Mechanical Behavior of Textile Composite Materials Using a Hybrid Finite Element Approach

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A novel analytical solution is presented to describe the mechanical behavior and failure of textile reinforced composites. This solution is constructed from two parts. First a geometrical model, based on a processing science approach coupled with graphical rendering, is utilized to quantify the spatial characteristics of the fabric preform inside the composite. A 3-D iterative hybrid finite element analysis is then used, utilizing data acquired from the geometrical model, to predict the stress-strain behavior of the composite. Inelastic behavior is modeled using an expanded Hahn and Tsai model and maximum stress criterion is used to predict initial failure. Damage progression is predicted based on a stiffness reduction approach. AS4 carbon/epoxy plain weave composite laminates, with a range of fiber volume fractions, have been tested. A 3-D woven E-glass/poly-vinyl-ester (PVE) angle interlock composite was also tested. Analytical results are compared with results from this experiment as well as other analytical models and experimental data in the literature. Theoretical model predictions are in agreement with most experimental data.

INTRODUCTION

Mechanical behavior and failure analysis of 2-D woven textile composites (e.g., plain weaves, twills, satins) have been studied by many investigators. Models available in literature can be divided into homogenization approaches and traditional finite element (FE) approaches.

An example of homogenization approaches for 2-D textile composites is found in the work by Ishakawa and Chou (1, 2), in which approximate 2-D “Mosaic” and “Fiber Undulation” models, based on the laminate theory, were extended to predict the composite strength. The inelastic behavior was attributed to: i) deformation of regions with crimped yarns due to off-axis loading ii) behavior of pure matrix regions and iii) transverse cracking in the warp region. The Hahn and Tsai model for inelastic behavior of unidirectional composites (3) was used for cases (i) and (ii). This piecewise superposition solution, although successful in predicting the properties of simple fabric preforms, could not be extended to model composites with complex structures.

Traditional finite element analysis was used by Woo and Whitcomb (4, 5) to model stress-strain behavior and by Blackketter (6) to model damage progression of plain weave composites. Both approaches used incremental iterating finite element algorithms to predict stress-strain behavior of plain weave composites. In the latter approach, the effect of damage progression was modeled by stiffness reduction. Although both models were able to predict with a good degree of accuracy, the behavior of plain weave composites, predicting properties of fabric with more complex structures (e.g., 3-D weaves, 3-D braids) would increase the number of finite elements needed to model the structure to an impractical value. Blackketter et al. reported that running traditional FE analysis for plain weaves using 256 elements took 30 min per iteration on a VAX 8800 computer.

Both models (i.e., the homogenization approach and the traditional FE analysis) could not be extended to model 3-D preform composites (e.g., 3-D woven and braided). The homogenization approach include high degree of approximation, while the traditional finite element analysis is bounded by the limit on the number of elements that could be used in the analysis.

Pastore and Bogdanovich (7, 8) developed a 3-D adaptive analysis based on a deficient spline displacement approximation approach for arbitrary anisotropic meso-volumes. This model was able to predict the stress-strain behavior of a laminated composite as well as damage initiation. Initial failure was predicted using a tensile polynomial failure criteria. Although this model could be extended to predict the mechanical behavior of 3-D preform composites, it did not take into consideration inelastic behavior, the effect of damage progression, and final composite failure.
TECHNICAL APPROACH

A model is constructed to predict the mechanical behavior of textile composites with arbitrary preform structures. The model utilizes, in the mechanical analysis, a small number of hybrid finite elements with fiber and matrix around each integration point. This drastically reduces the computational time needed to run the analysis. The model is based on the "Graphical Integrated Numerical Analysis (GINA)" (9, 10) developed earlier to predict the elastic and thermal properties of textile composites.

The Graphical Integrated Numerical Analysis (GINA)

This model was developed earlier (9, 10) to predict the elastic and thermal properties of textile composites. This is a two-part model. First a geometrical model is used to construct the textile preform and to characterize the relative volume fractions and spatial orientation of each yarn in the composite space. Data acquired from the geometrical analysis is then used by a hybrid finite element approach to model the composite elastic and thermal behavior.

The geometrical model (11) used in GINA starts by modeling the preform forming process in a typical textile machine. An ideal fabric geometrical representation is constructed by calculating the location of a set of spatial points, "knots," that can identify the yarn centerline path within the preform space. This is followed by incorporating a B-spline function to approximate a smooth yarn center-line path relative to the knots identified. The B-spline function is chosen as the approximation function due to its ability to minimize the radius of curvature along its path and its $C^2$ continuity (12, 13). The final step in this model is carried out by constructing a 3-D object (i.e., yarn) by sweeping a cross section along the smooth centerline forming the yarn surface.

A repeat unit cell of the modeled preform is identified from the geometric modeling and used to represent a complete yarn or tow pattern. A finite element approach is used to divide the unit cell into smaller subcells. Each subcell is a hexahedral brick element with fibers and matrix around each integration point. A virtual work technique is applied to the FE solution (14, 15) to calculate the properties of the repeat unit cell. The unit cell properties are considered to be representative of the composite properties.

The heterogeneous solution, although successful in cutting down the number of elements, has a limitation. This limitation is bounded by the difference in the fiber and matrix properties. To overcome such limitation a homogenization operation is carried out at the micro-scale level. This is done around each integration point in the FE mesh. Figure 1 shows a schematic presentation of the finite element division scheme and the micro-level homogenization.

INELASTIC BEHAVIOR, DAMAGE PROGRESSION AND FAILURE ANALYSIS

The Graphical Integrated Numerical Analysis, presented in the previous section, is modified and extended to model the inelastic behavior, damage progression and failure analysis of textile composites. The logical flow of the proposed approach is detailed in Fig. 2.

In this procedure, the geometrical model is first applied to construct the fabric. A unit cell is automatically identified from the geometrical model to represent the fabric construction. This unit cell is divided into smaller eight noded hexahedral brick elements called subcells. The stiffness matrix of each subcell is calculated.

The stiffness matrix of the unit cell is assembled from the stiffness matrices of different subcells. The boundary conditions for the unit cell as a whole, dictated by the assumption of its repeatability and structural continuity, is applied to reduce the size of the stiffness matrix.

The unit cell is then incrementally loaded at the edge nodes. Load can be applied in any direction, with various load distribution parameters (i.e., concentric...
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Fig. 2. A schematic representation of the logical flow of mechanical analysis.

Trated, distributed, parabolic, etc.). The displacement at each node in the unit cell continuum is calculated under the applied load. Strain and stresses at each node are then calculated from the knowledge of the node displacement, the shape function and the stiffness around each node.

The applied stresses at each node is compared with the strength around that node. If the node fails to carry the applied stresses, its contribution to its subcell stiffness matrix and consequently to the global stiffness matrix is eliminated and the load is re-distributed to other nodes. Otherwise, load iterations continue until the slope of the relative stress-strain curve approaches zero. At this point, load iterations are stopped and the stress value at the last load iteration is used as the strength value of the composite.

The displacement at each node is calculated using a Gaussian elimination with a back substitution technique (16), after reducing the size of the stiffness matrix by applying the boundary conditions. The strains at each node are calculated from the following equation:

\[ e_i = [B] \delta_i \]  

(1)

where \( e_i \) = strain vector at node \( i \), \([B]_i \) = first order derivative of the shape function at node \( i \) and \( \delta_i \) = displacement vector of node \( i \).

The stresses at each node are calculated in the global composite direction from the knowledge of the strain vector and the value of the stiffness tensor at this node. The inelastic component of the strain is then added to the elastic component by utilizing the Hahn and Tsai model (3). This model is modified to be used in a general form to account for all tension-shear interaction components of the compliance tensor.

These interaction components are highly evident in textile composites as a result of excessive yarn crimp:

\[ e_{ij} = S_{ijkl} \sigma_{kl} + S^{*}_{ijkl} \sigma_{kl}^2 \]  

(2)

where \( e_{ij} \) = shear strain (\( i \) is the loading direction), \( \sigma_{ij} \) = shear stress, \( S_{ijkl} \) = compliance tensor, and \( S^{*}_{ijkl} \) = material in-elastic behavior constant.

The strength of the subcell is calculated as the sum of the global contribution of the strengths of the impregnated yarns, pure matrix and interface in each subcell as follows:

\[ T_{n,kl} = \sum_{m=1}^{M} T_{n,kl} \alpha_{mn,il} \alpha_{m,ij} \]  

(3)

where \( T_{n,kl} \) = strength vector of subcell \( n \) in the \( kl \) global direction, \( T_{m,ij} \) = strength vector of material \( m \) (i.e. impregnated yarns, pure matrix or interface) obtained from experimental data in the \( ij \) local direction (i.e., this data could be obtained from unidirectional composites tests as well as single fiber and pure matrix tests) and \( \alpha_{mn} \) and \( \alpha_{m,ij} \) form the transformation tensor from local \( ij \) direction to global \( kl \) direction.

The stress-state at each node is evaluated in all spatial directions and compared with material strength at this node. It should be noted that the stress failure criterion is utilized in this approach because of the ease of implementation. Other tensor polynomial failure techniques [Tsai-Wu (17), Hoffman (18), etc.] could be implemented to predict initial failure provided that all experimental variables needed for the solution are available.

In any load iteration, if the applied stress exceeds the strength of a node, the node fails in that direction and its contribution is eliminated. This forms a dam-
age initiation site. During the next iteration, the stress distribution is altered. Subcells with damage sites, and accordingly reduced stiffness, will carry less load than subcells with fewer damage sites.

It was noticed that initial damage happens in either the fiber/matrix interface or in the out-of-plane shear direction. This typically occurred forming a knee point (i.e., a change in the slope) similar to that observed in (2), in the stress-strain curve. At this knee point, the composite modulus is reduced. It is typically the first point that the composite inelastic behavior starts to manifest its existence.

Final failure is determined by monitoring the slope of the stress-strain curve. If the slope of the stress-strain curve, between successive load iterations, approaches zero, the loading iterations are terminated and the strength and strain to failure are recorded.

**COMPARISON WITH OTHER EXPERIMENTAL AND ANALYTICAL WORK**

The analytical model presented in this paper was compared with the analytical and experimental work developed by Blackketter et al. (6) for balanced plain weave AS4/epoxy composite with volume fraction of 60%. Figure 3 shows the results of the experimental testing and analytical work from Blackketter (6) as well as the prediction using the current model with 64 subcells. All input data were obtained from Blackketter (6). The inelastic behavior constant for AS4/epoxy composites was taken as 7.29 (GPa)^{-3} (2, 3).

The time used to run the analysis was 20 sec per iteration on a SUN Sparc Station 10. Similar time was obtained on a Pentium P166 PC computer. This can be compared with the 30 min per iteration reported in (6).

It can be seen from Fig. 3 that the current model was able to predict the material mechanical behavior with good accuracy.

One of the major concerns of FE analysis is the convergence efficiency, especially for hybrid element models. To this end, a convergence study was conducted to evaluate the solution quality and efficacy. Figure 4 shows the fluctuation of strength values obtained with the current model with the total number of subcells (i.e., FE hybrid elements) for the same problem presented in Fig. 3. It can be seen from Fig. 4 that convergence was reasonably obtained at the 64 subcell level.

**EXPERIMENTAL WORK**

AS4 carbon/epoxy resin plain weave composite laminates were tested with a range of fiber volume fractions. The experimental program also included testing a 3-D E-glass/poly-vinyl-ester (PVE) angle interlock weave. The following sections give details as to materials used, manufacturing steps, testing procedure, and comparison with analytical results.

**AS4/Epoxy Plain Weave Composites**

The AS4 carbon balanced plain weave fabric was made from 5 ends/cm 3k yarns. The matrix system was Shell Epon resin 828 with V40 curing agent with a mix ratio of 3:1. The composite was manufactured using a compression molding machine. The composite was cured for 2 h at 100°C. Specimens were then post cured at 135°C. Four AS4 carbon/epoxy panels were manufactured with 27.5, 35.3, 41.4 and 47.2% total fiber volume fractions.

At least five samples were cut from each panel and prepared according to ASTM D3039 testing procedure. The samples were then tested in tension using an Instron machine model 4505. Figure 5 shows the average stress-strain behavior for the AS4/epoxy composites.

Figures 6 and 7 show the model prediction versus experimental results for strength and strain-to-failure values. The matrix properties were measured experimentally following the ASTM D3039 testing procedure. The matrix Young's modulus was 2.2 GPa, Poisson's ratio was 0.35, strength to failure was 0.159
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**Fig. 4. Convergence study.**

**Fig. 5. Experimental values of stress-strain diagrams for AS4/epoxy balanced plain weave composites with different total fiber volume fractions (Vf).**

**Fig. 6. Strength vs. volume fraction for AS4/epoxy balanced plain weave composites with different total fiber volume fractions (Vf).**

**Fig. 7. Strain to failure vs. Volume fraction for AS4/epoxy balanced plain weave composites with different total fiber volume fractions (Vf).**

GPa. The inelastic material constant used in the analysis was $7.29 \text{ (GPa)}^{-3}$ (2, 3). AS4 fiber properties were obtained from the manufacturer data sheet. All other composite mechanical properties were taken from Blackketter (6). The number of subcells used in the analysis was 64.

Figure 6 shows the strength values for AS4/epoxy plain weave composite for different composite fiber volume fractions. Figure 6 shows that the predicted and the experimental values for strength match very well. In Fig. 7, the strain to failure is plotted versus the composite fiber volume fraction for AS4/epoxy plain weave composite. The model prediction for the strain-to-failure has an average difference of 9% from the mean experimental value. It is expected that the difference between the prediction and experimental results could be reduced by experimentally evaluating the actual inelastic constant rather than using the value found in literature (2, 3). The chemical structure of the epoxy could have an effect on the inelastic behavior of the matrix.

**3-D E-Glass PVE Angle Interlock Composites**

The composite system under consideration is formed from three-dimensional angle interlock woven E-glass/poly-vinyl-ester composite. This composite was made of 12 end E-glass roving. The weft yarns are straight (without crimp) and perpendicular to the weaving axis. The warp yarns are woven with an average elevation angle of 15° to act as longitudinal re-
Table 1. Mechanical Properties for E-Glass and PVE Matrix.

<table>
<thead>
<tr>
<th>Material</th>
<th>E-Glass Fiber (GPa)</th>
<th>PVE Resin (19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile modulus</td>
<td>85.5</td>
<td>3.3</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>35.63</td>
<td>1.27</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Strength</td>
<td>2.82</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 2. Experimental Results and Analytical Predictions for 3-D Woven E-Glass/PVE Composite.

<table>
<thead>
<tr>
<th>Property</th>
<th>Experiment</th>
<th>Analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength in GPa (warp direction)</td>
<td>321.3 ± 42.3</td>
<td>330.0</td>
</tr>
<tr>
<td>Tensile strength in GPa (weft direction)</td>
<td>360.8 ± 22.3</td>
<td>450.0</td>
</tr>
</tbody>
</table>

inforcement (parallel to the weaving axis) as well as through the thickness reinforcement. Poly-vinyl-ester resin (Derakane 411-350 PA manufactured by Dow Plastics) with curing agent methyl ethyl ketone peroxide was infiltrated under vacuum at 80°C to form a 30 cm × 30 cm plate. The fiber volume fraction calculated was 33.6%. The straight weft yarns have the same relative volume fraction as the crimped yarns. Tensile testing was carried out in the longitudinal direction (perpendicular to the weft yarns) and in the transverse direction (parallel to the weft yarns) following the ASTM D3039 testing procedure. A Kawabata single fiber tester was used to evaluate the tensile modulus and strength of the E-glass fibers. Further details of this experimental work are found in Gowayed et al. (10).

The 3-D woven composite was modeled using 64 subcells (refer to Fig. 8). The mechanical properties used in the analysis for the fibers and matrix are shown in Table 1. E-glass fiber properties were obtained from the manufacturer data sheet. The transverse strength of the unidirectional composite was taken as 44 MPa (20), while the inelastic material constant was calculated from the literature (2, 3) as 100 (GPa)^-3.

Table 2 shows the experimental results and analytical predictions for the 3-D woven E-glass/PVE composites. The predictions matched very well the experimental value in the warp (crimped) direction. In the weft direction, where the yarns are straight, the model prediction was 20% higher than the experimental value.

The source of discrepancy between the experiment and prediction is not clear. Typically, the strength in the straight yarn direction in multi-direction composites follows closely the rule of mixtures. In this case, the experimental data is much lower than the "expected" value, while the prediction is very close to the rule of mixtures result. The authors believe that this could be an experimental abnormality rather a model deficiency.

**CONCLUSIONS**

A new model has been constructed to predict the inelastic behavior, damage progression, and failure of textile reinforced composites. A 3-D iterative hybrid finite element analysis, along with a science processing model for geometric characterization, was utilized to model the stress-strain behavior and the failure of the composite.

The computing time used by the new model is 90% less than the time required for comparable models described in literature. Unlike models that adopt a traditional finite element approach and are limited by the number of elements required to achieve a solution with good accuracy, this approach is able to model composites with arbitrary preform utilizing a hybrid FE solution. The model predictions were compared with experimental data and other models described in literature and provided a good correlation for most of the data.

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**REFERENCES**

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