy that ensure that changes in each country's imports are equal to changes in its exports.

The principle of non-discrimination, also called the Most-Favoured-Nation principle, forbids discrimination between GATT/WTO members. If a country grants to one of its trading

[^1]partners a special favour, such as a lower customs duty rate for one of its products, it has to grant the same favour to all WTO members.

The role of these fundamental principles has attracted much attention among trade lawyers, trade officials, political scientists and naturally economists, but until relatively recently there has not been any formal economic model analysing and explaining what these principles are for. Standard undergraduate textbook trade theory shows that there is a unilateral case for free trade. Therefore, from an economic point of view, there seems to be no room for the existence of any trade agreement and any economic analysis of the GATT/WTO is vain. Traditionally, the GATT/WTO principles have been interpreted as a result of mercantilist reasoning of the negotiating countries. This point has been famously stated by Krugman (1992): "There is no generally accepted label for the theoretical underpinnings of the GATT. I like to refer to it as 'GATT-think': a simple set of principles that is entirely consistent, explains most of what goes on in negotiations, but makes no sense in terms of economics. (...) In other words, GATT-think is enlightened mercantilism."

More recent developments in theoretical economic literature have challenged this traditional view. Bagwell and Staiger (1999) and Bagwell and Staiger (2002) have provided a seminal contribution to the analysis of trade agreements. By building on the work started by Johnson (1953-54), they show that when countries set their tariffs non-cooperatively, these tariffs are chosen inefficiently high because of a terms-of-trade externality. Countries want to manipulate their terms of trade in their favour via their tariffs and, because all countries do so, they end up in a Prisoner's Dilemma situation. Bagwell and Staiger (1999) thus identify a reason for trade agreements and explain the role of the fundamental GATT/WTO principles: trade agreements allow countries to overcome the terms-of-trade externality by agreeing to lower their tariffs according to the GATT/WTO principles.

The GATT/WTO theory of Bagwell and Staiger (1999) represents an important step forward in our understanding of trade agreements. Nevertheless, this theory has been criticised for its focus on the terms of trade. Bagwell and Staiger (2002) themselves point out that "many economists are skeptical as to the practical relevance of terms-of-trade considerations for actual trade policy negotiations". Krugman (1997) for example states that "this optimal tariff argument plays almost no role in real-world disputes over trade policy". A recent empirical analysis by Broda et al. (2006) concludes that countries do manipulate their terms of trade, but Regan (2006) argues that this evidence is unpersuasive. Also, critics
of the terms-of-trade theory point out that terms of trade are never mentioned in trade negotiations.

Furthermore, Wilfred Ethier, who is one of the strongest critics of the terms-of-trade theory (see for example Ethier (2004)), underlines that "actual multilateral agreements do not prevent countries from trying to influence their terms of trade". As it is well known, countries can affect their terms of trade by taxing either imports or exports. Trade negotiations have focused solely on bounding import taxes. Thus, if a country wished to manipulate its terms of trade, it could do so by imposing a set of export taxes. So a theory explaining trade agreements through countries' concern with terms-of-trade manipulation does not seem to provide the right insight into trade negotiations. Ossa (2007) provides an alternative motivation and analysis of trade agreements. In his 'new trade' model of trade negotiations, countries, when acting non-cooperatively, charge inefficiently high tariffs because they want to attract firms to locate in the Home country. Trade agreements enable countries to overcome the Prisoner's Dilemma driven by an industry location externality. Although this interpretation of trade agreements is also an interesting contribution to the literature, if we follow the rhetoric of trade (in trade agreements themselves, public speeches made by trade officials and political discussions) we will not find many references to industry location. ${ }^{2}$

Market access seems to play an important role in trade negotiations and more particularly firms' profits. Moreover, even though the current theories of GATT/WTO negotiations are useful for the understanding of the fundamental principles of reciprocity and nondiscrimination, many different aspects of trade negotiations have not yet been addressed in the literature. For example, countries negotiate tariff cuts according to formulas. How does formula-based trade liberalisation differ from trade liberalisation following the principles of reciprocity and non-discrimination? Also current theories do not seem to explain past success of GATT/WTO negotiations and its current stalemate.

This paper attempts to address these points. It builds on the work of Bagwell and Staiger (1999, 2002), but uses an oligopolistic model à la Brander (1981) to analyse trade negotiations. In this setting, firms make non-zero profits and supply socially sub-optimal quantities. Governments, when acting non-cooperatively, set inefficiently high tariffs for two reasons: to improve their terms of trade, but also to increase domestic output. In

[^2]the absence of domestic competition policy, trade policy is used to restore domestic output to socially optimal level. So trade agreements are partially used to neutralise a terms-of-trade externality, but also to neutralise a market access externality through a balanced exchange of market access concessions. This paper thus identifies a new rationale for trade agreements. In the absence of the terms-of-trade externality, trade agreements remedy the market inefficiency.

This paper also addresses Ethier's critique. In this oligopolistic framework, countries use tariffs on imports to improve their terms of trade and to increase domestic production. They could also use strategic trade policy on exports, i.e. if they could, they would subsidise exports to increase production. But export subsidies are explicitly forbidden by Article XVI of the GATT. So this model seems to provide a plausible rationale for trade agreements.

Furthermore, within this framework, I study aspects of trade negotiations that have not yet been addressed in the literature. I examine the role of asymmetries in the negotiation process in terms of asymmetric countries and asymmetric participation. Countries differ in number of firms. Some countries have more firms than others which gives them a kind of comparative advantage in the sense that their economies are more competitive and will export more. The paper shows that this kind of asymmetry has important implications for trade negotiations. The paper also studies the implications of free riding in the trade negotiations. This analysis provides a possible explanation for why multilateralism was successful in the past, but is currently having problems. The multilateral negotiations system can sustain a certain level of free-riding, but with the emergence of developing countries in world trade, free-riding has increased above the critical level. Further contribution of this paper is to compare the analysis of trade liberalisation based on ex post criterion of reciprocity with that based on ex ante tariff-reduction formulae, of the kind carried out in reality. This comparison throws further light on our understanding of the multilateral negotiations process.

The remainder of this paper proceeds as follows. Section 2 presents the basic underlying oligopolistic model of international trade in a multi-country setting where countries differ by their number of firms. The non-cooperative equilibrium is presented in Section 3. It is in Section 3 that the rationale for trade agreements is derived. Section 4 analyses the GATT/WTO fundamental principles and shows how they help countries reach a superior cooperative outcome. The distribution of the benefits from multilateral trade liberalisation is also examined in Section 4. Section 5 studies formula-based trade negotiations. Section 6 es-
tablishes a minimum participation constraint necessary for multilateral negotiations to work and explains the past success and present difficulties of the multilateral trade negotiations. Section 7 concludes.

## 2 The basic model

The model used to analyse trade negotiations and trade agreements is a Brander (1981) type oligopolistic model. The particular setting was derived by Yi (1996) to study customs union formation in a world with many symmetric countries. In this case, I ignore customs union formation and I adapt Yi's model to examine trade negotiations among asymmetric countries. ${ }^{3}$

There are $n$ countries of the same size, but they differ by their number of firms. Subscripts $i$ and $l$ designate countries and subscripts $j$ and $k$ designate firms. Country $i$ has $k_{i}$ firms, $i=1, \ldots n$. The set of firms located in Country $i$ will be denoted $K_{i}, i=1, \ldots n$. Each country can be identified by the number of its firms and with a slight abuse of notation, I will write the set of countries in the world as $C=\left\{k_{1}, k_{2}, \ldots, k_{n}\right\}$. There are $N=\sum_{i=1}^{n} k_{i}$ firms in total in the world.

Each firm produces one good at a constant marginal cost $c$ in terms of the numeraire good. Consumers have quasilinear-quadratic preferences of the form

$$
\begin{equation*}
u\left(\mathbf{q}_{i}, M_{i}\right)=v\left(\mathbf{q}_{i}\right)+M_{i}=a Q_{i}-\frac{\gamma}{2} Q_{i}^{2}-\frac{1-\gamma}{2} \sum_{j=1}^{N} q_{i j}^{2}+M_{i} \tag{1}
\end{equation*}
$$

where $q_{i j}$ is Country $i$ 's consumption of firm $j$ 's product, $\mathbf{q}_{i}=\left(q_{i 1}, q_{i 2}, \ldots, q_{i N}\right)$ is Country $i$ 's consumption profile, $Q_{i} \equiv \sum_{j=1}^{N} q_{i j}$ and $M_{i}$ is Country $i$ 's consumption of the numeraire good. $\gamma$ is a substitution index between goods which ranges from 0 (independent goods) to 1 (homogeneous products); as $\gamma$ increases, products become closer substitutes. An important feature of the model is that consumers have a taste for variety; for any given $Q_{i}$, the more balanced the consumption bundle is, the higher the utility. There are two sources of gains from trade: increased variety of goods and reduced market power of domestic industry.

Country $i$ 's inverse demand function for firm $j$ 's good is

[^3]\[

$$
\begin{equation*}
p_{i j}=a-(1-\gamma) q_{i j}-\gamma Q_{i}=a-q_{i j}-\gamma \sum_{\substack{k=1 \\ k \neq j}}^{N} q_{i k} \tag{2}
\end{equation*}
$$

\]

There are no transportation costs in this model. Countries impose specific tariffs on imports from other countries. $\tau_{i j}$ denotes Country $i$ 's tariff on imports from firm $j . \tau_{i j}=0$ if firm $j$ is located in Country $i$. Then firm $j$ 's effective marginal cost of exporting to Country $i$ is

$$
\begin{equation*}
c_{i j}=c+\tau_{i j} \tag{3}
\end{equation*}
$$

Firms compete by choosing quantities in each country (Cournot competition in segmented markets). In Country $i$, firm $j$ will solve

$$
\begin{equation*}
\max _{\left\{q_{i j}\right\}} \pi^{i j}=\left(p_{i j}-c_{i j}\right) q_{i j} \tag{4}
\end{equation*}
$$

The first order condition for this maximisation problem is

$$
\begin{equation*}
p_{i j}-c_{i j}-q_{i j}=0 \tag{5}
\end{equation*}
$$

As it is well know, in the Cournot equilibrium,

$$
\begin{equation*}
Q_{i}=\frac{N-T_{i}}{\Gamma(N)} \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
q_{i j}=\frac{\Gamma(0)+\gamma T_{i}-\Gamma(N) \tau_{i j}}{\Gamma(0) \Gamma(N)} \tag{7}
\end{equation*}
$$

where $\Gamma($.$) is defined as \Gamma(k)=2-\gamma+k \gamma ; T_{i}$ is the sum of tariffs charged by Country $i$ on all imported goods $T_{i}=\sum_{j=1}^{N} \tau_{i j}$; and where I have normalised $a-c=1$.

From (6) and (7), we can note that

$$
\begin{equation*}
\frac{d Q_{i}}{d \tau_{i j}}=-\frac{1}{\Gamma(N)}<0 \tag{8}
\end{equation*}
$$

and

$$
\begin{align*}
\frac{d q_{i k}}{d \tau_{i j}} & =\frac{\gamma}{\Gamma(0) \Gamma(N)}>0, \text { for } k \neq j  \tag{9}\\
\frac{d q_{i j}}{d \tau_{i j}} & =\frac{\gamma-\Gamma(N)}{\Gamma(0) \Gamma(N)}<0 \tag{10}
\end{align*}
$$

so if Country $i$ increases its tariff on imports from firm $j$, then the consumption of good $j$ and the total consumption in Country $i$ will fall, but the consumption of all other goods will increase.

Firm $j$ 's equilibrium export profit to Country $i$ can be obtained using the first order condition (5)

$$
\begin{equation*}
\pi^{i j}=\left(p_{i j}-c_{i j}\right) q_{i j}=q_{i j}^{2} \tag{11}
\end{equation*}
$$

so we can furthermore note that

$$
\begin{align*}
\frac{d \pi_{i k}}{d \tau_{i j}} & =2 q_{i k} \frac{d q_{i k}}{d \tau_{i j}}=\frac{\gamma q_{i k}}{\Gamma(0) \Gamma(N)}>0, \text { for } k \neq j  \tag{12}\\
\frac{d \pi_{i j}}{d \tau_{i j}} & =\frac{2[\gamma-\Gamma(N)] q_{i j}}{\Gamma(0) \Gamma(N)}<0 \tag{13}
\end{align*}
$$

When Country $i$ increases its tariff on imports from firm $j$, then firm $j$ 's export profit to Country $i$ falls and home firms' profits and all other firms' export profits to Country $i$ rise.

## 3 Non-cooperative trade policy

Country $i$ 's welfare $W^{i}$ is the sum of four components: the domestic consumer surplus ( $C S^{i}$ ), the domestic firms' profit in the home market ( $\sum \pi^{i j}$, for all firms $j$ located in country $i$ ), the tariff revenue ( $T R^{i}$ ), and the domestic firms' export profits ( $\sum \pi^{l j}$, for all domestic firms $j$ in all foreign countries $l$ ).

$$
\begin{equation*}
W^{i}=C S^{i}+\sum_{j \in K_{i}} \pi^{i j}+T R^{i}+\sum_{\substack{l=1 \\ l \neq i}}^{n} \sum_{j \in K_{i}} \pi^{l j} \tag{14}
\end{equation*}
$$

Countries set tariffs on imports to maximise their welfare, so country $i$ solves the following maximisation problem

$$
\begin{equation*}
\max _{\left\{\tau_{i j}\right\}_{j \notin K_{i}}} W^{i} \tag{15}
\end{equation*}
$$

where $\tau_{i j}=0$, for $j \in K_{i}$. The tariff balances the benefit of increasing profits of the home firm at the expense of foreign firms against the cost of lower consumer surplus. More fundamentally, the effect of a tariff on welfare can be decomposed into a terms-of-trade effect (ToT) and volume-of-trade effect (VoT) and a market access effect (MA). ${ }^{4}$

$$
\begin{equation*}
\frac{d W^{i}}{d \tau_{i h}}=-_{j \notin K_{i}} q_{i j} \frac{d p_{j}^{*}}{d \tau_{i h}}{ }_{T o T}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i h}} \sum_{V o T}+\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i h}}{ }_{M A} \tag{16}
\end{equation*}
$$

where $p_{j}$ is the mill price (the pre-tariff price), $p_{j}^{*}=p_{i j}-\tau_{i j}$. Note that the market access effect is due to the inefficiency of the market. In perfect competition, prices are equal to marginal cost and so the last term of (54) would be zero. This effect will provide a new rationale for trade negotiations which is different from the terms-of-trade manipulation externality identified by Bagwell and Staiger (1999).

### 3.1 Nash equilibrium tariff

The following proposition derives countries' optimal tariff.
Proposition 1. The unique Nash optimal tariff of a country with $k$ firms is

$$
\begin{align*}
\tau_{e}(k ; \gamma, N) & =\frac{[2-\gamma][2+(2 k-1) \gamma]}{\left[(2-\gamma)^{2}+(1-\gamma) k \gamma\right][2+(N-1) \gamma]+[2+(k-1) \gamma][2+(2 k-1) \gamma]} \\
& =\frac{\Gamma(0) \Gamma(2 k)}{D(k)} \tag{17}
\end{align*}
$$

with $D(k) \equiv \Psi(k) \Gamma(N)+\Gamma(k) \Gamma(2 k)$ and $\Psi(k) \equiv(\Gamma(0)+1) \Gamma(k)-\Gamma(2 k)$.
Proof. See Appendix page 40.
Nash equilibrium tariffs are non-zero because governments want to improve their terms of trade and because they care about their domestic firms' profits. Tariffs protect domestic firms from foreign competition and increase domestic output.

Note that the optimal tariff of a country with $k$ firms depends only on its number of firms $k$, the total number of firms in the world $N$ and the substitution index $\gamma$. In particular, it does not depend on the tariffs of the rest of the world. It is a dominant strategy and there is no strategic interdependence of optimal tariffs across countries. A country with

[^4]$k$ firms charges $\tau_{e}(k ; \gamma, N)$ on imports from any firm from any other country. This result is a consequence of the quasilinearity of the utility function, the assumption of segmented markets and of constant marginal cost.

Figure 1 illustrates the variations of the optimal tariff with $k, N$ and $\gamma^{5}$ First, we note that $d \tau_{e}(k ; \gamma, N) / d N<0$. When the number of domestic firms is constant, the more firms there are in the world, the greater the proportion of goods produced abroad, the lower the monopoly power of the home country and the greater the loss to consumers from a tariff (consumers prefer variety). Thus the equilibrium tariff is lower. Second, we note that $\tau_{e}(k ; \gamma, N)$ varies non-monotonically with $\gamma$ and with $k$. This non-monotonicity of $\tau_{e}(k ; \gamma, N)$ results from $\gamma$ 's impact on competition and from two conflicting effects on the equilibrium tariff: First, as consumers prefer variety, if there are fewer firms at home, the country wants to charge a lower tariff to assure a balanced consumption. As the proportion of the domestically produced goods rises, the negative effect of the tariff on consumer surplus is diminished and so the country wants to charge higher tariffs. Second, as governments care about firms' profits, if there are few firms in the Home country, they will suffer more from foreign competition than if the domestic market is very competitive. This tends to decrease the equilibrium tariff as the number of domestic firms increases. When $\gamma$ is low, goods are homogeneous and the second effect is more important. When goods become independent, firms do not compete with each other and the first effect outweighs the second.


Figure 1: Nash equilibrium tariff $\tau_{e}(k ; \gamma, N)$ as a function of the number of domestic firms.

To summarise, a country with fewer firms can charge both a higher or a lower tariff than a country with many firms depending on the parameters of the model $\gamma$ and $N$. For $\gamma=0$,

[^5]the tariff is independent of $k$ and all countries charge the same tariff $\tau_{e}(k ; 0, N)=1 / 3$. For very small $\gamma$ and any $N, \tau$ is an increasing function of $k$. For the majority of values of the parameters, $\tau$ is an increasing function of $k$ for small $k$ and a decreasing function of $k$ for larger $k$. For small $N$ and very hight $\gamma, \tau$ can be a monotonically decreasing function of $k$, but this case is rare.

As $N$ and $\gamma$ are identical for all countries, to simplify notation, in the remainder of this paper, I will denote $\tau_{e}\left(k_{i} ; \gamma, N\right)$ as $\tau_{e}\left(k_{i}\right)$.

### 3.2 Optimal tariff in the absence of terms-of-trade manipulation

Bagwell and Staiger (1999) show in their perfectly competitive framework that if governments did not value the terms-of-trade effects of their tariffs, the optimal tariff would be zero. This subsection shows that in the oligopolistic model of this paper, governments would still charge non-zero tariffs even if they did not care about their terms of trade. To establish this result, I proceed in the same way as Bagwell and Staiger (1999) and consider a hypothetical world where the terms-of-trade effects of the tariff are ignored. I.e. I calculate the tariff such that

$$
\begin{equation*}
\frac{d W^{i}}{d \tau_{i h}}=+\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i h}}{ }_{M I}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i h}} V_{V o T}=0 \tag{18}
\end{equation*}
$$

Proposition 2. The unique Nash optimal tariff of a country with $k$ firms in the absence of terms-of-trade effects is

$$
\begin{equation*}
\tau_{w T}(k ; \gamma, N)=\frac{\gamma \Gamma(0) k}{(N-k)\left[(\Gamma(N)-\gamma) \Gamma(0) \Gamma(N)-k \gamma^{2}\right]} \tag{19}
\end{equation*}
$$

Proof. See Appendix page 40.
Note that this tariff is strictly positive for $\gamma>0$. So even if there were no terms-of-trade effects, governments would still charge non zero tariffs (to correct the market inefficiency). Thus this model provides a new reason for trade negotiations.

This tariff $\tau_{w T}$ is a increasing function of $k$ (which is not the case for the Nash tariff). This is because when $k$ increases, the number of products produced domestically increases and the importance of trade decreases. Thus negative effects of a tariff coming from the volume-of-trade effect become less important while it becomes relatively more important to
correct for the domestic inefficiency. When $\gamma=0, \tau_{w T}=0$, goods are independent and firms do not compete with each other. A tariff thus cannot address the market inefficiency. So when $\gamma=0$, countries only use tariffs to manipulate their terms of trade.

### 3.3 Strategic trade policy on exports

The previous derivations show that governments use tariffs on imports to improve their terms of trade and to increase market access of the domestic firms. The remainder of this paper will show how trade agreements help countries overcome the terms-of-trade and marketaccess externalities and reach superior cooperative outcomes. But before moving to the next section, let us first consider strategic trade policy on exports. What would countries chose to do if they were to intervene strategically on exports?

The following equation shows the impact of a 'subsidy' on exports on country $i$ 's welfare [REPLACE BY $n$ COUNTRY CASE]

$$
\begin{align*}
\frac{d W^{1}}{d s_{21}} & =\frac{d}{d s_{21}}\left[\left(p_{21}-c\right) q_{21}\right] \\
& =q_{21} \frac{d p_{21}}{d s_{21}}+\left(p_{21}-c\right) \frac{d q_{21}}{d s_{21}} \tag{20}
\end{align*}
$$

There is a terms-of-trade effect and a market-access effect: a positive export subsidy would deteriorate country $i$ 's terms of trade, but would increase export sales of domestic firms whereas a negative export subsidy (tax) would improve country $i$ 's terms of trade and decrease export sales of domestic firms. Brander and Spencer (1985) show that in an oligopolistic market, governments would subsidize exports. The following proposition derives the optimal subsidy for the setting of this model.

## [ADD PROPOSITION]

The optimal subsidy is positive. If intervening on exports, countries would chose to subsidise them to increase domestic production. But countries are not allowed to intervene in this way. The Article XVI of the GATT forbids export subsidies. In the remainder of this paper, I will thus focus on import tariffs established in the previous subsections. These tariffs are inefficiently high and I will show how trade agreements help countries to set tariffs in a more efficient way.

### 3.4 Equilibrium characterisation

To analyse trade agreements in the next sections, I will start from the Nash equilibrium (with the Nash equilibrium tariffs given by (17)). The following expressions characterise this Nash equilibrium and introduce convenient notation.

From (7) we can express the consumption in Country $i$ which has $k_{i}$ firms of the domestically produced product (or sales of one of the $k_{i}$ firms in it's domestic market). This quantity will be denoted $q_{I}$ as in home market

$$
\begin{equation*}
q_{I}\left(k_{i}\right)=\frac{\Gamma(0)+\gamma\left(N-k_{i}\right) \tau_{e}\left(k_{i}\right)}{\Gamma(0) \Gamma(N)} \tag{21}
\end{equation*}
$$

and the consumption in Country $i$ which has $k_{i}$ firms of a product produced by a foreign country firm (sales of a foreign firm in Country $i$ ). $q_{O}$ as outside home market

$$
\begin{equation*}
q_{O}\left(k_{i}\right)=\frac{\Gamma(0)-\Gamma\left(k_{i}\right) \tau_{e}\left(k_{i}\right)}{\Gamma(0) \Gamma(N)} \tag{22}
\end{equation*}
$$

Furthermore, the total consumption $Q$ in Country $i$ which has $k_{i}$ firms, can be expressed from (6) as a function of its tariff $\tau_{e}\left(k_{i}\right)$

$$
\begin{equation*}
Q\left(k_{i}\right)=\frac{N-\left(N-k_{i}\right) \tau_{e}\left(k_{i}\right)}{\Gamma(N)} \tag{23}
\end{equation*}
$$

Note that when a country charges a non zero tariff, domestic firms have always higher sales than foreign firms $\left(q_{I}\left(k_{i}\right) \geq q_{O}\left(k_{i}\right), i=0, \ldots, n\right)$. Furthermore, $d q_{I}\left(k_{i}\right) / d \tau>0$ and $d q_{O}\left(k_{i}\right) / d \tau<0$. Tariffs protect domestic firms from foreign competition and so domestic firms' sales are an increasing function of tariffs. At the same time, tariffs restrict the market access of foreign firms and so foreign firms' sales are a decreasing function of tariffs.

As mentioned above, in this model profits are quantities squared, so saying that governments care about domestic firms' market access directly means that governments care about their firms' profits. This is a nice contact point of this model with real-world trade negotiations.

Finally, note that the sum of the first three terms of welfare (14) equals the total surplus in country $i$ from the consumption of $\mathbf{q}_{i}$ minus foreign profit and so the welfare of Country $i$ can be re-written as expressed in the following lemma.

Lemma 1. The welfare of Country $i$ can be written as

$$
\begin{equation*}
W^{i}=u\left(\mathbf{q}_{i}\right)-c Q_{i}-\sum_{j \notin K_{i}} \pi^{i j}+\sum_{\substack{l=1 \\ l \neq i}}^{n} \sum_{j \in K_{i}} \pi^{l j} \tag{24}
\end{equation*}
$$

Proof. See Appendix page 40.
The first two terms are the net benefits from consumption, the third term is foreign firms' profits in the home market and the final term is the export profits of the home firms abroad. This rewriting of the welfare function provides an intuitive link with mercantilism: an increase in trade surplus is good if causes of such an increase do not decrease consumers' net benefits from consumption. This expression of welfare will prove itself convenient in the next section to analyse the role of the principles of reciprocity and non-discrimination.

## 4 Cooperative trade policy

I will now study what happens when countries set their tariffs cooperatively according to the GATT/WTO principles of reciprocity and non-discrimination as usually defined in the literature.

### 4.1 The principle of reciprocity with bilateral liberalisation

The principle of reciprocity is a bilateral concept that has two real-world applications within the GATT/WTO negotiations. The first one that is encoded in GATT/WTO Articles enables countries to retaliate reciprocally, i.e. if a trading partner raises previously bound tariffs on imports from the Home country, the Home country is entitled to withdraw equivalent concessions from the trading partner. The second application that is not actually encoded in GATT/WTO Articles, but that is considered important in practice requires countries to exchange concessions when negotiating trade liberalisation.

I will use the following formal interpretation of Bagwell and Staiger (1999) of this principle: a tariff change is reciprocal if it leaves the bilateral trade balance constant.

Definition 1. Consider Country $i$ and Country $l$. Let $E X P_{i l}$ be the value of exports from Country $i$ to Country $l$ and $I M P_{i l}$ the value of imports from Country $l$ to Country $i$. The bilateral trade balance of Country $i$ and Country $l$ is $T B_{i l} \equiv E X P_{i l}-I M P_{i l}$. Then a tariff change $\left(d \tau_{i l}, d \tau_{l i}\right)$ is bilaterally reciprocal if it is such that $d T B_{i l}=0$.

As we can see, the definition of a reciprocal tariff change directly implies that when countries negotiate trade liberalisation, they exchange equal market access concessions. Reciprocal trade liberalisation has to keep the bilateral trade balance constant, so if exports from the Home country to a trading partner increase, the imports from the trading partner have to increase by the same value.

What does this principle of reciprocity imply in this model? To express the condition of reciprocity, we need to evaluate exports and imports of Country $i$ at mill prices. From the profit maximisation condition (5), we have the mill price $p_{j}^{*}$ of a good produced by firm $j$

$$
\begin{equation*}
p_{j}^{*}=p_{i j}-\tau_{i j}=c+q_{i j} \tag{25}
\end{equation*}
$$

For technical convenience, assume $c=0$. The trade balance for countries $i$ and $j$ can thus be written as

$$
\begin{equation*}
T B_{i l}=k_{i} q_{O}^{2}\left(k_{l}\right)-k_{l} q_{O}^{2}\left(k_{i}\right) \tag{26}
\end{equation*}
$$

Note that the assumption of zero marginal cost implies here that countries exchange equal "profit concessions". If trade liberalisation implies increased profits of domestic firms in the foreign country, profits of foreign firms in the domestic country have to increase by the same amount.

Total differentiation of (26) gives

$$
\begin{equation*}
d T B_{i l}=2 k_{i} q_{O}\left(k_{l}\right) d q_{O}\left(k_{l}\right)-2 k_{l} q_{O}\left(k_{i}\right) d q_{O}\left(k_{i}\right) \tag{27}
\end{equation*}
$$

The reciprocity condition $d T B_{i l}=0$ becomes thus

$$
k_{i} q_{O}\left(k_{l}\right) \Gamma\left(k_{l}\right) d \tau\left(k_{l}\right)=k_{l} q_{O}\left(k_{i}\right) \Gamma\left(k_{i}\right) d \tau\left(k_{i}\right)
$$

and so

$$
\begin{align*}
d \tau\left(k_{i}\right) & =\frac{k_{i} \Gamma\left(k_{l}\right) q_{O}\left(k_{l}\right)}{k_{l} \Gamma\left(k_{i}\right) q_{O}\left(k_{i}\right)} d \tau\left(k_{l}\right) \\
& =\frac{\left(\Gamma(0) k_{i}+\gamma k_{i} k_{l}\right) q_{O}\left(k_{l}\right)}{\left(\Gamma(0) k_{l}+\gamma k_{i} k_{l}\right) q_{O}\left(k_{i}\right)} d \tau\left(k_{l}\right) \tag{28}
\end{align*}
$$

Proposition 3. To respect the principle of reciprocity, a more competitive country has to liberalise more than a less competitive country.

Proof. Assume that there are more firms in Country $i$ than in Country l, i.e. $k_{i}>k_{l}$. From (22), we know that $q_{O}(k)$ is a decreasing function of $k$ : $\frac{d q_{O}(k)}{d k}=\frac{-\gamma}{D(k)^{2}}\{[2(2-\gamma)+1+4 k \gamma](2-$ $\left.\gamma)^{2}+2(1-\gamma) k^{2} \gamma^{2}\right\}<0$. And so $q_{O}\left(k_{i}\right)<q_{O}\left(k_{l}\right)$. Thus $\left(\Gamma(0) k_{i}+\gamma k_{i} k_{l}\right) q_{O}\left(k_{l}\right)>\left(\Gamma(0) k_{l}+\right.$ $\left.\gamma k_{i} k_{l}\right) q_{O}\left(k_{i}\right)$ and so it must be from (28) that to satisfy the principle of reciprocity, $d \tau\left(k_{i}\right)>$ $d \tau\left(k_{l}\right)$.

So the more competitive country has to liberalise more in order to offer same market access advantage as the less competitive liberalising partner. Reciprocity principle ensures that countries trade equal concessions in market access.

This result could be linked to the actual progress in trade liberalisation. Developed countries that can be considered more competitive (not only in the sense that more firms are located in these countries) have on average liberalised trade more than developing countries. In reality, there are provisions in the WTO agreements allowing developing countries to take more time to implement trade liberalisation and providing measures to increase their trading opportunities. But as my model shows, even without these provisions, with the given definition of reciprocity, developing countries - if less competitive - would be required to liberalise less.

A similar result was derived by Baldwin and Robert-Nicoud (2000) who study free trade agreements between asymmetric countries in an economic geography model. To motivate their analysis, they list several North-South type trade agreements where Northern (larger and more developed) countries were required to liberalise faster (e.g. free trade deals between the European Union and the Central and Eastern European countries, Asia-Pacific Economic Co-operation initiative or the ASEAN Free Trade Area). Their model explains this asymmetry as preventing firm delocation whereas here it assures the exchange of equivalent market access concessions.

How does a reciprocal liberalisation affect the welfare of participating countries? To answer this question, I will use a nice feature of this model which is that due to the simplicity of preferences, it is possible to obtain a closed-form solution for welfare of Country $i$ with $k_{i}$ firms, $i=1, \ldots, n$, in a given set of countries $C=\left\{k_{1}, k_{2}, \ldots, k_{n}\right\}$.

Lemma 2. The welfare of country with $k_{i}$ firms when the set of countries in the world is $C=\left\{k_{1}, k_{2}, \ldots, k_{n}\right\}$ is given by

$$
\begin{align*}
W\left(k_{i} ; C\right)= & Q\left(k_{i}\right)-\frac{\gamma}{2} Q\left(k_{i}\right)^{2}-\frac{1-\gamma}{2}\left\{k_{i}\left[q_{I}\left(k_{i}\right)\right]^{2}+\left(N-k_{i}\right)\left[q_{O}\left(k_{i}\right)\right]^{2}\right\} \\
& -\left(N-k_{i}\right)\left[q_{O}\left(k_{i}\right)\right]^{2}+k_{i} \sum_{\substack{l=1 \\
l \neq i}}^{n} q_{O}\left(k_{l}\right)^{2} \tag{29}
\end{align*}
$$

Proof. Directly follows from combining (24) with the expressions (23), (21) and (22).
Now, the first three terms are the net benefits from consumption, the fourth term is foreign firms' profits in the home market and the final term is the export profits of the home firms abroad. The last two terms represent the exchange of market access when countries cooperate and liberalise trade: as they lower tariffs, they give away market access in the domestic market and they gain market access abroad.

Proposition 4. 1. In a two country world, bilaterally reciprocal trade liberalisation monotonically increases welfare in both countries.
2. In an $n$ country world, $n>2$, bilaterally reciprocal trade liberalisation between two countries has an ambiguous impact on the welfare of these countries.

Proof. See Appendix page 41.
Intuitively, in a two country world, reciprocal trade liberalisation has the following impacts on countries' welfare: Tariff revenue decreases. Consumer surplus increases, because consumers can get foreign products at a lower price and also because the monopoly power of domestic firms is reduced. Furthermore, domestic firms can make higher profits in the foreign market, because they get a better access to this market, but at the same time they face more competition at home. The reciprocity condition ensures that the positive effects dominate.

In a $n$ country world $(n>2)$, the effect of trade liberalisation is more complex. When goods are independent (low $\gamma$ ), domestic firms will benefit from better access to foreign market, but they will not be harmed by more competition at home. So the welfare increasing effects dominate. For higher values of $\gamma$, firms will experience more competition in their home market. Moreover, the consumption pattern of the liberalising countries will be changed: consumers will now get only part of foreign products cheaper (those produced
in the partner country). This can make their consumption bundle less balanced. If trade liberalisation starts from a situation where Country $i$ is charging high tariffs on imports from other countries than Country $l$, the discriminatory liberalisation with Country $l$ can reduce Country $i$ 's welfare. This result illustrates the so-called "concertina rule" for tariff reform which aims to lower the variance of the tariff structure. ${ }^{6}$

### 4.2 The principle of non-discrimination

The previous subsection showed that reciprocal liberalisation can increase welfare of both participating countries, but does not always necessarily do so. In this subsection, I show that if two countries liberalise trade between themselves, outsiders are made unambiguously worse off. Thus the bilateral principle of reciprocity is not sufficient to ensure a monotonic increase in welfare of all countries. To do so, this principle has to be applied multilaterally which will be ensured by the non-discrimination principle.

Note that in this model, a given country is affected by other countries' tariffs only through its export profits to these countries. Suppose that a subset of $s$ countries decide to liberalise trade among themselves. $S$ firms in total are located in this subset of countries. How will this liberalisation affect the $n-s$ non involved countries? Suppose Country $i$ is involved in the trade liberalisation. The following expression gives the sales in Country $i$ of a firm located in one of the non involved countries

$$
\begin{equation*}
q_{O}\left(k_{i}\right)=\frac{\Gamma(0)-\gamma\left(N-S-k_{i}\right) \tilde{\tau}-\Gamma(N) \tilde{\tau}+\gamma\left(S-k_{i}\right) \tau}{\Gamma(0) \Gamma(N)} \tag{30}
\end{equation*}
$$

where $\tilde{\tau}$ is the tariff charged by Country $i$ on countries not involved in the liberalisation and is thus constant, and $\tau$ is the tariff that Country $i$ charges on its trading partners involved in the liberalisation. This tariff will be reduced in the trade liberalisation considered.

Proposition 5. A discriminatory liberalisation harms countries that are not involved in it.

Proof. To see what happens during this discriminatory liberalisation, we want to determine the impact of a decrease in $\tau$ on $q_{O}$ given by (30).

$$
\begin{equation*}
\frac{d q_{O}\left(k_{i}\right)}{d \tau}=\frac{\gamma\left(S-k_{i}\right)}{\Gamma(0) \Gamma(N)}>0 \tag{31}
\end{equation*}
$$

Thus not involved countries are hurt by a discriminatory trade liberalisation.

[^6]This phenomenon is well known in the literature and was named by Viner (1950) as trade diversion. The group of liberalising countries exchange market access concession among themselves. This diminishes the market access of not involved countries. (Note also that the bigger the subgroup of liberalising countries, the more harmful this liberalisation is to the non involved countries). The role of the non-discrimination principle is to prevent this to happen by forbidding discriminatory trade liberalisation.

### 4.3 Multilateral trade liberalisation under the GATT/WTO principles

In this subsection, I determine the impacts on welfare of a trade liberalisation that follows both the reciprocity and non-discrimination principles. The non-discrimination principle 'multilaterises' the reciprocity principle. A reciprocal tariff change leaves the trade balance between the two trading partners constant. The non-discrimination principle says that if a country decreases its trade barriers with respect to one of its trading partners, it has to do so with all other trading partners. The combination of these two principles can be formally stated as follows.
Definition 2. The multilateral trade balance of Country $i$ is $T B_{i} \equiv \sum_{\substack{l=1 \\ l \neq i}}^{n} E X P_{i l}-I M P_{i l}$. Then a tariff change is multilaterally reciprocal if it is such that $d T B_{i}=0$.

Still assuming for technical convenience $c=0$, the multilateral trade balance becomes in this case

$$
\begin{aligned}
T B_{i} & =k_{i} \sum_{l \neq i}^{n} q_{O}^{2}\left(k_{l}\right)-\sum_{l \neq i}^{n} k_{l} q_{O}^{2}\left(k_{i}\right) \\
& =k_{i} \sum_{l \neq i}^{n} q_{O}^{2}\left(k_{l}\right)-\left(N-k_{i}\right) q_{O}^{2}\left(k_{i}\right)
\end{aligned}
$$

So the multilateral reciprocity condition becomes

$$
\begin{equation*}
d T B_{i}=k_{i} \sum_{l \neq i}^{n} 2 q_{O}\left(k_{l}\right) d q_{O}\left(k_{l}\right)-\left(N-k_{i}\right) 2 q_{O}\left(k_{i}\right) d q_{O}\left(k_{i}\right)=0 \tag{32}
\end{equation*}
$$

Proposition 6. A trade liberalisation following both the principle of reciprocity and nondiscrimination monotonically increases the welfare of all countries.

Proof. See Appendix page 44.
The reciprocity principle together with the principle of non-discrimination ensure that no-one is made worse off. Note that the non-discrimination principle prevents situations where countries charge different tariffs on different trading partners and so prevents cases where reciprocal liberalisation could have been welfare reducing for the liberalising partners.

### 4.4 Distribution of benefits from multilateral trade liberalisation

The previous subsection showed that multilateral liberalisation according to the GATT/WTO principles unambiguously increases welfare of all countries. It might therefore seem puzzling why there is resistance towards trade liberalisation if everyone benefits. Here, I clarify this point by analysing the distribution of benefits from multilateral trade liberalisation.

### 4.4.1 Impacts of multilateral trade liberalisation on consumer surplus

Intuitively, consumers are better off when trade is liberalised as trade increases the variety of available products and restricts the monopoly power of domestic firms. The following study shows this formally.

Lemma 3. The consumer surplus of Country $i$ is given by

$$
\begin{equation*}
C S\left(k_{i}\right)=\frac{\gamma}{2} Q\left(k_{i}\right)^{2}+\frac{1-\gamma}{2}\left[k_{i} q_{I}\left(k_{i}\right)^{2}+\left(N-k_{i}\right) q_{O}\left(k_{i}\right)^{2}\right] \tag{33}
\end{equation*}
$$

Proof. Follows directly from (58) derived for welfare calculations in Lemma 1.
Note that as mentioned earlier, consumers are not affected in this model by tariffs charged by foreign countries. Consumer surplus depends only on tariffs charged by the home country.

Proposition 7. The consumer surplus is a decreasing function of the tariff charged by the domestic country.

Proof. See Appendix page 45
Consumers unambiguously gain from trade liberalisation. As consumer surplus does not depend on tariffs of foreign countries and it is a decreasing function of country's own tariff, consumers would be better off even in the case of a unilateral trade liberalisation.

### 4.4.2 Impacts of multilateral trade liberalisation on producer surplus

Producers are affected both by tariffs of their domestic country which protect them from foreign competition and by tariffs charged by foreign countries which limit their exports. In Country $i$, the sum of profits of the $k_{i}$ domestic firms is

$$
\begin{equation*}
\Pi=\sum_{j \in K_{i}} \pi^{i j}+\sum_{\substack{l=1 \\ l \neq i}}^{n} \sum_{j \in K_{i}} \pi^{l j}=k_{i} q_{I}\left(k_{i}\right)^{2}+k_{i} \sum_{\substack{l=1 \\ l \neq i}}^{n} q_{O}\left(k_{l}\right)^{2} \tag{34}
\end{equation*}
$$

Profits in the home country are an increasing function of the tariff charged by the domestic country, but profits in foreign countries are a decreasing function of the tariffs charged by the foreign countries. The following proposition summarises the impact on profits of a multilateral trade liberalisation following the GATT/WTO principles of reciprocity and non-discrimination.

Proposition 8. When countries liberalise multilaterally according to the principles of reciprocity and non-discrimination, Country $i$ 's firms' profits are a decreasing function of the tariff $\tau$ if and only if $\tau<\tau_{\text {PSmin }}$ with

$$
\begin{equation*}
\tau_{P S \min }=\frac{\Gamma(0)^{2}}{\Gamma\left(k_{i}\right)^{2}+k \gamma^{2}\left(N-k_{i}\right)} \tag{35}
\end{equation*}
$$

and $\tau_{\text {PSmin }}>\tau_{e}\left(k_{i}\right)$ for $\gamma<\gamma_{c}=2+k_{i}-\sqrt{k_{i}^{2}+4 k_{i}}$.
Proof. See Appendix page 47.
Profits are a quadratic function of tariffs, i.e. they vary non-monotonically with the tariff. Proposition 8 establishes that for low $\gamma$, profits are a decreasing function of the tariff. So for low $\gamma$, firms are made better off by multilateral liberalisation. On the other hand for high $\gamma$, profits are a non-monotonic function of the tariff. If countries liberalise from the initial tariff level $\tau_{e}$, profits will initially decrease and then increase. The following figure shows how profit varies for tariffs between 0 and $\tau_{e}$ for different values of $\gamma$.

The non-monotonicity of the profit function comes from two opposing effects of trade liberalisation. When countries lower their tariffs, firms get better access to foreign markets, but at the same time they face also more competition in the domestic market. For low $\gamma$ (below $\gamma_{c}$ ), goods are more independent and so access to foreign markets outweighs the disadvantages from more competition as consumers are not likely to substitute the products.


Figure 2: Normalised profit as a function of the tariff for $N=100$.

But when $\gamma$ is above $\gamma_{c}$ and goods are more substitutable, the losses from more competition in the domestic market outweigh the benefits from getting better access to foreign markets.

The critical value $\gamma_{c}$ below which gains in market access abroad outweigh losses in market access at home depends on the number of domestic firms $k_{i}$ : the more firms Country $i$ has, the lower this critical value and the smaller the range of parameters where the profits are a decreasing function of the tariff. This is a consequence of the reciprocity principle. The more firms Country $i$ has, the more it needs to liberalise to satisfy the reciprocity principle and firms will lose more in the domestic market than gain abroad. For example, when $k_{i}=1$, $\gamma_{c}=3-\sqrt{5} \approx 0.764$ whereas when $k_{i}=10, \gamma_{c}=2(6-\sqrt{35}) \approx 0.168$.

This result explains why certain sectors are more difficult to liberalise than others. When goods are independent, firms are not afraid of competition and are happy to gain better access to foreign markets. When goods are more substitutable, competition is more harmful than gains from foreign market access. So in this case, firms would be willing to lobby the government not to take part in liberalisation.

The typical example of this phenomenon is trade liberalisation in manufacturing versus trade liberalisation in agriculture. Liberalisation in manufacturing where goods can be seen as independent (because different) was highly successful. On the other hand, liberalisation in agriculture (where goods are close substitutes - a banana from Brazil or Costa Rica is still a banana) was and still is very difficult. As acknowledged on the WTO website
"The original GATT did apply to agricultural trade, but it contained loopholes. For example, it allowed countries to use some non-tariff measures such as import quotas, and to subsidize. Agricultural trade became highly distorted, especially
with the use of export subsidies which would not normally have been allowed for industrial products." ${ }^{7}$

## 5 Formula-based trade liberalisation

The previous section analysed cooperative trade liberalisation following the principles of reciprocity and non-discrimination as generally defined in the literature. It is important to note that the interpretation of the principle of reciprocity presented in the previous section ${ }^{8}$ is an elegant and convenient way of introducing this principle into models of international trade, but does not quite represent the reality of the trade liberalisation process. The reciprocity condition defined in the previous section is an ex post criterion and is thus difficult to implement in reality.

Tariff reductions are negotiated according to a variety of different methods. The simplest way of reducing tariffs is a single rate reduction: all tariffs are cut to a single rate. In practice, this is mainly used in regional trade agreements where the tariff among members is set to zero. A more frequent way of reducing tariffs in multilateral trade negotiations is a flat-rate percentage reduction: the same percentage reduction for all products, no matter whether the starting tariff is high or low. For example, all tariffs cut by $25 \%$ in equal steps over five years. More recent GATT/WTO rounds used formulas to cut tariffs. During the Kennedy Round the so-called linear cut formula was used for trade liberalisation. Francois and Martin (2003) note that thanks to the introduction of the formula approach it was possible to achieve a substantial cut in tariffs of about $35 \%$. The following Tokyo Round used a more sophisticated formula called the Swiss formula and reduced average tariffs by $30 \%$. In the Uruguay Round a variety of methods was used to negotiate tariff cuts and to reach a reduction average target comparable to that of the Tokyo Round ( $1 / 3$ cut). ${ }^{9}$

In this section, I study trade liberalisation following the simplest formula-based method: a flat-rate percentage reduction. The following proposition determines the impact on welfare of Country $i$ of a uniform marginal tariff cut $d \alpha$ from the tariff $\alpha \tau_{e}\left(k_{i}\right) .{ }^{10}$

[^7]Proposition 9. 1. When $\gamma=0$, a marginal flat-rate tariff cut is welfare increasing for countries that have more than $\bar{k}$ firms, where

$$
\begin{equation*}
\bar{k}=\frac{3(1-\alpha) N}{2(3-\alpha) n-3-\alpha} \tag{36}
\end{equation*}
$$

2. When $\gamma>0$, a marginal flat-rate tariff cut is welfare increasing for
(a) all countries when the asymmetry between countries (in terms of number of firms per country) is small,
(b) countries with many firms and countries with very few firms when the asymmetry between countries is intermediate,
(c) countries with many firms when the asymmetry between countries is large.

Proof. See Appendix page 48.
When countries cut tariffs multilaterally in a radial way, consumers gain (as shown in section 4 , consumers are always better off when trade is liberalised). Firms lose market access in the home country and gain market access in foreign countries. For trade liberalisation to be welfare increasing, firms must gain sufficient compensatory market access abroad. This is what happens when trade liberalisation follows the principles of reciprocity and non-discrimination. These principles ensure that firms get the right compensatory market access in foreign countries for what they lose at home. In a radial tariff cut, as this cut is independent of the market structure (i.e. number of firms), firms do not necessarily get the minimum compensatory market access.

In a perfectly symmetric world, countries will all charge initially the same Nash optimal tariff. A radial tariff cut will then be the same in all countries and domestic firms will gain exactly the same market access advantage in foreign countries as the home country will give away to foreign firms through this tariff cut. Each country will get exactly compensated for the offered market access in the domestic market through gained market access in other countries. And so in a perfectly symmetric world, a radial tariff cut is welfare increasing. ${ }^{11}$ The same reasoning applies when asymmetries between countries are small.

[^8]When countries are asymmetric, i.e. some countries have many firms and some have few firms, a flat rate tariff cut affects different countries differently. The proof in the appendix shows how the change in welfare depends on the variance between countries. Because a flat rate tariff cut is independent of the market structure (all countries cut by $\alpha$ irrespectively of how many firms they have), countries with many firms are making smaller market access concessions than countries with fewer firms. ${ }^{12}$ Furthermore, countries with many firms export more than countries with few firms and so they benefit more from increased market access abroad. Thus, for a country with few firms, the increased market access abroad is not sufficient to compensate market access lost in the home country and so countries with few firms may not benefit from a radial tariff cut.

The situation is different for countries with very few firms where consumers gain enormously from trade liberalisation so that the increase in consumer surplus outweighs firms' loss of market access. When the asymmetry between countries is very important, even in countries with few firms, the gain in consumer surplus is smaller than the loss of market access.

When $\gamma=0$, consumers do not gain as much from trade liberalisation and a minimum number of firms is necessary to benefit from a radial tariff cut.

Table 1 illustrates the evolution of the sign of the derivative of welfare when the variance of the number of firms per country changes. The columns present the sign of the derivative of welfare for each country in the set of countries $C=k_{1}, k_{2}, \ldots, k_{10}$ for the same parameter values of $N, n, \gamma$ and $\alpha$, but for situations where the number of firms per country changes starting in a symmetric world in the first two columns and then going towards a world that is more and more asymmetric as shown by the standard deviation in the first line.

This result provides another justification for why developing countries liberalise less and enjoy longer implementation periods for their tariff reductions than developed countries. For example, in the current Doha Round, it was agreed in the July 2004 package that developing countries can choose between 1) less than formula cuts for up to $10 \%$ of their tariff lines representing up to $10 \%$ of their import value; or 2) not apply formula cuts, or leave unbound

[^9]| St. Dev. | 0.0 | St. Dev. | 9.8 | St. Dev. | 16.0 | St. Dev. | 28.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $k_{i}$ | $d W / d \alpha$ | $k_{i}$ | $d W / d \alpha$ | $k_{i}$ | $d W / d \alpha$ | $k_{i}$ | $d W / d \alpha$ |
| 10 | - | 2 | - | 1 | - | 1 | + |
| 10 | - | 3 | - | 2 | - | 1 | + |
| 10 | - | 4 | - | 3 | - | 1 | + |
| 10 | - | 5 | - | 4 | - | 1 | + |
| 10 | - | 6 | - | 5 | + | 1 | + |
| 10 | - | 7 | - | 6 | + | 1 | + |
| 10 | - | 8 | - | 7 | + | 1 | + |
| 10 | - | 9 | - | 8 | + | 1 | + |
| 10 | - | 25 | - | 9 | + | 1 | + |
| 10 | - | 31 | - | 55 | - | 91 | - |

Table 1: Signs of the derivative $\frac{d W(\alpha \tau)}{d \alpha}$ for $N=100, n=10, \gamma=0.9, \alpha=0.5$ and different variances of the number of firms per country.
tariff lines, for up to $5 \%$ of their tariff lines representing up to $5 \%$ of their import value.

## 6 Partial participation: Why multilateralism worked and is now broken

Section 4 showed that multilateral trade liberalisation following the reciprocity and nondiscrimination principles is welfare increasing for all countries, nevertheless not all interest groups gain from this liberalisation in the same way and some might even lose. Section 5 showed that formula-based trade liberalisation can be welfare decreasing for some countries in certain circumstances. It is therefore understandable that it might be difficult to reach an agreement among all countries. This section explores what happens when all countries do not agree, but a subgroup does.

### 6.1 Participation constraint in reciprocity-based trade liberalisation

If all countries do not agree on a trade liberalisation, a subset of countries might. This subset of countries might be tempted to liberalise trade among themselves. This, as we have seen in subsection 4.2, would make the non involved countries worse off and it is forbidden by the non-discrimination principle. What if a subset of countries agree to lower their
tariffs unilaterally, i.e. on everyone else? ${ }^{13}$ The uninvolved countries, the free-riders, would unambiguously benefit as they would gain free market access in the liberalising countries. The following proposition states under which conditions a unilateral trade liberalisation by a subgroup of countries is welfare increasing for a country belonging to this subgroup. The liberalisation considered here follows the principles of reciprocity and non-discrimination: a subgroup of countries liberalise reciprocally and they extend this liberalisation unilaterally to all other countries.

Assume a subset of $p$ countries decide to liberalise trade unilaterally. $P$ firms are located in these countries.

Lemma 4. The more countries with more firms are involved in unilateral reciprocity-based trade liberalisation, the larger the increase in welfare of these countries. I.e.

$$
\frac{d}{d P}\left(\frac{d W}{d \tau}\right)<0
$$

Proof. See Appendix page 52.
The derivative of welfare with respect to the tariff is a linear decreasing function of the number of firms involved in the trade liberalisation $P$. There exists a minimum number of firms necessary for the trade liberalisation to be welfare increasing. The following proposition establishes the minimum participation constraint so that a marginal reciprocity-based trade liberalisation is welfare increasing for Country $i$ that has $k_{i}$ firms.

Proposition 10. A marginal unilateral tariff cut from $\alpha \tau_{e}(k)(\alpha \in[0,1])$ is welfare increasing for a liberalising country with $k$ firms if and only if the subgroup of trade liberalising countries has more than $\bar{P}_{r}$ firms, where

$$
\begin{equation*}
\bar{P}_{r}=\bar{P}_{r}(\alpha, k, N, \gamma)=\frac{D(k) A(k)-\alpha \Gamma(2 k) B(k)}{2 \Gamma(k)[D(k)-\alpha \Gamma(k) \Gamma(2 k)]} \tag{37}
\end{equation*}
$$

with $A(k)=N \Gamma(2 k)+k \Gamma(0)$ and
$B(k)=(N-k)^{2} \gamma\left[\Gamma(0)^{2}+(1-\gamma) k \gamma\right]+\Gamma(k)^{2}[(1-\gamma)(N-k)+2 N]$
Proof. See Appendix page 52.

[^10]Proposition 10 says that unilateral trade liberalisation following reciprocity and nondiscrimination principles can be welfare-increasing if the countries involved represent a sufficient part of world production. The larger the tariff cut, the more participation is needed to make this cut welfare increasing.

Note that the minimum participation constraint is a constraint on the number of firms involved. In the reciprocity-based trade liberalisation, if Country $i$ has relatively more firms than its liberalising partners, then it will have to liberalise more than its partners to satisfy the reciprocity condition. And so to benefit from trade liberalisation, it will have to gain sufficient compensatory market access and it will need many liberalising partners with few firms. If Country $i$ has relatively few firms than its partners, then it will liberalise less than its partners and to gain from trade liberalisation, it will need few countries with many firms.

The following corollaries study the unilateral trade liberalisation with partial participation in particular situations to better illustrate the minimum participation condition given by equation (37).

Corollary 1. Starting from Nash optimal tariffs, i.e. $\alpha=1$, a marginal unilateral reciprocitybased trade liberalisation is welfare increasing for a liberalising country with $k$ firms if and only if at least one other trading partner liberalises as well, $P>\bar{P}_{r}(1, k, N, \gamma)=k$.

Figure 3 shows how welfare changes as a function of the tariff when only one country is considering unilateral trade liberalisation $(P=k)$ and where several countries $(P>k)$ consider unilateral trade liberalisation following the GATT/WTO principles.

Corollary 2. Unilateral free trade $(\alpha=0)$ is welfare maximising for a participating country with $k$ firms if and only if the subgroup of trade liberalising countries has more than $\bar{P}_{r_{F T}}$ firms, where

$$
\begin{equation*}
\bar{P}_{r_{F T}}=\bar{P}_{r}(0, k, N, \gamma)=\frac{A(k)}{2 \Gamma(k)}=\frac{N \Gamma(2 k)+k \Gamma(0)}{2 \Gamma(k)} \tag{38}
\end{equation*}
$$

Figure 3 illustrates this result: when $N=100$ and $k=10$, unilateral free trade is welfare maximising if countries with at least 90 firms participate.

Note that $\Gamma(2 k)+\Gamma(0)=2 \Gamma(k)$ and so $\bar{P}_{r_{F T}}$ is a weighted average of the total number of firms in the world $N$ and the number of domestic firms $k$.

So $\bar{P}_{r_{F T}}$ is an increasing function of the total number of firms in the world ( $\frac{d \bar{P}_{r_{F T}}}{d N}>0$ ). A sufficient proportion of the total number of firms in the world is needed. If the total number of firms in the world increases, this proportion increases.


Figure 3: Welfare as a function of tariff with $N=100, \gamma=0.5, k=10$.
$\bar{P}_{r_{F T}}$ is also an increasing function of the number of domestic firms $\left(\frac{d \bar{P}_{r_{F T}}}{d k}=\frac{\Gamma(0) \Gamma(N)}{2 \Gamma(k)^{2}}>0\right)$. If the country considered has a lot of firms compared to its liberalising partners, it will have to liberalise a lot to satisfy the reciprocity condition and it will need a sufficient number of partner countries with few firms to make the trade liberalisation welfare increasing. If the country considered has few firms compared to its liberalising partners, it will liberalise less than its partners according to the reciprocity condition and so the minimum number of necessary partners is smaller.

Finally, $\bar{P}_{r_{F T}}$ is an increasing function of the substitution index $\gamma\left(\frac{d \bar{P}_{F T}}{d \gamma}=\frac{k(N-k)}{\Gamma(k)^{2}}>0\right)$. The more goods are substitutable, the more countries are needed to participate to offer enough market access to firms in foreign markets to compensate for more competition in the domestic market.

In a perfectly symmetric world where each country has only one firm, the minimum condition becomes a condition on the minimum number of countries involved.

Corollary 3. In a perfectly symmetric world, unilateral free trade is welfare maximising for liberalising countries if and only if more than $\bar{p}_{r_{S y m F T}}$ countries participate, where

$$
\begin{equation*}
\bar{p}_{r_{S y m F T}}=\frac{2 N+\gamma(N-1)+2}{4} \tag{39}
\end{equation*}
$$

Note that with $\gamma=0$, the number of countries that has to agree to liberalise (with
everyone else) has to be greater than $(N+1) / 2$ and when $\gamma=1$, this group has to have more than $(3 N+1) / 4$ for the liberalisation to be welfare enhancing.

Corollary 4. When $\gamma=0$, a marginal unilateral tariff cut from $\alpha \tau_{e}(k)(\alpha \in[0,1])$ is welfare increasing for a participating country with $k$ firms if and only if the subgroup of trade liberalising countries has more than $\bar{P}_{r_{\gamma}=0}$ firms, where

$$
\begin{equation*}
\bar{P}_{r_{\gamma=0}}=\bar{P}_{r}(\alpha, k, N, 0)=\frac{3(1-\alpha) N+(3+\alpha) k}{2(3-\alpha)} \tag{40}
\end{equation*}
$$

As previously, $\bar{P}_{r_{\gamma=0}}$ is an increasing function of the total number of firms in the world and an increasing function of the number of domestic firms.

### 6.2 Participation constraint in formula-based trade liberalisation

As discussed in Section 5, countries typically negotiate tariff cuts according to formulas. Does the previous result of minimum participation carry through in the flat-rate liberalisation case? More interestingly, Section 5 showed that some countries might not benefit from a formula-based trade liberalisation. What if only countries that would benefit from such a liberalisation decided to participate? Could they still achieve a higher level of welfare?

Assume again that a subset of $p$ countries with $P$ firms decide to liberalise trade unilaterally, but this time trade liberalisation is a flat tariff rate cut.

Lemma 5. The more countries with more firms are involved in unilateral formula-based trade liberalisation, the larger the increase in welfare of these countries. I.e.

$$
\frac{d}{d P}\left(\frac{d W(\alpha \tau)}{d \alpha}\right)<0
$$

Proof. See Appendix page 54.
The derivative of welfare with respect to the tariff is also in this case a decreasing function of the number of firms involved in the trade liberalisation $P$. There exists a minimum number of firms necessary for the trade liberalisation to be welfare increasing. The following proposition establishes the minimum participation constraint so that a marginal formulabased trade liberalisation is welfare increasing for Country $i$ that has $k_{i}$ firms. ${ }^{14}$

[^11]Proposition 11. A marginal unilateral radial tariff cut is welfare increasing for a liberalising country with $k_{i}$ firms if and only if countries with at least $\bar{P}_{f}$ firms participate, where

$$
\begin{equation*}
\bar{P}_{f}=\frac{\Gamma\left(2 k_{i}\right)\left[D\left(k_{i}\right) A\left(k_{i}\right)-\alpha \Gamma\left(2 k_{i}\right) B\left(k_{i}\right)\right]}{2 k_{i} D\left(k_{i}\right)^{2}\left[\bar{\Xi}_{p}-\alpha \bar{\Xi}_{p}^{2}-\alpha \operatorname{Var}_{p} \Xi\right]} \tag{41}
\end{equation*}
$$

with $\Xi\left(k_{i}\right)=\frac{\Gamma\left(k_{i}\right) \Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)}, \bar{\Xi}_{p}=\frac{1}{P} \sum_{l=1}^{p} \Xi\left(k_{l}\right)$ and $\operatorname{Var}_{p}(\Xi)=\frac{1}{P} \sum_{l=1}^{p} \Xi\left(k_{l}\right)^{2}-\bar{\Xi}_{p}^{2}$.
Proof. See Appendix page 55.
Proposition 11 establishes an equivalent result to Proposition 10. In the case of a radial trade liberalisation we can note that the minimum participation constraint is an increasing function of the variance of the number of firms per country within the subset of liberalising countries.

The following corollaries establish the minimum participation constraint in the same particular cases as discussed in the previous subsection.

Corollary 5. Starting from Nash optimal tariffs, i.e. $\alpha=1$, a marginal unilateral formulabased trade liberalisation is welfare increasing for a liberalising country with $k$ firms if and only if at least $\bar{P}_{f}(1, k, N, \gamma)$ firms participate, where

$$
\bar{P}_{f}(1, k, N, \gamma)=\frac{\Gamma\left(2 k_{i}\right)\left[D\left(k_{i}\right) A\left(k_{i}\right)-\Gamma\left(2 k_{i}\right) B\left(k_{i}\right)\right]}{2 k_{i} D\left(k_{i}\right)^{2}\left[\bar{\Xi}_{p}-\bar{\Xi}_{p}^{2}-\operatorname{Var}_{p} \Xi\right]}
$$

Corollary 6. Unilateral free trade is welfare maximising for a participating country with $k_{i}$ firms if and only if the subgroup of trade liberalising countries has more than $\bar{P}_{f_{F T}}$ firms, where

$$
\begin{equation*}
\bar{P}_{f_{F T}}=\frac{\Gamma\left(2 k_{i}\right)\left[N \Gamma\left(2 k_{i}\right)+k_{i} \Gamma(0)\right]}{2 k_{i} D\left(k_{i}\right) \bar{\Xi}_{p}} \tag{42}
\end{equation*}
$$

$\bar{P}_{f_{F T}}$ is an increasing function of the total number of firms in the world $N$, but it is a decreasing function of the number of domestic firms $k_{i}$.

In a perfectly symmetric world where each country has only one firm, the minimum condition becomes a condition on the minimum number of countries involved.

Corollary 7. In a perfectly symmetric world, unilateral free trade is welfare maximising for liberalising countries if and only if more than $\bar{p}_{\text {SymFT }}$ countries participate, where

$$
\begin{equation*}
\bar{p}_{f_{S y m F T}}=\frac{2 N+\gamma(N-1)+2}{4} \tag{43}
\end{equation*}
$$

Note that in the symmetric case, the minimum participation constraint in the reciprocitybased case and in the formula-based case is exactly the same. This is logic as in the symmetric case, the two liberalisations are perfectly equivalent.

Corollary 8. When $\gamma=0$, a marginal unilateral tariff cut from $\alpha \tau_{e}(k)(\alpha \in[0,1])$ is welfare increasing for a participating country with $k$ firms if and only if at least $\bar{p}_{f}$ countries participate, where

$$
\begin{equation*}
\bar{p}_{f_{\gamma=0}}=\frac{3(1-\alpha) N+3(1+\alpha) k}{2(3-\alpha) k} \tag{44}
\end{equation*}
$$

### 6.3 Comparing the reciprocity-based and formula-based trade liberalisation

How do the minimum participation constraints established for the two different ways of trade liberalisation compare? Due to the complexity of the expressions for $\bar{P}_{f}$ and $\bar{P}_{r}$, it is hard to compare them directly. They are both increasing functions of the total number of firms in the world $N$ : in both cases, a certain proportion of world production in necessary. The interesting difference between the two constraints is that $\bar{P}_{r}$ is independent of the distribution of firms among countries whereas $\bar{P}_{f}$ is an increasing function of the variance of the number of firms per country within the subset of liberalising countries. This difference is due to the fact that, in the reciprocity-based case, the actual tariff cut will depend on the distribution of firms between countries, whereas in the formula-based case, the tariff cut is independent of the structure of the markets. Therefore trade liberalisation will affect all countries in the same way in the reciprocity-based case, but in the formula-based case, the effect of trade liberalisation will depend on how many firms countries have.

In the free trade case, it is interesting to note that $\bar{P}_{r_{F T}}$ is an increasing function of the number of domestic firms $k$ whereas $\bar{P}_{f_{F T}}$ is a decreasing function of $k$. In the reciprocitybased case, the more firms a country has, the more is has to liberalise and so the larger compensatory market access it has to gain to benefit from trade liberalisation. In the flat rate trade liberalisation, the tariff cut is independent of the number of firms, all involved countries cut by the same proportion and so the more firms a country has the less is the home market affected and the more the country gains through exports.

When $\gamma=0$, the minimum constraint in the case of reciprocity-based trade liberalisation $\left(\bar{P}_{r_{\gamma=0}}\right)$ and the minimum constraint in the case of formula-based trade liberalisation ( $\bar{p}_{f_{\gamma=0}}$ ) are not completely comparable quantities for homogeneity reasons: $\bar{P}_{r_{\gamma=0}}$ is a minimum
number of firms whereas $\bar{p}_{f_{\gamma=0}}$ is a minimum number of countries and we have $\bar{p}_{f_{\gamma=0}}=\frac{\bar{P}_{\gamma}=0}{k}$. Why do we have a condition on the minimum number of firms in the first case and a condition on the minimum number of countries in the second? Note that when $\gamma=0$, export sales do not depend on the number of firms located in the foreign country. The difference comes from the different kinds of trade liberalisation considered. In the formula-based liberalisation, the tariff cut is given by a formula (an agreed percentage cut of the initial tariff) and so does not depend on the number of firms located in the given country. On the other hand, the tariff cut in the reciprocal liberalisation has to satisfy the reciprocity (and non-discrimination) condition and so depends on the number of firms of the liberalising country.
$\bar{P}_{r_{\gamma=0}}$ and $\bar{p}_{f_{\gamma=0}}$ vary in the same way with $\gamma$ and $N$, but because of the different kinds of trade liberalisation, they do not vary in the same way with $k$. We have seen that $\bar{P}_{r_{\gamma=0}}$ is an increasing function of the domestic number of firms $k$, because the higher $k$ the more the country has to liberalise according to the reciprocity condition. On the other hand, $\bar{p}_{f_{\gamma=0}}$ is a decreasing function of $k$. In the formula-based liberalisation, the tariff cut is independent of the number of domestic firms and so the more a country exports the more it benefits from trade liberalisation and so the smaller the minimum number of other participants needs to be.

When $\gamma>0$, both minimum participation conditions are conditions on the number of firms involved, but again these conditions do not vary in the same way with $k$. In the reciprocity-based trade liberalisation, the minimum number of firms necessary is an increasing function of $k$ whereas in the formula-based trade liberalisation, the minimum number of firms necessary is a decreasing function of $k$. This comes from the differences in the two liberalisations discussed above for the case $\gamma=0$. Furthermore, for the same reason, in the formula-based case, the minimum number of firms is an increasing function of the variance of the number of firms per country among the countries involved. In the reciprocity case, this variance does not matter.

### 6.4 Explaining success and failure of multilateralism

The minimum participation result explains an interesting feature of multilateral negotiations: the initial success of GATT. In its beginning, only a subgroup of developed countries was liberalising trade multilaterally on everyone else. This worked, because these participating countries represented a sufficient part of world trade.

This result also illustrates the motivations behind participation constraints in various international agreements. In trade negotiations, participation constraint has for example been required in the Information Technology Agreement (ITA): "Participants will implement the actions foreseen in the Declaration provided that participants representing approximately 90 per cent of world trade in information technology products have by then notified their acceptance, and provided that the staging has been agreed to the participants satisfaction." ${ }^{15}$

The minimum participation constraint also provides a possible explanation why the multilateral system is currently facing difficulties. In its beginning, when only a subgroup of countries were participating, trade liberalisation was feasible, because this subgroup of countries represented a sufficient part of world trade. But with the recent emergence of developing countries like Brazil, China and India in world trade, this situation has changed and it is not sufficient anymore for only the subgroup of developed countries to participate. All countries are now asked to make concessions in the Doha Round and it is more difficult to reach a unanimous agreement.

## 7 Conclusion

This paper analysed trade negotiations in an oligopolistic framework and identified a new rationale for trade agreements. Countries charge inefficiently high tariffs partially because of a terms-of-trade externality identified by Bagwell and Staiger (1999), but also because a market-access externality: in the absence of domestic competition policy, governments use trade policy to correct the oligopolistic inefficiency, i.e. they charge tariffs on imports to increase domestic production. This paper showed how trade agreements allow countries to overcome these externalities and reach a superior cooperative equilibrium by ensuring that countries exchange balanced profit concessions.

Furthermore, this rich but tractable model enabled me to analyse features of the multilateral trade negotiations that have not yet been addressed in the literature. I have studied formula-based trade liberalisation and derived a new reason for why less developed countries should liberalise less. My model also proposed a possible explanation for why GATT/WTO negotiations were successful in the past but are currently struggling, through a minimum participation constraint necessary to make trade liberalisation welfare increasing.

This paper makes a step forward in the study of trade negotiations in that it suggests a

[^12]model with new contact points with reality in which new questions can be addressed, but further work is needed in this direction to improve our understanding of this topic. One of the current pressing trade policy questions is: How should trade negotiations be designed to make them work? The minimum participation constraint established in this paper might be the way forward. The literature on international environmental treaties ${ }^{16}$ has established using non-cooperative coalition formation techniques that in the presence of uncertainty over net benefits of action, an agreement requiring only sufficient, but not necessarily full, participation might be preferable. This feature needs to be further explored in the context of international trade.

[^13]
## Appendix

## A Welfare decomposition

$$
\begin{align*}
W^{i} & =C S^{i}+\sum_{j \in K_{i}} \pi^{i j}+T R^{i}+\sum_{\substack{l=1 \\
l \neq i}}^{n} \sum_{j \in K_{i}} \pi^{l j}  \tag{45}\\
& =\sum_{j=1}^{N} \frac{1}{2}\left(a-p_{i j}\right) q_{i j}+\sum_{j \in K_{i}}\left(p_{i j}-c\right) q_{i j}+\sum_{\substack{j \notin K_{i}}} \tau_{i j} q_{i j}+\sum_{\substack{l=1 \\
l \neq i}}^{n} \sum_{j \in K_{i}}\left(p_{l j}-c-\tau_{l j}\right) q_{l j}
\end{align*}
$$

Country $i$ 's tariffs do not affect production or consumption decisions in other countries, so

$$
\begin{align*}
\frac{d W^{i}}{d \tau_{i p}}= & \sum_{j=1}^{N}-\frac{1}{2} \frac{d p_{i j}}{d \tau_{i p}} q_{i j}+\sum_{j=1}^{N} \frac{1}{2}\left(a-p_{i j}\right) \frac{d q_{i j}}{d \tau_{i p}} \\
& +\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i p}}+\sum_{j \in K_{i}} \frac{d p_{i j}}{d \tau_{i p}} q_{i j} \\
& +\sum_{j \notin K_{i}} \frac{d \tau_{i j}}{d \tau_{i p}} q_{i j}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i p}} \tag{46}
\end{align*}
$$

where the first line is the derivative of consumer surplus, the second is the derivative of producer surplus and the last line is the derivative of tariff revenue.

Country $i$ 's inverse demand function for firm $j$ 's good is

$$
\begin{equation*}
p_{i j}=a-q_{i j}-\gamma \sum_{\substack{k=1 \\ k \neq j}}^{N} q_{i k} \tag{47}
\end{equation*}
$$

Differentiating (47) gives

$$
\begin{equation*}
\frac{d p_{i j}}{d \tau_{i p}}=-\left(\frac{d q_{i j}}{d \tau_{i p}}+\gamma \sum_{\substack{k=1 \\ k \neq j}}^{N} \frac{d q_{i k}}{d \tau_{i p}}\right) \tag{48}
\end{equation*}
$$

Substituting (47) and (48) in (46) yields

$$
\begin{align*}
\frac{d W^{i}}{d \tau_{i p}}= & \sum_{j=1}^{N} \frac{1}{2}\left(\frac{d q_{i j}}{d \tau_{i p}}+\gamma \sum_{\substack{k=1 \\
k \neq j}}^{N} \frac{d q_{i k}}{d \tau_{i p}}\right) q_{i j}+\sum_{j=1}^{N} \frac{1}{2}\left(q_{i j}+\gamma \sum_{\substack{k=1 \\
k \neq j}}^{N} q_{i k}\right) \frac{d q_{i j}}{d \tau_{i p}} \\
& +\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i p}}+\sum_{j \in K_{i}}-\left(\frac{d q_{i j}}{d \tau_{i p}}+\gamma \sum_{\substack{k=1 \\
k \neq j}}^{N} \frac{d q_{i k}}{d \tau_{i p}}\right) q_{i j} \\
& +\sum_{j \notin K_{i}} \frac{d \tau_{i j}}{d \tau_{i p}} q_{i j}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i p}} \tag{49}
\end{align*}
$$

Terms from the first line and second term of the second line combine or cancel (pure transfers) to give

$$
\begin{align*}
\frac{d W^{i}}{d \tau_{i p}}= & \sum_{j \notin K_{i}} \frac{d q_{i j}}{d \tau_{i p}} q_{i j}+\frac{\gamma}{2} \sum_{j=1}^{N} q_{i j} \sum_{\substack{k=1 \\
k \neq j}}^{N} \frac{d q_{i k}}{d \tau_{i p}}+\frac{\gamma}{2} \sum_{j=1}^{N} \frac{d q_{i j}}{d \tau_{i p}} \sum_{\substack{k=1 \\
k \neq j}}^{N} q_{i k}-\gamma \sum_{j \in K_{i}} q_{i j} \sum_{\substack{k=1 \\
k \neq j}}^{N} \frac{d q_{i k}}{d \tau_{i p}} \\
& +\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i p}}+\sum_{j \notin K_{i}} \frac{d \tau_{i j}}{d \tau_{i p}} q_{i j}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i p}} \tag{50}
\end{align*}
$$

Now noting that

$$
\begin{gather*}
\sum_{j=1}^{N} q_{i j} \sum_{\substack{k=1 \\
k \neq j}}^{N} \frac{d q_{i k}}{d \tau_{i p}}=\sum_{j=1}^{N} \frac{d q_{i j}}{d \tau_{i p}} \sum_{\substack{k=1 \\
k \neq j}}^{N} q_{i k}  \tag{51}\\
\frac{d W^{i}}{d \tau_{i p}}=\sum_{j \notin K_{i}} \frac{d q_{i j}}{d \tau_{i p}} q_{i j}+\gamma \sum_{\substack{j \notin K_{i}}} q_{i j} \sum_{\substack{k=1 \\
k \neq j}}^{N} \frac{d q_{i k}}{d \tau_{i p}} \\
\\
+\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i p}}+\sum_{j \notin K_{i}} \frac{d \tau_{i j}}{d \tau_{i p}} q_{i j}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i p}} \\
=\sum_{j \notin K_{i}} q_{i j}\left(\frac{d q_{i j}}{d \tau_{i p}}+\gamma \sum_{\substack{k=1 \\
k \neq j}}^{N} \frac{d q_{i k}}{d \tau_{i p}}\right) \\
 \tag{52}\\
\quad+\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i p}}+\sum_{j \notin K_{i}} \frac{d \tau_{i j}}{d \tau_{i p}} q_{i j}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i p}} \\
=- \\
\quad-\sum_{j \notin K_{i}} q_{i j} \frac{d p_{i j}}{d \tau_{i p}}+\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i p}}+\sum_{j \notin K_{i}} \frac{d \tau_{i j}}{d \tau_{i p}} q_{i j}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i p}}
\end{gather*}
$$

which can be further simplified using mill prices

$$
\begin{equation*}
p_{i j}=p_{j}^{*}+\tau_{i j} \tag{53}
\end{equation*}
$$

to obtain

$$
\begin{align*}
\frac{d W^{i}}{d \tau_{i p}} & =-\sum_{j \notin K_{i}} q_{i j} \frac{d p_{j}^{*}}{d \tau_{i p}}-\sum_{j \notin K_{i}} q_{i j} \frac{d \tau_{i j}}{d \tau_{i p}}+\sum_{j \in K_{i}}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i p}}+\sum_{j \notin K_{i}} \frac{d \tau_{i j}}{d \tau_{i p}} q_{i j}+\sum_{j \notin K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i p}} \\
& =-\sum_{j \notin K_{i}} q_{i j} \frac{d p_{j}^{*}}{d \tau_{i p}}+\sum_{\text {ToT }}\left(p_{i j}-c\right) \frac{d q_{i j}}{d \tau_{i p}}+\sum_{j \neq K_{i}} \tau_{i j} \frac{d q_{i j}}{d \tau_{i p}} \tag{54}
\end{align*}
$$

## B Discussion of Nash tariff determinants

How does the optimal tariff vary with $k, N$ and $\gamma$ ? First, we can note that $d \tau_{e}(k ; \gamma, N) / d N<$ 0 . When the number of domestic firms is constant, the more firms there are in the world, the greater the proportion of goods produced abroad, the lower the monopoly power of the home country and the greater the loss to consumers from a tariff (consumers prefer variety). Thus the tariff is lower.
$\tau_{e}(k ; \gamma, N)$ varies non-monotonically with $\gamma$. These variations depend also on the parameters $k$ and $N$. For low values of $\gamma$ and any values of $k$ and $N$, the tariff is a decreasing function of $\gamma$. As mentioned above, there are two gains from trade in this model: increased variety and reduced monopoly power of the domestic firms. When $\gamma$ is low, firms do not compete with each other and the second gain from trade is negligible. It is therefore better for the country to charge higher tariff for lower values of $\gamma$ to get high tariff revenue. For higher values of $\gamma$ and low values of $k$ with respect to $N$, the tariff is an increasing function of $\gamma$. The higher is $\gamma$, the more firms compete with each other. If a country has only few domestic firms, these firms operate in a rather monopolistic market and they suffer more from a lower tariff. And so it is better for the country to charge higher tariffs for higher values of $\gamma$.

Finally, how does the tariff vary with $k$, i.e. what happens to the tariff when the proportion of firms located in the home country changes? The answer is ambiguous. The variations of the tariff with $k$ are studied in detail in Mrázová et al. (2008). For the completeness of the analysis, let us just note here that $\tau_{e}(k ; \gamma, N)$ increases with $k$ if and only if

$$
\begin{align*}
k<k^{*}=k^{*}(N, \gamma) & =\frac{\sqrt{\Gamma(0)[\Gamma(0)+1] \Gamma(N)}-\Gamma(0)}{2 \gamma} \\
& =\frac{\sqrt{(2-\gamma)(3-\gamma)(2+(N-1) \gamma)}-(2-\gamma)}{2 \gamma} \tag{55}
\end{align*}
$$

The non-monotonicity of $\tau_{e}(k ; \gamma, N)$ results from two conflicting effects on the optimal tariff of a change in the number of domestic firms.

1. Consumers prefer variety so it is beneficial to keep consumption balanced. If there are less firms at home, the country wants to charge a low tariff to assure a balanced consumption. If a bigger proportion of the products available on the world market is produced domestically, i.e. more firms in the home country, the negative effect on the consumer surplus is not so harmful, so the country wants to charge higher tariffs. This effect tends to increase the tariff as the number of domestic firms increases.
2. Governments care about domestic firms' profits. If there are only few firms in the Home country, their profits will decrease significantly if they face competition from the many foreign firms. On the other hand if there is already a lot of competition in the domestic market, additional competition from foreign firms does not change much. This effect tends to decrease the tariff as the number of domestic firms increases.

If goods are homogeneous, the competition effect is more important, because firms suffer more from competition when goods are highly substitutable. So a country with few firms will charge a higher tariff than a country with many firms. If goods are independent, competition does not harm firms as much. The consumers' effect is more important. So a country with few firms will charge a lower tariff than a country with many firms.

In conclusion, a country with fewer firms can charge both higher or lower tariff than a country with many firms depending on the parameters of the model $\gamma$ and $N$. For $\gamma=0$, the tariff is independent of $k$ and all countries charge the same tariff $\tau_{e}(k ; 0, N)=1 / 3$. For very small $\gamma$ and any $N, \tau$ is an increasing function of $k$. For the majority of values of the parameters, $\tau$ is an increasing function of $k$ for small $k$ and a decreasing function of $k$ for larger $k$. For small $N$ and very hight $\gamma, \tau$ can be a monotonically decreasing function of $k$, but this case is rare. The variations of $\tau$ are illustrated in Figure 1.

## C Proofs

Proof of Proposition 1. This proof proceeds exactly in the same way as Yi (1996)'s derivation of the optimum external tariff of a customs union.

Without loss of generality, consider Country 1 and assume that firms $1,2, \ldots, k_{1}$ are located in Country 1. It will choose its external tariffs to maximise its welfare

$$
\max _{\left\{\tau_{1 j}\right\}_{j>k_{1}}} W^{1}
$$

with

$$
W^{1}=u\left(\mathbf{q}_{1}\right)-c Q_{1}-\sum_{j=k_{1}+1}^{N} \pi^{1 j}+\sum_{l=2}^{n} \sum_{j \in K_{1}} \pi^{l j}
$$

As Country 1's tariffs do not affect production and consumption decisions in other countries, domestic firms' profits in foreign countries are independent of Country 1's tariff choices. So with the normalisation $a-c=1$, the maximisation problem becomes

$$
\max _{\left\{\tau_{1 j}\right\}_{j>k_{1}}}\left\{Q_{1}-\frac{\gamma}{2} Q_{1}^{2}-\frac{1-\gamma}{2} \sum_{j=1}^{N} q_{1 j}^{2}-\sum_{j=k_{1}+1}^{N} q_{1 j}^{2}\right\}
$$

Drop subscript 1. The first order condition with respect to $\tau_{p}, p=k+1, \ldots, N$, is

$$
\begin{equation*}
(1-\gamma) Q \frac{d Q}{d \tau_{p}}-(\Gamma(0)-1) \sum_{j=1}^{N} q_{j} \frac{d q_{j}}{d \tau_{p}}-2 \sum_{j=k+1}^{N} q_{j} \frac{d q_{j}}{d \tau_{p}}=0 \tag{56}
\end{equation*}
$$

Since the objective function is globally concave in $\tau_{p}$ 's, there exists a unique symmetric solution, denoted by $\tau(k)$. Substituting (8), (9) and (10) into (56), denoting a non-member country's sales by $q_{O}(k)$, total sales by $Q(k)$, and multiplying by $\Gamma(0) \Gamma(N)$,

$$
\begin{equation*}
-\Gamma(0)+\gamma Q(k)+[\Gamma(N)(\Gamma(0)+1)-2(N-k) \gamma] q_{O}(k)=0 \tag{57}
\end{equation*}
$$

Substituting (6) and (7) into (57) and rearranging the terms yields the formula.

## Proof of Proposition 2. ADD PROOF HERE.

Proof of Lemma 1. The welfare of Country $i$ with $k_{i}$ firms is given by

$$
W^{i}=C S^{i}+P S^{i}+T R^{i}+\sum_{\substack{l=1 \\ l \neq i}}^{n} \sum_{j \in K_{i}} \pi^{l j}
$$

where $P S^{i}$ is the home firms profit in the domestic market

$$
P S^{i}=\sum_{j \in K_{i}} \pi^{i j}
$$

By definition

$$
C S^{i j}=\frac{1}{2}\left(a-p_{i j}\right) q_{i j}=\frac{1-\gamma}{2} q_{i j}^{2}+\frac{\gamma}{2} Q_{i} q_{i j}
$$

so

$$
\begin{equation*}
C S^{i}=\sum_{j=1}^{N} C S^{i j}=\frac{\gamma}{2} Q_{i}^{2}+\frac{1-\gamma}{2} \sum_{j=1}^{N} q_{i j}^{2} \tag{58}
\end{equation*}
$$

On the other hand,

$$
\begin{aligned}
u\left(\mathbf{q}_{i}\right)-c Q_{i}-\sum_{j \notin K_{i}} \pi^{i j}= & a Q_{i}-\frac{\gamma}{2} Q_{i}^{2}-\frac{1-\gamma}{2} \sum_{j=1}^{N} q_{i j}^{2}-c Q_{i}-\sum_{j=1}^{N} \pi^{i j}+\sum_{j \in K_{i}} \pi^{i j} \\
= & a Q_{i}-\frac{\gamma}{2} Q_{i}^{2}-\frac{1-\gamma}{2} \sum_{j=1}^{N} q_{i j}^{2}-c Q_{i}-\sum_{j=1}^{N}\left(p_{i j}-c-t_{i j}\right) q_{i j}+P S^{i} \\
= & a Q_{i}-\frac{\gamma}{2} Q_{i}^{2}-\frac{1-\gamma}{2} \sum_{j=1}^{N} q_{i j}^{2}-\sum_{j=1}^{N}\left[a-(1-\gamma) q_{i j}-\gamma Q_{i}\right] q_{i j} \\
& +T R^{i}+P S^{i} \\
= & \frac{\gamma}{2} Q_{i}^{2}+\frac{1-\gamma}{2} \sum_{j=1}^{N} q_{i j}^{2}+T R^{i}+P S^{i} \\
= & C S^{i}+T R^{i}+P S^{i}
\end{aligned}
$$

The result follows.
Proof of Proposition 4. Consider a bilaterally reciprocal trade liberalisation between Country $i$ and Country $l$. In what follows, $\tau\left(k_{i}\right)$ (or simply $\tau$ ) will denote the tariff charged by Country $i$ on imports from Country $l$ and $\bar{\tau}\left(k_{i}\right)$ (or simply $\bar{\tau}$ ) will denote the tariff charged by Country $i$ on imports from all other countries. In the bilateral trade liberalisation, tariffs $\bar{\tau}$ will remain constant (trade on other countries is not being liberalised). I will now determine the impact of a variation in $\tau$ and the tariffs charged by Country $l$ on imports from Country $i\left(\tau\left(k_{i}\right)\right)$ such that they satisfy the condition of reciprocity on welfare of Country $i$.

For Country $i$ we have

$$
\begin{equation*}
Q\left(k_{i}\right)=\frac{N-\left(N-k_{i}-k_{l}\right) \bar{\tau}\left(k_{i}\right)-k_{l} \tau\left(k_{i}\right)}{\Gamma(N)} \tag{59}
\end{equation*}
$$

and

$$
\begin{equation*}
q_{I}\left(k_{i}\right)=\frac{\Gamma(0)+\gamma\left(N-k_{i}-k_{l}\right) \bar{\tau}\left(k_{i}\right)+\gamma k_{l} \tau\left(k_{i}\right)}{\Gamma(0) \Gamma(N)} \tag{60}
\end{equation*}
$$

Exports to Country $i$ from firms based in Country $l$ are

$$
\begin{equation*}
q_{O_{l}}\left(k_{i}\right)=\frac{\Gamma(0)+\gamma\left(N-k_{i}-k_{l}\right) \bar{\tau}\left(k_{i}\right)-\Gamma\left(N-k_{l}\right) \tau\left(k_{i}\right)}{\Gamma(0) \Gamma(N)} \tag{61}
\end{equation*}
$$

and exports to Country $i$ from firms based in other countries are

$$
\begin{equation*}
q_{O_{-l}}\left(k_{i}\right)=\frac{\Gamma(0)+\Gamma\left(k_{i}+k_{l}\right) \bar{\tau}\left(k_{i}\right)-\gamma k_{l} \tau\left(k_{i}\right)}{\Gamma(0) \Gamma(N)} \tag{62}
\end{equation*}
$$

The welfare of Country $i$ is

$$
\begin{align*}
W\left(k_{i}\right)= & Q\left(k_{i}\right)-\frac{\gamma}{2} Q\left(k_{i}\right)^{2} \\
& -\frac{1-\gamma}{2}\left\{k_{i}\left[q_{I}\left(k_{i}\right)\right]^{2}+\left(N-k_{i}-k_{l}\right)\left[q_{O_{-l}}\left(k_{i}\right)\right]^{2}+k_{l}\left[q_{O_{l}}\left(k_{i}\right)\right]^{2}\right\} \\
& -\left(N-k_{i}-k_{l}\right)\left[q_{O_{-l}}\left(k_{i}\right)\right]^{2}-k_{l}\left[q_{O_{l}}\left(k_{i}\right)\right]^{2}+k_{i}\left[q_{O}\left(k_{l}\right)^{2}+\sum_{\substack{p=1 \\
p \neq i \\
p \neq l}}^{n} q_{O}\left(k_{p}\right)^{2}\right] \tag{63}
\end{align*}
$$

Other countries than $i$ and $l$ keep their tariffs constant. Furthermore, the variation of tariffs charged by Country $i$ on imports from Country $l$ and of tariffs charged by Country $l$ on imports from Country $i$ is bilaterally reciprocal, so we have

$$
\begin{align*}
\frac{d W}{d \tau}\left(k_{i}\right)= & \frac{d Q}{d \tau}\left(k_{i}\right)-\gamma \frac{d Q}{d \tau}\left(k_{i}\right) Q\left(k_{i}\right) \\
& -(1-\gamma)\left[k_{i} \frac{d q_{I}}{d \tau}\left(k_{i}\right) q_{I}\left(k_{i}\right)+\left(N-k_{i}-k_{l}\right) \frac{d q_{O_{-l}}}{d \tau}\left(k_{i}\right) q_{O_{-l}}\left(k_{i}\right)+k_{l} \frac{d q_{O_{l}}}{d \tau}\left(k_{i}\right) q_{O_{l}}\left(k_{i}\right)\right] \\
& -2\left(N-k_{i}-k_{l}\right) \frac{d q_{O_{-l}}}{d \tau}\left(k_{i}\right) q_{O_{-l}}\left(k_{i}\right) \tag{64}
\end{align*}
$$

$$
\begin{equation*}
\frac{d W}{d \tau}\left(k_{i}\right)=\frac{k_{l}}{\Gamma(0)^{2} \Gamma(N)^{2}}[A+B \tau] \tag{65}
\end{equation*}
$$

with

$$
\begin{equation*}
A=\Gamma(0)^{2}\left[2 \gamma\left(N-k_{i}-k_{l}\right)-1\right]+\gamma\left(N-k_{i}-k_{l}\right)\left[4+\gamma\left(N(1-\gamma)+2\left(k_{i}+k_{l}\right)+\gamma-4\right)\right] \bar{\tau} \tag{66}
\end{equation*}
$$

and

$$
\begin{equation*}
B=(-1+\gamma) \Gamma(N)^{2}-\gamma^{2} k_{l}\left[N+(N-1) \gamma-2\left(k_{i}+k_{l}-1\right)\right] \tag{67}
\end{equation*}
$$

So $d W / d \tau$ is linear in $\tau$ and to determine its sign, I need to determine the sign of $A$ and $B$.

Sign of $B$ : the first term of $B$ is negative, the sign of the second term could be both negative or positive depending on $k_{i}+k_{l}$. To determine the sign of $B$, I will successively differentiate $B$ with respect to $\gamma$.

$$
\frac{d^{3} B}{d \gamma^{3}}=6(N-1)\left(N-k_{l}-1\right)>0
$$

So $d^{2} B / d \gamma^{2}$ is an increasing function of $\gamma$. Furthermore, $d^{2} B / d \gamma^{2}(1)=2\left[2\left(N-k_{l}\right)^{2}+k_{l}+2 k_{i} k_{l}-2\right]>0$ and $d^{2} B / d \gamma^{2}(0)=-2\left[N^{2}+N\left(k_{l}-6\right)+5-2 k_{l}\left(k_{i}+k_{l}-1\right)\right]$ which can be both positive or negative depending on $k_{i}$ and $k_{l}$. So $d B / d \gamma$, for $\gamma$ between 0 and 1 , can initially be increasing or decreasing and later becomes an increasing function. $d B / d \gamma(0)=-4 N+8<0$ and $d B / d \gamma(1)=(N+1)^{2}+\left[-5 N+4\left(k_{i}+k_{l}\right)-1\right] k_{l}$ which can again be both positive or negative depending on $k_{i}$ and $k_{l}$. So $B$ is an initially decreasing function of $\gamma$ and for $\gamma$ closer to 1 becomes either increasing or decreasing. Now note that $B(0)=-4<0$ and $B(1)=-2 k_{l}\left[N-\left(k_{i}+k_{l}\right)+1 / 2\right]<0$. So $B$ is always negative for any $\gamma$ between 0 and 1 .

Thus $d W / d \tau$ is a monotonically decreasing function of $\tau$ and its sign will depend on $A$.

Sign of A: the sign of the first term of $A$ depends on $\gamma$ and $k_{i}+k_{l}$, the second term is positive so $A$ can be both positive or negative depending on the different parameters.

1. $A$ is negative if $k_{i}+k_{l}$ is sufficiently large (in particular, note that $A=-\Gamma(0)^{2}$ when $k_{i}+k_{l}=N$ ). So, if countries $i$ and $l$ represent a large share of world production (the majority of firms is located in Country $i$ and $l$ ) bilaterally reciprocal trade liberalisation
is welfare increasing. This is because when countries $i$ and $l$ represent a large share of world production, the trade diversion created by the discriminatory trade liberalisation will be limited.
2. $A$ is negative for low values of $\gamma$ (in particular, note that $A(0)=-4$ ). So for low values of $\gamma, d W / d \tau$ is negative for any $\tau$ and bilaterally reciprocal trade liberalisation is also welfare increasing. Intuitively, this is because consumers will be better off through a better access to products from country $l$ and domestic firms will be better off through better access to country l's market. In the home market, they will not face more competition, because for low $\gamma$, goods are independent.
3. $A$ is an increasing function of $\bar{\tau}$, the higher the tariff on the other countries, the larger is the second term of $A$ and the larger is $A$. So if $\bar{\tau}$ is high, bilaterally reciprocal trade liberalisation will be welfare improving only if it starts from a very high tariff level and will become welfare reducing when it reaches lower tariff levels. This illustrates the "concertina rule" for tariff reform.
4. When $\bar{\tau}=0$, the second term of $A$ is zero, nevertheless, $A$ can still be both positive or negative. $A$ will be positive for high values of $\gamma$ and small values of $k_{i}+k_{l}$. So when goods are highly substitutable, and when Country $i$ is in free trade with all other countries of the world except Country $l$, bilateral trade liberalisation will be welfare increasing if starting from a sufficiently high tariff level, but it will not be optimal to go completely until free trade. This is because when Country $i$ is in free trade with the rest of the world and Countries $i$ and $l$ do not represent the majority of world's production, consumers in Country $i$ already have free trade access to the majority of products and liberalisation with Country $i$ would mean more competition for Country $i$ 's firms.

Proof of Proposition 6. From (29), we can see the impact on welfare of country with $k_{i}$ firms of a trade liberalisation satisfying the reciprocity and non-discrimination principles

$$
\begin{aligned}
\frac{d W}{d \tau}\left(k_{i}\right)= & -\frac{\left(N-k_{i}\right)}{\Gamma(N)}+\gamma \frac{\left(N-\left(N-k_{i}\right) \tau\right)}{\Gamma(N)} \frac{\left(N-k_{i}\right)}{\Gamma(N)} \\
& -(1-\gamma)\left[k_{i} \frac{\left(\Gamma(0)+\gamma\left(N-k_{i}\right) \tau\right)}{\Gamma(0) \Gamma(N)} \frac{\gamma\left(N-k_{i}\right)}{\Gamma(0) \Gamma(N)}-\left(N-k_{i}\right) \frac{\Gamma(0)+\Gamma\left(k_{i}\right) \tau}{\Gamma(0) \Gamma(N)} \frac{\Gamma\left(k_{i}\right)}{\Gamma(0) \Gamma(N)}\right]
\end{aligned}
$$

$$
\begin{equation*}
\frac{d W}{d \tau}\left(k_{i}\right)=\frac{N-k_{i}}{(\Gamma(0) \Gamma(N))^{2}}\{A+B \tau\} \tag{68}
\end{equation*}
$$

where $A=-\Gamma(0)^{2}$ and $B=-\Gamma(0)^{2} \gamma\left(N-k_{i}\right)-(1-\gamma) k_{i} \gamma^{2}\left(N-k_{i}\right)-(1-\gamma) \Gamma\left(k_{i}\right)^{2}$.
$A$ is always negative.

$$
B=-4+(8-4 N) \gamma+\left(-5+2 k_{i}+4 N-k_{i} N\right) \gamma^{2}+\left(1-k_{i}-N+k_{i} N\right) \gamma^{3}
$$

Drop subscript $i$.

$$
\begin{aligned}
\frac{d B}{d \gamma} & =8-4 N+2(-5+2 k+4 N-k N) \gamma+3(1-k-N+k N) \gamma^{2} \\
\frac{d^{2} B}{d \gamma^{2}} & =2(-5+2 k+4 N-k N)+6(1-k-N+k N) \gamma \\
\frac{d^{3} B}{d \gamma^{3}} & =6(k-1)(N-1)>0
\end{aligned}
$$

$\frac{d^{3} B}{d \gamma^{3}}>0$ so $\frac{d^{2} B}{d \gamma^{2}}$ is an increasing function of $\gamma \cdot \frac{d^{2} B}{d \gamma^{2}}(1)=2(2 N-1) k+2 N-4>0$ and $\frac{d^{2} B}{d \gamma^{2}}(0)=8 N-2(N-2) k-10$. $\frac{d^{2} B}{d \gamma^{2}}$ might be negative for low values of $\gamma$ (depending on $N$ and $k$ ), is monotonically increasing with $\gamma$ and become positive for higher values of $\gamma$. So $\frac{d B}{d \gamma}$ is either initially decreasing and then an increasing function of $\gamma$ or an increasing function of $\gamma \cdot \frac{d B}{d \gamma}(0)=8-4 N<0$ and $\frac{d B}{d \gamma}(1)=(1+k)(N+1)>0$, so in any case, $B$ is initially decreasing with $\gamma$ and later increasing with $\gamma . B(0)=-4<0$ and $B(1)=k-N<0$, so $B$ is negative for any value of $\gamma$.

Thus the welfare unambiguously increases as all countries liberalise according to the GATT/WTO principles.

Proof of Proposition 7. The idea behind Proposition 6 is clear: consumers prefer variety and they are better off when trade is freer. The proof of this result is however slightly tedious.

From (33) by differentiating with respect to tariff $\tau$, we have for Country $i$

$$
\begin{align*}
\frac{d C S}{d \tau}\left(k_{i}\right)= & -\frac{\left(N-k_{i}\right)}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{\Gamma(0)^{2}(1-\gamma+\gamma N)\right. \\
& \left.-\left[\gamma\left(N-k_{i}\right) \Gamma(0)^{2}+(1-\gamma) k_{i} \gamma^{2}\left(N-k_{i}\right)+\Gamma\left(k_{i}\right)^{2}(1-\gamma)\right] \tau\right\} \\
= & -\frac{\left(N-k_{i}\right)}{\Gamma(0)^{2} \Gamma(N)^{2}}\left(A^{\prime}+B^{\prime} \tau\right) \tag{69}
\end{align*}
$$

with

$$
\begin{aligned}
& A^{\prime}=\Gamma(0)^{2}(1-\gamma+\gamma N)>0 \\
& B^{\prime}=-\left[\gamma\left(N-k_{i}\right) \Gamma(0)^{2}+(1-\gamma) k_{i} \gamma^{2}\left(N-k_{i}\right)+\Gamma\left(k_{i}\right)^{2}(1-\gamma)\right]<0
\end{aligned}
$$

so the derivative of consumer surplus is a linear increasing function of the tariff. I now show that for any tariff smaller or equal to the Nash optimal tariff, $\tau \leq \tau_{e}$, the derivative of consumer surplus is negative.

Set $\tau_{\text {CSmin }}$ the tariff at which the derivative of consumer surplus is zero (and consumer surplus is minimum).

$$
\begin{aligned}
\tau_{C S m i n} & =\frac{\Gamma(0)^{2}(1-\gamma+\gamma N)}{\gamma\left(N-k_{i}\right) \Gamma(0)^{2}+(1-\gamma) k_{i} \gamma^{2}\left(N-k_{i}\right)+\Gamma\left(k_{i}\right)^{2}(1-\gamma)} \\
& =\frac{\Gamma(0)^{2}(1-\gamma+\gamma N)}{D(k)-2 \Gamma(k)^{2}}
\end{aligned}
$$

and

$$
\begin{aligned}
\tau_{C S m i n}-\tau_{e} & =\frac{\Gamma(0)^{2}(1-\gamma+\gamma N)}{D(k)-2 \Gamma(k)^{2}}-\frac{\Gamma(0) \Gamma(2 k)}{D(k)} \\
& =\frac{\Gamma(0)}{D(k)\left[D(k)-2 \Gamma(k)^{2}\right]}\left\{D(k)[\Gamma(0)(1-\gamma+\gamma N)-\Gamma(2 k)]+2 \Gamma(k)^{2} \Gamma(2 k)\right\}
\end{aligned}
$$

$\tau_{C S m i n}-\tau_{e}$ is of the same sign as $N u m=D(k)[\Gamma(0)(1-\gamma+\gamma N)-\Gamma(2 k)]+2 \Gamma(k)^{2} \Gamma(2 k)$.

$$
\frac{d^{2}}{d k^{2}} N u m=4 \gamma^{2} \Gamma(N) \geq 0
$$

and so the derivative of $N u m$ with respect to $k$ is a monotonically increasing function of $k$. Furthermore, at $k=1$, we have

$$
\frac{d}{d k} N u m(1)=\gamma \Gamma(N)\left[\gamma(1-\gamma)(2-\gamma) N+4-2 \gamma+4 \gamma^{2}-\gamma^{3}\right] \geq 0
$$

and so for any number of firms $k, \frac{d}{d k} N u m$ is positive. Hence $N u m$ is a monotonically increasing function of $k$.

Finally, at $k=1$

$$
\operatorname{Num}(1)=8(2+\gamma)+\gamma\left[\gamma(4-3 \gamma) N+12-8 \gamma+3 \gamma^{2}\right][\Gamma(0)(N-1)-2] \geq 0
$$

and so we have $\tau_{C S \text { min }} \geq \tau_{e}$ and the consumer surplus is a decreasing function of the tariff for any tariff between 0 and $\tau_{e}$.

## Proof of Proposition 8.

$$
\frac{d \Pi}{d \tau}\left(k_{i}\right)=2 k_{i} \frac{d q_{I}\left(k_{i}\right)}{d \tau} q_{I}\left(k_{i}\right)+2 k_{i} \sum_{\substack{l=1 \\ l \neq i}}^{n} \frac{d q_{O}\left(k_{l}\right)}{d \tau} q_{O}\left(k_{l}\right)
$$

The trade liberalisation considered satisfies the principles of reciprocity and non-discrimination and so we have

$$
k_{i} \sum_{l \neq i}^{n} 2 q_{O}\left(k_{l}\right) d q_{O}\left(k_{l}\right)=\left(N-k_{i}\right) 2 q_{O}\left(k_{i}\right) d q_{O}\left(k_{i}\right)
$$

and so

$$
\begin{aligned}
\frac{d \Pi}{d \tau}\left(k_{i}\right) & =2 k_{i} \frac{d q_{I}\left(k_{i}\right)}{d \tau} q_{I}\left(k_{i}\right)+2\left(N-k_{i}\right) \frac{d q_{O}\left(k_{i}\right)}{d \tau} q_{O}\left(k_{i}\right) \\
& =\frac{2\left(N-k_{i}\right)}{\Gamma(0)^{2} \Gamma(N)^{2}}\left[-\Gamma(0)^{2}+\left(k \gamma^{2}\left(N-k_{i}\right)+\Gamma\left(k_{i}\right)^{2}\right) \tau\right] \\
& =A+B \tau
\end{aligned}
$$

with $A<0$ and $B>0$ so $d \Pi / d \tau$ changes sign with $\tau$. $d \Pi / d \tau=0$ for

$$
\tau_{P S m i n}=\frac{\Gamma(0)^{2}}{k \gamma^{2}\left(N-k_{i}\right)+\Gamma\left(k_{i}\right)^{2}}
$$

So profit is a decreasing function of tariff for $\tau \leq \tau_{P S \min }$ and it is an increasing function of tariff for $\tau \geq \tau_{\text {PSmin }}$.

$$
\frac{d^{2} \Pi}{d \tau^{2}}=\frac{2\left(N-k_{i}\right)\left[k_{i}\left(N-k_{i}\right) \gamma^{2}+\Gamma\left(k_{i}\right)^{2}\right]}{\Gamma(0)^{2} \Gamma(N)^{2}}>0
$$

shows that profit function has a minimum at $\tau_{P S \text { min }}$.
So to determine what impact will a change in tariff have on profit, we have to know in which region is the tariff varying. In particular, we want to know what would be the impact
on profit if all countries started to lower their external tariffs from $\tau_{e}$. So we want to compare $\tau_{e}$ and $\tau_{P S m i n}$, and we have

$$
\begin{equation*}
\tau_{\text {PSmin }}>\tau_{e} \Leftrightarrow \gamma<2+k-\sqrt{k^{2}+4 k} \tag{70}
\end{equation*}
$$

which ends the proof.
Proof of Proposition 9. Consider Country $i$ in the set of countries $C=\left(k_{1}, k_{2}, \ldots, k_{n}\right)$. Differentiate $W\left(k_{i}, \alpha, C\right)$ with respect to $\alpha$ :

$$
\begin{aligned}
& \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}= \frac{d Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha}-\gamma \frac{d Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right) \\
&-(1-\gamma)\left[k_{i} \frac{d q_{I}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{I}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)+\left(N-k_{i}\right) \frac{d q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)\right] \\
&-2\left(N-k_{i}\right) \frac{d q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)+2 k_{i} \sum_{\substack{l=1 \\
l \neq i}}^{n} \frac{d q_{O}\left(k_{l}, \alpha \tau\left(k_{l}\right)\right)}{d \alpha} q_{O}\left(k_{l}, \alpha \tau\left(k_{l}\right)\right) \\
& \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}=\Lambda+\Upsilon
\end{aligned}
$$

with

$$
\Lambda=-\frac{\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{\Gamma(0)^{2}+\alpha \tau\left(k_{i}\right)\left[\gamma \Gamma(0)^{2}\left(N-k_{i}\right)+(1-\gamma) k_{i} \gamma^{2}\left(N-k_{i}\right)+(1-\gamma) \Gamma\left(k_{i}\right)^{2}\right]\right\}<0
$$

and

$$
\Upsilon=\frac{2}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{\left(N-k_{i}\right) \Gamma\left(k_{i}\right) \tau\left(k_{i}\right)\left[\Gamma(0)-\alpha \Gamma\left(k_{i}\right) \tau\left(k_{i}\right)\right]-k_{i} \sum_{\substack{l=1 \\ l \neq i}}^{n} \Gamma\left(k_{l}\right) \tau\left(k_{l}\right)\left[\Gamma(0)-\alpha \Gamma\left(k_{l}\right) \tau\left(k_{l}\right)\right]\right\}
$$

$\Lambda$ is the variation of the net benefits from consumption and it is always negative. Consumers are better off when tariff decreases.

On the other hand, $\Upsilon$ can be both positive and negative. It is the sum of the variation of foreign firms' profits in the home market and the export profits of home firms. Both foreign firms' profits in the home market and the export profits of home firms increase as tariffs decrease, but foreign firms' profits in the home market enter this expression negatively. This
term represents the market access concessions. The final sign of $\Upsilon$ depends on the parameters of the model.

$$
\begin{aligned}
\Upsilon= & \frac{2}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{\left(N-k_{i}\right) \Gamma\left(k_{i}\right) \frac{\Gamma(0) \Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)}\left[\Gamma(0)-\alpha \Gamma\left(k_{i}\right) \frac{\Gamma(0) \Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)}\right]\right. \\
& \left.-k_{i} \sum_{\substack{l=1 \\
l \neq i}}^{n} \Gamma\left(k_{l}\right) \frac{\Gamma(0) \Gamma\left(2 k_{l}\right)}{D\left(k_{l}\right)}\left[\Gamma(0)-\alpha \Gamma\left(k_{l}\right) \frac{\Gamma(0) \Gamma\left(2 k_{l}\right)}{D\left(k_{l}\right)}\right]\right\} \\
= & \frac{2 N}{\Gamma(N)^{2}}\left\{\frac{\Gamma\left(k_{i}\right) \Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)}\left[1-\alpha \frac{\Gamma\left(k_{i}\right) \Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)}\right]-\frac{k_{i}}{N} \sum_{l=1}^{n} \frac{\Gamma\left(k_{l}\right) \Gamma\left(2 k_{l}\right)}{D\left(k_{l}\right)}\left[1-\alpha \frac{\Gamma\left(k_{l}\right) \Gamma\left(2 k_{l}\right)}{D\left(k_{l}\right)}\right]\right\} \\
= & \frac{2 N}{\Gamma(N)^{2}}\left\{\Xi\left(k_{i}\right)\left[1-\alpha \Xi\left(k_{i}\right)\right]-\frac{k_{i}}{N} \sum_{l=1}^{n} \Xi\left(k_{l}\right)\left[1-\alpha \Xi\left(k_{l}\right)\right]\right\} \\
= & \frac{2 N}{\Gamma(N)^{2}}\left\{\Xi\left(k_{i}\right)\left[1-\alpha \Xi\left(k_{i}\right)\right]-k_{i}\left[\bar{\Xi}-\alpha \bar{\Xi}^{2}-\alpha \operatorname{Var}(\Xi)\right]\right\} \\
= & \frac{2 N}{\Gamma(N)^{2}}\left\{\Xi\left(k_{i}\right)\left[1-\alpha \Xi\left(k_{i}\right)\right]-k_{i} \bar{\Xi}[1-\alpha \bar{\Xi}]+\alpha k_{i} \operatorname{Var}(\Xi)\right\}
\end{aligned}
$$

with

$$
\begin{aligned}
\Xi\left(k_{i}\right) & =\frac{\Gamma\left(k_{i}\right) \Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)} \\
\bar{\Xi} & =\frac{1}{N} \sum_{l=1}^{n} \Xi\left(k_{l}\right) \\
\operatorname{Var}(\Xi) & =\frac{1}{N} \sum_{l=1}^{n} \Xi\left(k_{l}\right)^{2}-\bar{\Xi}^{2} \\
\Gamma\left(k_{i}\right) & =2-\gamma+k_{i} \gamma \\
D\left(k_{i}\right) & =\Psi\left(k_{i}\right) \Gamma(N)+\Gamma\left(k_{i}\right) \Gamma\left(2 k_{i}\right) \\
\Psi\left(k_{i}\right) & =(\Gamma(0)+1) \Gamma\left(k_{i}\right)-\Gamma\left(2 k_{i}\right) \\
\tau\left(k_{i}\right) & =\frac{\Gamma(0) \Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)} \\
N & =\sum_{i=1}^{n} k_{i}
\end{aligned}
$$

It is hard to sign $\Upsilon$ exactly in general due to the large number of parameters, but we can see that $\Upsilon$ depends on the variance of the number of firms per country through the variance of $\Xi .{ }^{17}$ The higher the variance of the number of firms per country, the higher the value of $\Upsilon$.

[^14]In a perfectly symmetric world where each country has $k$ firms, $\Upsilon$ is zero. If all countries have exactly the same number of firms, they will all charge initially the same Nash optimal tariff. A radial tariff cut will then be the same in all countries and domestic firms will gain exactly the same market access advantage in foreign countries as the home country will give away to foreign firms through this tariff cut. Each country will get exactly compensated for the offered market access in the domestic market through gained market access in other countries. Furthermore, consumers will gain through increased variety and lower prices ( $\Lambda$ always negative). And so in a perfectly symmetric world, a radial tariff cut is welfare increasing. Note that in a perfectly symmetric world, a radial cut is exactly equivalent to a tariff cut following the principles of reciprocity and non-discrimination discussed in the previous section where it was already shown that such a multilateral trade liberalisation is welfare increasing.

In an asymmetric world, $\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ as well as the welfare $W\left(k_{i}, \alpha, C\right)$ depends on the whole set of countries $C$, i.e. it depends on how many firms each country has through the term $\sum_{\substack{l=1 \\ l \neq i}}^{n} \Gamma\left(k_{l}\right) \tau\left(k_{l}\right)\left[\Gamma(0)-\alpha \Gamma\left(k_{l}\right) \tau\left(k_{l}\right)\right]$. Set

$$
O=\sum_{\substack{l=1 \\ l \neq i}}^{n} \Gamma\left(k_{l}\right) \tau\left(k_{l}\right)\left[\Gamma(0)-\alpha \Gamma\left(k_{l}\right) \tau\left(k_{l}\right)\right]
$$

$O \geq 0$ and we have

$$
\Upsilon=\frac{2}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{\left(N-k_{i}\right) \Gamma\left(k_{i}\right) \tau\left(k_{i}\right)\left[\Gamma(0)-\alpha \Gamma\left(k_{i}\right) \tau\left(k_{i}\right)\right]-k_{i} O\right\}
$$

To sign $\Upsilon$ and $\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ in a asymmetric world let us distinguish the two following cases depending on the values of $\gamma$.
$\gamma=0$ : How does the derivative of welfare with respect to $\alpha$ vary with $k$ ?

$$
\frac{d}{d k} \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}=-\frac{1}{24}[2(1-\alpha)+O] \leq 0
$$

$\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ is thus a monotonically decreasing function of $k$. For low values of $k, \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ is positive and for high values of $k$ it is negative. For the tariff cut to be welfare increasing, Country $i$ has to have a sufficient number of firms. For $\gamma=0$, tariffs and sales volumes are independent of the number of firms, welfare is independent of the set of countries $C$ and so
we can get an exact expression for $O$ and an explicit condition for the trade liberalisation to be welfare increasing:

$$
\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha} \leq 0 \Leftrightarrow k_{i} \geq \frac{3(1-\alpha) N}{2(3-\alpha) n-3-\alpha}
$$

So at $\alpha=1$, a marginal radial tariff reduction is welfare increasing for everyone (condition $k_{i}>0$ ). But for $\alpha<1$, a minimum number of firms is needed to benefit from trade liberalisation.

When $\alpha=0$, approximatively one third of the average number of firms per country is needed.

Note that in this calculation, we have reasoned around the Nash optimum tariff $\tau\left(k_{i}\right)$. If we assume that initially, countries charge arbitrarily high or low tariffs $\tau$ and we do the same calculation, we find the following condition:

$$
\frac{d W\left(k_{i}, \alpha\right)}{d \alpha} \leq 0 \Leftrightarrow k_{i} \geq \frac{(3 \alpha \tau-1) N}{1+\alpha \tau+2 n(\alpha \tau-1)}
$$

A radial tariff cut from an arbitrary tariff value $\tau$ is also welfare increasing if an only if the considered country has a sufficient number of firms. The minimum number is a decreasing function of $\tau$, i.e. if the initial tariff is very high, any country will benefit from trade liberalisation.

$$
\frac{d k_{\min }}{d \tau}=-\frac{4 \alpha N(n-1)}{(1+\alpha \tau+2 n(\alpha \tau-1))^{2}} \leq 0
$$

$\gamma>0$ : For $\gamma$ non zero, $\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ is a non-monotonic function of $k$. The variations of this derivative with $k$ are similar for all $\gamma$ such that $0<\gamma \leq 1$ and so to simplify the expressions, let us consider the case $\gamma=1$.

$$
\begin{equation*}
\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}=\frac{(1-\alpha)(2 N+1)}{(N+1)^{2}}-\frac{2(1-\alpha)}{N+k(2 k+3)+1}-\frac{2(1-\alpha+O) k}{(N+1)^{2}} \tag{71}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d}{d k} \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}=-\frac{2(1-\alpha+O)}{(N+1)^{2}}+\frac{2(1-\alpha)(4 k+3)}{(N+k(2 k+3)+2)^{2}} \tag{72}
\end{equation*}
$$

From (71) and (72) we can see that the variations with $k$ depend on $O$. If $O$ is sufficiently large, then for any $k, \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ will be negative. For intermediate values of $O, \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ will be negative only for large $k$ and small $k$ and for small values of $O, \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ will be negative only for large $k$.

Intuitively this makes sense, $O$ is linked to the variance of the number of firms, the smaller the variance, the larger $O$. We have seen that in a symmetric world, where the variance is zero (and so $O$ maximum), $\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}$ is negative. When asymmetries become important, only countries with a sufficient number of firms or very few firms will benefit from uniform trade liberalisation. Countries with many firms will benefit, because they will not be giving away a very important market access advantage and on the other hand, because they export a lot, they will benefit a lot from market access concessions of other countries. Countries with very few firms will benefit, because the increase in consumer surplus will outweigh insufficiently compensated market access concession. When asymmetries become very important, only countries with many firms benefit from radial trade liberalisation for reasons mentioned above.

Simulations: Simulations confirm the intuition and the fact that a variance is hidden somewhere in the condition. When the variance of the number of firms per country is small, the derivative is negative for everyone (all countries would benefit from a uniform trade liberalisation). When the variance is very large, only countries with many firms would benefit from a uniform trade liberalisation. For intermediate values of the variance, only countries with many firms and countries with very few firms would benefit. When a country has very few firms, consumers benefit a lot from trade liberalisation. The increase in consumer surplus is more important than the loss from reduced domestic market access of the few firms.

## Proof of Lemma 4.

$$
\frac{d}{d P}\left(\frac{d W}{d \tau}\right)=\frac{2 \Gamma(k)(-\Gamma(0)+\Gamma(k) \tau)}{\Gamma(0)^{2} \Gamma(N)^{2}}
$$

So $\frac{d}{d P}\left(\frac{d W}{d \tau}\right)$ is negative for $\tau \leq \frac{\Gamma(0)}{\Gamma(k)}$. $\tau_{e}(k)=\frac{\Gamma(0) \Gamma(2 k)}{D(k)}<\frac{\Gamma(0)}{\Gamma(k)}$ and so during trade liberalisation starting from $\tau_{e}$, we have always $\frac{d}{d P}\left(\frac{d W}{d \tau}\right)<0$. This means that the more firms are involved in the trade liberalisation, the smaller the derivative of $W$ with respect to $\tau$ and so the more likely is the liberalisation welfare increasing.

Proof of Proposition 10. Assume a subset of $p$ countries decide to liberalise trade unilaterally. $P$ firms are located in these countries. We want to determine the impact of such a
liberalisation on the welfare of Country $i$ that has $k_{i}$ firms.
These $p$ countries liberalise according to the reciprocity and non-discrimination principles. The tariff change will thus be such that $d T B=0$ where $T B=k_{i} \sum_{l \neq i}^{p} q_{O}^{2}\left(k_{l}\right)-\left(P-k_{i}\right) q_{O}^{2}\left(k_{i}\right)$. But non-involved countries will not change their tariffs. The derivative of the welfare function with respect to tariff is thus

$$
\begin{aligned}
\frac{d W}{d \tau}\left(k_{i}\right)= & \frac{d Q}{d \tau}\left(k_{i}\right)-\gamma \frac{d Q}{d \tau}\left(k_{i}\right) Q\left(k_{i}\right) \\
& -(1-\gamma)\left[k_{i} \frac{d q_{I}}{d \tau}\left(k_{i}\right) q_{I}\left(k_{i}\right)+\left(N-k_{i}\right) \frac{d q_{O}}{d \tau}\left(k_{i}\right) q_{O}\left(k_{i}\right)\right] \\
& -2(N-P) \frac{d q_{O}}{d \tau}\left(k_{i}\right) q_{O}\left(k_{i}\right) \\
= & -\frac{\left(N-k_{i}\right)}{\Gamma(N)}+\gamma \frac{\left(N-\left(N-k_{i}\right) \tau\right)}{\Gamma(N)} \frac{\left(N-k_{i}\right)}{\Gamma(N)} \\
& -(1-\gamma)\left[k_{i} \frac{\left(\Gamma(0)+\gamma\left(N-k_{i}\right) \tau\right)}{\Gamma(0) \Gamma(N)} \frac{\gamma\left(N-k_{i}\right)}{\Gamma(0) \Gamma(N)}-\left(N-k_{i}\right) \frac{\left(\Gamma(0)-\Gamma\left(k_{i}\right) \tau\right)}{\Gamma(0) \Gamma(N)} \frac{\Gamma\left(k_{i}\right)}{\Gamma(0) \Gamma(N)}\right] \\
& +2(N-P) \frac{\left(\Gamma(0)-\Gamma\left(k_{i}\right) \tau\right)}{\Gamma(0) \Gamma(N)} \frac{\Gamma\left(k_{i}\right)}{\Gamma(0) \Gamma(N)}
\end{aligned}
$$

Drop subscript $i$ to simplify notation.

$$
\begin{aligned}
\frac{d W}{d \tau}(k)= & \frac{1}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{-(N-k) \Gamma(0)^{2}+2(N-P) \Gamma(0) \Gamma\left(k_{i}\right)\right. \\
& \left.+\left[(N-k)\left(-\Gamma(0)^{2} \gamma(N-k)-(1-\gamma) k \gamma^{2}(N-k)-(1-\gamma) \Gamma(k)^{2}\right)-2(N-P) \Gamma(k)^{2}\right] \tau\right\} \\
= & \frac{1}{\Gamma(0)^{2} \Gamma(N)^{2}}\{A+B \tau\}
\end{aligned}
$$

with $A=-(N-k) \Gamma(0)^{2}+2(N-P) \Gamma(0) \Gamma\left(k_{i}\right)$ and $B=(N-k)\left(-\Gamma(0)^{2} \gamma(N-k)-(1-\right.$ $\left.\gamma) k \gamma^{2}(N-k)-(1-\gamma) \Gamma(k)^{2}\right)-2(N-P) \Gamma(k)^{2} . B$ is always negative and so the derivative of the welfare function is a linear decreasing function. Its sign will depend on $A$ (which depends on the parameters of the model) and the tariff $\tau$.

## Sign of the derivative at $\alpha \tau_{e}$ :

Let us evaluate the derivative of welfare at $\alpha \tau_{e}(k)$.

$$
\frac{d W}{d \tau}\left(k, \alpha \tau_{e}(k)\right)=\frac{1}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{A_{r}(k, N, \gamma)-\alpha B_{r}(k, N, \gamma)-2 P \Gamma(k)[\Gamma(0)-\alpha \Gamma(k) \tau(k)]\right\}
$$

with

$$
\begin{aligned}
& A_{r}(k, N, \gamma)=D(k)[N \Gamma(2 k)+k \Gamma(0)] \\
& B_{r}(k, N, \gamma)=\Gamma(2 k)\left\{(N-k)^{2} \gamma\left[\Gamma(0)^{2}+(1-\gamma) k \gamma\right]+\Gamma(k)^{2}[(1-\gamma)(N-k)+2 N]\right\}
\end{aligned}
$$

So

$$
\frac{d W}{d \tau}\left(k, \alpha \tau_{e}(k)\right) \leq 0 \Leftrightarrow P \geq \bar{P}_{r}=\frac{A_{r}(k, N, \gamma)-\alpha B_{r}(k, N, \gamma)}{2 \Gamma(k)[D(k)-\alpha \Gamma(k) \Gamma(2 k)]}
$$

Sign of the derivative at $\tau_{e}$, i.e. $\alpha=1$ :

$$
\bar{P}_{r}(1, k, N, \gamma)=k
$$

Also

$$
\begin{array}{lc}
\frac{d W}{d \tau}(k, \tau(k)) & = \\
& \frac{2 \Gamma(k)\left(\Gamma(0)^{2}+(1-\gamma) \gamma k\right)(P-k)}{\Gamma(0) \Gamma(N)(-12+\gamma(-4(-4+2 k+N)+\gamma(-7+\gamma(k-1)(N-1)+4 N-k(-6+2 k+N))))}
\end{array}
$$

This derivative is 0 for $P=k$ (the welfare is maximised at $\tau_{e}$ when tariff is set noncooperatively) and it is negative for $P>k$, i.e. marginal unilateral liberalisation will be welfare increasing at $\tau_{e}$ if at least some other partner liberalises as well.

Sign of the derivative at $\alpha=0$, i.e. unilateral free trade:
It will be negative for $\tau=0$ if and only if $A$ is also negative. And $A<0 \Leftrightarrow-(N-$ $k) \Gamma(0)^{2}+2(N-P) \Gamma(0) \Gamma(k)<0$, which implies

$$
P>\bar{P}_{r}(0, k, N, \gamma)=\frac{N \Gamma(2 k)+k \Gamma(0)}{2 \Gamma(k)}
$$

Which ends the proof.

## Proof of Lemma 5.

$$
\begin{aligned}
\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}= & \frac{d Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha}-\gamma \frac{d Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right) \\
& -(1-\gamma)\left[k_{i} \frac{d q_{I}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{I}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)+\left(N-k_{i}\right) \frac{d q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)\right] \\
& -2\left(N-k_{i}\right) \frac{d q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)+2 k_{i} \sum_{\substack{l=1 \\
l \neq i}}^{p} \frac{d q_{O}\left(k_{l}, \alpha \tau\left(k_{l}\right)\right)}{d \alpha} q_{O}\left(k_{l}, \alpha \tau\left(k_{l}\right)\right)
\end{aligned}
$$

Differentiate with respect to $p$ this expression or the one with variance?
Proof of Proposition 11. Assume that a subset of $p$ countries with $P$ firms decide to liberalise trade unilaterally through a flat rate tariff cut. The goal here is to determine the impact on welfare of a marginal variation in tariff $d \alpha$ from $\alpha \tau$.

Consider one country of the subset of countries $p$, Country $i$, in the set of countries $C=\left(k_{1}, k_{2}, \ldots, k_{n}\right)$. Differentiate $W\left(k_{i}, \alpha, C\right)$ with respect to $\alpha$ :

$$
\begin{aligned}
& \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}= \frac{d Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha}-\gamma \frac{d Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right) \\
&-(1-\gamma)\left[k_{i} \frac{d q_{I}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{I}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)+\left(N-k_{i}\right) \frac{d q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)\right] \\
&-2\left(N-k_{i}\right) \frac{d q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)}{d \alpha} q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)+2 k_{i} \sum_{\substack{l=1 \\
l \neq i}}^{p} \frac{d q_{O}\left(k_{l}, \alpha \tau\left(k_{l}\right)\right)}{d \alpha} q_{O}\left(k_{l}, \alpha \tau\left(k_{l}\right)\right) \\
& \frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}=\Lambda+\Upsilon_{p}
\end{aligned}
$$

with

$$
\Lambda=-\frac{\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{\Gamma(0)^{2}+\alpha \tau\left(k_{i}\right)\left[\gamma \Gamma(0)^{2}\left(N-k_{i}\right)+(1-\gamma) k_{i} \gamma^{2}\left(N-k_{i}\right)+(1-\gamma) \Gamma\left(k_{i}\right)^{2}\right]\right\}<0
$$

and

$$
\Upsilon_{p}=\frac{2}{\Gamma(0)^{2} \Gamma(N)^{2}}\left\{\left(N-k_{i}\right) \Gamma\left(k_{i}\right) \tau\left(k_{i}\right)\left[\Gamma(0)-\alpha \Gamma\left(k_{i}\right) \tau\left(k_{i}\right)\right]-k_{i} \sum_{\substack{l=1 \\ l \neq i}}^{p} \Gamma\left(k_{l}\right) \tau\left(k_{l}\right)\left[\Gamma(0)-\alpha \Gamma\left(k_{l}\right) \tau\left(k_{l}\right)\right]\right\}
$$

$\Lambda$ is the variation of the net benefits from consumption and it is always negative. Consumers are better off when tariff decreases.

On the other hand, $\Upsilon_{p}$ can be both positive and negative. It is the sum of the variation of foreign firms' profits in the home market and the export profits of home firms. As Country $i$ liberalises trade unilaterally, all foreign firms gains market access in the home country, but Country $i$ 's firms gain only market access in the $p-1$ foreign countries who agreed to liberalise.

Note that $\Upsilon_{p}$ is a decreasing function of $P$. In order to gain from trade liberalisation, Country $i$ has to be compensated for the offered market access. It has to gain a minimum compensatory market access which means that a certain minimum of other countries have to take part in this trade liberalisation. The minimum participation condition is derived as follows.

$$
\begin{aligned}
\frac{d W\left(k_{i}, \alpha, C\right)}{d \alpha}= & \frac{\Gamma\left(2 k_{i}\right)}{\Gamma(N)^{2} D\left(k_{i}\right)}\left\{N \Gamma(0)+k_{i} \Gamma(2 N)\right. \\
& -\alpha \frac{\Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)}\left(\gamma \Gamma(0)^{2}\left(N-k_{i}\right)^{2}+(1-\gamma) k_{i} \gamma^{2}\left(N-k_{i}\right)^{2}\right. \\
& \left.\left.+(1-\gamma) \Gamma\left(k_{i}\right)^{2}\left(N-k_{i}\right)+2 N \Gamma\left(k_{i}\right)^{2}\right)\right\} \\
& -\frac{2 P k_{i}}{\Gamma(N)^{2}}\left[\bar{\Xi}_{p}-\alpha \bar{\Xi}_{p}^{2}-\alpha V a r_{p} \Xi\right] \\
= & \frac{\Gamma\left(2 k_{i}\right)}{\Gamma(N)^{2} D\left(k_{i}\right)}[A(k)-\alpha B(k)]-\frac{2 P k_{i}}{\Gamma(N)^{2}}\left[\bar{\Xi}_{p}-\alpha \bar{\Xi}_{p}^{2}-\alpha V a r_{p} \Xi\right]
\end{aligned}
$$

with

$$
\begin{aligned}
A\left(k_{i}\right) & =N \Gamma\left(2 k_{i}\right)+k_{i} \Gamma(0) \\
B\left(k_{i}\right) & =\frac{\Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)}\left[\gamma \Gamma(0)^{2}\left(N-k_{i}\right)^{2}+(1-\gamma) k_{i} \gamma^{2}\left(N-k_{i}\right)^{2}+(1-\gamma) \Gamma\left(k_{i}\right)^{2}\left(N-k_{i}\right)+2 N \Gamma\left(k_{i}\right)^{2}\right] \\
\Xi\left(k_{i}\right) & =\frac{\Gamma\left(k_{i}\right) \Gamma\left(2 k_{i}\right)}{D\left(k_{i}\right)} \\
\bar{\Xi}_{p} & =\frac{1}{P} \sum_{l=1}^{p} \Xi\left(k_{l}\right) \\
\operatorname{Var}_{p}(\Xi) & =\frac{1}{P} \sum_{l=1}^{p} \Xi\left(k_{l}\right)^{2}-\bar{\Xi}_{p}^{2}
\end{aligned}
$$

and so

$$
\frac{d W\left(k_{i}, \alpha\right)}{d \alpha} \leq 0 \Leftrightarrow P \geq \bar{P}_{f}=\frac{\Gamma\left(2 k_{i}\right)\left[A\left(k_{i}\right)-\alpha B\left(k_{i}\right)\right]}{2 k_{i} D\left(k_{i}\right)\left[\bar{\Xi}_{p}-\alpha \bar{\Xi}_{p}^{2}-\alpha V a r_{p} \Xi\right]}
$$

We can see that $\bar{P}_{f}$ is an increasing function of the variance of the number of firms per country within the subset of trade liberalising countries. It is a decreasing function of the number of firms $k_{i}$ in Country $i$ considered.

Condition at $\tau_{e}\left(k_{i}\right)$, i.e. $\alpha=1$ :

$$
\frac{d W\left(k_{i}, 1\right)}{d \alpha} \leq 0 \Leftrightarrow P \geq \bar{P}_{f}=\frac{\Gamma\left(2 k_{i}\right)\left[A\left(k_{i}\right)-B\left(k_{i}\right)\right]}{2 k_{i} D\left(k_{i}\right)\left[\bar{\Xi}_{p}-\bar{\Xi}_{p}^{2}-\text { Var }_{p} \Xi\right]}
$$

Free trade: $\alpha=0$

$$
\frac{d W\left(k_{i}, \alpha\right)}{d \alpha} \leq 0 \Leftrightarrow P \geq \bar{P}_{f_{F} T}=\frac{\Gamma\left(2 k_{i}\right)\left[N \Gamma\left(2 k_{i}\right)+k_{i} \Gamma(0)\right]}{2 k_{i} D\left(k_{i}\right) \bar{\Xi}_{p}}
$$

Symmetric free trade: $k_{i}=1, i=1, \ldots, n$ and $\alpha=0$
When each country has only one firm, the minimum participation constraint becomes a constraint on the number of countries involved.

$$
\frac{d W\left(k_{i}, \alpha\right)}{d \alpha} \leq 0 \Leftrightarrow p \geq \bar{p}_{f_{S y m F T}}=\frac{2 N+\gamma(N-1)+2}{4}
$$

$\gamma=\mathbf{0}:$
When $\gamma=0$, firms' sales do not depend on the number of firms. The radial tariff cut is also independent of the number of firms and so the minimum participation condition of the flat rate tariff cut becomes a condition on the number of countries involved.

$$
\frac{d W\left(k_{i}, \alpha\right)}{d \alpha} \leq 0 \Leftrightarrow p \geq p_{f_{\gamma}=0}=\frac{3(1-\alpha) N+3(1+\alpha) k_{i}}{2(3-\alpha) k_{i}}
$$

which ends the proof. $\square$

## D Non-marginal flat rate tariff cut

Proposition 12. A flat-rate percentage tariff reduction is welfare increasing for competitive countries, i.e. for countries that have a sufficient number of firms.

The above proposition shows that a formula-based trade liberalisation is beneficial to countries with many firms. It can also be welfare increasing to countries with few firms, but not necessarily depending on the parameters of the model. Intuitively, consumers will benefit from trade liberalisation, firms will benefit through their exports, but lose due to more competition in the domestic market. So a country has to have a sufficient number of firms for the gains from exports to compensate the losses at home. Note that in a perfectly symmetric world where each country has the same number of firms $k$, all countries
will benefit from trade liberalisation (this is the case where formula-based liberalisation is strictly equivalent to trade liberalisation according to the reciprocity and non-discrimination principles). The proof discusses more in detail the conditions under which a radial trade liberalisation is welfare increasing. For example, when $\gamma=0$, a country needs to have approximately one third of the average number of firms per country to benefit from formulabased trade liberalisation.

Proof. Assume all countries cut their tariffs from the initial Nash levels $\tau_{e}\left(k_{i}\right)$ to $\alpha \tau_{e}\left(k_{i}\right)$ with $\alpha \in[0,1]$ and $i=1, \ldots n$. We want to determine the impact on welfare of Country $i$ of this multilateral trade liberalisation. The purpose of this proof is thus to determine the sign of $W\left(k_{i}, \alpha \tau_{e}\left(k_{i}\right)\right)-W\left(k_{i}, \tau_{e}\left(k_{i}\right)\right)$.

To simplify notation, drop subscript Nash.

$$
W\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)-W\left(k_{i}, \tau\left(k_{i}\right)\right)=\Xi+\Theta+\Upsilon
$$

with

$$
\begin{aligned}
& \Xi=Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)-Q\left(k_{i}, \tau\left(k_{i}\right)\right)-\frac{\gamma}{2}\left[Q\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)^{2}-Q\left(k_{i}, \tau\left(k_{i}\right)\right)^{2}\right] \\
& \Theta=-\frac{1-\gamma}{2}\left\{k_{i}\left[q_{I}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)^{2}-q_{I}\left(k_{i}, \tau\left(k_{i}\right)\right)^{2}\right]+\left(N-k_{i}\right)\left[q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)^{2}-q_{O}\left(k_{i}, \tau\left(k_{i}\right)\right)^{2}\right]\right\} \\
& \Upsilon=-\left(N-k_{i}\right)\left[q_{O}\left(k_{i}, \alpha \tau\left(k_{i}\right)\right)^{2}-q_{O}\left(k_{i}, \tau\left(k_{i}\right)\right)^{2}\right]+k_{i} \sum_{\substack{l=1 \\
l \neq i}}^{n}\left[q_{O}\left(k_{l}, \alpha \tau\left(k_{l}\right)\right)^{2}-q_{O}\left(k_{l}, \tau\left(k_{l}\right)\right)^{2}\right]
\end{aligned}
$$

Because of the complexity of these expressions, I analyse each of them separately. $\Xi+\Theta$ is the variation of the net benefits from consumption when tariff decreases from $\tau\left(k_{i}\right)$ to $\alpha \tau\left(k_{i}\right)$ and it is always positive. Consumers are better off when tariff decreases.

$$
\begin{aligned}
\Xi & =\frac{(1-\alpha)\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(N)}-\frac{\gamma}{2} \frac{(1-\alpha)\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(N)}\left[\frac{2 N-(1+\alpha)\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(N)}\right] \\
& =\frac{(1-\alpha)\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(N)}\left[\frac{2 \Gamma(0)+\gamma(1+\alpha)\left(N-k_{i}\right) \tau\left(k_{i}\right)}{2 \Gamma(N)}\right]>0
\end{aligned}
$$

and

$$
\begin{aligned}
\Theta= & -\frac{1-\gamma}{2} \frac{(\alpha-1) \gamma k_{i}\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(0) \Gamma(N)}\left[\frac{2 \Gamma(0)+(1+\alpha) \gamma\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(0) \Gamma(N)}\right] \\
& -\frac{1-\gamma}{2} \frac{(1-\alpha)\left(N-k_{i}\right) \Gamma\left(k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(0) \Gamma(N)}\left[\frac{2 \Gamma(0)-(1+\alpha) \Gamma\left(k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(0) \Gamma(N)}\right] \\
= & \frac{1-\gamma}{2} \frac{(1-\alpha)\left(N-k_{i}\right) \tau\left(k_{i}\right)}{\Gamma(0)^{2} \Gamma(N)^{2}}\left[-2 \Gamma(0)^{2}+(1+\alpha)\left(k_{i} \gamma^{2}+\Gamma\left(k_{i}\right)^{2}\right) \tau\left(k_{i}\right)\right]
\end{aligned}
$$

Combining the two we get

$$
\begin{aligned}
\Xi+\Theta= & \frac{(1-\alpha)\left(N-k_{i}\right) \tau\left(k_{i}\right)}{2 \Gamma(0)^{2} \Gamma(N)^{2}}\left\{\Gamma(0)^{2}(3-\gamma)\right. \\
& \left.+(1+\alpha)\left[\Gamma(0)^{2} \gamma\left(N-k_{i}\right)+(1-\gamma)\left(k_{i} \gamma^{2}+\Gamma\left(k_{i}\right)^{2}\right)\right] \tau\left(k_{i}\right)\right\}>0
\end{aligned}
$$

On the other hand, $\Upsilon$ can be both positive and negative. It is the sum of the variation of foreign firms' profits in the home market and the export profits of home firms. Both foreign firms' profits in the home market and the export profits of home firms increase as tariffs decrease, but foreign firms' profits in the home market enter this expression negatively and so the final sign of $\Upsilon$ depends on the parameters of the model. It is hard to sign $\Upsilon$ exactly due to the large number of parameters. Intuitively, $\Upsilon$ will be positive if Country $i$ is a net exporter, i.e. if a sufficient number of world firms is located in Country $i$, the increase in export profits from a tariff decrease (the second term of $\Upsilon$ ) will be greater than the increase of foreign firms' profits in the home market. In particular, note that in a perfectly symmetric world where each country has $k$ firms, $\Upsilon$ is zero.

$$
\begin{aligned}
\Upsilon & =-(N-k)\left[q_{O}(k, \alpha \tau)^{2}-q_{O}(k, \tau)^{2}\right]+k \sum_{\substack{l=1 \\
l \neq i}}^{n}\left[q_{O}(k, \alpha \tau)^{2}-q_{O}(k, \tau)^{2}\right] \\
& =-(N-k)\left[q_{O}(k, \alpha \tau)^{2}-q_{O}(k, \tau)^{2}\right]+k(n-1)\left[q_{O}(k, \alpha \tau)^{2}-q_{O}(k, \tau)^{2}\right] \\
& =0
\end{aligned}
$$

In an asymmetric world, very approximately, we can say that the variations of $q_{O}(k)$ are negligible compared to the variations of $k . q_{O}(k) \sim \frac{k}{k} \sim 1$, so we can approximately write $\Upsilon \sim-\left(N-k_{i}\right) \epsilon+k_{i}(n-1) \epsilon$ and so $\Upsilon \geq 0 \Leftrightarrow k \geq \frac{N}{n}$, if Country $i$ has more firms than the average number of firms per country, the formula-based multilateral liberalisation will be welfare increasing. This is of course very approximate, in this reasoning, I did not
take into account the positive impact of trade liberalisation on consumer surplus ( $\Xi+\Theta$ always positive) and I have only very grossly evaluated $\Upsilon$. To sign more precisely the whole expression $W\left(k_{i}, \alpha \tau_{e}\right)-W\left(k_{i}, \tau_{e}\right)$, I need to assign values to some of the parameters.

Assume $\gamma=0$ :

$$
\begin{aligned}
W\left(k_{i}, \alpha \tau_{e}\right)-W\left(k_{i}, \tau_{e}\right) & =\frac{1}{72}(1-\alpha)\left(k_{i}(2(5-\alpha) n-7-\alpha)-3(1-\alpha) N\right) \\
\frac{d}{d k} W\left(k_{i}, \alpha \tau_{e}\right)-W\left(k_{i}, \tau_{e}\right) & =\frac{1}{72}(1-\alpha)(2(5-\alpha) n-7-\alpha)>0
\end{aligned}
$$

So $W\left(k_{i}, \alpha \tau_{e}\right)-W\left(k_{i}, \tau_{e}\right)$ is an increasing function of $k_{i}$ and

$$
W\left(k_{i}, \alpha \tau_{e}\right)-W\left(k_{i}, \tau_{e}\right) \geq 0 \Leftrightarrow k_{i} \geq \frac{3(1-\alpha) N}{2(5-\alpha) n-7-\alpha}
$$

So when $\gamma=0$, a country needs to have approximately one third of the average number of firms per country to benefit from radial trade liberalisation.

When $\gamma \neq 0$, it becomes harder to sign the expression of interest, but we can note that as $\gamma$ increases, trade liberalisation becomes less profitable to firms and so we can expect the minimum number of firms necessary for a country to benefit from trade liberalisation to increase.

Assume now $n=2$ and any $\gamma$ :

$$
\begin{gathered}
\Upsilon=-(N-k)\left[q_{O}(k, \alpha \tau)^{2}-q_{O}(k, \tau)^{2}\right]+k\left[q_{O}(N-k, \alpha \tau)^{2}-q_{O}(N-k, \tau)^{2}\right] \\
\Upsilon \geq 0 \Leftrightarrow k \geq \frac{N}{2}
\end{gathered}
$$

As $\Xi+\Theta>0, \frac{N}{2}$ is an upper bound of the minimum of firms necessary. [Simulations for the remainder.]

## E Minimum participation and non-marginal flat rate tariff cut

Proposition 13. When $\gamma=0$, a unilateral radial tariff cut from $\tau_{e}$ to $\alpha \tau_{e}$ is welfare increasing for a country with $k$ firms if and only if at least $\bar{p}$ countries participate, where

$$
\begin{equation*}
\bar{p}=\frac{3(1-\alpha) N+(7+\alpha) k}{2(5-\alpha) k} \tag{73}
\end{equation*}
$$

Proof. For $\gamma=0, q_{O}$ does not depend on $k$. The tariff cut from $\tau_{\text {nash }}$ to $\alpha \tau_{e}$ thus induces the following change in welfare

$$
\begin{aligned}
W\left(k, \alpha \tau_{e}\right)-W\left(k, \tau_{e}\right)= & Q\left(k, \alpha \tau_{e}\right)-Q\left(k, \tau_{e}\right) \\
& -\frac{\gamma}{2}\left[Q\left(k, \alpha \tau_{e}\right)^{2}-Q\left(k, \tau_{e}\right)^{2}\right] \\
& -\frac{1-\gamma}{2} k\left[q_{I}\left(k, \alpha \tau_{e}\right)^{2}-q_{I}\left(k, \tau_{e}\right)^{2}\right] \\
& -\frac{1-\gamma}{2}(N-k)\left[q_{O}\left(\alpha \tau_{e}\right)^{2}-q_{O}\left(\tau_{e}\right)^{2}\right] \\
& -(N-k)\left[q_{O}\left(\alpha \tau_{e}\right)^{2}-q_{O}\left(\tau_{e}\right)^{2}\right] \\
& +k(p-1)\left[q_{O}\left(\alpha \tau_{e}\right)^{2}-q_{O}\left(\tau_{e}\right)^{2}\right]
\end{aligned} \quad \begin{aligned}
W\left(k, \alpha \tau_{e}\right)-W\left(k, \tau_{e}\right)= & \frac{(1-\alpha)}{72}[k(2(5-\alpha) p-7-\alpha)-3(1-\alpha) N]
\end{aligned}
$$

and we have $W\left(k, \alpha \tau_{e}\right)-W\left(k, \tau_{e}\right) \geq 0 \Leftrightarrow p \geq \frac{3(1-\alpha) N+(7+\alpha) k}{2(5-\alpha) k}$.

The following table summarises the conditions for a uniform tariff cut to be welfare increasing in the marginal and non-marginal case when $\gamma=0$ :

|  | Marginal | Non-Marginal |
| :---: | :---: | :---: |
| All participate | $k_{i} \geq \frac{3(1-\alpha) N}{2(3-\alpha-\alpha-\alpha-\alpha}$ | $k_{i} \geq \frac{3(1-\alpha) N}{2(5-\alpha) n-7-\alpha}$ |
| Subgroup of $p$ countries participate | $p \geq \frac{3(1-\alpha) N+3(1+\alpha) k_{i}}{2(3-\alpha) k_{i}}$ | $p \geq \frac{3(1-\alpha) N+(7+\alpha) k_{i}}{2(5-\alpha) k_{i}}$ |

The non-marginal case yields less strict conditions.

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[^1]:    ${ }^{1}$ Rose (2004a) and Rose (2004b) estimated the effects of GATT/WTO on international trade and found no correlation between GATT/WTO membership and more liberal trade policy. According to these findings, there seemed to be no evidence that the WTO has increased world trade. However, Subramanian and Wei (2007) and Tomz et al. (2007) challenge Rose (2004a)'s conclusion about the ineffectiveness of the WTO and offer empirical evidence supportive of the important role played by the GATT/WTO in trade liberalisation.

[^2]:    ${ }^{2} \mathrm{~A}$ different branch of the economic literature on trade agreements provides a commitment motivation for trade negotiations: trade agreements help governments to make commitments to their private sectors (see for example Maggi and Rodriguez-Clare (1998)). These models study trade negotiations from a different perspective from the literature started by Johnson (1953-54) and therefore I do not discuss them here.

[^3]:    ${ }^{3}$ A two-country version of this model was recently used by Fujiwara (2008) to analyse welfare effects of free trade.

[^4]:    ${ }^{4}$ The derivation of this decomposition is given in the Appendix page 36 .

[^5]:    ${ }^{5}$ For a detailed discussion of the Nash tariff determinants see Appendix page 38.

[^6]:    ${ }^{6}$ For a discussion of the "concertina rule" for tariff reform see for example Neary (1998).

[^7]:    ${ }^{7}$ http://www.wto.org/english/theWTO_e/whatis_e/tif_e/agrm3_e.htm
    ${ }^{8}$ For a detailed discussion of the interpretation of the principle of reciprocity see Bagwell and Staiger (2002).
    ${ }^{9}$ The distribution of the tariff cut across sectors was left to negotiations between trading partners in the Uruguay Round. The result of this was that this round reduced more tariffs that were already relatively low than higher tariffs.
    ${ }^{10}$ For policy considerations, it may be interesting to determine the impact on welfare of a non-marginal tariff cut which is possible in this framework thanks to the tractability of the model. This calculation yields

[^8]:    a similar, but less strict condition as the marginal analysis. For details see Appendix page 57.
    ${ }^{11}$ Note that in a perfectly symmetric world, a radial cut is exactly equivalent to a tariff cut following the principles of reciprocity and non-discrimination discussed in the previous section where it was already shown that such a multilateral trade liberalisation is welfare increasing.

[^9]:    ${ }^{12}$ Note that in most cases, countries with fewer firms will be charging higher tariffs in this setting which means that they will be cutting tariffs more than countries with many firms. Nevertheless the argument developed here holds even if countries with few firms charge initially lower tariffs than countries with many firms. In a radial cut liberalisation, countries with few firms will be offering a more important market access advantage because of the structure of their market.

[^10]:    ${ }^{13}$ Article XXIV of the WTO provides an exception to the non-discrimination principle and allows formation of regional trade agreements. But one of the conditions for these agreements is that internal barriers should be completely removed. Partial discriminatory trade liberalisation is not permitted.

[^11]:    ${ }^{14}$ For non-marginal analysis see Appendix page 57 .

[^12]:    ${ }^{15}$ Ministerial declaration on trade in information technology products

[^13]:    ${ }^{16}$ See for example Black et al. (1993) or Carraro et al. (2004).

[^14]:    ${ }^{17}$ For the variance of a function we have approximately $\operatorname{Var}[f(X)] \approx\left(f^{\prime}(E[X])\right)^{2} \operatorname{Var}[X]$.

