Putting Per-Capita Income back into Trade Theory

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Abstract

A major role for per-capita income in international trade, as opposed to simply country size, was persuasively advanced by Linder (1961). Yet this crucial element of Linder's story was abandon by most later trade theorists in favor of the analytically tractable assumption that all countries share identical and homothetic preferences. Much empirical analysis in international economics persists in relying on this assumption in spite of the fact that it is clearly counter-empirical. This paper collects and unifies a number of disjoint points in the existing literature and builds further on them using a simple and tractable generic alternative to the assumption of homothetic preferences. Adding non-homothetic preferences to a traditional competitive Heckscher-Ohlin model helps explain such diverse phenomenon as growing wage gaps, the mystery of the missing trade, home bias in consumption, and the role of intra-country income distribution, solely from the demand side of general equilibrium. In a free-entry oligopoly model with increasing returns to scale, we can explain higher markups and higher price levels in higher per-capita income countries, something often assumed to reflect quality differences. In both competitive and imperfect-competition cases (a) the effects of growth are quite different depending on whether it is growth in productivity or in factor accumulation. (b) when per-capita incomes differ between countries, there is inter-industry trade even with identical aggregate incomes and relative factor endowments. The paper concludes with some suggestions for calibration, estimation, and gravity equations.

First Draft, February 2, 2010

Comments, suggestions and added citations welcome

1. Introduction

All international trade economists understand that many things can cause trade. However, our models and empirical analyses typically and appropriately tend to focus on one cause of trade at a time in order to understand how a particular basis for trade contributes to explaining trade patterns, determines gains from trade, and impacts on income distribution. Theory consists of a portfolio of specialized models such as the Ricardian and Heckscher-Ohlin models. Behind this diversity is the understanding that all of these causes of trade are working simultaneously: the question is never whether one theory is right or wrong, the question is the importance of each approach in explaining aggregate flows.

That having been said, it is probably appropriate to suggest that most trade theory focuses on production side determinants of trade. These are generally either based on differences between countries, such as differences in technologies or factor endowments, or factors such as increasing returns to scale and imperfect competition which can lead to trade and gains from trade even between identical countries. It is typically assumed that consumers have identical and homothetic preferences across countries. This gives the modeler the powerful property that the ratio in which goods are demand and consumed depends only on relative prices and that this relationship is the same across countries. In addition, this property justifies perfect aggregation within countries: aggregate demand depends only on commodity prices and aggregate income, it is independent of the distribution of income.

I also believe that it is appropriate to suggest that no one thinks that this is a good empirical assumption and that it is made for analytical convenience and tractability and that it justifies aggregation. Any budget study I have ever seen makes it very clear that budget shares across goods and services depend very much on per capita income and that the income elasticities of demand for many groups of goods and services differ substantially from one. Soon China will have the same total GNP as the United States, but I don't think anyone would seriously hypothesize that China's budget shares across consumption categories are going to match those in the US for decades to come.[1]

If we control for differences in prices across countries, the observation of different budget shares can indicate either that preferences differ and/or that they are non-homothetic. Two pure cases can be distinguished: one in which countries have homothetic but non-identical preferences and one in which countries have identical but non-homothetic preferences. Another of my assertions is that trade economists are uncomfortable with the assumption that preferences simply differ across countries in some "random" way, but this is in fact what is assumed in most applied general-equilibrium models: observed budget shares and relative prices are used to calibrate homothetic preference functions across countries. An almost final assertion (I am sure that there will be more) is that we are more comfortable

[1]An even more egregious assumption is made in much of the strategic trade-policy literature: quasi-linear preferences in which the income elasticity of demand for increasing-returns good(s) is zero. Yet the industries offered as examples, such as aircraft and electronics are surely goods with income elasticities greater than one! The present author has of course been guilty himself of this atrocity.

with a model in which preferences are identical but non-homothetic across countries. Then demand differences are not only systematic but the hypothesis is testable and falsifiable.

The purpose of this paper is collect, synthesize, and build on fragmented results from existing research in order to offer a generic model of identical but non-homothetic preferences in order to present a unified and testable set of results. By generic I mean or perhaps hope that the paper is both simple yet general enough to be useful as a teaching tool. In section two, the preferences are presented and analyzed and then place on top of a standard two-good, two-factor, two-country Heckscher-Ohlin model. I show that the resulting model offers alternative explanations for such diverse phenomenon as growing wage gaps, the mystery of the missing trade, home bias in consumption, and a role for intra-country income distribution solely from the demand side of general equilibrium. In section 3, I add scale economies and imperfect competition and show that the model can offer alternative explanations for higher price levels and higher markups in high-productivity economies, something often attributed to quality differences. In both competitive and imperfect-competition cases (a) the effects of growth are quite different depending on whether it is growth in productivity or in factor accumulation. (b) when per-capita incomes differ between countries, there is inter-industry trade even with identical aggregate incomes and relative factor endowments.

Before proceeding, a short literature review is in order and I should first point out that many of the ideas here are found in earlier papers focusing on specific issues. Several papers, obviously beginning with Linder, focused on monopolistic competition and the impact of non-homothetic preferences on intra versus inter-industry trade. Paper by Markusen (1986), Bergstrand (1990), and Francois and Kaplan (1996) draw out implications for intra-industry and total trade volumes, with good supporting empirical results in the latter paper. Matsuyama (2000) uses a competitive Ricardian model in which the South's comparative-advantage goods are low-income-elasticity-of-demand goods to derive a number of results relate to findings later in the present paper.

Papers by Hunter and Markusen (1989) and Hunter (1991) proposed that high income-elasticity goods are also capital intensive goods and if so, the volume of trade should be less than predicted by the standard Heckscher-Ohlin model. Table 1 gives some income-elasticity estimates from broad categories of consumption goods from Markusen and Hunter (1989). As described in Table 1, it is clear that the income elasticities of demand vary across goods. This means that the shares of a country's national expenditure spent on particular goods vary with levels of per-capita income. For example, Hunter (1991) listed figures showing that countries with annual per-capita income less than $750 in 1973 spent 50 percent of income on food, five percent on house furnishings, and four percent on medical care. In contrast, countries with per-capita incomes between $3,800 and $5,200 spent 17 percent on food, nine percent on house furnishings, and nine percent on medical care. Using a different set of commodity categories, Cassing and Nishioka (2009) demonstrated that a set of high-income developed economies in 2000 devoted about two percent of income to agricultural goods, seven percent to food products, and 23 percent to various business services. For a selection of low-income developing countries those shares were nine percent, 15 percent, and 14 percent respectively.

Evidence in Hunter (1991) gives strong evidence in support of non-homothetic preferences and also shows that this influence is in the direction of reducing the volume of trade. Put the other way around, Hunter did a counter-factual analysis of her econometric results by neutralizing the non-homotheticity and found that for the average country and commodity category the effect of consumption neutralization by imposing homogeneity was to raise trade flows by 29 percent. It is not clear why her results were later dismissed without evidence by Trefler (1995) as an explanation for missing trade. Cassing and Nishioka (2009) use a neutralization exercise similar to Hunter's and find that developing countries do consume relatively more labor-intensive goods (largely agriculture and food in their data) than would be expected under preference homogeneity. Second, they find that preference biases between rich and poor countries explain a larger proportion of missing factor trade than do differences in technology, though preference differences are not distinguished from non-homotheticity.

Results in the section introducing imperfect competition are similar to those found in Wong (2003) and Simonovska,(2009) which is that markups and hence the price level will be higher in the high per-capita-income country. Simonovska gets strong empirical support for this relationship, also found in earlier papers including Hsieh and Klendow (2007). I should also note that Simonovska carefully considers identical products, which eliminates quality issues which could be an alternative explanation for systematic price differences by per-capita income. Essentially the same result was found by Wong for pricing of identical pharmaceutical products.

An area where per-capita income does play an important role is in the analysis of product quality. If a consumer is going to buy only one unit of a good or zero, then the quality demanded is likely to depend on per-capita income. This makes the average level of per-capita income important for trade but also the intra-country distribution of income matters for inter-country trade. I have always felt that the reason there are so many Mercedes in Africa is due to the highly unequal distribution of income: if everyone had the average income there would be none. Because the issues connected with product quality are somewhat better understood and because they require a quite different analytical approach, they will not feature in this paper despite their importance. For analyses of product quality and/or the importance of intra-country income distribution (which I will treat briefly) see Choi, Hummels and Xiang (2009), Dalgin, Trindade and Mitra (2008),Fajgelbaum, Grossman and |Helpman (2009), Flam and Helpman (1987), Francois and Kaplan (1996), Hallak (2006), Manova and Zhang (2009), Mitra and Trindade (2005), Schott (2004), Shaked and Sutton (1983, 1984), and Stokey (1991).

2. A Generic Model

The preferences we will use are variation on a standard Stone-Geary utility function, to be introduced shortly. The production side of the model is deliberately Heckscher-Ohlin to permit an easy comparison with traditional results. There are two good (X and Y), two factors of production (K and L) and two countries home and foreign (h and f).

Throughout the paper, the following assumptions are made.

(1) good X is relatively capital intensive, and Y is relatively labor intensive
(2) good X has an income elasticity of demand greater than one

(3) the labor supply to production is identical to the number of households
(4) country h is relatively capital abundant and f is labor abundant, except when relative endowments are assumed identical
(5) country h has higher productivity when productivities differ across countries.

Most of these assumptions are without loss of generality, except for assumption (2): combining (1) and (2) will mean that the capital-intensive good has the high income-elasticity of demand. This strongly influences the results, but it will generally be obvious what happens if the assumption is reversed. Empirical support for this assumption is found in Bergstrand (1990) and indirectly in Hunter (1991) and Cassing and Nishioka (2009). Matsuyama (2000) uses an equivalent assumption in his Ricardian model: the South's comparative advantage goods are low-income-elasticity goods.

Since we will focus on a limited number of experiments, some short-hand terminology is used throughout. *Productivity advantage* or growth, or higher productivity refers to an equal proportional Hicks-neutral productivity advantage or growth in *both* sectors in one country (always taken to be country h). Country h may have an absolute advantage over f, but there is *no comparative advantage* based on productivity differences. *Factor accumulation* refers to a equal proportional growth in factor endowments of one or both countries: factor accumulation increases the number of households in the same proportion.

In addition to *X* and *Y*, there is a parameter which can be thought of as an endowment good denoted by *Z*. There is a fixed number of units of *Z* per household, denoted *z*, with aggregate *Z* then being strictly proportional to the country's labor endowment. Under the interpretation that *Z* is an endowment good, we assume that *households cannot buy or sell Z*. Using lower-case letters for per-household quantities, preferences are given as follows.

$$u = (x + z)^{\beta} y^{1-\beta} \qquad (1)$$

The assumption that *x* and *z* are additive has little to with the results of this paper, but has the advantages that (a) there is an explicit solution for demands and (b) perfect aggregation within an economy holds: aggregate demand does not depend on the distribution of income (with a qualification noted below). Again thinking of *Z* as an endowment good that cannot be exchanged, *X* could be televisions and *Z* could be watching a sunset (non-rivaled and non-excludable: sitting on a dock on the bay as Otis Redding might say).

It is more common to see Stone-Geary written with *(y - z)* instead of *(x + z)*, with *z* then referred to as a "minimum consumption requirement". But this leads to a problem if someone's income is insufficient to purchase the minimum consumption requirement (they die?) and no household (having at least the minimum income) will ever be observed to purchase only good *Y*. In addition, the formulation in (1) will mean that the price elasticity of demand for *X* will be falling in per-capita income, a property exploited in Simonovska, as we shall see.

Let m^i denote the income of household i and let p_x and p_y denote the prices of *X* and *Y*. The households budget constraint is given by:

$$m^i = p_x x + p_y y \tag{2}$$

Maximization of (1) subject to (2) gives the following Marshallian demand function for *X* at the household (*x*).   .

$$x^i = \max\left[0,\ (\beta - 1)z + \frac{\beta m^i}{p_x}\right] \qquad y^i = \min\left[\frac{m^i}{p_y},\ \frac{(1-\beta)(m^i + p_x z)}{p_y}\right] \tag{3}$$

$$x^i > 0 \quad iff \quad m^i > \frac{(1-\beta)}{\beta} p_x z \tag{4}$$

At low levels of income, the household buys only good *Y*, and above the threshold income indicated in (4), begins to buy *X*.   This is an interesting and surely realistic point, and it makes aggregate demand depend on the distribution of income. This has been dealt with in a number of papers as I have indicated and I will treat it relatively briefly here. I will assume for much of the paper that (4) holds with strict inequality for all households. Given this assumption, perfect aggregation holds within an economy and aggregate demand is independent of the distribution of income.

Let *L* denote *both* the country's labor supply *and* household measure so that $Z = zL$ denotes the economy-wide "endowment" of *Z*: *z* is a parameter, while *Z* is strictly proportional to the number of households. Given that (4) holds for all households, aggregate demand for X is given by

$$X = \sum_1^L x_i = (\beta - 1)Z + \frac{\beta M}{p_x} \qquad Z = zL \qquad M = \sum_{i=1}^L m^i \tag{5}$$

Now consider the income elasticity of demand for *X* and assume that income grows through a productivity increase, holding the number of households *L* and therefore *Z* constant.

$$\left[\frac{M}{X}\frac{dX}{dM}\right]_{dZ=0} = \frac{\beta M}{\beta M + (\beta - 1)p_x Z} = \frac{\beta M}{p_x X} = \frac{\beta}{p_x X/M} = \frac{MPC}{APC} > 1 \tag{6}$$

(growth through productivity improvement)

where *MPC* denotes the marginal propensity to consume *X*, equal to β, while *APC* is the average propensity to consume *X*, an observable. If growth instead occurs through factor accumulation, so that *Z* is strictly proportional to *M*, then

$$\left[\frac{M}{X}\frac{dX}{dM}\right]_{dZ/Z = dM/M} = 1 \qquad \text{(growth through factor accumulation)} \tag{7}$$

The price elasticity of demand (Marshallian: holding income constant) is given by

$$\frac{dX}{dp_x} = -\frac{\beta M}{p_x^2}$$

$$\left[\frac{p_x}{X}\frac{dX}{dp}\right] = -\frac{\beta M}{\beta M + (\beta - 1)p_x Z} = -\frac{\beta M}{p_x X} = -\frac{\beta}{p_x X/M} = -\frac{MPC}{APC} < -1 \quad (8)$$

Thus the income (from productivity growth) and price elasticities of demand for *X* are (locally) the same.

The properties of preferences and aggregate demand for the economy, holding prices constant are shown in Figure 1. Let Z_0 denotes the initial value of *Z*. Hold *Z* constant but allowing aggregate income to vary either through productivity or through capital accumulation, holding *L* constant. This leads to an Engel's income-consumption curve that starts at the origin and moves up the *Y* axis (consuming only *Y*) to point Y_0 after which higher income will result in positive *X* demand. At incomes above that which allows point Y_0 to be reached, the Engel's curve is linear through A at income level M_0 and reaching B at higher income level M_1. As noted earlier, we largely restrict ourselves to incomes and *z* levels such that all households can reach Y_0.[2]

Consider point A and income level M_0 in Figure 1. Now suppose instead we let the economy grow through proportional factor accumulation, adding households in strict proportion to the increase in income, so *Z* and *M* grow to Z_1 and M_1 respectively. Now the Engel's curve will be given by a ray through the origin and points A and C and aggregate demand is homothetic with respect to aggregate income in the case where income changes purely through a proportional change in endowments.

From this point on, I will present results in terms of simulations. All of the results are intuitive, some are found formally in earlier papers and I am quite sure that all of the qualitative properties of all results have no dependence on the specific parameters or other assumptions used in these specific examples. Figures 2 and 3 show the properties of a model when we vary the productivity level of two identical countries (essentially a one-country case if you like). An initial "calibration" point is at productivity one, which gives an income and price elasticity of 1.333 and a share of *X* in consumption of 0.5; the value of β = 2/3 is used in this example and throughout the paper. As productivity grows without bound the income and price elasticity approaches one and the consumption share approaches its marginal value of 2/3 in Figure 2.

With the neutral and equal productivity growth in both sectors, the production frontier of the economy is growing radially, but demand is shifting toward good *X*. This generates a movement around the frontier toward *X*, so the relative price of *X* rises as shown in Figure 3.

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[2]Though not particularly important here, a proportional increase in productivity across sectors is going to change relative prices, and hence per-capita income will not change in strict proportion to productivity.

But this generates the usual Stolper-Samuelson effect on relative factor prices, so the rental (*r*) - wage (*w*) ratio *r/w* is rising as shown in Figure 3. Suppose we interpret capital as skilled labor or human capital and *L* as unskilled labor. A neutral productivity growth generates an increase in the wage gap between skilled and unskilled labor. Thus we can get a wage gap phenomenon driven entirely by the demand side of the general-equilibrium model without appealing to trade or to skill-biased technical change.

Now consider differences in relative endowments, beginning with the two countries identical, under the assumption of costless trade. Now move capital from f to h and labor from h to f, implying the *Z* rises in f and falls in h by an equal and opposite amount. Their Engel's curves will move apart in Figure 1 but they remain parallel. The effect on the volume of trade is shown in Figure 4 both for the case of non-homothetic demand and an alternative case in which *Z*s are held at their initial and equal levels, which we label as homothetic demand. Units are chosen such that free trade under homothetic demand generates a trade volume of 1.0 when both countries are fully specialized.

The effect of non-homotheticity under identical productivities can be to reduce the volume of trade, an effect noted in Markusen (1986) and again in several later papers. This reduction is proportional within the factor-price equalization set (production is the same in each country with and without homotheticity because world demand and prices are not changing). At the boundary of the FPE set, which occurs at $K_h = 0.8$, $L_h = 0.2$ and vice versa for country f (the factor shares in sectors *X* and *Y* are $(K/L)_x = 80/20$ and $(K/L)_y = 20/80$) both countries specialize. Further spread in the endowment ratios leads to a fall in trade volume because incomes are falling. We are sometimes sloppy about this in maintaining that trade volume is an increasing function of endowment differences: this is clearly only true within the FPE set. Figure 4 shows that the divergence in the trade volume between the homothetic and non-homothetic cases gets bigger outside of the FPE with incomes falling.

Figure 5 is the same experiment of widening the endowment differences but presents the information in a different way. It graphs the share of world consumption and production in each country. The consumption shares in this exercise would be constant at 0.5 under homothetic demand. But under our assumption that the capital intensive good is the high-income-elasticity good, the consumption shares are positively correlated with their respective country's production share. Figure 5 gives a demand-side explanation for two phenomenon that have previously been identified and attributed to production-side causes. The positive correlation between production and consumption shares has been one (of several) definition of "home bias". Secondly, the volume of trade is less under our assumptions than is predicted under a standard Heckscher-Ohlin model and thus offers a demand-side explanation for the empirical puzzle of "missing trade" in Trefler's (1995) terminology. The amount of missing trade is identified in Figure 5 and note that it continues to grow in imporrtance once countries are specialized: production specialization cannot continue to increase but consumption specialization can. As noted earlier, non-homotheticity as a cause of missing trade is empirically verified in Hunter (1991) and conjectured in Cassing and Nishioka (2009).

Identical but non-homothetic preferences as a pure basis for trade is shown in Figure 6. In this experiment, both countries are initially identical. Then we raise the productivity level in country h by a certain proportion and reduce its factor endowment by the inverse proportion. The opposite is done in country f: its factor endowment grows at the same proportion as country h's productivity and country f's productivity falls by the same proportion as country h's endowment. In a Heckscher-Ohlin world, both countries would have the same aggregate income and there would be no trade. In our case, demand shifts toward *X* in country h and toward *Y* in country f as we see in Figure 6. Trade is created purely through the demand side of the economies: at all point in the graph, the countries produce the same relative proportions of *X* and *Y*. Per-capita income becomes an independent basis for trade or, if we like, absolute advantage (productivity differences) become a basis for trade. Closely related points in the Ricardian context are found in Matsuyama (2000).

Figure 7 shows a role for intra-country income distribution, which I deal with briefly because it has been noted before.[3] If each consumer in a country has enough income as given in (4) to want positive amounts of *X*, then the linear property of the Engel's curve means that redistribution of income within the country (subject to (4) continuing to hold for all households) does not affect aggregate demand. But if redistribution puts some households on the vertical section of the curve in Figure 1 where they only buy *Y* (points below Y^0), then it does matter.

Figure 7 conducts an experiment in which two identical household types have the same identical amounts of *L* and *K*, but we scale up and the productivity of one type (high) and down the productivity of the other, which holds aggregate production possibilities constant. The share of income spent on *X* increases for high and decreases for low as shown. Eventually, the low-income type stops consuming *X* while the high-income type continues to devote a larger share of income to *X*. Thus aggregate *X* production and consumption (a simple closed-economy is shown) increase even though the production frontier is unchanged. As in Figure 3, the relative prices of *X* and *K* must rise. As has been noted before, an increasing spread in the income distribution leads to a larger aggregate demand for luxury goods (those Mercedes in Africa).

3. Imperfect competition, prices and markups

In this section, we add scale economies, imperfect competition, and free entry and exit of firms in the *X* industry in a standard model of Cournot competition, continuing with the assumption that *X* is a homogeneous good. *Y* is produced with constant returns under perfect competition. We assume segmented markets simply because the results are more interesting and in line with Simonovska for example, so the model is similar to Venables (1985) or Markusen and Venables (1988), the latter contrasting segmented and integrated markets cases. This is placed on top of our 2x2x2 Heckscher-Ohlin structure as before.

In order to keep things simple and the results sharp, we will continue to assume that trade is costless and work only within the factor-price-equalization set. Thus any firm supplying country i will have the same marginal cost and the same market share when

[3] For example, Choi et. al. (2009), Daglin et. al. (2008), Francois and Kaplan (1996), Hallak (2006), and Mitra and Trindade (2005).

countries differ in endowments. This will remain true when countries differ in productivity as we have defined it: the country with higher productivity will have a proportionately higher price for each factor, so marginal costs will be the same in the two countries.[4]

Revenue for a Cournot firm i and selling in country j is given by the price in j times quantity of the firm's sales. Price is a function of all firms' sales.

$$R_{ij} = p_j(X_j)X_{ij}. \text{ where } X_j \text{ is total sales in market j by all firms: } X_j = \sum_i X_{ij} \qquad (9)$$

Cournot conjectures imply that $\partial X_j / \partial X_{ij} = 1$; that is, a one-unit increase in the firm's own supply is a one-unit increase in market supply. Marginal revenue is then given by the derivative of revenue in (9) with respect to firm i's output (sales) in j.

$$\frac{\partial R_{ij}}{\partial X_{ij}} = p_j + X_{ij}\frac{\partial p_j}{\partial X_j}\frac{\partial X_j}{\partial X_{ij}} = p_j + X_{ij}\frac{\partial p_j}{\partial X_j} \qquad \text{since } \frac{\partial X_j}{\partial X_{ij}} = 1 \text{ (Cournot)} \qquad (10)$$

Now multiple and divide the right-hand equation by total market supply and also by the price.

$$\frac{\partial R_{ij}}{\partial X_{ij}} = p_j + X_{ij}\frac{\partial p_j}{\partial X_j} = p_j + p_j\frac{X_{ij}}{X_j}\left[\frac{X_j}{p_j}\frac{\partial p_j}{\partial X_j}\right] \qquad (11)$$

The term in square brackets in (11) is just the inverse of the price elasticity of demand. Following convention, we will denote minus the elasticity of demand by the Greek letter $\eta > 0$. We can then write (11) as

$$\frac{\partial R_{ij}}{\partial X_{ij}} = p_j\left[1 - \frac{X_{ij}}{X_j}\frac{1}{\eta_j}\right] \qquad \eta_j \equiv -\left[\frac{p_j}{X_j}\frac{\partial X_j}{\partial p_j}\right] \qquad (\textit{elasticity of demand}) \qquad (12)$$

The term X_{ij}/X_j in (12) is just firm i's market share in market j, which we can denote by s_{ij}. Marginal revenue = marginal cost is given by:

$$mr_{ij} = p\left[1 - \frac{s_{ij}}{\eta_j}\right] = mc_i \qquad (13)$$

Marginal revenue in Cournot competition turns out to have a fairly simple form as shown in (13). The term s_{ij}/η_j is the markup, with the price elasticity η given by (minus) (8).

The interesting thing about the segmented markets case is that the markup can differ between countries even with costless trade if the countries' per-capita incomes differ leading

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[4]Nominal factor-price equalization holds, the price index differs between countries. But with costless trade, each firm sells the same amount to each market at the same prices as any other firm regardless of location, and so the revenue of each firm is equal, regardless of country of location. With free entry and exit, marginal costs will also be equal (assuming firms in both countries).

to a difference in their price elasticities of demand. Figure 8 presents a case similar to Figures 3 and 4 earlier: starting with two identical economies, capital is transferred from f to h and labor from h to f, preserving the totals. Trade is costless and we will look just within the factor-price equalization set which remains the same. In the initial benchmark with countries identical and trade costs zero, trade volume is indeterminate and this indeterminacy is broken by a very small trade cost of 0.01 percent: firms in both countries engage in "reciprocal dumping" and have (almost) the same market share in both countries.

In Figure 8, proportional differences between the non-homothetic versus the homothetic case are plotted. Because country h has an increasingly higher per-capita income as we move to the right and relative endowments become increasingly unequal, the price elasticity in country h falls relative to f and thus the markup in country h rises relative to country f as suggested by (13). With the price of Y equalized between countries, this means that the price index is greater in country h as shown in Figure 8. I also run a counter factual with homothetic preferences to derive the difference in trade volume from the non-homothetic case and the result shown in Figure 8 is consistent with the result in the competitive model: trade volume is lower in the non-homothetic case as countries' shares of production and consumption are correlated. The results on prices and markups is consistent with those in Wong (2003), Simonovska (2009) and Hsieh and Klendow (2007).

Figure 9 conducts the same experiment as in the competitive case of Figure 6. Both countries are initially identical. Then we raise the productivity in h and lower its endowment in the inverse proportion, and we do the opposite for country f. Thus per-capita income rises in country h as we move to the right and per-capita income in country f falls. At all points on the horizontal axis, the countries have roughly (discussed shortly) the same total income and the same relative factor endowments. In the traditional approach with homothetic demand, there would be reciprocal dumping trade that is balance, in other words balanced intra-industry trade and no inter-industry trade. In our case, inter-industry trade is generated as we move to the right in Figure 9 since X consumption shifts to country h and away from country f. This outweighs any loss of intra-industry trade and total trade volume is now *higher* under non-homothetic demand as it was in the similar experiment with the competitive model in Figure 6. The markup is higher in country h and hence the price level must be higher in country h.

One final result worth noting is that the higher markup and X price in country h in both the experiments in Figures 8 and 9 mean that the aggregate and per-capita income differences between the countries are somewhat smaller than in the homothetic-demand case. In the latter case, total incomes of the two countries would be the same as we move to the right in Figures 8 and 9. In the non-homothetic demand case, aggregate real income of country h divided by country f is 0.95 at the value 0.80 on the horizontal axis of Figure 8. The income ratio is 0.88 at the value 3 on the horizontal axis of Figure 9. This price elasticity and markup effect leads to a transfer of income from the high to the low per-capita-income country.

4. Thoughts on calibration, estimation and gravity

I want to conclude by offering some thought on the implications of the analysis for calibrated modeling (applied general-equilibrium analysis) and econometric estimation. The usual procedure in AGE modeling is to assume a homothetic functional form such as Cobb-Douglas for example, and then used observed expenditure shares in the data to calibrate the share parameters of the Cobb Douglas. In the numerical case used in Figures 2-6, the initial expenditure shares observed in the data at point 1.0 on the horizontal axes of Figures 2, 3, and 6 is 0.5.

Refer back to Figure 1 and think of expenditure shares at point A as being 0.5. The standard AGE calibration method then assumes that any expansions of the economy will be on the Engel's curve through A and C. However, the marginal expenditure share per household used in these experiments is 0.667 ($\beta = 2/3$). Thus if some counterfactual leads to an increase in per-capita income, the actual Engel's curve is that through A and B in Figure 1. Furthermore, if two countries are observed to have different initial budget shares, such as one country at A and one at B in Figure 1, then they will be calibrated as having different but homothetic preferences when in fact they might have identical but homothetic preferences.

There is a rather simple procedure that GE modelers can use to recalibrate their models to identical but non-homothetic preferences of the type used here. First, their data can be used as a cross-section data set to estimate the type of Stone-Geary utility function used here: the Stone-Geary yields the familiar linear expenditure system, which is the approach used in Hunter and Markusen (1989). This could, for example, generate a common value of β in our two-good case, or more generally a set of βs across consumption goods. Then in each country the *z* or *Z*s could be calibrated by using the observed consumption shares in the data with the estimated βs. In our two-good case, a rearrangement of (5) gives us

$$s_x = \frac{p_x X}{M} = \beta + \frac{(\beta - 1) p_x \bar{Z}}{M} = \beta + (\beta - 1) s_z \tag{14}$$

where s_x is the *observed* share of *X* in expenditure and s_z is the *calibrated* share of (unobserved) *Z* in income using an econometric estimate of β. *Z* is then treated as a non-traded endowment good In counterfactual experiments not involving a change in the number of households *Z* is held constant while it is allowed to vary with the number of households when that occurs. This procedure then distinguishes between movements along Engel's curves AC and AB in Figure 1. Failing to do so as in standard models will lead to a mis-prediction about the effects of world productivity growth or capital deepening.

There is a long history of fitting Heckscher-Ohlin theory (Leamer 1980, Maskus 1985, Davis and Weinstein 1991, Hakura 2001, Trefler 1995). A much simplified description of this literature is that it examines the relationship $E = X - C$, where *E* is the net export vector, *X* is the production vector and *C* is the consumption vector. E is then converted into the "factor-content of trade". A simple procedure, for example, is to estimate a common technology matrix [*A*] where a_{ij} is good j's use of factor i. Then the *measured* factor content of trade for a country is [*A*]*E*. The *predicted* factor content of the production vector is just the country's endowment vector, *V*.

A considerable amount of effort has gone into trying to improve the fit through modifications of the basic model but I think that it is reasonable to say that almost all of that effort has gone into improving the measured factor content of production. In most all cases I am aware of, *C* is in fact not really fitted at all nor is it given by data: *C* is *assumed* to be given by sX^w, where *s* is the country's share of world income and X^w is the observed world production vector, equal to the world consumption vector. So the estimators *impose* the assumption of identical and homothetic preferences across countries and of course equal relative prices. The factor content of consumption is then $s[A]X^w = sV^w$. This gives the relataionship $[A]E = (V - sV^w)$, where the left-hand side is the measured factor content of trade and the right-hand side is the predicted value. The typical result in the simplest formulations is that there is substantial "missing trade": measured value is much less than the predicted value.

The findings of Hunter (1991) and Cassing and Nishioka (2009) should encourage researchers to devote some effort to improving the estimation of the consumption vector. Let *C* continue to denote the consumption vector and assume identical but non-homothetic preferences across countries and assume that commodity prices are the same everywhere (this is not a new assumption here, it is used in virtually all of the literature referred to). Let β_i denote the marginal share of good i in consumption and $z_i^k = z_i L^k$ be the fixed "endowment" of good z_i. in country k. The hypothesized demand for good C_i in country k is given by

$$C_i^k = -Z_i^k + \beta_i (M^k + \sum_j p_j Z_j^k)/p_i \tag{15}$$

These demand values can be converted into shares of consumption σ_i^k by multiplying through by the price of good i and dividing by income

$$\sigma_i^k = \frac{p_i C_i^k}{M^k} = \beta_i + \frac{\beta_i \left(\sum_j p_j Z_j^k \right) - p_i Z_i^k}{M^k} \qquad \text{where} \tag{16}$$

$$\sum_i \sigma_i^k = \sum_i \beta_i = 1 \quad \text{and} \quad \sum_k \frac{M^k}{M^w} \sigma_i^k = \sum_k s^k \sigma_i^k = \sigma_i^w \tag{17}$$

where superscript w denote world aggregate values. Note that this demand system satisfies adding up: summing (16) over i (goods), the final terms on the right-hand side sum to zero and the sum of the shares equals the sum over β which is one as noted in (17). The second equation of (17), which will be used shortly, is an implication that our demand system in (15) also satisfies global adding up ((15) applies to global consumption and income as well), implying that the share of i in world consumption σ_i^w is the income-share weighted sum of the consumption shares across countries ($s^k = M^k/M^w$).

Now introduce our assumption we have used throughout that $Z_i^k = z_i L^k$ is strictly proportional to the labor supply L^k and that $M^k = m^k L^k$, where m is average per-capita income.[5] The L^k can be factored out of both the numerator and denominator of (16) and cancelled out. (16) can be written as:

$$\sigma_i^k = \beta_i + \frac{\beta_i(\sum p_j z_j^k) - p_i z_i^k}{m^k} = \beta_i + \frac{\beta_i(\sum p_j z_j) - p_i z_i}{m^k} \qquad z_i^k = z_i \ \forall k \qquad (18)$$

In our hypothesized preferences and demand system, the z_i's are *common parameters* across countries and hence the country superscript can be dropped. The numerator of the right-hand term in (18) is identical across countries, and so (18) can be written as

$$\sigma_i^k = \beta_i + \frac{\alpha_i}{m^k} \qquad \sum_i \alpha_i / m^k = 0 \qquad (19)$$

where the right-hand equation follow from (16). Since our demand system imposes identical preferences across countries, the σ_i^k's only differ from world average values by differences across countries in per-capita income, which is easily observed and calculated. These α and β parameters can be estimated across countries. Then let (19) give the fitted or predicted value of $\sigma = \sigma_i^k$ and let σ_i^w denote the *observed* share of good i in world consumption, which in turn is equal to its share in world production X^w. Let M^k denote the country's income and M^w denote world income (both scalars). Then the *predicted* demand for good i in our country is given by

$$C_i^k = \sigma_i^k M^k = \frac{\sigma_i^k}{\sigma_i^w} \sigma_i^w M^k \qquad (20)$$

where

$$\sigma_i^w M^k = \frac{X_i^w}{M^w} M^k = \frac{M^k}{M^w} X_i^w = s^k X_i^w \qquad (21)$$

Substituting (21) into (20), we have the predicted value of consumption.

$$C_i^k = \left[\frac{\sigma_i^k}{\sigma_i^w}\right] s^k X_i^w \qquad C^k = s^k [\Sigma^k] X^w \qquad \sum_k C^k = [I] X^w \qquad (22)$$

---

[5]If all households have sufficient income to satisfy (4) and buy positive amounts of X, then total income is just the average per-capita income times L: suming over i households we get $M = \sum (m_i / L) L = \bar{m} L$ where $\bar{m}$ is average per-capita income, an observable.

where Σ is a diagonal matrix with its ith diagonal element equal to σ_i^k/σ_i^w and I is the identity matrix. The equations in (22) satisfy adding up over countries, which follows from applying the right-hand equation of (17) to the second equation in (22). Let the predicted factor content of world consumption be given by

$$V^w = [A]X^w \qquad \text{so} \qquad s^k[A]X^w - s^kV^w = 0 \tag{23}$$

Adding the right-hand equation of (23) to (X - C), the predicted factor-content of trade is then

$$[A](X^k - s^k\Sigma^k X^w) = (V^k - s^kV^w) + s^k[A][I - \Sigma^k]X^w \tag{24}$$

where the last term on the right-hand side is the "correction" for non-homotheticity (I is the identity matrix). Once again, adding up is satisfied when summing over countries k: the first bracketed term in (24) is zero by construction of V^w; the second term is zero as noted in the second and third equations of (22).

The expression on the left-hand side of (24) gives the intuition about how per-capita income differences due to different relative endowments combined with the assumption that labor intensity and low income elasticity are correlated tends to lower the predicted volume of trade. Under these assumptions, there should be a systematic correlation such that goods in which the country is relatively specialized in production $(X_i/sX_i^w) > 1$ are goods in which the country is relatively specialized in consumption $(\sigma_i^k/\sigma_i^w) > 1$ (or the ith element of $[I - \Sigma^k]$ is $(1 - \sigma_i^k/\sigma_i^w) < 0$). Then (24) tends tor reduce the predicted value of trade for any vector of world outputs. The difference between actual and fitted values of trade would then be reduced, reducing missing trade, given that trade volumes are over-predicted in the standard analysis where $(\sigma_i^k/\sigma_i^w) = 1$ by assumption. However, I emphasize that such an outcome is by no means trivial or guaranteed. If differences in per-capita incomes reflect differences in productivity rather than differences in capital / labor endowments, then predicted trade could be even larger than under homothetic preferences as we noted in Figures 6 and 8.

A final point has to do with standard gravity equations. A common practice is to put in separate terms for both aggregate income (or population) and per-capita incomes, yet there is rarely if ever justification for this. With homothetic preferences, aggregate income should soak up all the explanatory power. Yet estimates of this type invariably show an important, positive and independent role for per-capita income. If (and it is not an obvious if) traded goods are income elastic goods, then this positive dependence makes sense.

There are a great many formulations of the gravity equation in the literature. Let me give a quick and simple example provided by Frankel et. al. (1998). Let M_i denote aggregate income, P_i denote population for countries 1 and 2, and T_{12} the trade between countries i and j. The Frankel et. al. formulations is

$$\ln T_{12} \; = \; \alpha \ln(M_1 * M_2) \; + \; \beta \ln\left(\frac{M_1}{P_1} * \frac{M_2}{P_2} \right) \; + \; .... \tag{25}$$

In levels, this is equivalent to

$$T_{12} \; = \; (M * M_2)^{\alpha} \left(\frac{M_1}{P_1} * \frac{M_2}{P_2} \right)^{\beta} \; = \; (M * M_2)^{\alpha \, + \, \beta} (P_1 * P_2)^{-\beta} \tag{26}$$

If the world is characterized by homothetic preferences, then aggregate income is all that matters, so the hypothesis is $\beta = 0$ and indeed unity for α: $\alpha = 1$.

If preferences are non-homothetic, then we should find β to be non-zero, although in what direction is not obvious. If traded goods are income elastic (we should really include traded services here), then we should find $\beta > 0$, $\alpha = 1$, and $\alpha + \beta > 1$. Frankel et. al.'s results are that $\alpha = 0.72$ and $\beta = 0.23$ with both coefficients significant at the one-percent level.

The hypothesis that $\beta > 0$ and that per-capita income is important is clearly supported and suggests that traded goods are income elastic. But some puzzle remains over the fact that the hypothesis $\alpha = 1$ and $\alpha + \beta > 1$ are not supported (by the point estimates). This could just reflect growing intra-country trade as a country grows in size but not per-capita income, long felt to be an explanation as to why larger countries tend to have a lower ratio of trade to GDP.

In general, I feel that the gravity literature gives good support to the notion that per-capita income has an independent and positive influence on trade, but it is beyond the scope of this paper to give a more comprehensive literature review.

5. Summary

As suggested in the introduction, there are bits-and-pieces of theoretical and empirical analysis about the role or roles for per-capita income in determining trade flows. But there is little unity and by and large per-capita income is not given much of a place as an important determinant of trade. This paper tries to unify and connect the bits, and to offer some further ideas about how per-capita income might matter. I offer a "generic" model that I am hoping might prove to be useful for graduate teaching, a sort of all-in-one model that nests a number of other contributions.

The model imposes a variant of Stone-Geary preferences (used before by a number of authors) on top of a traditional 2x2x2 Heckscher-Ohlin model. Maintained hypotheses are that labor endowments in the HO model are strictly proportional to the number of households and that the capital-intensive good in the HO model is the high income-elasticity-of-demand good. The latter assumption is testable and fasifiable. Results from the model offer a strictly demand-side explanation for a range of phenomena including (a) home bias in consumption, (b) the mystery of the missing trade, (c) a growing wage gap in an environment of growing productivity and (d) a role for the intra-country distribution of income similar to that found in the product-quality literature (high inequality - more demand for luxury goods).

I then add an assumption of increasing returns to scale in the capital-intensive, high-income-elasticity industry with free entry and exit of firms, Cournot pricing and segmented markets: a common framework in the so-called new trade theory and strategic trade-policy literatures. This generates some interesting and testable results, in particular higher markups and higher price levels in higher per-capita-income countries. As in the case of the competitive examples, most of the implications have already received good empirical support.

In both competitive and imperfect-competition cases (a) the effects of growth are quite different depending on whether it is growth in productivity or in factor accumulation. (b) when per-capita incomes differ between countries, there is inter-industry trade even with identical aggregate incomes and relative factor endowments.

The paper concludes with a couple of suggestions about how the model might be useful in further calibration and estimation research and for including and interpreting per-capita income coefficients in gravity models.

REFERENCES

Bergstrand, Jeffrey H. (1990), "The Heckscher-Ohlin Samuelson Model, the Linder Hypothesis and the Determinants of Bilateral Intra-Industry Trade", *The Economic Journal* 100, 1216-1229.

Cassing, James and Shuichiro Nishioka (2009), "Nonhomothetic Tastes and Missing Trade in Factor Services", working paper.

Choi, Yo Chul, David Hummels and Chong Xiang (2009), "Explaining Import Quality: The Role of Income Distribution", *Journal of International Economics* 77, 265-275.

Dalgin, M., Vitor Trindade and Devashish Mitra (2008), "Inequality, Non-homothetic Preferences and Trade: A Gravity Approach", *Southern Economic Journal* 74, 747-774.

Davis, Donald R. And David E. Weinstein (2001), "An Account of Global Factor Trade", *American Economic Review* 91, 1423-1453.

Fajgelbaum, Pablo, Gene M. Grossman and Elhanan Helpman (2009), "Income Distribution, Product Quality, and International Trade", working paper.

Flam, Harry and Elhanan Helpman (1987), "Vertical Product Differentiation and North-South Trade ", *American Economic Review* 77, 244-822.

Francois, Joseph F. and S. Kaplan (1996), "Aggregate Demand Shifts, Income Distribution, and the Linder Hypothesis", *Review of Economics and Statistics* 78, 244-250.

Hakura, Dalia S. (2001), "Why Does HOV Fail? The Role of Technological Differences within the EC", *Journal of International Economics* 54, 361-382.

Hallak, Juan-Carlos (2006), "Product Quality and the Direction of Trade", *Journal of International Economics* 68, 238-265.

Hsieh, C. and P. Klenow (2007): "Relative prices and relative prosperity," *American Economic Review*, 97, 562–585.

Hunter, Linda (1991), "The Contribution of Non-homothetic Preferences to Trade", *Journal of International Economics* 30, 345-358.

Hunter, Linda and James R. Markusen (1988), "Per-Capita Income as a Determinant of Trade," in Robert Feenstra (editor), *Empirical Methods for International Economics*, Cambridge: MIT Press 89-109.

Leamer, Edward E. (1980), "The Leontief Paradox Reconsidered", *Journal of Political Economy* 88, 495-503.

Linder, Staffan B. (1961). *An Essay on Trade and Transformation.* Stockholm: Almqvist &

Wiksell.

Manova, Kalina and Zhiwei Zhang (2009), "Export Prices and Heterogeneous Firm Models", Stanford University working paper.

Markusen, J. R. (1986). "Explaining the Volume of Trade: An Eclectic Approach." *American Economic Review* 76, 1002-1011.

Markusen, James R. and Anthony Venables (1988), "Trade Policy with Increasing Returns and Imperfect Competition: Contradictory Results from Competing Assumptions," *Journal of International Economics* 24, 299-316.

Maskus, Keith E. (1985), "A test of the Heckscher-Ohlin-Vanek Theorem: the Leontief Commonplace", *Journal of International Economics* 19, 201-212.

Matsuyama, Kiminori (2000), "A Ricardian Model with a Continuum of Goods under Non-Homothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade", *Journal of Political Economy* 108, 1093-2000.

Mitra, Devashish and Vitor Trindade (2005), " Inequality and Trade", *Canadian Journal of Economics* 38, 1253-1271.

Schott, Peter K. (2004), "Across-Product versus Within-Product Specialization in International Trade," *Quarterly Journal of Economics* 119, 647-678.

Shaked, A. and John Sutton (1983), "Natural Oligopolies", *Econometrica* 51, 1469-1483.

Shaked, A. and John Sutton (1984), "Natural Oligopolies and International Trade", in Henryk Kierzkowski (editor), *Monopolistic-Competition and International Trade*, Oxford: Oxford University Press.

Simonovska, Ina (2009), "Income Differences and Prices of Tradables", working paper.

Stokey, Nancy L. (1991), "The Volume and Composition of Trade between Rich and Poor Countries", *Review of Economic Studies* 58, 63-80.

Trefler, Daniel (1995), "The Case of Missing Trade and Other Mysteries", *American Economic Review* 85, 1029-1046.

Venables, Anthony J. (1985), "Trade and trade-policy with imperfect competition - the case of indentical products and free entry", *Journal of International Economics* 19, 1-19.

Wong, Eina (2003), "Pharmaceutical Drug Prices in the International Market", unpublished PhD thesis, University of Colorado, Boulder.

Table 1: Income elasticities of demand, budget shares for various consumption goods and services

Food	0.45
Household furniture	0.76
Fuel and power	0.81
Education	0.87
Clothing and footwear	1.00
Beverages and tobacco	1.23
Other	1.25
Recreation	1.42
Transport, communication	1.72
Gross rent	1.74
Medical	1.91

Hunter and Markusen (1989)

Hunter (1991) budget shares

	low income	high income
food	0.50	0.17
furnishing	0.05	0.09
medical	0.04	0.09

Cassing-Nishioka (2009)

	low income	high income
agricultural	0.09	0.02
food prod	0.15	0.07
bus serv	0.14	0.23

Hunter (1991) counterfactual: homogeneous demand, calculate trade flows

Result: raises trade flows by 29%

Figure 1: Growth through productivity versus factor accumulation

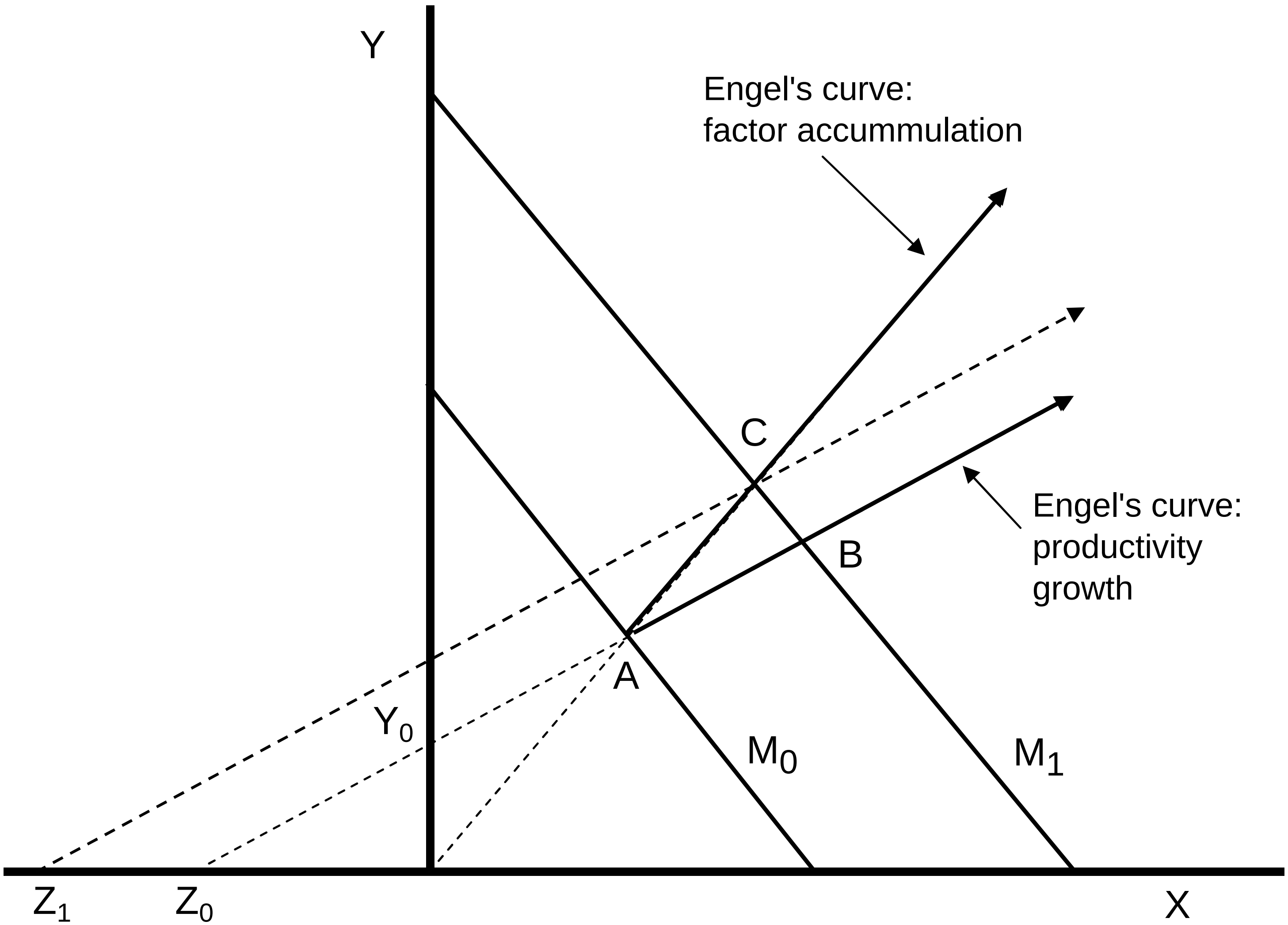


Figure 2: Identical countries: income elasticity, X share

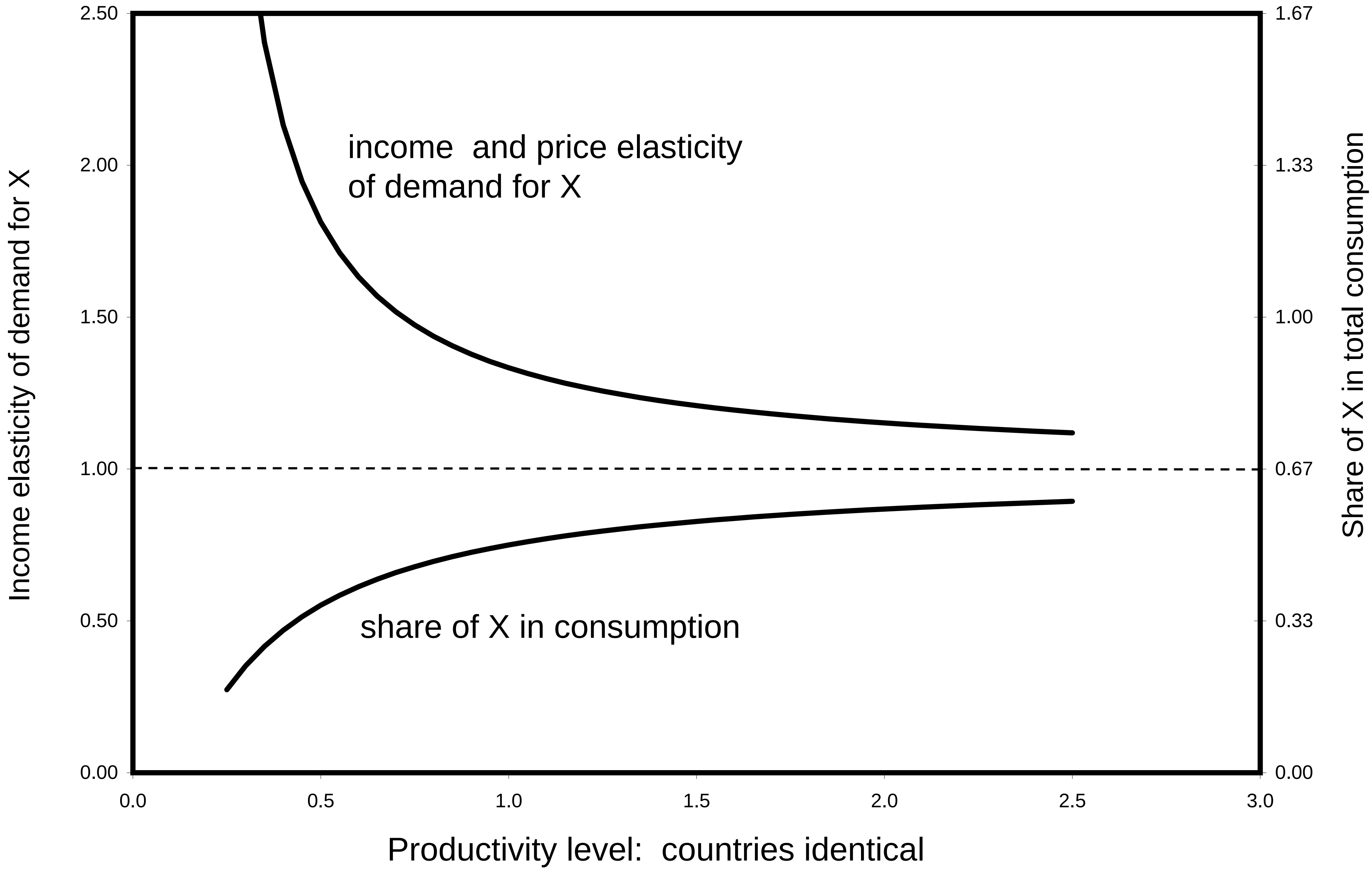


Figure 3: Wage gap from the demand side (identical countries)

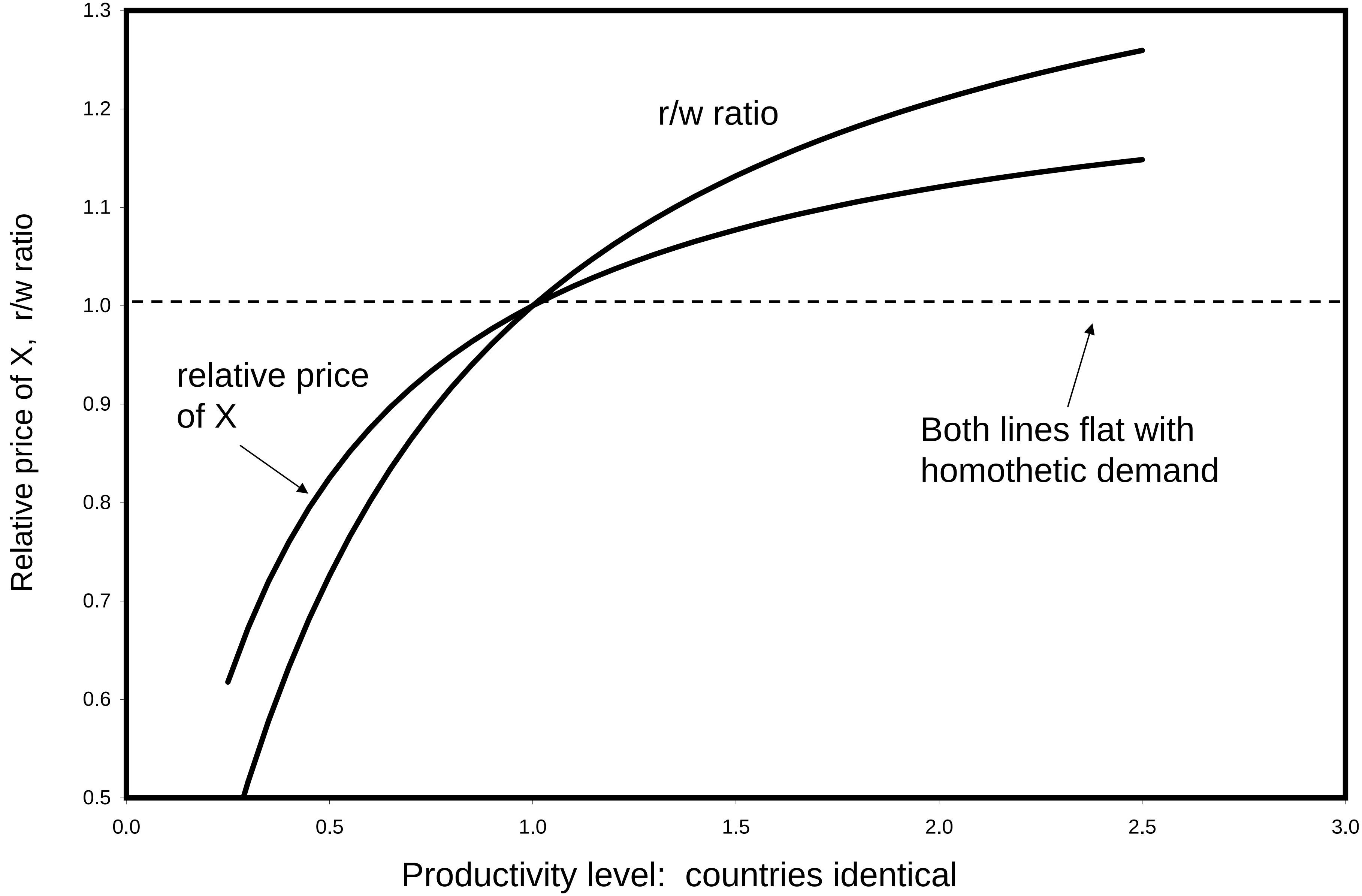


Figure 4: Classic Heckscher-Ohlin model: volume of trade

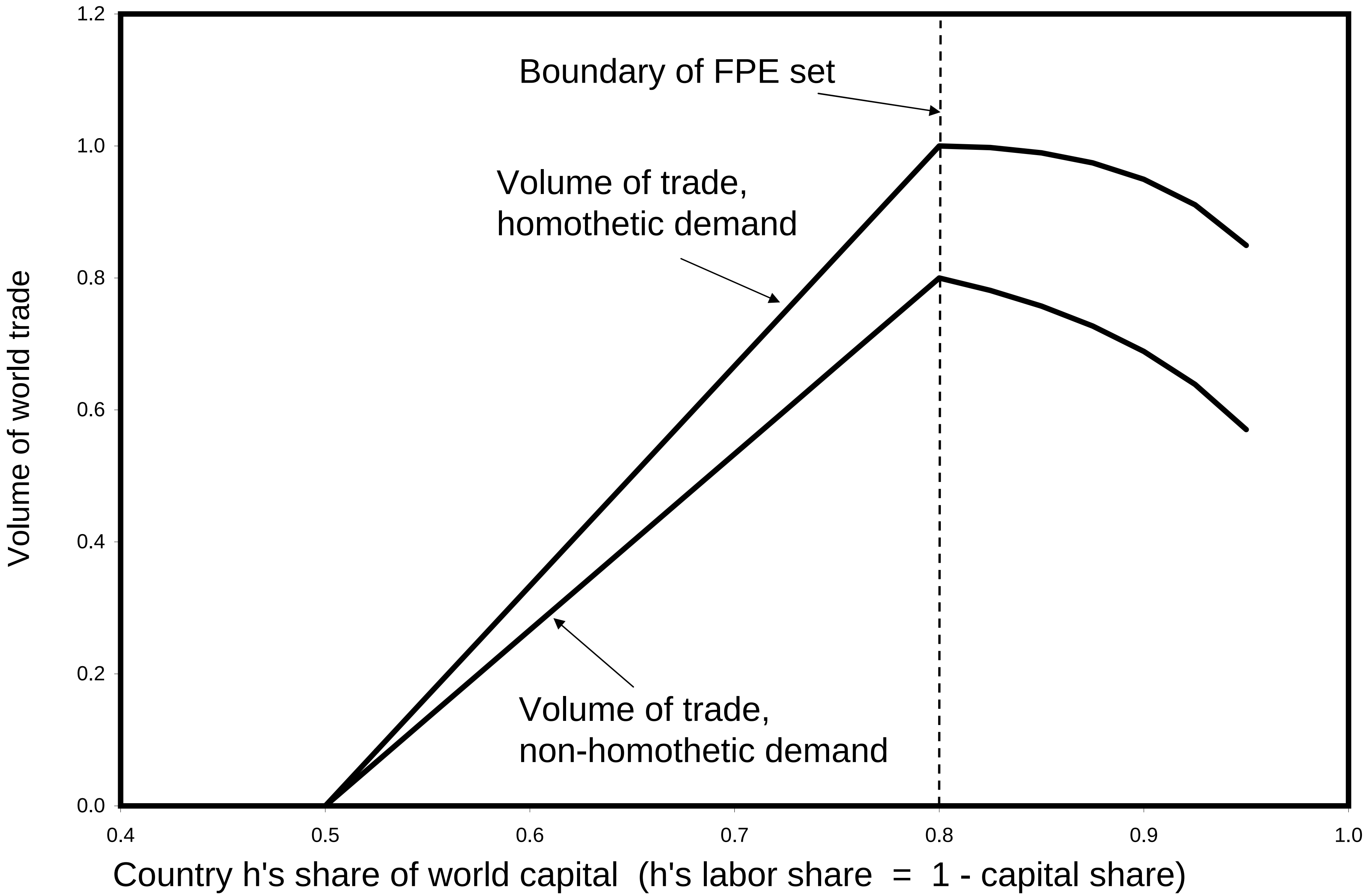


Figure 5: Home bias and the mystery of the missing trade

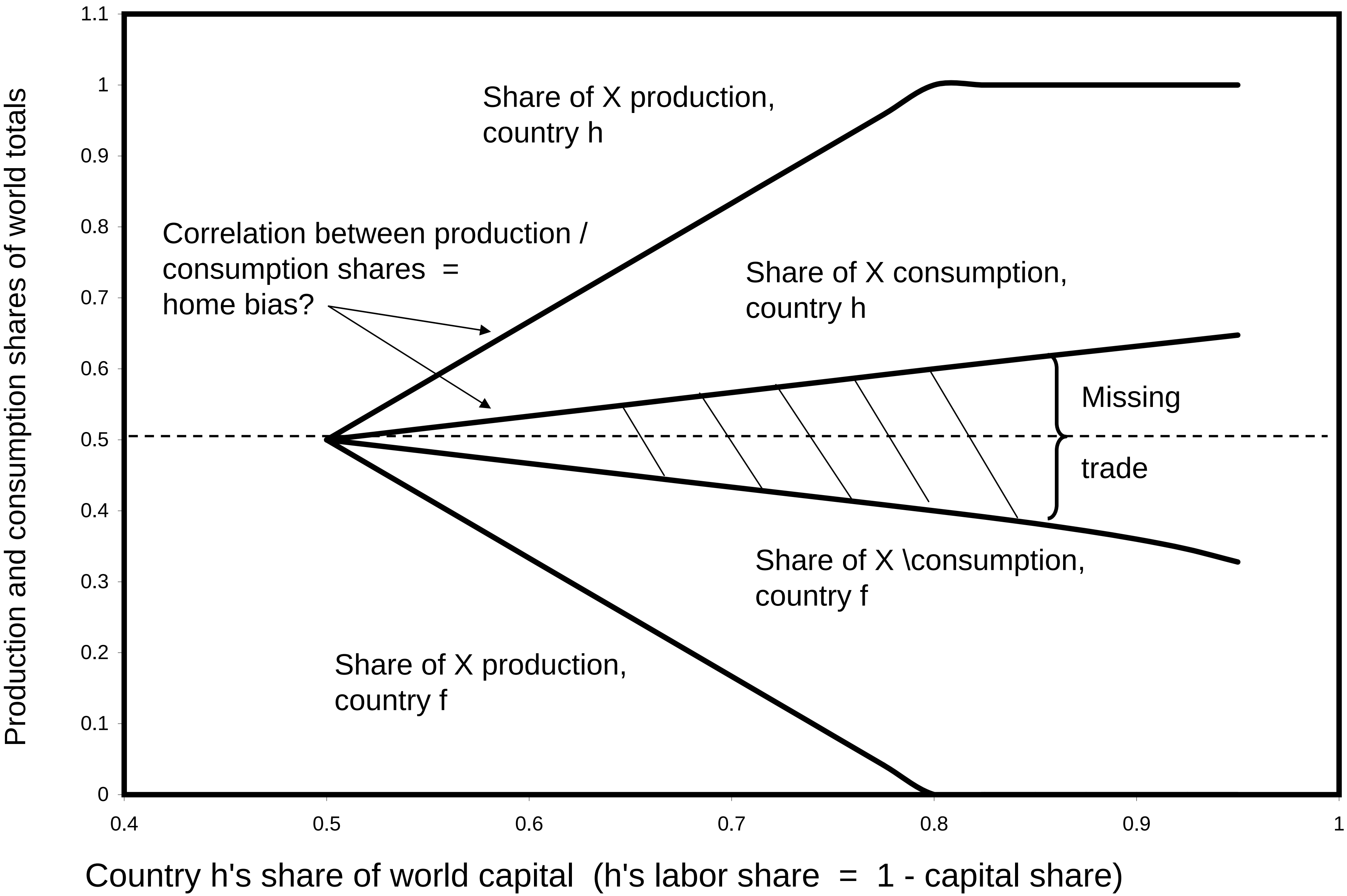


Figure 6: Per-capita income as a basis for trade: productivity versus absolute endowments (identical relative endow and country sizes)

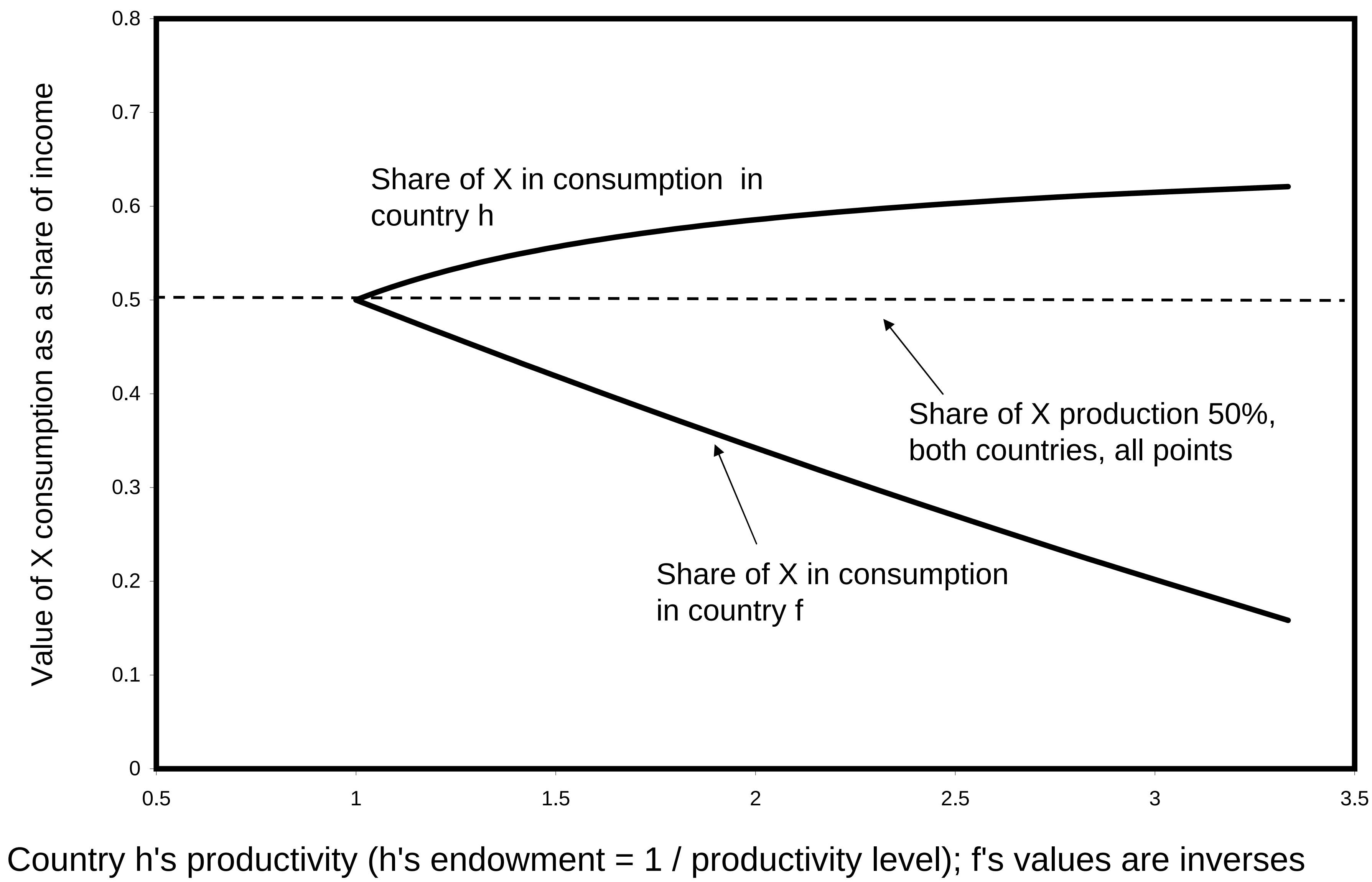


Figure 7: Intra-country income distribution and aggregate demand

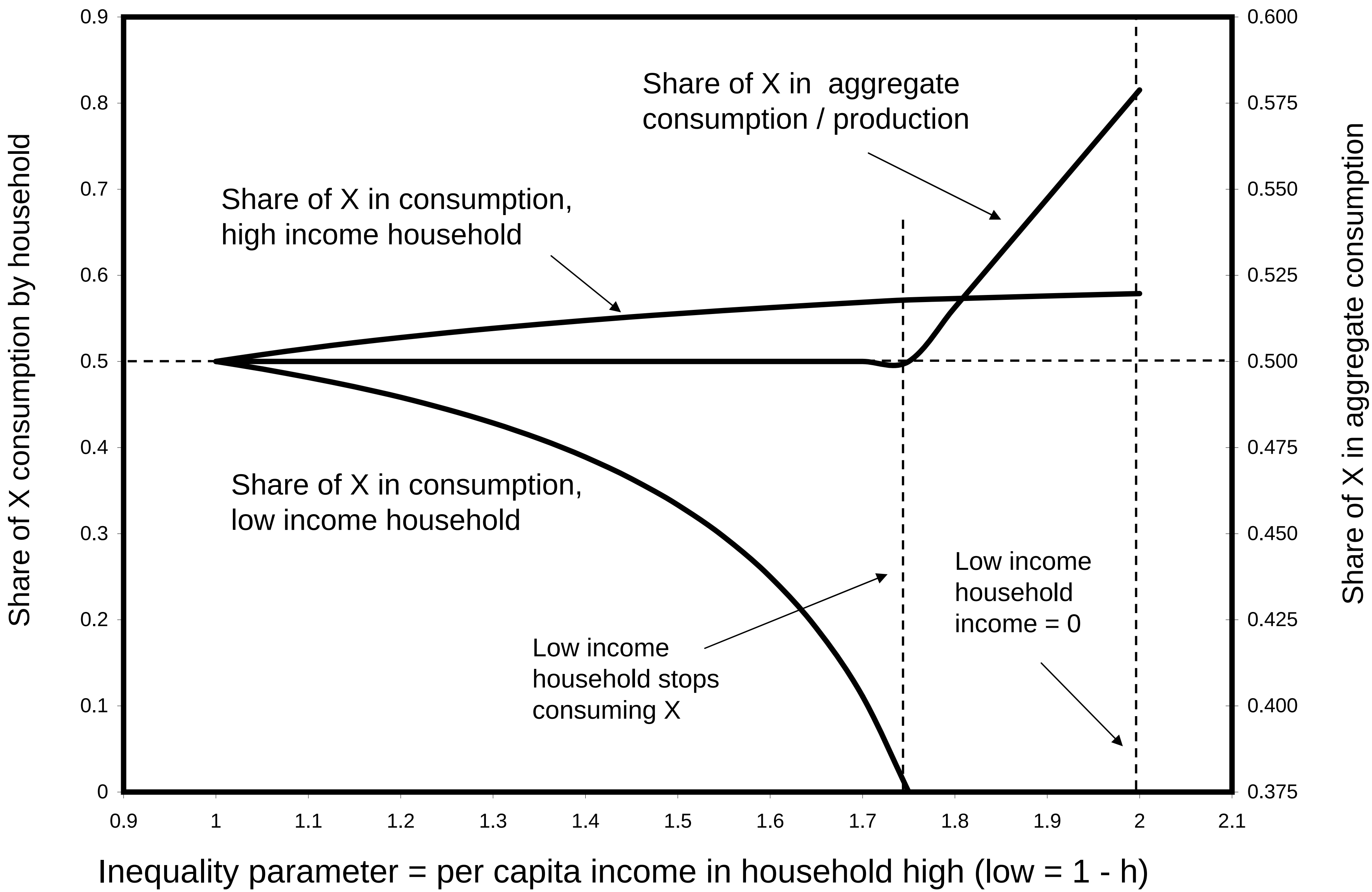


Figure 8: Markups, prices due to per-capita income differences 1: relative endowment differences

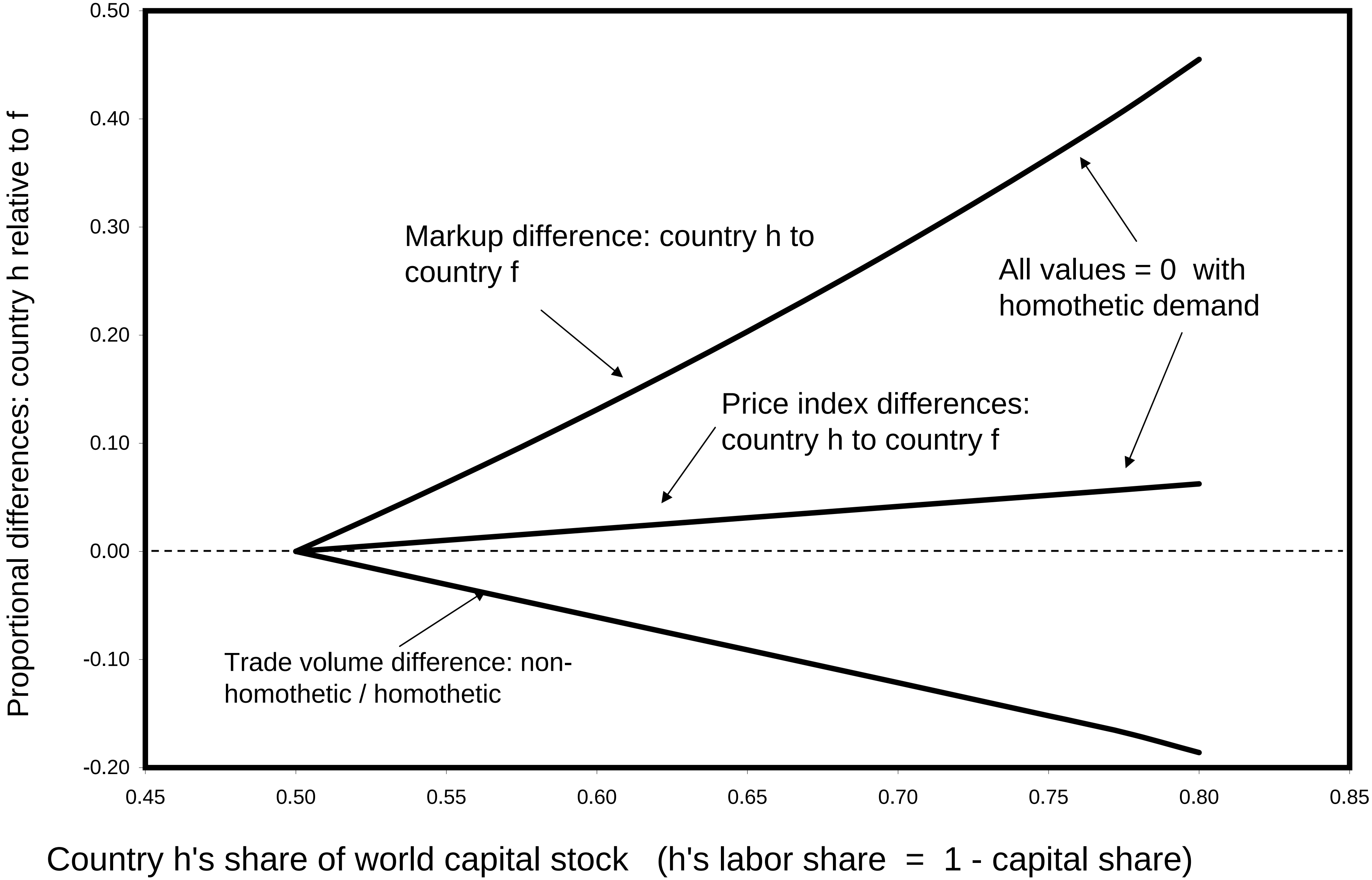
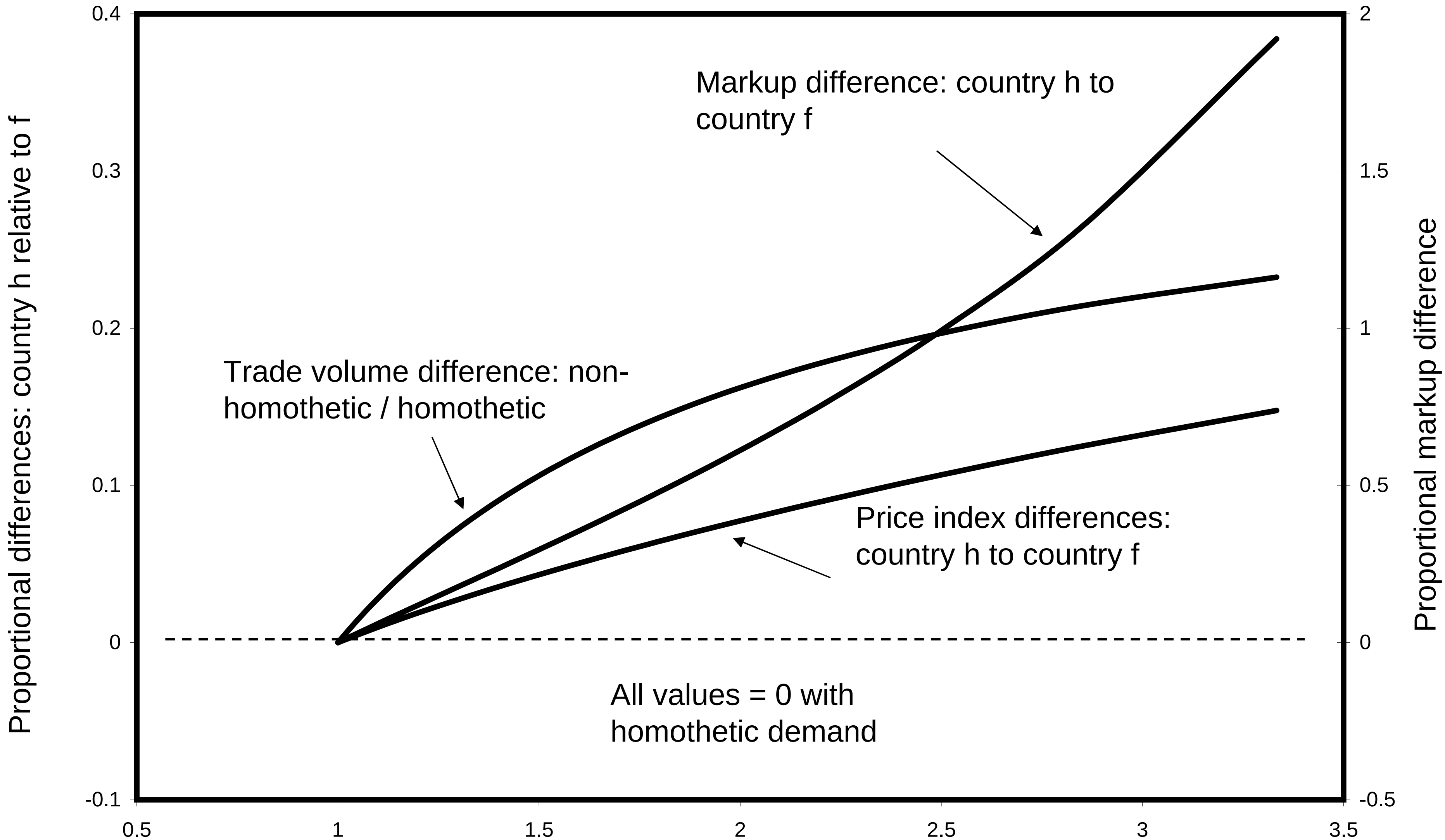


Figure 9: Markups, prices due to per-capita income differences 2: productivity differences



Country h's productivity (h's endowment = 1 / productivity level); f's values are inverses