Reciprocity, World Prices, and Welfare

By

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Abstract: We examine in detail the circumstances under which reciprocity, as defined in Bagwell and Staiger (1999), leads to fixed world prices. We show that a change of tariffs satisfying reciprocity does not necessarily imply constant world prices in a world of many goods and countries. While it is possible to find tariff reforms that are *consistent* with both reciprocity and constant world prices (as Bagwell and Staiger, 1999, argue), these reforms do not follow from the reciprocity condition, but rather from the requirement of unchanged world prices. More importantly, however, we show that even if these reforms keep world prices unchanged, they do not necessarily raise welfare in a multi-good, multi-country model. We propose an alternative reciprocity rule that performs better on this account.

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1. Introduction

In an important paper in the *American Economic Review*, Kyle Bagwell and Robert Staiger (1999) propose a general equilibrium theory of GATT that rationalizes the use of reciprocity and non-discrimination as the two main pillars of GATT negotiations. Bagwell and Staiger show that multilateral tariff reforms that are based on the rules of reciprocity and non-discrimination remove the well-known terms-of-trade externalities and allow countries to enjoy the positive efficiency gains of their tariff reforms. The main mechanism that leads to this advantageous situation is based on their conclusion that multilateral tariff reforms that adhere to the rule of reciprocity leave *world prices unchanged*. Clearly, an understanding of this argument is essential in appreciating the Bagwell and Staiger theory of GATT negotiations.

Bagwell and Staiger define a tariff reform satisfying reciprocity as one that keeps the value of each country's trade unchanged, where the evaluation is at the initial world prices. Within the context of a two-country, two-product model in which the initial equilibrium is a Nash equilibrium, they demonstrate that a reciprocity-based tariff reform leads to a new equilibrium at which world prices are unchanged. Moreover, such a tariff change will move the countries to a point within the set of Pareto improvements and so both countries gain from the tariff changes. This powerful consequence of reciprocity, viz. that world prices remain unchanged, makes the analysis of the negotiation game between GATT countries more transparent.

Holding world prices fixed has previously appeared as an important insight in the literature of regional trading-clubs. See, for example, Ohyama (1972) and Kemp and Wan (1976) for early contributions, and Kowalczyk and Sjöström (2000), Panagariya and Krishna (2002), Ohyama (2002), Raimondos-Møller and Woodland (2006) and Grinols and Silva (2007) for more recent uses of this insight. However, none of these papers characterize the tariff reforms that preserve the world prices to the old level. The contribution of Bagwell and Staiger (1999) is quite different as it is argued to provide a simple, and policy relevant, characterization of how to keep world prices constant in a multilateral trading model, viz. by imposing reciprocity.

The present paper looks at the conditions needed for this powerful consequence of reciprocity to be extended to a multi-country, multi-good case. Bagwell and Staiger (1999) argue that it does. However, while their rigorous proof of it is based on a 2×2 version of the model (see p. 224), their footnote 16 on page 225 considers an extension to many goods and many countries that left us with some open questions.[1] We hope our way of presenting the Bagwell and Staiger

[1] The same holds for the appendix to chapter 5 in their book (Bagwell and Staiger, 2002), where a more detailed

main idea will be helpful in clearing up issues that readers may have when reading such an important paper as "An economic theory of GATT".

We show that, in general, reciprocity by itself *does not imply* fixed world prices. Moreover, we also show that even if there exist some tariff reforms that are *consistent* with both reciprocity and fixed world prices, these reforms can only be derived by imposing the extra constraint of fixed world prices. This in essence takes us back to the Kemp-Wan-Ohyama insight and its practical difficulties for guiding tariff reforms. We put emphasis on these issues because there seems to be a generally acceptance in the literature that Bagwell and Staiger (1999) show that trade liberalizing reciprocity fixes the world prices and thereby leads to welfare gains. Examples of statements to this effect may be found, amongst others, in Anderson and Neary (2007),[2] Epifani and Vitaloni (2006),[3] and Matoo and Olarreaga (2004).[4] In addition, the language in the writings of Bagwell and Staiger on this topic seem to emphasize this power of reciprocity. For example, Staiger (2008) states that "*reciprocity describes a fixed-terms-of-trade rule to which mutual tariff changes must conform*".

More importantly, however, we are also able to demonstrate that these reciprocity reforms may not lead to welfare improvements in this general multi-good, multi-country environment — more information is simply needed before that is guaranteed.[5] We proceed by proposing an alternative reciprocity rule that guarantees welfare improvement without any demand on the world price vectors. We require countries to increase their volume of trade (and not their value of trade) proportionally. As we show, such a reciprocity rule delivers what we want, viz. strict Pareto improvements in a multi country, multi good setting.

2. Model of International Trade

To provide a rigorous analysis of the issue, we consider a perfectly competitive general equilibrium model of the world consisting of K nations trading in L internationally tradeable com-

discussion of the many goods case is provided.

[2] Quote from p. 187: "*Bagwell and Staiger (1999) note that reciprocity, which they interpret as trade policy "concessions" that yield equal increases in market access and so keep world prices constant, is one of the foundational principles of GATT.*"

[3] Quote from p. 428: *"A key observation of BS99 (p. 224) is that, as long as changes in import volumes are measured at existing world prices, mutual changes in trade policy that conform to reciprocity leave the relative world price unchanged."*

[4] Quote from p.1: "*In an important recent paper, Bagwell and Staiger (1999) show that reciprocity can be given a more direct positive economic interpretaion: it serves to neutralize the adverse terms of trade effects accosiated with unilateral reductions in protection, and therefore, leads to greater liberalization.*"

[5] It should be noted that Bagwell and Staiger do not claim total generality as far as it concerns their welfare results.

modities. Following Turunen-Red and Woodland (1991), the model may be expressed as

$$\sum_{k\in K} E_p^k(p^k, u^k) = 0, \tag{1}$$

$$p^\intercal E_p^k(p^k, u^k) = 0,\ k \in K, \tag{2}$$

in terms of the world price vector p ($p^\intercal$ denotes the transpose of a vector), the domestic price vectors p^k for each country $k \in K$ and the utility levels u^k for each country $k \in K$.[6] In this specification, $E^k(p^k, u^k) \equiv e^k(p^k, u^k) - g^k(p^k)$ is the net or trade expenditure function, being the difference between the gross domestic product function g^k and the consumer expenditure function e^k. Also, $E_p^k(p^k, u^k) \equiv m^k$ denotes the gradient of the trade expenditure function with respect to prices and represents the country-specific vector of compensated net import functions.[7]

Equations (1)-(2) consist of the market equilibrium conditions and the budget constraints for each country. The market equilibrium conditions express the requirement that the net imports of countries, m^k, sum to the zero vector, meaning that world markets clear. The national budget constraints state that the value (at world prices) of net imports (the balance of trade) must be zero.

It is implicit in this formulation of the model that there is just one consumer in each country, who receives a transfer of the tariff revenue from the government and has utility u^k.[8] Moreover, with the focus being on reciprocity, our model encompasses the MFN principle, i.e. a good is facing the same tariff independent of the country of origin. The model is expressed in terms of domestic and world prices. These are connected by tariffs, which may be expressed in specific terms, whence we may write $p^k = p^k(p, t^k) \equiv p + t^k$.[9]

Reciprocity in Bagwell and Staiger (1999) is defined in terms of the outcome of tariff negotiations. In particular, it is required that the initial world price value of the change in the net trade vector of each country remains zero. The changes in tariffs, which are the policy instruments, have to ensure that the reciprocity conditions hold after all general equilibrium effects

[6] The notation K is used to denote the set of countries as well as the number of countries.

[7] Woodland (1982) spells out the properties of the revenue and expenditure functions.

[8] It is relatively straightforward to extend the model, at the cost of added notational complexity, to handle multi-household economies. In the case of multiple households, Pareto improvements may be ensured by assuming the existence of lump sum income transfers between households and the national governments. Alternatively, under appropriate assumptions, commodity taxes may be used to carry out internal Pareto-improving redistributions. See, for example, Diewert, Turunen-Red and Woodland (1989, 1991).

[9] The model may also be specified in terms of ad valorem tariff rates rather than specific (unit) tariff rates. Nothing of essence is altered by this choice. Indeed, our example introduced further below is specifiied using a valorem rates.

have taken place. In our notation, this definition can be formally written as follows.

Definition 1 [**Reciprocity**]. *A set of tariff changes conforms to the principle of reciprocity if*

$$p_0^{\intercal}(m_1^k - m_0^k) = 0, \qquad k \in K, \tag{3}$$

where subscripts 0 *and* 1 *denote, respectively, pre- and post-tariff reform values.*

Bagwell and Staiger examine the implications of using this reciprocity rule in trade negotiations within a two-good, two-country model. At the same time, they comment about the issues involved in moving to higher dimensions and it is these issues that occupy us in the remainder of this paper. Before moving on to these issues, however, it is useful to provide a diagrammatic account of the essence of their results in the two-dimensional setting and, thereby, provide a platform for our own results.

Figure 1 depicts the offer curves of two countries (1 and 2) that trade two goods internationally and choose tariffs non-cooperatively. The initial equilibrium point is the Nash equilibrium, N_0, where the offer curves intersect and each country's indifference curve is tangential to the other country's offer curve. The shaded 'cigar-shaped' area contained by these indifference curves is the set of trade vectors that yield a Pareto improvement over point N_0.

Figure 1: about here

Bagwell and Staiger define a tariff reform satisfying reciprocity as one that keeps the value of each country's trade unchanged, where the evaluation is at the initial world prices. In the figure, the ray passing through the origin and N_0 indicates the trade vectors that are consistent with a zero trade balance at the initial world prices, p_0. Thus, any expansion of trade along this ray is consistent with the reciprocity rule of Bagwell and Staiger. Thus, it is easy to see in this figure that a negotiated tariff reform that is consistent with reciprocity and market equilibrium expands the trade volumes out along this ray to a point such as N_1, at which the equilibrium world prices are unaltered. Moreover, such a tariff change will definitely move the countries within the 'cigar' area of Pareto improvements and so both countries gain from the tariff changes.[10]

[10] The fact that the starting point is a Nash equilibrium is important. If tariffs were not optimally set, the cigar-shaped area of Pareto improvements may be totally to the right or left of the world price ray. For example,

The outcomes of the reciprocity rule in this two-dimensional context are clear. Reciprocity implies that the new world prices equal the initial world prices and that, provided the reform is small enough to prevent trade going out of the cigar shaped region, both countries gain in welfare. Bagwell and Staiger argue that the reciprocity reform neutralizes the terms of trade externality by ensuring unchanged world prices. We now proceed to determine how robust these clear-cut results are to the dimensions of the model by considering the general L good, K country case.

3. Reciprocity and World Prices

Using the reciprocity definition of Bagwell and Staiger, our first task is to determine the conditions under which a tariff reform that satisfies reciprocity ensures that the new world price vector is exactly equal to the initial world price vector. To this end, we proceed in several steps.

First, we can write the reciprocity condition in an alternative, but equivalent, way. Using balance of trade conditions $p_0^\intercal m_0^k = 0,\ k \in K$ (see (2)) at the initial equilibrium, equation (3) implies that $p_0^\intercal m_1^k = 0,\ k \in K$. This means that the new trade vectors, evaluated at the initial world prices, also take a value of zero. Combining this last equation with the balance of trade condition $p_1^\intercal m_1^k = 0,\ k \in K$ (see again (2)) in the post-reform situation, we can write $(p_0 - p_1)^\intercal m_1^k = 0, k \in K$. In matrix form, this set of conditions may be written as

$$(p_0 - p_1)^\intercal M_1 = 0, \tag{4}$$

where M_1 is the $L \times K$ matrix of national net imports vectors, m_1^k, in the post-reform situation 1. Accordingly, we have re-written the reciprocity conditions (3) in the form of equation (4). Thus, the reciprocity condition implies that the product of the $L \times 1$ vector, $p_0 - p_1$, and the $L \times K$ world trade matrix, M_1, is the zero vector.

Focusing on (4), we now determine the circumstances under which a solution to this set of equations necessarily implies unchanged world prices, i.e. $p_0 = p_1$.[11] Defining $v \equiv p_0 - p_1$, this equation system may be written as $v^\intercal M_1 = 0$ and the question is whether $v = 0$ is the only solution. If $v \equiv p_0 - p_1 = 0$ is the only solution to equation system (4), then reciprocity

if one of the two countries adopted a free trade policy then its indifference curve would be tagential to the ray through the origin and so the Pareto-improving area would be to one side of the ray. In this case, reciprocity would lower the welfare of the free-trading country.

[11] From (4) is straightforward to see that tariff policy adjustments that preserve world prices will always lead to reciprocity. However, the question that we examine here is the opposite: viz. whether the tariff policy adjustments that satisfy reciprocity necessarily imply fixed world prices.

necessarily implies that world prices do not change as a result of the tariff reform. On the other hand, if solutions with $v \equiv p_0 - p_1 \neq 0$ exist, then reciprocity does not necessarily imply unchanged world prices and, moreover, will generally imply different world prices as a result of the tariff reform. In this case, terms of trade changes occur and these need to be taken into account in the analysis of reciprocity reforms.

The following Proposition establishes the necessary and sufficient conditions for reciprocity to imply unchanged world prices.

Proposition 1. *A multilateral change of tariffs satisfying reciprocity implies that world prices remain unchanged if, and only if, the world trade matrix in the post tariff-change situation has maximal row rank,* $L-1$ *(that is,* $rank(M_1) = L-1$*). Equivalently, a change in world prices is consistent with a tariff reform satisfying reciprocity if, and only if,* $rank(M_1) < L-1$.

The proof of this proposition has been relegated to the appendix 1.[12]

There are just two possibilities regarding the rank condition: (1) $rank(M_1) = L-1$ and (2) $rank(M_1) < L-1$. First, if $rank(M_1) = L-1$ Proposition 1 states that a tariff change obeying the reciprocity conditions ensures that the resulting world price vector is exactly the same as in the initial equilibrium ($p_1 = p_0$). This outcome means that there are no terms of trade externalities arising from the multilateral tariff reform. The second possibility is that $rank(M_1) < L-1$. In this case, Proposition 1 states that reciprocity is not sufficient to ensure unchanged world prices as a result of the tariff change. In other words, under this rank condition, the equation system $(p_1 - p_0)^\intercal M_1 = 0$ *always* has a solution such that $p_1 \neq p_0$ (in addition to the solution with $p_1 = p_0$, which always exists). When this situation arises, terms of trade externalities do, indeed, arise from the multilateral tariff reform.

Using the above results, we now consider two possible cases based upon the relative numbers of goods and countries.

Case α: $L \leq K$, i.e. the number of goods is smaller than, or equal to, the number of countries. In this case $rank(M_1) \leq \min(L-1, K-1) \leq L-1 \leq K-1$, which is consistent with either possibilities in the above Proposition. If the trade matrix M_1 has maximal row rank

[12] The essence of the proof relates to the assumed properties of the trade matrix and well-known results from linear algebra concerning the existence of solutions to a set of homogeneous equations, such as (4). Equation system (4) is a set of K homogeneous linear equations in L unknown variables (elements of $v \equiv p_0 - p_1$). The trade matrix cannot have rank greater than $L-1$ (due to the balance of trade or reciprocity conditions) and this accounts for the reference to the world trade matrix having maximal row rank $L-1$.

($rank(M_1) = L-1$) then reciprocity does imply the world prices are unchanged as a result of the tariff reform.[13]

Case β: $L > K$, i.e. the number of goods is larger than the number of countries. Here $rank(M_1) \leq \min(L-1, K-1) \leq K-1 < L-1$ and this inequality clearly violates the rank condition in Proposition 1. As a consequence, a multilateral tariff reform that obeys reciprocity does not necessarily imply unchanged world prices. Indeed, in this case equilibrium solutions with post-reciprocity-reform world prices different from the initial world prices always exist.

The above sequence of arguments shows that reciprocity implies (4), but (4) does not always imply unchanged world prices. This result is now recorded as a corollary to Proposition 1 as follows.

Corollary 2. *Reciprocity in trade negotiations does not necessarily lead to constant world prices for the (empirically relevant) case where the number of goods is larger than the number of countries. Indeed, in this case there always exists an equilibrium solution for world prices that differ from initial world prices as a result of a tariff reform satisfying reciprocity.*

Bagwell and Staiger are aware of the possibility that reciprocity might not be sufficient to ensure unchanged world prices. They argue, however, that in such cases a supplementary tariff reform will be able to reconstruct the initial world prices and the new domestic prices with no effect on the equilibrium.[14] Let us explain their argument.

As we have seen above (see the discussion prior to (4)), reciprocity implies

$$p_0^\intercal m^k(p_1^k, u_1^k) = 0, k \in K, \qquad p_1^k = p_1 + t_1^k. \tag{5}$$

[13] However, it is possible that the trade matrix has lower rank ($rank(X_1) < L-1$), in which case a difference between initial and new world prices may ensue. It is noteworthy that this rank situation is only consistent with a trade matrix that is degenerate in the sense that it has less than maximal rank determined by theory. If such degeneracies are ruled out, then reciprocity necessarily implies unchanged world prices when $L \leq K$.

[14] Specifically, Bagwell and Staiger (1999) provide footnote 16, where they write: *"In the many-good case, however, is also possible that reciprocity can be satisfied even when world prices change. To evaluate this possibility, we note that the restriction of reciprocity can be rewritten as* (in our notation) $(p_1 - p_1^k)^\intercal x_1^k = (p_0 - p_1^k)^\intercal x_1^k$. *This indicates that any trade-policy adjustment giving rise to the price vectors p_1 and p_1^k results in the same aggregate tariff revenue as would an alternative tariff-policy adjustment that gave rise to the price vector p_0 and p_1^k, when each adjustment is consistent with the restriction of reciprocity. Since world prices affect welfare only through tariff revenue, we may therefore restrict attention to tariff policy adjustments that preserve the world prices. These properties of reciprocity also extend naturally to a many-country case."*

However, (5) can be rewritten as

$$p_0^{\intercal} m^k(p_1^k, u_1^k) = 0, k \in K, \qquad p_1^k = p_0 + t_2^k, \quad t_2^k \equiv p_1 + t_1^k - p_0, \tag{6}$$

where now we define another tariff vector, t_2^k, that replicates the same domestic prices p_1^k and the old world prices p_0.

Clearly, equations (5) and (6) are welfare equivalent, as each set of equations has the same post-reform domestic prices. The only thing that has changed is the definition of domestic prices, with new tariff vectors separating the domestic price vectors from the initial world prices. Thus, all real economic variables are the same and the only thing that has happened is a re-definition of tariffs in one particular way, viz. replicating the new domestic prices and the old world prices. Clearly, since domestic prices and world prices are connected by tariffs, one can *always* choose a particular tariff vector that replicates particular values of domestic and world prices (and thereby tariff revenues).

This result may be formalized as follows.

Proposition 3. *If a reciprocity-compliant tariff reform causes world prices to change, it is always possible to create an additional tariff reform that moves equilibrium world price back to initial world prices without affecting real variables, including utility levels.*

Does this result vindicate reciprocity as a mechanism for tariff reforms? The answer to this question is "no" if the objective is to obtain a tariff reform that also preserves world prices at their initial values. Because of the results in the previous section, reciprocity failed to preserve world prices at their initial levels. To obtain unchanged world prices an additional tariff reform was necessary and it was constructed explicitly with unchanged world prices as the objective. Accordingly, the important point here is that it is not reciprocity that leads to these new tariffs, but *the imposition of the constraint that world prices should remain the same.*

4. RECIPROCITY AND WELFARE

The discussion above was focused on the effects of reciprocity on world prices. Clearly, our interest is on the welfare effects of tariff reforms and trade agreements. If it were the case that the welfare effects of a reciprocity-based tariff reform were independent of whether world prices changed then changing world prices would be of limited interest.[15]

[15] Bagwell and Staiger (1999, footnote 16) seem to suggest that this is, indeed, the case. Following their demonstration that any equilbrium in which world prices change can be written as an equilibrium with unchanged

To analyse the welfare impication of reciprocity we adopt revealed preference arguments. These arguments identify conditions that distinguish equilibrium outcomes that do and do not generate Pareto improvements. We will be comparing the effects on welfare from changes in trade policy. It is important to note that the following analysis is for the general model and does not depend in any way upon the dimension of the economy — whether there are many goods and countries or whether there are different numbers of goods and countries.

Equivalent variation. We will be comparing the effects on welfare from changes in trade policy. To evaluate this welfare change we first apply an equivalent variation measure of the welfare gain. We define the equivalent variation for country k by

$$EV^k = E(p_0^k, u_1^k) - E(p_0^k, u_0^k) \tag{7}$$

where situation 0 is the pre-reform equilibrium and situation 0 is the post-reform equilibrium. Country k's welfare has increased if $u_1^k > u_0^k$ or, equivalently, if $EV^k > 0$.

Since it is always true that $p_0^{k\intercal} m_1^k \geq E(p_0^k, u_1^k)$ (since m_1^k is feasible for the trade expenditure minimization problem but not necessarily optimal under p_0^k) and $p_0^{k\intercal} m_0^k = E^k(p_0^k, u_0^k)$, we can state that a welfare gain necessarily implies that

$$p_0^{k\intercal} m_1^k \geq E^k(p_0^k, u_1^k) > E^k(p_0^k, u_0^k) = p_0^k m_0^k. \tag{8}$$

Thus, a *necessary* condition for a welfare increase ($u_1^k > u_0^k$) is that

$$p_0^{k\intercal} \left(m_1^k - m_0^k \right) > 0. \tag{9}$$

This inequality defines the set of new import vectors, m_1^k, that contains all strict Pareto improving import outcomes. Geometrically, it states that the change in the import vectors, $\left(m_1^k - m_0^k\right)$, must be at an acute angle to the initial domestic price vectors, p_0^k. In figure 2, this region is the cone emanating from N_0.

Imposing Bagwell and Staiger reciprocity, i.e. $p_0^\intercal (m_1^k - m_0^k) = 0$, and subtracting it from the

world price (Proposition 3 above), they state that "Since world prices affect welfare only through tariff revenue, we may therefore restrict attention to tariff policy adjustments that preserve the world prices."

right hand side of (9), we obtain that

$$\left(p_0^k - p_0\right)^{\intercal} (m_1^k - m_0^k) = t_0^{k\intercal}(m_1^k - m_0^k). \tag{10}$$

The positivity of this expression provides a characterization of the region of the import space in which it is necessary for the import vectors to lie to allow the possibility of a welfare gain. Outside this region, a Pareto improvement in welfare is not possible. It is also interpreted literally as the change in tariff revenue, evaluated at the initial tariff rates. Theory provides no presumption concerning the sign of $t_0^{k\intercal}(m_1^k - m_0^k)$. Being a necessary condition only, knowledge that this term is positive does not guarantee a welfare gain. Rather, the result is useful in identifying regions in which welfare cannot have increased, even though reciprocity holds.

Compensating variation. We now apply a compensating variation measure of the welfare gain. We define the compensating variation for country k by

$$CV^k = E^k(p_1^k, u_1^k) - E^k(p_1^k, u_0^k) \tag{11}$$

where situation 0 is the pre-reform equilibrium and situation 1 is the post-reform equilibrium. Country k's welfare has increased if $u_1^k > u_0^k$ or, equivalently, if $CV^k > 0$.

Since it is always true that $p_1^{k\intercal} m_0^k \geq E^k(p_1^k, u_0^k)$ (since m_0^k is feasible for the trade expenditure minimization problem but not necessarily optimal under p_1^k) and $p_1^{k\intercal} m_1^k = E^k(p_1^k, u_1^k)$, we can rewrite (11) as

$$\begin{aligned} CV^k &= E^k(p_1^k, u_1^k) - E^k(p_1^k, u_0^k) \\ &= p_1^{k\intercal} m_1^k - p_1^{k\intercal} m_0^k + p_1^{k\intercal} m_0^k - E^k(p_1^k, u_0^k) \\ &= p_1^{k\intercal} \left(m_1^k - m_0^k\right). \end{aligned} \tag{12}$$

An unambiguous welfare increase prevails if, and only if, the last term in the right hand side of (12) is positive. However, in general, we have no theoretical presumption about the sign of $p_1^{k\intercal}(m_1^k - m_0^k)$.[16]

Imposing Bagwell and Staiger reciprocity, i.e. $p_0^{\intercal}(m_1^k - m_0^k) = 0$, and subtracting it from the

[16] By considering particular tariff reforms we are able to sign (12). For example, Fane (1991) proves that for a proportional tariff reduction within a small open economy (12) is unambigously positive. Moreover, Konishi et al. (2003) show that (12) is positive if a (large) country adopts a free trade policy and, at the same time, receives transfers according to the so–called Grinols rule.

right hand side of (12), we obtain that

$$CV^k = \left(p_1^k - p_0\right)^{\intercal} (m_1^k - m_0^k) = t_2^k (m_1^k - m_0^k). \tag{13}$$

where $p_1^k - p_0 = t_2^k$ is the tariff vector that we need to impose in order to reproduce the old world prices and the new domestic prices (see equation (6) above). Again, we have no theoretical presumption concerning the sign of $t_2^{k\intercal}(m_1^k - m_0^k)$ — which evaluates the change in tariff revenues using the tariff rates, t_2^k, that preserve initial world prices.

Thus, in general, the Bagwell and Staiger reciprocity rule does not impose enough structure on tariff reforms in order to guarantee welfare increases for each single country in a multi-good set up.

4.1. Example. As an illustartion of the issues involved we provide a simple numerical example. In this example, there are $K = 2$ countries trading $L = 3$ goods.[17] The countries have fixed endowments of goods and no production. The endowment vectors of the two countries are assumed to be $(0.50, 0.15, 0.05)$ and $(0.25, 0.70, 0.05)$. There is a single consumer in each country, each with the same Cobb-Douglas preferences. The utility functions are $U(c_1, c_2) = (c_1 c_2)^{1/3}$. All tariff revenue is distributed to the consumer as a lump sum. Without loss of generality, good 1 is taken as the numeraire with price equal to unity and it is further assumed that there are no tariffs imposed on this good by any country.

Table 1 presents the equilibria for several different scenarios. Column (1) presents the Nash equilibrium, which we assume is the initial situation prior to the tariff reform. The equilibrium trade pattern involves Country 1 importing goods 1 and 3 and exporting good 2. In this equilibrium, country 1 imposes an ad valorem tariff rate of 54.77% ($t_{21} = 0.5477$) on its imports of good 2 and an export subsidy of 25.6% ($t_{31} = 0.2560$) on good 3. Country 2 taxes its exports of good 2 at the rate 49.34% ($t_{22} = -0.4934$) and has a subsidy rate of 11.19% ($t_{32} = -0.1119$) on imports of good 3.[18] Both countries are worse off in the Nash equilibrium than at free trade (the free trade values are not reported here for simplicity).

The equilibrium corresponding to a tariff reform that obeys the B&S reciprocity condition

[17] This example is drawn from Table A1 of Kennan and Riezman (1990). Their example has three countries and three goods, so we simply remove the third country to get our example.

[18] By Lerner symmetry we can convert the tariff rates for country 2 such that it imposes no tax on its export good (good 2). These tariff rates for country 2 on the three goods are 0.9740 0.0000 0.7531. Thus, country 2 effectively imposes duties of 97.4% and 75.31% on imports of goods 1 and 3.

Table 1: Example: Equilibia Under Alternative Tariff Policies

		Reciprocity Tariff Reform A		Reciprocity Tariff Reform B	
Variable	(1) Nash	(2) Reciprocity	(3) Price Preservation	(4) Reciprocity	(5) Price Preservation
p_2	1.0911	1.0933	1.0911	1.4734	1.0911
p_3	7.0711	7.1237	7.0711	11.1093	7.0711
u_1	0.5067	0.5107	0.5107	0.4915	0.4915
u_2	0.6608	0.6691	0.6691	0.6801	0.6801
t_{21}	0.5477	0.3320	0.3347	-0.2526	0.0093
t_{31}	0.2560	0.2560	0.2654	0.2560	-0.3458
t_{22}	-0.4934	-0.4441	-0.4429	-0.5156	0.9733
t_{32}	-0.1119	-0.1119	-0.1053	-0.5643	-0.3155
m_{12}	0.0835	0.0994	0.0994	0.0927	0.0927
m_{22}	-0.0966	-0.1251	-0.1251	-0.2198	-0.2198
m_{32}	0.0031	0.0052	0.0052	0.0208	0.0208
TR_1	0.0521	0.0358	0.0358	-0.1410	-0.1410
TR_2	0.0496	0.0565	0.0565	0.0365	0.0365

is presented in column (2) of Table 1 labelled "Reciprocity Tariff Reform A".[19] To obtain the results presented, we keep the tariff rates on good 3 as in the Nash equilibrium, and we alter the tariff rate imposed on good 2 by country 2 from $t_{22} = -0.4934$ to $t_{22} = -0.4441$ (a 10% change) and solve the equilibrium conditions and one reciprocity condition for the world prices, utility levels and the tariff rate t_{21}. The resulting tariff reform (only involving good 2 by assumption) obeys the reciprocity conditions for each of the two countries.

It is clearly seen that this reciprocity-compliant reform results in world prices that are different from those observed in the Nash equilibrium. The prices of goods 2 and 3 have both increased as a result of the tariff reform. This result is consistent with Proposition 1 and Corollary 2; since we have that $K = 2 < 3 = L$, a solution with different world prices is assured.

Column 3 of Table 1 provides the equilibrium values of the "constant world price" reform, where new tariff rates are derived *to ensure that world prices remain unchanged*. That is, the new tariffs are obtained as $t_2^k \equiv p_1 + t_1^k - p_0$, where p_0 is the Nash world price vector and p_1 is the world price vector from the reciprocity reform. Naturally, this new equilibrium yields the same real variables (e.g. utility levels and trade flows) and the same tariff revenues (last two rows) as the reciprocity equilibrium. Thus, this column provides an illustration of Proposition 3.

[19] There are two reciprocity conditions — one for each country — but one of these conditions is redundant in view of the market equilibrium conditions.

It should be emphasized that reciprocity by itself does not completely specify the required tariff reforms but merely the conditions that the tariff reform must meet. When there are more goods than countries (the empirically relevant case), the reciprocity requirements leave degrees of freedom in the selection of changes to tariffs. We illustrate this point by some further simulations that, for the sake of exposition, we report in Appendix 2.

However, in this multi-good setup there can exist other reciprocity-consistent tariff reforms that do not lead to welfare gains for both countries. Such a reciprocity-compliant tariff reform is presented in Table 1 and labelled "Reciprocity Tariff Reform B". Column (4) reports the reciprocity equilibrium, while column (5) reproduces the same equilibrium but for the corresponding "constant world price" reform.

Although the tariff reform illustrated in column (4) of Table 1 satisfies reciprocity, it is evident that it does not represent a Pareto improvement over the initial Nash equilibrium reported in column (1). The utility of country 1 has declined while that of country 2 has risen.

5. An Alternative Reciprocity Rule

It has been argued above that the reciprocity rule of Bagwell and Staiger, but itself, is not sufficient to specify tariff reforms that yield strict Pareto improvement in welfare. We have used revealed preference inequalities to delineate regions in import space where welfare gains do and do not occur. In the present section, we offer an alternative reciprocity rule that is guaranteed to yield strict Pareto improvements in the neighbourhood of the initial Nash equilibrium.

This reciprocity rule is defined as follows.

Definition 2 [**Quantity Reciprocity**]**.** *A set of tariff changes conforms to the principle of quantity reciprocity if*

$$m_1^k = \lambda m_0^k,\ \lambda > 1, \qquad k \in K, \tag{14}$$

where subscripts 0 *and* 1 *denote, respectively, pre- and post-tariff reform values.*

This is called quantity reciprocity (QR) to distinguish it from that of Bagwell and Staiger, which is a value based rule. Here, we simply require that all countries increase their net import vectors in the same proportion, defined by the scalar λ.

Clearly, quantity reciprocity implies value reciprocity. This is because (14) implies that $p_0^\intercal(m_1^k - m_0^k) = (\lambda - 1)p_0^\intercal m_0^k = 0$, since the initial balance of trade requires that $p_0^\intercal m_0^k = 0$. The reverse is not generally true as should be evident from the results above. The reverse is true in

the two-good, two-country case considered by Bagwell and Staiger.

First, we compare the variation results obtained above for the value reciprocity rule with those for our quantity reciprocity rule. Using the equivalent variation measure we obtain that

$$T_0^k \equiv p_0^{k\intercal}\left(m_1^k - m_0^k\right) = (\lambda - 1)p_0^{k\intercal}m_0^k = (\lambda - 1)t_0^{k\intercal}m_0^k.$$

This expression will be positive if $p_0^{k\intercal}m_0^k = t_0^{k\intercal}m_0^k > 0$, since $\lambda > 1$. This inequality means that the initial tariff revenue is positive. This, as is well known, is an implication of a Nash equilibrium which is our assumed initial equilibrium. Thus, our quantity reciprocity rule satisfies the necessary condition for a strict Pareto improvement. It ensures that the new equilibrium import vectors are in the cone illustrated in Figure 2 for the three-good, two-country case. Such an outcome is not guaranteed for a tariff reform based upon the value reciprocity rule.

The compensating variation inequality using the quantity reciprocity rule may be expressed as

$$CV^k = p_1^{k\intercal}\left(m_1^k - m_0^k\right) = (\lambda - 1)p_1^{k\intercal}m_0^k = (\lambda - 1)t_2^{k\intercal}m_0^k. \tag{15}$$

This expression will be positive if the initial import vector were to generate positive tariff revenue, using the supplementary tariff rates, t_2^k, that were determined to preserve initial world prices at the new equilibrium. This can be contrasted with the result for value reciprocity, which was for $t_2^{k\intercal}(m_1^k - m_0^k) > 0$. One is a difference in a tariff revenue, while the other is a level.

Finally, we show that, in the neighbourhood of the initial Nash equilibrium, any quantity reciprocity reform unambiguously improves welfare in the sense of creating a strict Pareto improvement in welfare. First, we observe that the first order conditions for utility maximization by country k at the initial Nash equilibrium is that $\nabla U^k(m_0^k) = \theta_0^k p_0^k$, where $\theta_0^k > 0$ is the Lagrange multiplier; that is, the gradient of the direct trade utility function is proportional to the domestic price ratio.[20] Second, we note that the change in the utility for country k in the direction of the initial import vector is given by $D(m_0^k) = m_0^{k\intercal}\nabla U^k(m_0^k)$. This directional derivative indicates the gradient of the utility function as imports move out along a ray defined by m_0^k and is evaluated at m_0^k. Third, using these two results we obtain that $D(m_0^k) = m_0^{k\intercal}\nabla U^k(m_0^k) = \theta_0^k p_0^{k\intercal}m_0^k$. Fourth, we note that $p_0^{k\intercal}m_0^k = t_0^{k\intercal}m_0^k$ is the tariff revenue that accrues at the initial Nash

[20] See Woodland (1980) for further details on the trade utility function and its dual.

equilibrium, which is positive. Thus, we have that

$$D(m_0^k) = \theta_0^k p_0^{k\intercal} m_0^k > 0$$

for any quantity reciprocity reform $m_1^k = \lambda m_0^k,\ \lambda > 1,\ k \in K$, with λ sufficiently close to unity. Accordingly, welfare improves for every country for a small quantity reciprocity reform.

Proposition 4. *A quantity reciprocity tariff reform of the form $m_1^k = \lambda m_0^k,\ \lambda > 1,\ k \in K$, yields a strict Pareto improvement in welfare for λ sufficiently close to unity.*

This result is very strong. Its only requirements are that the reform is sufficiently small and that the initial equilibrium has every country with positive tariff revenue. The former requirement is, of course, needed because very large quantity reciprocity reforms could easily take the countries well beyond the Pareto improving "cigar", just as is the case for a revenue reciprocity reform. The second requirement is satisfied at any Nash equilibrium and is needed to avoid other situations such as when one country is initially a free trader, in which case any expansion of trade will be clearly welfare reducing.

The advantage of this result is that it shows that, under minimal conditions, a quantity reciprocity tariff reform is guaranteed to yield welfare gains for every country. Whether world prices change as a result of the reform is of no consequence.[21] Whether there are more goods than countries is also of no consequence. This is in contrast to the value reciprocity reform of Bagwell and Staiger, where having more goods than factors can lead to changed world prices and, more importantly, to reforms that are not welfare improving even in the neighbourhood of the Nash equilibrium.

Finally, the quantity reciprocity reform is still within the spirit of the WTO principle of "equal concessions". Under the value reciprocity reform, tariffs are altered so that, at initial world prices, an increase in a countries import values are matched by an increase in its export values, which have to be matched by an increase in the values of imports by other counties. In the case of the quantity reciprocity tariff reform, the concessions are expressed in quantities. An increase in a countries imports have to be matched by equi-proportional increases in its exports, which have to be matched by an equal increase in the quantities of imports by other countries.

[21] The quantity reciprocity reform can be readily constructed to have unchanged world prices, but this is not a requirment.

6. Conclusions

Influenced by the important work of Bagwell and Staiger (1999, 2002) on the economics of GATT, we focus on whether tariff reforms based upon the reciprocity condition imply unchanged world prices can be extended from the two-good, two-country case to a more general model with many goods and many countries. We show that, in general, it does not. In the empirically relevant context where there are more goods than countries, it is generally the case that a reciprocity-based tariff reform will lead to changes in the terms of trade. This result is important since it is widely accepted in the literature that tariff reforms based on reciprocity imply constant world prices. The present paper provides a clarification of the conditions under when this perception is correct and when it is not. It also provides further commentary and clarification on the reciprocity rule and attempts, consequently, to place Bagwell and Staiger's important results in context.

The remedy to changed world prices arising from a reciprocity-based tariff reform that Bagwell and Staiger mention in their writings, namely that there exists a particular (supplementary) tariff reform that is consistent with both reciprocity and fixed world prices, is, of course, correct. Its value is in showing that implementation of this supplementary tariff reform leads to an equilibrium with unchanged world prices that is welfare equivalent to the reciprocity-based equilibrium. However, this does not constitute a simple rule that policy makers can use in finding this tariff reform. The reform is based completely upon the target of reproducing the original world prices, not upon the reciprocity rule. Thus, interpreting the important work of Bagwell and Staiger as saying that "reciprocity fixes the world prices" is not accurate.

More importantly though, when the number of goods is larger than the number of countries it is possible that the imposition of reciprocity will not be sufficient for ensuring welfare gains. We have provided a general proof that shows that the reciprocity condition does not provide enough structure on the tariff changes needed in order to ensure welfare improvements. Moreover, we have provided an alternative reciprocity rule that delivers strict Pareto improvements. Our rule is based on reciprocity in the volume of trade rather than in the value of trade.

Appendix 1

Proof of Proposition . To prove this proposition, we make use of some well-known results from linear algebra that depend upon the properties of the world trade matrix M_1. To put the resulting condition in context, it is useful to note the general rank properties of this world trade matrix.

Remark 1. Lemma 5. *Properties of the world trade matrix. The world trade matrix, M, has the following rank properties: (i) $rank(M) \leq K-1$ due to the world market equilibrium conditions,*[22] *(ii) $rank(M) \leq L-1$ due to the balance of trade conditions in the new situation (or the reciprocity conditions)*[23] *and (iii) $rank(M) \leq \min(L-1, K-1)$, since the rank of any matrix must be smaller than or equal to the lower of its dimensions.*

Lemma 6. *(Hadley, 1965, p. 193) Do we put in his result here?*

Using a result from Hadley (1965, p. 173), a necessary and sufficient condition for the system of K homogeneous linear equations $v^{\intercal}M_1 = 0$ to have a non-trivial solution $v \neq 0$ is that $rank(M_1) < L$. Since $rank(M_1) \leq L-1 < L$ from property (ii) above, it follows that a non-trivial solution $v = p_0 - p_1 \neq 0$ always exists. To illustrate, the reciprocity conditions for situation 1 imply that $v = p_0$ is a solution and so $v = \lambda p_0$ is also a solution for any $\lambda > 0$. Thus, the equilibrium solution for p_1 can be written as $p_1 = (1-\lambda)p_0$. However, this is uninteresting from an economics viewpoint, since it says that one price vector is a multiple of the other. Normalizing the price of one good to be unity (the numeraire), $\lambda = 0$ is required and so the solution for v becomes trivial. Accordingly, we restrict attention to price systems that contain a numeraire good whose price is set to unity and which is not subject to a tariff in either situation.

Without loss of generality, we choose the first good as the numeraire, normalize the first element of vector $v = p_0 - p_1$ to 0 and assume that no tariffs are allowed to be imposed on the numeraire good. Since $v_1 = 0$, the equation system $v^{\intercal}M_1 = 0$ may be written as $\widetilde{v}^{\intercal}\widetilde{M_1} = 0$, where $\widetilde{v}$ is the $(L-1)$-dimensional vector of price differences for non-numeraire goods and $\widetilde{M_1}$ is the $(L-1)\times K$ dimensional trade matrix for non-numeraire goods. Hadley's result quoted above now implies that $\widetilde{v}^{\intercal}\widetilde{M_1} = 0$ has a non-trivial solution $\widetilde{v} \neq 0$ if and only if $rank(\widetilde{M_1}) < L-1$. In

[22] *With X_1 being the world net export matrix, the sum of the columns must be the zero vector due to market equilibrium. Therefore, the maximum rank of this matrix – i.e. the maximum number of independent columns – must be $K-1$.*

[23] *The balance of trade conditions (2) for the new situation may be expressed as $p_1^{\intercal}X_1 = 0$, meaning that X_1 has less than full row rank.*

words, the necessary and sufficient condition for reciprocity to allow $p_1 \neq p_0$ is that the rank of the trade matrix in the new situation is less than the number of traded goods less one (that is, less than the number of non-numeraire goods).

A corollary to Hadley's theorem quoted above is that a necessary and sufficient condition for $\widetilde{v}^{\intercal}\widetilde{M}_1 = 0$ to *only* have the trivial solution $\widetilde{v} = 0$ is that $rank(M_1) = L$. Applying this result to the problem at hand, we can write that

$$\widetilde{v}^{\intercal}\widetilde{M}_1 = 0 \Rightarrow \widetilde{v}^{\intercal} = 0 \;\; (p_1 = p_0) \text{ if, and only if, } rank(\widetilde{M}_1) = L - 1. \tag{16}$$

This completes the proof. ■

Appendix 2

We made the point in the text that when there are more goods than countries (the empirically relevant case), the reciprocity requirements leave degrees of freedom in the selection of changes to tariffs. This point can be effectively made by referring back to the simulation example.

To implement the reciprocity-compliant tariff reform in the above example, the tariff rates t_{21} and t_{22} were altered, leaving other tariff rates unchanged. We needed to change (at least) one tariff and needed another tariff to change endogenously to ensure that reciprocity was satisfied. There was a choice of which tariffs to leave unchanged, which to change endogenously and which to be the policy change tariff. Put another way, there may be many (an infinity of) tariff reforms that satisfy reciprocity and, hence, there is some arbitrariness in a reciprocity-compliant tariff reform.

This important point may be further illustrated by extending the above example. To this end, we allow the t_{22} rate to fall from the Nash rate towards zero and compute the corresponding t_{21} values that satisfy reciprocity. By performing this calculation over a grid of t_{22} rates, we thus obtain a *reciprocity locus* in tariff rate space, i.e. a (t_{22}, t_{21}) locus that satisfies reciprocity but without preserving the world prices at their initial Nash equilibrium values (see Figure 2). We also plot the utility levels relative to their Nash values for the two countries along the reciprocity locus in Figure 3.

Figure 2 and 3: (about here)

As the subsidy rate t_{22} is reduced from the Nash value of $t_{22} = -0.4934$ to $t_{22} = -0.2$ moving from left to right in Figure 3, the tariff rate t_{21} falls from its Nash value of $t_{21} = .5477$ towards

$t_{21} = -0.2045$. Figure 4 shows that both countries gain from the reciprocity-based tariff reforms along the reciprocity locus until around $t_{22} \simeq -.38$, after which country 1's utility falls as the locus moves out of the Pareto improvement set.

Figure 4: (about here)

For each equilibrium on the reciprocity locus we now compute the supplementary tariff reform needed to preserve the world prices at their Nash values. It is important to note that this supplementary tariff reform requires all tariff rates to change, and is therefore more complex than the reciprocity reform alone. A complete summary of these results is provided in Figure 4, which plots the four tariff rates required to preserve world prices at the initial Nash levels relative to their values along the reciprocity-based tariff reform locus against the grid of reciprocity values for t_{22}. Each curve provides a measure of the additional (supplementary) tariff reform needed on top of the initial reciprocity-based reform to preserve world pries at initial Nash levels. The graph shows that t_{21} and t_{22} have to be further increased, while t_{31} and t_{32} have to be decreased (recalling that they were held constant in our chosen reciprocity-based reform).

In short, these figures illustrate the important points that (i) the implementation of reciprocity generally does not completely define the required tariff reform but allows degrees of freedom in the selection of which tariffs to change and by how much, and (ii) that this supplementary tariff reform requires all tariffs to be altered. This supplementary reform is not an implication of reciprocity, but is defined by the requirement of fixed world prices.

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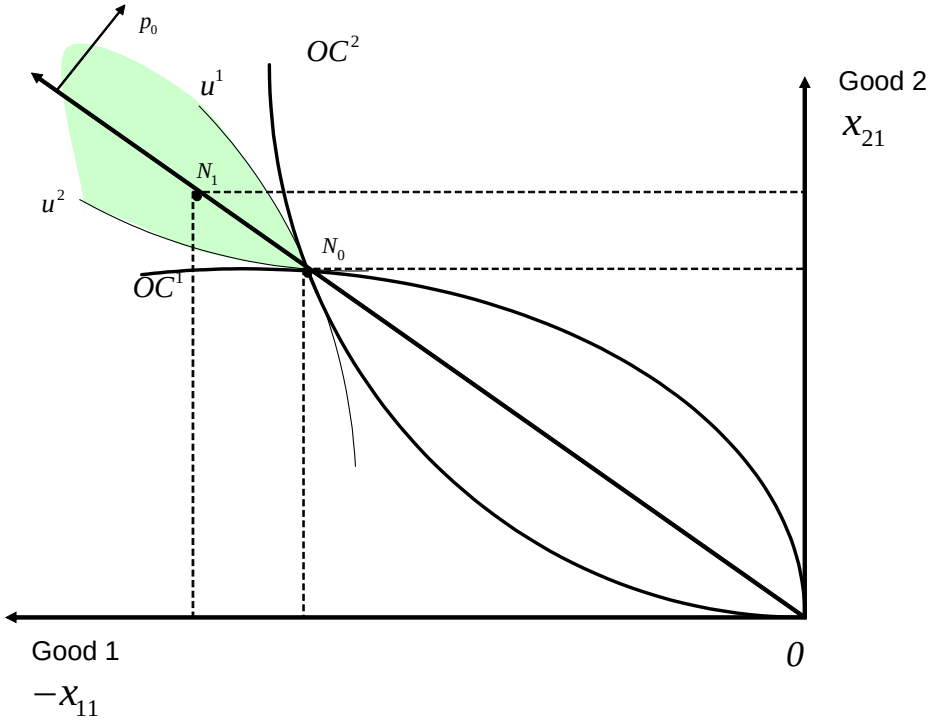

Figure 1: World price preserving reciprocity

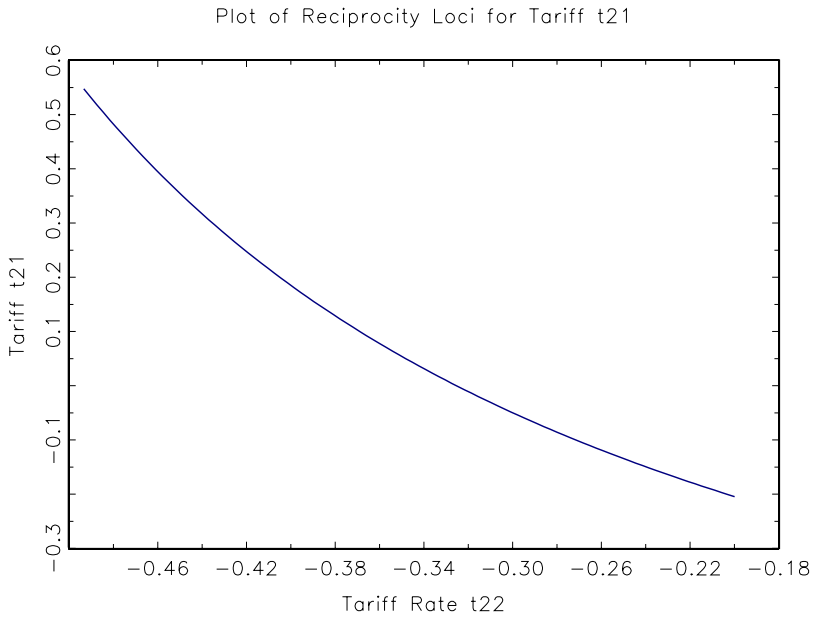

Figure 2: Reciprocity Locus

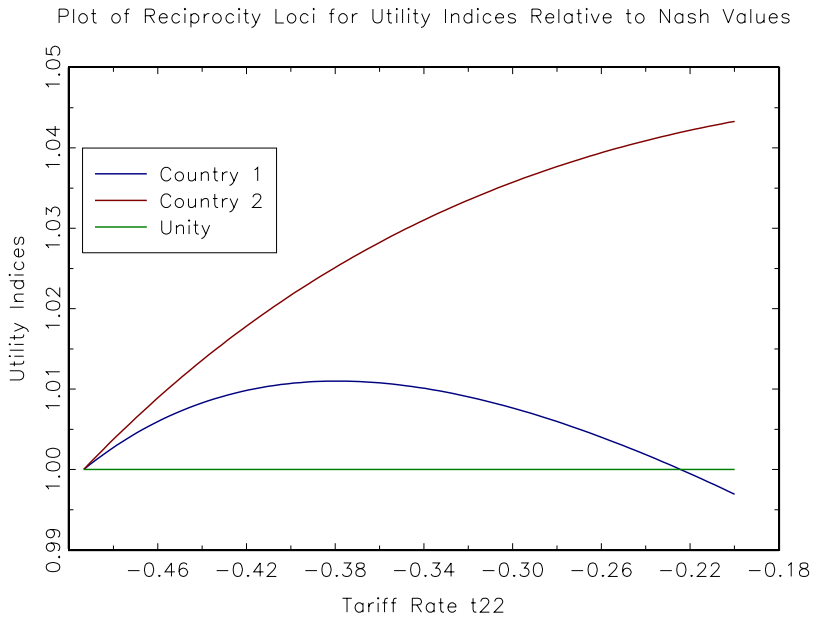

Figure 3: Utilities along Reciprocity Locus Relative to Nash Levels

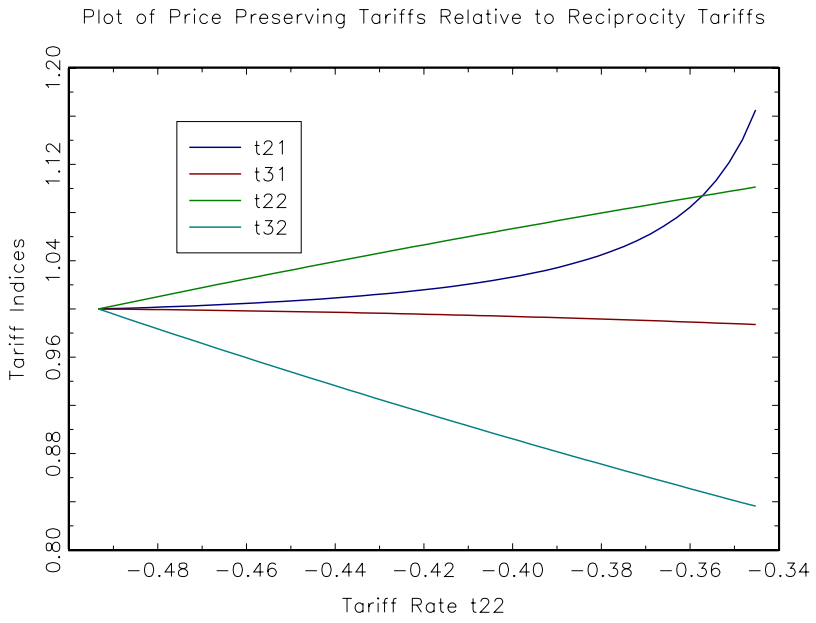

Figure 4: World Price Preserving Tariff Rates Relative to Reciprocity Rates