Maths-for-Life

Welcome to the Maths-for-Life resources for Teachers. The materials here are intended to support a small network of teachers working as a Maths-for-Life "lesson-study" group.

Hence, the lesson resources include a description of each lesson and questions on some of the key pedagogical decisions. Anyone using these resources should at the very least consider their responses to these questions before teaching the lessons.

The Maths-for-Life programme involves:

- Local networks of teachers collaborating together providing one of the most effective forms of professional development available. Through these groups, professionals seek to better understand the learning of students, the maths we teach and their pedagogical practices.
- Dialogic learning which research shows is helpful in improving students' mathematical understanding, confidence and outcomes. It harnesses the power of talk to stimulate and extend students' thinking and advance their learning and understanding. Throughout the programme we are seeking to develop learning that is collective, reciprocal, supportive, cumulative and purposeful.
- Five key lessons that focus on some of the most fundamental mathematical concepts that underpin GCSE mathematics.
- Five key pedagogies that support students' learning of mathematics.

This file of resources is designed to support teachers adopting this Maths-for-Life approach. It is the intention that Maths-for-Life Teacher groups will be supported by a Lead Teacher and a separate file of materials is available to support them. All resources for teachers, Lead Teachers and classroom lessons are available at: https://www.nottingham.ac.uk/maths-for-life/.

We hope that you enjoy working with colleagues adopting the Maths-for-Life approach in ways that support your students enjoy maths more than in the past and importantly achieve a higher grade at GCSE.

Our thanks go to the EEF and J P Morgan for funding this project and to the Behavioural Insights Team that worked to evaluate the programme. Additionally, we thank all the Lead Teachers, teachers and students who took part in the Maths-for-Life research project (2017-18). All these groups of participants provided valuable insights and feedback which helped us improve and refine the resources.

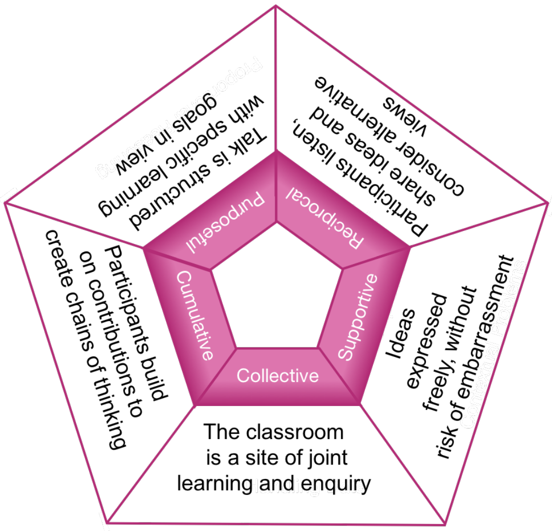
Finally, the research team extend their heartfelt thanks to their colleagues in the research support team led by Kanchana Minson that made sure that the Maths-for-Life project was delivered on time and to a high level.

Geoff Wake, Matt Woodford, Sheila Evans, Michael Adkins and Marie Joubert

Underpinning Principles

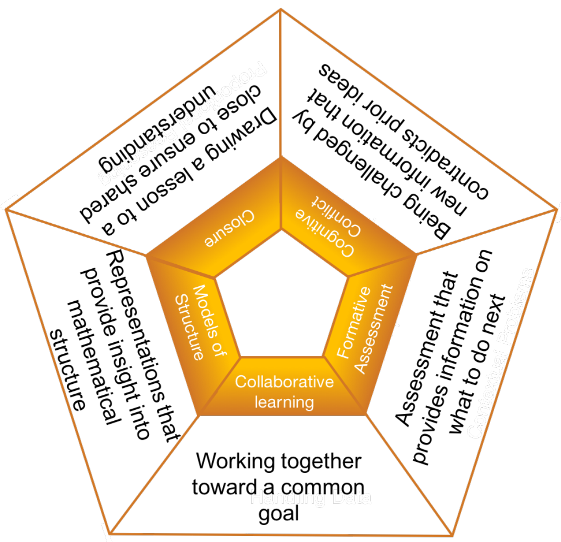
Dialogic Learning

Fundamental to these lessons is the belief that dialogic learning is essential to improving students' confidence and outcomes. The pink pentagon in Maths-for-Life summarises the five principles of dialogic learning that we are seeking to develop in classrooms. These define the behaviours that we would expect to see develop over the year in all members of the classroom – including teachers, teaching assistants and students.



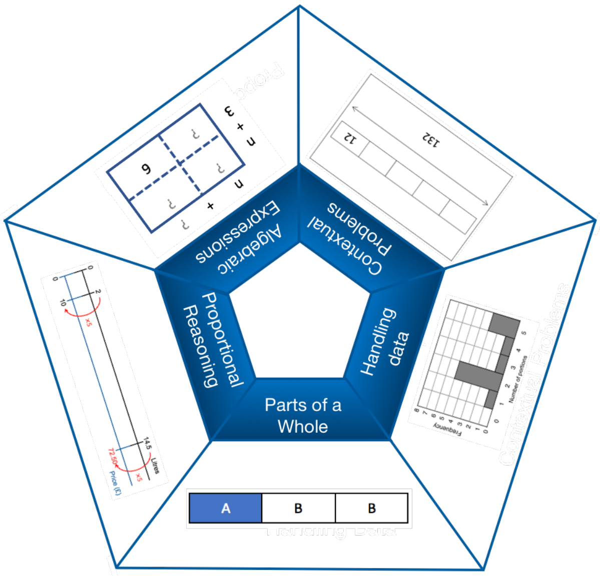
Pedagogies

Five key pedagogical ideas underpin the design of each of the five lessons. Each pedagogy will be studied in turn and the effect that it has on one of the principles of dialogic learning examined. These pedagogies are seen throughout all lessons in the actions of teachers and are supported by the design of the resources.



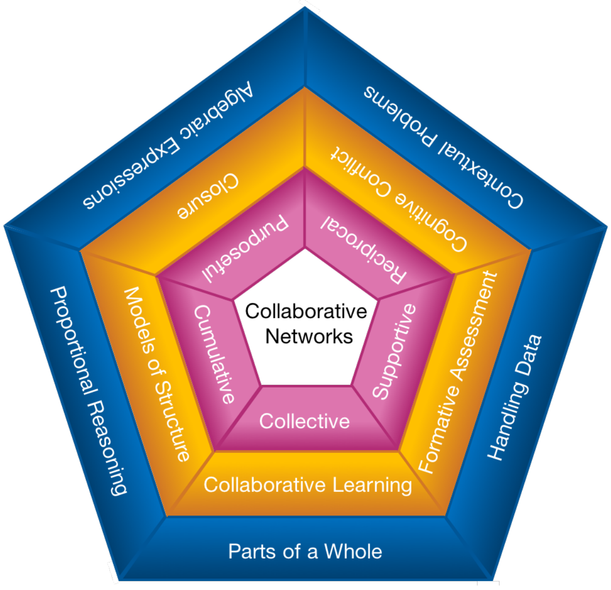
Lessons

The Maths-for-Life lessons have evolved from resources and principles used in the Improving Learning in Mathematics (Standards Unit) box. These new resources have been designed and trialled so that teachers are able to focus on exploiting anticipated learning opportunities.



The Maths-for-Life Pentagon

Together, the three individual pentagons combine to give the Maths-for-Life Pentagon. Each section could be rotated to line up with any other section. However, the orientations have been chosen by the designers working with the project's Lead Teachers as a best fit for the focus of each research lesson.

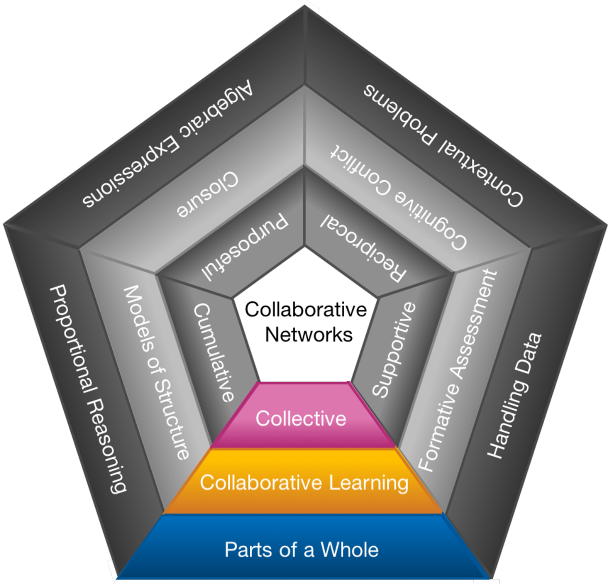


Lesson 1 Parts of a Whole

Introduction

Overview

The focus in the lesson, **Parts of a Whole,** is on how to develop **collaborative learning** and to see how this contributes towards creating a **collective** classroom.



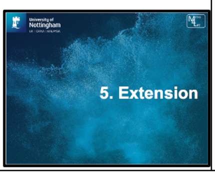
Collaborative learning is when students work together in both pairs and small groups, and as a whole class, towards common goals.

A **collective** classroom is one in which both the students and the teacher see their lessons as being based around joint learning and enquiry.

Research Question

How does collaborative learning (through the design of resources and the actions of the teacher) promote collective endeavour?

Lesson Summary

Phase	Timings (minutes)	Notes
	**5**	The initial problem is explained, clarifying why Ali and Blair split the payment in the given ratio.
	**5 - 10**	Students match the fractions cards to the ratios on the grid. *It is important that students are given the time to follow their thinking through without teacher correction.*
	10 - 15	Students work with the representation cards. *Students resolve misconceptions and cognitive conflict through the insight that the representations provide.*
	10 - 15	Students work with the word description cards. *These cards allow students to make links between fractions, ratios and the ways that they are often described in exam questions.*
	**5 – 10**	Check understanding of the activity using the review slides in the electronic presentation.
	**10 – 15**	Return to the original Ali and Blair problem. *Ensure a common understanding of the misconception and its resolution.* *Examine how given amounts can be shared using fractions and ratios.*
		Extension questions used if appropriate. *These slides allow formative assessment with a variety of possible questions and answers.*

L1.1 Lesson Outline: Parts of a Whole

Mathematical goals

To help students:

- understand ways of mathematically describing a part-part relationship;
- understand ways of mathematically describing a part-whole relationship;
- understand how to use representations to give insight into solving problems.

Starting points

There is often confusion about the connections and differences between fractions and ratio. Many students assume that the ratio 1:2 is equivalent to the fraction ½.

Diagrams can be a powerful representation that allow students to understand the relationship between ratio and fraction.

Materials required

For each group of students, you will need:

- L1.2 Cards (fraction cards, diagram cards and word cards all separate);
- L1.3 Template ***enlarged*** onto A3 paper;
- glue sticks.

L1.4 PowerPoint Presentation

Time needed

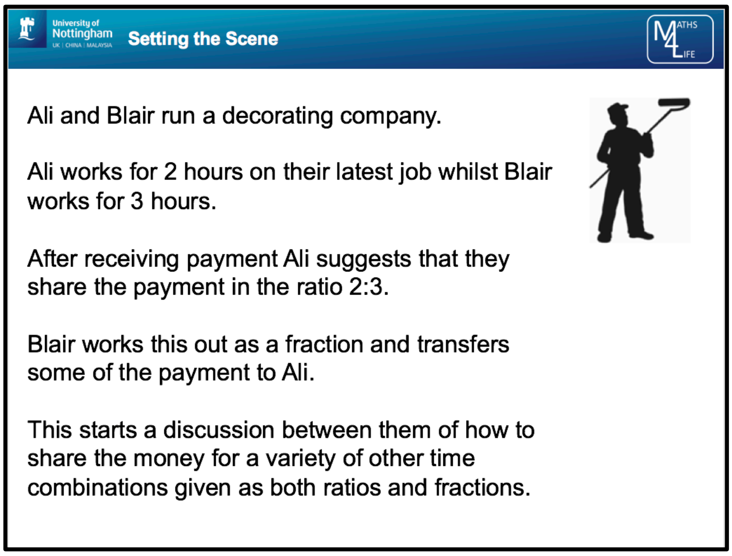
Approximately 1 to 1½ hours.

Lesson Structure

Setting the Scene

Introduce the problem using the PowerPoint presentation. This allows students to understand why there is the suggestion to share the money in the ratio 2:3.

1. What assumption must be made for the money to be shared in the ratio 2:3?

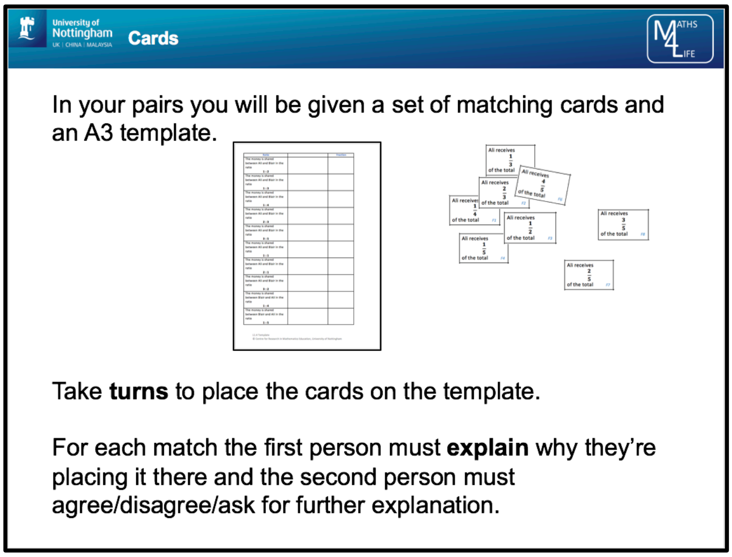


Collaborative Learning using Cards

Arrange students into pairs. Explain to the students that each pair will be given a template and a set of fraction matching cards. (You must only give out the fraction cards at this point. Save the cards with representations on them for later in to the activity).

2. Why is it important to hold on to the representation cards when the students first tackle the question?

Students must take turns to match a pair, and to explain their reasons for putting them together. The other person in the pair must agree to the match, disagree or ask for further explanation.



3. What mistakes and difficulties do you expect?

"Don't intervene too early"

4. When is the optimum time for handing out the diagram cards?

Once students are part way through the task of matching, give them the representation cards. They should be asked to see whether the diagrams change their thinking or support it.

Remind students to complete the blank cards and to explain to one another their reasoning.

"Use mistakes and miscon-ceptions to encourage dialogue"

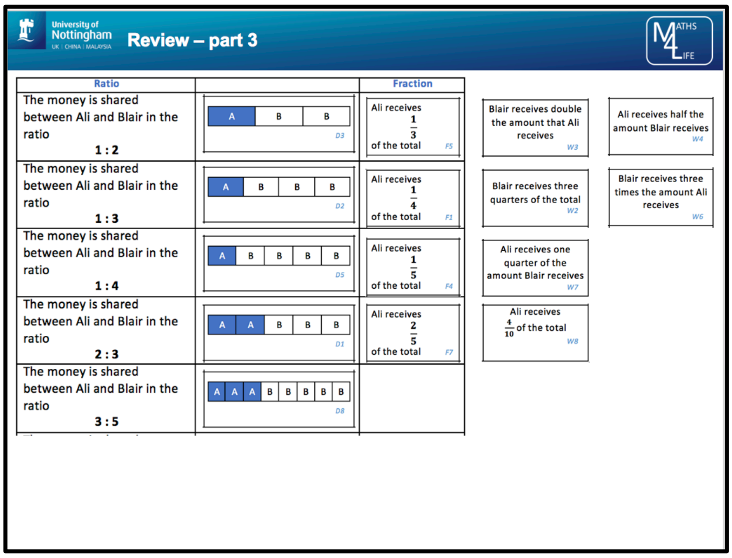
5. Which word cards will be particularly important to highlight in the whole group learning phase?

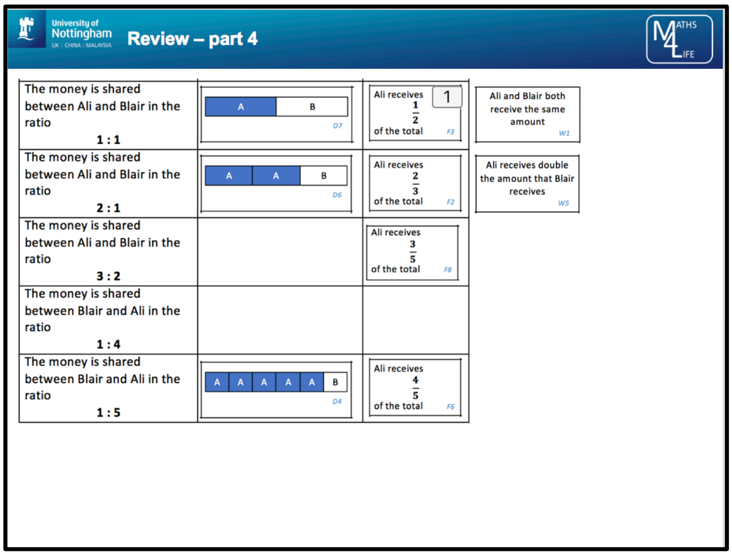
Once students have completed the ratio-diagram-fraction matchings they can be given the set of word cards. Note that these cards will have to be placed on the edge of the template. Also note that some rows have more than one word card and some rows don't have any. Students are not expected to create cards for rows that have no word cards.

To extend the task further, groups of students could be asked to calculate how much Ali and Blair receive in each situation if the total sum to be shared is £120. (Their answers could be written on the picture cards).

Review

The presentation includes slides to show how the cards should have been matched.





Before proceeding with the PowerPoint presentation examine any interesting mistakes or misconceptions that have taken place in the classroom.

All the time, it is important to keep re-phrasing student explanations using the language of part to part and part to whole.

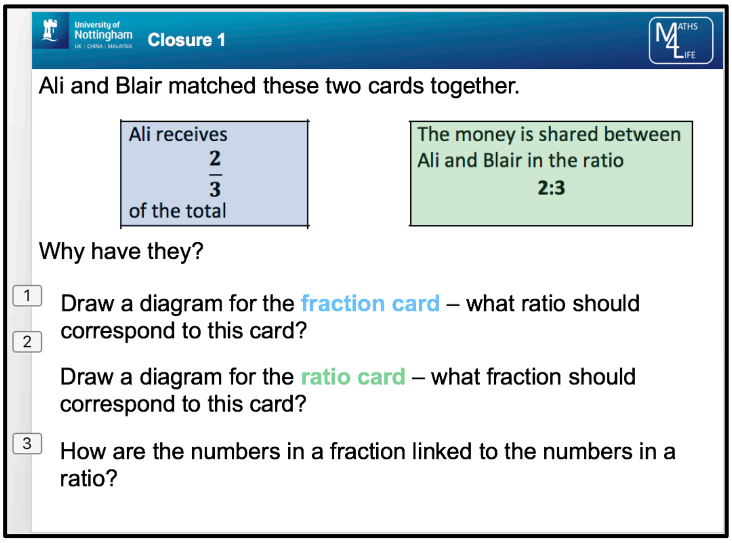
For example,

Ali receives 1 part for every 2 parts that Blair receives.
Ali receives 1 part out of 3 parts of the whole.

Closure

Allow approximately 15 minutes for this section of the lesson. Using the PowerPoint presentation as a guide, bring closure to the lesson by asking students to work in a pair to answer the questions on the next slide.

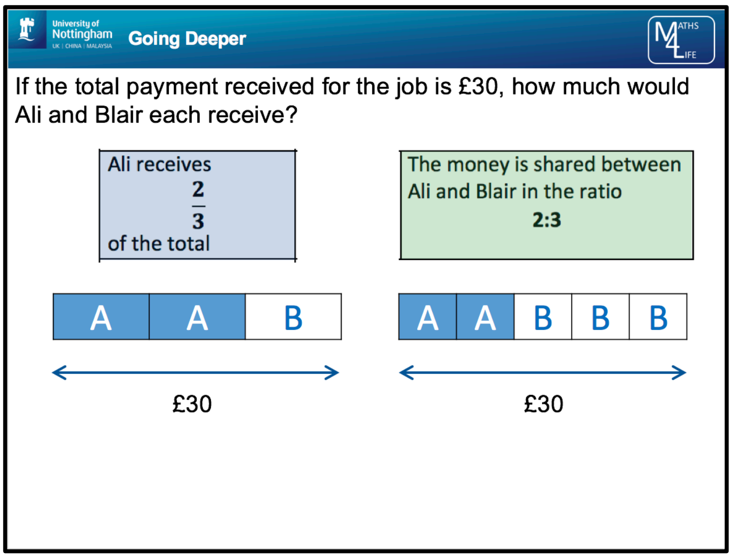
"Students need closure in a lesson"



6. How can the link between fractions and ratios be explained succinctly? What do you expect students to say?

Ask students to identify the link between information provided as a fraction and information provided as a ratio. Re-phrase student explanations to draw out an understanding of ratio showing part to part whilst fractions show a part of a whole.

Now ask students to calculate how much each person would receive if the total money paid was £30. Doing this for both the ratio 2:3 and the fraction $\frac{2}{3}$ emphasises the importance of understanding these calculations.

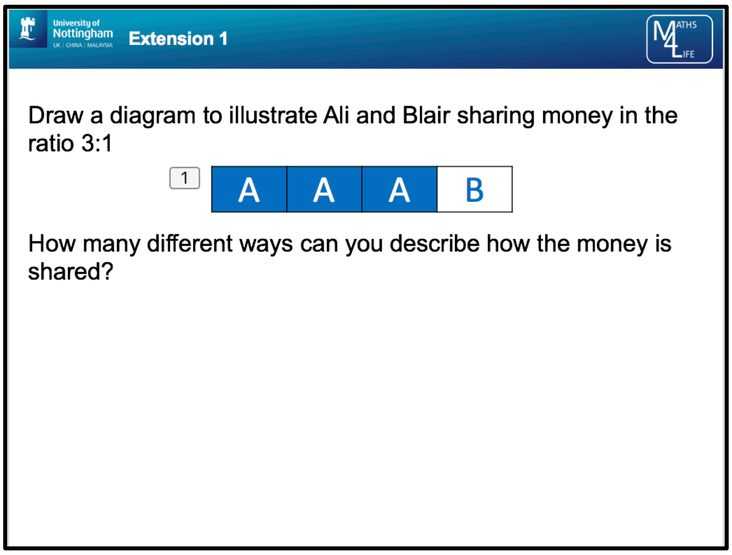


7. Which features of the representation cards are important to highlight to students?

Extension

The next slides ask open-ended questions that give the opportunity for formative assessment to take place.

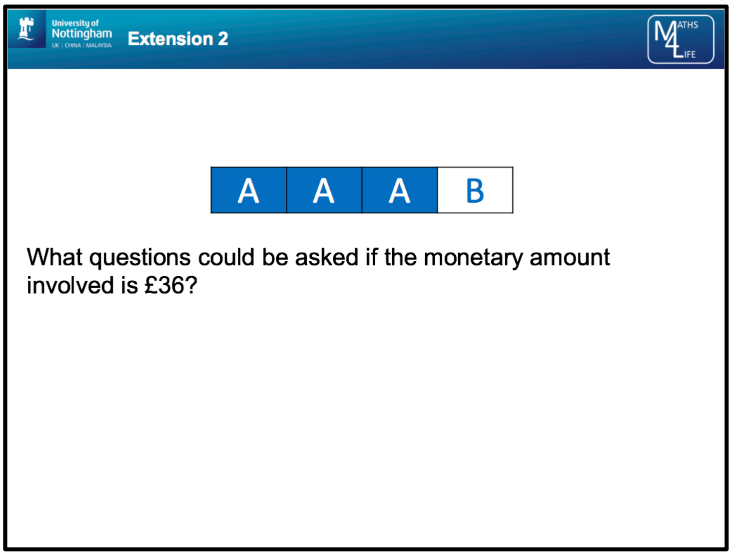
8. Which of these ways are most important to stress with your class?



"It's about what we do and what we don't do"

"We can affect the way students work together through the way we structure a task"

9. What are the key messages from the lesson that you expect to draw out?



"We want to see students developing both questions and ideas together"

Ali receives $\frac{1}{4}$ of the total *F1*	A A B B B *D1*	Ali and Blair both receive the same amount *W1*
Ali receives $\frac{2}{3}$ of the total *F2*	A B B B *D2*	Blair receives three quarters of the total *W2*
Ali receives $\frac{1}{2}$ of the total *F3*	A B B *D3*	Blair receives double the amount that Ali receives *W3*
Ali receives $\frac{1}{5}$ of the total *F4*	A A A A A B *D4*	Ali receives half the amount Blair receives *W4*
Ali receives $\frac{1}{3}$ of the total *F5*	A B B B B *D5*	Ali receives double the amount that Blair receives *W5*
Ali receives $\frac{4}{5}$ of the total *F6*	A A B *D6*	Blair receives three times the amount Ali receives *W6*
Ali receives $\frac{2}{5}$ of the total *F7*	A B *D7*	Ali receives one quarter of the amount Blair receives *W7*
Ali receives $\frac{3}{5}$ of the total *F8*	A A A B B B B B *D8*	Ali receives $\frac{4}{10}$ of the total *W8*

One set of these cards should be printed for each group

Ali receives — of the total	To be completed
Ali receives — of the total	To be completed

One set of these cards should be printed for each group

Ali receives — of the total	To be completed
Ali receives — of the total	To be completed

One set of these cards should be printed for each group

Ali receives — of the total	To be completed
Ali receives — of the total	To be completed

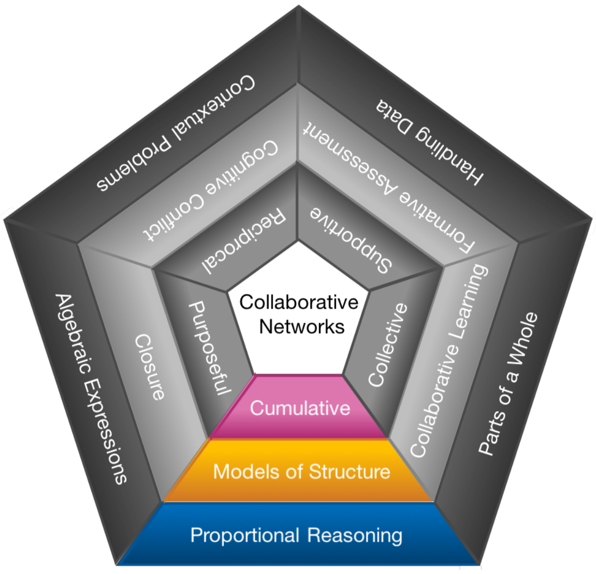
Ratio		Fraction
The money is shared between Ali and Blair in the ratio **1 : 2**		
The money is shared between Ali and Blair in the ratio **1 : 3**		
The money is shared between Ali and Blair in the ratio **1 : 4**		
The money is shared between Ali and Blair in the ratio **2 : 3**		
The money is shared between Ali and Blair in the ratio **3 : 5**		
The money is shared between Ali and Blair in the ratio **1 : 1**		
The money is shared between Ali and Blair in the ratio **2 : 1**		
The money is shared between Ali and Blair in the ratio **3 : 2**		
The money is shared between Blair and Ali in the ratio **1 : 4**		
The money is shared between Blair and Ali in the ratio **1 : 5**		

Lesson 2 Proportional Reasoning

Introduction

Overview

The focus in the lesson, **Proportional Reasoning**, is on how **models of structure** can support **cumulative** dialogue in the classroom.



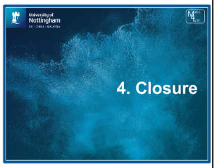
Models of Structure are representations that provide insight into mathematical structure.

Cumulative dialogue is seen when both students and teachers build on each other's contributions to create chains of thinking.

Research Question

How do models of structure help to facilitate cumulative dialogue and insight into mathematical structure?

Lesson Summary

Phase	Approximate Timings (minutes)	Notes
	**10**	The initial problem is explained. *Ensure students have enough time to consider the strategy that could be used to find the cost of using 4.54 litres.*
	10	Review methods. *Review strategies used by the class and then discuss the methods shown in the PowerPoint presentation.*
	**20 - 30**	Students work with the problem cards. *Support group work – this may include asking students to explain their thinking on a double number line.* *An extension question is available in the PowerPoint presentation.*
	**15 - 20**	Check understanding using the review slides in the PowerPoint presentation. *Encourage students to share alternative ways of thinking and illustrate them on double number lines.*
	**10 - 15**	Students work on the final problem. *Students work out which decorator charges the least.* *Ensure clarity of what has been covered in this lesson is shared.*
		Extension questions used if appropriate.

L2.1 Lesson Outline: Proportional Reasoning

Mathematical goals

To help students:

- reflect on the reasoning they currently use when solving proportion problems;
- understand the power and efficiency of a multiplicative structure through the concept of rate.

Starting points

Proportional reasoning is notoriously difficult for many students. There is a tendency to use additive thinking based around doubling and halving. This session aims to expose and build on this prior understanding.

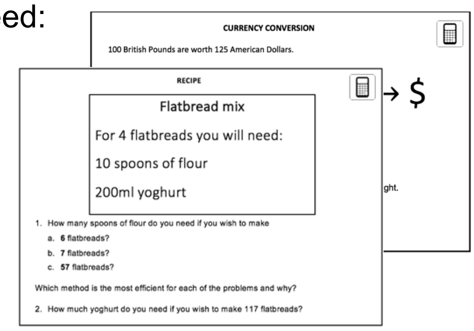
Students are given problems to work on in small groups aided by careful questioning from the teacher. This leads to discussions that compare the use of thinking that requires adding, doubling and halving with the use of more sophisticated thinking that use multiplication.

Materials required

For each group of students, you will need:

- mini-whiteboards;
- L2.2 – Cards (cut individually);
- a calculator.

L2.3 PowerPoint Presentation.



Time needed

Approximately 1- 1½ hours.

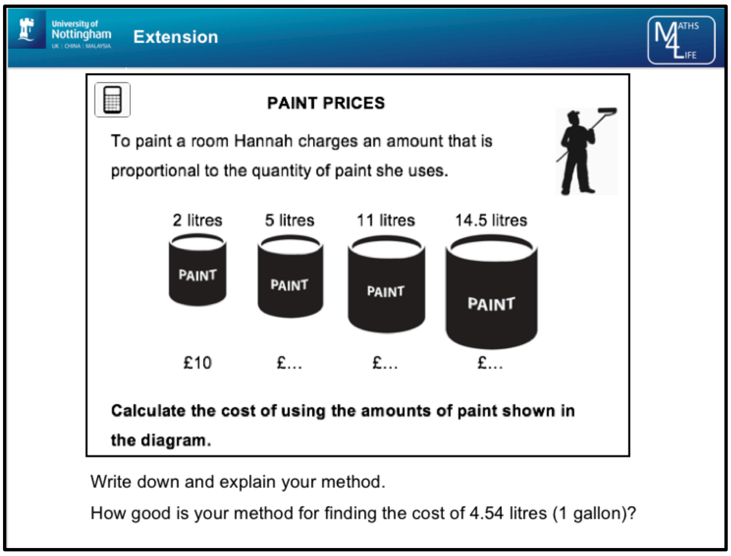
Lesson structure

Setting the Scene

Project and explain the Paint Prices problem from the PowerPoint presentation. Give students a short time to consider how they would tackle this problem on their own and then ask them to work in pairs on mini-whiteboards to try and solve the problem.

"The double number line can show different ways of mathematical thinking"

1. What strategies do you expect students to use?



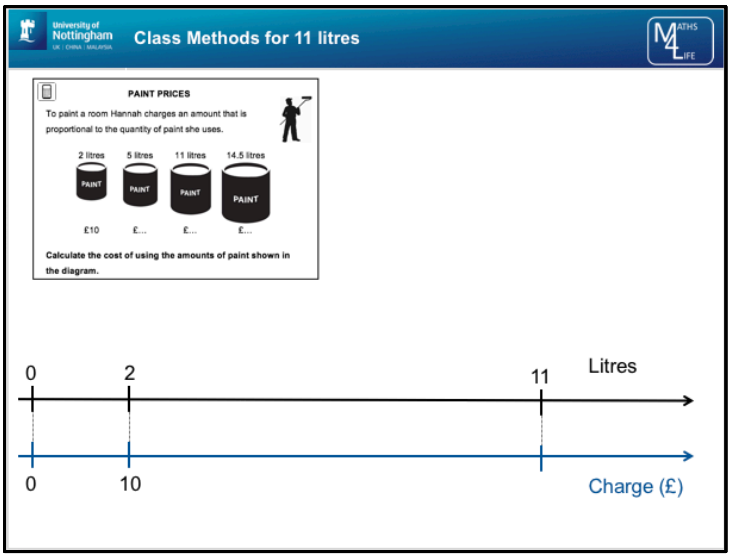
Note that the extension question about explaining their method and thinking about 4.54 litres should only be shown after students have attempted the main problem.

Bring the class together and ask students to explain their methods for finding the charge for 11 litres.

"Encourage discussion – rather than try to fix"

"Use the double number line to illustrate student thinking"

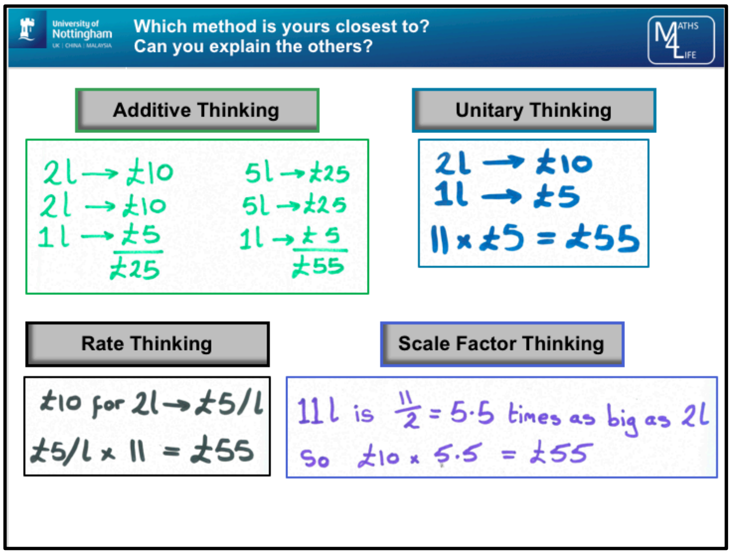
2. Why has 11 litres been chosen to demonstrate students thinking rather than 5 litres or 14.5 litres?



It is vital that the teacher captures their different methods and understanding on the double number line. This is about valuing and making sense of different ways of thinking – not about imposing a method.

Project the next slide and ask students to identify which ways of thinking their work most closely resembles and to try to make sense of other methods.

3. Is it essential that every way of thinking is gone through with students?

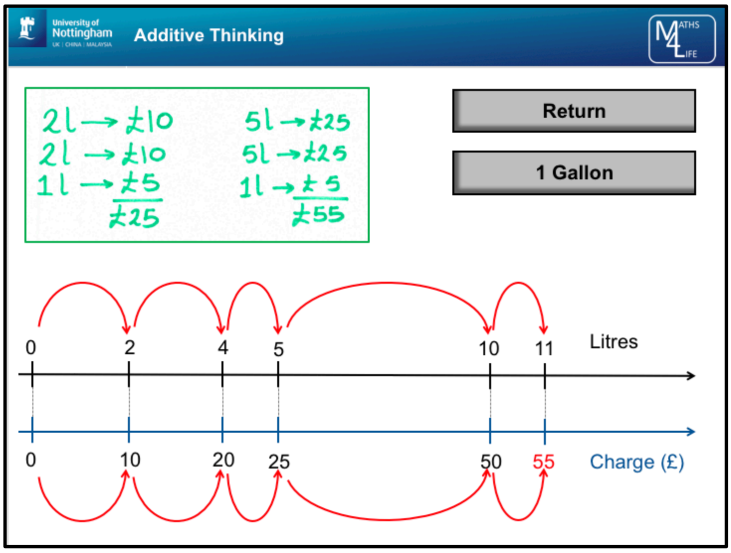


Note that the emphasis is not on the answer, but on making sense of the four different ways of thinking.

Click on each of the buttons in turn to illustrate the thinking of the student on a double number line. The four ways of thinking are illustrated on the following four slides.

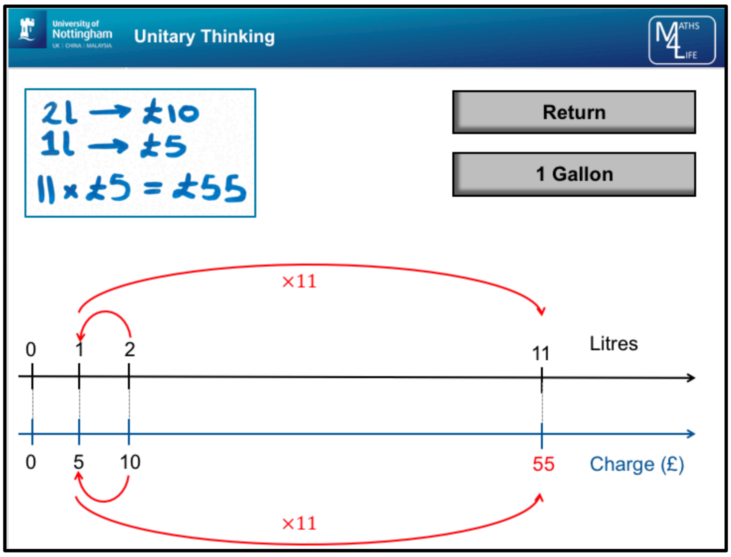
The first slide shows an additive method.

4. What are the features of the thinking that should be brought out to students?



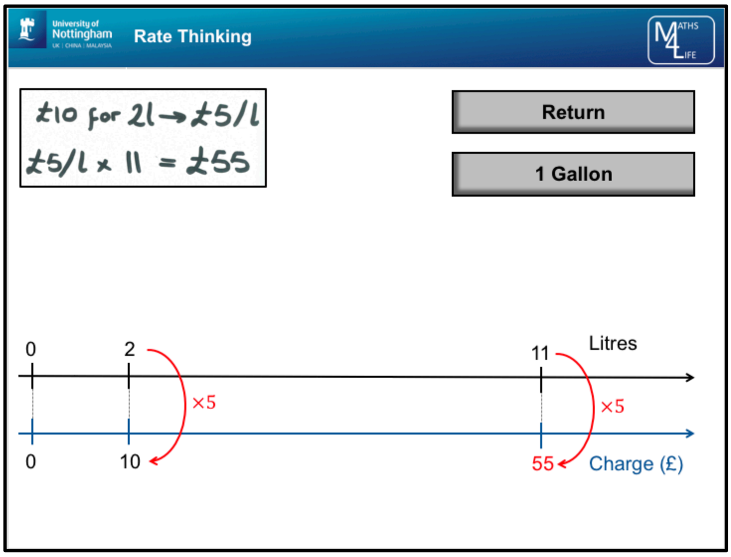
The second slide shows the unitary method - based on finding the cost of 1 litre, then being able to multiply to get the cost of any amount.

5. What are the features of the thinking that should be brought out to students?



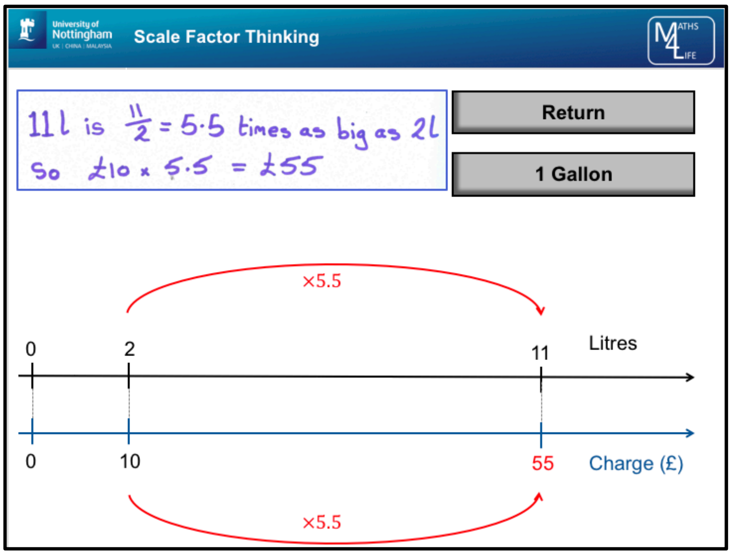
The third slide shows the ratio method. This is where the amount is multiplied by the rate of £5/l - a relationship that is true for all corresponding points on the number lines.

6. What are the features of the thinking that should be brought out to students?



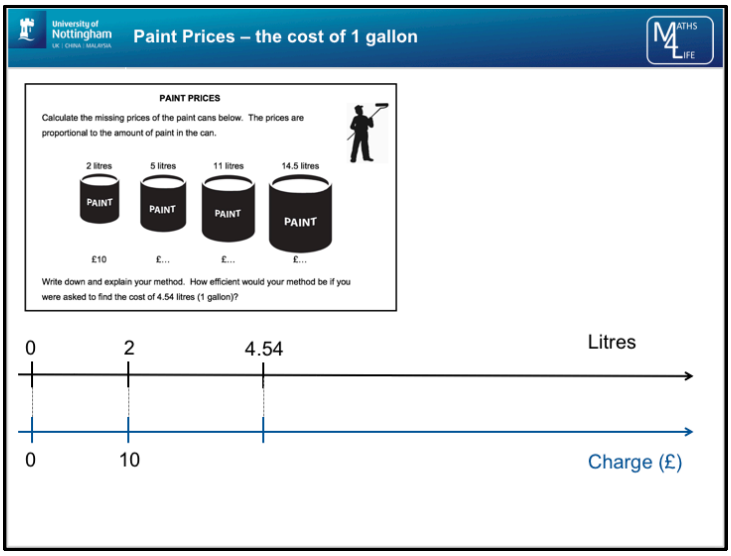
The fourth slide shows the idea of a scale factor – that the 11 litre can is 5.5 times as large, so it should cost 5.5 times as much.

7. What are the features of the thinking that should be brought out to students?



Now look at the next slide and ask students to work in pairs to find the cost of 1 gallon using as many of the four ways of thinking as possible.

8. Which of the ways of thinking are most helpful in this example?



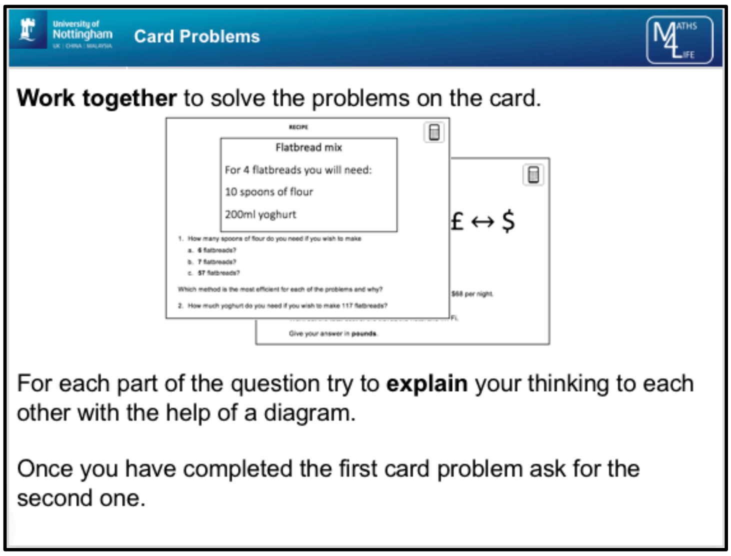
Ask students to identify which ways of thinking were most helpful for this question. This emphasises that methods (such as additive) are fine and can be used to obtain the correct answer. However, other thinking (such as unitary and rate) can be more efficient in some problems.

Collaborative Learning using Cards

Give out copies of the second problem (Flatbread Mix) to pairs of students and ask them to work together to solve. Remind students of the need to work together to agree a solution and a method.

"Predict what students might do and how to make the most of their thinking"

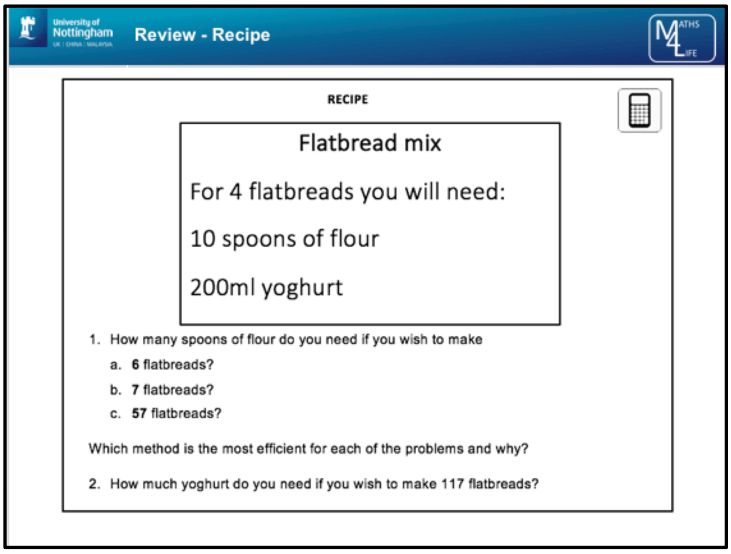
9. Why is it important to set out expectations for how students will attempt the questions?



The following two slides show the questions that the students will be asked to complete. Encourage students to explain their thinking to you on a double number line and encourage them to consider alternative ways of thinking.

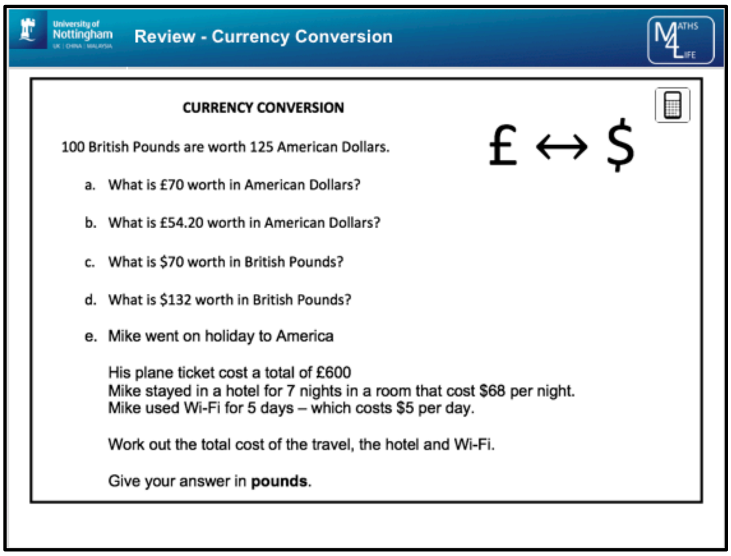
10. How do you expect students to tackle these questions?

11. Why have the numbers 6, 7, 57 and 117 been chosen?



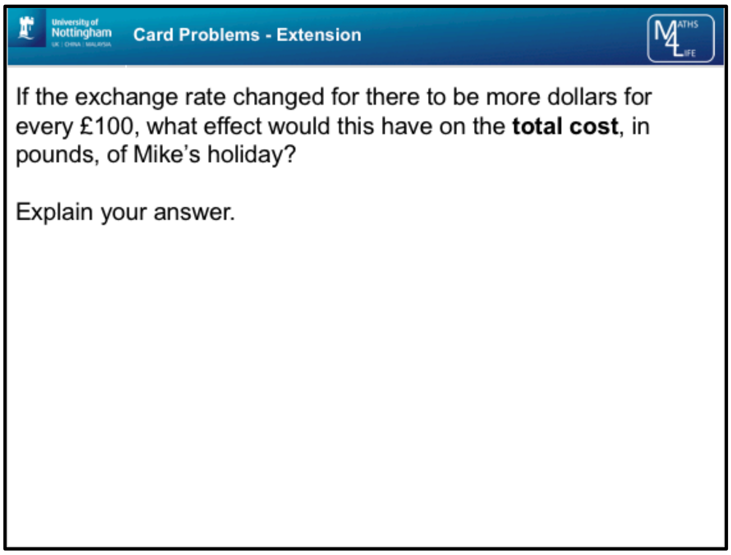
12. How do you expect students to tackle these questions?

13. What do you anticipate will cause difficulties?



Whilst students are working pay particular attention to any thinking that will be helpful to share in the Closure section of the lesson.

Note that an extension questions can be projected if required.



Review

Once all groups have attempted the questions draw them together to summarise the learning.

First clarify the concept of these kinds of proportional reasoning problems.

The problems we have looked at in this session have involved two quantities, help me list them.

Quantity of paint	*Cost of paint*
Number of flatbreads	*Spoons of flour*
Number of flatbreads	*Quantity of yoghurt*
Pounds	*Dollars*

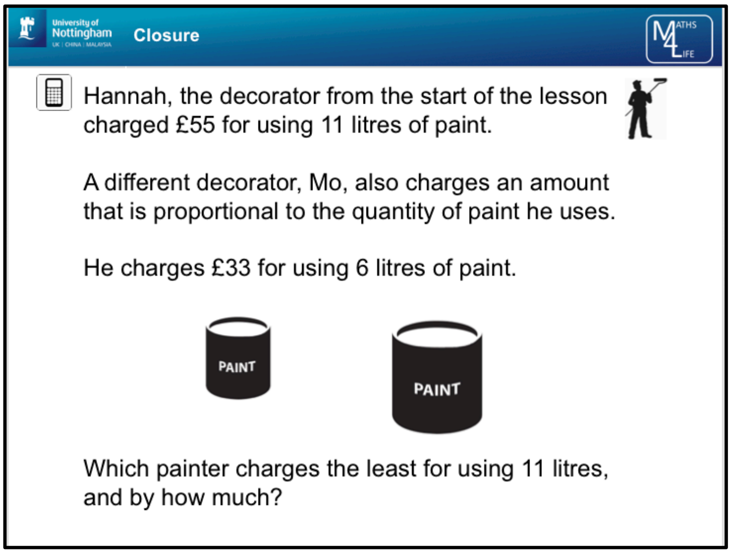
These are proportional situations. If we double the first quantity, we double the second. If there were none of the first quantity, there would be none of the second quantity.

Now capture the ways of thinking for each of the problems of various groups on the double number lines in the electronic presentation. Once again, it is important to make sense and capture students' ways of thinking – not to prescribe a best method.

Closure

14. What are the key messages that you expect to draw out?

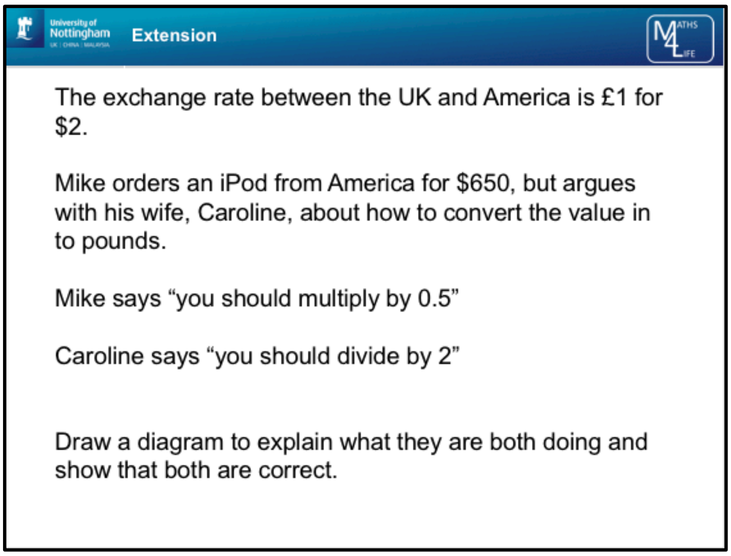
Now ask students to look at the next slide on the PowerPoint presentation. This links back to the original problem and asks students to work out which decorator charges the least for 11 litres, and by how much.



15. What is the purpose of the double number line?

Extension

The following PowerPoint slide can be used to help students make sense of the equivalence between multiplying by 0.5 and dividing by 2 using the double number line.



16. How can the double number line be used to help students to understand this question?

"The double number line gives students something to structure their conversations around"

"The aim is for students to build on one another's ideas"

RECIPE

Flatbread mix

For 4 flatbreads you will need:

10 spoons of flour

200ml yoghurt

1. How many spoons of flour do you need if you wish to make
   a. **6** flatbreads?
   b. **7** flatbreads?
   c. **57** flatbreads?

Which method is the most efficient for each of the problems and why?

2. How much yoghurt do you need if you wish to make 117 flatbreads?

CURRENCY CONVERSION

100 British Pounds are worth 125 American Dollars.

£ ↔ $

a. What is £70 worth in American Dollars?

b. What is £54.20 worth in American Dollars?

c. What is $70 worth in British Pounds?

d. What is $132 worth in British Pounds?

e. Mike went on holiday to America

   His plane ticket cost a total of £600
   Mike stayed in a hotel for 7 nights in a room that cost $68 per night.
   Mike used Wi-Fi for 5 days – which costs $5 per day.

   Work out the total cost of the travel, the hotel and Wi-Fi.

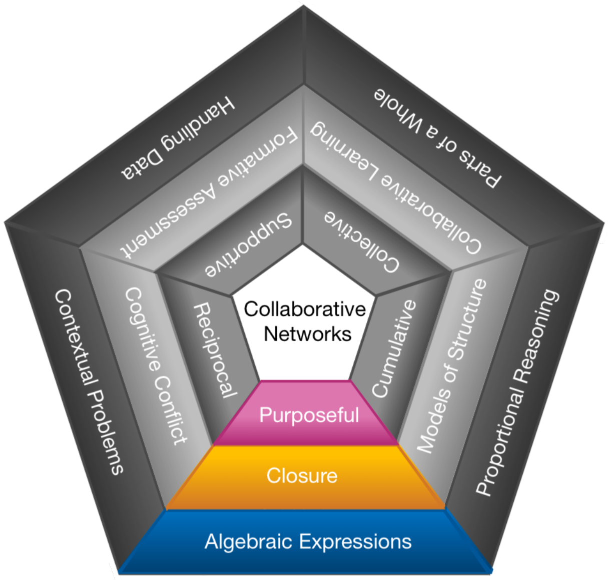
   Give your answer in **pounds**.

Lesson 3 Algebraic Expressions

Introduction

Overview

The focus in the lesson, **Algebraic Expressions**, is on how **closure** can help to develop **purposeful** dialogue in the classroom.



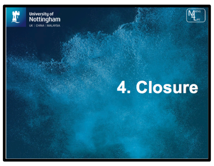
Closure is when part of a lesson is drawn together to ensure a shared understanding.

Purposeful dialogue is when talk is structured with specific learning goals in view.

Research Question

How does purposeful dialogue contribute to student understanding during the closure phase of the lesson?

Lesson Summary

Phase	Timings (minutes)	Notes
	**5 - 10**	The initial problem is explained and students work through the stages. *Key messages include;* *1. that n represents a variable.* *2. the convention of drawing a fixed length looking rectangle, but labelling it with an n indicates that it is a variable.*
	**10 - 15**	Students match the factorised expressions and expanded expressions to the area representations on the grid. *It is important that students are encouraged to use the speaking frame.* *An extension is available in the PowerPoint presentation that will help with factorising double brackets*
	**10 - 15**	Check understanding of the activity using the review slides in the electronic presentation.
	**10 – 15**	Check understanding of multiplying out double brackets using the PowerPoint slides.
	**5 - 20**	Extension questions used if appropriate. *Additional time may be spent on:* *expanding single brackets,* *factorising single brackets,* *double brackets,* *simplifying expressions.*

L3.1 Lesson Outline: Algebraic Expressions

Mathematical goals

To help students:

- understand that *n* can represent a variable;
- understand multiplicative algebraic structure using an area representation;
- create algebraic expressions from area representations.

Starting points

Students often see letters in algebra as representing specific unknowns that must be found. Hence an area of 8*n* (for a rectangle with variable width and length 8) is a difficult concept to make sense of. This lesson begins by establishing that *n* represents a variable.

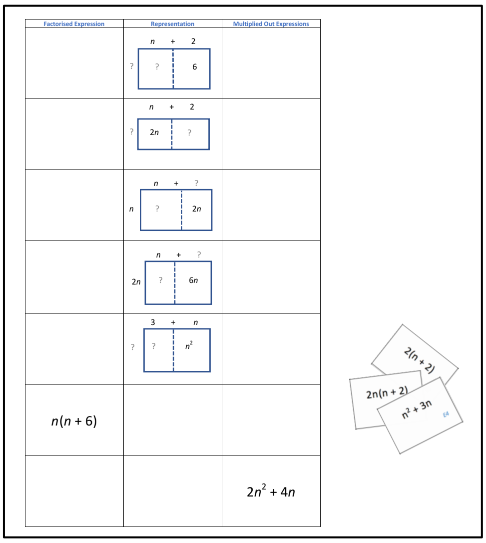
Students then extend to making sense of factorising and expanding brackets using an area representation. Previously many students will only have learned this in a procedural way.

Materials required

For each group of students, you will need:

- L3.2 Cards;
- L3.3 Template (copied on to A3 paper);
- glue sticks.

L3.5 Presentation;
mini-whiteboards and pens.



Time needed

Approximately 1 – 1.5 hours.

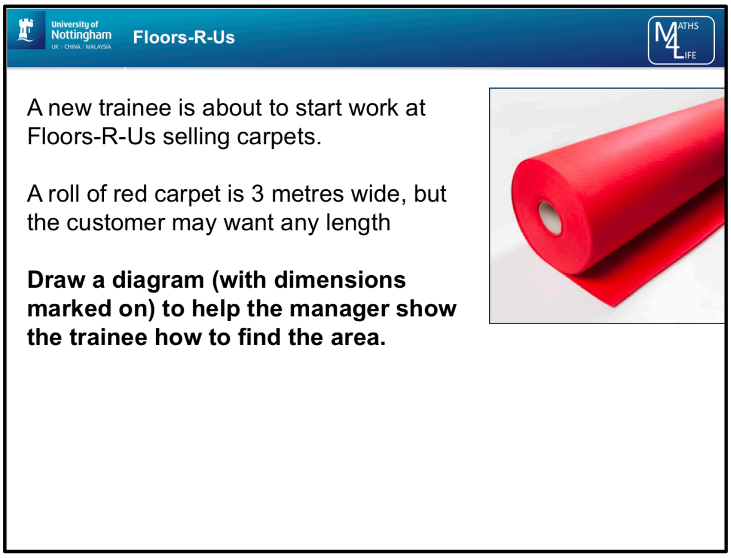
Lesson structure

Setting the scene

"The opening task has been carefully crafted"

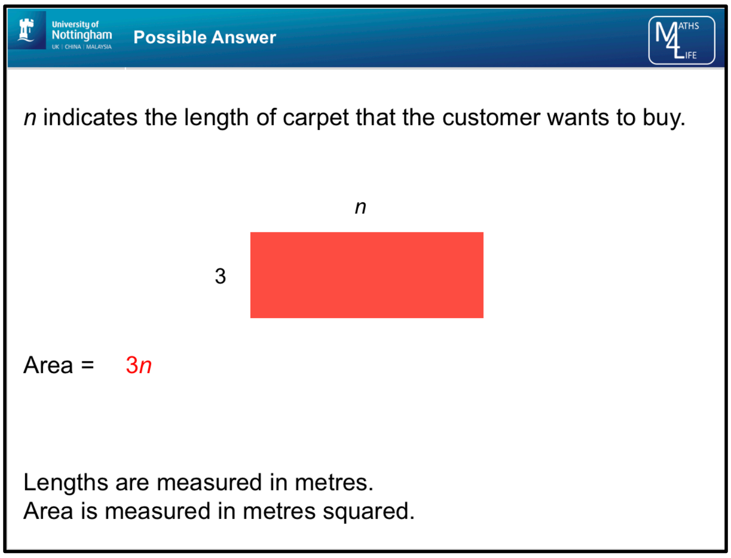
Introduce the initial task to students using the PowerPoint slide provided.

1. What diagrams (and dimensions) might students draw?

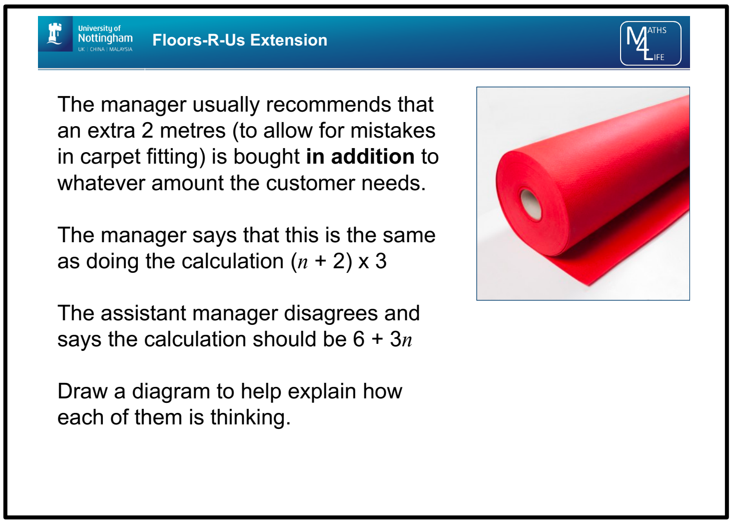


Note that area is being found to help calculate the cost of the carpet. To maintain task clarity, this cost implication has not been included in the problem.

Using the animated slide draw students' attention to the fact that *n* is used to indicate a variable length.

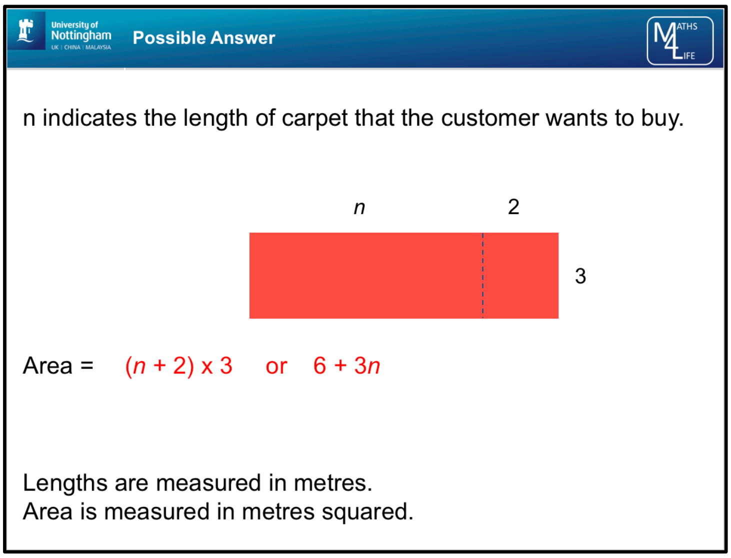


The convention, in this lesson, is that we draw a representation of fixed looking length but the *n* indicates that it could vary. Ask students to work in pairs on a shared mini-whiteboard for the next task.



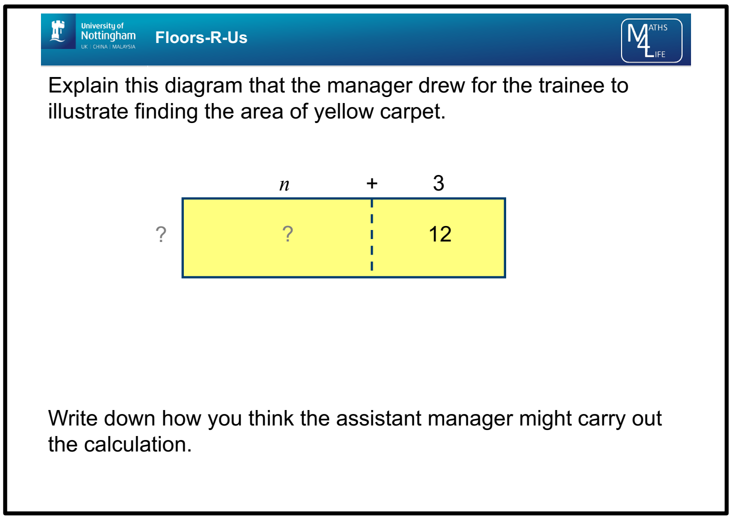
Again, use the animated slide to illustrate that *n* varies but our convention is to draw a static looking diagram.

2. What difficulties might students have with this question?



Help students to see that the manager is describing a calculation involving multiplying two lengths whilst the assistant manager is describing a calculation involving adding two areas. At this point there is no need to show students the algebraic equivalence of the two expressions.

In pairs ask students to note down what they can understand from the next diagram.

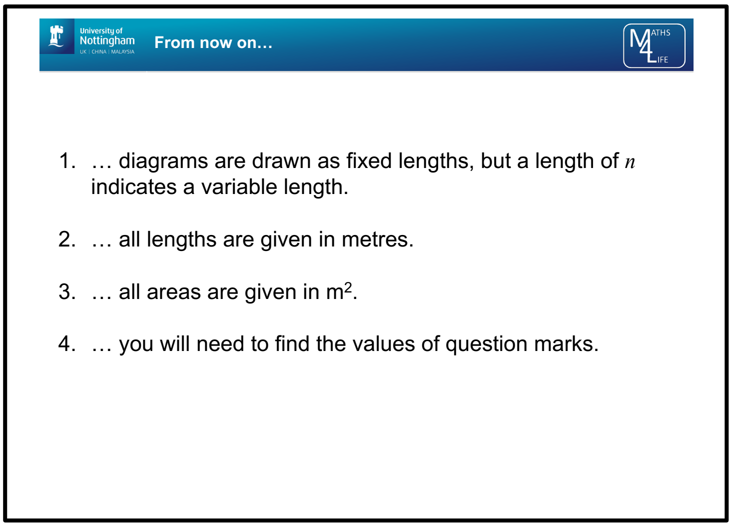


Discussion could include the following points:

- the yellow carpet has width of 4 metres.
- the manager encourages customers to buy an additional 3 metres.
- the manager would describe this calculation as $(n + 3) \times 4$
- the assistant manager would describe the calculation as $12 + 4n$

Use the next to establish the conventions that will be used during the rest of this lesson.

3. Why is it important to share this slide with students?



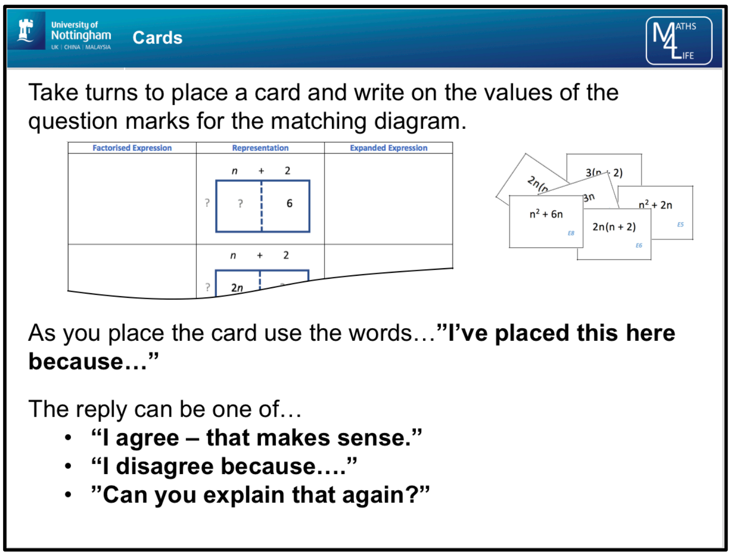
Collaborative Learning using Cards

Arrange students into pairs and give each pair both the template and the matching cards.

Present the task to students using the following slide from the electronic presentation.

4. What is the purpose of giving the students a speaking frame?

5. Where do you expect difficulties in this task?

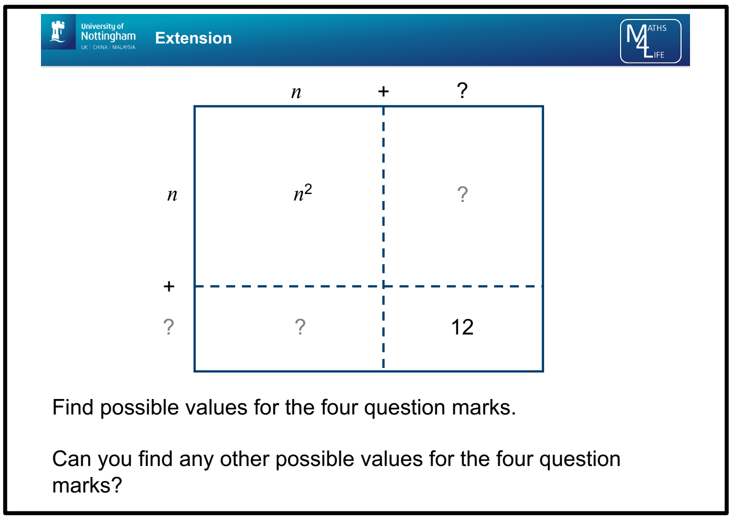


"Use the task to encourage discussion"

As students are working, encourage them to discuss and explain their thinking to each other. Check understanding by asking them to explain how each of the three items in the row relate to each other. In particular, make sure that students can explain how to go from the expanded expression to the factorised expression.

6. Ensure your questions check student understanding, not act to correct their understanding.

An extension task is provided on the following slide.

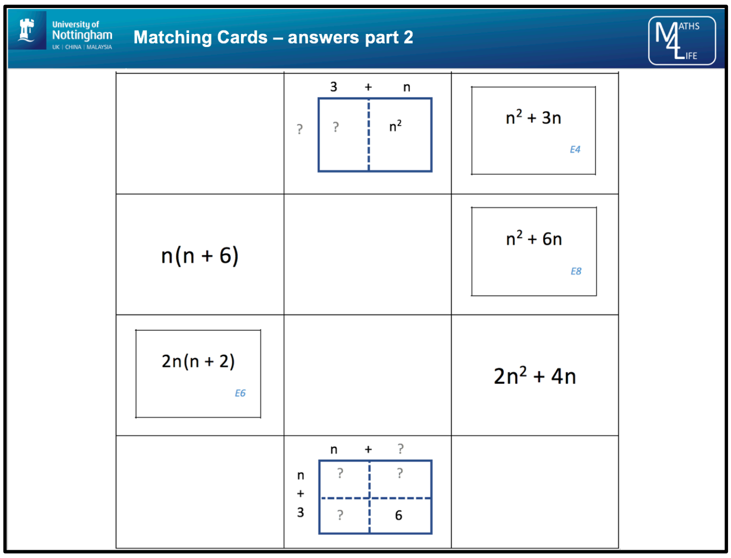


Note that non-integer answers such as side lengths of 8 and 1.5 are also possible.

Review

Student confidence should be built by allowing them to see the correct matchings. Encourage students to explain how each of the columns relate to the other two columns. It is important that time is spent understanding the written answers to blanks (not shown in the PowerPoint presentation) to ensure that there is a common understanding within the class of how these answers relate to each of the other cards.

7. Which of these rows do you expect to emphasise with your class?

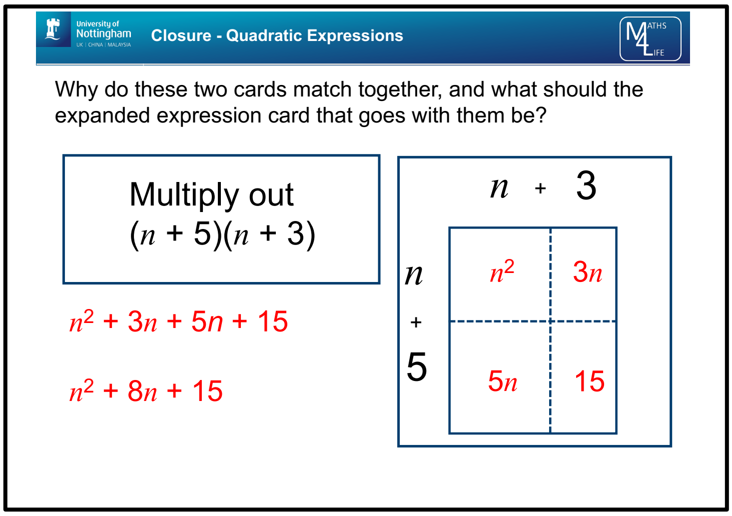


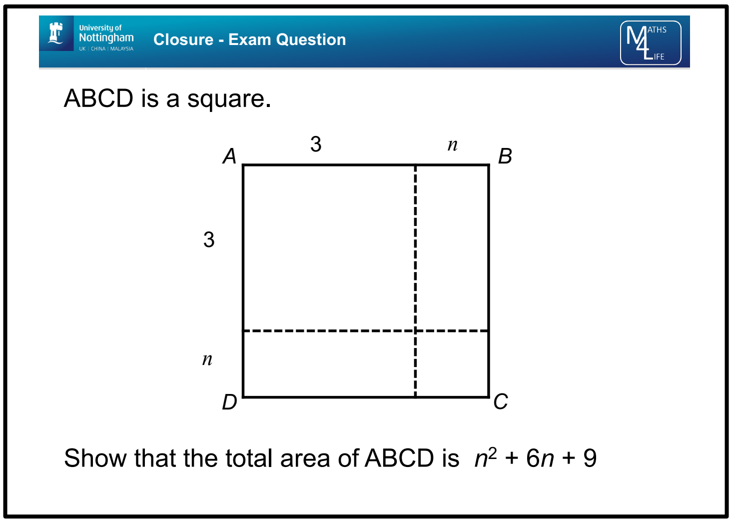
Continue to stress to students that the factorised expressions represent the calculation of multiplying two lengths, whilst the multiplied-out expressions represent the addition of area values.

"Avoid closing a lesson by turning what has been covered into a procedure"

Closure

Students should work in pairs to explain the following two slides.





"Closure is more effective when the students shape the thinking"

8. What are the key messages from the lesson that you expect to draw out?

Extension

Depending on the task selected (see slide below), students may now have additional practice with expanding single brackets, factorising single brackets, working with double brackets or looking at an extension on combining expressions.

"Look for discussion that contributes to the aims of the lesson"

Where next?

	Questions Only	Correcting Student Work
Expanding Single Brackets	Go	Go
Factorising Single Brackets	Go	Go
Double Brackets	Go	
Simplifying Expressions	Go	

Expanding single brackets

Note that for the expanding single brackets section you can choose between asking the questions only or asking students to spot corrections/improvements.

Expanding Single Brackets

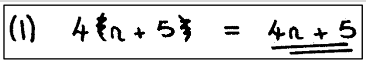
1. Multiply out 4(+ 5)
2. Multiply out $n(n + 5)$
3. Draw a rectangle diagram to represent an area of $3n^2 + 12n$, remember to give the lengths of the sides.
4. Expand and simplify
   $5(n + 2) + 2(n + 2)$

   Can you draw a rectangle diagram to illustrate question 4?

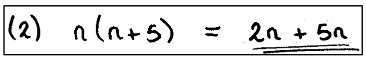
Return

Expanding Single Brackets – what do you think?

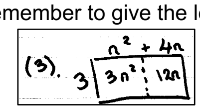
1. Multiply out $4(n + 5)$



2. Multiply out $n(n + 5)$

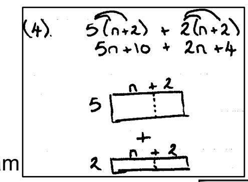


3. Draw a rectangle diagram to represent an area of $3n^2 + 12n$, remember to give the lengths of the sides.



4. Expand and simplify
   $5(n + 2) + 2(n + 2)$

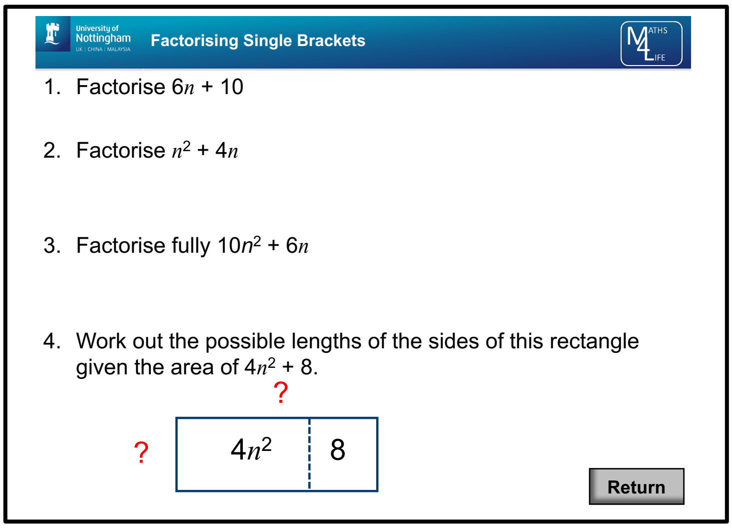
   Can you draw a rectangle diagram to illustrate question 4?

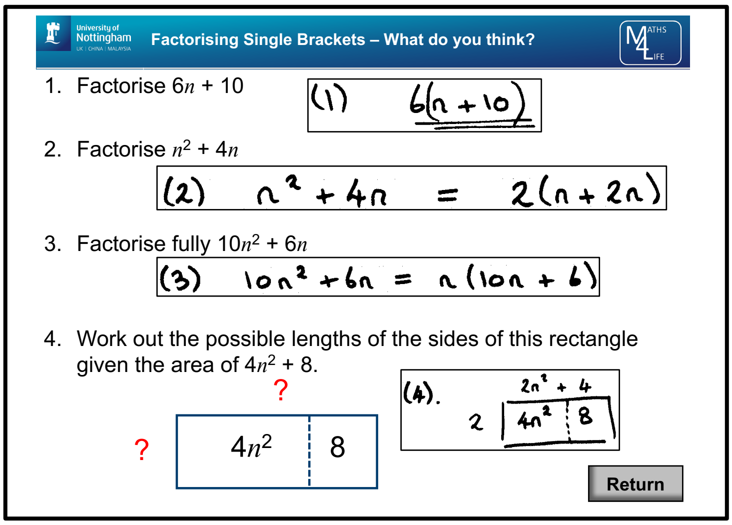


Return

Factorising Single Brackets

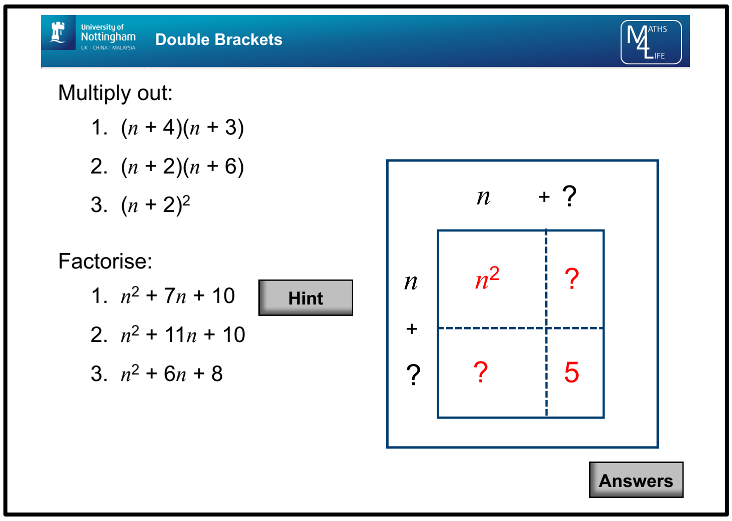
Note that in the factorising single brackets section you can chose between asking the questions only or asking students to spot corrections/improvements.





Double Brackets

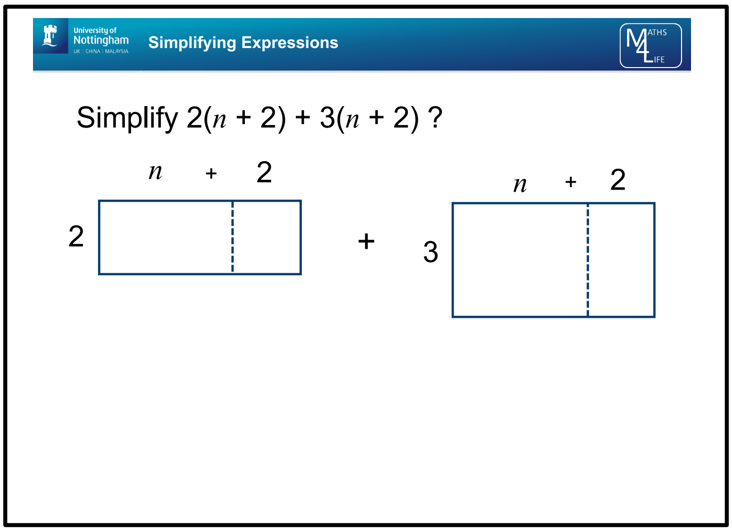
Note that a hint (showing the representation) is available for factorising double brackets if required.



Simplifying expressions

The aim of the next set of slides is to help students to understand (with the help of a representation) that $2(n + 2) + 3(n + 2)$ can be simplified straight away to $5(n + 2)$ without the need for expanding and factorising.

9. What misconception does this explanation risk creating?



Use the animation to help students see that the expression can be simplified without needing to multiply out, add like terms and then factorise.

In pairs, students should use the true/false slide and representations to consider when this way of simplifying makes sense.

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Simplifying Expressions

Can you explain whether the following are true or false with the help of an area representation?

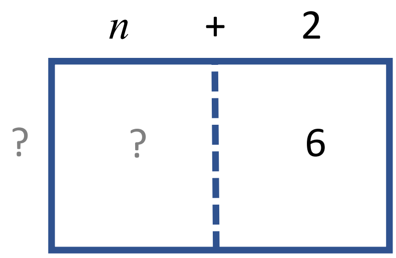
1. $4(n + 3) + 5(n + 3) = 9(n + 3)$
2. $4(n + 2) + 5(n + 3) = 9(n + 2)(n + 3)$
3. $4(n + 3) + n\,(n + 3) = 4n(n + 3)$
4. $n(n + 3) + 5(n + 3) = (n + 5)(n + 3)$

Return

L3.2 Card Set E - Expressions

$3(n + 2)$ *E1*	$2n(n + 3)$ *E2*
$2(n + 2)$ *E3*	$n^2 + 3n$ *E4*
$n^2 + 2n$ *E5*	$2n(n + 2)$ *E6*
$3n + 6$ *E7*	$n^2 + 6n$ *E8*

L3.3 Template

<table>
<tr><th>Factorised Expression</th><th>Representation</th><th>Multiplied Out Expressions</th></tr>
<tr><td></td><td>
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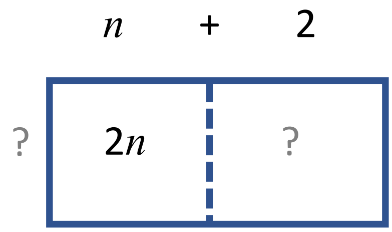
<tr><td></td><td>
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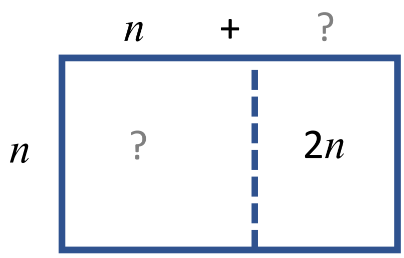
<tr><td></td><td>
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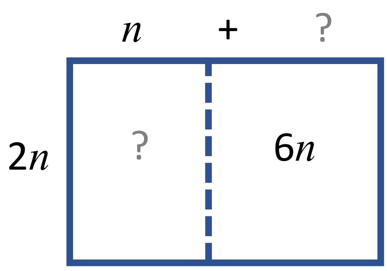
<tr><td></td><td>
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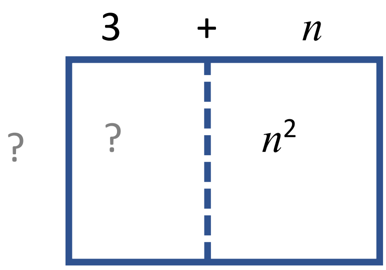
<tr><td></td><td>
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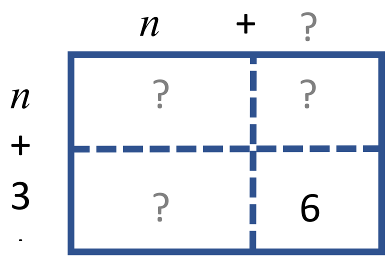
</table>

$n(n + 6)$		
		$2n^2 + 4n$

Extension

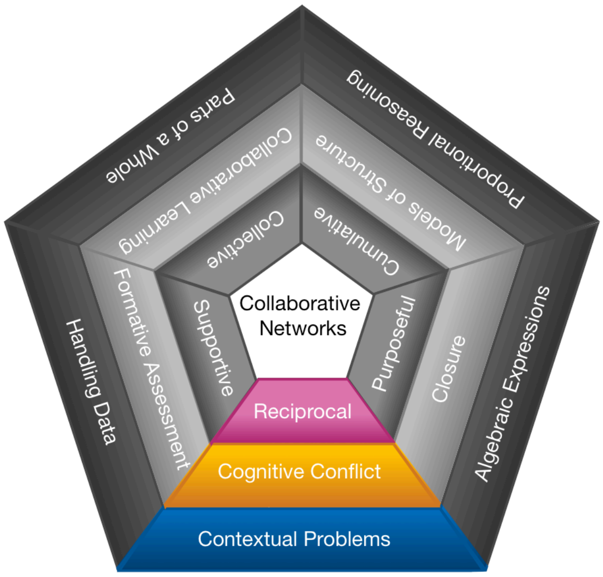
		

Lesson 4 Contextual Problems

Introduction

Overview

The focus in the lesson, **Contextual Problems**, is on how **cognitive conflict** contributes towards **reciprocal** dialogue.



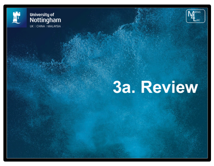
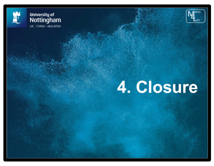
Dialogic learning should be **reciprocal** – that is students and teachers listen to each other, share ideas and consider alternative viewpoints.

Cognitive Conflict can occur at any time in the classroom when a learner is wrestling with uncertainty. It may occur when a predicted course of events is disrupted by a contradiction and existing methods and tools are found to be insufficient. It is in this struggling to resolve the conflict new learning takes place.

Research Question

How does the management of **cognitive conflict** help to develop a culture of **reciprocal** dialogic learning?

Lesson Summary

Phase	Timings (minutes)	Notes
	5	The initial problem is explained and worked on by students. *Highlight the link to ratios and the mathematical structure revealed by the diagrammatic representation.*
	10 - 15	Students work on questions 1 and 2. *These problems focus on two different types of relationships between the unknowns.*
	10	Check understanding of the activity using the review slides in the electronic presentation.
	5 - 10	A further problem that may prompt cognitive conflict is explained and worked on by students.
	10 - 15	Students work on questions 3 and 4. *These problems focus on students establishing both additive and multiplicative relationships between the unknowns.*
	10	Check understanding of the activity using the review slides in the electronic presentation.
	10 - 15	Highlight the difference between ‘times more’ and ‘more’.
		Extension questions used if appropriate.

L4.1 Lesson Outline: Contextual Problems

Mathematical goals

To help students:

- determine the unknown in a problem;
- translate statements that relate to an unknown in a problem into a mathematical form;
- determine the value of an unknown in a problem.

Starting points

Many students have followed procedural methods for solving linear equations. However, they may have had little practice at setting up and solving such equations in a way that encourages a more deeper understanding. The contextual problems in this lesson aim to do just that.

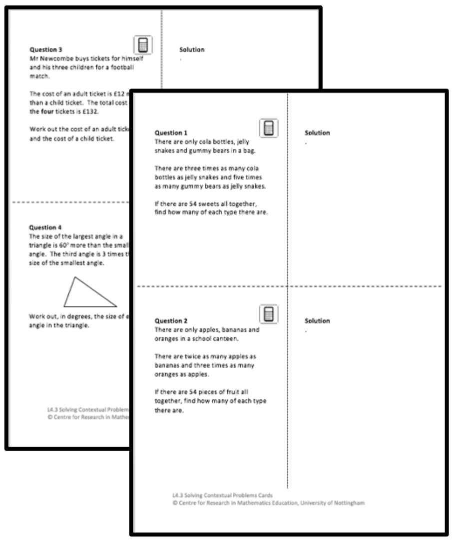
Some of these can be solved by trial and improvement. Whilst students can be successful using this method, it may not be efficient and may not be practical with 'difficult' numbers.

Materials required

- L4.3 Presentation;
- mini-whiteboards and pens;
- L4.4 Exam questions (for a follow-on lesson).

For each group of students, you will need:

- L4.2 Cards (Problems 1 and 2 together, problems 3 and 4 together);
- a calculator.



Time needed

Approximately 1 – 1.5 hours.

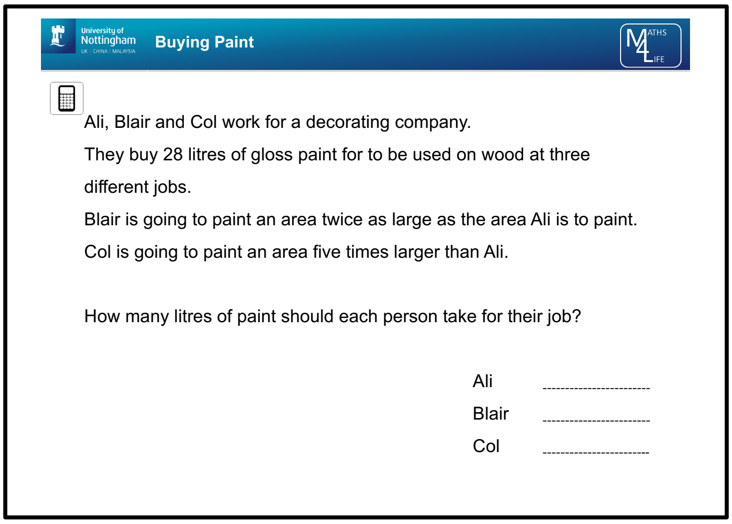
Lesson structure

1a. Setting the Scene

On mini-whiteboards ask students to attempt the first question on their own. If students are struggling to get started, suggest they use a trial and improvement strategy.

"The opening task sets the tone for the lesson and signals expectations of students"

1. What are the advantages of using trial and improvement, and when might it be problematic?

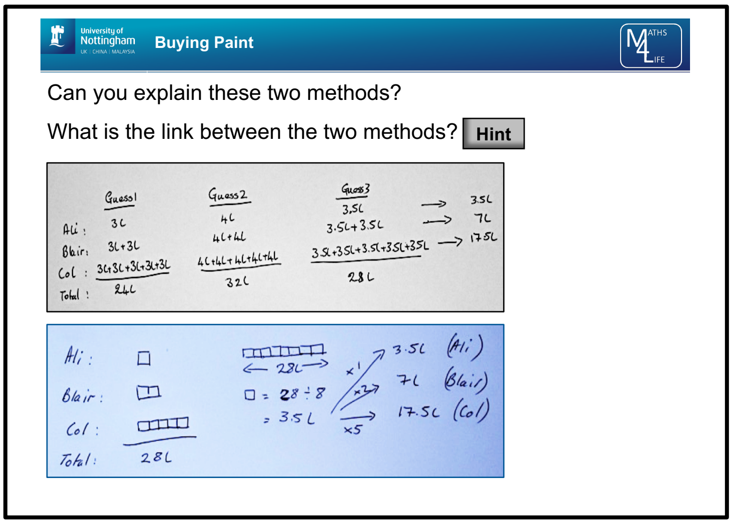


Ideally, ask a member of the class to explain their trial and improvement strategy and to identify which quantity they started with and why.

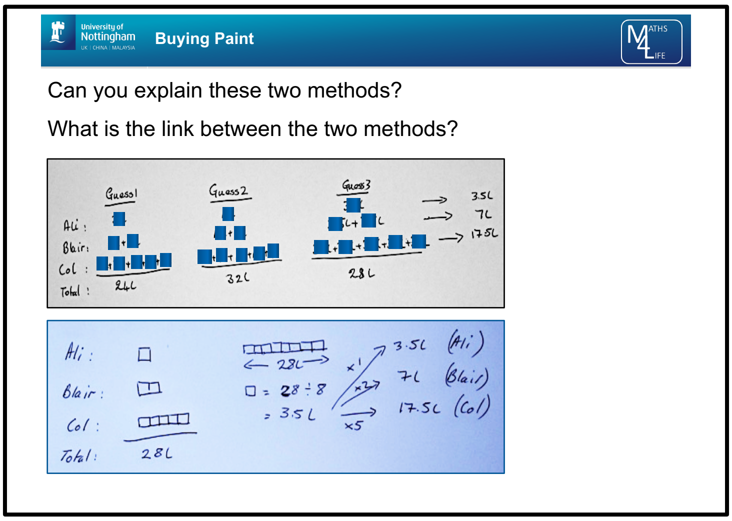
"Trial and error can help lead to understanding the algebraic structure"

Ask students to explain the following two methods, and ask whether they can identify the link between them. A hint is provided by clicking on the button.

2. What does a box in the blue method represent?



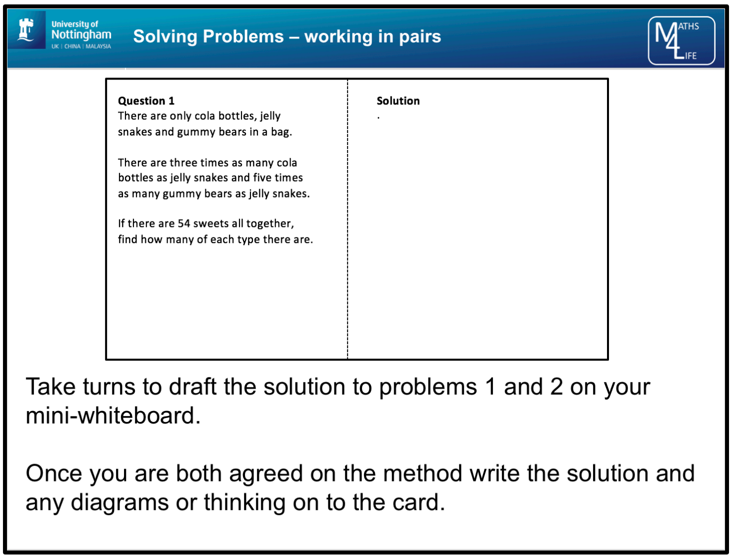
Note that the hint shows how the mathematical structure of the two methods is the same. In the first method, the value of one part is guessed until the total of 8 parts is 28 litres. In the second method, the total of 28 litres is divided by 8 to find the value of one (equal) part.



Also note the importance of identifying, in this question, that the quantity of paint is for Ali. It provides the base value that everything else is compared to.

2a. Cards: Collaborative Learning

For each group/pair, hand out the card for Questions 1 and 2. Ask one member of the group/pair to work on the mini-whiteboard for Question 1. When they are agreed on the method and solution, the other person should transfer their thinking on to the card. They should then swap roles for Question 2.



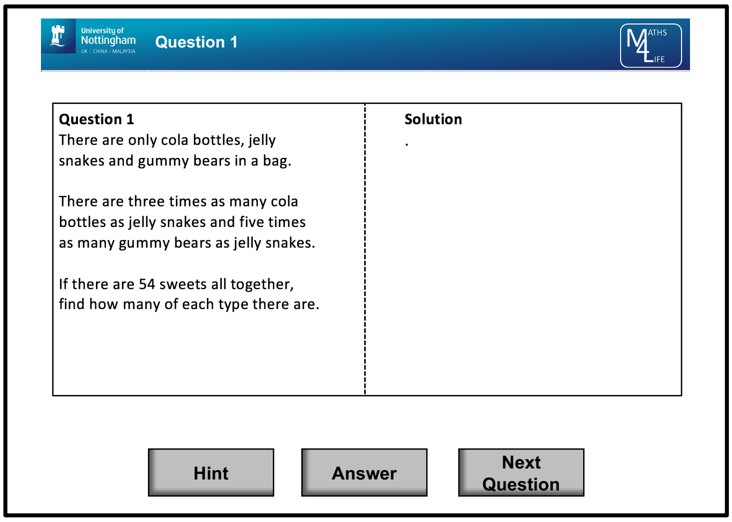
"Encourage students to share their own ideas and listen to their partner's ideas"

3a. Review

3. How do you expect students to tackle this question?

4. How can you encourage students to check whether their answer is sensible?

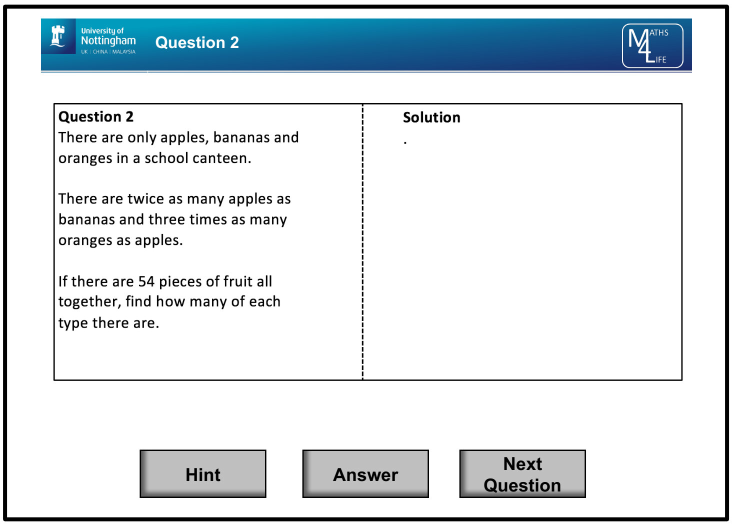
Bring the class together and go through each of the two questions in turn.



Note: Pressing the *Hint* button takes you to slide to support a discussion on a diagrammatic approach. You may want to ask students how the boxes shown on the slide could be used to represent the different types of sweets. Only use this slide if you think students are struggling to use a diagrammatic approach.

5. What do you expect students to find difficult in this question?

6. What is the best way to help them?

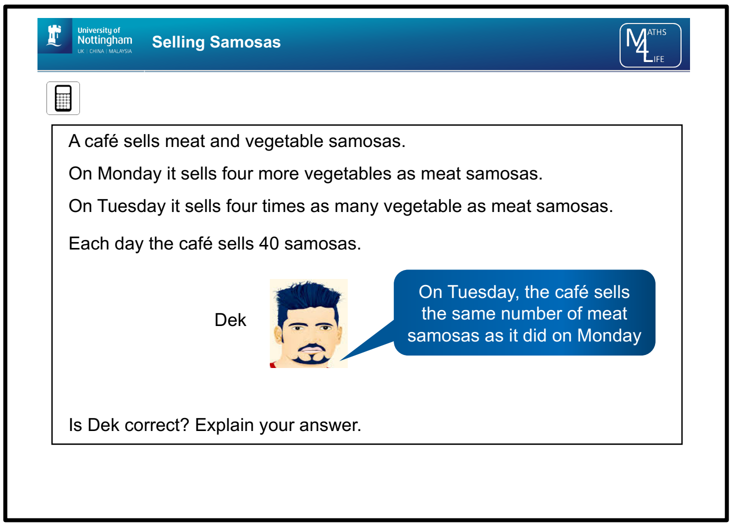


"Anticipate likely misconceptions and how you will respond to move students on"

It is important to notice that for both questions, the final stages of the solution involve the same mathematics, however, the answers to the two questions are different. This is because the relationships between the unknowns are different. Highlight the importance of checking that the final answers 'fit' the original problems. Depending on students method, checking the total may be insufficient. Students may need to also check relationships. If a group/pair completes these questions, ask them to start thinking about questions 3 and 4.

1b. Setting the Scene

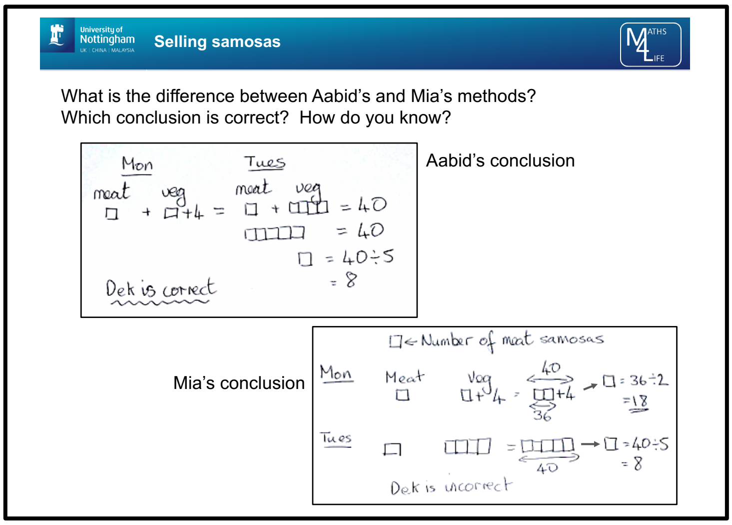
Now ask students to attempt a new problem.



"We mustn't jump in too early to try to fix problems'

Some students' conclusions may conflict with Dek's, others may not. Rather than resolve any issues, show the next slide. Ask students to identify which of the two methods shown on the slide are correct and any mistakes. You may want to also ask students to think about how they can check which is correct.

7. What are the important differences in the two methods that should be highlighted to students?



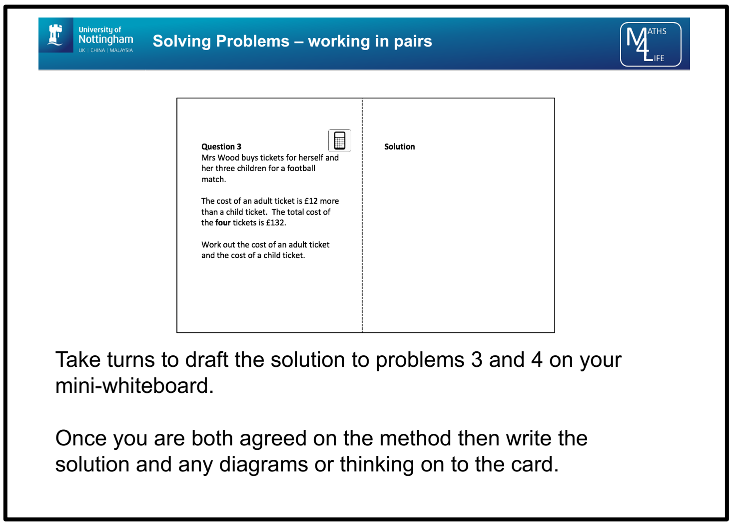
Draw attention to the difference between 'four times as many and 'four more'. The first means four lots of the same unknown, the second means one unknown plus four. Emphasise how not attending to such differences can result in very different answers.

Students may struggle with the notion that a box can represent different values. Point out that there are two different situations, and the box simply represents the unknown in the two situations.

2b. Cards: Collaborative Learning

For each pair, hand out the card for problems 3 and 4.

As before, ask person 1 to write on the mini-whiteboard for problem 3 and the other to write it on the card when they are both agreed. They should then swap roles for problem 4.

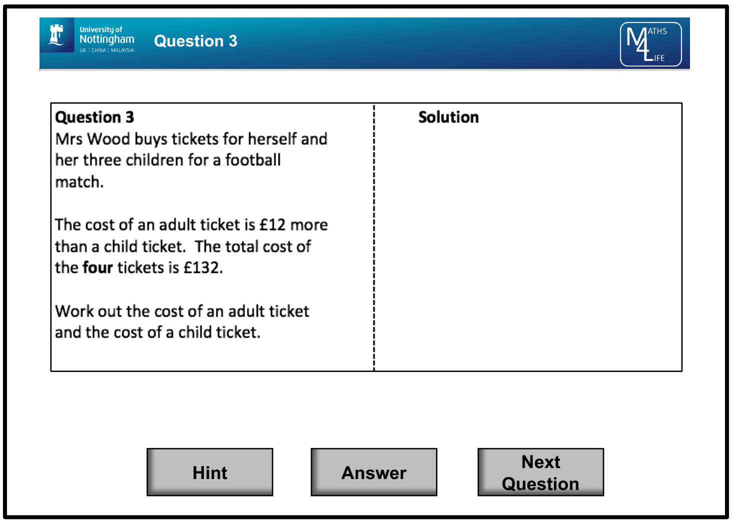


An extension question is provided in the presentation for any group that completes problems 3 and 4.

3b. Review

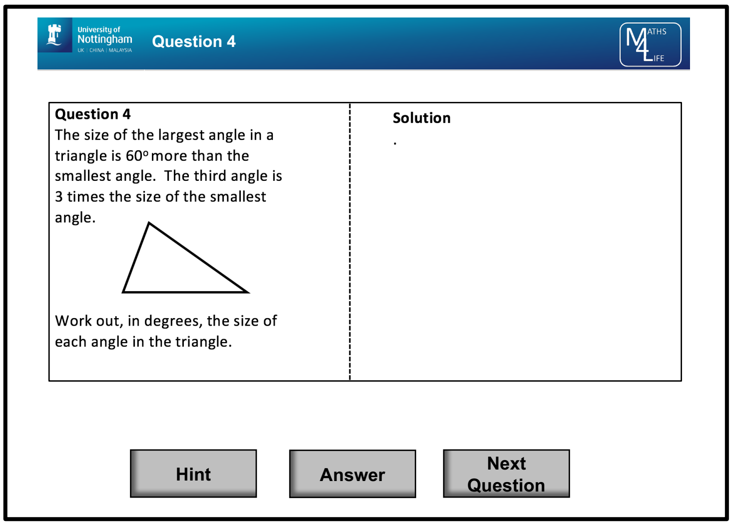
Bring the class together and go through each of the two problems in turn.

8. What do you expect students to find difficult in this question?



“We should encourage students to share their thinking with the whole class”

9. What do you expect students to find difficult in this question?



“Allow plenty of time to ‘close’ the lesson”

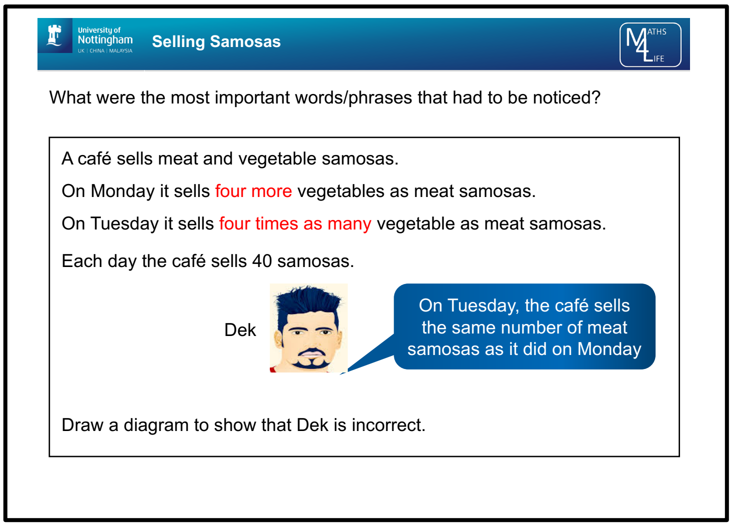
Once again, notice that the *Hint* buttons can be used to show a possible diagrammatic representation if required.

Closure

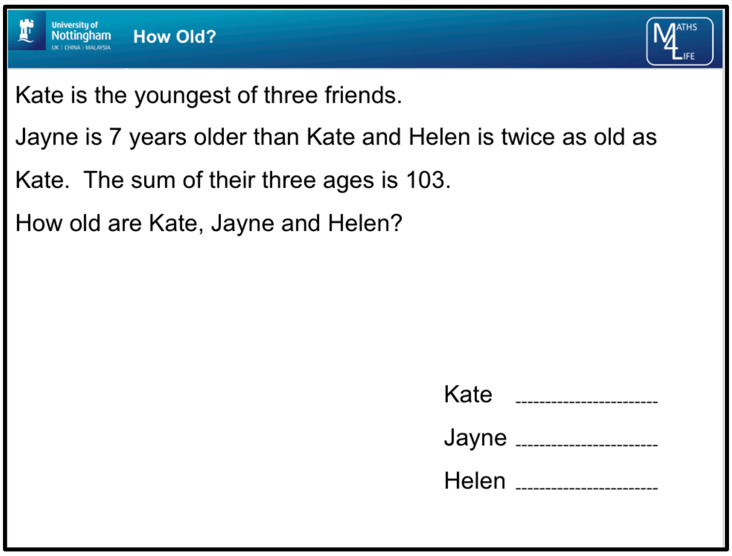
10. What are the key messages from the lesson that you expect to draw out?

These tasks have required students to identify a ‘base’ value that all other values relate to. Once that is identified a representation can be drawn that captures the structure of the relationships between values in a way that allows first one unknown, and then the others, to be evaluated.

Ask students to notice the key phrases on the task that resulted in different answers for the two days (Selling Samosas problem).



With this key point in mind, students should attempt the next question.



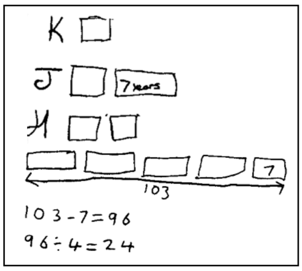
Once students have attempted the question, reveal the two methods on the next slide and ask them to explain how they relate to one another.

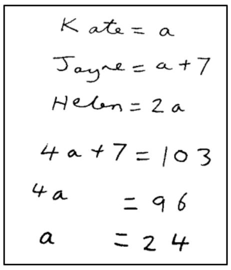
How Old?

Kate is the youngest of three friends.

Jayne is 7 years older than Kate and Helen is twice as old as Kate. The sum of their three ages is 103.

How old are Kate, Jayne and Helen?





"Continue to draw on student thinking and share this with the class."

One final task is provided with the intention of helping students to see how to tackle questions that provide an algebraic equation taken from the context.

Going deeper…

Fran has a friend, James, who is 4 years older than her.

Paul is three times as old as Fran.

The sum of their three ages is 89.

If x represents the age of Fran, show that **5x = 85** and hence find how old Paul, James and Fran are.

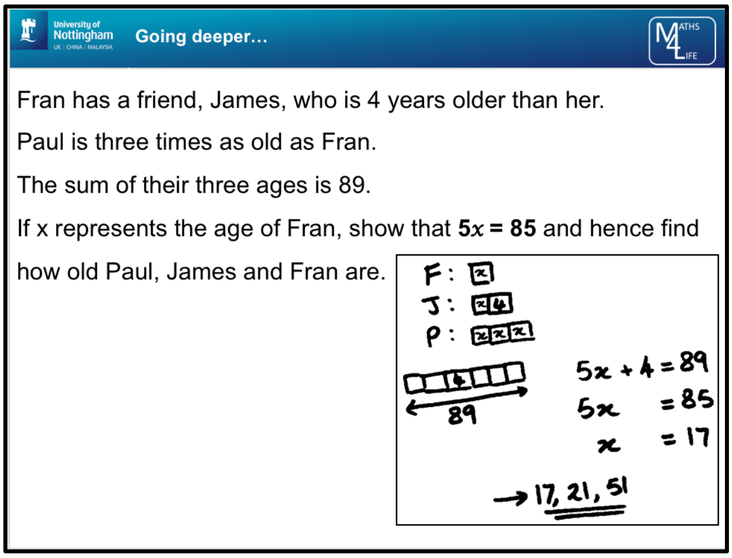
Paul ....

James ....

Fran ....

If necessary, ask students to start by solving the problem using a diagrammatic approach and then to consider how it links to the algebra. They can then compare their solution to the student work provided and see how the algebra can be brought out from the diagram.

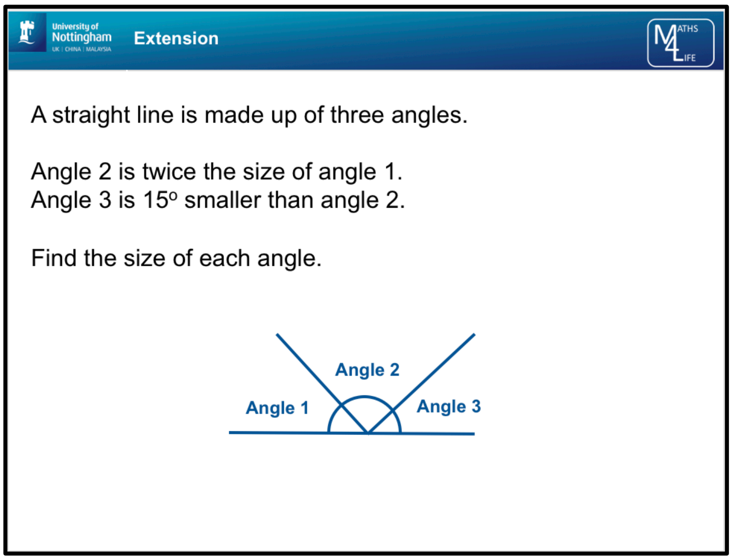
"It's useful to share a range of different diagrams with the whole class"



Extension

If not already used, students can extend their thinking by looking at a problem that involves negative values.

11. What is different about this extension question?



L4.2 Cards

Cards for the students to use are on the following pages.

Question 1
There are only cola bottles, jelly snakes and gummy bears in a bag.

There are three times as many cola bottles as jelly snakes and five times as many gummy bears as jelly snakes.

If there are 54 sweets all together, find how many of each type there are.

Solution

Question 2
There are only apples, bananas and oranges in a school canteen.

There are twice as many apples as bananas and three times as many oranges as apples.

If there are 54 pieces of fruit altogether, find how many of each type there are.

Solution

Question 3

Mrs Wood buys tickets for herself and her three children for a football match.

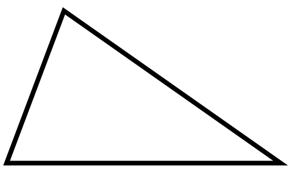
The cost of an adult ticket is £12 more than a child ticket. The total cost of the **four** tickets is £132.

Work out the cost of an adult ticket and the cost of a child ticket.

Solution

Question 4

The size of the largest angle in a triangle is 60° more than the smallest angle. The third angle is 3 times the size of the smallest angle.



Work out, in degrees, the size of each angle in the triangle.

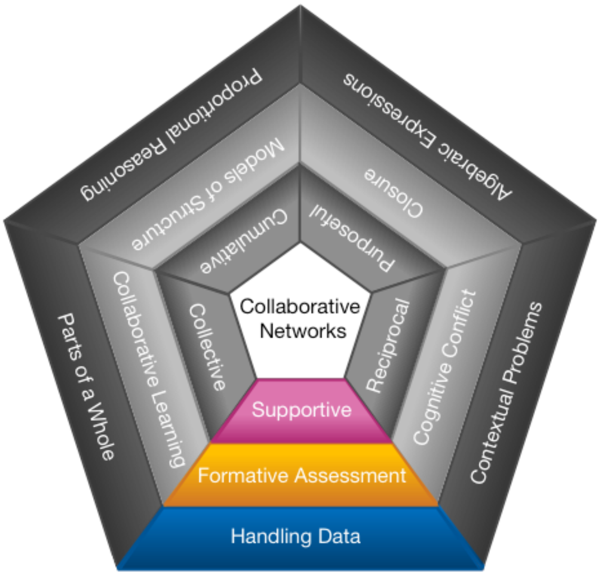
Solution

Lesson 5 Handling Data

Introduction

Overview

The focus in the lesson, **Handling Data**, is on how **formative assessment** contributes towards **supportive** dialogue.



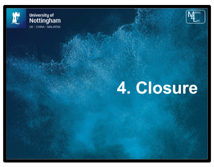
Formative assessment is assessment that provides information on what to do next.

Supportive dialogue is when ideas are expressed freely, without risk of embarrassment.

Research Question

How does the use of **formative assessment** help to develop an environment of **supportive** dialogic learning?

Lesson Summary

Phase	Timings (minutes)	Notes
		Students must complete **L5.5 Pre-Lesson Assessment** and feedback provided prior to this lesson.
	**15 - 20**	Students work on the initial problem. *Students should have the opportunity to consider the various definitions of averages. Ensure that students understand how to construct a table and bar chart from their own class data on fruit and vegetables eaten. Also, students together, should establish how to work out averages from the bar chart.*
	**10**	Students match tables to bar charts on the template. *Observe what students are doing, ask students questions. The purpose of these questions should be to both support students learning and understand better their thinking. The Common Issues table can be used as a supportive teaching tool.*
	**5 - 10**	Check understanding of the activity using the review slides in the electronic presentation.
	**10 - 15**	The purpose of the activity is to both check students' understanding of key concepts, and extend their thinking. As such Closure provides an opportunity for both teacher and students to consider the extent the learning goals have been met.
	**10**	The extension consists of one problem. Students first provide an answer, and then consider the mistakes other students have made.

L5.1 Lesson Outline: Handling Data

Mathematical goals

To help students:

- understand the relationship between data and its representations;
- understand and be able to find measures of location and spread.

Starting points

Too often students will have been asked to simply find measures of location and spread from a decontextualised set of numbers. This can lead to a procedural understanding of finding summary statistics.

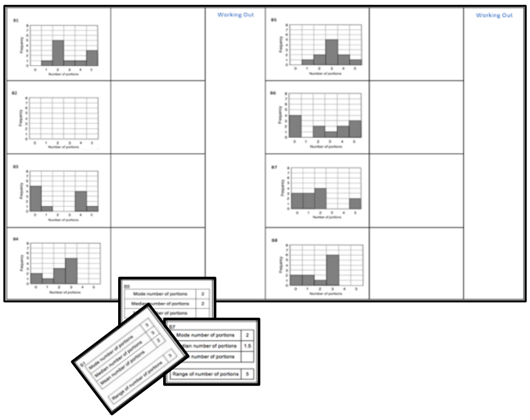
This problem is often compounded when data is displayed in the form of a frequency diagram. Students have particular difficulty in understanding how the data in the frequency diagram relates to an actual event.

Materials required

- L5.6 Presentation;
- L5.7 Spreadsheet.

For each group of students, you will need:

- a calculator;
- L5.2 Template printed on to A3 paper;
- L5.3 Cards;
- glue sticks;
- L5.4 Data Summary Sheet;
- L5.5 Pre-lesson Assessment;
- mini-whiteboards and pen.



Time needed

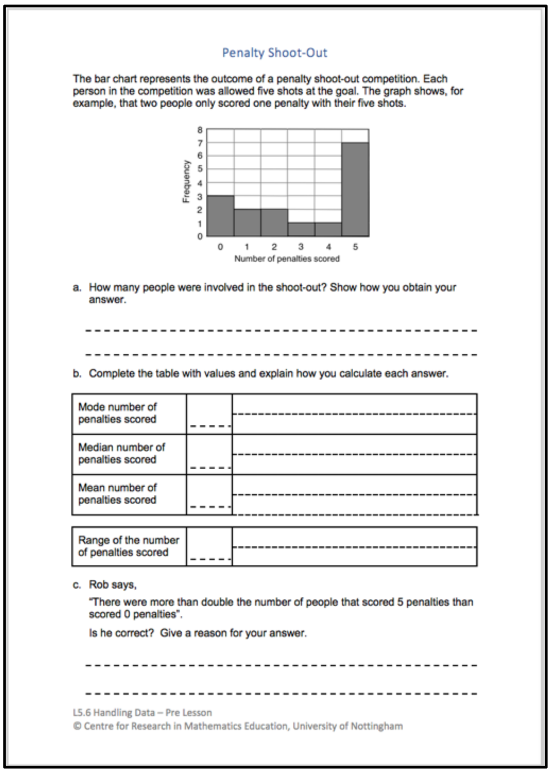
Approximately 1 – 1 ½ hours.

Lesson structure

Before the lesson

"The assessment task helps to motivate students to really want to understand."

Have students attempt **L5.5 Pre-lesson Assessment**, in class or for homework, a few days before the lesson. This will give you an opportunity to assess the work, to find out the kinds of difficulties students have and help you target your lesson effectively.



Penalty Shoot-Out

The bar chart represents the outcome of a penalty shoot-out competition. Each person in the competition was allowed five shots at the goal. The graph shows, for example, that two people only scored one penalty with their five shots.

a. How many people were involved in the shoot-out? Show how you obtain your answer.

b. Complete the table with values and explain how you calculate each answer.

Mode number of penalties scored		
Median number of penalties scored		
Mean number of penalties scored		
Range of the number of penalties scored		

c. Rob says,

"There were more than double the number of people that scored 5 penalties than scored 0 penalties".

Is he correct? Give a reason for your answer.

L5.6 Handling Data – Pre Lesson
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It is important that, as far as possible, students are allowed to answer the questions without your assistance. Students should not worry too much if they cannot understand or do everything, because in the next lesson they will work on a similar task, which should help them. Explain that by the end of the next lesson, they should be able to answer questions such as these confidently.

1. How will the information you find out about the students' understanding affect the way you teach this lesson?

Before the main lesson, collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different approaches.

It is suggested that you do not put a mark on student's work. Research shows that this will be counter-productive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics. Instead, help students to progress by summarising their difficulties with one or two questions.

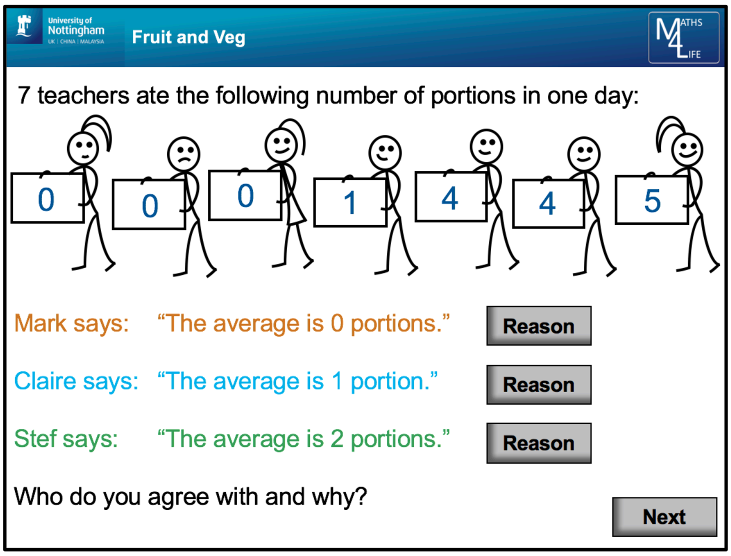
Some suggestions for these are given in the common issues table on the next page. These have been drawn from common difficulties observed in trials of this unit. Although the issues concern the specifics of the pre-lesson task, they are common to other lessons.

Common Issues	**Suggested questions and prompts**
Misinterprets the axes on the bar chart For example: The student states that there were five people involved in the shoot-out. Or: The student does not understand the term 'Frequency'.	• *What does the term 'frequency' mean?* • *How many people scored two goals? How many scored three?*
Reads off the frequency of the tallest bar as the mode, rather than the score For example: The student gives the mode as 7.	• *How many penalties did each person take?* • *Which score happened the most? How can you tell?*
Confuses the position of the median with the value for the median For example: The student adds one to the total frequency and divides by two to give a median of 8.5. Or: The student just halves the frequency. Or: The student assumes the median is 2.5, half way between 0 and 5. Or: The student writes two values for the median, 3 and 4.	• *The median is the middle score when all the scores are in order. Is this what you have found?* • *Try writing the scores in order: 0, 0, 0, 0, 1, 1, 2, …Which is the middle score?* • *How could you do this directly from the frequency graph without writing a list?*
Uses incorrect values to calculate the mean For example: The student finds the total of the frequencies rather than the total number of goals. Or: The student divides by six rather than the total frequency. Or: The student adds the scores (0+1+2+3+4+5) and divides this total by six.	• *How many goals were scored?* • *Five goals were scored seven times. So, what is the total number of goals? Compare this to your total, what do you notice?* • *Imagine writing the scores out as a list. From this list, how would you work out the mean?*
Presents the range as two figures, the highest and the lowest scores	• *What calculation is needed to obtain the range?*
Calculates the range in frequencies rather than the range of goals scored	• *What was the highest number of goals scored?* • *What was the lowest number of goals scored?*
Completes the task The student needs an extension task.	• *Can you produce a different bar chart that would have the same statistical measures? What is the same and what is different?*

Setting the scene

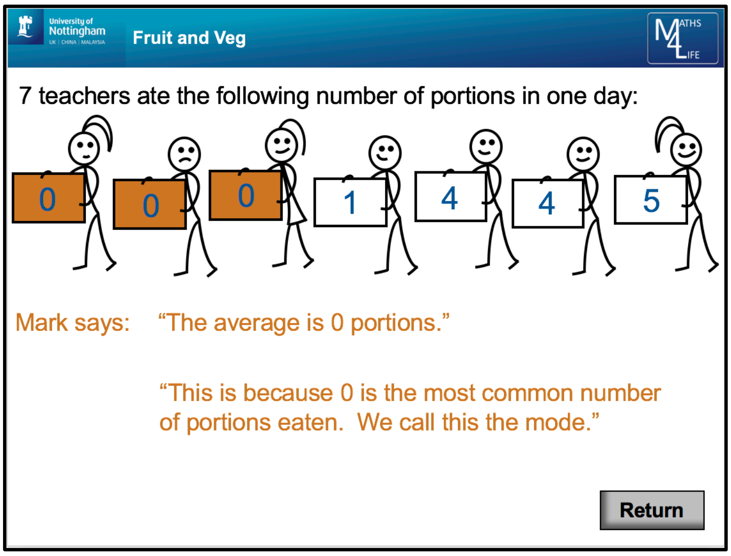
First provide a general overview of how students' work on the pre-lesson assessment has helped guide the learning goals of the lesson. Then introduce the lesson by displaying the following slides:

"Be sensitive to students' mistakes. Point out that they are in line with what many students think."

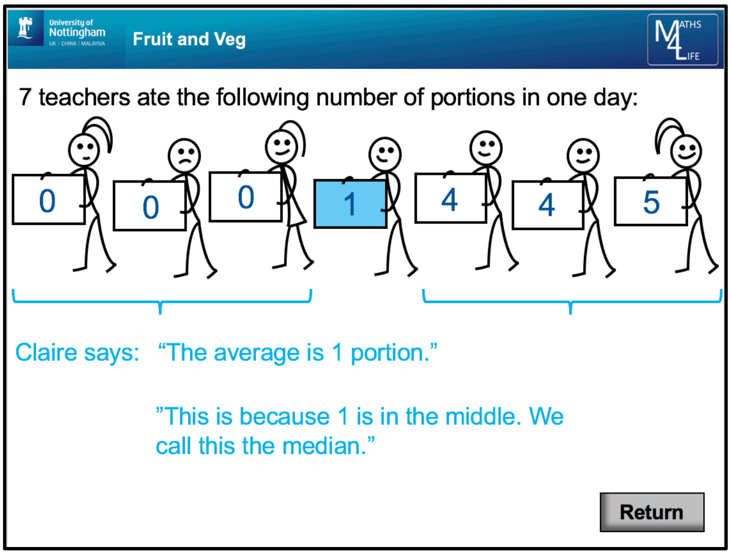


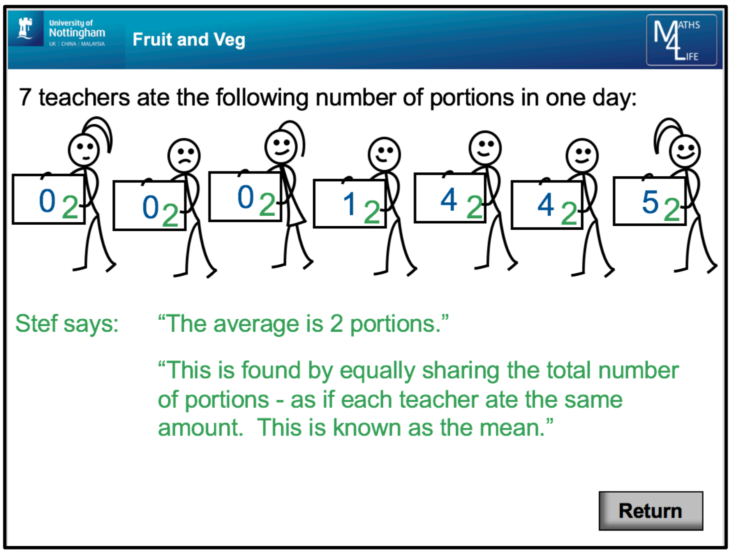
Ask students to decide on their own who they agree with and why, before sharing their ideas with a partner.

Ask students to share their ideas with the whole class before clicking on the appropriate button to see an explanation and definition. Be sure to emphasise that average is the value representing a data set. Although it could be argued that all three are correct, presented separately could be misleading.



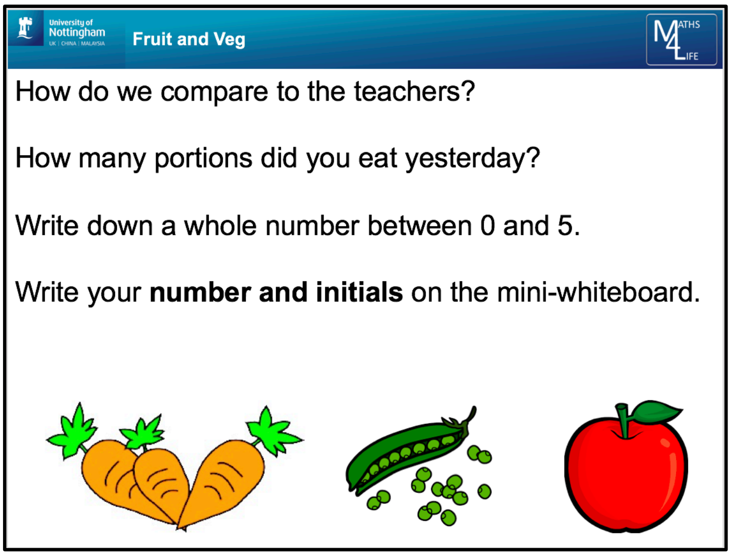
2. Is this the 'best' definition of the median?





Now ask students to write the number of portions of fruit or vegetables that they ate yesterday on their mini-whiteboards.

3. How can you ensure that the data you collect from students and yourself is as helpful as possible?



"Involve students personally in the data they will work with"

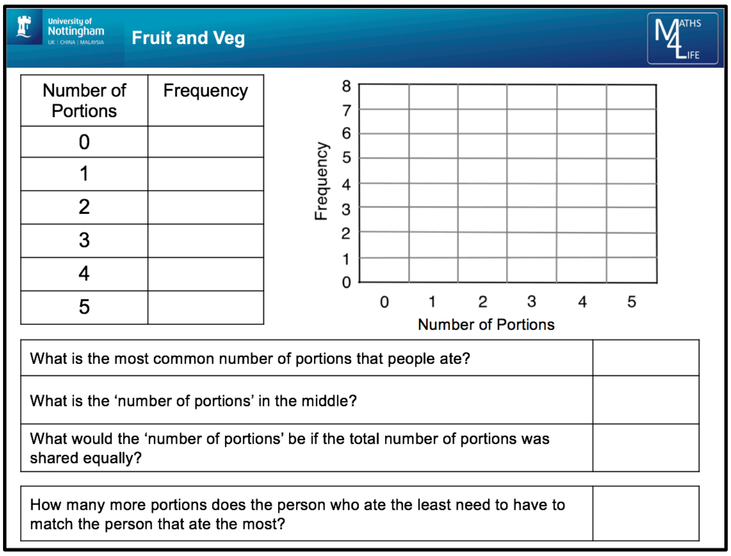
"Use A4 sheets of paper if mini whiteboards are not available"

On your whiteboard show me how many portions of fruit or vegetables you ate yesterday.
It must be a whole number between 0 and 5, so 2½ would not be allowed.
If you ate more than 5 then just write 5 for what we're doing.
Please also write your initials on the whiteboard.

Collect the mini-whiteboards from everyone who said 0, then everyone who said 1, and so on. Spread the mini-whiteboards out on the floor in a line from the least to the most portions.

Data can now be recorded on the electronic presentation (both the table and the bar chart), or on L5.7 Spreadsheet which will automatically generate the bar chart. For each student, add their number of portions into the section of the spreadsheet 'Raw Scores'. Then, if you would like them sorted in numerical order, press the 'Sort' button. The 'Show' buttons will show the respective statistics. Although this is a very straightforward spreadsheet, it may be useful to practice your understanding of it before the lesson.

Students may record the data on L5.4 Data Summary Sheet if required.



4. Why do we use the word 'frequency'?

Check that students understand the term 'Frequency'.
In this case, can you think of an equivalent phrase?
Why do we use 'Frequency' instead of (the equivalent phrase)?

Ask students for the average number of portions eaten. This will allow comparison with the teacher data.

The first measure of location suggested by students is usually the mode. Point out how this can be seen easily on both the frequency table and the bar chart.

5. What different ways can be used to help students make sense of finding the median?

Using the mini-whiteboards, find the median number of portions by counting to the middle from each side. Now re-arrange the mini-whiteboards on the floor to form a bar chart. Maintain the order of the whiteboards by keeping the left most whiteboard in the line as the base of the bar. Ask students how they would be able to identify the median directly from a bar chart.

Now ask students to identify what would be the number of portions if everyone ate the same.

Finally, ask students to identify the range of the number of portions eaten.

"Involve students by asking them questions about where they are in relation to the statistical measures."

Can you say how many more portions the person that eats the lowest number would need to eat to match the person that eats the most?

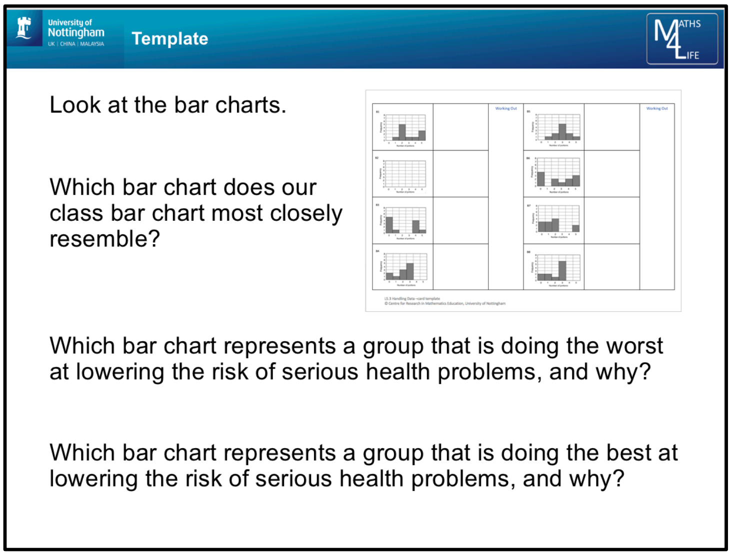
Note that visually this is moving the lowest bar the number' of columns to get to the highest bar.

Collaborative Learning using Cards

Organise the class into pairs of students and give each group the card template, the matching cards and a glue stick.

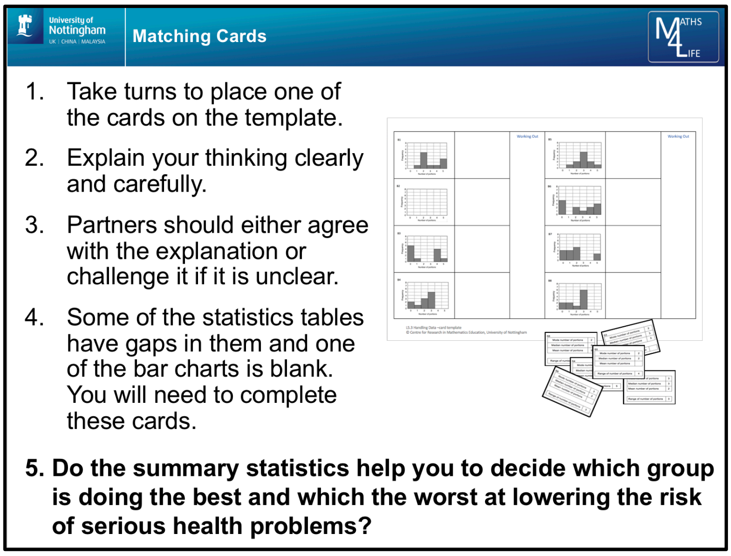
Ask students to look at the bar charts and make comments using the following slide.

6. Why is this an important stage in the lesson?



Give students a couple of minutes to discuss and listen to a few ideas. Explain that we will now look at the summary statistics to see if they support their suggestions. Remind students that a purpose of finding summary statistics is to allow comparisons. Furthermore, it is still possible to compare, when we have a different number of pieces of data.

Explain how students are to work collaboratively using these instructions.

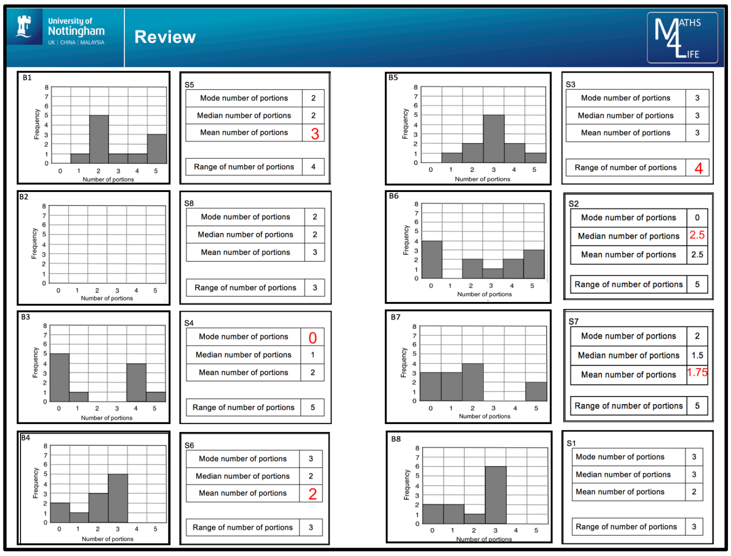


7. How will you react if students are finding the task difficult?

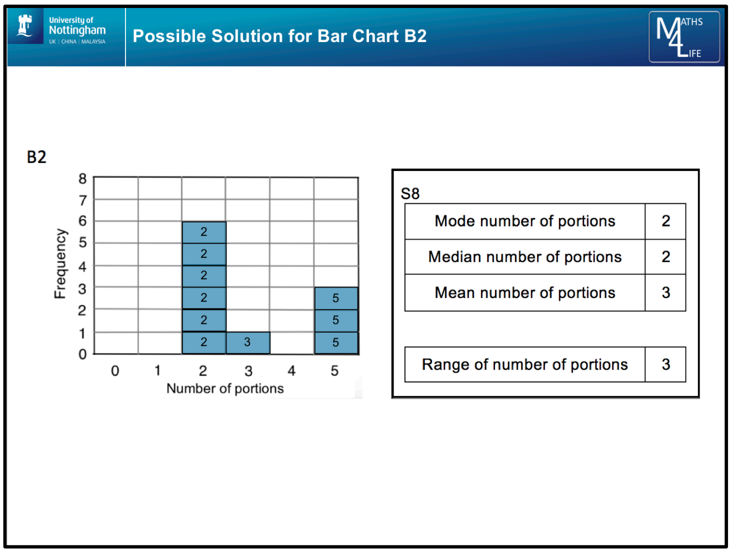
"Anticipate students' likely misconceptions and how you will respond"

Review

Draw the learning together by projecting the matchings to students and checking any questions or issues that they have.



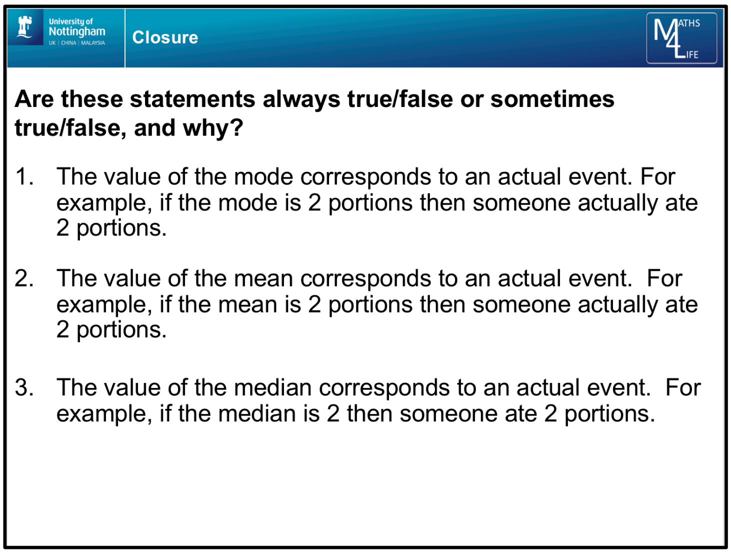
A possible answer for bar chart B2 is also included in the PowerPoint presentation.



Closure

Use the following slide in whole class discussion to develop a better understanding of the measures of location and the measure of spread.

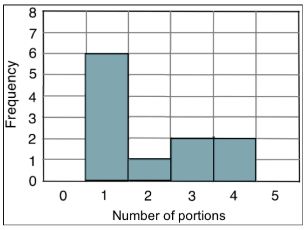
8. Which bar charts in the matching can be referred to in these questions?



The next three slides are designed to help students consider what-if style questions. Leaners should be encouraged to recalculate values and offer a reasoned explanation. (The slides are animated to aid understanding).

Closure

4. What would happen to the the bar chart below if everyone represented on it ate 1 more portion each?

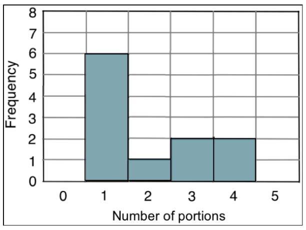


Mode number of portions	1
Median number of portions	1
Mean number of portions	2
Range of number of portions	3

9. How can you convince students of this?

Closure

5. How would the mode, median, mean and range change if everyone represented on the bar chart ate 1 more portion each?

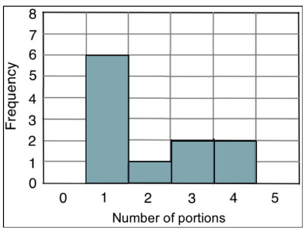


Mode number of portions	1
Median number of portions	1
Mean number of portions	2
Range of number of portions	3

10. What are the key messages from the lesson that you expect to draw out?

Closure

6. What would happen if everyone represented on the bar chart below ate 1 less portion each?



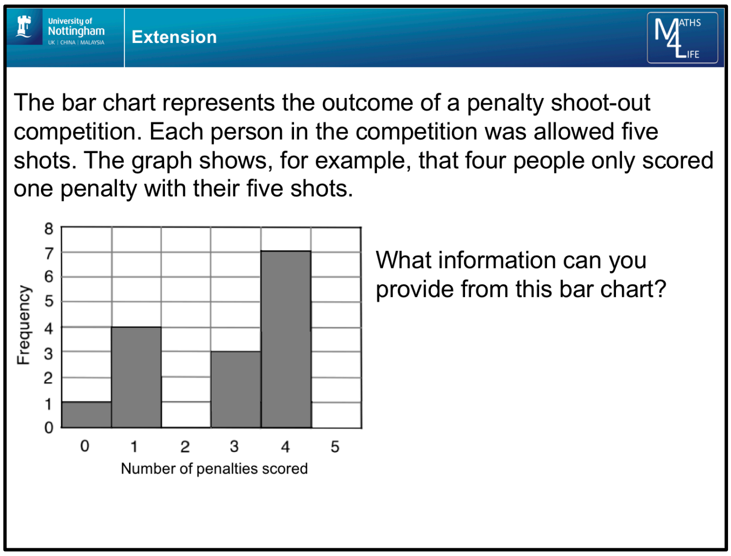
Mode number of portions	1
Median number of portions	1
Mean number of portions	2
Range of number of portions	3

"Perhaps have a whole-class discussion about what they have learnt."

Extension

The final set of slides may be used during this lesson or in a subsequent lesson to review understanding. They give the opportunity for some further formative assessment by asking the students to write down anything they can tell from the bar chart. Subsequent slides are more structured with a number of common mistakes and misconceptions included.

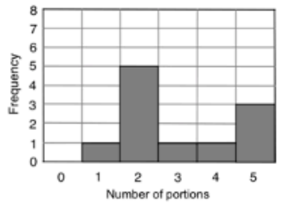
11. Can you identify the mistakes and misconceptions that have been made in the electronic presentation?



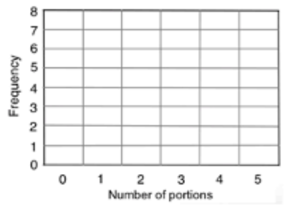
L5.2 Template

Template for students to use is on the following page.

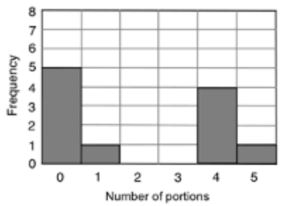
B1



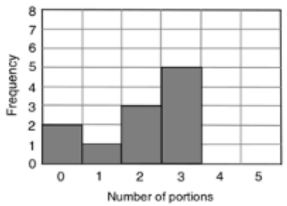
B2



B3

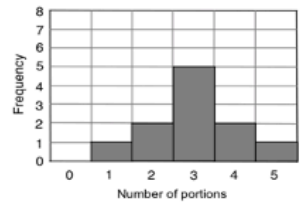


B4

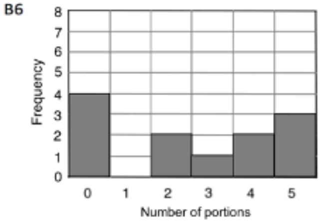


Working Out

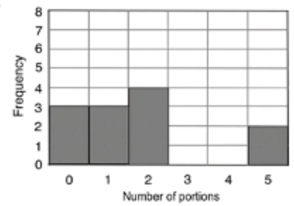
B5



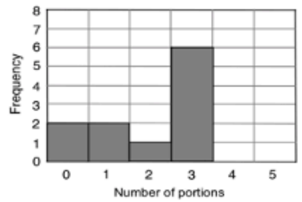
B6



B7



B8



Working Out

L5.3 Cards: Statistics Tables (S1 – S8)

S1

Mode number of portions	3
Median number of portions	3
Mean number of portions	2
Range of number of portions	3

S2

Mode number of portions	0
Median number of portions	
Mean number of portions	2.5
Range of number of portions	5

S3

Mode number of portions	3
Median number of portions	3
Mean number of portions	3
Range of number of portions	

S4

Mode number of portions	
Median number of portions	1
Mean number of portions	2
Range of number of portions	5

S5

Mode number of portions	2
Median number of portions	2
Mean number of portions	
Range of number of portions	4

S6

Mode number of portions	3
Median number of portions	2
Mean number of portions	
Range of number of portions	3

S7

Mode number of portions	2
Median number of portions	1.5
Mean number of portions	
Range of number of portions	5

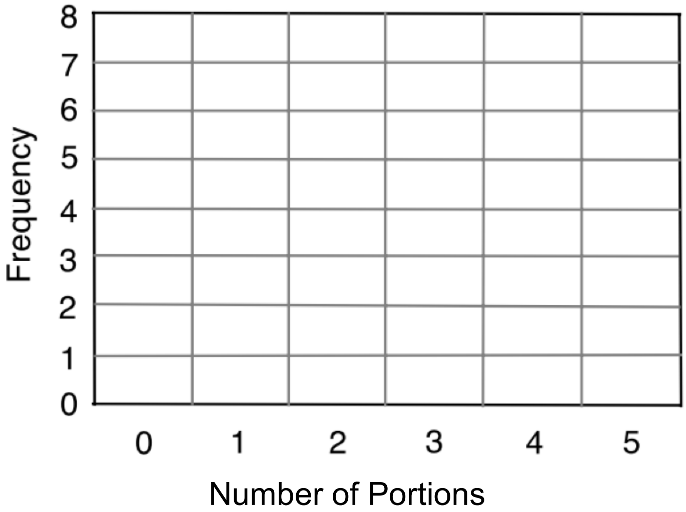
S8

Mode number of portions	2
Median number of portions	2
Mean number of portions	3
Range of number of portions	3

L5.4 Data summary sheets

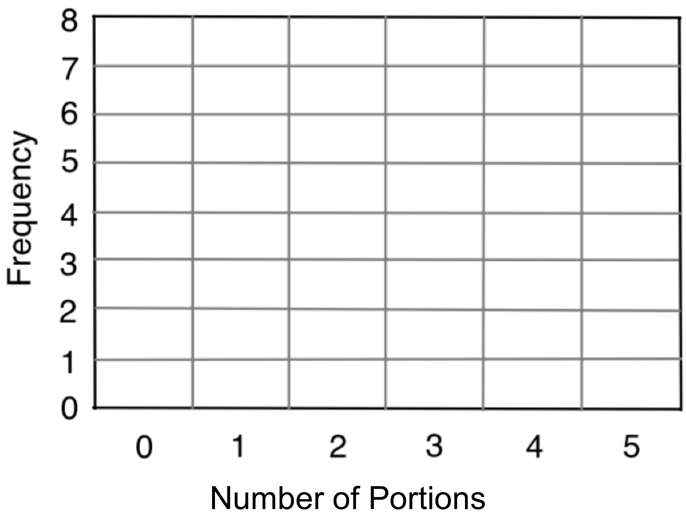
Data summary sheet 1

Number of Portions	Frequency
0	
1	
2	
3	
4	
5	



Data summary sheet 2

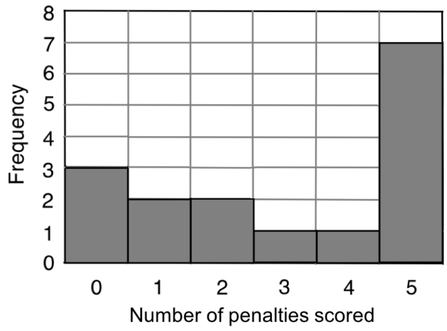
Number of Portions	Frequency
0	
1	
2	
3	
4	
5	



L5.5 Pre-lesson assessment

Penalty Shoot-Out

The bar chart represents the outcome of a penalty shoot-out competition. Each person in the competition was allowed five shots at the goal. The graph shows, for example, that two people only scored one penalty with their five shots.



a. How many people were involved in the shoot-out? Show how you obtain your answer.

b. Complete the table with values and explain how you calculate each answer.

Mode number of penalties scored		
Median number of penalties scored		
Mean number of penalties scored		
Range of the number of penalties scored		

c. Rob says,

"There were more than double the number of people that scored 5 penalties than scored 0 penalties."

Is Rob correct? Give a reason for your answer.