



**University of
Nottingham**

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Problem Solving Taster Lecture

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The mathematician's main reason for existence is to solve problems, [...], what mathematics really consists of is problems and solutions.

Paul Halmos (*The Heart of Mathematics*, 1980, p. 519).

Mathematics largely consists of *questions*. Questions are tasks that mathematicians carry out and, at some later stage, submit an account of their answer.

Questions can be categorised into two types:

- A mathematical *exercise* is a question whose solution involves only routine procedures. Having learnt the relevant techniques, we can systematically apply them to reach a correct solution.
- A mathematical *problem* is a question whose process for answering it is unclear.

Important!

Whether a question is an exercise or a problem depends on who is answering it!

Calculate

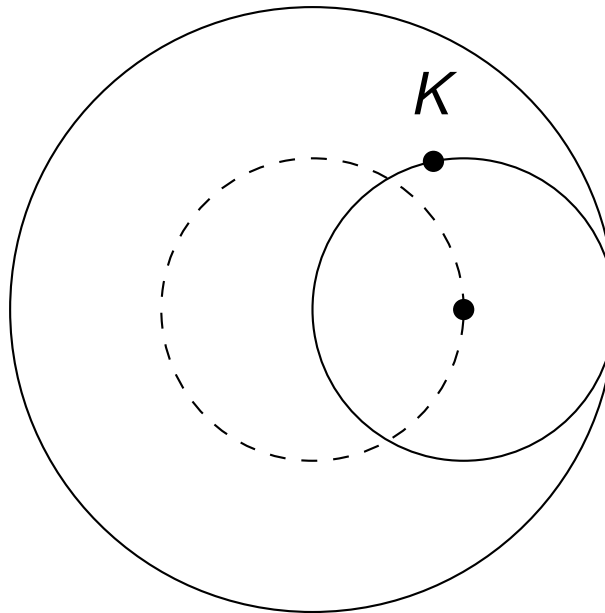
$$\int_{-\pi}^{\pi} \sin(2x) \, dx.$$

Question: Is this an exercise or a problem?

Answer: Go to <https://pingo.coactum.de> then enter the code 786850

Example 2

Let K be a point on a circle sitting inside and touching a stationary circle of twice its diameter. Describe the path of K as the smaller circle rolls around the larger one without sliding.



Question: Is this an exercise or a problem?

Answer: Go to <https://pingo.coactum.de> then enter the code 786850

- The first part of solving a problem is translating the problem into mathematical language.
- By doing so, we gain a concrete and detailed understanding of the problem in mathematical terms.
- Having done this, we can find and apply appropriate mathematical techniques to attack the problem.
- At this stage, we should think about the form of the solution.
 - Numerical answer? If so, a single number or a range of values?
 - Governing equation?
 - Mathematical argument/proof?
 - Diagram/graph?
 - Written conclusion?

Writing down “the answer” to a problem is not enough. You need to also provide clear and thorough justification of your solution, including the steps you took to arrive at it.

That said, I’m going to ask you to provide some answers to problems for the remainder of this lecture. You should, however, take some notes on how you got to your answers for the benefit of others (and your future self).

Write an equation to express the statement

“There are six times as many students as professors at University of Nottingham.”

Use S for the number of students and P for the number of professors.

Answer: Go to <https://pingo.coactum.de> then enter the code 786850

Some problems are easy to state and understand, but are surprisingly difficult to solve.

Goldbach's Conjecture

Every even integer greater than 2 can be expressed as the sum of two primes.

The statement of this conjecture is easy enough to understand, but its proof has remained an unsolved problem for over 250 years!

- Where do we begin?
- What constitutes a solution?
- Which routine mathematical techniques could we use?

The mathematician and educationist George Pólya suggests the following process for solving problems:

- 1 Understand the problem.
- 2 Make a plan.
- 3 Carry out the plan.
- 4 Looking back.

Problem solving is an iterative process that only terminates once the reviewed plan is confirmed to work.

The plan we make and carry out in steps 2 and 3 depend on the problem ... and also the type of mathematician we are.

Various approaches include (but are not limited to):

- Design and develop a mathematical (or statistical) model.
 - Mathematical modelling is (at least) an entire lecture course in its own right!
- Use of existing mathematical theorems (or devise and prove one or two theorems of our own!)
- Design and develop a computer algorithm using an appropriate programming language (e.g. Python, R, C++).
- Collect and analyse relevant data.
- A combination of the above (almost always necessary for complex problems).

Now it's your turn to try solving a problem. This one dates back to around 775AD!

A dog starts in pursuit of a hare at a distance of thirty of its own leaps from the hare. If the dog covers as much ground in two leaps as the hare does in three, in how many of the dog's leaps will the hare be caught?

Answer: Go to <https://pingo.coactum.de> then enter the code 786850

Hint: What assumption(s), if any, do we make?

Tom and Charlotte are out for a walk with their dog, Lara. At a particular moment, Tom and Charlotte are 50 m apart and are walking at 5 km h^{-1} directly towards each other.

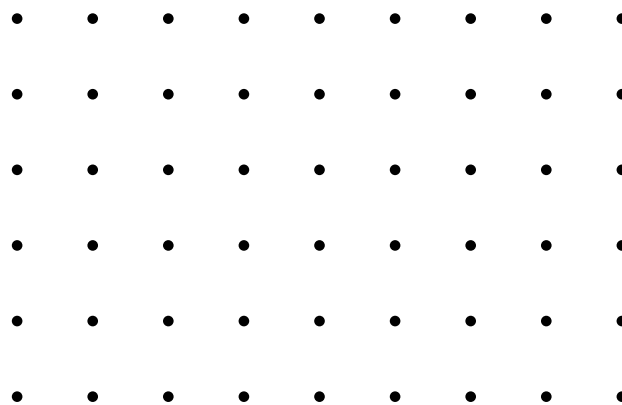
At that moment, Lara sets off from Tom at 10 km h^{-1} directly towards Charlotte. If Lara instantaneously reverses her direction every time she reaches Charlotte or Tom, how far will she have travelled (in metres) before they meet?

Answer: Go to <https://pingo.coactum.de> then enter the code 786850

Hint: What assumption(s), if any, do we make?

Here is a final problem for you to take away with you.

Below is part of an infinite integer lattice. A *lattice triangle* is a triangle where the coordinates of all vertices are integers (i.e. all vertices are located at lattice points).



What is the size of the smallest **equilateral** lattice triangle?

Big Hint: There are no equilateral lattice triangles! To solve this problem, we must prove this.

- Problem solving is central to all mathematical pursuits.
- A mathematical **exercise** is a question that can be solved routinely.
- A mathematical **problem** is a question whose process for answering it is unclear.
- Gain a good understanding of a problem before attempting to solve it.
- Follow Pólya's iterative process to solve the problem and check the solution.
- Many problems are surprising as we develop solutions. Often, the “solution” is to prove that there are no solutions!



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Maths Courses at the University of Nottingham

Joel Feinstein

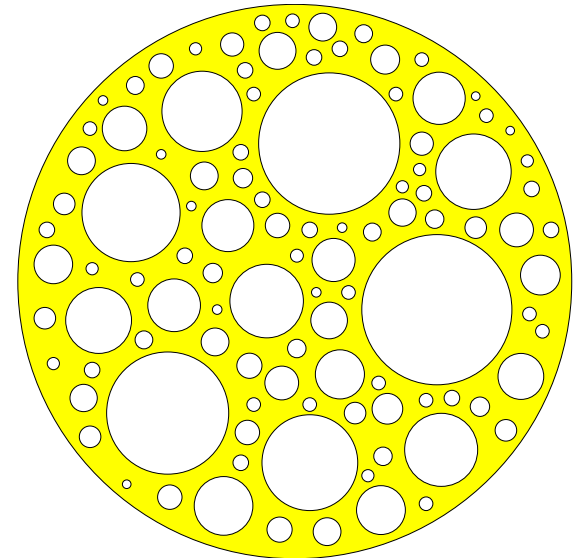
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About me – Joel Feinstein

- Associate Professor, Pure Mathematics
- Outreach Officer
- Teaching Support Officer
- I teach the first-year module **Foundations of Pure Mathematics**
- My research includes work on Swiss cheeses!





School of Mathematical Sciences

- Department of Mathematics formed in 1919
- School of Mathematical Sciences formed in 1998
- Moved to current, purpose-built, home in 2011
- Situated in a lovely campus with great facilities
- Over 70 academic staff





Maths Courses at Nottingham

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- **Mathematics MMath (4 years)**
- **Mathematics (International Study) BSc (4 years)**
- **Mathematics with a Year in Industry BSc (4 years)**
- **Mathematics with a Year in Industry MMath (5 years)**
- **Statistics BSc (3 years)**



Maths Courses at Nottingham

Joint Degrees

- **Financial Mathematics BSc (3 years)**
 - with Nottingham University Business School
- **Mathematics and Economics BSc (3 years)**
 - with School of Economics
- **Mathematical Physics BSc/MSci (3/4 years)**
 - coordinated by School of Physics & Astronomy
- **Natural Sciences BSc/MSci (3/4 years)**
 - coordinated across schools involved
 - available with a year abroad



Careers with Mathematics

The most popular employment sectors nationally for maths graduates are*:

- Business, HR and finance professionals (42%)
e.g., Consultant, Actuarial Graduate, Analyst, Strategic Consultant, Accountant
- IT professionals (12%)
e.g., Software Engineer, Data Analyst, Cyber Security Associate, Technology Analyst
- Education professionals (9%)
e.g., Teacher of Mathematics, Teaching Assistant

*Source: *What do graduates do?* (HECSU 2018)

Top four employers for our graduates:

- Deloitte
- PwC
- Ernst & Young
- KPMG



Some useful links

University of Nottingham, School of Mathematical Sciences and our maths courses:

<https://tinyurl.com/mathsuon>



<https://tinyurl.com/mathscourseuon>



Complete sets of videos for the first-year module **Foundations of Pure Mathematics**:

<https://tinyurl.com/uonfpm>



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Any questions?

Please give us feedback on this session using the link in the Q&A chat

Future Maths Taster Sessions: <https://tinyurl.com/uonmathstaster>