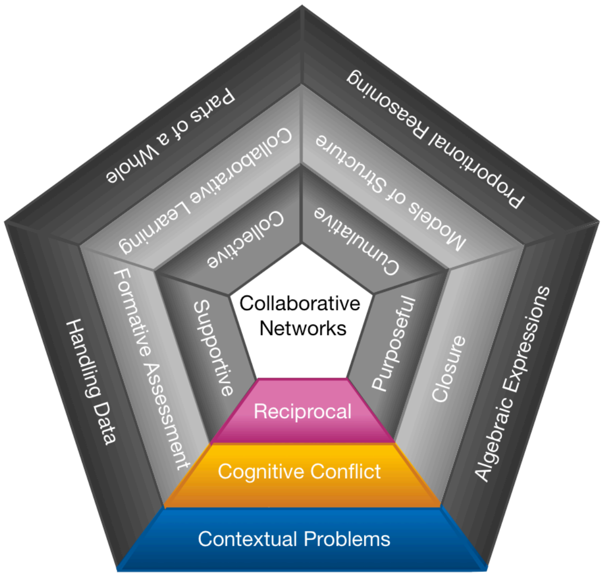
Overview

The focus in the lesson, **Contextual Problems**, is on how **cognitive conflict** contributes towards **reciprocal** dialogue.



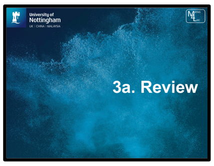
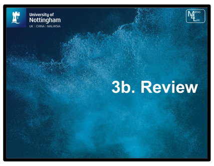
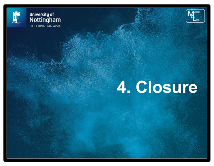
Dialogic learning should be **reciprocal** – that is students and teachers listen to each other, share ideas and consider alternative viewpoints.

Cognitive Conflict can occur at any time in the classroom when a learner is wrestling with uncertainty. It may occur when a predicted course of events is disrupted by a contradiction and existing methods and tools are found to be insufficient. It is in this struggling to resolve the conflict new learning takes place.

Research Question

How does the management of **cognitive conflict** help to develop a culture of **reciprocal** dialogic learning?

Lesson Summary

Phase	Timings (minutes)	Notes
	5	The initial problem is explained and worked on by students. *Highlight the link to ratios and the mathematical structure revealed by the diagrammatic representation.*
	10 - 15	Students work on questions 1 and 2. *These problems focus on two different types of relationships between the unknowns.*
	10	Check understanding of the activity using the review slides in the electronic presentation. .
	5 - 10	A further problem that may prompt cognitive conflict is explained and worked on by students.
	10 - 15	Students work on questions 3 and 4. *These problems focus on students establishing both additive and multiplicative relationships between the unknowns.*
	10	Check understanding of the activity using the review slides in the electronic presentation.
	10 - 15	Highlight the difference between ‘times more’ and ‘more’.
		Extension questions used if appropriate.

L4 Lesson Outline: Contextual Problems

Mathematical goals

To help students:

- determine the unknown in a problem;
- translate statements that relate to an unknown in a problem into a mathematical form;
- determine the value of an unknown in a problem.

Starting points

Many students have followed procedural methods for solving linear equations. However, they may have had little practice at setting up and solving such equations in a way that encourages a more deeper understanding. The contextual problems in this lesson aim to do just that.

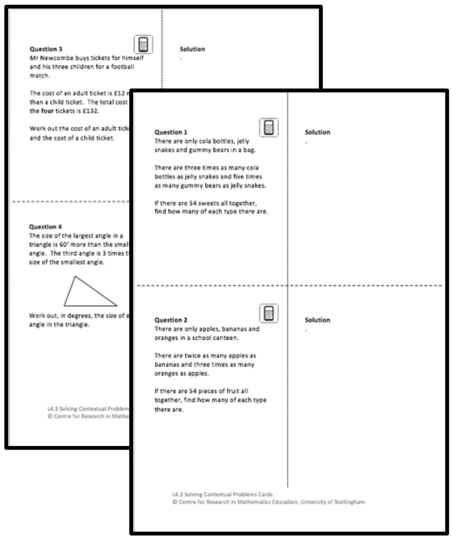
Some of these can be solved by trial and improvement. Whilst students can be successful using this method, it may not be efficient and may not be practical with ‘difficult’ numbers.

Materials required

- L4.3 Presentation;
- mini-whiteboards and pens;
- L4.4 Exam questions (for a follow-on lesson).

For each group of students, you will need:

- L4.2 Cards (Problems 1 and 2 together, problems 3 and 4 together);
- a calculator.



Time needed

Approximately 1 – 1.5 hours.

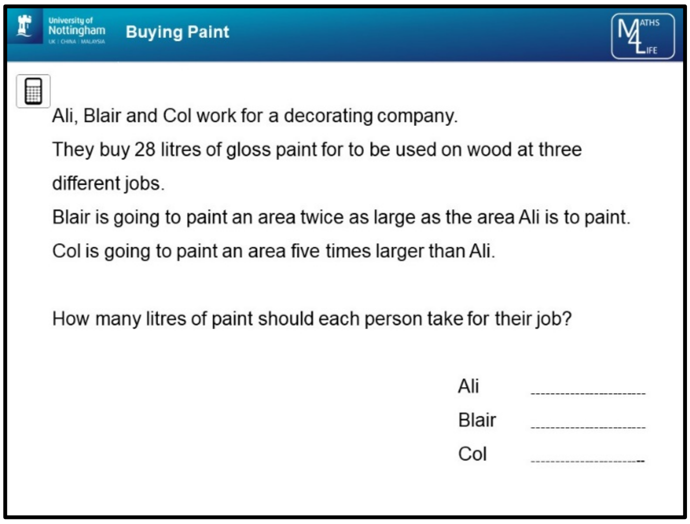
Lesson structure

1a. Setting the Scene

On mini-whiteboards ask students to attempt the first question on their own. If students are struggling to get started, suggest they use a trial and improvement strategy.

"The opening task sets the tone for the lesson and signals expectations of students"

1. What are the advantages of using trial and improvement, and when might it be problematic?

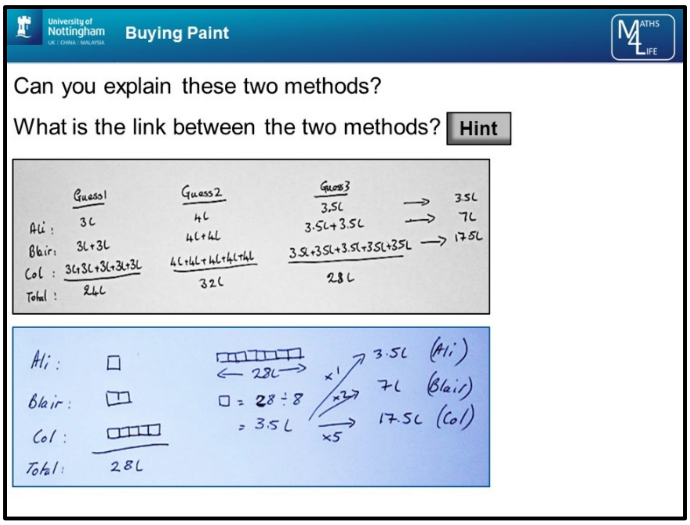


Ideally, ask a member of the class to explain their trial and improvement strategy and to identify which quantity they started with and why.

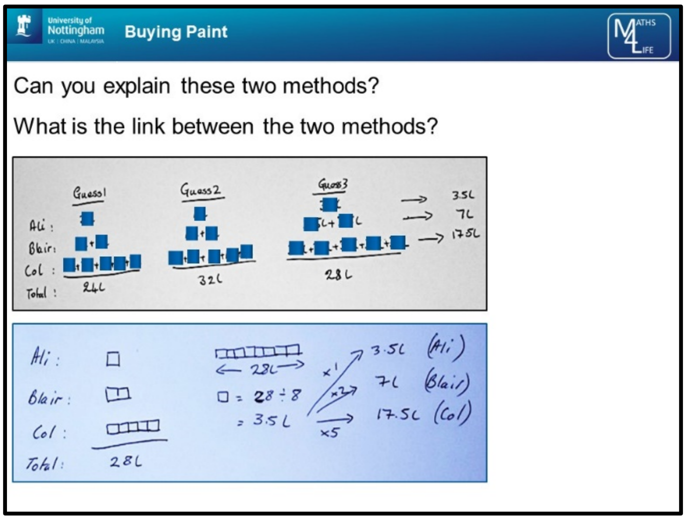
"Trial and error can help lead to understanding the algebraic structure"

Ask students to explain the following two methods, and ask whether they can identify the link between them. A hint is provided by clicking on the button.

2. What does a box in the blue method represent?



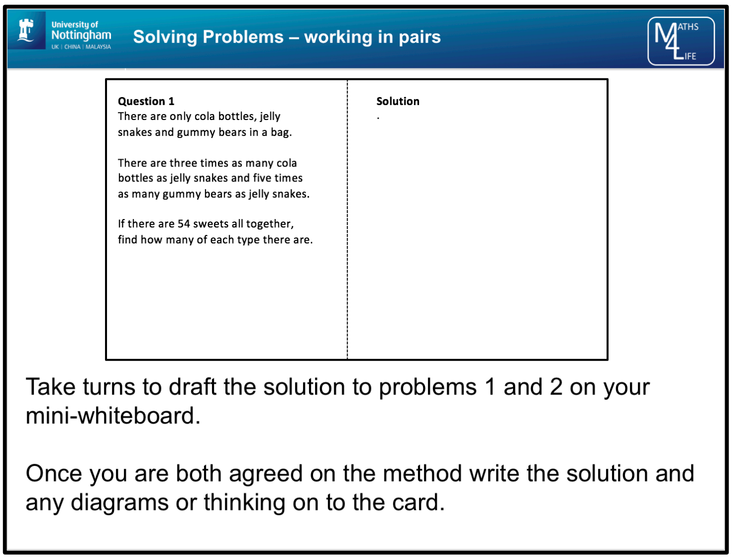
Note that the hint shows how the mathematical structure of the two methods is the same. In the first method, the value of one part is guessed until the total of 8 parts is 28 litres. In the second method, the total of 28 litres is divided by 8 to find the value of one (equal) part.



Also note the importance of identifying, in this question, that the quantity of paint for Ali. It provides the base value that everything else is compared to.

2a. Cards: Collaborative Learning

For each group/pair, hand out the card for Questions 1 and 2. Ask one member of the group/pair to work on the mini-whiteboard for Question 1, when they are agreed on the method and solution, the other person should transfer their thinking on to the card. They should then swap roles for Question 2.



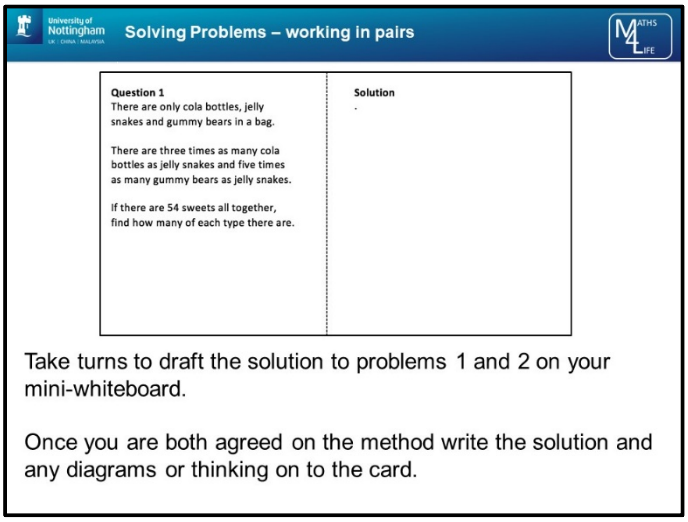
"Encourage students to share their own ideas and listen to their partner's ideas"

3a. Review

3. How do you expect students to tackle this question?

4. How can you encourage students to check whether their answer is sensible?

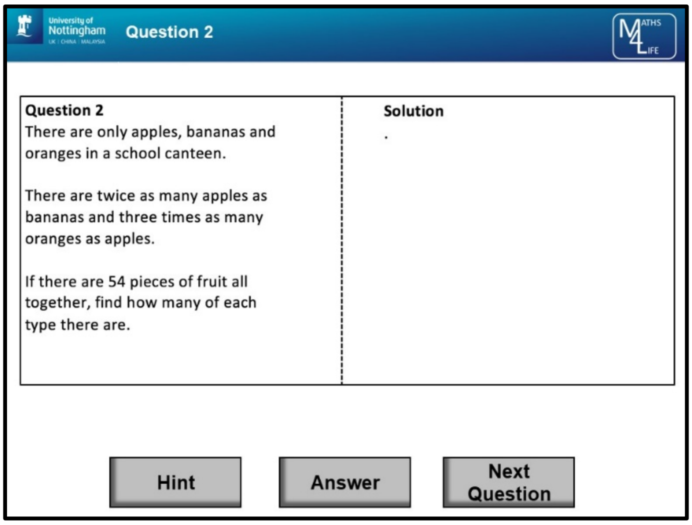
Bring the class together and go through each of the two questions in turn.



Note: Pressing the *Hint* button takes you to slide to support a discussion on a diagrammatic approach. You may want to ask students how the boxes shown on the slide could be used to represent the different types of sweets. Only use this slide if you think students are struggling to use a diagrammatic approach.

5. What do you expect students to find difficult in this question?

6. What is the best way to help them?

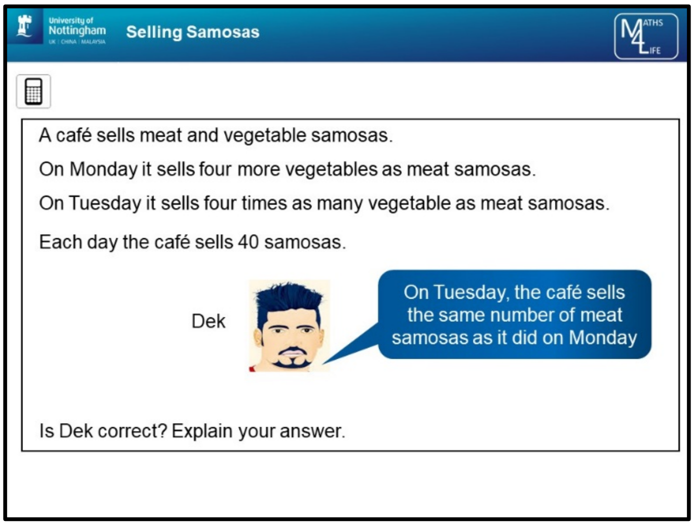


"Anticipate likely misconceptions and how you will respond to move students on"

It is important to notice that for both questions, the final stages of the solution involve the same mathematics, however, the answers to the two questions are different. This is because the relationships between the unknowns are different. Highlight the importance of checking that the final answers 'fit' the original problems. Depending on students method, checking the total may be insufficient. Students may need to also check relationships. If a group/pair completes these questions, ask them to start thinking about questions 3 and 4.

1b. Setting the Scene

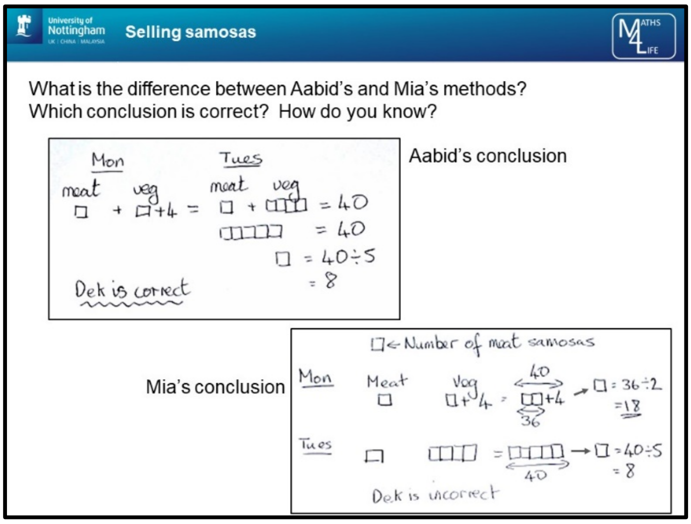
Now ask students to attempt a new problem.



"We mustn't jump in too early to try to fix problems'

Some students' conclusions may conflict with Dek's, others may not. Rather than resolve any issues, show the next slide. Ask students to identify which of the two methods shown on the slide are correct and any mistakes. You may want to also ask students to think about how they can check which is correct.

7. **What are the important differences in the two methods that should be highlighted to students?**



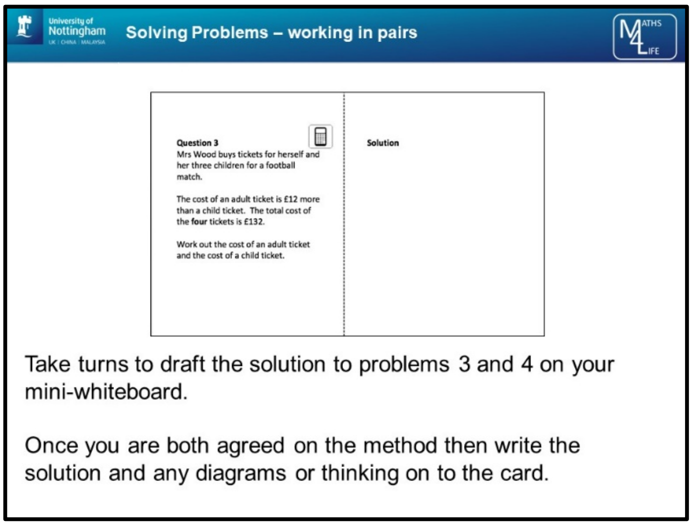
Draw attention to the difference between 'four times as many and 'four more'. The first means four lots of the same unknown, the second means one unknown plus four. Emphasise how not attending to such differences can result in very different answers.

Students may struggle with the notion that a box can represent different values. Point out that there are two different situations, and the box simply represents the unknown in the two situations.

2b. Cards: Collaborative Learning

For each pair, hand out the card for problems 3 and 4.

As before, ask person 1 to write on the mini-whiteboard for problem 3 and the other to write it on the card when they are both agreed. They should then swap roles for problem 4.

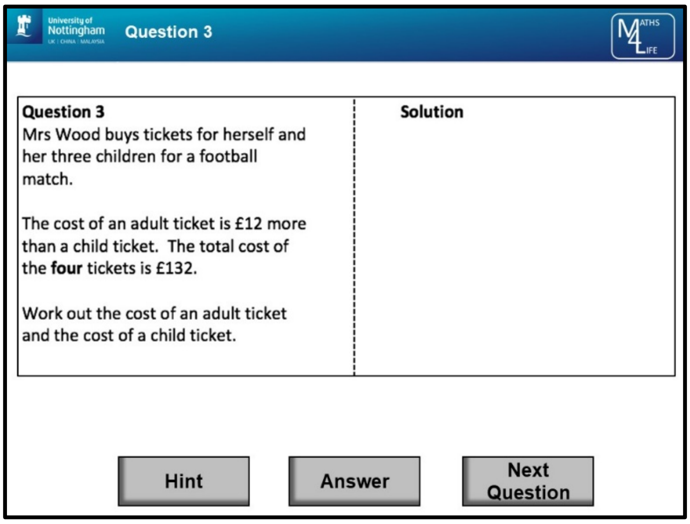


An extension question is provided in the presentation for any group that completes problems 3 and 4.

3b. Review

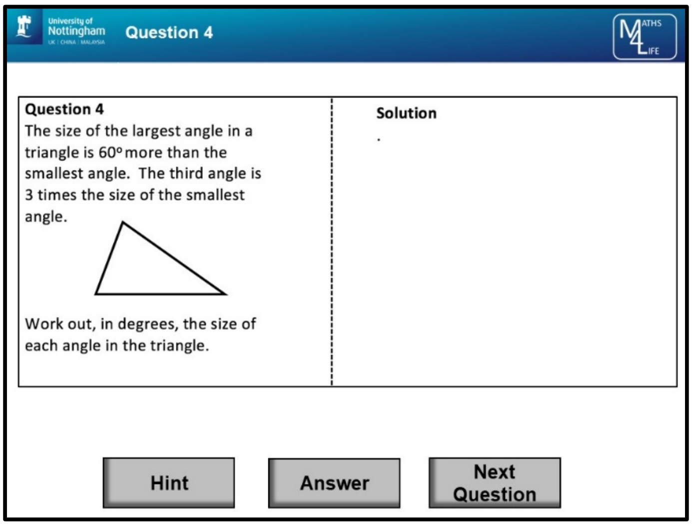
Bring the class together and go through each of the two problems in turn.

8. What do you expect students to find difficult in this question?



"We should encourage students to share their thinking with the whole class"

9. What do you expect students to find difficult in this question?



Once again, notice that the *Hint* buttons can be used to show a possible diagrammatic representation if required.

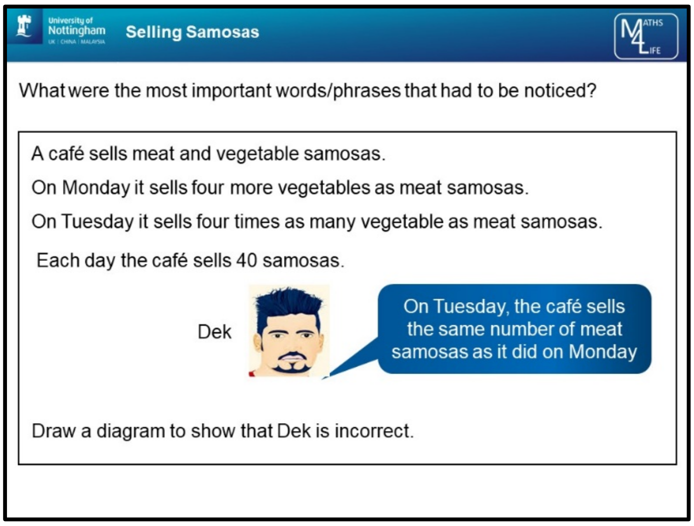
Closure

These tasks have required students to identify a 'base' value that all other values relate to. Once that is identified a representation can be drawn that captures the structure of the relationships between values in a way that allows first one unknown, and then the others, to be evaluated.

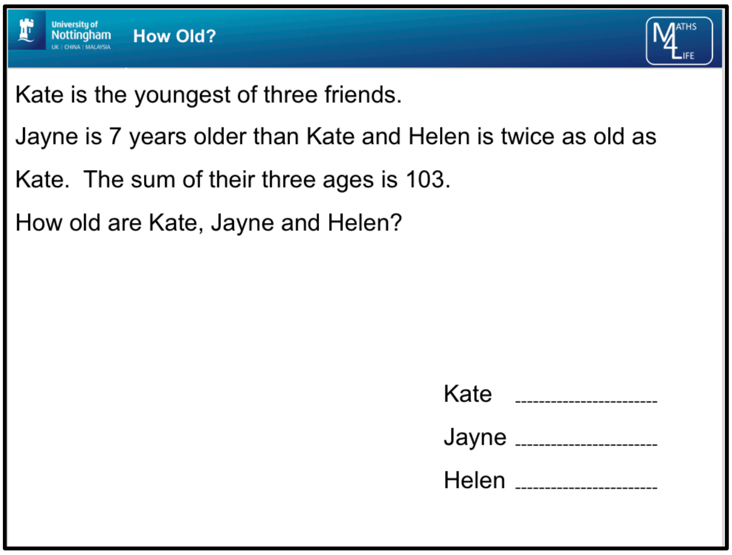
"Allow plenty of time to 'close' the lesson"

10. What are the key messages from the lesson that you expect to draw out?

Ask students to notice the key phrases on the task that resulted in different answers for the two days (Selling Samosas problem).



With this key point in mind, students should attempt the next question.



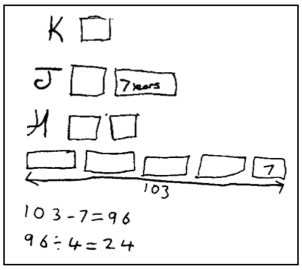
Once students have attempted the question, reveal the two methods on the next slide and ask them to explain how they relate to one another.

How Old?

Kate is the youngest of three friends.

Jayne is 7 years older than Kate and Helen is twice as old as Kate. The sum of their three ages is 103.

How old are Kate, Jayne and Helen?



Kate = a

Jayne = $a + 7$

Helen = $2a$

$4a + 7 = 103$

$4a = 96$

$a = 24$

"Continue to draw on student thinking and share this with the class."

One final task is provided with the intention of helping students to see how to tackle questions that provide an algebraic equation taken from the context.

Going deeper…

Fran has a friend, James, who is 4 years older than her.

Paul is three times as old as Fran.

The sum of their three ages is 89.

If x represents the age of Fran, show that **5x = 85** and hence find how old Paul, James and Fran are.

Paul ....

James ....

Fran ....

If necessary, ask students to start by solving the problem using a diagrammatic approach and then to consider how it links to the algebra. They can then compare their solution to the student work provided and see how the algebra can be brought out from the diagram.

"It's useful to share a range of different diagrams with the whole class"

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If x represents the age of Fran, show that **$5x$ = 85** and hence find how old Paul, James and Fran are.

F: $\boxed{x}$

J: $\boxed{x}\boxed{4}$

P: $\boxed{x}\boxed{x}\boxed{x}$

$\underbrace{\boxed{}\boxed{}\boxed{4}\boxed{}\boxed{}\boxed{}}_{89}$

$$5x + 4 = 89$$
$$5x = 85$$
$$x = 17$$

→ 17, 21, 51

Extension

If not already used, students can extend their thinking by looking at a problem that involves negative values.

11. What is different about this extension question?

Extension

A straight line is made up of three angles.

Angle 2 is twice the size of angle 1.
Angle 3 is 15º smaller than angle 2.

Find the size of each angle.