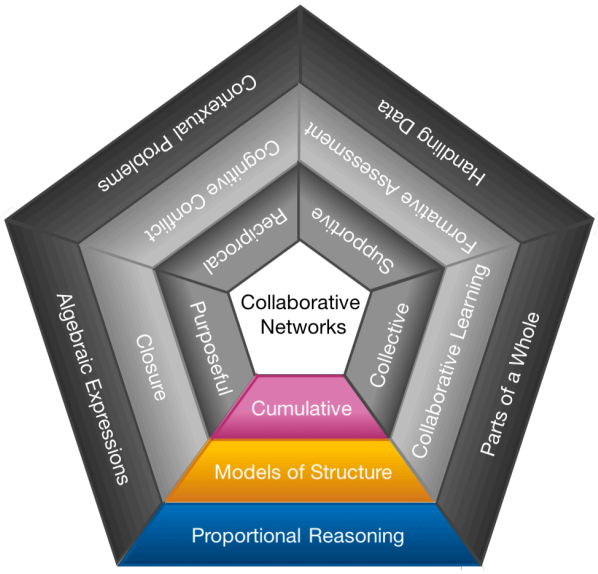
Overview

The focus in the lesson, **Proportional Reasoning**, is on how **models of structure** can support **cumulative** dialogue in the classroom.



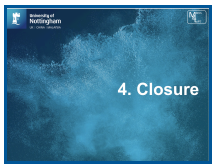
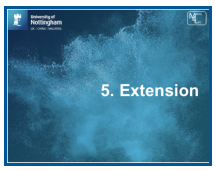
Models of Structure are representations that provide insight into mathematical structure.

Cumulative dialogue is seen when both students and teachers build on each other's contributions to create chains of thinking.

Research Question

How do models of structure help to facilitate cumulative dialogue and insight into mathematical structure?

Lesson Summary

Phase	Approximate Timings (minutes)	Notes
	**10**	The initial problem is explained. *Ensure students have enough time to consider the strategy that could be used to find the cost of using 4.54 litres.*
	10	Review methods. *Review strategies used by the class and then discuss the methods shown in the PowerPoint presentation.*
	**20 - 30**	Students work with the problem cards. *Support group work – this may include asking students to explain their thinking on a double number line.* *An extension question is available in the PowerPoint presentation.*
	**15 - 20**	Check understanding using the review slides in the PowerPoint presentation. *Encourage students to share alternative ways of thinking and illustrate them on double number lines.*
	**10 - 15**	Students work on the final problem. *Students work out which decorator charges the least.* *Ensure clarity of what has been covered in this lesson is shared.*
		Extension questions used if appropriate.

L2 Lesson Outline: Proportional Reasoning

Mathematical goals

To help students:

- reflect on the reasoning they currently use when solving proportion problems;
- understand the power and efficiency of a multiplicative structure through the concept of rate.

Starting points

Proportional reasoning is notoriously difficult for many students. There is a tendency to use additive thinking based around doubling and halving. This session aims to expose and build on this prior understanding.

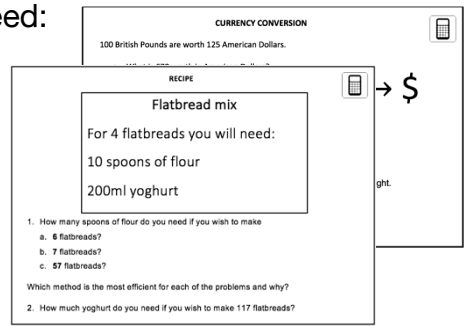
Students are given problems to work on in small groups aided by careful questioning from the teacher. This leads to discussions that compare the use of thinking that requires adding, doubling and halving with the use of more sophisticated thinking that use multiplication.

Materials required

For each group of students, you will need:

- mini-whiteboards;
- L2.2 – Cards (cut individually);
- a calculator.

L2.3 PowerPoint Presentation.



Time needed

Approximately 1- 1½ hours.

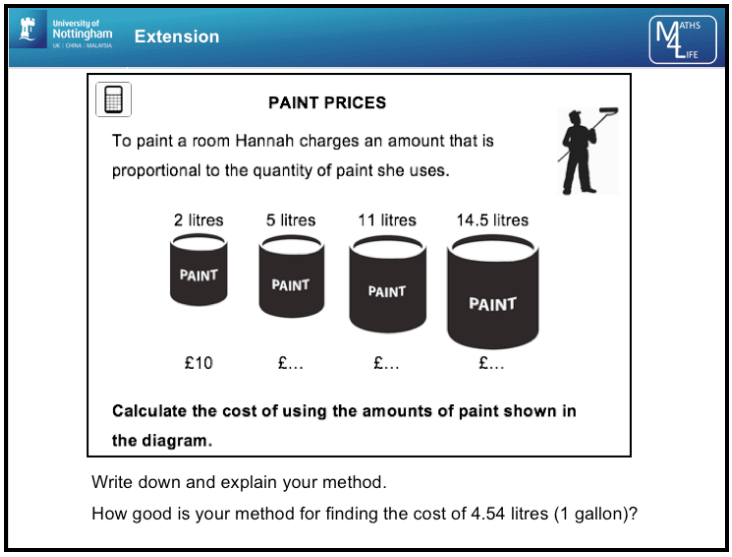
Lesson structure

Setting the Scene

Project and explain the Paint Prices problem from the PowerPoint presentation. Give students a short time to consider how they would tackle this problem on their own and then ask them to work in pairs on mini-whiteboards to try and solve the problem.

"The double number can show different ways of mathematical thinking"

1. What strategies do you expect students to use?



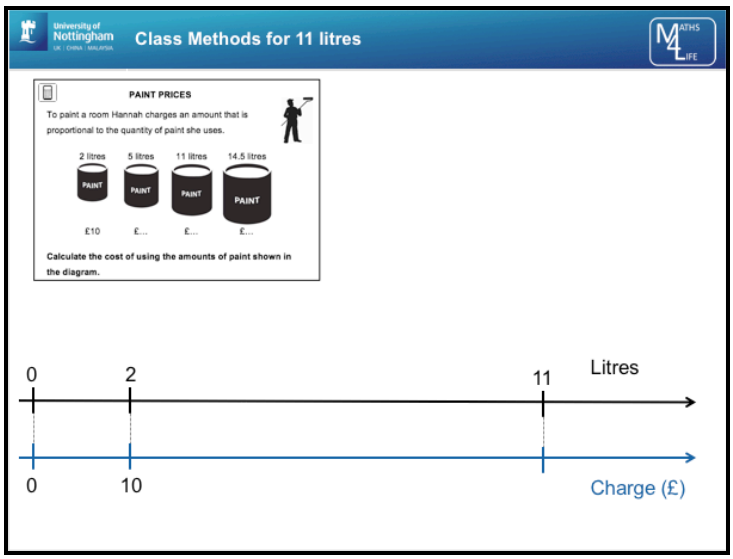
Note that the extension question about explaining their method and thinking about 4.54 litres should only be shown after students have attempted the main problem.

"Encourage discussion – rather than try to fix"

Bring the class together and ask students to explain their methods for finding the charge for 11 litres.

"Use the double number line to illustrate student thinking"

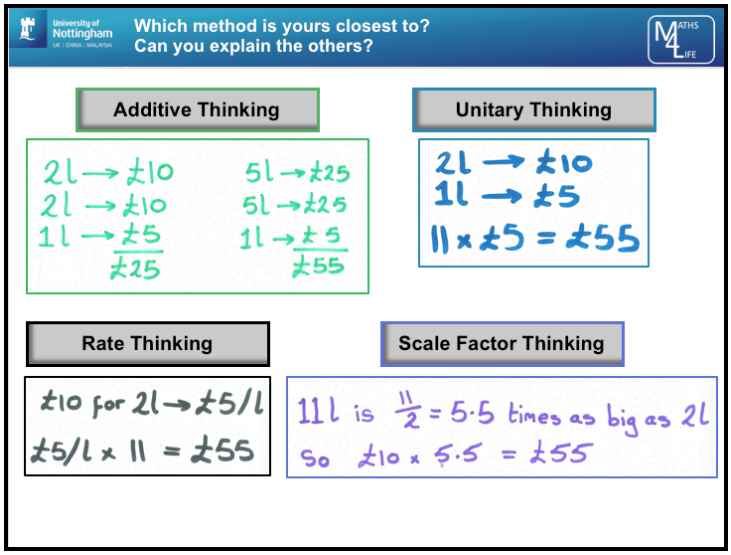
2. Why has 11 litres been chosen to demonstrate students thinking rather than 5 litres or 14.5 litres?



It is vital that the teacher captures their different methods and understanding on the double number line. This is about valuing and making sense of different ways of thinking – not about imposing a method.

Project the next slide and ask students to identify which ways of thinking their work most closely resembles and to try to make sense of other methods.

3. Is it essential that every way of thinking is gone through with students?

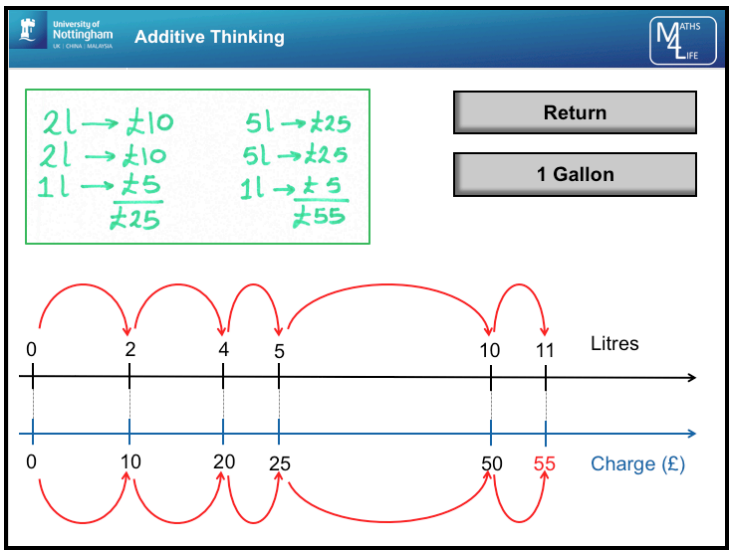


Note that the emphasis is not on the answer, but on making sense of the four different ways of thinking.

Click on each of the buttons in turn to illustrate the thinking of the student on a double number line. The four ways of thinking are illustrated on the following four slides.

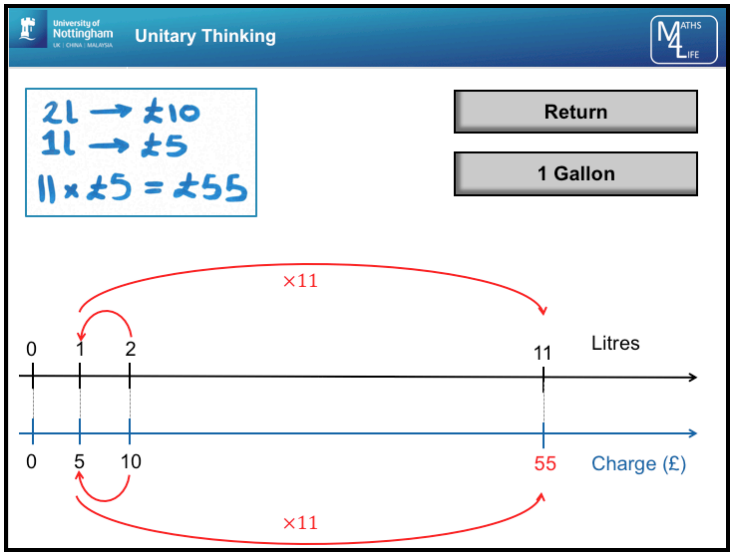
The first slide shows an additive method.

4. What are the features of the thinking that should be brought out to students?



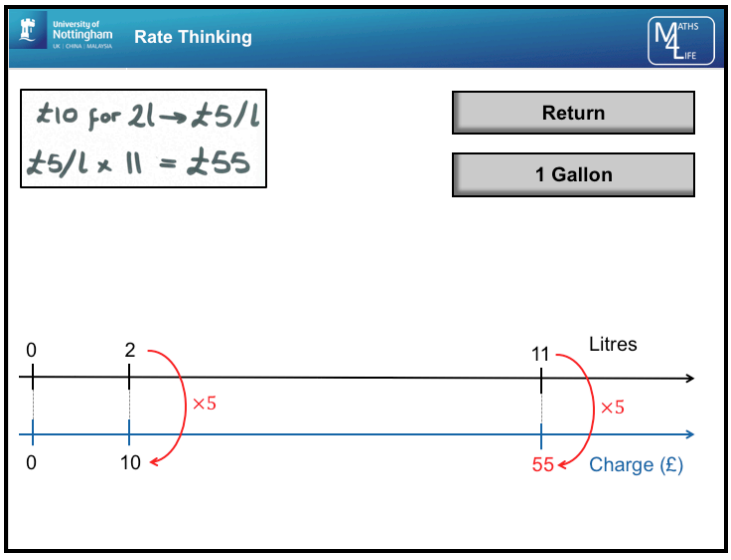
The second slide shows the unitary method - based on finding the cost of 1 litre, then being able to multiply to get the cost of any amount.

5. What are the features of the thinking that should be brought out to students?



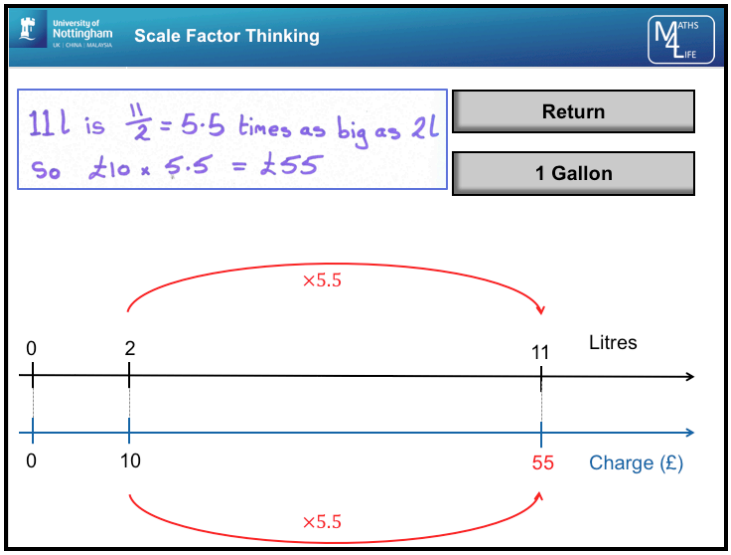
The third slide shows the ratio method. This is where the amount is multiplied by the rate of £5/l - a relationship that is true for all corresponding points on the number lines.

6. What are the features of the thinking that should be brought out to students?



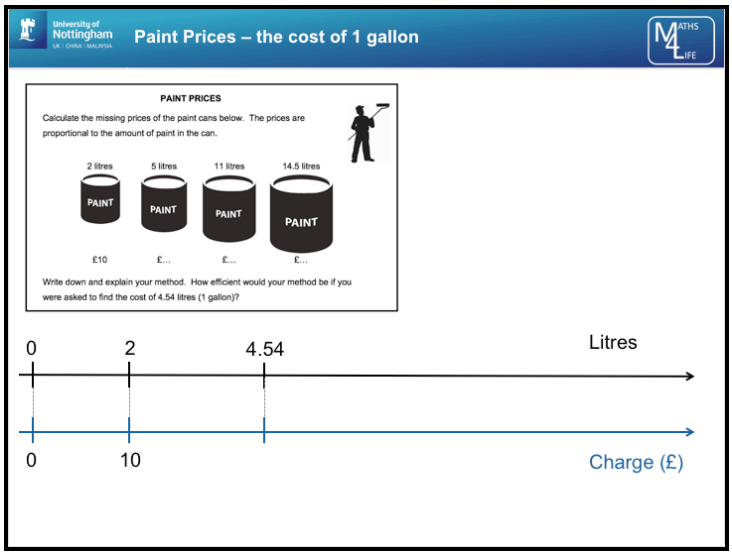
The fourth slide shows the idea of a scale factor – that the 11 litre can is 5.5 times as large, so it should cost 5.5 times as much.

7. What are the features of the thinking that should be brought out to students?



Now look at the next slide and ask students to work in pairs to find the cost of 1 gallon using as many of the four ways of thinking as possible.

8. Which of the ways of thinking are most helpful in this example?



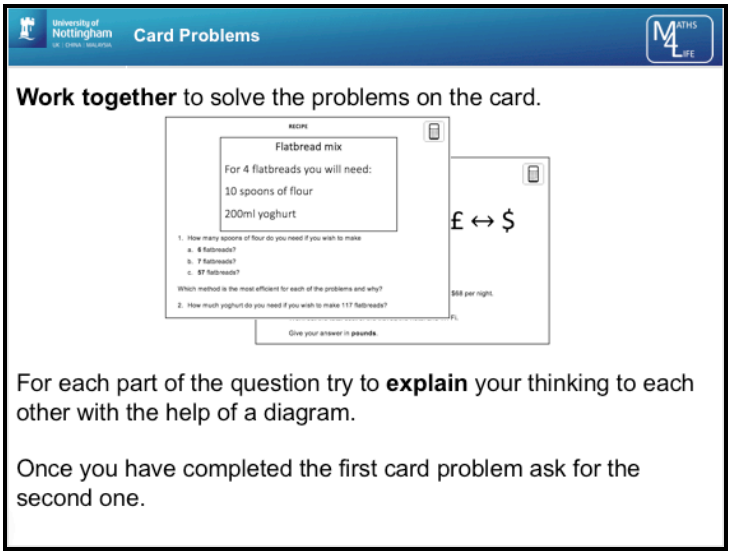
Ask students to identify which ways of thinking were most helpful for this question. This emphasises that methods (such as additive) are fine and can be used to obtain the correct answer. However, other thinking (such as unitary and rate) can be more efficient in some problems.

9. Why is it important to set out expectations for how students will attempt the questions?

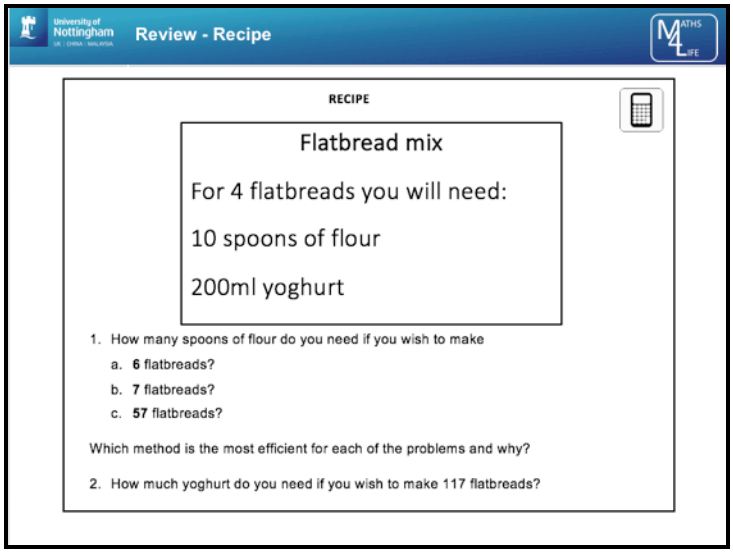
Collaborative Learning using Cards

Give out copies of the second problem (Flatbread Mix) to pairs of students and ask them to work together to solve. Remind students of the need to work together to agree a solution and a method.

"Predict what students might do and how to make the most of their thinking"



The following two slides show the questions that the students will be asked to complete. Encourage students to explain their thinking to you on a double number line and encourage them to consider alternative ways of thinking.



Whilst students are working pay particular attention to any thinking that will be helpful to share in the Closure section of the lesson.

Note that an extension questions can be projected if required.

Review

Once all groups have attempted the questions draw them together to summarise the learning.

First clarify the concept of these kinds of proportional reasoning problems.

The problems we have looked at in this session have involved two quantities, help me list them.

Quantity of paint	*Cost of paint*
Number of flatbreads	*Spoons of flour*
Number of flatbreads	*Quantity of yoghurt*
Pounds	*Dollars*

These are proportional situations. If we double the first quantity, we double the second. If there were none of the first quantity, there would be none of the second quantity.

Now capture the ways of thinking for each of the problems of various groups on the double number lines in the electronic presentation. Once again, it is important to make sense and capture students' ways of thinking – not to prescribe a best method.

Closure

Now ask students to look at the next slide on the PowerPoint presentation. This links back to the original problem and asks students to work out which decorator charges the least for 11 litres, and by how much.

Extension

The following PowerPoint slide can be used to help students make sense of the equivalence between multiplying by 0.5 and dividing by 2 using the double number line.