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Applied Mathematics Taster Lecture

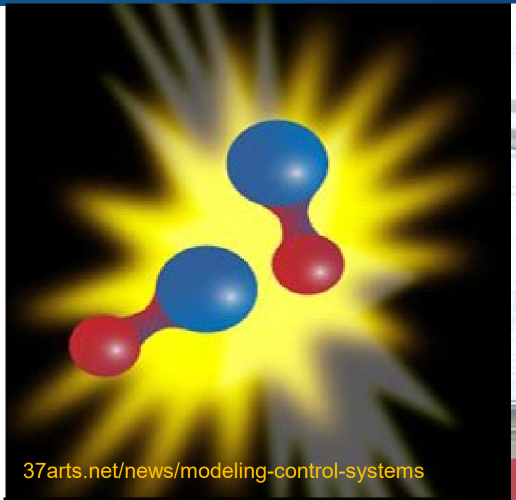
Stephen Creagh

Applied Maths: what is it?

What’s applied maths?

Mechanics

- Molecular dynamics
- Relativity, black holes
- Quantum Mechanics



37arts.net/news/modeling-control-systems

www.nasa.gov/image-feature/fast-flying-black-hole

nasa.gov/directorates/spacetech/niac/kwiat_entanglement_assisted.html

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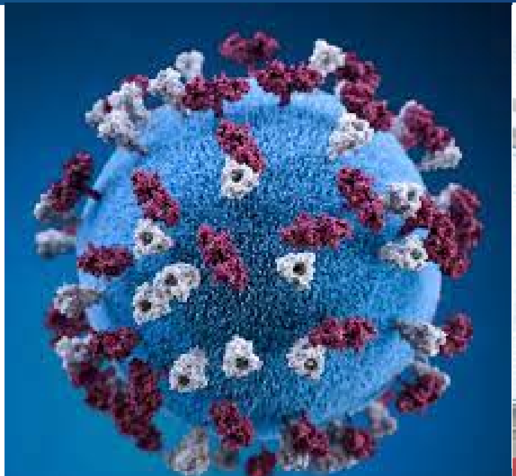
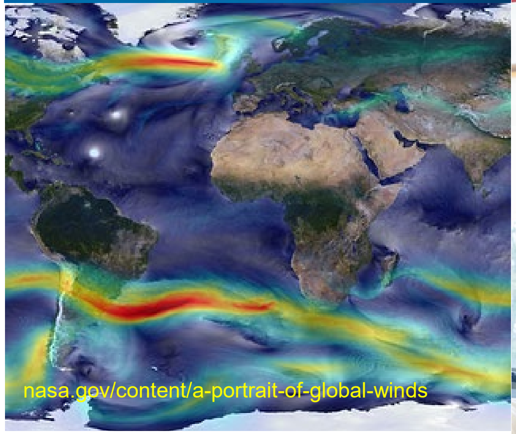
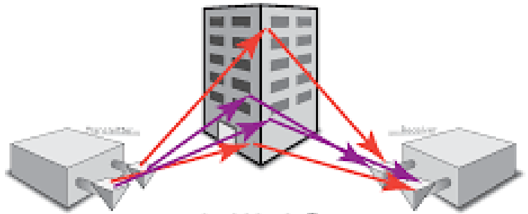
What’s applied maths?

Modelling

- Epidemics, Biology
- Fluids, weather
- Mobile comms, acoustics


https://pxhere.com/en/photo/1608799


nasa.gov/content/a-portrait-of-global-winds



en.wikipedia.org/wiki/File:MIMO_with_building.png

QUIZ: go to pingo.coactum.de/542220

Practice question: go to pingo.coactum.de/542220) and click all that apply:

A. I hate mechanics.

B. I love mechanics

C. Mechanics and modelling both sound so wonderful I can't tell which I'll prefer!

D. Mechanics is meh but I'm interested in other applications.

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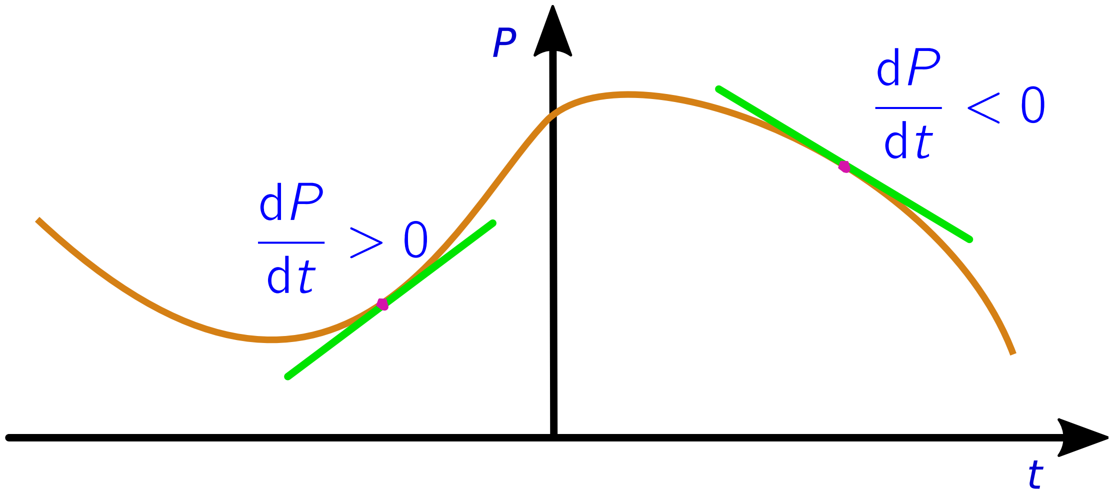
Applied Maths: what should I know?

What do we need from calculus?

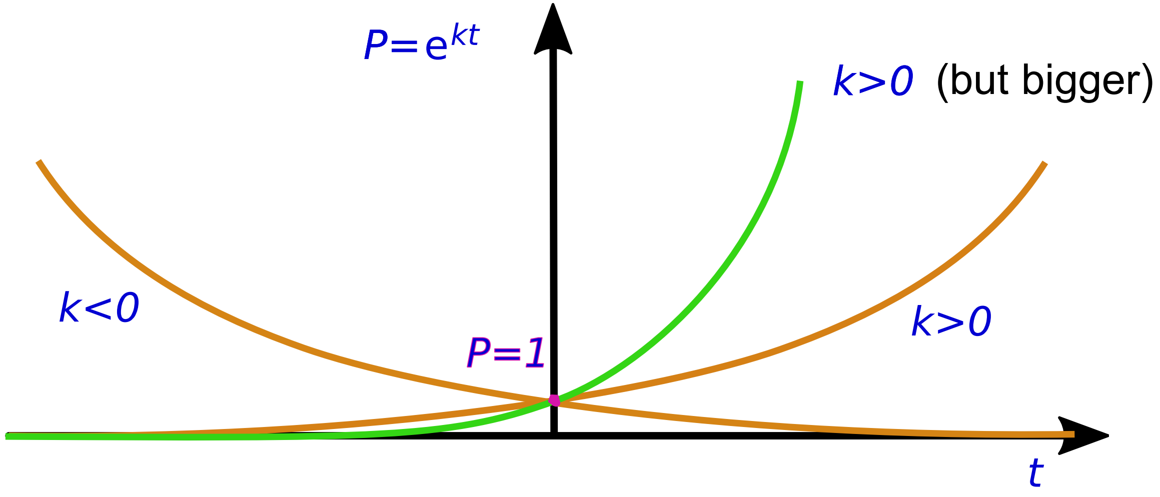
- Derivatives are rates of change



- Important special case

$$P(t) = e^{kt} \quad \Rightarrow \quad \frac{dP}{dt} = ke^{kt} = kP$$

- Graphs of exponentials



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Applied Maths: modelling

Modelling a pandemic

What do we see in COVID models?

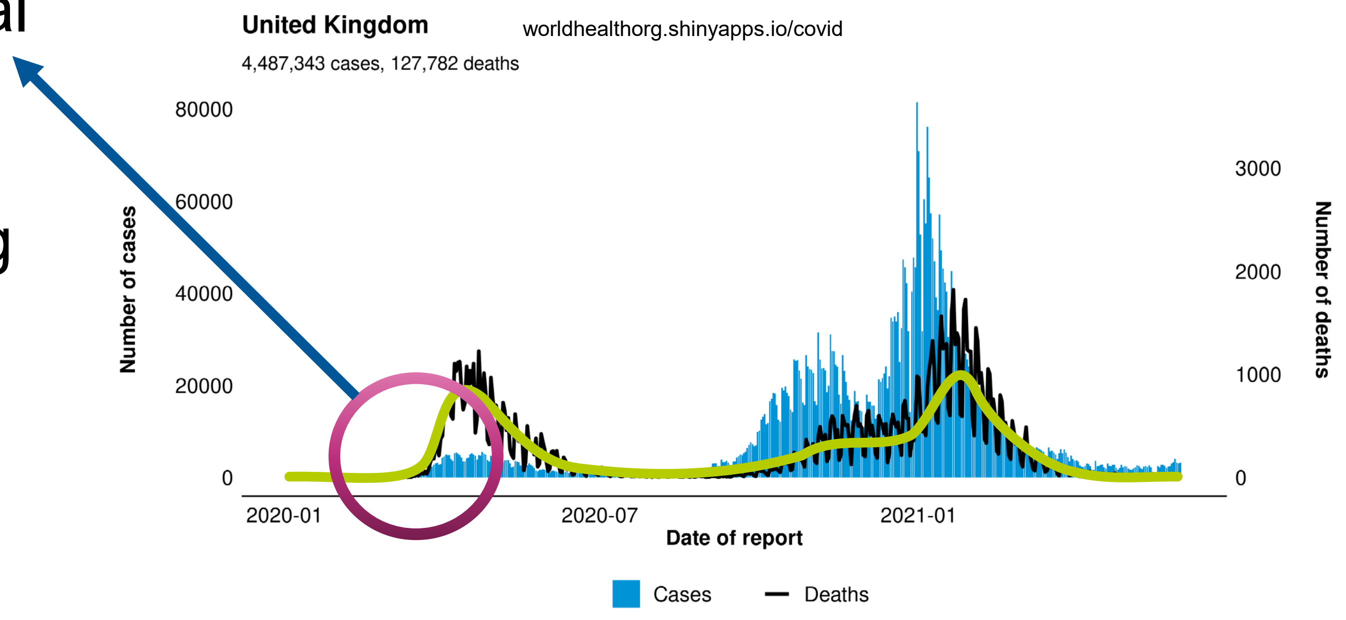
Initially, there was a rapid rise which looks a lot like our graphs of exponential functions!

Later, get different outcomes depending on:

- the natural evolution of the disease (eg immunity)
- changes in policy (eg social distancing)

Can we account for things like this?

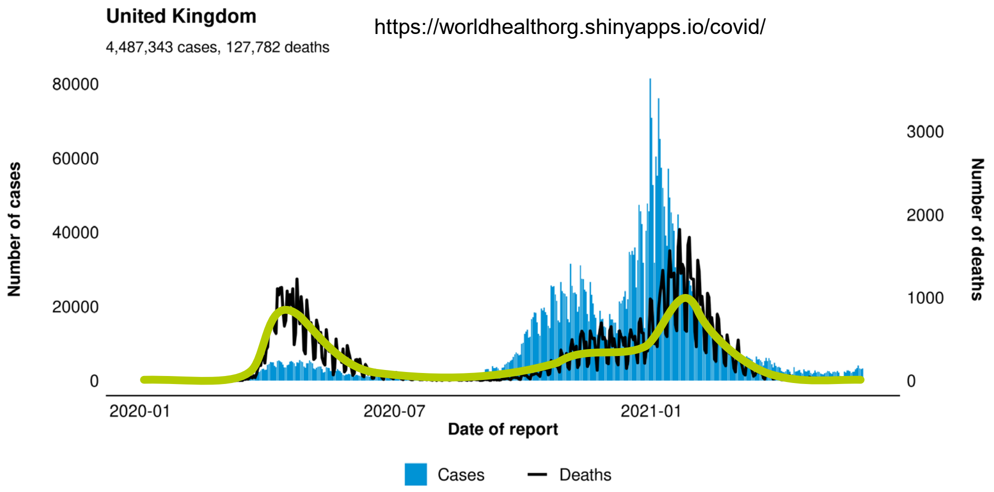
Models



Creating a simplified model for a NOTCOVID epidemic

Simplifying assumptions

- Even though real data is discrete (people are not fractions!) our model will regard the population of infected people as being a continuous variable. $P(t)$



- Assume the rate of new infections is proportional to the current population $\frac{\mathrm{d}P}{\mathrm{d}t} \sim bP$

- A fixed fraction of the infected population recovers every day $\frac{\mathrm{d}P}{\mathrm{d}t} \sim -cP$

- Recovered people can be reinfected (**NOT**COVID) - $P(t)$ is enough.

Hear about real COVID models in the taster lecture of Katie Severn from May 6 2021.

Creating a simplified model

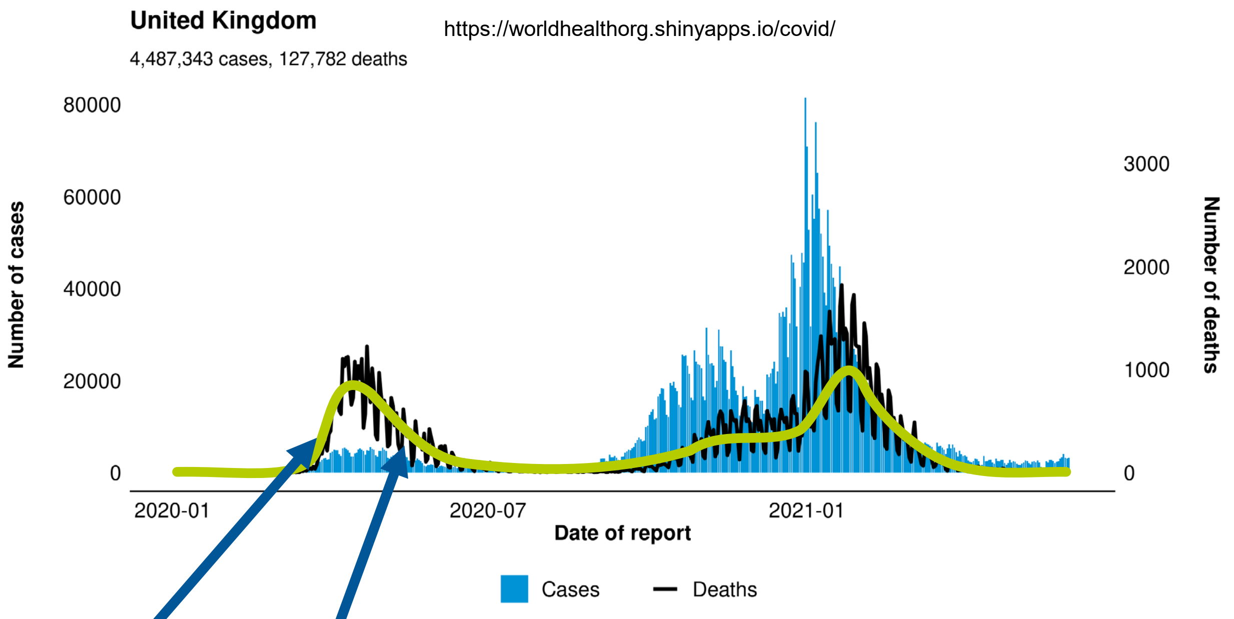
Simplified model

$$\frac{\mathrm{d}P}{\mathrm{d}t} = bP - cP$$
$$= (b-c)P$$
$$= kP \qquad k = b - c$$

Everything depends critically in the **sign** of k!

If $k > 0$ then the population $P(t)$ is **increasing**

If $k < 0$ then the population $P(t)$ is **decreasing**



An equation for the solution

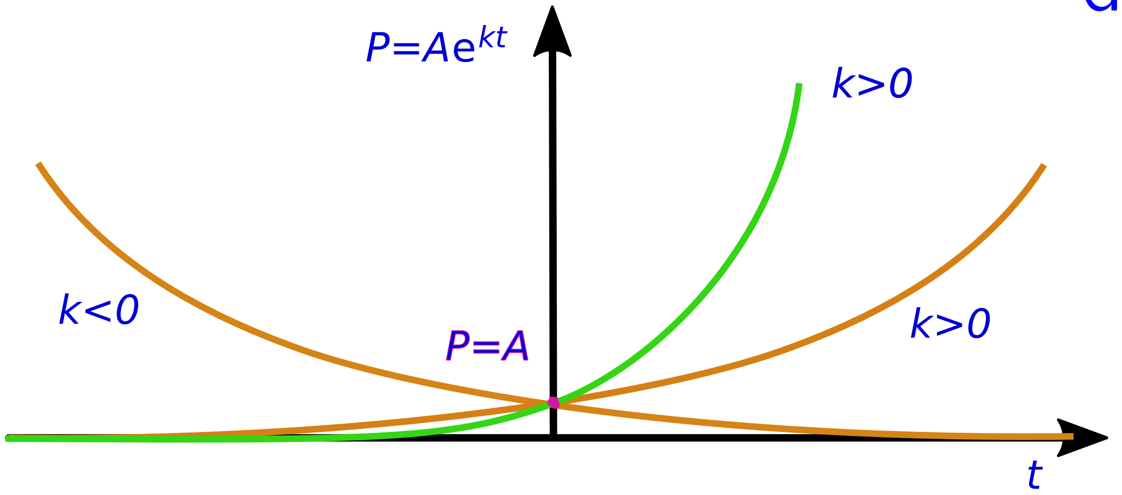
Suppose k is a constant.

What functions of t satisfy

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP \text{ ?}$$

This is a **differential equation**

We've seen this before! $P = \mathrm{e}^{kt} \Rightarrow \dfrac{\mathrm{d}P}{\mathrm{d}t} = kP$



Or more generally

$$P = A\mathrm{e}^{kt}$$

$$A = P(t = 0)$$

QUIZ: go to pingo.coactum.de/105308

The model and solution

$$\frac{dP}{dt} = kP \qquad P = Ae^{kt}$$

falls down because (choose all that apply on pingo.coactum.de/105308):

A. This solution can only grow, whereas sometimes P is seen to decrease.

B. If growth was exponential for all time, infected people would soon fill the solar system.

C. When P is large, there are fewer uninfected people to provide new hosts, so the assumption k = constant is unrealistic.

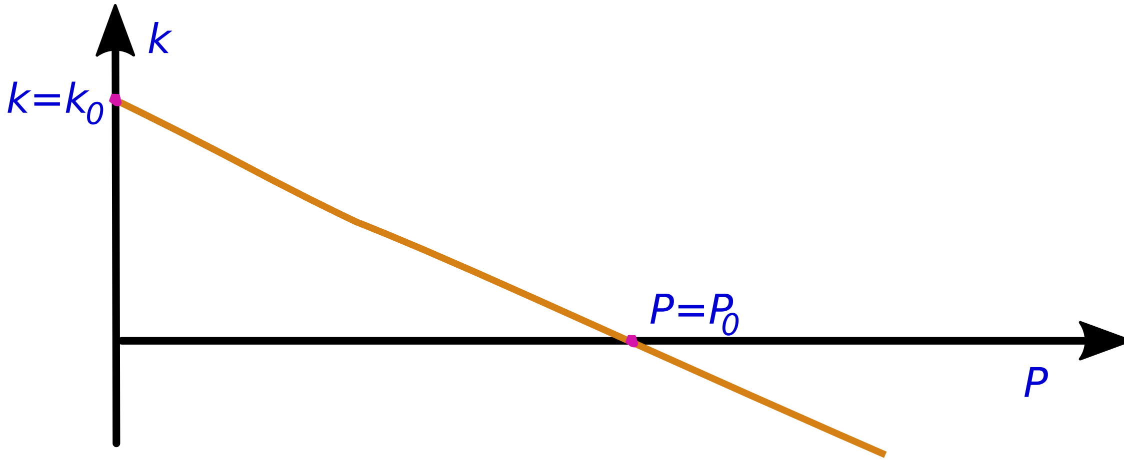
D. We don’t know what A is.

A more realistic model

In practice, as a greater proportion of people are infected there is less opportunity for the virus to spread

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP \quad \text{but} \quad k \neq \text{constant}$$

k should get **smaller**, as P gets bigger. For example $k = \dfrac{k_0}{P_0}(P_0 - P)$



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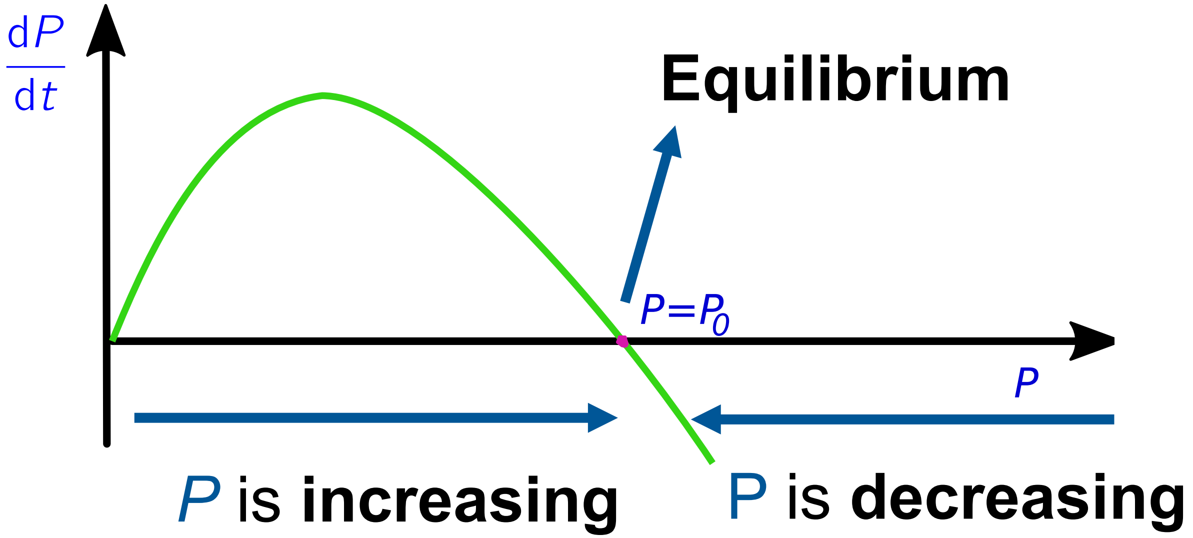


$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP = \frac{k_0}{P_0}(P_0 - P)P$$

QUIZ: go to pingo.coactum.de/705554

Let

$$\frac{dP}{dt} = 2P - P^2$$

and let the starting value of P be $P(0)=1$. Then (put your answer on pingo.coactum.de/705554):

A. P increases and approaches the value $P=2$.

B. P decreases and approaches the value $P=0$.

C. P stays the same forever – $P=1$ is an equilibrium.

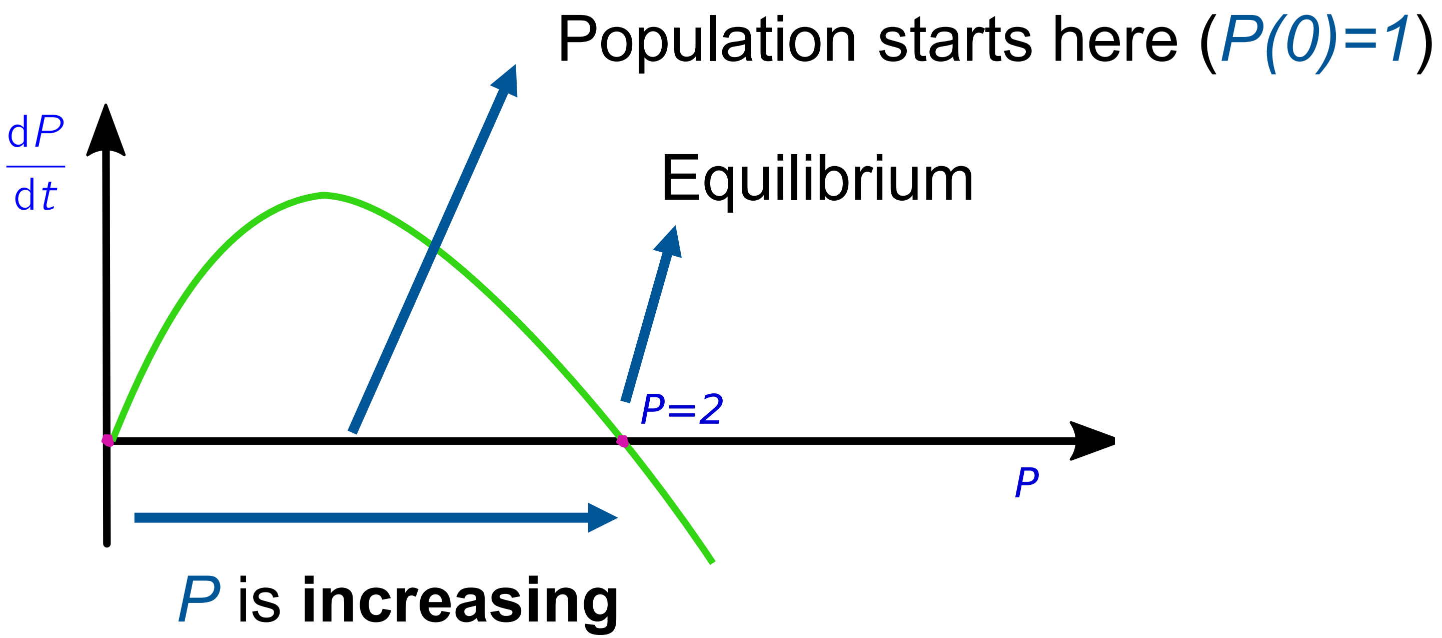
D. P keeps growing forever, and can get arbitrarily large.

A worked example

Let

$$\frac{dP}{dt} = 2P - P^2 = (2 - P)P$$

Equilibria $P = 0$ or $P = 2$



Conclusion: because the system starts in the interval where

$$\frac{dP}{dt} > 0$$

The infected population *increases.* It keeps increasing as it approaches the equilibrium value $P = 2$.

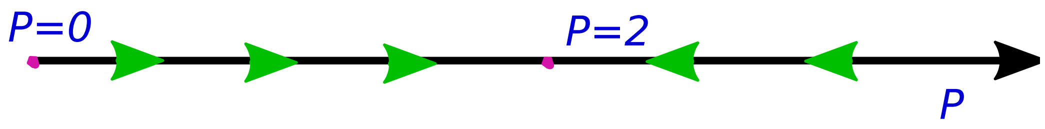
A worked example

Let

$$\frac{dP}{dt} = 2P - P^2 = (2 - P)P$$

Equilibria $P = 0$ or $P = 2$

Phase line:



Conclusion: because the system starts in the interval where

$$\frac{dP}{dt} > 0$$

The infected population *increases*. It keeps increasing as it approaches the equilibrium value $P = 2$.

Conclusion

- We don’t need complicated formulas to know what’s going on!
- Qualitative information (pictures, diagrams) can be very valuable in understanding the solutions of differential equations.
- Features such as equilibria tell us a lot about the global structure.

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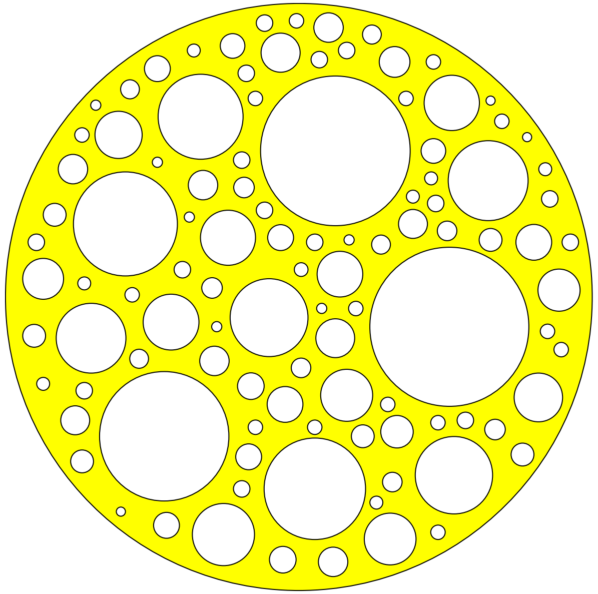
Maths Courses at the University of Nottingham

Joel Feinstein

School of Mathematical Sciences

About me – Joel Feinstein

- Associate Professor, Pure Mathematics
- Outreach Officer and Teaching Support Officer
- I teach the first-year module **Foundations of Pure Mathematics**
- You can find my teaching blog at https://explainingmaths.wordpress.com/
- My research includes work on Swiss cheeses!



School of Mathematical Sciences

- Department of Mathematics formed in 1919
- School of Mathematical Sciences formed in 1998
- Moved to current, purpose-built, home in 2011
- Situated in a lovely campus with great facilities
- Over 70 academic staff

Maths Courses at Nottingham

Single-Subject Degrees

- **Mathematics BSc** (3 years)
- **Mathematics MMath** (4 years)
- **Mathematics (International Study) BSc** (4 years)
- **Mathematics with a Year in Industry BSc** (4 years)
- **Mathematics with a Year in Industry MMath** (5 years)
- **Statistics BSc** (3 years)

Maths Courses at Nottingham

Joint Degrees

- **Financial Mathematics BSc** (3 years)
  - with Nottingham University Business School
- **Mathematics and Economics BSc** (3 years)
  - with School of Economics
- **Mathematical Physics BSc/MSci** (3/4 years)
  - coordinated by School of Physics & Astronomy
- **Natural Sciences BSc/MSci** (3/4 years)
  - coordinated across schools involved
  - available with a year abroad

Careers with Mathematics

The most popular employment sectors nationally for maths graduates are*:

- Business, HR and finance professionals (42%)

  e.g., Consultant, Actuarial Graduate, Analyst, Strategic Consultant, Accountant

- IT professionals (12%)

  e.g., Software Engineer, Data Analyst, Cyber Security Associate, Technology Analyst

- Education professionals (9%)

  e.g., Teacher of Mathematics, Teaching Assistant

*Source: *What do graduates do?* (HECSU 2018)

Top four employers for our graduates:

- Deloitte
- PwC
- Ernst & Young
- KPMG

Some useful links

Links and QR codes for the University of Nottingham, School of Mathematical Sciences and for more details about our maths courses

https://tinyurl.com/mathsuon

https://tinyurl.com/mathscourseuon

Complete sets of videos for the first-year module **Foundations of Pure Mathematics**:

https://tinyurl.com/uonfpm

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Any questions?

Please give us feedback on this session using the link in the Q&A chat!

To see slides and videos from our taster sessions so far, or to sign up to our mailing list, visit https://tinyurl.com/uonmathstaster