A theory of trade in a global production network

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Abstract

This paper argues that the determinants of the welfare gains from trade are fundamentally different in the presence of international production linkages. We highlight the salient features of supply chain trade by means of three novel counterfactual exercises in a simple Armington-type trade model. Towards this end, we develop new tools to derive closed-form solutions for arbitrary counterfactuals that allow us to split the compound welfare effects into several meaningful channels. Our main finding shows that, in a global production network, the gains from trade do not so much depend on a country's own geography and technology, but on its access to other countries with superior geography and technology of their own. In particular, our analysis highlights a few key countries that tie the global production network together by intermediating other countries' value added. (JEL codes: C67, F12, F63).

Keywords: global supply chains, gains from trade, trade intermediation.

1 Introduction

Global supply chains are one of the defining characteristics of today's production processes. They carry many potential benefits. Most importantly, they allow countries to specialize in tasks in which they have comparative advantage. This increases overall productivity and the size of world welfare. However, it is not immediately clear that all countries benefit alike. In fact, production fragmentation might even hurt countries –developing and developed alike– that do not manage to participate in the global production network.

In this paper, we investigate this relationship between international production fragmentation and worldwide income inequality. For this purpose, we develop a simple model

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of trade in a global production network.[1] Of course, we are not the first to study the effects of production fragmentation. Already the early theories of Ethier (1979, 1982), and later Eaton and Kortum (2002), Yi (2003), Alvarez and Lucas (2007), and Costinot et al. (2013), have made clear that this has important implications for the sensitivity of national incomes to changes in trade barriers and factor costs.[2] Also, it is widely recognized since Feenstra and Hanson (1996) that production fragmentation has different effects on the economic fortunes of distinct groups of laborers within a nation (e.g. Antràs et al., 2006; Grossman and Rossi-Hansberg, 2008). All these studies stress, however, the very same factors determining the gains and losses of a further increase in production fragmentation that were already highlighted in older trade theories: a country's own technology and its own geographic location.

What has received little attention is that, in a global production network, a country does not only have access to the products and markets of its direct trading partners. It also, indirectly, has access to the products and markets of all the other countries in the supply chains that its trading partners are part of. In other words, in a global production network, the fortunes of a nation also depend on the geography and technology of its trading partners. In fact, our main result shows that this is the single-most important factor for understanding the effects of increased production fragmentation on world income inequality: a country's welfare gain is exclusively determined by its access to other countries with superior technology and geography of their own (Theorem 3).

So, why did prior research fail to stress this salient feature of supply chain trade? A simple answer is that solving a general equilibrium model of trade in a global supply chain is a complicated matter in itself. As a result, prior research has mainly focussed on rather stylized counterfactual exercises –such as the move to autarky or to a state of frictionless trade–, uses simplifying assumptions –such as uniform trade costs around the world–, or resorts to simulations of the old and the new equilibrium under different trade cost regimes. The main methodological contribution of our paper is to show that closed-form solutions for arbitrary counterfactuals are feasible in modern trade models. This allows us, on the one hand, to separate the compound welfare effects of increased production fragmentation into

[1]Throughout the paper, we will use the terms "global supply chain" and "global production network" interchangeably to denote a multi-stage, multi-country spanning production process leading to a final output.

[2]For empirical evidence in support of these theoretical models, see Bems et al. (2011), Baldwin and Taglioni (2011), and Altomonte et al. (2012). Caliendo and Parro (2014), di Giovanni et al. (2014), and Aichele et al. (2014) use them to simulate the impact of policy changes. Another stream in the international economics literature investigates the effect of input-output linkages on the location of industries (Krugman and Venables, 1995; Baldwin and Venables, 2013).

several meaningful channels, whose relative size can be easily compared between countries. On the other hand, it opens up the scope for a whole new range of counterfactual exercises that could previously only be looked at using simulations.[3]

We illustrate most of our ideas by means of a simple Armington-type trade model (Anderson, 1979) with tradable final and intermediate goods, and labor as the sole original production factor. Moreover, producers employ a constant elasticity (CES) production function in our model that subsumes all inputs under the same aggregator. The extent of fragmentation arises endogenously in this model. It depends on the size of a coordination cost that accrues, on top of the usual trade costs, when a firm uses a foreign intermediate good instead of domestic labor for production. Because the equilibrium characterization is relatively easy, this model is ideal for our case, as we can focus on the analysis of the counterfactuals. However, we do also show how our methods and insights can be readily applied to a broader class of trade models, at the example of the Ricardian model of Eaton and Kortum (2002).[4]

We first show that the combination of Armington-type trade and CES technology admits for a unique equilibrium, in which all product and labor markets clear. Conveniently, the model admits for closed-form solutions for the prices and trade volumes of all the tradable final and intermediate goods. This means that, unlike in earlier related models (e.g., Eaton and Kortum, 2002; Yi, 2003), only each country's wage rate is implicitly defined. Interestingly, our expressions for prices and trade volumes bear a close resemblance with the coefficients of a Leontief matrix that is commonplace in input-output models (Leontief, 1986). In our context, this matrix relates a country's prices and trade volumes to each and every other country's inverse wage rate, discounted by the trade and coordination costs involved in getting access to these countries via all direct and indirect trade routes in the global supply chain. Hereby, the matrix ties down a country's overall welfare to its exact

[3]Hereby our paper is closely related to Arkolakis et al. (2012) that show that the overall welfare gains from any trade cost reduction can be inferred from two simple statistics in any model that predicts the gravity equation, including our model. Their results are however silent about where these gains come from, whether they are based on a country's own technological or trade cost advantages, or that of its trading partners. Our closed-form solutions do allow us to exactly pinpoint the important determinants of the gains from trade. This focus also sets our study apart from the literature on the adjustment margins to a trade cost reduction, e.g. new goods or services or the exit of inefficient firms (Melitz and Redding, 2015; Arkolakis et al., 2008; Feenstra, 2010).

[4]We have motivated this paper with the argument that, in a global supply chain, countries can specialize in tasks in which they have comparative advantage. Obviously, this argument calls for a Ricardian model. The Armington model is, however, nothing but a special case, where each country is endowed with an exogenous set of unique capabilities and where a trade cost reduction only changes the extent to which a country takes advantage of its capabilities.

position in the world trade network.

Next, we expand on and generalize results from the network literature, notably Ballester et al. (2006) and Temurshoev (2010), and show that we can quantify changes in the input-output matrix entirely based on its original state. This allows us to do comparative statics analysis for any variation of countries' geography or technology. To also pinpoint the effects on wages, we re-introduce classic comparative statics analysis for general equilibrium into the realm of trade theory. It turns out that the wage effects in the different nations are determined by the immediate changes in the input-output matrix and a multi-country extension of the Harrod foreign-trade multiplier (Harrod, 1936). The latter is responsible for the transmission of income shocks throughout the entire world economy.

We then illustrate the power of our comparative statics approach in three counterfactual exercises. Together they highlight the salient features of supply chain trade. We focus on a global, a country-specific, and a country-pair specific change in the trade or coordination costs, respectively. Our main findings are the following:

(i) It has been argued that –by functioning as containers for foreign technologies– the intermediate goods shipped between the different stages of the global supply chain mitigates country-specific productivity differentials. Consequently, production fragmentation might reduce world income inequality (Baldwin, 2011). We investigate this hypothesis by considering a worldwide trade or coordination cost reduction. Our main result, Theorem 3, shows that whether or not a poor country catches up to a richer country solely depends on its access to key intermediaries, i.e. other countries with a superior supplier and market access (as defined in Redding and Venables, 2004). Because access to these intermediaries is unrelated to the level of per capita income in our model (this is much more determined by a country's own market and supplier access), this finding suggests that a catch up is principally possible.

(ii) Next, we consider the externality that a trade cost reduction along a single trade route between two countries imposes on the welfare of third countries whose trade costs remain unchanged. Common intuition tells us that the labor demand in third countries is negatively affected, because of the more intense competition from the two, now better connected, countries. Decomposing the overall effect into a wage and a consumer price externality, we show that this is a clear cut case only in a world without international production linkages (Theorem 4). In an integrated supply chain, the same cost reduction implies an additional positive externality on foreign labor demand, because of the trading partners' increased demand for intermediate inputs.

(iii) In our third counterfactual we identify those countries that are the most valuable trading partners in a global supply chain. Towards this end, we introduce a novel concept into the realm of international trade. We borrow the Ballester et al. (2006) concept of a "key player" –introduced also earlier in Rasmussen (1956) and Hirschman (1958)– that identifies the importance of a country by looking at how its isolation would affect other nations' welfare. In other words, we are looking at the flipside of the often studied gains from trade, where the focus is on the isolated country's welfare itself (e.g. Arkolakis et al., 2012). Our findings highlight a few countries that derive their importance for the global supply chain not by adding value themselves, but by intermediating the value added produced in other nations. Being connected to those key intermediaries, is the single-most important determinant of a country's welfare gains from a further deepening of the global supply chain (see counterfactual (i)).

To get a feeling for the importance of these salient features of supply chain trade vis-à-vis the classic gains from trade, we also take our model to the data. We develop a simple novel empirical strategy for this purpose. It uses the structure of our model in combination with readily available trade and production data to estimate three empirical constructs that are sufficient to perform all our counterfactual exercises. Here too does our classic comparative statics approach pay off, because our closed-form solutions tell us exactly which constructs of variables and parameters we need. Our results show that intermediation is indeed an important feature of today's world economy. The average country derives its importance, measured by the welfare it generates in other countries, for 30% from its role as an intermediary.

Next to the theoretical articles that we already mentioned, our paper closely relates to a series of empirical studies that, following Hummels et al. (2001), aims to attribute the value generated in a global supply chain to the participating nations and sectors.[5] The contribution of our paper is that we provide a sound theoretical foundation for the measures developed in these papers. In particular, our results on the welfare implications of trade intermediation (counterfactual (i)) add substantive relevance to the measure of vertical specialization trade (Hummels et al., 2001). Moreover, the measures we develop in this paper (counterfactual (iii)) do not only apply to the observed input-output linkages in a global supply chain. Our analysis explicitly takes commodity and factor substitution into account when asking the counterfactual question: is a country is really indispensable or can other countries easily replace its position in the global production network?

[5] Important recent studies along this line are Koopman et al. (2010), Antràs et al. (2012), Johnson and Noguera (2012a), Baldwin and Lopez-Gonzalez (2014), and Los et al. (2014).

Finally, our paper is related in spirit and methodology to a growing literature emphasizing the consequences of interdependent decision making in social and economic networks. It is most closely related to two recent studies on the impact of the network structure of a national supply chain on macroeconomic outcomes (Acemoglu et al., 2012; Oberfield, 2013). One of the common findings is that an idiosyncratic productivity shock to a sector propagates unlike faster, if that sector is proximate to a sector that is a key intermediary to the economy. The mechanisms behind our findings are very related. But unlike these earlier studies, we are interested in the impact of network structure on income inequality in the world economy.

The remainder of the paper is organized in five sections. In Section 2, we present our variant of the Armington model and derive its predictions concerning equilibrium trade volumes, prices, and income levels. Section 3 sets out our empirical strategy. Subsequently, in Section 4, we turn to the comparative statics analysis. Besides deriving several counterfactual predictions analytically, we also illustrate them numerically based on a combination of data and the estimates from Section 3. Section 5 shows how our comparative statics approach can be applied to other trade models, at the example of the Eaton and Kortum (2002) model. Section 6 concludes.

2 Global Supply Chains in an Armington model

Consider a world of $i = 1, 2, ..., n$ trading countries, where each country $i \in \mathcal{N}$ hosts a number of people $L_i > 0$. One category of products traded in our model are the varieties of a final product, which are indexed by $v^{fin} \in \mathcal{V}^{fin}$. Their producers use labor as the sole original production factor. Labor is inelastically supplied and immobile across countries; the total labor endowment is given by L_i, and $w_i > 0$ denotes the wage rate in country i. The other carriers of (foreign) labor value added in our model are the varieties of an intermediate product. They are indexed by $v^{int} \in \mathcal{V}^{int}$ and are produced by a distinct set of firms selling domestically and abroad. Moreover, as these firms themselves use tradable intermediates in their production, all countries are embedded in an integrated production network.

In the following, we further specify our model and solve for its equilibrium:

Final goods market: in every country i, there is a homogeneous group of consumers with Dixit-Stiglitz preferences, who maximize their utility subject to the constraint that expenditures must not exceed w_i. A consumer's indirect utility can be written as $U_i = w_i/P_i^c$, where P_i^c denotes the consumer price index.

The following assumptions tie down our model to the Armington case:[6]

Assumption 1 (Market structure)**.** *For every variety* $v \in \{\mathcal{V}^{fin} \cup \mathcal{V}^{int}\}$ *there is one, and only one, country with a perfectly competitive sector able to produce this variety. The number of varieties producible in country* i *are* $v_i^{fin} \geq 0$ *and* $v_i^{int} \geq 0$ *(with at least one inequality being strict),* $\sum_{i\in\mathcal{N}} v_i^{fin} = |\mathcal{V}^{fin}|$ *and* $\sum_{i\in\mathcal{N}} v_i^{int} = |\mathcal{V}^{int}|$*.*

Assumption 2 (Technology 1)**.** *The unit costs of production and shipment –and thus the competitive equilibrium prices– of all final goods varieties* $v^{fin} \in \mathcal{V}_j^{fin}$ *shipped from country* $j \in \mathcal{N}$ *to country* $i \in \mathcal{N}$ *are identical. The same holds for the intermediate goods varieties* $v^{int} \in \mathcal{V}_j^{int}$ *shipped from exporter* j *to importer* i*.*

The consumer price index can then be written as

$$P_i^c = \left[\sum_{j\in\mathcal{N}} v_j^{fin} (p_{ji}^{fin})^{1-\sigma}\right]^{1/(1-\sigma)} \tag{1}$$

where $p_{ji}^{fin} > 0$ denotes the destination-specific sales price of a typical final goods variety from country j sold in country i, and $\sigma > 1$ the elasticity of substitution between varieties.

From this, the demand in country i for an individual variety produced in j can be obtained from Roy's identity, $q_{ji}^{fin} = -P_i^c(\partial U_i/\partial p_{ji}^{fin})$. The total value of all final goods shipments is then given by

$$X_{ji}^{fin} \equiv v_j^{fin} p_{ji}^{fin} q_{ji}^{fin} L_i = \frac{v_j^{fin}\left(p_{ji}^{fin}\right)^{1-\sigma}}{(P_i^c)^{1-\sigma}} w_i L_i \tag{2}$$

We will call this equation henceforth the *final goods trade equation.*

Intermediate goods market: turning to the producer demand for domestic and foreign intermediate inputs, we make –next to the assumptions introduced above– the following assumption:

Assumption 3 (Technology 2)**.** *Every (final and intermediate goods) producer operates with the same CES production technology that subsumes all input factors under a single*

[6] Assumptions 1-3 will be relaxed in Section 5.

aggregator. Also, producers substitute between input factors with the same elasticity $\sigma > 1$, with which consumers substitute between final goods varieties.[7]

Specifically, to sell $q_{ik} > 0$ units to country $k \in \mathcal{N}$, a producer from country i requires inputs according to

$$l_i \geq 0, \left(q_{ji}^{int} \geq 0\right)_{j \in \mathcal{N}} \quad \textit{such that: } q_{ik} = \frac{\kappa_i}{\tau_{ik}} \left[l_i^{(\sigma-1)/\sigma} + \theta \sum_{j \in \mathcal{N}} v_j^{int} (q_{ji}^{int})^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{3}$$

where l_i denotes the number of workers employed and q_{ji}^{int} the amount of intermediate inputs purchased from one of the $v_j^{int} \geq 0$ competitive upstream sectors in country j. The parameters $\kappa_i > 0$, $\tau_{ik} \geq 1$, and $0 \leq \theta < 1$ depict the total factor productivity in country i, a country pair-specific transportation cost, and the relative factor productivity of intermediate inputs, respectively.

With this specification, we depart from a convention in the international economics literature (e.g. Krugman and Venables, 1995; Eaton and Kortum, 2002; Yi, 2003). Prior studies have typically assumed that the varieties of intermediates are first agggregated under a CES function before they enter, together with labor, a Cobb-Douglas aggregator of the form

$$q_{ik} = \frac{\kappa_i}{\tau_{ik}} l_i^{\beta} \left[\theta \int_{v \in \mathcal{V}^{int}} \left(q_i(v)\right)^{(\sigma-1)/\sigma} dv \right]^{\frac{(1-\beta)\sigma}{\sigma-1}} \tag{4}$$

where $\beta > 0$ and $1 - \beta > 0$ measure the fixed labor and intermediate goods cost shares, respectively. There are two major advantages of our specification. First, it turns out that technology (3) allows us to derive closed-form solutions for equilibrium prices and outputs of all the tradable goods in our model. Thereby, we can move a signification step ahead in the solution of an n-country general equilibrium model with international input-output linkages. Second, by giving up the assumption of constant input shares in production, technology (3) can accommodate the stylized empirical fact of a declining labor cost share (Karabarbounis and Neiman, 2014) and an increasing intermediate input share (Feenstra, 1998) in all major economies over the course of the past thirty years. A common explanation for this trend are the major ICT breakthroughs of the mid 1980s that

[7] We relax this assumption in Section 5. None of our results hinge on it. Assuming different elasticities in our model would still allow us to establish a unique equilibrium and to conduct all the desired comparative statics analyses. Only the empirical implementation of our model in Section 3 would become more cumbersome, as it would require us to a-priori assume specific values for the different elasticities.

have lowered the costs of outsourcing and thereby facilitated the substitution of (foreign) intermediate inputs for domestic labor. This trend is well-captured by an increase in the parameter θ of production function (3).

To see this, note that cost-minimization yields the following (destination-specific) minimum variable costs for a producer

$$c_{ik} = \frac{P_i^p}{\kappa_i} \tau_{ik} q_{ik} \tag{5}$$

where the associated producer price index is given by

$$P_i^p = \left[w_i^{1-\sigma} + \theta^\sigma \sum_{j \in \mathcal{N}} v_j^{int} (p_{ji}^{int})^{1-\sigma} \right]^{1/(1-\sigma)} \tag{6}$$

Using Shephard's lemma to obtain the input demand functions, $l_i = \partial(\sum_{k \in \mathcal{N}} c_{ik})/\partial w_i$ and $q_{ji}^{int} = \partial(\sum_{k \in \mathcal{N}} c_{ik})/\partial p_{ji}^{int}$, we arrive at the following expression for the labor cost share

$$\frac{w_i l_i}{w_i l_i + \sum_{j \in \mathcal{N}} v_j^{int} p_{ji}^{int} q_{ji}^{int}} = \frac{w_i^{1-\sigma}}{(P_i^p)^{1-\sigma}} \tag{7}$$

Hence, our model predicts that –when keeping wage rates and input prices fixed– an increase of θ lowers the share of labor costs and increases the share of domestic and foreign intermediate inputs in production.

Continuing with the solution of the model, in a competitive equilibrium, the country-pair specific prices need to equalize with the unit costs of production and shipment

$$p_{ik}^{fin} = \frac{P_i^p}{\kappa_i} \tau_{ik}^{fin} \quad \text{and} \quad p_{ik}^{int} = \frac{P_i^p}{\kappa_i} \tau_{ik}^{int} \tag{8}$$

where, as of now, we allow for possibly different trade costs for final goods, τ_{ik}^{fin}, and intermediate goods, τ_{ik}^{int}.

Furthermore, combining the input demand function, $q_{ji}^{int} = \partial(\sum_{k \in \mathcal{N}} c_{ik})/\partial p_{ji}^{int}$, with the identities (2) and (8) results in our *intermediate goods trade equation*, which measures the value of all intermediate goods shipped from country j to i:

$$\begin{aligned} X_{ji}^{int} &\equiv v_j^{int} p_{ji}^{int} q_{ji}^{int} \left[v_i^{fin} + v_i^{int} \right] \\ &= \theta^{\sigma} v_j^{int} (p_{ji}^{int})^{1-\sigma} \kappa_i^{-1} (P_i^p)^{\sigma} \sum_{k \in \mathcal{N}} \left[v_i^{fin} \tau_{ik}^{fin} q_{ik}^{fin} L_k + v_i^{int} \tau_{ik}^{int} q_{ik}^{int} \left(v_k^{fin} + v_k^{int} \right) \right] \\ &= \theta^{\sigma} \frac{v_j^{int} (p_{ji}^{int})^{1-\sigma}}{(P_i^p)^{1-\sigma}} \sum_{k \in \mathcal{N}} \left[X_{ik}^{fin} + X_{ik}^{int} \right] \end{aligned} \tag{9}$$

This equation nicely formalizes two distinctive features of trade in a global supply chain. First, the demand for intermediate inputs produced in country j does not only increase in the size of the importer market i, but also in the distance of country i to its own export markets $k \neq i$ and the size of the latter (a *pull factor*). In other words, equation (9) states that the gravity of nations $k \neq i, j$ has a trade-enhancing effect on the intermediate goods flows between j and i, instead of the distorting forces emphasized in earlier theories (see e.g. Anderson and van Wincoop, 2003) that can also be found in our final goods trade equation (2).

Second, equation (9) suggests that –despite the fact that all suppliers at the same stage of the global supply chain compete for the downstream demand– a price reduction by any one of them, $dp_{li}^{int} < 0$ for $l \neq j$, has a countervailing positive effect on the demand of country l's competitor j. The reason is that the input price reduction leads to a cost reduction at the downstream stage, which in equilibrium results in a price reduction for the downstream output. But this means that the demand for the composite good and thus the demand for all its inputs rise (a *push factor*). Formally, this can be seen from the fact that the augmented price index, $(P_i^p)^{\sigma-1}$, in equation (9), which captures the intensity of upstream competition, appears in its inverted form in X_{ik}^{fin} and X_{ik}^{int} (after substituting (8) for p_{ik}^{fin} and p_{ik}^{int}). There, it reflects the productivity of country i's producers. The lower p_{li}^{int} is the higher that productivity, and thus the higher the demand in country i for intermediate inputs.[8]

Returning to the solution of the model, the systems of trade equations and producer price indices need to clear in equilibrium. In the following, we show that this requirement, combined with our technology specification (3), allows for a closed-form solution for all prices and trade volumes up to the wage rates, which remain the only implicitly defined

[8] In total, we find $\partial X_{ji}^{int} / \partial p_{li}^{int} = 0$ for $l \neq j$, which indicates that the higher demand for inputs is just offset by the intensified competition at the upstream stage. Yet, a comparable price reduction in the final goods market yields $\partial X_{ji}^{fin} / \partial p_{li}^{fin} > 0$.

variables in our model.

Substituting the competitive prices (8) into (6) and taking both sides to the power of $(1-\sigma)$, we obtain

$$(P_i^p)^{1-\sigma} = w_i^{1-\sigma} + \sum_{j\in\mathcal{N}} (P_j^p)^{1-\sigma}\theta^\sigma v_j^{int}\kappa_j^{\sigma-1}(\tau_{ji}^{int})^{1-\sigma} \tag{10}$$

Similarly, substituting (8) into (9) we get

$$\begin{aligned}(P_i^p)^{\sigma-1}\sum_{j\in\mathcal{N}}\left[X_{ij}^{fin}+X_{ij}^{int}\right] &= (P_i^p)^{\sigma-1}\sum_{j\in\mathcal{N}}X_{ij}^{fin}+\theta^\sigma v_i^{int}\kappa_i^{\sigma-1}\sum_{j\in\mathcal{N}}(\tau_{ij}^{int})^{1-\sigma}\\ &\times\ (P_j^p)^{\sigma-1}\left[\sum_{k\in\mathcal{N}}X_{jk}^{fin}+\sum_{k\in\mathcal{N}}X_{jk}^{int}\right]\end{aligned} \tag{11}$$

Equation (10) shows the often encountered interdependence between the producer price indices in different nations (e.g., Eaton and Kortum, 2002; Alvarez and Lucas, 2007). Given the functional form of the formerly studied price indices, an explicit solution was considered impossible because of their non-linear interdependence. However, by virtue of our CES specification (3), we are able to express the row vector of augmented price indices in matrix notation as

$$\mathbf{1}^T(P^p)^{1-\sigma} = \mathbf{1}^T W^{1-\sigma} + \mathbf{1}^T(P^p)^{1-\sigma}A \tag{12}$$

where $\mathbf{1}$ is a column vector of ones and $\mathbf{1}^T$ its transpose, $W^{1-\sigma}$ and $(P^p)^{1-\sigma}$ are $n\times n$ diagonal matrices with elements $w_i^{1-\sigma}$, respectively $(P_i^p)^{1-\sigma}$, on their diagonals, and A is the $n\times n$ full matrix of *trade intensities*

$$A = \theta^\sigma V^{int}K^{\sigma-1}(T^{int})^{1-\sigma} \tag{13}$$

Here, θ^σ is a scalar, V^{int} and $K^{\sigma-1}$ are $n\times n$ diagonal matrices with elements v_i, respectively $\kappa_i^{\sigma-1}$, on their diagonals, and $(T^{int})^{1-\sigma}$ is the $n\times n$ full matrix $\left((\tau_{ij}^{int})^{1-\sigma}\right)$, where all entries in row i of this matrix belong to the exporting country i and entries in column j belong to importer j.

Similarly, denote by $\bar{M}$ and $\bar{M}^c$ the $n\times n$ diagonal matrices with entries $(P_i^p)^{\sigma-1}\sum_{j\in\mathcal{N}}[X_{ij}^{fin}+X_{ij}^{int}]$ respectively $(P_i^p)^{\sigma-1}\sum_{j\in\mathcal{N}}X_{ij}^{fin}$ on their diagonals. The system of equations (11) can be written in matrix notation as

$$\bar{M}\mathbf{1} = \bar{M}^c\mathbf{1} + A\bar{M}\mathbf{1} \tag{14}$$

To solve these systems, suppose the following:

Assumption 4 (Existence of Leontief inverse)**.** *The largest, real-valued component of all characteristic roots of* A*,* $\rho(A)$*, satisfies* $0 < \rho(A) < 1$.

It follows from spectral theory (e.g., Debreu and Herstein, 1953) that the matrix inverse $[I - A]^{-1}$ exists and is given by $[I - A]^{-1} = \sum_{h=0}^{\infty} A^h$, with $A^0 = I$, $A^1 = A$, $A^2 = AA$, etc. In other words, in order to ensure convergence of the matrix series, Assumption 4 imposes an upper bound on the number of traded varieties and the factor, country, and country-pair specific productivity parameters. To see this, note that an alternative sufficient condition for existence of $[I - A]^{-1}$ is that either $\max_{i\in\mathcal{N}}\{\sum_{j\in\mathcal{N}} a_{ij}\} < 1$ or $\max_{j\in\mathcal{N}}\{\sum_{i\in\mathcal{N}} a_{ji}\} < 1$. This alternative condition shows also more directly that all we require is a sufficiently small value for θ. The economic interpretation is that, for producer prices to be strictly positive and import volumes to be bounded from above, the intermediate goods available on world markets need to be relatively unproductive in comparison to the domestically available labor.[9]

Given that Assumption 4 is met, the market-clearing price indices and trade volumes of our model can be summarized in the following matrix expressions[10]

$$\begin{aligned} \mathbf{1}^T(P^p)^{1-\sigma} &= \mathbf{1}^T W^{1-\sigma}[I - A]^{-1} = \mathbf{1}^T W^{1-\sigma}\sum_{h=0}^{\infty} A^h \\ \mathbf{1}^T(P^c)^{1-\sigma} &= \mathbf{1}^T W^{1-\sigma}\sum_{h=0}^{\infty} A^h B \\ \bar{M}\mathbf{1} &= \sum_{h=0}^{\infty} A^h \bar{M}^c\mathbf{1} = \sum_{h=0}^{\infty} A^h BWL(P^c)^{\sigma-1}\mathbf{1} \end{aligned} \tag{16}$$

[9]Variants of this condition are common in models of supply chain trade. For the very same reason, Eaton and Kortum (2002, p. 1747) and Alvarez and Lucas (2007, p. 1729) impose an upper bound on the variability of the sector-specific productivity draws in their models, such that highly efficient intermediate goods are very unlikely.

[10]Expanding on the matrices in (16), we can furthermore express the intermediate goods trade equation (9) by

$$X_{ji}^{int} = (P_j^p)^{1-\sigma} a_{ji}\left[\sum_{k\in\mathcal{N}}\sum_{h=0}^{\infty} a_{ik}^{[h]}\sum_{l\in\mathcal{N}} b_{kl} w_l L_l (P_l^c)^{\sigma-1}\right] \tag{15}$$

where $(P^c)^{1-\sigma}$, W, and L are the diagonal matrices of consumer price indices, wage rates, and population sizes, respectively, and where in analogy to (13) the full matrix B contains the pairwise *trade intensities for final goods*

$$B = V^{fin} K^{\sigma-1} (T^{fin})^{1-\sigma} \tag{17}$$

To add meaning to the matrices in (16), the entries i in the vectors $\mathbf{1}^T(P^p)^{1-\sigma}$ and $\mathbf{1}^T(P^c)^{1-\sigma}$ correspond to the *supplier access* of the producers and consumers in country i respectively, as defined in Redding and Venables (2004). The entries in $\bar{M}\mathbf{1}$, on the other hand, correspond to their definition of *market access*. It is straightforward to show that $U_i L_i = (\bar{M}_i)^{1/\sigma}/P_i^c$, i.e. the real labor income in country i is the product of its supplier and market access.

The new dimension of the matrices in (16) that we exploit is that they clearly show how a country's welfare is tied down to its exact position in the world trade network. The inverse matrix $[I - A]^{-1} = \sum_{h=0}^{\infty} A^h$ plays a key role here. This matrix has long been of interest to regional and development economists who, dating back to Leontief (1936), have studied the flow of factor content in a national supply chain.[11] In the context of our stylized two-sector economy, the interpretation is as follows: the vectors of *supplier access*, $\mathbf{1}^T(P^p)^{1-\sigma}$ and $\mathbf{1}^T(P^c)^{1-\sigma}$, state that every input-producing nation contributes with its labor force to the productivity, respectively the consumption, in other nations. The value added of country i is $w_i^{1-\sigma}$. This value added is used by all foreign manufacturers: some of them employ it directly, while others use it indirectly, embodied in the intermediate products of yet another nation. The matrices

$$S^p \equiv W^{1-\sigma} \sum_{h=0}^{\infty} A^h \quad \text{and} \quad S^c \equiv W^{1-\sigma} \sum_{h=0}^{\infty} A^h B \tag{18}$$

keep track of all the direct and indirect value flows between any pair of countries. The summand $W^{1-\sigma}A$ captures the value added of each country embodied in its intermediate good shipments to other countries. The summands $W^{1-\sigma}A^h$, for $h > 1$, on the other hand, keep track of each country's value added that reaches other countries only after another

[11] Two recent examples are Temurshoev (2010) and Acemoglu et al. (2012). While the majority of studies in this field assumes Leontief's original fixed technical coefficients, for which the existence of a solution to the linear relationships (10) and (11) is obvious, Burres (1994) and Acemoglu et al. (2012) show that also with a Cobb-Douglas technology, value flows and prices are (log-)linearly interrelated. This homeomorphism between Leontief and Cobb-Douglas model is not surprising given the fact that both models are nested in our CES specification (3).

$h - 1$ intermediate production steps.

Similar, the *market access* vector $\bar{M}\mathbf{1}$ suggests that the demand for goods produced in country i depends on the size of every other country's market. The real size of market j is $w_j L_j (P_j^c)^{\sigma-1}$. The final goods producers from country i approach this market directly; their access depends on the trade intensity b_{ij}. The intermediate goods producers, in contrast, access market j indirectly, through all the final products that incorporate their intermediate outputs. These direct and indirect market accesses are contained in matrix

$$M \equiv \sum_{h=0}^{\infty} A^h BLW(P^c)^{\sigma-1} \tag{19}$$

In other words, the matrices in (16) show that a country that is directly or indirectly better connected, i.e. takes in a more central position in the global production network, experiences lower production costs, a higher demand for its products, and a larger welfare. In support of this view, the matrices in (16) are closely related to the Katz-Bonacich centrality vector corresponding to the world trade network (Katz, 1953; Bonacich, 1987).

Labor market and general equilibrium: So far, we have found that the prices and quantities of all tradable goods in our model can be expressed as functions of only exogenous parameters and wage rates. We now turn to the labor market, pin down the wages, and thereby close the entire world economy.

Workers are immobile between countries but free to move between their domestic intermediate and final goods sectors. So, each country has a uniform wage rate w_i. Since all of a country's firms make use of the same labor share $w_i^{1-\sigma}/(P_i^p)^{1-\sigma}$, the economy-wide labor demand is given by

$$w_i L_i = \frac{w_i^{1-\sigma}}{(P_i^p)^{1-\sigma}} \sum_{j\in\mathcal{N}} \left[X_{ij}^{fin} + X_{ij}^{int} \right] = w_i^{1-\sigma} \bar{M}_i \tag{20}$$

From (16), it follows that $\bar{M}_i$ can be expressed as a function of only parameters and wages. Hence, an equilibrium for our model is given by a wage vector $\mathbf{w} = (w_1, w_2, ..., w_m)$ that solves (20). The following result is inspired by Alvarez and Lucas (2007) and establishes existence of a unique, positive wage vector with this property:

Theorem 1. *There is a unique* $\mathbf{w} \in \mathfrak{R}_{++}^n$ *such that* $\lambda\mathbf{w}$*, with* $\lambda \in \mathfrak{R}_{++}$*, establishes an equilibrium for equation system (20).*

The proof can be found in the appendix. There, we show that the sufficient conditions for existence of a Walrasian wage equilibrium are satisfied. Moreover, the wage vector is unique up to a common scale factor λ. Combined with the trade and price equations (1)-(16), it then follows that all available goods are traded in all markets at a positive and finite price. Also, since $U_i = w_i/P_i^c$ is homogeneous of degree zero with respect to $\mathbf{w}$, the per capita income is uniquely determined in our model.

In the following sections, we expand on this result and conduct various comparative statics analyses for our equilibrium. Together, they highlight the salient features of trade in a global production network. Besides deriving several general propositions, we complement our analytic predictions by illustrating them numerically for the global trade network of 2005. We do this based on a simple empirical strategy.

3 Empirical strategy

Our empirical strategy uses the structure of our model in combination with readily available data on final and intermediate goods trade flows (UN COMTRADE)[12], total industrial value added and production (WDI and UNIDO), and various observed trade cost determinants (CEPII). It provides us with three empirical constructs that are sufficient to numerically perform all the comparative statics exercises introduced in Section 4. These constructs are estimates of each country's producer price index, $(P_i^p)^{1-\sigma}$, and the A and B matrices that capture the final and intermediate goods trade intensities respectively, i.e. (6), (13) and (17). Each of them is not directly observable in any readily available dataset.

We obtain these empirical constructs by estimating the following simple gravity equation, which is similar to that estimated in Costinot et al. (2012):

$$\ln \tilde{X}_{ij}^k = s_i^k + m_j^k + \boldsymbol{\Lambda_{ij}}\boldsymbol{\delta^k} + \epsilon_{ij}^k \tag{21}$$

[12] All data we use is for the year 2005. The coverage of bilateral trade flows is most comprehensive in this year, allowing us to include as many countries as possible (100). We use the UN's BEC classification to distinguish between final and intermediate goods: the BEC class "consumption goods" is considered as final goods; the combined BEC classes "capital goods" and "intermediate goods" are our intermediate goods. We consider capital goods as intermediate inputs as they are used in the production of other goods. Results excluding capital goods are available upon request.

where $k \in \{fin, int\}$ and

$$\begin{aligned}
\tilde{X}_{ij}^{int} &= \frac{X_{ij}^{int}}{\sum_{k\in\mathcal{N}}[X_{jk}^{fin} + X_{jk}^{int}]} = \frac{X_{ij}^{int}}{(Ind.Prod)_j} \\
\tilde{X}_{ij}^{fin} &= \frac{X_{ij}^{fin}}{w_j L_j} = \frac{X_{ij}^{fin}}{\sum_{k\in\mathcal{N}}[X_{jk}^{fin} + X_{jk}^{int} - X_{kj}^{int}]} = \frac{X_{ij}^{fin}}{(Ind.VA)_j}
\end{aligned}$$

The parameters s_i^k and m_j^k are sector-specific exporter- and importer- fixed effects, $\boldsymbol{\Lambda_{ij}}$ is a vector of observable bilateral trade cost determinants (distance, and sharing a common border, language, or colonial history), and $\boldsymbol{\delta^k}$ the corresponding sector-specific coefficient vector measuring the relative importance of each trade cost determinant. On top of this, ϵ_{ij}^k captures any sector-specific unobserved bilateral trade cost components that are assumed to be orthogonal to all included regressors.[13]

Based on the theoretical equivalents of (21), i.e. equations (2) and (9), in combination with (8), and (16), we can use the estimated parameters from (21) to immediately obtain

[13] Costinot et al. (2012) avoid the need to include observable trade cost determinants. They capture trade costs by a full set of ij-specific fixed effects (while the error term contains any sector-specific trade cost component). Such ij-specific effects do, however, not only capture trade costs, but also any exporter- and importer-specific market size and productivity differentials. This is no problem in Costinot et al. (2012), where the exporter fixed effects are the main focus of study (see their footnotes 18 and 32). In our case, however, trade costs are a crucial ingredient for our counterfactual exercises. The fact that their estimates are confounded by the sector averages of the s_i^k and m_j^k effects, for $k \in \{fin, int\}$, makes the Costinot et al. (2012) "dummy-approach" unsuitable for our purposes. It makes it e.g. impossible to calculate the Leontief matrix $(I - A)^{-1}$.

the three constructs we are after:

$$
\begin{aligned}
\left(\tilde{P}_i^p\right)^{1-\sigma} &= \exp(\hat{m}_R^{int} - \hat{m}_i^{int}) \\
&= \underbrace{\left(\frac{\theta^\sigma \upsilon_R^{int} \kappa_R^{\sigma-1} (P_R^p)^{1-\sigma}}{(P_R^p)^{1-\sigma}}\right)}_{\exp(\hat{m}_R^{int})} \underbrace{\left(\frac{(P_i^p)^{1-\sigma}}{\theta^\sigma \upsilon_R^{int} \kappa_R^{\sigma-1} (P_R^p)^{1-\sigma}}\right)}_{1/\exp(\hat{m}_i^{int})} = \left(\frac{P_i^p}{P_R^p}\right)^{1-\sigma} \\
a_{ij} &= \exp(\hat{s}_i^{int} + \hat{m}_i^{int} + \boldsymbol{\Lambda_{ij}} \boldsymbol{\hat{\delta}^{int}} + e_{ij}^{int}) \\
&= \underbrace{\left(\frac{\upsilon_i^{int} \kappa_i^{\sigma-1} (P_i^p)^{1-\sigma}}{\upsilon_R^{int} \kappa_R^{\sigma-1} (P_R^p)^{1-\sigma}}\right)}_{\exp(\hat{s}_i^{int})} \underbrace{\left(\frac{\theta^\sigma \upsilon_R^{int} \kappa_R^{\sigma-1} (P_R^p)^{1-\sigma}}{(P_i^p)^{1-\sigma}}\right)}_{\exp(\hat{m}_i^{int})} \underbrace{\left(\tau_{ij}^{int}\right)^{1-\sigma}}_{\exp(\boldsymbol{\Lambda_{ij}} \boldsymbol{\hat{\delta}^{int}} + e_{ij}^{int})} = \theta^\sigma \upsilon_i^{int} \kappa_i^{\sigma-1} \left(\tau_{ij}^{int}\right)^{1-\sigma} \\
\tilde{b}_{ij} &= \exp(\hat{s}_i^{fin} - (\hat{m}_R^{int} - \hat{m}_i^{int}) + \boldsymbol{\Lambda_{ij}} \boldsymbol{\hat{\delta}^{fin}} + e_{ij}^{fin}) \\
&= \underbrace{\left(\frac{\upsilon_i^{fin} \kappa_i^{\sigma-1} (P_i^p)^{1-\sigma}}{\upsilon_R^{fin} \kappa_R^{\sigma-1} (P_R^p)^{1-\sigma}}\right)}_{\exp(\hat{s}_i^{fin})} \underbrace{\left(\frac{P_R^p}{P_i^p}\right)^{1-\sigma}}_{\left(\tilde{P}_i^p\right)^{\sigma-1}} \underbrace{\left(\tau_{ij}^{fin}\right)^{1-\sigma}}_{\exp(\boldsymbol{\Lambda_{ij}} \boldsymbol{\hat{\delta}^{fin}} + e_{ij}^{fin})} = \frac{\upsilon_i^{fin} \kappa_i^{\sigma-1} \left(\tau_{ij}^{fin}\right)^{1-\sigma}}{\upsilon_R^{fin} \kappa_R^{\sigma-1}}
\end{aligned}
\tag{22}
$$

where R refers to a reference country whose two exporter dummies (one for each sector) we exclude when estimating (21) to avoid perfect multicollinearity.[14] Furthermore, we avoid the need to specify countries' internal trade costs, τ_{ii}^{int} and τ_{ii}^{fin}, by calculating the trade intensities a_{ii} and $\tilde{b}_{ii}$ directly using (9) and (8), and (2), (8), (16) and (22), respectively:

$$
\begin{aligned}
a_{ii} &= \tilde{X}_{ii}^{int} \\
\tilde{b}_{ii} &= \tilde{X}_{ii}^{fin} \left(\frac{\sum_{j \neq i} \left[\left(\tilde{P}_j^p\right)^{1-\sigma} \tilde{b}_{ji} \right]}{\left(\tilde{P}_i^p\right)^{1-\sigma}} \right)
\end{aligned}
\tag{23}
$$

Unlike internal trade costs, the internal flows X_{ii}^{fin} and X_{ii}^{int} can be immediately retrieved from our trade and production data: $X_{ii}^{fin} = (Ind.VA)_i - \sum_{j \neq i}[X_{ij}^{fin} + X_{ij}^{int} - X_{ji}^{int}]$ and $X_{ii}^{int} = (Ind.Prod)_i - (Ind.VA)_i - \sum_{j \neq i} X_{ji}^{int}$.

Three remarks are in place: first, it is important to note that our empirical strategy works by virtue of our production function specification (3), where all input factors are

[14]This is why our estimates of $(P_i^p)^{1-\sigma}$ and B are confounded by the reference country's exporter-effect in the same sector, explaining all the R-terms in (22). Note, however, that this does not impose any problem for our counterfactual exercises, where these terms will always cancel out. It makes the choice of reference country irrelevant - we choose Germany.

subsumed under the same CES aggregator. Making this assumption in other trade models generating the gravity equation could also prove useful from an empirical perspective, as it facilitates meaningful combinations of a country's estimated importer- and exporter-fixed effects.

Second, by including the residuals when calculating a_{ij} and $\tilde{b_{ij}}$, we take ϵ_{ij}^k in (21) as an integral part of overall trade costs. In this way, we can reconcile the zero trade flows reported for 20% of our exporter-importer pairs by assuming prohibitively high trade costs on these trade routes. We could also exclude the residuals from (22), hereby implicitly interpreting the zero trade flows as measurement error.[15] Results when excluding the residuals from (22) are available upon request. Of course, we could also do our analysis on a selected sample of countries where each country exports to, and imports from, each other country. Results for this much smaller sample of 40 mainly developed countries are also available upon request.

Finally, the largest characteristic root of our estimated matrix A is well below one: $\rho(A) = 0.76$. This ensures, see Assumption 4, existence of a unique equilibrium in our model and justifies the comparative statics analysis that we do in the remainder of the paper.

4 Counterfactual analysis

In the following, we perform various counterfactual variations of the *trade intensity matrices* A and B, as defined in (13) and (17), and investigate the implications around the unique equilibrium point in our model. Our focus is on the central components of these matrices: the coordination cost parameter θ and the trade cost matrices T^{fin} and T^{int}. However, the methods developed in this section readily lend themselves to the analysis of other parameter variations, such as consumer preferences, population sizes, or total factor productivity.

Our approach is that of classic comparative statics analysis. This means that we focus on *small shocks* to A and B so that, on the one hand, Assumption 4 is satisfied even in the new Leontief matrix $[I - A']^{-1}$, where A' indicates the new matrix. On the other hand, we approximate the changing wage rates by the total differential of system (20). The novelty of our approach with regard to prior comparative statics analyses of similar trade models (e.g., Eaton and Kortum, 2002; Arkolakis et al., 2012; Costinot et al., 2013) is that our

[15] See Silva and Tenreyro (2006), e.g. footnote 3 and 4, for a more detailed discussion on how to reconcile the existence of zero trade flows. See also e.g. Helpman et al. (2008) for a paper that unlike our Armington model explicitly considers the selection into trade, both theoretically and empirically.

approach allows us to start from any initial parameter constellation and to study the effect of any (small) variation of the same. An alternative approach has been developed in Dekle et al. (2008) and Caliendo and Parro (2014). They define an *equilibrium in changes* that is –unlike our comparative statics approach– independent of many time-constant parameters that are difficult to identify in the data, such as bilateral trade costs. However, they need to rely on numerical simulations to determine the effects of a shock, because all variables are only implicitly defined in their equilibrium. The advantage of our approach is its closed-form solutions. Based on this, we are not only able to derive several general propositions on the overall welfare effect of a shock, but we can also arbitrarily decompose it into several meaningful components. Moreover, our approach is less data intensive in other respects, because the explicit solutions tell us exactly which data and estimates are sufficient for our numerical counterfactuals.

We assess all comparative statics effects in terms of real income per capita, $U_i = w_i/P_i^c$. Since we hold population sizes fixed and assume full employment, our results also apply to other welfare measures, such as income per worker or per capita or total labor income. For a small shock to matrices A or B, the relative income change in country i can be written as $dU_i/U_i = dw_i/w_i - dP_i^c/P_i^c$. Since we do have explicit solutions for consumer prices, the second effect is easily obtained. Yet, we still need to understand how a shock to A (or B) translates into a wage change and, importantly, how it affects $[I - A]^{-1}$.

The following lemma, which is proven in the appendix, characterizes the total differential of the system of labor market equations (20). Because (20) and U_i are homogeneous of degree zero, we fix the wage rate in any one country and investigate the changes in the remaining ones:

Lemma 1. *The effect on* $\mathbf{w}$ *of a small shock* $dA = A' - A$ *(or* $dB = B' - B$*) is given by*

$$\frac{\mathbf{dw}}{\mathbf{w}} = \frac{1}{\sigma}\left\{\sum_{h=0}^{\infty}(\Phi^{-i^*})^h\right\}^{+i^*}\frac{\mathbf{d\bar{M}}}{\mathbf{\bar{M}}} \tag{24}$$

where $\mathbf{dw}/\mathbf{w}$ *is such that* $dw_{i^*}/w_{i^*} = 0$ *for any one* $i^* \in \mathcal{N}$*. Moreover, the superindex* $-i^*$ *indicates that row* i^* *and column* i^* *is deleted from a matrix, whereas* $+i^*$ *means that a vector of zeros is inserted in row* i^* *and column* i^* *in a matrix. Finally,* $d\bar{M} = \bar{M}' - \bar{M}$ *and* Φ^{-i^*} *denotes the* $(n-1)\times(n-1)$ *matrix*

$$\Phi^{-i^*} \equiv \frac{1}{\sigma}\left\{\left[\bar{M}\right]^{-1} M\left[I + (\sigma - 1)\left[S^c(P^c)^{\sigma-1}\right]^T\right]\right\}^{-i^*} \tag{25}$$

where $\bar{M}$ and $(P^c)^{\sigma-1}$ are the diagonal matrices defined in (16), and S^c and M are given in (18) and (19), respectively.

The matrix $\sum_{h=0}^{\infty}(\Phi^{-i^*})^h$ is closely related to the Harrod foreign trade multiplier (Harrod, 1936). Our version is a multi-country extension that captures all the feedbacks of a single idiosyncratic wage shock throughout the entire world economy. A positive wage shock in any one country raises labor demand in all other nations by, on the one hand, increasing demand for their products and, on the other hand, weakening the competitive position of the country whose labor costs went up (see (25)). And since this leads to higher wage/income levels around the globe, it in turn starts another round of demand increases, etc.

Based on Lemma 1, we can summarize the per capita income changes in the following column vector

$$\frac{\mathbf{dU}}{\mathbf{U}} = \underbrace{\frac{1}{\sigma}\,\Psi\,\frac{\mathbf{d\bar{M}}}{\mathbf{\bar{M}}}}_{\text{wage effect}} - \underbrace{\frac{1}{1-\sigma}\,\frac{\mathbf{d(P^c)^{1-\sigma}}}{\mathbf{(P^c)^{1-\sigma}}}}_{\text{price effect}} \tag{26}$$

where the $n \times n$ full matrix Ψ, with positive elements on its diagonal and negative off-diagonal elements, is given by

$$\Psi \equiv \left(I - \underbrace{\left[S^c(P^c)^{\sigma-1}\right]^T}_{\substack{\text{wage-induced}\\ \text{price effect}}}\right)\left[\sum_{h=0}^{\infty}(\Phi^{-i^*})^h\right]^{+i^*} \tag{27}$$

Next, we derive for three different changes to matrix A an expression for the *new* Leontief matrix, $[I-A']^{-1}$, where in the spirit of comparative statics analysis we relate this matrix to its initial state, $[I-A]^{-1}$. The following lemma expands on established results on the exact solution for the inverse of a matrix sum (Henderson and Searle, 1981), and generalizes previous applications of these results in the economics literature by Minabe (1966, p. 58), Ballester et al. (2006, Lemma 1, p. 1411), and Temurshoev (2010, Lemma 2, p. 877):

Lemma 2. *Consider square matrices A and A' and scalars $x, y \in \mathfrak{R}$, such that $0 < \rho(A) < 1$ and $0 < \rho(A') < 1$:*

1. *For $x \to 0^+$ and $A' = A + x\tilde{A}$, where $\tilde{A}$ is identical to A except that an arbitrary number*

of entries is set to zero (i.e. for all $ij \in \mathcal{N} \times \mathcal{N}$ *either* $\tilde{a}_{ij} = a_{ij}$ *or* $\tilde{a}_{ij} = 0$*)*

$$\left[I - A - x\tilde{A}\right]^{-1} = \left[I - A\right]^{-1} + x\left[I - A\right]^{-1}\tilde{A}\left[I - A\right]^{-1} \tag{28}$$

2. *For* $A' = A + xI_{ij}$*, where* I_{ij} *is a square matrix with a one in cell* ij *and zero everywhere else, and the scalar* $\sum_{h=0}^{\infty} a_{ij}^{[h]}$ *denotes cell* ij *of matrix* $[I - A]^{-1}$

$$\left[I - A - xI_{ij}A\right]^{-1} = \left[I - A\right]^{-1} + \frac{x}{1 - x\sum_{h=0}^{\infty} a_{ji}^{[h]}}\left[I - A\right]^{-1} I_{ij}\left[I - A\right]^{-1} \tag{29}$$

3. *For* $A' = I_{xi}AI_{yi}$*, where* $I_{xi} = (I + xI_{ii})$*,* $I_{yi} = (I + yI_{ii})$*, and* I_{ii} *denotes a square matrix with a one in cell* ii *and zero everywhere else, and where the scalar* $\sum_{h=1}^{\infty} a_{ii}^{[h]}$ *denotes cell* ii *of matrix* $[I - A]^{-1}A$

$$\begin{aligned}\left[I - I_{xi}AI_{yi}\right]^{-1} &= I_{xi}\left[I - A\right]^{-1}AI_{yi} - \frac{(x + y + xy)}{1 - (x + y + xy)\sum_{h=1}^{\infty} a_{ii}^{[h]}} \\ &\times I_{xi}\left[I - A\right]^{-1}AI_{ii}\left[I - A\right]^{-1}AI_{yi}\end{aligned} \tag{30}$$

The proof can be found in the appendix. Property (1.) provides the foundation for a proportional change to an arbitrary number of cells in matrix A, as long as this change is marginally *small*. Properties (2.) and (3.), in contrast, describe the impact of a *large* proportional change to matrix A. Here, we focus however on a single cell ij, respectively a proportional variation of row i and column i.[16] In the following, we exploit Properties

[16] Expanding on Property (2.) of Lemma 2, one can also trace back any arbitrary *large* variation of $[I - A]^{-1}$ to the original matrix in a sequence of $k \geq 1$ functional mappings. Let $I_{i_s j_s}$ be a square matrix with a one in cell $i_s j_s$ and zero everywhere else. Moreover, let $y_{i_s j_s}$ be a well-defined scalar of the endomorphic function

$$f_{i_s j_s}\left([X - Y]^{-1}\right) = [X - Y]^{-1} + y_{i_s j_s}[X - Y]^{-1} I_{i_s j_s}[X - Y]^{-1}$$

with the property that $f_{i_s j_s}$ maps a matrix of the space of square matrices of the form $[X - Y]^{-1}$ into another element of the same space, and where X is non-singular matrix and Y is a square matrix of the same dimension. Based on this definition, we can write

$$\left[I - A - \sum_{s=1}^{t} x_s I_{i_s j_s}\right]^{-1} = f_{i_t j_t}\left(f_{i_{t-1} j_{t-1}}\left(f_{i_{t-2} j_{t-2}}\left(\ldots f_{i_1 j_1}\left([I - A]^{-1}\right)\right)\right)\right)$$

whereby at any step $s > 1$, $f_{i_{s+1} j_{s+1}}\left(f_{i_s j_s}\right)$ can be written as $f_{i_{s+1} j_{s+1}} = [I - A]^{-1} + [I - A]^{-1} D_{i_s j_s}[I - A]^{-1}$ for some well-defined $n \times n$ matrix $D_{i_s j_s}$.
An alternative approach to investigate the effect of arbitrary large changes to matrix $[I - A]^{-1}$ is proposed in Sonis and Hewings (1992).

(1.), (2.) and (3.) in turns.

4.1 A global trade cost reduction

Our first counterfactual exercise focuses on a proportional increase of all cells in the trade intensity matrices: $A' = \delta_A A$ and $B' = \delta_B B$, with $\delta_A, \delta_B > 1$. This might be triggered by a worldwide improvement of transportation technologies, or in the case of a change in A, a reduction in the costs of coordinating remote production processes, i.e. an increase in θ. The aim of this exercise is not only to answer the question which countries benefit (or lose) from such a cost reduction, but also to explain why. That is, we want to identify the model parameters and variables that predict the gains from trade.

A motivation for our analysis comes from the interesting hypothesis that the emergence of a global supply chain leads to a convergence of income levels (Baldwin, 2011). As the argument goes, the access to foreign intermediate inputs enables every nation to take advantage of the technologies developed in other parts of the world. At the same time, the fragmentation of production increases the scope for specialization. Thus, countries only need to contribute incremental value to an existing supply chain to make their products an export success. Does the emergence of a global production network inevitably lead to income convergence? And if not, how does this depend on a country's position in the network? Our model allows us to look at these questions from a general equilibrium perspective and to compare the relative gains (or losses) across countries.

For comparison, we first investigate the gains from increasing the trade intensity of only final goods shipments:

Theorem 2. *Consider a proportional increase in all cells of matrix B such that $B' = (1+\delta_B)B$, with $\delta_B > 0$, and $A' = A$: It follows* $\mathbf{dw/w} = \mathbf{0}$. *Moreover, the price effect in any country $i \in \mathcal{N}$ is given by the column vector*

$$\frac{\mathbf{dU}}{\mathbf{U}} = \frac{\delta_B}{\sigma - 1} \frac{\mathbf{d(P^c)^{1-\sigma}}}{\mathbf{(P^c)^{1-\sigma}}} = \frac{\delta_B}{\sigma - 1}\mathbf{1} > 0 \tag{31}$$

A first insight from the result, which is proven in the appendix, is that the gains from increasing the intensity of final goods trade are solely determined by its impact on consumer prices. Labor demand, in contrast, and thus wages are entirely unaffected. Second, the per capita income in every nation grows at the exact same rate.

The logic behind the first part lies in our CES specification for consumer preferences plus the assumption of a constant price markup over trade costs, i.e. $p_{ij}^{fin} = x\tau_{ij}^{fin}$ with

$x > 0$, both of which are however common in the literature (e.g. Krugman, 1980; Eaton and Kortum, 2002; Melitz, 2003). As a consequence, the final goods trade equation (2) and the labor demand equation (20) are homogeneous of degree zero with regard to a proportional change in matrix B. Thus, labor demand and wages remain unaffected. The intuition is that the final goods producers from all nations gain from an improved access to their overseas (and domestic) markets. As the increase in trade intensity experienced by a firm is proportional to its original trade intensity, every firm gains in proportion to its original market share. So, no firm yields any competitive advantage. On the other hand, consumers from all nations benefit from their access to cheaper products. And because the *supplier access* vector is homogeneous of degree one with respect to a common change in B, as shown in (16), the price reduction experienced by consumers is proportional to their original supplier access, which explains part two of the result.

Hence, when only final goods shipments are affected, such as in a world where only final goods are traded (i.e. $\theta = 0$), the welfare gains from a global trade cost reduction can –in absolute terms– be perfectly predicted from the trade cost elasticity, $1 - \sigma$, and the level of consumer prices, P_i^c, in a country.

We move on to investigate a trade or coordination cost reduction in the intermediate goods sector, i.e. a proportional increase in all cells of matrix A.[17] Based on Property (i) of Lemma 2 for $\tilde{A} = A$, we obtain the following result:

Theorem 3. *Consider a small and proportional increase in all cells of matrix A, such that $A' = (1 + \delta_A)A$, with $\delta_A \to 0^+$, and $B' = B$. Also, fix $dw_{i^*} = 0$ for $i^* \in \mathcal{N}$: The per capita income change in any $i \in \mathcal{N}$ is given by the column vector*

$$\begin{aligned} \frac{\mathbf{dU}}{\mathbf{U}} &= \frac{\delta_A}{\sigma - 1} (P^c)^{\sigma-1} \Big[\underbrace{S^p \sum_{h=1}^{\infty} A^h B}_{\substack{\text{supplier access} \\ \text{effect (i)}}} \Big]^T \mathbf{1} \\ &+ \frac{\delta_A}{\sigma} \Psi \left[\bar{M}\right]^{-1} \Big[\underbrace{\sum_{h=1}^{\infty} A^h \bar{M}}_{\substack{\text{market access} \\ \text{effect (ii)}}} - \underbrace{M\Big[S^p \sum_{h=1}^{\infty} A^h B (P^c)^{\sigma-1}\Big]^T}_{\text{competition effect (iii)}} \Big] \mathbf{1} \end{aligned} \tag{32}$$

where $(P^c)^{1-\sigma}$ and $\bar{M}$ are the diagonal matrices defined in (16) and S^p, M, and Ψ are the

[17] Our exposition focuses on the effect of a coordination cost reduction, i.e. increasing θ. A global reduction in the cost of shipping intermediate goods has the exact same effects.

full matrices defined in (18), (19), and (28), respectively.

The result, which is proven in the appendix, states that consumers unambiguously gain from a coordination cost reduction (effect (i) in equation (32)). Yet, they do so only indirectly, because in a first instance it improves their final goods suppliers' access to intermediate inputs (see equation (6)). Thus, unlike in Theorem 2, the price reduction experienced by consumers is not proportional to their initial supplier access, but to the supplier access of the nations producing their goods. The benefit of a coordination cost reduction to consumers is therefore dependent on the *trade intensity* of their home country with the nations hosting their final goods suppliers. Furthermore, it depends on the latters' trade intensity with their own suppliers of intermediate inputs. Formally, while in Theorem 2 the absolute effect of an increase in B on consumer prices can simply be written as $\mathbf{d(P^c)^{1-\sigma}} = \delta_B\,[S^p\,B]^T\mathbf{1}$, the counterpart of Theorem 3 is given by

$$\begin{aligned}\mathbf{d(P^c)^{1-\sigma}} &= \Bigg[\underbrace{\big(\delta_A\,S^pA\big)B}_{\substack{\text{discounted cost red.}\\ \text{final goods prod.}}} + \underbrace{\big(\delta_A\,S^pA\big)AB + \big(\delta_A\,S^pA\big)A^2B + \ldots}_{\substack{\text{discounted cost reduction input producers}\\ \text{at upstream stages 1, 2, ...}}}\Bigg]^T\mathbf{1} \qquad (33)\\ &= \delta_A\left[S^p\sum_{h=1}^{\infty}A^hB\right]^T\mathbf{1}\end{aligned}$$

Turning to the effect on *wages*, the terms (ii) and (iii) in equation (32) show that –unlike in Theorem 2– labor demand is no longer unaffected. The explanation lies in a fundamental difference between the demand for final and intermediate goods, which we already pointed out earlier (below equation (9)): a cost reduction on all the intermediate goods shipped from any country i reduces the production costs, and hence increases the sales volume, of all countries that use country i's intermediate goods in subsequent production steps. Hence, country i's workers benefit from an increased demand for their intermediate products. Moreover, as an increase in θ affects the coordination costs at all stages of the global supply chain, every input producing nation experiences another indirect demand increase, as their downstream customers also increase their sales volumes. This is summarized in the following expression, which is equivalent to effect (ii) in (32)

$$\mathbf{d\bar{M}} = \underbrace{\delta_A\,A\,\mathbf{\bar{M}}}_{\substack{\text{discounted sales incr.}\\ \text{at downstr. stage 1}}} + \underbrace{A\big(\delta_A\,A\,\mathbf{\bar{M}}\big) + A^2\big(\delta_A\,A\,\mathbf{\bar{M}}\big) + \ldots}_{\substack{\text{discounted sales increases}\\ \text{at downstream stages 2, 3, ...}}} = \delta_A\sum_{h=1}^{\infty}A^h\bar{M}\mathbf{1} \qquad (34)$$

Equation (34) suggests that not all laborers from all nations benefit alike. Instead, the labor demand increase experienced in a country depends on its direct and indirect trade connections with all countries at the downstream stages of the global supply chain. Thus, some countries actually gain a competitive edge over others. In fact, effect (iii) of equation (32) shows that for the very same reason the workers of some nations might actually need to accept wage cuts, because the coordination cost reduction intensifies competition in their sales markets.

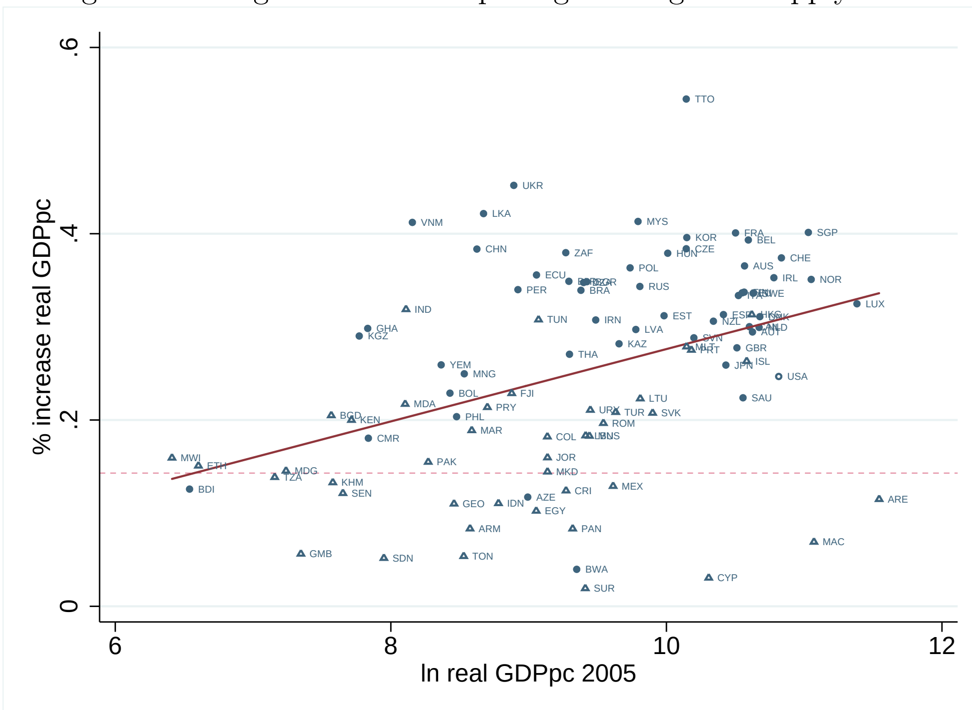
To sum up, Theorem 3 shows that the predicted changes in prices and wages are not at all related to a country's own initial supplier and market access, i.e. the country's initial level of welfare. Instead, these changes are solely dependent on the supplier and market access of the country's trading partners. Thus, whether or not a country gains or loses from a further integration of the global production network depends entirely on its connections to countries who themselves have a superior supplier and market access. Put differently, a country benefits from being well-connected to important trade intermediaries.

An interesting implication of this result is that it provides hope for convergence of income levels around the globe. Access to important trade intermediaries only determines the welfare gains from trade, but not the current level of welfare (this is determined by a country's own market and supplier access). A further integration of the global production network might therefor help developing countries catch up with developed countries' living standards.

To put perspective on this possibility, Figure 1 plots the per capita income gains predicted by our model as a result of a 1% proportional increase of all cells in matrices A and B against the level of GDP per capita in 100 countries in 2005. The predictions are calculated using (31) and (32) in combination with our estimated empirical constructs for $(P_i^p)^{1-\sigma}$, A, and B from (22). We also need to fix a value for σ at this point of the paper. For all our counterfactual exercises we chose $\sigma = 8$.[18] Our choice directly influences the magnitude of the overall welfare effects: the larger σ the smaller the welfare effects. However, very important for our purposes, changing σ leaves the relative size of the different welfare channels and the relative effects in different countries largely untouched. For

[18] We could instead estimate σ using e.g. the method developed in Caliendo and Parro (2014). Yet, their approach uses bilateral tariff data that is typically available at a much finer-grained level than the broad sector classification, intermediates and final goods, used in this paper. Of course, we could aggregate the available tariff data up to the two sectors that we use. This aggregation is, however, not straightforward and can be done in several different ways, each using different assumptions, and each resulting in a different estimate of σ. We therefore decided to keep things simple and fix σ at a value that lies in the middle of the range of existing estimates (see e.g. Caliendo and Parro (2014), Eaton and Kortum (2002) or Romalis (2007)).

Figure 1: The gains from a deepening of the global supply chain



NOTES: The figure shows the welfare gains from a 1% proportional increase of all cells in the trade intensity matrices A and B against real GDP per capita for 100 countries in 2005. The increase in real GDP per capita is calculated using (31) and (32) in combination with the estimated empirical constructs for $(P_i^p)^{1-\sigma}$, A, and B from (22). Moreover, we fix $\sigma = 8$ and $dw_{i^*} = 0$ for $i^* = USA$, and use a country's total industrial value added (WDI) for $w_i L_i$. As a first step in calculating (32), we obtain each country's $(P_i^c)^{1-\sigma}$, augmented wages, $w_i^{1-\sigma}$, supplier and market access, as well as the Ψ matrix using (16), (18), (19), and (28), respectively. A $\bullet$ ($\triangle$) shaped marker indicates that country i experiences a larger (smaller) wage increase than the USA, $dw_i > dw_{USA}$ ($dw_i \leq dw_{USA}$). The USA is marked with a $\circ$.

example, choosing $\sigma = 5$ in the calculations underlying Figure 1 increases each country's welfare gain by roughly 75%. However, the (rank)correlation between countries' welfare gains when fixing σ at 5 or 8 is 0.999.

Figure 1 shows an average country's welfare gain of about 0.25%. This is almost twice as large as the predicted 0.14% welfare increase of a trade cost reduction at only the final stage of the global supply chain (depicted by the dashed horizontal line in the figure). Much more interesting, however, are the large differences between countries. Overall, a further integration of the global supply chain is predicted to result in a further divergence of per capita incomes. Rich countries gain on average more than poor countries. This average picture does however hide quite some heterogeneity. Many low-middle income countries actually experience larger gains than the US or the large European economies. Generally, we find a diverse geographic spread of both the most and the least gaining countries. An exception are the Sub-Saharan African and Middle-Eastern countries: almost all of them are among the least-benefiting countries.

Interestingly, the most important determinant of the overall welfare effect is the impact on a country's labor demand. In Figure 1, a triangle marks those countries that –in order to stay competitive in the now more integrated global supply chain– have to accept a wage drop relative to the US. With only a few exceptions, these are also the countries that experience the smallest overall welfare gains.[19] It suggests that the fortunes of any country in a further integrated global supply chain will mainly depend on whether it attracts more labor demand than other nations, or not. As argued in Theorem 3, this is entirely dependent on how well that country is connected to countries that are important intermediaries in the global production network. These intermediaries will be identified in Section 4.3.

4.2 A unilateral trade cost reduction

In our second counterfactual, we explore the welfare effects of a cost reduction along a single trade route. Because of the distinct effects from the facilitation of a country's in- and outgoing trade flows, we focus on a one-sided trade cost reduction, such that $A' = A + \delta_{ij}^A I_{ij}$ and $B' = B + \delta_{ij}^B I_{ij}$ for an exporting nation i and an importer $j \neq i$.[20] Naturally, this raises exports and the demand for labor in country i and improves the supplier access of the importer j. Another interesting question is, however, whether welfare in third countries is positively or negatively affected, also because it is well-understood that the sign of this externality is an important factor for the success of multilateral trade negotiations (Panagariya, 2000; Aghion et al., 2007). Here, we show that the sign of the externality depends nontrivially on the extent of production fragmentation.

We start by considering a unilateral export cost reduction in a world without international production linkages:

Theorem 4. *Suppose that $\theta = 0$ and $dw_i = 0$. Consider a unilateral increase in the final*

[19] This is also shown by the fact that the correlation of the overall welfare effect, dU_i/U_i, with the wage effect, dw_i/w_i, is 0.81, whereas it is only −0.002 with the overall price effect, dP_i^c/P_i^c, including the wage-induced price effect (as defined in equation (27)).

[20] As an example for this type of intervention, one might think of a preferential export procedure installed in country i or an import tariff reduction negotiated with country j. Note, however, that we do not consider tariff revenues in our welfare analysis and also abstract from optimal tariff policies.

goods trade intensity b_{ij}, such that $B' = B + \delta_B I_{ij}$ with $\delta_B > 0$. It follows:

$$\textit{(i) for all } k \in \mathcal{N}\textit{: } d(P_k^c)^{1-\sigma} > 0 \quad \textit{and (ii) for all } k \in \mathcal{N}\backslash\{i\}\textit{: } dw_k < 0.$$

The result, which is proven in the appendix, suggests an unambiguously positive externality on consumer prices, but a negative externality on foreign wages. The latter is simply explained by the fact that a better access of exporter i to importer j intensifies competition for all the other final goods manufacturers from countries $k \neq i$, including the producers from j. Thus, their demand for domestic labor, and hence wage rates, decline. This in turn produces another round of wage drops governed by the Harrod-Keynes multiplier (27) resulting in unambiguously lower equilibrium wage rates in countries $k \neq i$ (as compared to country i where we have fixed $dw_i = 0$). In contrast, part (i) of the result states that consumers in all nations benefit from a price reduction for their consumption goods. This is easily explained for importer j due to its better access to country i's products. For the remaining nations, including i, the reason is the above mentioned wage declines that trigger a worldwide price reduction on consumer goods.

Hence, in a world without production linkages, whether or not a third country gains or loses depends essentially on the importance of the world market for the supply of consumption goods for that country versus its role as a sales market. This result is not only specific to our model, but applies to a much broader class of trade models (e.g. Krugman, 1980; Eaton and Kortum, 2002; Melitz, 2003).[21]

The difference of introducing a fragmented production process into the picture is that there are additional positive externalities of a comparable export cost reduction. These might be strong enough to turn the negative result of Theorem 4 Part (ii) around. To see this, suppose again $dw_i = 0$ but now $\theta > 0$. Consider a unilateral increase in both trade intensity matrices A and B, such that $A' = A + \delta_A I_{ij}$ and $B' = B + \delta_B I_{ij}$. According to equation (26), we can decompose the total per capita income effect in a third country into a wage and a price effect. More interesting, however, based on Property (2.) of Lemma 2 that provides us with an explicit solution for matrix $[I - A - \delta_A I_{ij}]^{-1}$, we can further

[21] The externality on third countries' welfare is essentially the same in a more realistic bilateral trade agreement, i.e. when $B' = B + \delta_{ij}^B I_{ij} + \delta_{ji}^B I_{ji}$. The difference is that both countries i and j can expect an increase in their labor demand. Fixing $dw_i = 0$, this means that dw_j might become positive. And since a wage increase in country j is accompanied by a higher demand for foreign products, labor demand, and thus wages, in a third country $k \in \mathcal{N}\backslash\{i, j\}$ might actually rise, if country j is among its preferred sales markets. Nevertheless, fixing the wage rate in the country i or j that expects the largest labor demand increase yields unambiguously lower wages in all other nations. Thus, the results of Theorem 4 carry immediately over.

decompose the wage and price effects into several meaningful channels.

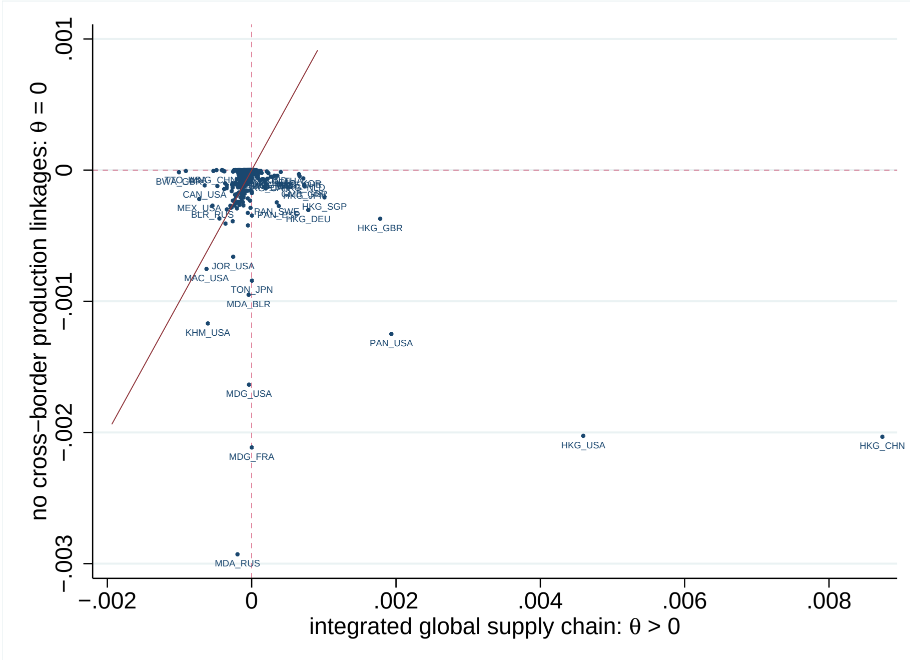
The individual effects of a unilateral export cost reduction between countries i and j on the per capita income in a third country $k \in \mathcal{N} \backslash \{i\}$ are shown in cell k of the following column vector

$$
\begin{aligned}
\frac{\mathbf{dU}}{\mathbf{U}} &= \frac{1}{\sigma - 1}(P^c)^{\sigma-1}\Bigg[\underbrace{(P_i^p)^{1-\sigma}\delta_B\, I_{ij}}_{\text{supplier access effect (i)}} + \underbrace{\frac{(P_i^p)^{1-\sigma}\delta_A}{1-\delta_A\sum_{h=0}^{\infty}a_{ji}^{[h]}} I_{ij}\sum_{h=0}^{\infty}A^h B}_{\text{supplier access externality (ii)}}\Bigg]^T \mathbf{1} \\
&+ \frac{1}{\sigma}\Psi[\bar{M}]^{-1}\Bigg\{\underbrace{\sum_{h=0}^{\infty}A^h\,\delta_B I_{ij} w_j L_j (P_j^c)^{\sigma-1}}_{\text{market access externality (iii)}} + \underbrace{\sum_{h=0}^{\infty}A^h\frac{\delta_A}{1-\delta_A\sum_{h=0}^{\infty}a_{ji}^{[h]}} I_{ij}\bar{M}}_{\text{market access externality (iv)}} \\
&\qquad - \underbrace{M\Bigg[(P_i^p)^{1-\sigma}\Bigg(\delta_B\, I_{ij} + \frac{\delta_A}{1-\delta_A\sum_{h=0}^{\infty}a_{ji}^{[h]}} I_{ij}\sum_{h=0}^{\infty}A^h B\Bigg)(P^c)^{\sigma-1}\Bigg]^T}_{\text{competition externality (v)}}\Bigg\}\mathbf{1}
\end{aligned} \tag{35}
$$

Breaking down the total income effect, the unilateral export cost reduction results –just as in a world without production linkages– in a price reduction on consumer goods sold in country j (effect (i)) and fiercer competition for all other countries $k \neq i$'s producers (effect (v)). What makes the presence of a global supply chain very different are the three additional positive externalities: consumers in all countries benefit from the fact that country j's producers gain access to cheaper inputs from country i's intermediate goods industry. This cost reduction is partially passed on to the rest of the world (effect (ii)). Moreover, foreign workers benefit due to an increased demand for their intermediate products from country i's final and intermediate goods industry (effects (iii) and (iv) respectively). As already mentioned earlier below equation (9), the origin of these positive welfare channels lies in the fact that a more intense trade relationship between countries i and j facilitates the flow of intermediate goods through the entire global supply chain. Depending on the size of these two positive externalities vis-à-vis the negative competition effect (v), wages may now even go up in (some) third countries. This would never happen in a world without production linkages, see Theorem 4.

Equation (35) shows that the sign (and size) of the overall externality on a third country's wages depend on that country's precise position in the world trade network vis-à-vis the two trading partners. Hence, to get a feeling for whether our findings play a role in reality, we predict the strength of the average wage externality for each of the 9,900

Figure 2: Avg. wage externality of a unilateral export cost reduction



NOTES: The figure plots, for each of the 9,900 exporter-importer pair in the trade network of 2005, the average effect of a 1% export intensity increase along the trade link ij on the wage rates in nations $k \in \mathcal{N}\backslash\{i\}$ against the average wage effect of a corresponding 1% export intensity increase of ij assuming a world without production linkages ($\theta = 0$). The average wage effect on the x-axis is calculated using the second and third line of (35) in combination with our estimated empirical constructs from (22). The average wage effect on the y-axis is based on the same equation (35) and on the same empirical constructs, but under the assumption that $\sum_{h=0}^{\infty} A^h = 0$ in (35). In all calculations we fix $\sigma = 8$, $\delta_A = 0.01a_{ij}$, $\delta_B = 0.01b_{ij}$, use a country's total industrial value added (WDI) for $w_i L_i$, and for every exporter-importer pair ij we set $dw_i = 0$.

exporter-importer pairs (100 countries times 99 links) in our 2005 dataset. Towards this end, we use (35) in combination with our empirical constructs from (22) to predict the wage effect on every country $k \in \mathcal{N}\backslash\{i\}$ as a result of a 1% increase in the export intensity for both the final and intermediate goods shipped from country i to country j, i.e. $\delta_A = 0.01a_{ij}$ and $\delta_B = 0.01b_{ij}$. Figure 2 plots, for each exporter-importer pair, the resulting average wage externality in third countries in a world without input-output linkages ($\theta = 0$) against that predicted in the integrated global production network of 2005 ($\theta > 0$).[22]

In a world without production linkages these wage externalities are, as expected, always negative. Things are quite different in 2005's integrated global supply chain: for 28% of the exporter-importer pairs, the negative wage externality is mitigated. These are the pairs

[22]We calculate the wage changes in a world without input-output linkages by simply setting $\theta = 0$ or equivalently $\sum_{h=0}^{\infty} A^h = 0$ in (35). Yet, strictly speaking, it is impossible to quantify these changes, simply because our empirical constructs and data at hand are from a world with significant input-output linkages.

shown to the right of the solid 45°-degree line in Figure 2. We even find positive wage externalities for 4.6% of the exporter-importer pairs, i.e. those in the positive domain of the x-axis. Remarkably, since we always fix $dw_i = 0$ for the exporting nation i, these are exporter-importer pairs, where the average third country benefits more than the country reducing its exports cost. On the exporter's side of these pairs, we always find the same few countries. Among them Hong Kong and Panama stand out. As becomes clear in the following section, the reason is that these countries primarily facilitate the flow of value added through the global supply chain, while at the same time adding little value of their own. In other words, they are "pure" intermediaries of other countries' value added; their own industry poses no substantial competitive threat. On the importer's side of these pairs, we typically find the large economies, like the US, China, or Great Britain, with an appetite for consumption goods. A country's improved access to the final goods shipped by these "pure" intermediaries does not, unlike improved access to the intermediate goods that they intermediate, make it a fiercer competitor on world markets.

4.3 Identifying key countries

Here, we expand on the analysis from the previous sections and show who these countries are that contribute most to other nations' gains from trade. That is, we identify the key intermediaries in the global supply chain. For this purpose, we draw on the analytical concept of *hypothetical extraction* developed in the regional economics literature (Rasmussen, 1956; Hirschman, 1958) and the literature on network robustness (Ballester et al., 2006). This concept identifies the importance of a node for a network by the impact of its isolation on the utility of all remaining nodes. Accordingly, we isolate one nation after the other from the world trade network and calculate the magnitude, but more importantly also the sources, of the income losses inflicted on the remaining nations.[23]

Formally, denote by B^{-i} and $[I - A^{-i}]^{-1}$ the trade intensity matrices obtained from B and $[I - A]^{-1}$, respectively, after isolating country i from them. This corresponds to replacing row i and column i in matrices A and B by vectors of zeros, i.e. $B^{-i} = (I - I_{ii})B(I - I_{ii})$

[23]This counterfactual exercise is not only of hypothetical value. Several recent tragic events have made clear that our world economy is vulnerable to idiosyncratic shocks hitting any one nations, which have led not only to significant drops in the welfare of the afflicted nation, but also of its direct and indirect trading partners. Moreover, even though we interpret the isolation of a country as a hypothetical trade embargo, the findings from this section do also predict the consequences from the destruction of a nation's productive capacities. This becomes clear from our expressions for the trade intensity matrices A and B, (13) and (17). In case of both these matrices, the modification of row i in $(T^{int})^{1-\sigma}$ or $(T^{fin})^{1-\sigma}$ is equivalent to a modification of cell ii in matrices V or K.

and, by substitution of $x = y = -1$ in Property (3.) of Lemma 2, it follows

$$\sum_{h=0}^{\infty} \left(A^{-i}\right)^h = I_{ii} + \sum_{h=0}^{\infty} A^h - \frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \Big(\sum_{h=0}^{\infty} A^h \Big) I_{ii} \Big(\sum_{h=0}^{\infty} A^h \Big) \tag{36}$$

where $\sum_{h=0}^{\infty} a_{ii}^{[h]}$ denotes cell ii of matrix $\sum_{h=0}^{\infty} A^h$. The key player nation is defined as follows:

Definition 1. *The key player nation is the nation i^* whose isolation causes the largest per capita income loss in all remaining nations $j \in \mathcal{N} \backslash \{i^*\}$. Equivalently:*

$$i^* = \arg\min_{i \in \mathcal{N}} \left\{ \sum_{j \neq i} \frac{U_j(A^{-i}, B^{-i}) - U_j(A, B)}{U_j(A, B)} \right\} \tag{37}$$

Before we identify this nation and provide a detailed breakdown of the overall effect of its isolation on the welfare in other countries, we briefly elaborate on formula (37).

Here, we define the key player country in terms of its contribution to the per capita income of a representative inhabitant in every other nation. Plausible alternative definitions can be easily implemented: by summing up the income losses for a subset of nations one can, for example, identify the key country for a certain world region. Or, by weighting the per capita income losses with the respective population sizes, one can determine the total welfare losses inflicted on other nations.[24] Moreover, Property (3.) of Lemma 2 allows for less extreme versions of the key player analysis presented here, where the in- and out-going trade flows of a nation would be reduced by less than 100%.

Note furthermore that our key player analysis is closely related to two strands in the international economics literature. On the one hand, there is the obvious link to the standard gains from trade analysis (e.g., Arkolakis et al., 2012) that looks at the welfare losses in the isolated nation itself[25]. On the other hand, our analysis is closely related to an empirical literature that decomposes the observed trade flows in a global supply chain with the aim of identifying the value added that certain sectors and/or countries contribute to it. Related to the *upstreamness measure* of Antràs et al. (2012), our decomposition of the overall welfare effects of isolating a country (shown in equation (38) below) allows us to investigate whether the country is more active at the top or the bottom end of the global

[24]Note that, in comparison to our unweighted formula, such a modification would render smaller nations more prominent key players, because of the larger weight on the total losses incurred in more populous nations.

[25]Also in our model these gains can be readily inferred using the two sufficient statistics stressed in (Arkolakis et al., 2012).

supply chain. Also, like in Hummels et al. (2001) and Johnson and Noguera (2012a), we can break down the observed exports of a country into *value added* and *vertical specialization* trade.

Our analysis contributes to this literature in two important ways: first, the theoretical foundation of formula (37) in a general equilibrium framework –combined with our empirical implementation of Section 3– allows for a meaningful interpretation of our decomposition of observed trade flows. It identifies directly how important a country is for other nations' welfare based on one of the six channels presented in (38). Second, unlike earlier decompositions, our formula does not only apply to the observed input-output linkages in a supply chain. It also allows to ask the counterfactual question: is a country really indispensable or can other countries fill in its position? Based on its theoretical foundation, formula (37) takes commodity and factor substitution into account.[26]

The following column vector shows how the effect of isolating country $i \in \mathcal{N}$ on the per capita income in any $j \in \mathcal{N} \backslash \{i\}$ can be decomposed into several meaningful channels

$$
\begin{aligned}
\frac{\mathbf{dU}}{\mathbf{U}} = & -\frac{1}{\sigma-1}\left(P^c\right)^{\sigma-1} \Bigg[\underbrace{\left(P_i^p\right)^{1-\sigma} I_{ii} B}_{\text{final goods supply (i)}} + \underbrace{I_{ii}\, S^p \big(I - I_{ii}\big) B}_{\text{intermediate goods supply (ii)}} \\
& + \underbrace{\frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \big(I - I_{ii}\big) S^p\, I_{ii} \sum_{h=1}^{\infty} A^h \big(I - I_{ii}\big) B}_{\text{intermediated supply (iii)}} \Bigg]^T \mathbf{1} \\
& -\frac{1}{\sigma} \Psi \left[\bar{M}\right]^{-1} \Bigg[\underbrace{M\, I_{ii}}_{\substack{\text{final goods} \\ \text{demand (iv)}}} + \underbrace{\frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \sum_{h=1}^{\infty} A^h I_{ii}\, M \left(I - I_{ii}\right)}_{\text{intermediated demand (v)}} \\
& - \underbrace{M \Big[\frac{\left(P_i^p\right)^{1-\sigma}}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} I_{ii} \sum_{h=0}^{\infty} A^h\, B \left(P^c\right)^{\sigma-1} + S^c I_{ii} \left(P^c\right)^{\sigma-1} \Big]^T}_{\text{competition (vi)}} \Bigg] \mathbf{1}
\end{aligned}
\tag{38}
$$

whereby, for the isolated nation i, we fix $dw_i = 0$.

Equation (38), which is developed in the appendix, highlights six channels through which welfare is affected in other nations: on the supply side, there is (i) the foregone access to the final goods produced in country i, and (ii) the foregone value added by country

[26] A drawback of formula (38) is that it is based on a simple model with a very stylized supply chain, whereas earlier trade flow decompositions are suited for realistic supply chains involving multiple sectors. However, it should be noted that our model readily lends itself to extensions involving more than two sectors.

i's intermediate goods industry. The latter reduces consumer access to cheaper products only indirectly, since in a first instance it lowers producer productivity in the remaining nations $j \neq i$. Finally, consumers lose access to the value added by the intermediate goods producers from countries $j \neq i$, which was incorporated in the products exported by country i before the shock (effect (iii)). These three channels have already been set apart in earlier studies: a comparison of the combined channels (ii) and (iii) with channel (i) shows the *upstreamness* of a country, similar to the measure by Antràs et al. (2012). Moreover, channel (ii) measures the *value added* trade of country i in the upstream stages of the global supply chain, whereas channel (iii) captures its *vertical specialization* trade according to Hummels et al. (2001) or Johnson and Noguera (2012a).

Equation (38) highlights three additional channels on the demand side, however: channel (iv) and (v) show that wages in the remaining nations are negatively affected by the lost access to the demand for final and intermediate products respectively that originated from country i. As the only positive effect, channel (vi) measures the degree to which a country's isolation relaxes competition in world markets, with positive effects on labor demand in other nations. In other words, it captures the possibility that some countries might not be indispensable, because others can easily fill the gap that their isolation would leave behind.

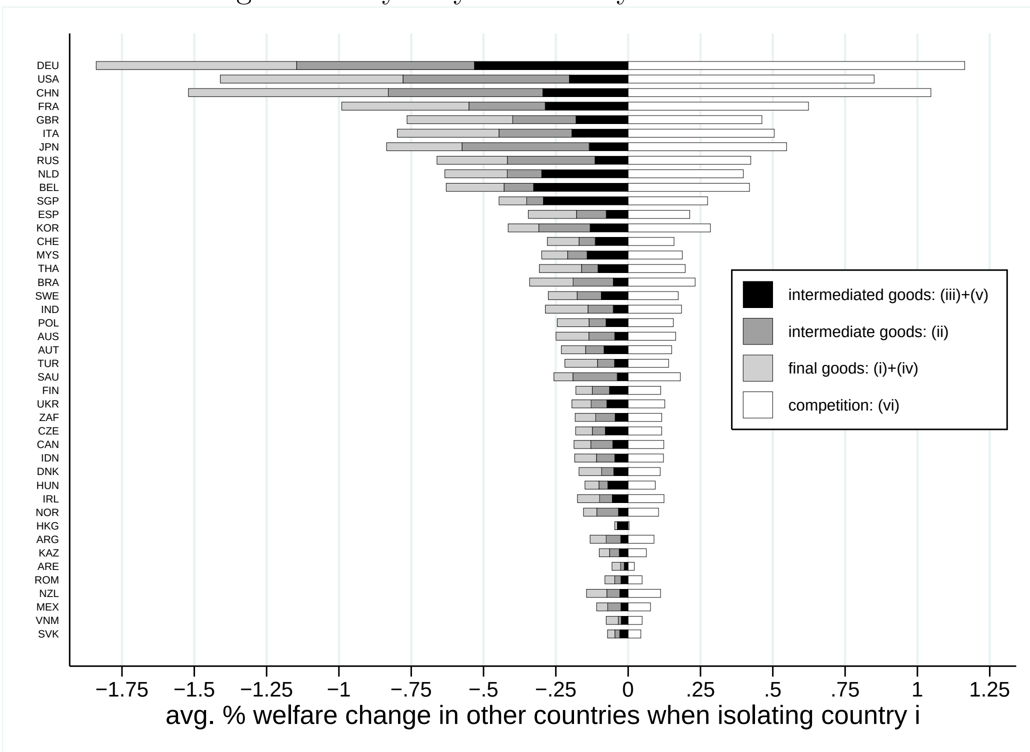
Overall, the (relative) size of channels (i)-(vi) allow us to pinpoint the different roles that countries can take in the global production network. Figure 3 illustrates this for the 2005 trade network. It shows the 43 countries whose isolation is predicted to cause an average per capita income loss of more than 0.025% in the remaining 99 countries.[27] In addition, the figure uses channels (i)-(v) of (38) to show in how much this total loss is due to losing the country as a player in final goods markets, (i)+(iv), as a producer of intermediate goods, (ii), and as an intermediary of other countries' value added, (iii)+(v). Moreover, it shows to what extent a country's isolation can be mitigated by other countries, i.e. channel (vi).[28]

The average per capita income loss is largest when severing all trade ties with Germany, the overall key player: 0.7%. Isolating the average nation shown in Figure 3 causes a loss of only 0.14%. Splitting up this total effect shows that 60% of this overall welfare loss is

[27] Results for the other 57 countries are available upon request.

[28] In our discussion of Figure 3, we focus primarily on the relative size of the channels (i)-(vi), but less so on the total effect of isolating a country. The absolute size of the latter is inversely related to the value we fix for σ. The larger σ, the more substitutable are the different final and intermediate goods varieties produced around the world. This makes it easier for other nations to fill the gap left by the isolated nation, thereby yielding a lower per capita income loss.

Figure 3: Key Players and Key Intermediaries



NOTES: The figure shows the 43 countries in our 2005 dataset whose isolation is predicted to cause an average per capita income loss of 0.14% in the remaining 99 nations. We calculate the per capita income effects using (38) in combination with our empirical constructs from (22). Moreover, we fix $\sigma = 8$, use a country's total industrial value added (WDI) for $w_i L_i$, and set $dw_i = 0$ for the isolated country i. The 43 countries are sorted in ascending order of overall key player ranking. Distinguishing by the four channels in the legend, the average per capita income loss of 0.14% consists of an average 0.15%, 0.13%, resp. 0.11% per capita income loss due to losing a country as a provider of final, intermediate, and intermediated goods, respectively. This average loss of 0.39% is mitigated by a 0.25% per capita income gain due to weaker competition.

due to the isolated country's foregone contribution to the upstream stages of the global supply chain, channels (ii)+(iii)+(v) (see also the notes below Figure 3). Only an average 50% of a country's contribution to the upstream stages consists of the value it adds itself, (ii); the other half stems from its role as an intermediary of other nations' value added, (iii)+(v).

These averages however hide substantive variation between countries. Some countries, e.g. China, Spain, Thailand, India, Vietnam or Turkey, are primarily active in the final stage of the global supply chain, channels (i)+(iv). Others, e.g. Japan, South Korea, Canada, Russia or Saudi Arabia, are more specialized in contributing value added to the upstream stages, channel (ii).[29] And, yet other countries derive their importance from

[29]Our inclusion of "capital goods" in our definition of intermediate goods (see footnote 12) exacerbates

being an intermediary, channelling the value added generated in one part of the world to the demand for it in another part, i.e. channels (iii)+(v). Singapore, Belgium, The Netherlands, Malaysia, and Hong Kong are the most noteworthy examples.

This latter group of countries is of particular interest to us. In Section 4.2, we argued that "pure" intermediaries, i.e. intermediaries that add little value of their own to the global supply chain, generate the largest positive wage externality in other countries when lowering their export costs. Figure 3 highlights Hong Kong (on position 35 of the overall ranking) as the country that stands out in this respect. The per capita income loss in other nations associated with the country's isolation is for 81% determined by its role as an intermediary. Also, there is very little scope for other countries' to mitigate this welfare loss, which is indicated by the negligible competition effect (vi) for Hong Kong.

Furthermore, as we have seen in Section 4.1, a country's access to key intermediaries is the single-most important predictor of the welfare gains from a further integration of the global production network. Based on the analysis in this section, we can identify these countries. To see this, note that when we consider only a small shock to the trade connections of the isolated nation, i.e. $x = y \to 0_-$ in equation (36), and sum up channels (iii), (v), and the intermediation part of (vi), over all nations $i \in \mathcal{N}$ in (38), we retain equation (32) of Theorem 3. In Figure 3, Germany, Belgium, the Netherlands, China, and Singapore are the five countries with the largest average contribution to the per capita income in other nations based on their role as intermediary.

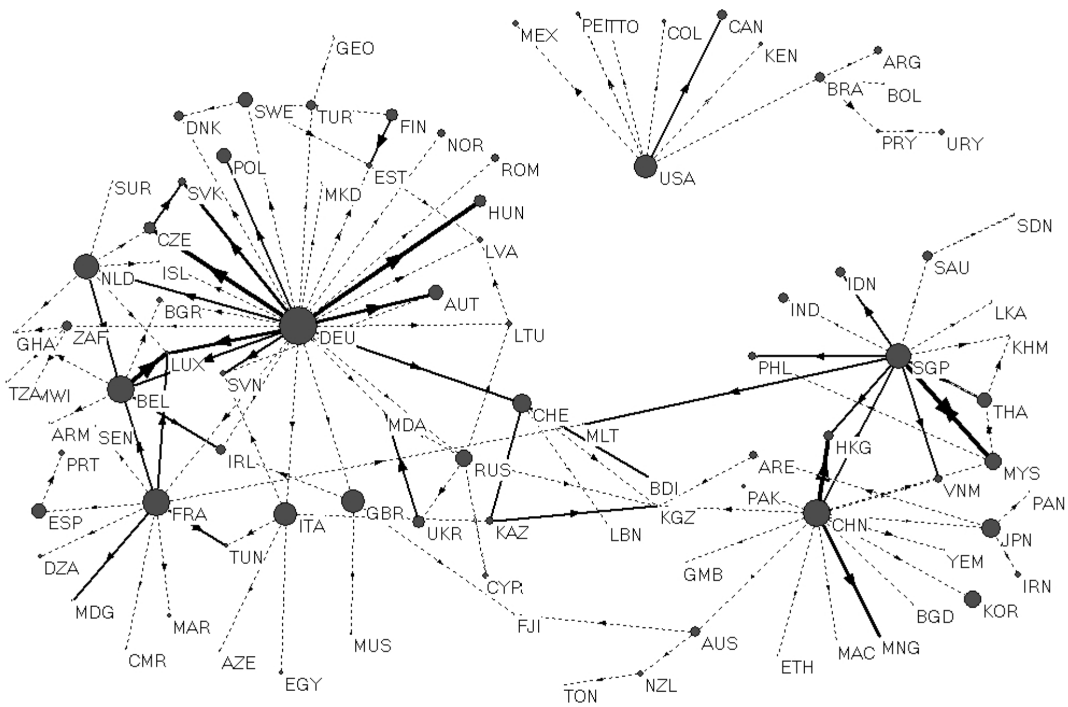
However, the averages shown in Figure 3 do not reveal for which particular countries these key intermediates are so important. Figure 4 provides this missing piece of information. Here, we position all countries in our dataset in a network graph. An arrow from country i to j in this graph indicates that country i is either the single-most important intermediary of country j or that, at least, the trade intermediation through country i contributes to a significant extent to country j's per capita income (at least 0.45%).

Figure 4 clearly shows that trade intermediation is a geographically very confined phenomenon.[30] In the Americas, the US is the single most important trade intermediary for

these numbers in case of Russia and Saudi Arabia. However, also when excluding "capital goods", they remain among the countries whose importance is to a large extent derived from the value they add to the upstream sector.

[30]This is also supported by the empirical evidence provided in Johnson and Noguera (2012b). They find a negative correlation between geographical distance and the share of vertical specialization trade in the gross trade flows between countries. As already mentioned earlier, vertical specialization trade is immediately related to channel (ii) of our equation (38).
Note, furthermore, that if we would plot a network of gross trade flows between countries, the picture looks quite different, with some significant trade links across the Atlantic and Pacific Oceans. Such a picture of

Figure 4: Key intermediaries



NOTES: The figure positions all 100 countries in our dataset in a network graph. For each country, we show its most important trade intermediary in terms of the welfare channels (iii), (v), and the intermediation part of (vi) as defined in (38). On top, we add the 1% most important intermediation links among all the 9,900 country pairs in our dataset that each contribute by at least 0.45% to the per capita income in the receiving nation. All intermediation links are indicated by an arrow pointing from the intermediary to the receiving country. The thicker an arrow, the larger the per capita income contribution on the range of 0.45–2.2%. Moreover, the size of each country's node reflects the country's overall importance as an intermediary (as also shown in Figure 3). To calculate all effects underlying this Figure we use (iii), (v), and the intermediation part of (vi), as defined in (38), in combination with our empirical constructs from (22). Moreover, we fix $\sigma = 8$, use a country's total industrial value added (WDI) for $w_i L_i$, and set $dw_i = 0$ for the isolated country i.

almost all other countries on the American continent. With the single exception of Kenya, the US shares no intermediation links with any other country outside the American continent. A similar picture holds for the key intermediaries located in Europe and South-East Asia (Germany, the Netherlands, Belgium and Singapore). These countries contribute significantly to the welfare in their respective regions, but they play virtually no role elsewhere. Noteworthy exceptions are France, Italy, and Great Britain in Europe and China in Asia. These countries are also important intermediaries for a number countries outside their immediate neighborhood.

This geographically localized nature of trade intermediation also sheds light on our finding in Section 4.1 that a further integration of the world economy is expected to only widen the worldwide income gap (see Figure 1). Figure 4 shows that many developing countries in Africa, Central Asia, and the Middle East are the countries with the worst access to the key intermediaries in today's production network. These key intermediaries are mostly located in the rich parts of the world, explaining why the already rich countries benefit most from a further deepening of the global supply chain. Figure 4 does however also identify the most important intermediary for each individual developing country. In order to realize the largest welfare benefits from participating in the global production network, improved access to these intermediaries is key for developing countries. For many African countries the strong historical ties to Europe are still very important in this respect. In the Americas these have been entirely replaced by the United States. Finally, China plays an important role here: it is the key intermediary for some of the poorest countries in South- and Central Asia as well as for some African nations, like Ethiopia and Gambia.

5 Global supply chains in a Ricardian model

In this section, we show in how far our main results, Theorems 2-4, carry over to the Eaton and Kortum (2002) model of Ricardian trade, that relaxes our Armington Assumptions 1-3.

In the Eaton and Kortum model, henceforth the EK model, every country is in principal able to produce all varieties $v \in \mathcal{V}$. However, a country only exports a certain variety, if it has a destination-specific comparative advantage in its production. This advantage is endogenously determined. In particular, the probability that country i has at most effi-

a more integrated world economy can also be found in a network graph similar to Figure 4, where we take all six welfare channels of equation (38) into account. It can be found in the Appendix.

ciency z in the production of variety v is, in our notation, given by the Fréchet distribution function, $F_i(z) = \exp[-\kappa_i z^{-\mu}]$, where $\kappa_i > 0$ and $\mu > 1$ are shape parameters. Consumers and firms from country $j \in N$ then purchase variety v from the lowest cost producer, trade costs included. Thus, a comparative static shock to any one link in the world trade network has additional implications for the number of goods exported by a nation, on top of the intensive margin adjustments considered in our Armington model. Moreover, unlike our assumption of a CES production technology (3), the EK model builds on the nested Cobb-Douglas technology (4).

Although the EK model starts from this richer micro-foundation, it arrives at the same two aggregate constructs that also fix an economy's welfare in our Armington model: the price index and the labor market equation. In the EK model, the logarithm of the producer price index for tradable intermediate inputs (equation (16) on p. 1756) is, in our notation, given by

$$\ln\left[P_i^p\right] = \ln\left[\gamma\right] - \frac{1}{\mu}\ln\left[\sum_{j\in\mathcal{N}} w_j^{-\mu\beta}(P_j^p)^{-\mu(1-\beta)}a_{ji}\right] \tag{39}$$

where $\gamma > 0$ is a construct of exogenous parameters (equation (9) on p. 1749), β and $(1-\beta)$ are the labor-cost and intermediate input-cost shares in production, respectively, and $a_{ji} = \kappa_j(\tau_{ji}^{int})^{-\mu}$ is the bilateral trade intensity between exporter j and importer i. To stay close to our setup, let us slightly modify the EK model by assuming that consumers buy from a separate sector that produces only final consumption goods, which are shipped at the bilateral trade cost τ_{ji}^{fin}. Otherwise, this sector operates under the same conditions. Thus, there is a distinct consumer price index P_i^c, whose logarithm is similar to (39), apart from a_{ji} being replaced by $b_{ji} = \kappa_j(\tau_{ji}^{fin})^{-\mu}$.

The EK labor-market equation (on p. 1757) needs to be adjusted accordingly

$$\ln\left[w_i\right] \;=\; \ln\left[\sum_{j\in\mathcal{N}} \beta v_{ij}^c w_j L_j + (1-\beta)v_{ij}^p w_j L_j\right] - \ln\left[L_i\right] \tag{40}$$

Here, v_{ij}^p and v_{ij}^c are the probabilities that the lowest price vendors for the firms, respectively the consumers, in country j are the producers from country i. These probabilities are given by

$$\begin{aligned}\ln\left[v_{ij}^p\right] &= -\mu\ln\left[\gamma\right] - \mu\ln\left[w_i^\beta(P_i^p)^{1-\beta}\right] + \mu\ln\left[P_j^p\right] + \ln\left[a_{ij}\right] \\ \ln\left[v_{ij}^c\right] &= -\mu\ln\left[\gamma\right] - \mu\ln\left[w_i^\beta(P_i^p)^{1-\beta}\right] + \mu\ln\left[P_j^c\right] + \ln\left[b_{ij}\right]\end{aligned} \tag{41}$$

As already mentioned earlier, the EK model does not admit any more explicit solution than that. In particular, the system of interdependent price indices (39) can only be solved numerically. However, as shown by Alvarez and Lucas (2007), system (39) implicitly defines a functional mapping of wages into the price vector $\ln \mathbf{P^p}(\mathbf{w}, \cdot)$. In addition, the same conditions for existence of a unique Walrasian equilibrium are satisfied as in our Theorem 1.

Turning to the comparative statics for this equilibrium, it follows from Theorem 1 of Alvarez and Lucas (2007) and Proposition 17.G.3 of Mas-Collel et al. (1995, p. 618) that an arbitrary, but small, change to matrix A (or B) has the following effect on the system of price and wage vectors

$$\begin{aligned}
\mathbf{d}\ln[\mathbf{P^p}] &= \sum_{h=0}^{\infty}\big((1-\beta)K\big)^h\Big(\mathbf{d}_A\ln[\mathbf{P^p}] + \mathbf{d}_{\ln[\mathbf{w}]}\ln[P^p]\times\mathbf{d}\ln[\mathbf{w}]\Big)\\
\mathbf{d}\ln[\mathbf{P^c}] &= \mathbf{d}_A\ln[\mathbf{P^c}] + \mathbf{d}_{\ln[\mathbf{P^p}]}\ln[P^c]\times\mathbf{d}\ln[\mathbf{P^p}] + \mathbf{d}_{\ln[\mathbf{w}]}\ln[P^c]\times\mathbf{d}\ln[\mathbf{w}]\\
\mathbf{d}\ln[\mathbf{w}] &= \left\{\sum_{h=0}^{\infty}(\Phi^{-i^*})^h\right\}^{+i^*}\Big(\mathbf{d}_A\ln[\mathbf{w}] + \mathbf{d}_{\ln[\mathbf{P^p}]}\ln[w]\times\mathbf{d}\ln[\mathbf{P^p}] + \mathbf{d}_{\ln[\mathbf{P^c}]}\ln[w]\times\mathbf{d}\ln[\mathbf{P^c}]\Big)
\end{aligned} \tag{42}$$

where K is an $n \times n$ matrix with elements

$$\xi_{ij} = \frac{w_j^{-\mu\beta}(P_j^p)^{-\mu(1-\beta)}a_{ji}}{\sum_{k\in\mathcal{N}} w_k^{-\mu\beta}(P_k^p)^{-\mu(1-\beta)}a_{ki}} \tag{43}$$

and $\sum_{h=0}^{\infty}(\Phi^{-i^*})^h$ is the Harrod multiplier matrix similar to equation (25) in our model. Moreover, $\mathbf{d}_A\ln[\mathbf{P^p}], \mathbf{d}_A\ln[\mathbf{P^c}]$, and $\mathbf{d}_A\ln[\mathbf{w}]$ denote the direct effects of a shock to matrix A on prices and wages, respectively, whereas the remaining summands in (42) show the indirect effects caused by these changes in the other equations of the system.

Due to the more complex interdependencies between the wage and price vectors in (42), a general characterization of arbitrary comparative static variations is much more difficult than in our Armington model. However, we can still derive clear cut results for the counterfactuals underlying our Theorems 2-4:

A global trade cost reduction: first, consider a proportional increase of all cells in matrix B such that $B' = (1+\delta_B)B$ with $\delta_B > 0$. Because the EK model assumes CES consumer preferences and constant price markups over trade costs, we obtain the same result as in our Theorem 2: an increase in the intensity of final goods shipments only affects consumer prices, but not the wage rates, in any one nation, whereby each country's welfare gain is proportional to its initial supplier access.[31]

Next, suppose $A' = (1+\delta_A)A$ with $\delta_A \to 0^+$. It follows $\mathbf{d}_A \ln[\mathbf{P^p}] = -(\delta_A/\mu)\mathbf{1}$, $\mathbf{d}_A \ln[\mathbf{P^c}] = \mathbf{0}$, and $d_A \ln[V^p] = \delta_A \times 1$, where 1 denotes the $n \times n$ full matrix of ones. To find a fixed point of system (42), let us assume that $\mathbf{d}\ln[\mathbf{w}] = \mathbf{0}$. Since the row sum of matrix K in (42) is one (as verified in (43)), it follows that $\mathbf{d}\ln[\mathbf{P^p}] = -(\delta_A/(\mu\beta))\mathbf{1}$. Substituting this into the second equation of (42) gives $d_{\ln[P^p]} \ln[P^c] \times \mathbf{d}\ln[\mathbf{P^p}] = -(\delta_A(1-\beta)/(\mu\beta))\mathbf{1}$. Furthermore, after substitution into the third equation, we find indeed that $\mathbf{d}\ln[\mathbf{w}] = \mathbf{0}$ is part of a fixed point of (42). However, the interpretation of this fixed point is partially inconsistent with our Theorem 3: both the EK model as well as our Armington model predict that the effect on consumer prices is amplified, when matrix A changes in comparison to an equivalent change in matrix B. Yet, in the EK model, the price effect is independent of the position of a country in the global production network: just as in Theorem 2, consumers from all countries gain in proportion to their initial supplier access. Moreover, the change in matrix A does not materialize in a wage effect.

To explore the nature of these diverting findings, we also solved a variant of the EK model with our preferred CES production technology (3) that relaxes the Cobb-Douglas assumption of a constant labor cost share in production. In that case, a proportional change in matrix A produces the exact same picture as our Armington model: a country's gains from trade depend on how well it is connected to key intermediaries. As such, our main result, Theorem 3, crucially hinges on the stylized empirical fact that the gradual international fragmentation of production over the past thirty years has indeed substantively reduced labor cost shares in production (Feenstra, 1998; Karabarbounis and Neiman, 2014).

A unilateral trade cost reduction: consider a unilateral increase in the export intensity of the final goods shipments from country i to country j, such that $B' = B + \delta_B b_{ij} I_{ij}$ where $\delta_B > 0$. Suppose, in a first instance, a world without production linkages, i.e. $\beta = 1$, $A = 0$. If we fix $dw_i = 0$ we immediately arrive at our Theorem 4 stating that $\mathbf{d}\ln[\mathbf{P^c_k}] < \mathbf{0}$ and $d\ln[w_k] < 0$ for all $k \neq i$.[32]

Consider, now, the same cost reduction, but $0 < \beta < 1$ and $A > 0$. Then, contrary to the above, foreign wages might increase. The reason is the additional effect caused by a wage

[31] To verify this, note that it holds $\mathbf{d}_B \ln[\mathbf{P^p}] = \mathbf{0}$, $\mathbf{d}_B \ln[\mathbf{P^c}] = -(\delta_B/\mu)\mathbf{1}$, and $d_B \ln[V^c] = \delta_B \times 1$, where 1 denotes the $n \times n$ full matrix of ones. Thus, by (40) and (41), it must be $\mathbf{d}\ln[\mathbf{w}] = \mathbf{0}$.

[32] To verify this, the direct effects of the export cost reduction are $\mathbf{d}_B ln[\mathbf{P^p}] = \mathbf{0}$ and $d_B \ln[P^c_k] = 0$ as well as $d_B \ln[v^c_{kl}] = 0$ for all $k \neq j$ and $kl \neq ij$, whereas $(-1/\mu) < d_B \ln[P^c_j] < 0$ and $d_B \ln[v^c_{ij}] = \delta_B$. This immediately implies $d\ln[w_k] < 0$ for all $k \neq i$ and, in turn, $\mathbf{d}\ln[\mathbf{P^c}] < \mathbf{0}$.

reduction, $d\ln[w_k] < 0$ for any $k \neq i$, on producer prices. According to the first equation in (42), producer prices fall, which in turn implies a wage increasing effect governed by (41). Thus, the EK model preserves an important result of our paper that the presence of a global supply chain moderates the welfare effects of a unilateral trade cost reduction by imposing a positive externality on foreign labor markets.

6 Conclusion

In this paper, we present novel theoretical results concerning the effects of an emerging global supply chain on the distribution of incomes around the world. The main difference to prior studies on the topic is that ours stresses a unique feature of supply chain trade: the well-being of any one nation depends not so much on its own technology and geography, but much more on the technology and geography of all other nations that are part of the global production network. We highlight this salient feature by means of three novel counterfactual exercises in a simple Armington-type trade model. We introduce a novel set of methods for this purpose that allows us to look at the closed-form expressions for these counterfactuals (where, up to now, only simulations were thought to be feasible). Beyond the theoretical implications of our paper, and the potential usefulness of our novel methods in other modern trade or economic geography models, the network perspective that we advocate here has important practical implications. For example, our insights can be of use for the development of optimal trade policies, or for the identification of countries that are of systemic importance for the world economy. At the very least, our network perspective further challenges the mercantilistic view of trade by showing that in today's highly integrated global production network, we are all in the same boat.

Even though our mathematical results are based on the assumptions of a simple Armington type trade model, the same insights can also be found in other modern trade models, which we confirm in Section 5 at the example of the Eaton and Kortum (2002) model. Another very valuable extension of our model is to incorporate a supply chain of more than just two production stages, such as in Caliendo and Parro (2014). This will bring the model much closer to reality and improve its suitability for quantitative exercises based on one of the world input-output datasets that are currently under development.

7 Appendix

7.1 Proof of Theorem 1

Proof. To prove existence of at least one equilibrium, we verify that there is a $\mathbf{w} = (w_1, w_2, ..., w_n) \in \mathfrak{R}^n_{++}$ such that the transformed equation system (20)

$$z_i(\mathbf{w}) = \frac{w_i^{-\sigma}}{\left(P_i^p(\mathbf{w})\right)^{1-\sigma}} \sum_{j\in\mathcal{N}} \left[X_{ij}^{fin}(\mathbf{w}) + X_{ij}^{int}(\mathbf{w})\right] - L_i \tag{44}$$

satisfies the following properties: for all $i \in \mathcal{N}$ and vectors $\mathbf{w}$

i) $z_i(\mathbf{w})$ is continuous,

ii) $z_i(\mathbf{w})$ is homogeneous of degree zero,

iii) $\sum_{i\in\mathcal{N}} w_i z_i(\mathbf{w}) = 0$ for all $\mathbf{w} \in \mathfrak{R}^n_{++}$ (Walras' Law),

iv) for $k = \max_j L_j > 0$, $z_i(\mathbf{w}) > -k$ for all $\mathbf{w} \in \mathfrak{R}^n_{++}$ and

v) if $\mathbf{w} \to \mathbf{w}^0$, where $w^0_{-i} \neq 0$ and $w^0_i = 0$ for some i, then $\max_j z_j(\mathbf{w}) \to \infty$.

Existence then follows from Proposition 17.C.1 of Mas-Collel et al. (1995, p. 585). A smiley (☺) indicates the end of the proof.

(i) The continuity of $z_i(\mathbf{w})$ follows immediately from Assumption 4, which ensures that some continuous functions $X_{ij}^{fin}(\mathbf{w})$, $X_{ij}^{int}(\mathbf{w})$, and $P_i^p(\mathbf{w})$ exist, which are given by (2), (15), and (16), respectively.

(ii) Since (2), (15), and (16) are all homogeneous of degree one, it follows that $z_i(\mathbf{w})$ is homogeneous of degree zero.

(iii) To verify Walras' Law, we expand equation (20) to get

$$\sum_{i\in\mathcal{N}} w_i z_i(\mathbf{w}) = \sum_{i\in\mathcal{N}} \left[\sum_{j\in\mathcal{M}} X_{ij}^{fin}(\mathbf{w}) + \sum_{j\in\mathcal{N}} X_{ij}^{int}(\mathbf{w}) - \sum_{j\in\mathcal{N}} X_{ji}^{int}(\mathbf{w}) - L_i w_i\right]$$

so that $\sum_{i\in\mathcal{N}} w_i z_i(\mathbf{w}) = 0$ follows from the fact that $\sum_{j\in\mathcal{N}} X_{ji}^{fin}(\mathbf{w}) = L_i w_i$.

(iv) A lower bound on $z_i(\mathbf{w})$ is implied by $z_i(\mathbf{w}) > -L_i$ for all $\mathbf{w} \in \mathfrak{R}^n_{++}$. Thus, let $k = \max_{j\in\mathcal{N}} L_j$. It holds $z_i(\mathbf{w}) > -k$ for all $i \in \mathcal{N}$.

Finally, to prove part (v) suppose that $\mathbf{w} \to \mathbf{w}^0$, where $w^0_{-i} \neq 0$ and $w^0_i = 0$. For any country i with $v_i^{fin} > 0$, countries $j, k \in \mathcal{N}$, and $\mathbf{w} \in \mathfrak{R}^n_{++}$ it holds

$$\begin{aligned} z_i(\mathbf{w}) \quad &> \quad \max_{j\in\mathcal{N}} \frac{w_i^{-\sigma}}{\left(P_i^p(\mathbf{w})\right)^{1-\sigma}} X_{ij}^{fin}(\mathbf{w}) \; - \; \max_{k\in\mathcal{N}} L_k \\ &= \quad \max_{j\in\mathcal{N}} \frac{b_{ij} L_j w_j}{w_i^{\sigma} \sum_{k\in\mathcal{N}} \left(P_k^p(\mathbf{w})\right)^{1-\sigma} b_{kj}} - \max_{k\in N} L_k \end{aligned} \tag{45}$$

By inspection of (16), it immediately becomes clear that the denominator of (45) approaches zero in the limit as w_i goes to zero (a similar argument can be made for a country with $v_i^{fin} = 0$ but $v_i^{int} > 0$). This implies that $\lim_{\mathbf{w} \to \mathbf{w}^0} z_i(\mathbf{w}) \to \infty$ and therefore establishes existence of an equilibrium. In fact, since $z_i(\mathbf{w})$ is homogeneous of degree zero, if $\mathbf{w}$ is an equilibrium so is $\lambda \mathbf{w}$ for any $\lambda \in \mathfrak{R}_{++}$.

To prove existence of exactly one equilibrium wage vector, we verify that $z_i(\mathbf{w})$ has the gross substitutes property

$$\frac{\partial z_i(\mathbf{w})}{\partial w_j} > 0 \quad \text{for all } j \neq i \quad \text{and for all } \mathbf{w} \in \mathfrak{R}^n_{++}.$$

Uniqueness then follows from Proposition 17.F.3 in Mas-Collel et al. (1995, p. 613).

For any $j \neq i$, the partial derivatives of (44) are given by

$$\frac{\partial z_i}{\partial w_j} = \frac{w_i^{-\sigma}}{w_j} \left[M_{ij} + (\sigma - 1) \sum_{k \in \mathcal{N}} M_{ik} S^c_{jk} (P^c_k)^{\sigma - 1} \right] \tag{46}$$

where M_{ij} is the ij'th entry in matrix (19) and S^c_{jk} the jk'th entry in (18). Since $\sigma > 1$, it immediately follows that $\partial z_i / \partial w_j > 0$. ■

7.2 Empirical Framework

Table 1: Estimation results - estimating our gravity equation (21)

Trade cost determinant	final goods	intermediates
ln Distance	-1.75	-1.60
	(0.05)	(0.06)
Common Border	0.98	0.55
	(0.15)	(0.17)
Common Official Language	0.48	0.42
	(0.15)	(0.13)
Common Spoken Language	0.33	0.21
	(0.12)	(0.13)
Colonial Link	1.27	1.12
	(0.11)	(0.19)
Common Colonizer	0.95	0.81
	(0.13)	(0.13)
Observations	14920	11246

Notes: A full set of importer- and exporter-dummies is added to the regression. The dependent variable in the regression is as specified below (21). The data used to do this regression stem from UN COMTRADE, WDI, and CEPII. We use all available bilateral trade flows of final and intermediate goods, where we use the UN's BEC classification to classify each trade flow as involving either intermediate or final goods. It means that we include more countries in this regression than the 100 countries that we eventually use to numerically perform all counterfactuals. To be included in our counterfactuals, we need to observe a country at least once as an importer and once as an exporter of final and intermediate goods respectively. For countries not meeting this data requirement we do not get an estimate of all four sector-specific importer- and exporter-effects needed for our counterfactual exercises. Standard errors, clustered at the importer level, are shown in parentheses.

7.3 Proof of Lemma 1

Proof. Suppose the inverse of the $(n-1)\times(n-1)$ matrix $\left\{J\right\}^{-i^*}$ exists, where J denotes the Jacobian corresponding to equation system (20) and $\left\{J\right\}^{-i^*}$ the Jacobian of the reduced system with equation i^* left out.[33]

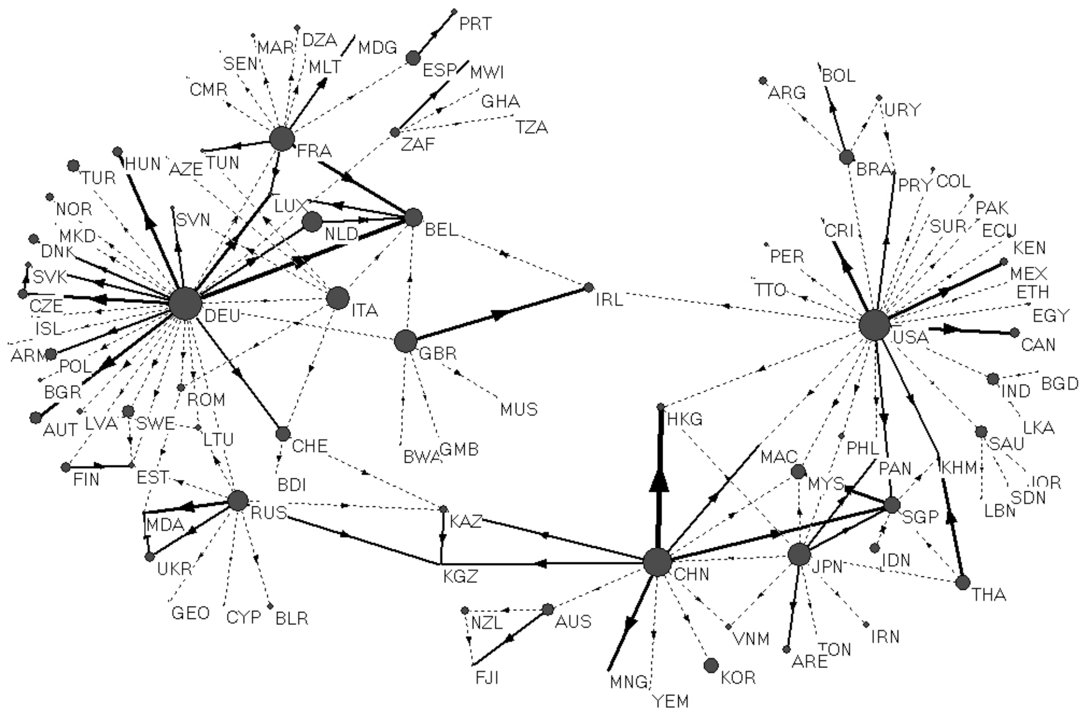
Moreover, fix $dw_{i^*} = 0$. The vector of wage changes in the remaining nations is then given by the total differential of system (20)

$$\left\{\left\{J\,\mathbf{dw}\right\}^{-i^*}\right\}^{+i^*} = -\left\{\left\{W^{1-\sigma} d\bar{M}\,\mathbf{1}\right\}^{-i^*}\right\}^{+i^*} \tag{47}$$

where $W^{1-\sigma}$ denotes a diagonal matrix with entries $w_i^{1-\sigma}$ on its diagonal. This equation

[33] In the notation of the proof for Theorem 1, where equation system (20) is written as $WZ(\mathbf{w}) - WL = 0$, J is the Jacobian matrix of $WZ(\mathbf{w})$.

Figure 5: Key players



NOTES: The figure positions all 100 countries in our dataset in a network graph. For each country, we show its most important trade partner in terms of the all six welfare channels defined in (38). On top, we add the 1% most important links among all the 9,900 country pairs in our dataset that each contribute by at least 1.81% to the per capita income in the receiving nation. All links are indicated by an arrow pointing from the country contributing to the welfare of the receiving country. The thicker an arrow, the larger the per capita income contribution on the range of 1.81–8.9%. Moreover, the size of each country's node reflects the country's overall importance (as also shown in Figure 3). To calculate all effects underlying this Figure we use (38), in combination with our empirical constructs from (22). Moreover, we fix $\sigma = 8$, use a country's total industrial value added (WDI) for $w_i L_i$, and set $dw_i = 0$ for the isolated country i.

can be rewritten as

$$\begin{aligned}
\left\{\left\{J\,W\,\mathbf{dw/w}\right\}^{-i^*}\right\}^{+i^*} &= -\left\{\left\{W^{1-\sigma} d\bar{M}\,\mathbf{1}\right\}^{-i^*}\right\}^{+i^*} \\
\left\{\left\{\mathbf{dw/w}\right\}^{-i^*}\right\}^{+i^*} &= -\left\{\left[\left\{J\,W\right\}^{-i^*}\right]^{-1}\right\}^{+i^*} \left\{\left\{W^{1-\sigma} d\bar{M}\,\mathbf{1}\right\}^{-i^*}\right\}^{+i^*} \\
\mathbf{dw/w} &= -\left\{\left[\left\{J\,W\right\}^{-i^*}\right]^{-1}\right\}^{+i^*} W^{1-\sigma} d\bar{M}\,\mathbf{1}
\end{aligned} \tag{48}$$

where W denotes a diagonal matrix with entries w_i on its diagonal and where, in the third line, we have made use of the assumption $dw_{i^*} = 0$ in vector $\mathbf{dw/w}$.

Let us, next, prove that $\left\{J\,W\right\}^{-i^*}$ is actually invertible. Note that the diagonal matrix

$Z(\mathbf{w}, \cdot)$, as defined in (44), is homogenous of degree zero with regard to a common change in all elements of vector $\mathbf{w}$. It follows that the row sum norm of $J\,W = 0$.

Moreover, since $Z(\mathbf{w}, \cdot)$ has the gross substitutes property -and the off-diagonal elements of $J\,W$ are therefore larger zero-, it follows that the row sum norm of $\{J\,W\}^{-i^*} < 0$. We can then apply Proposition 17.G.3 of Mas-Collel et al. (1995, p. 618) ensuring existence of an inverse of this matrix, which has moreover all its entries negative.

In fact, the inverse Jacobian matrix can be explicitly determined as follows: the partial derivatives on the off-diagonal of $WZ(\mathbf{w}, \cdot)$, $\partial w_i z_i / \partial w_j$ for $j \neq i$, are given in (46). On the diagonal, the derivatives are

$$\begin{aligned} \frac{\partial w_i z_i}{\partial w_i} &= (1-\sigma)\frac{w_i^{1-\sigma}}{(P_i^p)^{1-\sigma}} \sum_{j \in \mathcal{N}} \left[X_{ij}^{fin} + X_{ij}^{int} \right] - L_i \\ &+ w_i^{1-\sigma}\left[M_{ii} + (\sigma - 1) \sum_{k \in \mathcal{N}} M_{ik} S_{ik}^c (P_k^c)^{\sigma-1} \right] \\ &= -\sigma L_i w_i + w_i^{1-\sigma}\left[M_{ii} + (\sigma - 1) \sum_{k \in \mathcal{N}} M_{ik} S_{ik}^c (P_k^c)^{\sigma-1} \right] \end{aligned} \tag{49}$$

where, for the second equality, we have made use of the fact that in equilibrium $w_i z_i = 0$. Hence, we get

$$\begin{aligned} \left[\{J\,W\}^{-i^*}\right]^{-1} &= \left[\left\{ -\sigma L W + W^{1-\sigma} M \big(I + (\sigma-1)\big[S^c (P^c)^{\sigma-1}\big]^T \big) \right\}^{-i^*} \right]^{-1} \\ &= -\frac{1}{\sigma} \left[\left\{ I - \frac{1}{\sigma} [\bar{M}]^{-1} M \big(I + (\sigma-1)\big[S^c (P^c)^{\sigma-1}\big]^T \big) \right\}^{-i^*} \right]^{-1} \left\{ W^{\sigma-1} [\bar{M}]^{-1} \right\}^{-i^*} \\ &\equiv -\frac{1}{\sigma} \left[\left\{ I - \Phi \right\}^{-i^*} \right]^{-1} \left\{ W^{\sigma-1} [\bar{M}]^{-1} \right\}^{-i^*} \end{aligned} \tag{50}$$

where, for the second equality, we use the relationship $LW = W^{1-\sigma}\bar{M}$. Matrix $\sum_{h=0}^{\infty} (\Phi^{-i^*})^h$ in Lemma 1 is then nothing but a Neumann's series expansion of the inverse matrix in the third line of (50). Finally, note that

$$\begin{aligned} \mathbf{dw}/\mathbf{w} &= \frac{1}{\sigma} \left\{ \sum_{h=0}^{\infty} (\Phi^{-i^*})^h \left\{ W^{\sigma-1} [\bar{M}]^{-1} \right\}^{-i^*} \right\}^{+i^*} W^{1-\sigma} d\bar{M}\, \mathbf{1} \\ &= \frac{1}{\sigma} \left\{ \sum_{h=0}^{\infty} (\Phi^{-i^*})^h \right\}^{+i^*} [\bar{M}]^{-1}\, d\bar{M}\, \mathbf{1} \end{aligned} \tag{51}$$

which completes the proof. ■

7.4 Proof of Lemma 2

Before we proof the claim, let us review some well-established results on how to determine the inverse of a sum of matrices.

Henderson and Searle (1981): let X be a nonsingular square matrix, and U,Y and V be (possibly rectangular) matrices such that UYV is a square matrix. It holds

$$\left[X + UYV\right]^{-1} = X^{-1} - X^{-1}U\left[I + YVX^{-1}U\right]^{-1}YVX^{-1} \tag{52}$$

The following results are useful special cases of (52):

Minabe (1966, p.58): by successive application of (52) for a nonsingular square matrix X and a square matrix Y

$$\left[X - Y\right]^{-1} = X^{-1} + \sum_{h=1}^{\infty}\left(X^{-1}Y\right)^{h}X^{-1} \tag{53}$$

Neumann's series expansion: expanding on Minabe (1966) we get for $X = I$

$$[I - Y]^{-1} = I + Y[I - Y]^{-1} = I + [I - Y]^{-1}Y = \sum_{h=0}^{\infty} Y^{h} \tag{54}$$

Sherman and Morrison (1949, 1950): for $y \in \mathfrak{R}$, a column vector $\mathbf{u}$, and a row vector $\mathbf{v}^T$ of identical length

$$\left[X + y\mathbf{u}\mathbf{v}^T\right]^{-1} = X^{-1} - \frac{y}{1 + y\mathbf{v}^T X^{-1}\mathbf{u}} X^{-1}\mathbf{u}\mathbf{v}^T X^{-1} \tag{55}$$

Equipped with these results, we are ready to the proof the claims:

Proof of part (1.). Applying (53) for $X = I - A$ and $Y = x\tilde{A}$, where $\tilde{A}$ is such that for all $ij \in \mathcal{N} \times \mathcal{N}$ either $\tilde{a}_{ij} = a_{ij}$ or $\tilde{a}_{ij} = 0$, we get

$$\left[I - A - x\tilde{A}\right]^{-1} = \left[I - A\right]^{-1} + \sum_{h=1}^{\infty}\left(\left[I - A\right]^{-1}x\tilde{A}\right)^{h}\left[I - A\right]^{-1} \tag{56}$$

Suppose, now, that $x \to 0^+$. We get

$$\lim_{x\to 0^+}\frac{1}{x}\sum_{h=1}^{\infty}\left(\left[I - A\right]^{-1}x\tilde{A}\right)^{h}\left[I - A\right]^{-1} = \left[I - A\right]^{-1}\tilde{A}\left[I - A\right]^{-1} \tag{57}$$

Expression (28) of Lemma 2 is thus a first-order Taylor approximation around $[I - A]^{-1}$.

Proof of part (2.). Expression (29) of Lemma 2 is an immediate corollary of the result by Sherman and Morrison (1949, 1950), where we fix $X = I - A$, $y = -x$, $\mathbf{u} = (u_1 = 0, u_2 = 0, ..., u_i = 1, u_{i+1} = 0, u_n = 0)$, and $\mathbf{v} = (v_1 = 0, v_2 = 0, ..., v_j = 1, v_{j+1} = 0, v_n = 0)$.

Proof of part (3.). Applying (52) for $X = I$, $Y = -A$, $U = I_{xi}$, and $V = I_{yi}$, we get

$$\begin{aligned}\left[I - I_{xi}AI_{yi}\right]^{-1} &= I + I_{xi}\left[I - AI_{xi}I_{yi}\right]^{-1} AI_{yi} \\ &= I + I_{xi}\left[I - A - A(x+y+xy)I_{ii}\right]^{-1} AI_{yi}\end{aligned} \tag{58}$$

Applying (52) again, this time for $X = I - A$, $Y = I$, $U = -A(x+y+xy)I_{ii}$, and $V = I$, we find

$$\begin{aligned}\left[I - A - A(x+y+xy)I_{ii}\right]^{-1} &= \left[I - A\right]^{-1} + \left[I - A\right]^{-1} A(x+y+xy) \\ &\times \; I_{ii}\left[I - \left[I - A\right]^{-1} A(x+y+xy)I_{ii}\right]^{-1} I_{ii}\left[I - A\right]^{-1}\end{aligned} \tag{59}$$

where we have made use of the fact that I_{ii} is idempotent ($I_{ii} = I_{ii}I_{ii}$). Finally, let us rewrite $I_{ii} = \mathbf{e}\mathbf{e}^T$, where $\mathbf{e}$ is a column vector of zeros with a single one in the proper position. We can then write

$$\begin{aligned}I_{ii}\left[I - \left[I - A\right]^{-1} A(x+y+xy)I_{ii}\right]^{-1} I_{ii} &= \mathbf{e}\left[1 - \sum_{h=1}^{\infty} a_{ii}^{[h]}(x+y+xy)\right]^{-1} \mathbf{e}^T \\ &= \left[1 - \sum_{h=1}^{\infty} a_{ii}^{[h]}(x+y+xy)\right]^{-1} I_{ii}\end{aligned} \tag{60}$$

The expression in (30) of Lemma 2 follows immediately from the combined (58)-(60). ■

7.5 Proof of Theorem 2

Proof. Consider a proportional increase in all cells of matrix B, such that $B' = (1 + \delta_B)B$, with $\delta_B > 0$, and $A' = A$. We verify that consumer prices reduce at the same rate in any $i \in N$ and that nominal wages stay put.

Concerning the price effects, note that $(P_i^c)^{1-\sigma} = \sum_{j \in N}(P_j^p)^{1-\sigma} b_{ji}$ is homogenous of degree one with regard to a common change in all b_{ji}. Thus, for any $i \in N$ it is:

$$\frac{d(P_i^c)^{1-\sigma}}{(P_i^c)^{1-\sigma}} = \delta_B > 0 \tag{61}$$

Turning to the wage effects, for a marginally small $x \to 0^+$ the wage increase in any $i \in N$ is determined by equation (24) of Lemma 1. In particular, this depends on the direct effect of a change in matrix B on the market access term

$$\bar{M}_i = \sum_{j,k\in\mathcal{N}} \sum_{h=0}^{\infty} a_{ik}^{[h]} \frac{b_{kj}}{\sum_{l\in N} (P_l^p)^{1-\sigma} b_{lj}} L_j w_j \tag{62}$$

As the right-hand side of this equation is homogenous of degree zero with regard to a common change in all b_{kj}, for $k \in N$, it immediately follows $d\bar{M}_i = 0$, and thus by (24), $\mathbf{dw} = \mathbf{0}$ for a small $x \to 0^+$. Taking the integral of the wage effect over $x \in [0; \delta_B]$, we find $\mathbf{dw} = \mathbf{0}$ for any $\delta_B > 0$. ■

7.6 Proof of Theorem 3

Proof. Consider a proportional increase in all cells of matrix A, such that $A' = (1+\delta_A)A$, with $\delta_A \to 0^+$, and $B' = B$. From Property (1.) of Lemma 2 it follows

$$\left[I - (1+\delta_A)A\right]^{-1} B - \left[I - A\right]^{-1} B = \delta_A \sum_{h=0}^{\infty} A^h \sum_{h=1}^{\infty} A^h B \tag{63}$$

Combined with the supplier access matrices in (18), the effect on consumer prices in $i \in \mathcal{N}$ can then be summarized in the row vector

$$\mathbf{1^T} dS^c = \delta_A \mathbf{1^T} W^{1-\sigma} \sum_{h=0}^{\infty} A^h \sum_{h=1}^{\infty} A^h B = \delta_A \mathbf{1^T} S^p \sum_{h=1}^{\infty} A^h B \tag{64}$$

which is identical to effect (i) in equation (32) of Theorem 3.

Combining (63) with the market access matrix (19), the effect on market access in $i \in \mathcal{N}$ is given by

$$\begin{aligned} dM\mathbf{1} &= \delta_A \sum_{h=0}^{\infty} A^h \sum_{h=1}^{\infty} A^h BWL(P^c)^{\sigma-1}\mathbf{1} - \sum_{h=0}^{\infty} A^h BWL\left[(P^c)^{\sigma-1} d(P^c)^{1-\sigma} (P^c)^{\sigma-1}\right]^T \mathbf{1} \\ &= \delta_A \sum_{h=1}^{\infty} A^h M - \delta_A M \left[S^p \sum_{h=1}^{\infty} A^h B (P^c)^{\sigma-1}\right]^T \mathbf{1} \end{aligned} \tag{65}$$

where, for the first equality, we make use of the rule for the derivative of an inverse matrix, $d(P^c)^{\sigma-1} = -(P^c)^{\sigma-1} d(P^c)^{1-\sigma} (P^c)^{\sigma-1}$. For the second equality, we use the identities

$$\left[(P^c)^{\sigma-1} d(P^c)^{1-\sigma} (P^c)^{\sigma-1}\right]^T \mathbf{1} = (P^c)^{\sigma-1} \left[\mathbf{1^T} d(P^c)^{1-\sigma} (P^c)^{\sigma-1}\right]^T \tag{66}$$

and $\mathbf{1^T} d(P^c)^{1-\sigma} = \mathbf{1^T} dS^c$. The last of these expressions is defined in (64). Moreover, we use $\sum_{h=0}^{\infty} A^h \sum_{h=1}^{\infty} A^h = \sum_{h=1}^{\infty} A^h \sum_{h=0}^{\infty} A^h$ and our definition $\sum_{h=0}^{\infty} A^h BWL(P^c)^{\sigma-1} \equiv M$. From the final identity in (65) we immediately arrive at *effects (ii) and (iii)* of (32), when additionally considering that $\bar{M}\mathbf{1} \equiv M\mathbf{1}$. ■

7.7 Proof of Theorem 4

Proof. Suppose that $\theta = 0$ (i.e. $A = 0$) and $dw_i = 0$. Consider a change in cell ij of the final goods trade intensity matrix, such that $B' = B + \delta_B I_{ij}$ with $\delta_B > 0$.

Since the matrix of supplier access (18) reduces to $S^c = W^{1-\sigma}B$ in this case, the effect on consumer prices can be retrieved from the following row vector

$$\mathbf{1^T} dS^c \quad = \quad \mathbf{1^T} W^{1-\sigma} \delta_B I_{ij} \tag{67}$$

Cell k in this vector is strictly positive if and only if $k = j$, and zero otherwise.

Turning to the wage effects, we first need to investigate the direct effect on the matrix of market access, $\bar{M} = BWL(P^c)^{\sigma-1}\mathbf{1}$. The wage adjustments are then given by equation (24) of Lemma 1.

The direct effect is given by the column vector

$$dM\mathbf{1} = \delta_B I_{ij} WL(P^c)^{\sigma-1}\mathbf{1} \; - \; M\left[\mathbf{1^T} dS^c \, (P^c)^{\sigma-1}\right]^T \tag{68}$$

where we have made use of the rule for the derivative of an inverse matrix to obtain the second summand. Since we fix $dw_i = 0$, we can omit row i from the first summand. Thus, the direct effect reduces to the second (negative) summand in (68).

Substituting equation (67) for $\mathbf{1^T} dS^c$ and making use of equation (25) of Lemma 1, this verifies part (ii) of Theorem 4 that $dw_k < 0$ for all $k \in \mathcal{N} \backslash \{i\}$. Furthermore, since the *wage-induced price effect* of equation (27) becomes equal to $-\left[S^c(P^c)^{\sigma-1}\right]^T \frac{\mathbf{dw}}{\mathbf{w}} > 0$, this verifies part (i) of the theorem stating that $d(P^c_k)^{1-\sigma} > 0$ for all $k \in \mathcal{N}$. ■

7.8 The key player formula

In order to derive the column vector (38), we first need to determine the impact of isolating country i on matrix $[I - A]^{-1}B$:

From the product rule for matrices, combined with equation (36), the impact can be written as

$$\begin{aligned}
[I - A^{-i}]^{-1}B^{-i} - [I - A]^{-1}B \quad &= \quad \left(\sum_{h=0}^{\infty} A^h \; - \; \frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \sum_{h=0}^{\infty} A^h I_{ii} \sum_{h=0}^{\infty} A^h \; + \; I_{ii} \right) \\
&\quad \times \quad \left(B - I_{ii}B - BI_{ii} + I_{ii}BI_{ii} \right) - \sum_{h=0}^{\infty} A^h B \\
&= \quad - \frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \sum_{h=0}^{\infty} A^h I_{ii} \sum_{h=0}^{\infty} A^h B \big(I - I_{ii}\big) \; - \; \sum_{h=0}^{\infty} A^h B I_{ii}
\end{aligned} \tag{69}$$

where, in the final line, we have made use of the fact that I_{ii} is an idempotent matrix, i.e. $I_{ii} = I_{ii}I_{ii}$, and the fact that $I_{ii}\sum_{h=0}^{\infty} A^h I_{ii} = I_{ii}\sum_{h=0}^{\infty} a_{ii}^{[h]}$.

We are now ready to determine the price and wage effects of isolating country i: applying (69) to the supplier access matrix (18), the price effect on any $j \in \mathcal{N}$ can be summarized in the row vector

$$
\begin{aligned}
\mathbf{1}^{\mathbf{T}} dS^c &= -\mathbf{1}^{\mathbf{T}} \frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} W^{1-\sigma} \sum_{h=0}^{\infty} A^h I_{ii} \sum_{h=0}^{\infty} A^h B \big(I - I_{ii}\big) - \mathbf{1}^{\mathbf{T}} W^{1-\sigma} \sum_{h=0}^{\infty} A^h B I_{ii} \\
&= -\mathbf{1}^{\mathbf{T}} \Bigg[\underbrace{\frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} (I - I_{ii}) W^{1-\sigma} \sum_{h=0}^{\infty} A^h I_{ii} \sum_{h=0}^{\infty} A^h (I - I_{ii}) B}_{\text{intermediated supply (iii)}} \\
&\qquad + \underbrace{\frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} I_{ii} W^{1-\sigma} \sum_{h=0}^{\infty} A^h I_{ii} \sum_{h=0}^{\infty} A^h (I - I_{ii}) B}_{\text{intermediate goods supply (ii)}} + \underbrace{W^{1-\sigma} \sum_{h=0}^{\infty} A^h I_{ii} B}_{\text{final goods supply (i)}} \Bigg] \big(I - I_{ii}\big) \\
&\quad - \mathbf{1}^{\mathbf{T}} W^{1-\sigma} \sum_{h=0}^{\infty} A^h B \, I_{ii}
\end{aligned}
\tag{70}
$$

where, in the second and third line, we simply decompose the first summand from line one. Effects (i)-(iii) of equation (37) follow immediately from (70), when considering our definitions of $W^{1-\sigma} \sum_{h=0}^{\infty} A^h \equiv S^p$ and $W^{1-\sigma} \sum_{h=0}^{\infty} A^h B \equiv S^c$, and $W^{1-\sigma} \sum_{h=0}^{\infty} A^h = (P^p)^{1-\sigma}$ and the additional fact that the final summand in line four of (70) can be omitted, because we ignore the effect on country i itself.

Combining (69) with the market access matrix (19), the wage effect on any $j \in \mathcal{N}$ is summarized in the column vector

$$
\begin{aligned}
dM\mathbf{1} &= -\Bigg[\underbrace{\frac{1}{\sum_{h=0}^{\infty} a_{ii}^{[h]}} \sum_{h=0}^{\infty} A^h I_{ii} \sum_{h=0}^{\infty} A^h B \, W \, L \, (P^c)^{\sigma-1} \big(I - I_{ii}\big)}_{\text{intermediated demand (v)}} \\
&\qquad + \underbrace{\sum_{h=0}^{\infty} A^h B \, I_{ii} \, W \, L \, (P^c)^{\sigma-1}}_{\text{final goods demand (iv)}} \Bigg] \mathbf{1} \\
&\quad - \underbrace{\sum_{h=0}^{\infty} A^h B \, W \, L \, (P^c)^{\sigma-1} \big[\mathbf{1}^{\mathbf{T}} dS^c (P^c)^{\sigma-1}\big]^T}_{\text{competition (vi)}}
\end{aligned}
\tag{71}
$$

where we make use of the rule for the derivative of an inverse matrix and the identity $[(P^c)^{\sigma-1} d(P^c)^{1-\sigma} (P^c)^{\sigma-1}]^T \mathbf{1} = (P^c)^{\sigma-1} [\mathbf{1}^{\mathbf{T}} dS^c (P^c)^{\sigma-1}]^T$.

The demand effects (iv)-(vi) in equation (37) follow immediately, after substituting (70) for $\mathbf{1}^{\mathbf{T}} dS^c$, and after considering our definition $\sum_{h=0}^{\infty} A^h B W L (P^c)^{\sigma-1} \equiv M$ and the fact that $WL(P^c)^{\sigma-1}$ is a diagonal matrix so that $I_{ii} W L (P^c)^{\sigma-1} = W L (P^c)^{\sigma-1} I_{ii}$.

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