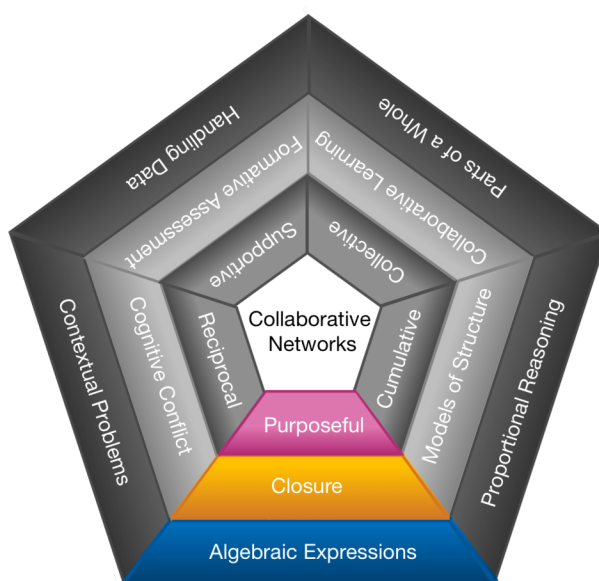


Lesson 3 Algebraic Expressions



Overview

The focus in the lesson, **Algebraic Expressions**, is on how **closure** can help to develop **purposeful** dialogue in the classroom.



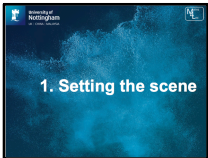

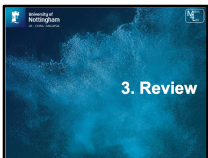
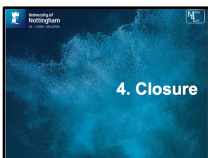

Closure is when part of a lesson is drawn together to ensure a shared understanding.

Purposeful dialogue is when talk is structured with specific learning goals in view.

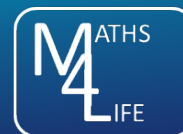
Research Question

How does **purposeful** dialogue contribute to student understanding during the **closure** phase of the lesson?

Lesson Summary

Phase	Timings (minutes)	Notes
 <p>1. Setting the scene</p>	5 - 10	<p>The initial problem is explained and students work through the stages.</p> <p><i>Key messages include;</i></p> <ol style="list-style-type: none"> 1. that n represents a variable. 2. the convention of drawing a fixed length looking rectangle, but labelling it with an n indicates that it is a variable.
 <p>2. Cards</p>	10 - 15	<p>Students match the factorised expressions and expanded expressions to the area representations on the grid.</p> <p><i>It is important that students are encouraged to use the speaking frame.</i></p> <p><i>An extension is available in the PowerPoint presentation that will help with factorising double brackets</i></p>
 <p>3. Review</p>	10 - 15	<p>Check understanding of the activity using the review slides in the electronic presentation.</p>
 <p>4. Closure</p>	10 - 15	<p>Check understanding of multiplying out double brackets using the PowerPoint slides.</p>
 <p>5. Extension</p>	5 - 20	<p>Extension questions used if appropriate.</p> <p><i>Additional time may be spent on:</i></p> <p><i>expanding single brackets,</i></p> <p><i>factorising single brackets,</i></p> <p><i>double brackets,</i></p> <p><i>simplifying expressions.</i></p>

L3 Lesson Outline: Algebraic Expressions



Mathematical goals

To help students:

- understand that n can represent a variable;
- understand multiplicative algebraic structure using an area representation;
- create algebraic expressions from area representations.

Starting points

Students often see letters in algebra as representing specific unknowns that must be found. Hence an area of $8n$ (for a rectangle with variable width and length 8) is a difficult concept to make sense of. This lesson begins by establishing that n represents a variable.

Students then extend to making sense of factorising and expanding brackets using an area representation. Previously many students will only have learned this in a procedural way.

Materials required

For each group of students, you will need:

- L3.2 Cards;
- L3.3 Template (copied on to A3 paper);
- glue sticks.

L3.5 Presentation;
mini-whiteboards and pens.

Time needed

Approximately 1 – 1.5 hours.

Factored Expression	Representation	Multipled Out Expression
$n(n+6)$		
		$2n^2 + 4n$



Lesson structure

Setting the scene

Introduce the initial task to students using the PowerPoint slide provided.

1. What diagrams (and dimensions) might students draw?


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M4 MATHS LIFE

A new trainee is about to start work at Floors-R-Us selling carpets.

A roll of red carpet is 3 metres wide, but the customer may want any length

Draw a diagram (with dimensions marked on) to help the manager show the trainee how to find the area.



“The opening task has been carefully crafted”


Note that area is being found to help calculate the cost of the carpet. To maintain task clarity, this cost implication has not been included in the problem.

Using the animated slide draw student’s attention to the fact that n is used to indicate a variable length.

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Possible Answer

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n indicates the length of carpet that the customer wants to buy.




Area = $3n$

Lengths are measured in metres.
Area is measured in metres squared.


The convention, in this lesson, is that we draw a representation of fixed looking length but the n indicates that it could vary.

Ask students to work in pairs on a shared mini-whiteboard for the next task.



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Floors-R-Us Extension




The manager usually recommends that an extra 2 metres (to allow for mistakes in carpet fitting) is bought **in addition** to whatever amount the customer needs.

The manager says that this is the same as doing the calculation $(n + 2) \times 3$


The assistant manager disagrees and says the calculation should be $6 + 3n$

Draw a diagram to help explain how each of them is thinking.




Again, use the animated slide to illustrate that n varies but our convention is to draw a static looking diagram.

2. What difficulties might students have with this question?




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Possible Answer



n indicates the length of carpet that the customer wants to buy.



Area = $(n + 2) \times 3$ or $6 + 3n$

Lengths are measured in metres.
Area is measured in metres squared.

Help students to see that the manager is describing a calculation involving multiplying two lengths whilst the assistant manager is describing a calculation involving adding two areas. At this point there is no need to show students the algebraic equivalence of the two expressions.

In pairs ask students to note down what they can understand from the next diagram.

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M4
L1

Explain this diagram that the manager drew for the trainee to illustrate finding the area of yellow carpet.

Write down how you think the assistant manager might carry out the calculation.

Discussion could include the following points:

- the yellow carpet has width of 4 metres.
- the manager encourages customers to buy an additional 3 metres.
- the manager would describe this calculation as $(n + 3) \times 4$
- the assistant manager would describe the calculation as $12 + 4n$

Use the next to establish the conventions that will be used during the rest of this lesson.

3. Why is it important to share this slide with students?

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From now on...

M4
L1

1. ... diagrams are drawn as fixed lengths, but a length of n indicates a variable length.
2. ... all lengths are given in metres.
3. ... all areas are given in m^2 .
4. ... you will need to find the values of question marks.

Collaborative Learning using Cards

Arrange students into pairs and give each pair both the template and the matching cards.

Present the task to students using the following slide from the electronic presentation.

4. What is the purpose of giving the students a speaking frame?

5. Where do you expect difficulties in this task?

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Cards

Take turns to place a card and write on the values of the question marks for the matching diagram.

Factorised Expression	Representation	Expanded Expression
	$\begin{array}{ c c c } \hline n & + & 2 \\ \hline ? & & 6 \\ \hline \end{array}$	
	$\begin{array}{ c c c } \hline n & + & 2 \\ \hline ? & & 2n \\ \hline \end{array}$	

As you place the card use the words...**"I've placed this here because..."**

The reply can be one of...

- "I agree – that makes sense."
- "I disagree because...."
- "Can you explain that again?"

"Use the task to encourage discussion"

6. Ensure your questions check student understanding, not act to correct their understanding.

As students are working, encourage them to discuss and explain their thinking to each other. Check understanding by asking them to explain how each of the three items in the row relate to each other. In particular, make sure that students can explain how to go from the expanded expression to the factorised expression.

An extension task is provided on the following slide.

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Extension

	n	$+$	$?$
n	n^2		$?$
$+$			
$?$	$?$		12

Find possible values for the four question marks.


Can you find any other possible values for the four question marks?

Note that non-integer answers such as side lengths of 8 and 1.5 are also possible.

Review

Student confidence should be built by allowing them to see the correct matchings. Encourage students to explain how each of the columns relate to the other two columns. It is important that time is spent understanding the written answers to blanks (not shown in the PowerPoint presentation) to ensure that there is a common understanding within the class of how these answers relate to each of the other cards.

7. Which of these rows do you expect to emphasise with your class?

<div>  <div> <div>University of Nottingham</div> <div>Maths Education</div> </div> </div> <div>Matching Cards – answers part 2</div> <div> <div>M</div> <div>MATHS</div> <div>LIFE</div> </div>			
		<div> <div>3 + n</div> <div> <div>?</div> <div>?</div> <div>n²</div> </div> </div>	<div>n² + 3n</div> <div>EG</div>
	n(n + 6)		<div>n² + 6n</div> <div>EB</div>
	<div>2n(n + 2)</div> <div>EG</div>		2n ² + 4n
		<div> <div>n + ?</div> <div> <div>?</div> <div>?</div> <div>?</div> <div>6</div> </div> </div>	

Continue to stress to students that the factorised expressions represent the calculation of multiplying two lengths, whilst the multiplied-out expressions represent the addition of area values.

Closure

Students should work in pairs to explain the following two slides.

“Avoid closing a lesson by turning what has been covered into a procedure”

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Closure - Quadratic Expressions

Why do these two cards match together, and what should the expanded expression card that goes with them be?

Multiply out
 $(n + 5)(n + 3)$

$n^2 + 3n + 5n + 15$

$n^2 + 8n + 15$

$n + 3$

n	n^2	$3n$
$+ 5$	$5n$	15

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Closure - Exam Question

ABCD is a square.

Show that the total area of ABCD is $n^2 + 6n + 9$



8. What are the key messages from the lesson that you expect to draw out?

Extension

Depending on the task selected (see slide below), students may now have additional practice with expanding single brackets, factorising single brackets, working with double brackets or looking at an extension on combining expressions.



“Closure is more effective when the students shape the thinking”

“Look for discussion that contributes to the aims of the lesson”

 Where next? 		
	Questions Only	Correcting Student Work
Expanding Single Brackets	<input type="button" value="Go"/>	<input type="button" value="Go"/>
Factorising Single Brackets	<input type="button" value="Go"/>	<input type="button" value="Go"/>
Double Brackets	<input type="button" value="Go"/>	
Simplifying Expressions	<input type="button" value="Go"/>	

Expanding single brackets

Note that for the expanding single brackets section you can choose between asking the questions only or asking students to spot corrections/improvements.

 Expanding Single Brackets 	
1. Multiply out $4(+5)$	
2. Multiply out $n(n+5)$	
3. Draw a rectangle diagram to represent an area of $3n^2 + 12n$, remember to give the lengths of the sides.	
4. Expand and simplify $5(n+2) + 2(n+2)$	
Can you draw a rectangle diagram to illustrate question 4?	
<input type="button" value="Return"/>	

Expanding Single Brackets – what do you think?

1. Multiply out $4(n + 5)$ (1) $4(n + 5) = 4n + 20$
2. Multiply out $n(n + 5)$ (2) $n(n + 5) = n^2 + 5n$
3. Draw a rectangle diagram to represent an area of $3n^2 + 12n$, remember to give the lengths of the sides.

(3)

(4) $5(n+2) + 2(n+2)$
 $5n+10 + 2n+4$
4. Expand and simplify $5(n + 2) + 2(n + 2)$
 Can you draw a rectangle diagram to illustrate question 4? Return

Factorising Single Brackets

Note that in the factorising single brackets section you can choose between asking the questions only or asking students to spot corrections/improvements.

Factorising Single Brackets

1. Factorise $6n + 10$
2. Factorise $n^2 + 4n$
3. Factorise fully $10n^2 + 6n$
4. Work out the possible lengths of the sides of this rectangle given the area of $4n^2 + 8$.

?

$4n^2$

8

Return

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Factorising Single Brackets – What do you think?

1. Factorise $6n + 10$ (1) $6(n+10)$

2. Factorise $n^2 + 4n$ (2) $n^2 + 4n = 2(n+2n)$

3. Factorise fully $10n^2 + 6n$ (3) $10n^2 + 6n = n(10n + 6)$

4. Work out the possible lengths of the sides of this rectangle given the area of $4n^2 + 8$.

?

?

4n² 8

(4). 2 $\begin{array}{|c|c|} \hline 2n^2 + 4 & \\ \hline 4n^2 & 8 \\ \hline \end{array}$

Return

Double Brackets

Note that a hint (showing the representation) is available for factorising double brackets if required.

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Double Brackets

Multiply out:

- $(n + 4)(n + 3)$
- $(n + 2)(n + 6)$
- $(n + 2)^2$

Factorise:

- $n^2 + 7n + 10$
- $n^2 + 11n + 10$
- $n^2 + 6n + 8$

Hint

?

n + ?

n

+

?

n² ?

? 5

Answers

Simplifying expressions

The aim of the next set of slides is to help students to understand (with the help of a representation) that $2(n + 2) + 3(n + 2)$ can be simplified straight away to $5(n + 2)$ without the need for expanding and factorising.

9. What misconception does this explanation risk creating?

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Simplifying Expressions
MATHS
LIFE

Simplify $2(n + 2) + 3(n + 2)$?

2

n	+	2

+

3

n	+	2

Use the animation to help students see that the expression can be simplified without needing to multiply out, add like terms and then factorise.

In pairs, students should use the true/false slide and representations to consider when this way of simplifying makes sense.

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Simplifying Expressions
MATHS
LIFE

Can you explain whether the following are true or false with the help of an area representation?

1. $4(n + 3) + 5(n + 3) = 9(n + 3)$
2. $4(n + 2) + 5(n + 3) = 9(n + 2)(n + 3)$
3. $4(n + 3) + n(n + 3) = 4n(n + 3)$
4. $n(n + 3) + 5(n + 3) = (n + 5)(n + 3)$

Return