Observatory for Mathematical Education

Validation of secondary pupil attitudes survey

Technical Report 2025/1

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Abstract

This technical report describes the validation of a survey of secondary school pupils' attitudes towards mathematics. The survey will be used in a 7-year longitudinal study of pupils in England which is being conducted by the Observatory for Mathematical Education. It is designed to measure pupils' attitudes in four dimensions: perceived value of learning mathematics, confidence in mathematics, enjoyment of mathematics, and anxiety about mathematics, using 32 items adapted from previously validated scales. The validation was conducted with a sample of Year 7 pupils (N = 392). Confirmatory factor analysis was used to assess the convergent validity, discriminant validity and internal consistency of the scales. The survey was found to be sufficiently reliable and valid for the proposed use.

1. Introduction

The Observatory for Mathematical Education will be carrying out a 7-year longitudinal study with a starting cohort of Year 7 pupils in England. The programme aims to track the learning of mathematics of secondary school pupils, including their development of attitudes towards mathematics and relationships between attitudes and other characteristics of the pupils related to the learning of mathematics. To do this, a reliable and valid instrument for attitudes towards mathematics is essential for quality data collection, and hence, a pupil survey was developed to serve this purpose.

1.1 The survey

The full survey consists of 61 items, of which 32 items are designed to measure pupils' attitudes in four dimensions: *perceived value of learning mathematics*, *confidence in mathematics*, *enjoyment of mathematics*, and *anxiety about mathematics*. The items relating to perceived value of learning mathematics and enjoyment of mathematics were adapted from the Attitudes Toward Mathematics Inventory (Tapia & Marsh, 2004), which was validated by the developers (and others) and used with, but not limited to, undergraduate students in the United States (Tapia & Marsh, 2002) and junior secondary school pupils in Australia (Majeed et al., 2013). The items relating to confidence in mathematics were adapted from the Fennema-Sherman Mathematics Attitudes Scales (Fennema & Sherman, 1976), which was validated by the developers (and others) and used with, but not limited to, secondary school pupils in the United States (Fennema & Sherman, 1976) and primary school teachers and pre-service teachers in the United States (Ren et al., 2016). The items relating to anxiety about mathematics were adapted from the mathematics anxiety scale from the OECD Programme for International Student Assessment (PISA) 2022 (OECD, 2024), which was validated by the developers and used with 15-year-old school pupils internationally (OECD, 2024).

There was a need to validate the 32 adapted items for use with secondary school pupils in England, given that the items have been amended and used in a different population (Andrews & Diego-Mantecón, 2015). The remaining 29 survey items are designed to gather data of pupils' perception of their learning experience in mathematics and are to be analysed item-wise in the main study, and hence, they are not considered in the validation reported here.

1.2 The technical report

This report describes the processes and results of the validation of the pupil survey. This report focuses on its factorial validity and internal consistency (with reference to other measures, such as response missingness and distributions) and serves to give technical details of the analysis. In the next section, the analysis methods are discussed, followed by text that guides readers through tables and figures of the analysis, where appropriate. Finally, the conclusion summarises the results and discusses possible future research. To establish the validity and reliability of the analysis in this report, sensitivity analysis was conducted with exploratory factor analysis and item response theory, with the results reported in Appendices 6.2 Exploratory factor analysis and 6.3 Item response theory.

2. Methods

The analysis aimed to examine convergent validity, discriminant validity and internal consistency of the scales; in other words, to evaluate whether the groups of items measure what is intended. Convergent validity refers to ‘the agreement between two attempts to measure the same trait through maximally ***different*** methods’ (Campbell & Fiske, 1959, p. 83), and discriminant validity refers to a trait that ‘can be meaningfully differentiated from other traits’ (ibid., p. 100). Internal consistency refers to ‘the agreement between two efforts to measure the same trait through maximally ***similar*** methods’ (ibid., p. 83); in other words, how well the items within a scale measure different aspects of the same trait.

To address these aims of the analysis, confirmatory factor analysis (CFA) was conducted to test models that assume different relationships among the items (factor models), with the use of correlations between the items. The analysis was conducted with R Version 4.4.1 (R Core Team, 2024) and its packages ***lavaan*** (Rosseel, 2012) and ***semTools*** (Jorgensen et al., 2022). A link to the code is provided in Appendix 6.1 R code. This section is divided into three parts: the first part discusses participants involved and administration of the survey in this study, the second part provides descriptive statistics of the data collected, and the third part discusses the analysis of convergent validity, discriminant validity and internal consistency using CFA and the factor models tested in this analysis.

2.1 Participants and administration of the survey

Four secondary schools in England were recruited in a pilot study. The schools were different in number and age of pupils, and location and type of the schools. Table 1 shows the characteristics of the four schools.

	School 1	School 2	School 3	School 4
Location	Conurbation	Urban	Rural	Conurbation
Type	Non-selective, academy converter	Non-selective, foundation school	Non-selective, religious, academy converter	Non-selective, academy converter
School size	800-900	1100-1200	1300-1400	700-800
Age range of pupils	11-16	11-18	11-18	11-16
% pupils entitled to free school meals	20-30	10-20	10-20	20-30

Table 1 Characteristics of the participating schools.

The administration of the survey was conducted by the schools: after administering paper copies of the surveys to their pupils, the schools collected and sent the copies to Cleardata UK, commissioned by the Observatory to perform data entry. A total of 484 copies of the survey were returned to the Observatory, of which 37 were spare copies not assigned to pupils, 39 were not completed due to pupil absence on the day of survey administration, and a further 6 were assigned to pupils but not completed. Of the remaining 402 copies in which pupils’ responses have been recorded, 10 copies indicated that the pupils chose not to participate in the pilot study. As a result, responses of 392 pupils were used in the analysis.

2.2 Survey response missingness and distributions

The participating schools replied that pupils were able to complete the survey in 10-20 minutes and did not express any negative feelings about it. Of the 392 pupils, about 77% answered all survey items. Out of 23,912 possible responses (= 392 pupils × 61 items), 29 responses were invalid due to multiple options being chosen for a single item. For each item, the missingness, including invalid responses, ranged between 0% and 5% with a mean of 2% and a median of 2%. Figure 1 shows that the missingness increased with the item index, but it remained low throughout the survey. The increase in missingness was not 'strictly' monotonic, indicating that some pupils skipped some individual items. The pattern of the missingness of the valid responses was not different from 'missing completely at random', as evidenced by a non-parametric analysis of the type of missingness (Jamshidian & Jalal, 2010), in which 290 pupils provided valid responses to all items whereas data of the remaining 102 pupils resulted in a total of 74 different missingness patterns; $p = 0.13$, excluding the response patterns that had only one observation (pupil), N = 333.

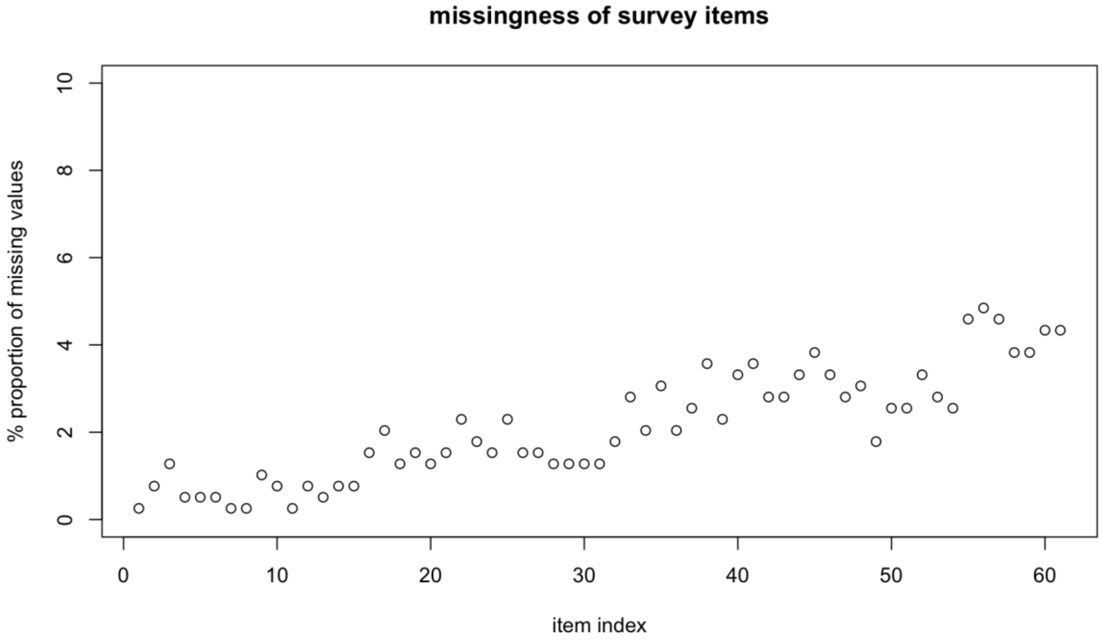

Figure 1 Missingness of survey items.

For each item, all response categories were used and almost all items had all response categories endorsed by at least 10 pupils. Regarding the response distributions, not all items displayed domination of a single category over others, and some items had two peaks. The response distributions showed that the response categories were meaningful to the pupils and the items captured individual variability.

2.3 Convergent validity, discriminant validity and internal consistency

Focusing on the items relating to pupils' perceived value of learning mathematics, confidence in mathematics, enjoyment of mathematics and anxiety about mathematics, confirmatory factor analysis (CFA) was used to evaluate the factorial validity and the internal consistency of the survey. Traditional CFA assumes data to be continuous or interval in nature, however in this case item responses were ordered categorical in nature. The analysis therefore applied ordinal CFA, involving polychoric correlations

(Olsson, 1979) and unweighted least squares estimation of model parameters (Jöreskog, 1994; Muthén, 1984) with robust standard errors and a mean-and-variance-adjusted test statistic (Satorra & Bentler, 2001; Savalei, 2014). Given that the missingness was minimal, pairwise deletion was used. Each polychoric correlation between two items were estimated with an assumption that responses of each item were discretisation of an underlying normality $\mathcal{N}(0,1)$ with four thresholds as the cut-off boundaries between any two neighbouring response categories (given five response categories for each item).

			Threshold			
Item	Statement	N	1	2	3	4
VAL1	Maths is a very useful subject.	391	-1.610	-1.200	-0.697	0.138
VAL2	I want to improve my maths skills.	391	-1.835	-1.393	-0.649	0.256
VAL3	Maths helps to develop thinking skills.	388	-1.992	-1.050	-0.356	0.634
VAL4	Maths is one of the most important subjects.	389	-1.607	-1.051	-0.515	0.264
VAL5	I can think of ways to use maths outside of school.	391	-1.313	-0.781	-0.331	0.649
VAL6	Maths helps with learning other subjects.	389	-1.197	-0.685	-0.100	1.051
VAL7	Being good at maths will help me in my future job.	390	-1.542	-1.123	-0.481	0.314
CON1	I feel okay about trying new maths problems.	387	-1.497	-1.094	-0.533	0.410
CON2	I am sure that I could do the hardest Year 7 maths.	387	-0.640	-0.186	0.172	0.818
CON3	I am sure that I can learn maths.	385	-1.944	-1.419	-0.938	-0.167
CON4	I think I could handle more difficult maths.	381	-1.096	-0.636	-0.109	0.660
CON5	I can get good grades in maths.	384	-1.578	-1.113	-0.533	0.332
CON6	I am confident in maths.	380	-1.238	-0.769	-0.322	0.494
CON7	I am no good at maths.	384	-0.305	0.438	0.868	1.350
CON8	I don’t think I could do the hardest Year 7 maths.	382	-0.670	-0.026	0.313	0.662
CON9	I’m not the type to do well in maths.	378	-0.317	0.248	0.695	1.279
CON10	I find maths difficult, even when I work hard.	383	-0.819	-0.141	0.234	0.987
CON11	I do better in other subjects than in maths.	379	-1.155	-0.632	-0.217	0.362
CON12	Maths is my worst subject.	378	-0.053	0.409	0.857	1.167
ENJ1	I feel good when I solve a maths problem.	381	-1.552	-1.171	-0.489	0.573
ENJ2	I enjoy maths.	381	-0.825	-0.353	0.122	0.843
ENJ3	I would rather do maths than write a story.	379	-0.426	-0.096	0.347	0.664
ENJ4	I am happier in maths than in other lessons.	377	-0.603	-0.030	0.564	1.192
ENJ5	Maths is a very interesting subject.	379	-0.938	-0.376	0.163	0.908
ENJ6	I feel okay sharing my ideas with others in maths.	381	-1.345	-0.727	-0.256	0.719
ENJ7	I feel okay answering maths questions from teachers.	380	-1.328	-0.814	-0.343	0.465
ANX1	I often worry that it will be difficult for me in maths.	385	-0.805	-0.200	0.301	1.068
ANX2	I get very tense when I have to do maths homework.	382	-0.536	-0.066	0.412	0.985
ANX3	I get very nervous doing maths problems.	382	-0.272	0.272	0.864	1.452
ANX4	I feel helpless when doing a maths problem.	379	-0.136	0.433	1.001	1.527
ANX5	I worry that I will get poor marks in maths.	381	-0.588	-0.082	0.283	0.941
ANX6	I feel anxious about failing in maths.	382	-0.575	-0.152	0.293	0.923

For each item, response underlying distribution $\sim\mathcal{N}(0,1)$. Threshold 1: threshold between ‘disagree a lot’ and ‘disagree a little’; threshold 2: threshold between ‘disagree a little’ and ‘neither’; threshold 3: threshold between ‘neither’ and ‘agree a little’; threshold 4: threshold between ‘agree a little’ and ‘agree a lot’.

Table 2 shows, for each item, its statement, the number of responses, and the thresholds of the underlying distributions of the items (assuming the responses are discretisation of a normal distribution); Table 3 is the pairwise polychoric correlation matrix between the items involved in the factor models tested. Using the said tables can solve the equations of the factor models by numerical methods, to produce estimates of model parameters.

Item	Statement	N	Threshold 1	2	3	4
VAL1	Maths is a very useful subject.	391	-1.610	-1.200	-0.697	0.138
VAL2	I want to improve my maths skills.	391	-1.835	-1.393	-0.649	0.256
VAL3	Maths helps to develop thinking skills.	388	-1.992	-1.050	-0.356	0.634
VAL4	Maths is one of the most important subjects.	389	-1.607	-1.051	-0.515	0.264
VAL5	I can think of ways to use maths outside of school.	391	-1.313	-0.781	-0.331	0.649
VAL6	Maths helps with learning other subjects.	389	-1.197	-0.685	-0.100	1.051
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ENJ3	I would rather do maths than write a story.	379	-0.426	-0.096	0.347	0.664
ENJ4	I am happier in maths than in other lessons.	377	-0.603	-0.030	0.564	1.192
ENJ5	Maths is a very interesting subject.	379	-0.938	-0.376	0.163	0.908
ENJ6	I feel okay sharing my ideas with others in maths.	381	-1.345	-0.727	-0.256	0.719
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ANX5	I worry that I will get poor marks in maths.	381	-0.588	-0.082	0.283	0.941
ANX6	I feel anxious about failing in maths.	382	-0.575	-0.152	0.293	0.923

For each item, response underlying distribution $\sim\mathcal{N}(0,1)$. Threshold 1: threshold between 'disagree a lot' and 'disagree a little'; threshold 2: threshold between 'disagree a little' and 'neither'; threshold 3: threshold between 'neither' and 'agree a little'; threshold 4: threshold between 'agree a little' and 'agree a lot'.

Table 2 Items and distributions.

Link to Table 3 spreadsheet

Table 3 Bivariate polychoric correlation.

We evaluated convergent validity, discriminant validity and internal consistency of the items using confirmatory factor analysis (CFA). Cheung et al. (2023) reviewed literature on convergent validity, discriminant validity and internal consistency under the framework of CFA and discussed strengths and concerns about different evaluation criteria in detail. In this study, recommendations on evaluation of factor models (Brown, 2006; Cheung et al., 2023; Khine, 2013) were considered, whilst being aware that many of the existing evaluation criteria are designed for continuous or interval data, rather than ordinal data.

A number of models were tested, for various purposes. The items were grouped into: perceived value of learning mathematics (VAL), confidence in mathematics (CON), enjoyment of mathematics (ENJ) and anxiety about mathematics (ANX). For each group, a 1-factor model was constructed that assumed the responses to the items were dependant on a latent variable:

$$y_{ij} = \lambda_j x_i + \varepsilon_{ij}$$

where, for each pupil i, y_{ij} is the observed response to item j, λ_j is the factor loading for item j, x_i is the value of the factor (latent variable) for pupil i, and ε_{ij} is an individual error term.

To evaluate convergent validity, we examine the 1-factor models, focusing on standardised item factor loadings and average variance extracted (AVE), which is computed by:

$$\text{AVE} = \frac{\sum \lambda_j}{k}$$

where k is the number of items.

For each group, the items should have a standardised factor loading of about 0.5 or above each and an average of 0.7, with AVE of about 0.5 or above (Cheung et al., 2023). This indicates that on average, about 50% or above of the variance of the items are shared with or can be explained by the latent variable.

When evaluating the factor models, different types of goodness-of-fit indices were examined (Brown, 2006; Khine, 2013). The indices used in the analysis consisted of chi-square (χ^2), standardised root mean square residual (SRMR), root mean square error of approximation (RMSEA) and comparative fit index (CFI). We used conventional cut-offs of the indices for model evaluation: χ^2 should be non-significant, SRMR and RMSEA should be close to or below 0.08, and CFI should be above 0.9 (Brown, 2006; Khine, 2013). When a model did not reach desirable goodness-of-fit indices, model modification was carried out driven by both theories and data; the modification involved adding parameters for correlation of error terms between some items. Only outputs of the initial and the final models are presented in this report.

To evaluate discriminant validity, the items of any pair of the four groups were fitted to two models: the first model assumed that the items of two groups were dependant on one latent variable (in other words, the items together measured only a single latent variable), whereas the second model assumed that the two groups were dependant on two separate latent variables. Whilst separate, the two latent variables were expected to be correlated as constructed. The two models were compared; the second model should be significantly better than the first as the items should not measure a latent variable other than what they were intended to measure (Cheung et al., 2023). In the second model, each AVE of the two latent variables should be greater than the squared correlation between the two latent variables (Cheung et al., 2023).

To evaluate internal consistency, coefficient omega for ordinal variables (Green & Yang, 2009) was computed for each group of the items. Each group should have a coefficient omega of at least 0.7 (Cheung et al., 2023). To compute coefficient omegas, a 4-factor model was used, in which the VAL, CON, ENJ and ANX items were specified to four different latent variables.

2.4 Additional robustness analyses

To evaluate the robustness of the analysis, apart from CFA, two further analyses were conducted: exploratory factor analysis (EFA; Bartholomew, 1980; Mislevy, 1986) and item response theory (IRT; Bock & Gibbons, 2010; Ostini & Nering, 2006). EFA is a data-driven statistical technique that aims to identify the number of latent variables underlying the observed variables, whereas IRT uses full-information approach to examine item response models, in a similar manner to factor models in CFA or EFA. The results of the analyses are presented in Appendices 6.2 Exploratory factor analysis and 6.3 Item response theory, respectively.

3. Results

This section reports the analysis of convergent validity, discriminant validity and internal consistency of the survey items.

3.1 Convergent validity

3.1.1 Perceived value of learning mathematics

The 1-factor models of the items relating to perceived value of learning mathematics showed that the items correlated with each other and measured the assumed latent variable with about 50% of shared variance, on average. The initial and final models differed in that in the final model there was a correlation specified between the error terms of VAL1 and VAL4. In the final model the correlated error terms showed that there was a positive but moderate correlation between VAL1 and VAL4 that was not explained by the latent variable. This correlation was not strong and could be explained by the fact that the two items were similarly worded.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.872	(0.022)
VAL2	I want to improve my maths skills.	0.718	(0.033)
VAL3	Maths helps to develop thinking skills.	0.699	(0.033)
VAL4	Maths is one of the most important subjects.	0.766	(0.029)
VAL5	I can think of ways to use maths outside of school.	0.700	(0.030)
VAL6	Maths helps with learning other subjects.	0.640	(0.036)
VAL7	Being good at maths will help me in my future job.	0.647	(0.036)
	Average variance extracted	0.524	

Table 4 Model parameters of the initial 1-factor model of perceived value of learning mathematics.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.838	(0.027)
VAL2	I want to improve my maths skills.	0.726	(0.033)
VAL3	Maths helps to develop thinking skills.	0.709	(0.033)
VAL4	Maths is one of the most important subjects.	0.730	(0.035)
VAL5	I can think of ways to use maths outside of school.	0.708	(0.030)
VAL6	Maths helps with learning other subjects.	0.648	(0.036)
VAL7	Being good at maths will help me in my future job.	0.652	(0.036)
	Average variance extracted	0.524	
	Correlation between unique variances of VAL1 and VAL4	0.350	(0.071)

Table 5 Model parameters of the final 1-factor model of perceived value of learning mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
initial	43.511	14	<0.001	0.037	0.092	0.964
final	32.734	13	0.002	0.032	0.064	0.984

Table 6 Goodness-of-fit indices of the 1-factor models of perceived value of learning mathematics.

3.1.2 Confidence in mathematics

The 1-factor models of the items relating to confidence in mathematics showed that the items correlated with each other and measured the assumed latent variable with about 70% of shared variance, on average, which is above 50%. The initial and final models differed in that the final model specified correlations between the error terms of five pairs of items. In the final model the correlated error terms showed that there were moderate correlations between several items that were not explained by the latent variable. These correlations were not strong and could be explained by consecutive order of the items and the fact that the items were similarly worded. The strongest correlation between the error terms of CON2 and CON8 could be explained by the fact that CON8 was a negatively worded version of CON2.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
CON1	I feel okay about trying new maths problems.	0.833	(0.019)
CON2	I am sure that I could do the hardest Year 7 maths.	0.836	(0.019)
CON3	I am sure that I can learn maths.	0.801	(0.023)
CON4	I think I could handle more difficult maths.	0.854	(0.017)
CON5	I can get good grades in maths.	0.881	(0.015)
CON6	I am confident in maths.	0.908	(0.012)
CON7	I am no good at maths.	-0.821	(0.021)
CON8	I don’t think I could do the hardest Year 7 maths.	-0.835	(0.018)
CON9	I’m not the type to do well in maths.	-0.875	(0.016)
CON10	I find maths difficult, even when I work hard.	-0.796	(0.020)
CON11	I do better in other subjects than in maths.	-0.686	(0.029)
CON12	Maths is my worst subject.	-0.834	(0.019)
	Average variance extracted	0.692	

Table 7 Model parameters of the initial 1-factor model of confidence in mathematics.

Item	Statement	Factor Loading Estimate	(S.E.)
CON1	I feel okay about trying new maths problems.	0.826	(0.020)
CON2	I am sure that I could do the hardest Year 7 maths.	0.822	(0.021)
CON3	I am sure that I can learn maths.	0.793	(0.024)
CON4	I think I could handle more difficult maths.	0.850	(0.018)
CON5	I can get good grades in maths.	0.878	(0.016)
CON6	I am confident in maths.	0.915	(0.011)
CON7	I am no good at maths.	-0.815	(0.022)
CON8	I don't think I could do the hardest Year 7 maths.	-0.821	(0.019)
CON9	I'm not the type to do well in maths.	-0.870	(0.018)
CON10	I find maths difficult, even when I work hard.	-0.802	(0.020)
CON11	I do better in other subjects than in maths.	-0.675	(0.030)
CON12	Maths is my worst subject.	-0.827	(0.020)
	Average variance extracted	0.683	
	Correlation between unique variances of		
	CON1 and CON3	0.325	(0.067)
	CON2 and CON8	-0.524	(0.038)
	CON4 and CON5	0.305	(0.064)
	CON7 and CON9	0.354	(0.047)
	CON11 and CON12	0.340	(0.058)

Table 8 Model parameters of the final 1-factor model of confidence in mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
initial	332.495	54	<0.001	0.046	0.169	0.880
final	208.956	49	<0.001	0.035	0.131	0.934

Table 9 Goodness-of-fit indices of the 1-factor models of confidence in mathematics.

3.1.3 Enjoyment of mathematics

The 1-factor models of the items relating to enjoyment of mathematics showed that the items correlated with each other and measured the assumed latent variable with about 50% of shared variance, on average. The initial and final models differed in that the final model specified correlations between the error terms of two pairs of consecutive items. In the final model the correlated error terms showed that there were positive correlations between several items that were not explained by the latent variable. Whilst the correlations could be explained by the fact that the items were similarly worded, the relatively strong correlation between the error terms of ENJ6 and ENJ7 and the relatively low factor loadings of the two items indicated that revision of these items might be needed. Also, ENJ1 had a relatively low factor loading.

Item	Statement	Factor Loading Estimate	(S.E.)
ENJ1	I feel good when I solve a maths problem.	0.543	(0.041)
ENJ2	I enjoy maths.	0.917	(0.017)
ENJ3	I would rather do maths than write a story.	0.628	(0.040)
ENJ4	I am happier in maths than in other lessons.	0.798	(0.027)
ENJ5	Maths is a very interesting subject.	0.799	(0.027)
ENJ6	I feel okay sharing my ideas with others in maths.	0.577	(0.038)
ENJ7	I feel okay answering maths questions from teachers.	0.664	(0.036)
	Average variance extracted	0.511	

Table 10 Model parameters of the initial 1-factor model of enjoyment of mathematics.

Item	Statement	Factor Loading Estimate	(S.E.)
ENJ1	I feel good when I solve a maths problem.	0.555	(0.041)
ENJ2	I enjoy maths.	0.961	(0.017)
ENJ3	I would rather do maths than write a story.	0.579	(0.045)
ENJ4	I am happier in maths than in other lessons.	0.763	(0.030)
ENJ5	Maths is a very interesting subject.	0.831	(0.027)
ENJ6	I feel okay sharing my ideas with others in maths.	0.494	(0.044)
ENJ7	I feel okay answering maths questions from teachers.	0.591	(0.042)
	Average variance extracted	0.490	
	Correlation between unique variances of		
	ENJ3 and ENJ4	0.449	(0.059)
	ENJ6 and ENJ7	0.620	(0.037)

Table 11 Model parameters of the final 1-factor model of enjoyment of mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
initial	197.985	14	<0.001	0.088	0.230	0.817
final	24.437	12	0.018	0.029	0.026	0.998

Table 12 Goodness-of-fit indices of the 1-factor models of enjoyment of mathematics.

3.1.4 Anxiety about mathematics

The 1-factor models of the items relating to anxiety about mathematics showed that the items correlated with each other and measured the assumed latent variable with more than 65% of shared variance, on average. The initial and final models differed in that the final model specified correlations between the error terms of ANX1, ANX5 and ANX6. In the final model the correlated error terms showed that there were positive correlations between several items that were not explained by the latent variable. Whilst being statistically significant, the correlations between the error terms of ANX1 and ANX5 and between the error terms of ANX1 and ANX6 were weak. The relatively strong correlation between the error terms of ANX5 and ANX6 could be explained by the fact that the items were similarly worded regarding ‘failing in maths’.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
ANX1	I often worry that it will be difficult for me in maths.	0.848	(0.018)
ANX2	I get very tense when I have to do maths homework.	0.753	(0.025)
ANX3	I get very nervous doing maths problems.	0.881	(0.016)
ANX4	I feel helpless when doing a maths problem.	0.821	(0.023)
ANX5	I worry that I will get poor marks in maths.	0.870	(0.016)
ANX6	I feel anxious about failing in maths.	0.805	(0.021)
	Average variance extracted	0.690	

Table 13 Model parameters of the initial 1-factor model of anxiety about mathematics.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
ANX1	I often worry that it will be difficult for me in maths.	0.822	(0.022)
ANX2	I get very tense when I have to do maths homework.	0.779	(0.025)
ANX3	I get very nervous doing maths problems.	0.918	(0.015)
ANX4	I feel helpless when doing a maths problem.	0.850	(0.022)
ANX5	I worry that I will get poor marks in maths.	0.787	(0.024)
ANX6	I feel anxious about failing in maths.	0.704	(0.031)
	Average variance extracted	0.660	
	Correlation between unique variances of		
	ANX1 and ANX5	0.191	(0.055)
	ANX1 and ANX6	0.292	(0.061)
	ANX5 and ANX6	0.697	(0.030)

Table 14 Model parameters of the final 1-factor model of anxiety about mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
initial	137.032	9	<0.001	0.051	0.247	0.891
final	9.485	6	0.148	0.012	0.040	0.998

Table 15 Goodness-of-fit indices of the 1-factor models of anxiety about mathematics.

3.2 Discriminant validity

For each pair of the item groups, 1- and 2-factor models were tested and compared to examine the relationship between the items relating to perceived value of learning mathematics, confidence in mathematics, enjoyment of mathematics and anxiety about mathematics, pairwise. The items of any two groups were not expected to measure the same latent variable, but two separate latent variables.

3.2.1 Perceived value of learning mathematics and confidence in mathematics

The results showed that the 2-factor model of the items relating to perceived value of learning mathematics and confidence in mathematics described the data better than the 1-factor model. The 1-factor model showed that the items relating to perceived value of learning mathematics had a lower factor loading than the items relating to confidence in mathematics, although on average, the items had about 50% of shared variance with the assumed latent variable. On the other hand, the 2-factor model showed that the two sets of the items measured their corresponding latent variable with about 50% and 70%

of shared variance on average. The two latent variables were correlated moderately, with a squared correlation of 0.339, which is below AVE_{VAL} = 0.521 and AVE_{CON} = 0.692.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.566	(0.039)
VAL2	I want to improve my maths skills.	0.489	(0.042)
VAL3	Maths helps to develop thinking skills.	0.588	(0.037)
VAL4	Maths is one of the most important subjects.	0.536	(0.040)
VAL5	I can think of ways to use maths outside of school.	0.576	(0.037)
VAL6	Maths helps with learning other subjects.	0.506	(0.039)
VAL7	Being good at maths will help me in my future job.	0.408	(0.046)
CON1	I feel okay about trying new maths problems.	0.847	(0.017)
CON2	I am sure that I could do the hardest Year 7 maths.	0.798	(0.022)
CON3	I am sure that I can learn maths.	0.829	(0.024)
CON4	I think I could handle more difficult maths.	0.824	(0.019)
CON5	I can get good grades in maths.	0.859	(0.017)
CON6	I am confident in maths.	0.897	(0.013)
CON7	I am no good at maths.	-0.784	(0.024)
CON8	I don’t think I could do the hardest Year 7 maths.	-0.776	(0.023)
CON9	I’m not the type to do well in maths.	-0.840	(0.020)
CON10	I find maths difficult, even when I work hard.	-0.740	(0.024)
CON11	I do better in other subjects than in maths.	-0.696	(0.029)
CON12	Maths is my worst subject.	-0.833	(0.021)
	Average variance extracted	0.519	

Table 16 Model parameters of the 1-factor model of perceived value of learning mathematics and confidence in mathematics.

Item	Statement	Factor Loading Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.801	(0.034)
VAL2	I want to improve my maths skills.	0.681	(0.041)
VAL3	Maths helps to develop thinking skills.	0.781	(0.036)
VAL4	Maths is one of the most important subjects.	0.741	(0.037)
VAL5	I can think of ways to use maths outside of school.	0.768	(0.037)
VAL6	Maths helps with learning other subjects.	0.678	(0.040)
VAL7	Being good at maths will help me in my future job.	0.579	(0.048)
CON1	I feel okay about trying new maths problems.	0.862	(0.017)
CON2	I am sure that I could do the hardest Year 7 maths.	0.820	(0.022)
CON3	I am sure that I can learn maths.	0.840	(0.024)
CON4	I think I could handle more difficult maths.	0.846	(0.019)
CON5	I can get good grades in maths.	0.881	(0.016)
CON6	I am confident in maths.	0.917	(0.013)
CON7	I am no good at maths.	-0.806	(0.023)
CON8	I don't think I could do the hardest Year 7 maths.	-0.802	(0.022)
CON9	I'm not the type to do well in maths.	-0.863	(0.019)
CON10	I find maths difficult, even when I work hard.	-0.765	(0.023)
CON11	I do better in other subjects than in maths.	-0.707	(0.029)
CON12	Maths is my worst subject.	-0.850	(0.020)
	Average variance extracted of		
	VAL	0.521	
	CON	0.692	
	Correlation between VAL and CON	0.582	(0.040)

Table 17 Model parameters of the 2-factor model of perceived value of learning mathematics and confidence in mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
1-factor	1346.599	152	<0.001	0.114	0.166	0.752
2-factor	445.043	151	<0.001	0.058	0.119	0.874

Table 18 Goodness-of-fit indices of the models of perceived value of learning mathematics and confidence in mathematics.

3.2.2 Perceived value of learning mathematics and enjoyment of mathematics

The results showed that the 2-factor model of the items relating to perceived value of learning mathematics and enjoyment of mathematics described the data better than the 1-factor model. The 1-factor model showed that on average, the items had about 45% of shared variance with the assumed latent variable, which is below 50%, although all items had a factor loading of at least 0.5. On the other hand, the 2-factor model showed that the two sets of items measured their corresponding latent variable with a slightly higher factor loading and about 50% of shared variance each, on average. The two latent variables were correlated strongly, with a squared correlation 0.591, similar to $AVE_{VAL} = 0.523$ and $AVE_{ENJ} = 0.512$. Results showed that the two latent variables were different, but not distinct; inspection of the polychoric correlation matrix showed that ENJ2, ENJ4 and ENJ5 had a correlation above 0.4 with several VAL items (Table 3).

Item	Statement	Factor Loading Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.766	(0.028)
VAL2	I want to improve my maths skills.	0.690	(0.032)
VAL3	Maths helps to develop thinking skills.	0.706	(0.031)
VAL4	Maths is one of the most important subjects.	0.684	(0.033)
VAL5	I can think of ways to use maths outside of school.	0.679	(0.031)
VAL6	Maths helps with learning other subjects.	0.604	(0.036)
VAL7	Being good at maths will help me in my future job.	0.605	(0.039)
ENJ1	I feel good when I solve a maths problem.	0.567	(0.038)
ENJ2	I enjoy maths.	0.867	(0.019)
ENJ3	I would rather do maths than write a story.	0.570	(0.043)
ENJ4	I am happier in maths than in other lessons.	0.742	(0.031)
ENJ5	Maths is a very interesting subject.	0.760	(0.029)
ENJ6	I feel okay sharing my ideas with others in maths.	0.510	(0.042)
ENJ7	I feel okay answering maths questions from teachers.	0.573	(0.040)
	Average variance extracted	0.453	

Table 19 Model parameters of the 1-factor model of perceived value of learning mathematics and enjoyment of mathematics.

Item	Statement	Factor Loading Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.825	(0.027)
VAL2	I want to improve my maths skills.	0.733	(0.032)
VAL3	Maths helps to develop thinking skills.	0.747	(0.031)
VAL4	Maths is one of the most important subjects.	0.733	(0.033)
VAL5	I can think of ways to use maths outside of school.	0.721	(0.031)
VAL6	Maths helps with learning other subjects.	0.642	(0.037)
VAL7	Being good at maths will help me in my future job.	0.643	(0.039)
ENJ1	I feel good when I solve a maths problem.	0.596	(0.039)
ENJ2	I enjoy maths.	0.934	(0.018)
ENJ3	I would rather do maths than write a story.	0.611	(0.043)
ENJ4	I am happier in maths than in other lessons.	0.797	(0.030)
ENJ5	Maths is a very interesting subject.	0.815	(0.029)
ENJ6	I feel okay sharing my ideas with others in maths.	0.547	(0.042)
ENJ7	I feel okay answering maths questions from teachers.	0.618	(0.040)
	Average variance extracted of		
	VAL	0.523	
	ENJ	0.512	
	Correlation between VAL and ENJ	0.769	(0.032)

Table 20 Model parameters of the 2-factor model of perceived value of learning mathematics and enjoyment of mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
1-factor	483.668	77	<0.001	0.081	0.158	0.767
2-factor	287.881	76	<0.001	0.060	0.118	0.872

Table 21 Goodness-of-fit indices of the models of perceived value of learning mathematics and enjoyment of mathematics.

3.2.3 Perceived value of learning mathematics and anxiety about mathematics

The results showed that the 2-factor model of the items relating to perceived value of learning mathematics and anxiety about mathematics described the data better than the 1-factor model. The 1-factor model showed that the items relating to perceived value of learning mathematics had a relatively lower factor loading than the items relating to anxiety about mathematics and, on average, the items had less than 40% of shared variance with the assumed latent variable. On the other hand, the 2-factor model showed that the two sets of items measured their corresponding latent variable with about 50% and 70% of shared variance, on average, respectively. The two latent variables were correlated weakly, with a squared correlation of 0.073, which is below AVE_{VAL} = 0.524 and AVE_{ANX} = 0.690.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.569	(0.039)
VAL2	I want to improve my maths skills.	0.440	(0.043)
VAL3	Maths helps to develop thinking skills.	0.527	(0.041)
VAL4	Maths is one of the most important subjects.	0.544	(0.040)
VAL5	I can think of ways to use maths outside of school.	0.583	(0.038)
VAL6	Maths helps with learning other subjects.	0.458	(0.041)
VAL7	Being good at maths will help me in my future job.	0.412	(0.045)
ANX1	I often worry that it will be difficult for me in maths.	-0.682	(0.029)
ANX2	I get very tense when I have to do maths homework.	-0.669	(0.030)
ANX3	I get very nervous doing maths problems.	-0.786	(0.023)
ANX4	I feel helpless when doing a maths problem.	-0.825	(0.024)
ANX5	I worry that I will get poor marks in maths.	-0.663	(0.032)
ANX6	I feel anxious about failing in maths.	-0.560	(0.036)
	Average variance extracted	0.367	

Table 22 Model parameters of the 1-factor model of perceived value of learning mathematics and anxiety about mathematics.

Item	Statement	Factor Loading Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.857	(0.025)
VAL2	I want to improve my maths skills.	0.685	(0.037)
VAL3	Maths helps to develop thinking skills.	0.716	(0.034)
VAL4	Maths is one of the most important subjects.	0.771	(0.030)
VAL5	I can think of ways to use maths outside of school.	0.744	(0.031)
VAL6	Maths helps with learning other subjects.	0.642	(0.036)
VAL7	Being good at maths will help me in my future job.	0.625	(0.039)
ANX1	I often worry that it will be difficult for me in maths.	0.832	(0.021)
ANX2	I get very tense when I have to do maths homework.	0.768	(0.026)
ANX3	I get very nervous doing maths problems.	0.907	(0.016)
ANX4	I feel helpless when doing a maths problem.	0.887	(0.021)
ANX5	I worry that I will get poor marks in maths.	0.833	(0.022)
ANX6	I feel anxious about failing in maths.	0.743	(0.027)
	Average variance extracted of		
	VAL	0.524	
	ANX	0.690	
	Correlation between VAL and ANX	-0.270	(0.053)

Table 23 Model parameters of the 2-factor model of perceived value of learning mathematics and anxiety about mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
1-factor	1354.288	65	<0.001	0.212	0.263	0.486
2-factor	259.832	64	<0.001	0.076	0.128	0.880

Table 24 Goodness-of-fit indices of the models of perceived value of learning mathematics and anxiety about mathematics.

3.2.4 Confidence in mathematics and enjoyment of mathematics

The results showed that the 2-factor model of the items relating to confidence in mathematics and enjoyment of mathematics described the data better than the 1-factor model. The items relating to enjoyment of mathematics had a lower factor loading in the 1-factor model than in the 2-factor model. In the 1-factor model, ENJ1 had a factor loading less than 0.4, although on average the items had 56% of shared variance with the assumed latent variable. On the other hand, the 2-factor model showed that the two sets of items measured their corresponding latent variable with about 70% and about 50% of shared variance, on average, respectively. The two latent variables were correlated strongly, with a squared correlation of 0.558, which is below AVE_{CON} = 0.691 but above AVE_{ENJ} = 0.509. The results showed that the two latent variables were different, but not distinct; inspection of the polychoric correlation matrix showed that CON1 and CON11 had a correlation above 0.4 with several ENJ items and ENJ2 and ENJ7 had a correlation above 0.4 with several CON items (Table 3).

Item	Statement	Factor Loading	
		Estimate	(S.E.)
CON1	I feel okay about trying new maths problems.	0.849	(0.017)
CON2	I am sure that I could do the hardest Year 7 maths.	0.781	(0.023)
CON3	I am sure that I can learn maths.	0.827	(0.023)
CON4	I think I could handle more difficult maths.	0.816	(0.020)
CON5	I can get good grades in maths.	0.861	(0.017)
CON6	I am confident in maths.	0.913	(0.012)
CON7	I am no good at maths.	-0.797	(0.023)
CON8	I don't think I could do the hardest Year 7 maths.	-0.774	(0.022)
CON9	I'm not the type to do well in maths.	-0.828	(0.021)
CON10	I find maths difficult, even when I work hard.	-0.757	(0.023)
CON11	I do better in other subjects than in maths.	-0.717	(0.027)
CON12	Maths is my worst subject.	-0.860	(0.018)
ENJ1	I feel good when I solve a maths problem.	0.356	(0.045)
ENJ2	I enjoy maths.	0.776	(0.023)
ENJ3	I would rather do maths than write a story.	0.534	(0.041)
ENJ4	I am happier in maths than in other lessons.	0.623	(0.034)
ENJ5	Maths is a very interesting subject.	0.665	(0.032)
ENJ6	I feel okay sharing my ideas with others in maths.	0.530	(0.040)
ENJ7	I feel okay answering maths questions from teachers.	0.663	(0.032)
	Average variance extracted	0.556	

Table 25 Model parameters of the 1-factor model of confidence in mathematics and enjoyment of mathematics.

Item	Statement	Factor Loading	
		Estimate	(S.E.)
CON1	I feel okay about trying new maths problems.	0.861	(0.017)
CON2	I am sure that I could do the hardest Year 7 maths.	0.799	(0.022)
CON3	I am sure that I can learn maths.	0.837	(0.023)
CON4	I think I could handle more difficult maths.	0.833	(0.020)
CON5	I can get good grades in maths.	0.877	(0.017)
CON6	I am confident in maths.	0.928	(0.012)
CON7	I am no good at maths.	-0.812	(0.023)
CON8	I don't think I could do the hardest Year 7 maths.	-0.793	(0.021)
CON9	I'm not the type to do well in maths.	-0.847	(0.020)
CON10	I find maths difficult, even when I work hard.	-0.773	(0.022)
CON11	I do better in other subjects than in maths.	-0.725	(0.027)
CON12	Maths is my worst subject.	-0.871	(0.018)
ENJ1	I feel good when I solve a maths problem.	0.433	(0.048)
ENJ2	I enjoy maths.	0.921	(0.020)
ENJ3	I would rather do maths than write a story.	0.625	(0.043)
ENJ4	I am happier in maths than in other lessons.	0.741	(0.033)
ENJ5	Maths is a very interesting subject.	0.786	(0.031)
ENJ6	I feel okay sharing my ideas with others in maths.	0.616	(0.042)
ENJ7	I feel okay answering maths questions from teachers.	0.767	(0.033)
	Average variance extracted of		
	CON	0.691	
	ENJ	0.509	
	Correlation between CON and ENJ	0.747	(0.027)

Table 26 Model parameters of the 2-factor model of confidence in mathematics and enjoyment of mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
1-factor	1214.685	152	<0.001	0.094	0.175	0.750
2-factor	790.618	151	<0.001	0.073	0.149	0.820

Table 27 Goodness-of-fit indices of the models of confidence in mathematics and enjoyment of mathematics.

3.2.5 Confidence in mathematics and anxiety about mathematics

The results showed that the 2-factor model of the items relating to confidence in mathematics and anxiety about mathematics described the data better than the 1-factor model. In the 1-factor model, the items relating to anxiety about mathematics had a lower factor loading than in the 2-factor model, although all items correlated with each other and measured the assumed latent variable with a factor loading above 0.5 (in magnitude) and about 60% of shared variance, on average. On the other hand, the 2-factor model showed the two sets of items measured their corresponding latent variable with about 70% of shared variance each, on average. The two latent variables were correlated strongly, but the squared correlation of 0.581 was below AVE_{CON} = 0.691 and AVE_{ANX} = 0.689, indicating that the two latent variables were reasonably distinct.

Item	Statement	Factor Loading	
		Estimate	(S.E.)
CON1	I feel okay about trying new maths problems.	0.802	(0.021)
CON2	I am sure that I could do the hardest Year 7 maths.	0.800	(0.023)
CON3	I am sure that I can learn maths.	0.745	(0.028)
CON4	I think I could handle more difficult maths.	0.799	(0.022)
CON5	I can get good grades in maths.	0.846	(0.019)
CON6	I am confident in maths.	0.874	(0.015)
CON7	I am no good at maths.	-0.826	(0.021)
CON8	I don't think I could do the hardest Year 7 maths.	-0.831	(0.020)
CON9	I'm not the type to do well in maths.	-0.871	(0.018)
CON10	I find maths difficult, even when I work hard.	-0.826	(0.018)
CON11	I do better in other subjects than in maths.	-0.705	(0.028)
CON12	Maths is my worst subject.	-0.819	(0.020)
ANX1	I often worry that it will be difficult for me in maths.	-0.756	(0.024)
ANX2	I get very tense when I have to do maths homework.	-0.648	(0.032)
ANX3	I get very nervous doing maths problems.	-0.782	(0.022)
ANX4	I feel helpless when doing a maths problem.	-0.810	(0.023)
ANX5	I worry that I will get poor marks in maths.	-0.706	(0.028)
ANX6	I feel anxious about failing in maths.	-0.614	(0.034)
	Average variance extracted	0.615	

Table 28 Model parameters of the 1-factor model of confidence in mathematics and anxiety about mathematics.

Item	Statement	Factor Loading	
		Estimate	(S.E.)
CON1	I feel okay about trying new maths problems.	0.821	(0.021)
CON2	I am sure that I could do the hardest Year 7 maths.	0.820	(0.022)
CON3	I am sure that I can learn maths.	0.765	(0.027)
CON4	I think I could handle more difficult maths.	0.820	(0.021)
CON5	I can get good grades in maths.	0.867	(0.018)
CON6	I am confident in maths.	0.895	(0.014)
CON7	I am no good at maths.	-0.842	(0.021)
CON8	I don't think I could do the hardest Year 7 maths.	-0.848	(0.019)
CON9	I'm not the type to do well in maths.	-0.889	(0.017)
CON10	I find maths difficult, even when I work hard.	-0.840	(0.018)
CON11	I do better in other subjects than in maths.	-0.716	(0.028)
CON12	Maths is my worst subject.	-0.837	(0.020)
ANX1	I often worry that it will be difficult for me in maths.	0.867	(0.022)
ANX2	I get very tense when I have to do maths homework.	0.743	(0.030)
ANX3	I get very nervous doing maths problems.	0.898	(0.019)
ANX4	I feel helpless when doing a maths problem.	0.922	(0.022)
ANX5	I worry that I will get poor marks in maths.	0.816	(0.025)
ANX6	I feel anxious about failing in maths.	0.714	(0.032)
	Average variance extracted of		
	CON	0.691	
	ANX	0.689	
	Correlation between CON and ANX	-0.762	(0.023)

Table 29 Model parameters of the 2-factor model of confidence in mathematics and anxiety about mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
1-factor	1303.637	135	<0.001	0.086	0.187	0.757
2-factor	628.475	134	<0.001	0.056	0.149	0.846

Table 30 Goodness-of-fit indices of the models of confidence in mathematics and anxiety about mathematics.

3.2.6 Enjoyment of mathematics and anxiety about mathematics

The results showed that the 2-factor model of the items relating to enjoyment of mathematics and anxiety about mathematics described the data better than the 1-factor model. The 1-factor model showed that the items had a lower factor loading than in the 2-factor model, ENJ1 had a factor loading less than 0.4 and, on average, the items had 43% of shared variance with the assumed latent variable, which is below 50%. On the other hand, the 2-factor model showed that the two sets of items measured their corresponding latent variable with about 50% and about 70% of shared variance, on average, respectively. The two latent variables were correlated moderately, with a squared correlation of 0.226, which is below AVE_{ENJ} = 0.508 and AVE_{ANX} = 0.690.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
ENJ1	I feel good when I solve a maths problem.	0.310	(0.047)
ENJ2	I enjoy maths.	0.694	(0.030)
ENJ3	I would rather do maths than write a story.	0.531	(0.043)
ENJ4	I am happier in maths than in other lessons.	0.588	(0.036)
ENJ5	Maths is a very interesting subject.	0.582	(0.036)
ENJ6	I feel okay sharing my ideas with others in maths.	0.527	(0.041)
ENJ7	I feel okay answering maths questions from teachers.	0.672	(0.031)
ANX1	I often worry that it will be difficult for me in maths.	-0.736	(0.025)
ANX2	I get very tense when I have to do maths homework.	-0.670	(0.030)
ANX3	I get very nervous doing maths problems.	-0.809	(0.021)
ANX4	I feel helpless when doing a maths problem.	-0.818	(0.023)
ANX5	I worry that I will get poor marks in maths.	-0.724	(0.028)
ANX6	I feel anxious about failing in maths.	-0.647	(0.032)
	Average variance extracted	0.425	

Table 31 Model parameters of the 1-factor model of enjoyment of mathematics and anxiety about mathematics.

		Factor Loading	
Item	Statement	Estimate	(S.E.)
ENJ1	I feel good when I solve a maths problem.	0.450	(0.046)
ENJ2	I enjoy maths.	0.896	(0.021)
ENJ3	I would rather do maths than write a story.	0.646	(0.040)
ENJ4	I am happier in maths than in other lessons.	0.760	(0.031)
ENJ5	Maths is a very interesting subject.	0.756	(0.031)
ENJ6	I feel okay sharing my ideas with others in maths.	0.623	(0.040)
ENJ7	I feel okay answering maths questions from teachers.	0.772	(0.031)
ANX1	I often worry that it will be difficult for me in maths.	0.836	(0.021)
ANX2	I get very tense when I have to do maths homework.	0.753	(0.028)
ANX3	I get very nervous doing maths problems.	0.906	(0.018)
ANX4	I feel helpless when doing a maths problem.	0.890	(0.021)
ANX5	I worry that I will get poor marks in maths.	0.834	(0.023)
ANX6	I feel anxious about failing in maths.	0.752	(0.028)
	Average variance extracted of		
	ENJ	0.508	
	ANX	0.690	
	Correlation between ENJ and ANX	-0.475	(0.045)

Table 32 Model parameters of the 2-factor model of enjoyment of mathematics and anxiety about mathematics.

model	χ^2	df	p-value	SRMR	RMSEA	CFI
1-factor	1128.777	65	<0.001	0.164	0.257	0.552
2-factor	375.783	64	<0.001	0.086	0.169	0.811

Table 33 Goodness-of-fit indices of the models of enjoyment of mathematics and anxiety about mathematics.

3.3 Internal consistency

Coefficient omegas were computed to examine how well the items of each of the four groups were correlated with each other and measured different aspects of the same latent variable. To compute coefficient omegas, a 4-factor model in which the items were loaded on their corresponding scales was used. The 4-factor model showed that the coefficient omegas of the items relating to perceived value of learning mathematics, confidence in mathematics, enjoyment of mathematics and anxiety about mathematics were 0.850, 0.943, 0.836 and 0.883 respectively, which are all above 0.7. In other words, for each scale more than 50% of the variance of the items were shared within the scale, on average.

VAL	CON	ENJ	ANX
0.850	0.943	0.846	0.883

Table 34 Coefficient omegas of the items relating to perceived value of learning mathematics, confidence in mathematics, enjoyment of mathematics and anxiety about mathematics.

Item	Statement	Factor Loading Estimate	(S.E.)
VAL1	Maths is a very useful subject.	0.791	(0.034)
VAL2	I want to improve my maths skills.	0.687	(0.042)
VAL3	Maths helps to develop thinking skills.	0.790	(0.036)
VAL4	Maths is one of the most important subjects.	0.732	(0.038)
VAL5	I can think of ways to use maths outside of school.	0.777	(0.037)
VAL6	Maths helps with learning other subjects.	0.666	(0.040)
VAL7	Being good at maths will help me in my future job.	0.587	(0.049)
CON1	I feel okay about trying new maths problems.	0.864	(0.018)
CON2	I am sure that I could do the hardest Year 7 maths.	0.788	(0.024)
CON3	I am sure that I can learn maths.	0.826	(0.025)
CON4	I think I could handle more difficult maths.	0.810	(0.022)
CON5	I can get good grades in maths.	0.868	(0.018)
CON6	I am confident in maths.	0.919	(0.014)
CON7*	I am no good at maths.	0.820	(0.023)
CON8*	I don’t think I could do the hardest Year 7 maths.	0.793	(0.023)
CON9*	I’m not the type to do well in maths.	0.858	(0.020)
CON10*	I find maths difficult, even when I work hard.	0.791	(0.022)
CON11*	I do better in other subjects than in maths.	0.751	(0.027)
CON12*	Maths is my worst subject.	0.873	(0.018)
ENJ1	I feel good when I solve a maths problem.	0.455	(0.046)
ENJ2	I enjoy maths.	0.925	(0.019)
ENJ3	I would rather do maths than write a story.	0.628	(0.042)
ENJ4	I am happier in maths than in other lessons.	0.749	(0.033)
ENJ5	Maths is a very interesting subject.	0.786	(0.031)
ENJ6	I feel okay sharing my ideas with others in maths.	0.606	(0.042)
ENJ7	I feel okay answering maths questions from teachers.	0.751	(0.033)
ANX1	I often worry that it will be difficult for me in maths.	0.851	(0.025)
ANX2	I get very tense when I have to do maths homework.	0.747	(0.032)
ANX3	I get very nervous doing maths problems.	0.912	(0.021)
ANX4	I feel helpless when doing a maths problem.	0.960	(0.024)
ANX5	I worry that I will get poor marks in maths.	0.795	(0.029)
ANX6	I feel anxious about failing in maths.	0.680	(0.036)
	Average variance extracted of		
	VAL	0.521	
	CON	0.691	
	ENJ	0.510	
	ANX	0.688	
	Correlation between		
	VAL and CON	0.583	(0.040)
	VAL and ENJ	0.763	(0.032)
	VAL and ANX	-0.279	(0.053)
	CON and ENJ	0.746	(0.027)
	CON and ANX	-0.762	(0.023)
	ENJ and ANX	-0.476	(0.045)

*CON7-CON12 reverse coded for coefficient omega computation.

Table 35 Model parameters of the 4-factor model.

4. Conclusion

The results showed that each group of the items measured their intended latent variable and did not directly measure the other latent variables. However, the results also indicated that some items might be redundant, and revision was needed for some items to further improve convergent validity and discriminant validity. As a result, three items relating to enjoyment of mathematics have been revised for the main study (Table 36).

Item	Original Statement	Revised Statement
ENJ1	I feel good when I solve a maths problem.	I feel satisfied when I solve a maths problem.
ENJ6	I feel okay sharing my ideas with others in maths.	I feel comfortable sharing my ideas in maths.
ENJ7	I feel okay answering maths questions from teachers.	I feel comfortable answering teacher questions in maths.

Table 36 Revised items.

The results were also cross-checked by sensitivity analysis with exploratory factor analysis and item response theory (see Appendices 6.2 Exploratory factor analysis and 6.3 Item response theory). The sensitivity analysis showed that different analysis methods yielded consistent results, establishing that the CFA results were valid and reliable.

4.1 Future research

Given that the research design was cross-sectional in this pilot study, in which we aimed to establish the factorial validity and internal consistency of the survey items, test-retest reliability was not examined. Also, group invariance (e.g., sex, ethnicity) was not evaluated as relevant data were not collected. These questions will be addressed in the main study.

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6. Appendices

6.1 R code

This linked document records the R code of the analyses of confirmatory factor analysis, exploratory factor analysis and item response theory.

6.2 Exploratory factor analysis

An exploratory factor analysis (EFA) was conducted for sensitivity analysis in order to examine the robustness of the results of the CFA reported above. Polychoric correlations were estimated with pairwise deletion, and unweighted least squares with mean-and-variance-adjusted test statistics and geomin were used as estimation and rotation method. In the analysis, five models were tested, assuming there were one to five factors, as their eigenvalues were greater than 1.

The EFA results were consistent with the CFA results. Under the framework of EFA, the items could be grouped into the four intended sets, measuring perceived value of learning mathematics, confidence in mathematics, enjoyment of mathematics and anxiety about mathematics. The models showed that three items relating to enjoyment of mathematics (ENJ1, ENJ6 and ENJ7) should be revised, consistent with the CFA results. However, the CFA results and the EFA results were not entirely identical, as in the 4- and 5-factor EFA models, rather than relating to confidence in mathematics, CON11 appeared to measure enjoyment of mathematics and anxiety about mathematics.

	Factor 1	
	Factor Loading	
Item	Est.	S.E.
VAL1	**0.541**	(0.039)
VAL2	**0.466**	(0.042)
VAL3	**0.567**	(0.037)
VAL4	**0.513**	(0.040)
VAL5	**0.566**	(0.037)
VAL6	**0.473**	(0.039)
VAL7	**0.395**	(0.045)
CON1	**0.843**	(0.018)
CON2	**0.761**	(0.025)
CON3	**0.809**	(0.025)
CON4	**0.785**	(0.023)
CON5	**0.842**	(0.019)
CON6	**0.893**	(0.015)
CON7	**-0.793**	(0.024)
CON8	**-0.763**	(0.024)
CON9	**-0.829**	(0.021)
CON10	**-0.762**	(0.023)
CON11	**-0.733**	(0.027)
CON12	**-0.851**	(0.019)
ENJ1	**0.360**	(0.044)
ENJ2	**0.775**	(0.023)
ENJ3	**0.536**	(0.041)
ENJ4	**0.625**	(0.034)
ENJ5	**0.658**	(0.032)
ENJ6	**0.525**	(0.040)
ENJ7	**0.659**	(0.031)
ANX1	**-0.648**	(0.030)
ANX2	**-0.579**	(0.035)
ANX3	**-0.708**	(0.027)
ANX4	**-0.765**	(0.025)
ANX5	**-0.598**	(0.034)
ANX6	**-0.497**	(0.039)
Proportion of variance explained	0.456	

Factor loadings above 0.3 or below -0.3 are in bold

Table 37 Model parameters of the 1-factor EFA model.

Item	Factor 1 Factor Loading Est.	S.E.	Factor 2 Factor Loading Est.	S.E.
VAL1	**0.787**	(0.033)	0.030	(0.034)
VAL2	**0.765**	(0.035)	0.095	(0.053)
VAL3	**0.674**	(0.042)	-0.083	(0.057)
VAL4	**0.679**	(0.038)	-0.021	(0.034)
VAL5	**0.595**	(0.044)	-0.141	(0.052)
VAL6	**0.602**	(0.047)	-0.037	(0.050)
VAL7	**0.652**	(0.043)	0.085	(0.052)
CON1	**0.367**	(0.054)	**-0.613**	(0.048)
CON2	0.125	(0.056)	**-0.718**	(0.041)
CON3	**0.444**	(0.058)	**-0.515**	(0.053)
CON4	0.212	(0.051)	**-0.672**	(0.040)
CON5	0.240	(0.053)	**-0.712**	(0.041)
CON6	**0.310**	(0.046)	**-0.711**	(0.038)
CON7	-0.107	(0.043)	**0.767**	(0.031)
CON8	-0.016	(0.034)	**0.809**	(0.028)
CON9	-0.115	(0.036)	**0.800**	(0.028)
CON10	0.008	(0.029)	**0.828**	(0.023)
CON11	-0.257	(0.049)	**0.583**	(0.045)
CON12	**-0.327**	(0.048)	**0.653**	(0.044)
ENJ1	**0.604**	(0.047)	0.087	(0.061)
ENJ2	**0.710**	(0.040)	-0.278	(0.057)
ENJ3	**0.421**	(0.060)	-0.243	(0.066)
ENJ4	**0.632**	(0.048)	-0.176	(0.063)
ENJ5	**0.651**	(0.043)	-0.197	(0.059)
ENJ6	**0.337**	(0.056)	-0.296	(0.057)
ENJ7	**0.314**	(0.061)	**-0.458**	(0.055)
ANX1	0.259	(0.043)	**0.913**	(0.027)
ANX2	0.131	(0.047)	**0.730**	(0.037)
ANX3	0.128	(0.042)	**0.869**	(0.026)
ANX4	-0.005	(0.029)	**0.820**	(0.026)
ANX5	**0.301**	(0.048)	**0.891**	(0.029)
ANX6	**0.383**	(0.051)	**0.848**	(0.033)
Proportion of variance explained	0.219		0.356	
Correlation between Factor 1 and Factor 2	-0.438	(0.048)		

Factor loadings above 0.3 or below -0.3 are in bold

Table 38 Model parameters of the 2-factor EFA model.

	Factor 1		Factor 2		Factor 3	
	Factor Loading		Factor Loading		Factor Loading	
Item	Est.	S.E.	Est.	S.E.	Est.	S.E.
VAL1	**0.792**	(0.041)	<0.001	(0.057)	0.072	(0.083)
VAL2	**0.741**	(0.045)	0.018	(0.058)	0.149	(0.083)
VAL3	**0.638**	(0.057)	0.156	(0.106)	0.079	(0.147)
VAL4	**0.713**	(0.055)	-0.032	(0.083)	-0.008	(0.064)
VAL5	**0.569**	(0.053)	0.169	(0.122)	0.027	(0.170)
VAL6	**0.540**	(0.058)	0.180	(0.101)	0.140	(0.153)
VAL7	**0.710**	(0.061)	-0.151	(0.086)	0.001	(0.057)
CON1	0.262	(0.088)	**0.642**	(0.229)	-0.070	(0.417)
CON2	-0.089	(0.115)	**0.934**	(0.288)	0.037	(0.567)
CON3	0.288	(0.102)	**0.699**	(0.221)	0.075	(0.442)
CON4	-0.013	(0.115)	**0.939**	(0.270)	0.091	(0.562)
CON5	0.085	(0.100)	**0.811**	(0.268)	-0.043	(0.509)
CON6	0.227	(0.089)	**0.657**	(0.234)	-0.157	(0.423)
CON7	-0.056	(0.080)	**-0.612**	(0.232)	0.258	(0.395)
CON8	0.121	(0.106)	**-0.819**	(0.269)	0.146	(0.502)
CON9	-0.024	(0.091)	**-0.721**	(0.247)	0.205	(0.449)
CON10	0.016	(0.080)	**-0.557**	(0.217)	**0.366**	(0.363)
CON11	**-0.399**	(0.060)	-0.082	(0.138)	**0.492**	(0.127)
CON12	**-0.329**	(0.074)	**-0.433**	(0.193)	0.275	(0.308)
ENJ1	**0.594**	(0.053)	-0.010	(0.071)	0.110	(0.080)
ENJ2	**0.767**	(0.052)	0.090	(0.110)	-0.161	(0.130)
ENJ3	**0.582**	(0.075)	-0.162	(0.103)	**-0.342**	(0.083)
ENJ4	**0.761**	(0.057)	-0.124	(0.099)	-0.232	(0.074)
ENJ5	**0.685**	(0.054)	0.083	(0.099)	-0.090	(0.119)
ENJ6	0.280	(0.068)	**0.328**	(0.160)	-0.010	(0.263)
ENJ7	0.290	(0.077)	**0.362**	(0.171)	-0.143	(0.274)
ANX1	0.109	(0.057)	-0.248	(0.207)	**0.715**	(0.224)
ANX2	-0.013	(0.041)	-0.150	(0.174)	**0.607**	(0.166)
ANX3	-0.021	(0.038)	-0.229	(0.204)	**0.681**	(0.216)
ANX4	-0.132	(0.054)	-0.255	(0.206)	**0.603**	(0.232)
ANX5	0.076	(0.059)	-0.054	(0.179)	**0.866**	(0.127)
ANX6	0.145	(0.064)	0.014	(0.158)	**0.885**	(0.086)
Proportion of variance explained	0.215		0.242		0.159	

Correlation between		
Factor 1 and Factor 2	0.550	(0.144)
Factor 1 and Factor 3	-0.249	(0.094)
Factor 2 and Factor 3	-0.602	(0.297)

Factor loadings above 0.3 or below -0.3 are in bold

Table 39 Model parameters of the 3-factor EFA model.

Item	Factor 1 Factor Loading Est.	 S.E.	Factor 2 Factor Loading Est.	 S.E.	Factor 3 Factor Loading Est.	 S.E.	Factor 4 Factor Loading Est.	 S.E.
VAL1	**0.969**	(0.104)	-0.125	(0.169)	-0.045	(0.080)	-0.012	(0.049)
VAL2	**0.652**	(0.069)	-0.005	(0.025)	0.140	(0.083)	0.139	(0.065)
VAL3	**0.514**	(0.082)	0.154	(0.100)	0.166	(0.078)	0.086	(0.066)
VAL4	**0.826**	(0.085)	-0.133	(0.154)	0.009	(0.028)	-0.075	(0.062)
VAL5	**0.659**	(0.098)	0.102	(0.144)	-0.016	(0.064)	-0.023	(0.062)
VAL6	**0.525**	(0.086)	0.149	(0.118)	0.049	(0.082)	0.115	(0.068)
VAL7	**0.628**	(0.085)	-0.182	(0.115)	0.166	(0.073)	-0.007	(0.034)
CON1	0.122	(0.063)	**0.668**	(0.060)	0.159	(0.073)	-0.041	(0.057)
CON2	-0.002	(0.065)	**0.913**	(0.049)	-0.108	(0.067)	0.005	(0.044)
CON3	0.152	(0.074)	**0.727**	(0.071)	0.128	(0.083)	0.097	(0.073)
CON4	-0.033	(0.065)	**0.948**	(0.054)	-0.016	(0.056)	0.087	(0.072)
CON5	0.030	(0.066)	**0.825**	(0.057)	0.053	(0.067)	-0.033	(0.069)
CON6	-0.018	(0.053)	**0.721**	(0.061)	0.263	(0.067)	-0.090	(0.064)
CON7	-0.015	(0.047)	**-0.620**	(0.065)	-0.084	(0.069)	0.241	(0.071)
CON8	0.081	(0.072)	**-0.812**	(0.058)	0.032	(0.060)	0.155	(0.067)
CON9	-0.083	(0.055)	**-0.701**	(0.071)	0.023	(0.043)	0.220	(0.074)
CON10	0.033	(0.054)	**-0.559**	(0.075)	-0.076	(0.056)	**0.350**	(0.069)
CON11	-0.005	(0.030)	-0.155	(0.067)	**-0.507**	(0.055)	**0.384**	(0.060)
CON12	-0.002	(0.030)	**-0.509**	(0.067)	**-0.379**	(0.060)	0.185	(0.067)
ENJ1	**0.304**	(0.091)	0.046	(0.084)	**0.303**	(0.086)	0.169	(0.062)
ENJ2	0.148	(0.103)	0.220	(0.066)	**0.688**	(0.081)	-0.003	(0.007)
ENJ3	-0.030	(0.093)	-0.053	(0.093)	**0.719**	(0.069)	-0.190	(0.074)
ENJ4	0.054	(0.110)	0.003	(0.030)	**0.812**	(0.071)	-0.060	(0.061)
ENJ5	0.127	(0.102)	0.205	(0.073)	**0.604**	(0.076)	0.052	(0.050)
ENJ6	0.115	(0.091)	**0.368**	(0.080)	0.172	(0.079)	0.030	(0.067)
ENJ7	0.067	(0.090)	**0.417**	(0.079)	0.250	(0.076)	-0.081	(0.066)
ANX1	-0.012	(0.053)	-0.205	(0.101)	-0.016	(0.046)	**0.723**	(0.066)
ANX2	-0.223	(0.088)	-0.067	(0.133)	0.067	(0.065)	**0.658**	(0.070)
ANX3	-0.202	(0.077)	-0.154	(0.126)	0.028	(0.042)	**0.721**	(0.068)
ANX4	-0.255	(0.075)	-0.194	(0.116)	-0.021	(0.029)	**0.630**	(0.064)
ANX5	0.008	(0.040)	-0.023	(0.072)	-0.099	(0.070)	**0.849**	(0.048)
ANX6	0.120	(0.061)	0.022	(0.051)	-0.133	(0.077)	**0.848**	(0.045)
Proportion of variance explained	0.136		0.248		0.119		0.144	

Correlation between		
Factor 1 and Factor 2	0.552	(0.117)
Factor 1 and Factor 3	0.632	(0.077)
Factor 1 and Factor 4	-0.110	(0.138)
Factor 2 and Factor 3	0.465	(0.080)
Factor 2 and Factor 4	-0.547	(0.080)
Factor 3 and Factor 4	-0.145	(0.099)

Factor loadings above 0.3 or below -0.3 are in bold

Table 40 Model parameters of the 4-factor EFA model.

	Factor 1		Factor 2		Factor 3		Factor 4		Factor 5	
	Factor Loading		Factor Loading		Factor Loading		Factor Loading		Factor Loading	
Item	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.	Est.	S.E.
VAL1	**0.978**	(0.124)	-0.106	(0.153)	-0.062	(0.134)	-0.019	(0.079)	-0.016	(0.063)
VAL2	**0.652**	(0.095)	0.003	(0.020)	0.106	(0.118)	0.048	(0.076)	0.140	(0.080)
VAL3	**0.511**	(0.094)	0.148	(0.110)	0.120	(0.112)	0.082	(0.078)	0.083	(0.085)
VAL4	**0.843**	(0.103)	-0.089	(0.139)	0.014	(0.067)	-0.073	(0.081)	-0.046	(0.076)
VAL5	**0.652**	(0.118)	0.033	(0.130)	-0.088	(0.109)	0.148	(0.082)	-0.101	(0.094)
VAL6	**0.535**	(0.092)	0.185	(0.114)	0.030	(0.101)	-0.004	(0.036)	0.147	(0.087)
VAL7	**0.630**	(0.103)	-0.153	(0.119)	0.163	(0.101)	-0.022	(0.058)	0.016	(0.047)
CON1	0.117	(0.066)	**0.634**	(0.063)	0.087	(0.070)	0.156	(0.059)	-0.045	(0.070)
CON2	0.012	(0.051)	**0.984**	(0.061)	-0.122	(0.056)	-0.020	(0.051)	0.097	(0.071)
CON3	0.141	(0.076)	**0.654**	(0.072)	0.019	(0.051)	0.245	(0.061)	0.049	(0.086)
CON4	-0.022	(0.053)	**1.013**	(0.056)	-0.042	(0.065)	0.015	(0.045)	0.178	(0.062)
CON5	0.033	(0.059)	**0.823**	(0.046)	0.001	(0.045)	0.100	(0.049)	-0.004	(0.041)
CON6	-0.017	(0.051)	**0.743**	(0.052)	0.226	(0.051)	0.073	(0.055)	-0.026	(0.050)
CON7	-0.020	(0.056)	**-0.640**	(0.051)	-0.066	(0.063)	-0.021	(0.049)	0.190	(0.054)
CON8	0.065	(0.068)	**-0.930**	(0.058)	0.007	(0.048)	0.114	(0.054)	0.012	(0.051)
CON9	-0.095	(0.058)	**-0.762**	(0.056)	0.019	(0.042)	0.046	(0.056)	0.135	(0.057)
CON10	0.026	(0.053)	**-0.599**	(0.061)	-0.086	(0.060)	0.037	(0.063)	0.277	(0.060)
CON11	-0.003	(0.014)	**-0.330**	(0.078)	**-0.648**	(0.063)	0.268	(0.072)	0.161	(0.119)
CON12	-0.005	(0.047)	**-0.561**	(0.066)	**-0.372**	(0.059)	-0.005	(0.028)	0.090	(0.075)
ENJ1	0.269	(0.107)	-0.149	(0.099)	0.154	(0.103)	**0.408**	(0.078)	-0.009	(0.015)
ENJ2	0.133	(0.118)	0.183	(0.071)	**0.607**	(0.082)	0.197	(0.083)	0.003	(0.009)
ENJ3	-0.045	(0.105)	-0.012	(0.057)	**0.732**	(0.078)	0.010	(0.060)	-0.103	(0.093)
ENJ4	0.037	(0.122)	0.005	(0.041)	**0.771**	(0.074)	0.116	(0.090)	-0.011	(0.057)
ENJ5	0.113	(0.115)	0.168	(0.071)	**0.527**	(0.079)	0.188	(0.078)	0.052	(0.051)
ENJ6	0.018	(0.053)	-0.008	(0.020)	-0.100	(0.084)	**0.793**	(0.078)	**-0.333**	(0.178)
ENJ7	-0.039	(0.089)	0.056	(0.098)	0.014	(0.026)	**0.749**	(0.088)	**-0.420**	(0.161)
ANX1	-0.014	(0.051)	-0.185	(0.074)	-0.032	(0.099)	0.029	(0.070)	**0.716**	(0.058)
ANX2	-0.220	(0.079)	0.008	(0.032)	0.081	(0.090)	-0.044	(0.087)	**0.722**	(0.052)
ANX3	-0.200	(0.071)	-0.054	(0.082)	0.061	(0.081)	-0.087	(0.098)	**0.804**	(0.069)
ANX4	-0.252	(0.073)	-0.151	(0.078)	-0.010	(0.022)	-0.030	(0.081)	**0.654**	(0.064)
ANX5	0.005	(0.031)	-0.017	(0.067)	-0.137	(0.118)	0.076	(0.065)	**0.827**	(0.060)
ANX6	0.127	(0.059)	0.065	(0.089)	-0.157	(0.131)	0.018	(0.049)	**0.859**	(0.067)
Proportion of variance explained	0.133		0.242		0.102		0.061		0.140	
							Correlation between			
							Factor 1 and Factor 2		0.525	(0.120)
							Factor 1 and Factor 3		0.636	(0.101)
							Factor 1 and Factor 4		0.488	(0.102)
							Factor 1 and Factor 5		-0.128	(0.138)
							Factor 2 and Factor 3		0.484	(0.070)
							Factor 2 and Factor 4		0.354	(0.164)
							Factor 2 and Factor 5		-0.639	(0.072)
							Factor 3 and Factor 4		0.425	(0.134)
							Factor 3 and Factor 5		-0.233	(0.168)
							Factor 4 and Factor 5		0.039	(0.209)

Factor loadings above 0.3 or below -0.3 are in bold

Table 41 Model parameters of the 5-factor EFA model.

6.3 Item response theory

Another analysis was conducted with item response theory (IRT; Bock & Gibbons, 2010; Ostini & Nering, 2006) in order to examine the robustness of the results of the CFA reported above. Graded response models (Samejima, 2010) were fitted with an expectation-maximisation estimation algorithm to mimic the models tested in CFA.

A graded response model has the following characteristics: For each item *j*, *M* boundary functions are modelled, where *M* + 1 is the number of possible response categories (in this case, *M* + 1 = 5). The functions act in a similar manner to the mapping of the ordered categorical responses to an underlying distribution in the estimation of a polychoric correlation. The underlying distribution is often known as a function of the trait level θ. The functions are two-parameter logistic functions of the cumulative operating characteristic specified by:

$$P^*_{jm}(\theta) = \frac{1}{1 + e^{-Da_j(\theta - b_{jm})}}$$

where a_j is the item discrimination parameter, b_j is the item difficulty parameter, *D* is a scaling factor usually set equal to 1.7, and *m* = 1, …, *M* + 1.

Then the probability of responding with a given response category *m* is obtained by subtracting the adjacent functions:

$$\begin{cases} P^*_{j1}(\theta) = 1 \\ P_{jm}(\theta) = P^*_{jm}(\theta) - P^*_{j(m+1)}(\theta) \\ P^*_{j(M+1)}(\theta) = 0 \end{cases}$$

The analysis was conducted with R Version 4.4.1 and its package ***mirt*** (Chalmers, 2012). The IRT results were consistent with the CFA: within each group the items measured the same latent variable, whereas between any pair of the groups the latent variables measured by the items of the two groups were not identical, but distinct and correlated.

	Discrimination		Difficulty 1		Difficulty 2		Difficulty 3		Difficulty 4	
Item	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)
VAL1	3.377	(0.397)	-1.815	(0.135)	-1.367	(0.104)	-0.794	(0.081)	0.150	(0.071)
VAL2	1.761	(0.182)	-2.600	(0.252)	-1.955	(0.179)	-0.912	(0.108)	0.358	(0.092)
VAL3	1.717	(0.177)	-2.826	(0.292)	-1.423	(0.143)	-0.467	(0.093)	0.909	(0.112)
VAL4	2.269	(0.229)	-2.037	(0.170)	-1.321	(0.116)	-0.657	(0.086)	0.318	(0.082)
VAL5	1.703	(0.171)	-1.869	(0.174)	-1.085	(0.119)	-0.442	(0.093)	0.939	(0.113)
VAL6	1.451	(0.151)	-1.865	(0.187)	-1.043	(0.127)	-0.146	(0.096)	1.629	(0.167)
VAL7	1.513	(0.160)	-2.356	(0.234)	-1.660	(0.169)	-0.710	(0.107)	0.452	(0.103)

Table 42 Item response parameters of the 1-factor model of perceived value of learning mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
VAL1	0.893	27.492	29	0.545	<0.001
VAL2	0.719	38.129	36	0.373	0.012
VAL3	0.710	29.831	35	0.716	<0.001
VAL4	0.800	36.173	39	0.600	<0.001
VAL5	0.707	50.240	44	0.240	0.019
VAL6	0.649	49.518	44	0.262	0.018
VAL7	0.664	66.752	43	0.012	0.038
				Average variance extracted	0.546

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 43 Model parameters and item fit statistics of the 1-factor model of perceived value of learning mathematics.

	Discrimination		Difficulty 1		Difficulty 2		Difficulty 3		Difficulty 4	
Item	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)
CON1	2.731	(0.232)	-1.794	(0.135)	-1.295	(0.105)	-0.621	(0.079)	0.459	(0.080)
CON2	2.887	(0.244)	-0.748	(0.083)	-0.248	(0.072)	0.180	(0.073)	0.961	(0.092)
CON3	2.618	(0.247)	-2.390	(0.196)	-1.740	(0.133)	-1.127	(0.099)	-0.207	(0.075)
CON4	3.121	(0.261)	-1.224	(0.100)	-0.690	(0.079)	-0.110	(0.070)	0.743	(0.083)
CON5	3.524	(0.308)	-1.737	(0.126)	-1.231	(0.095)	-0.597	(0.074)	0.346	(0.072)
CON6	3.923	(0.343)	-1.324	(0.099)	-0.808	(0.078)	-0.333	(0.068)	0.527	(0.074)
CON7	-2.831	(0.245)	0.355	(0.076)	-0.462	(0.075)	-0.945	(0.090)	-1.528	(0.121)
CON8	-2.981	(0.257)	0.785	(0.085)	0.053	(0.071)	-0.337	(0.073)	-0.738	(0.083)
CON9	-3.444	(0.297)	0.315	(0.073)	-0.323	(0.069)	-0.771	(0.078)	-1.380	(0.106)
CON10	-2.290	(0.192)	1.019	(0.102)	0.172	(0.078)	-0.269	(0.078)	-1.206	(0.110)
CON11	-1.662	(0.157)	1.684	(0.158)	0.905	(0.111)	0.309	(0.092)	-0.508	(0.098)
CON12	-2.635	(0.239)	0.034	(0.075)	-0.499	(0.078)	-1.026	(0.095)	-1.401	(0.115)

Table 44 Item response parameters of the 1-factor model of confidence in mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
CON1	0.849	40.290	36	0.286	0.018
CON2	0.861	53.838	42	0.104	0.028
CON3	0.838	24.093	31	0.807	<0.001
CON4	0.878	38.983	39	0.471	<0.001
CON5	0.900	42.150	36	0.222	0.022
CON6	0.917	30.671	40	0.856	<0.001
CON7	-0.857	46.464	45	0.412	0.010
CON8	-0.868	65.583	40	0.007	0.043
CON9	-0.896	35.107	47	0.899	<0.001
CON10	-0.803	42.228	41	0.418	0.009
CON11	-0.699	38.792	35	0.303	0.017
CON12	-0.840	59.845	44	0.056	0.032
				Average variance extracted	0.727

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 45 Model parameters and item fit statistics of the 1-factor model of confidence in mathematics.

	Discrimination	Difficulty 1	Difficulty 2	Difficulty 3	Difficulty 4

Item	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)
ENJ1	1.138	(0.133)	-2.847	(0.330)	-2.051	(0.244)	-0.793	(0.139)	1.035	(0.152)
ENJ2	5.810	(0.945)	-0.847	(0.079)	-0.344	(0.067)	0.135	(0.064)	0.871	(0.077)
ENJ3	1.339	(0.148)	-0.670	(0.120)	-0.135	(0.102)	0.567	(0.112)	1.071	(0.140)
ENJ4	2.291	(0.212)	-0.747	(0.093)	-0.030	(0.078)	0.707	(0.089)	1.495	(0.127)
ENJ5	2.958	(0.264)	-1.087	(0.097)	-0.410	(0.076)	0.209	(0.072)	1.041	(0.094)
ENJ6	1.057	(0.127)	-2.591	(0.309)	-1.355	(0.183)	-0.469	(0.126)	1.345	(0.184)
ENJ7	1.340	(0.146)	-2.152	(0.228)	-1.250	(0.153)	-0.510	(0.110)	0.734	(0.121)

Table 46 Item response parameters of the 1-factor model of enjoyment of mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
ENJ1	0.556	66.539	54	0.118	0.025
ENJ2	0.960	43.259	32	0.088	0.031
ENJ3	0.618	70.349	60	0.170	0.022
ENJ4	0.803	42.839	42	0.435	0.007
ENJ5	0.867	66.509	41	0.007	0.041
ENJ6	0.527	53.294	59	0.685	<0.001
ENJ7	0.619	51.348	57	0.686	<0.001
				Average variance extracted	0.524

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 47 Model parameters and item fit statistics of the 1-factor model of enjoyment of mathematics.

	Discrimination		Difficulty 1		Difficulty 2		Difficulty 3		Difficulty 4	
Item	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)
ANX1	2.932	(0.266)	-0.929	(0.091)	-0.223	(0.073)	0.338	(0.073)	1.210	(0.103)
ANX2	2.017	(0.191)	-0.682	(0.096)	-0.057	(0.082)	0.560	(0.089)	1.298	(0.124)
ANX3	3.282	(0.331)	-0.304	(0.074)	0.307	(0.071)	0.950	(0.087)	1.637	(0.125)
ANX4	2.711	(0.259)	-0.146	(0.076)	0.503	(0.078)	1.156	(0.102)	1.838	(0.145)
ANX5	3.289	(0.338)	-0.673	(0.081)	-0.090	(0.070)	0.324	(0.071)	1.034	(0.093)
ANX6	2.586	(0.254)	-0.674	(0.087)	-0.146	(0.075)	0.373	(0.077)	1.089	(0.103)

Table 48 Item response parameters of the 1-factor model of anxiety about mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
ANX1	0.865	35.419	34	0.401	0.011
ANX2	0.764	47.943	46	0.394	0.011
ANX3	0.888	34.610	31	0.300	0.018
ANX4	0.847	35.443	34	0.400	0.011
ANX5	0.888	28.104	34	0.751	<0.001
ANX6	0.835	31.597	39	0.794	<0.001
				Average variance extracted	0.721

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 49 Model parameters and item fit statistics of the 1-factor model of anxiety about mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
VAL1	0.495	60.261	49	0.130	0.026
VAL2	0.433	58.437	55	0.350	0.013
VAL3	0.529	62.577	52	0.150	0.024
VAL4	0.477	65.227	51	0.087	0.028
VAL5	0.524	64.907	59	0.278	0.017
VAL6	0.464	79.538	61	0.056	0.030
VAL7	0.353	67.853	54	0.097	0.027
CON1	0.859	63.000	47	0.059	0.031
CON2	0.852	148.840	58	<0.001	0.067
CON3	0.849	66.977	44	0.014	0.039
CON4	0.873	92.170	55	0.001	0.044
CON5	0.896	107.964	52	<0.001	0.056
CON6	0.918	103.356	53	<0.001	0.053
CON7	-0.845	65.160	47	0.041	0.034
CON8	-0.852	94.422	66	0.012	0.035
CON9	-0.887	92.275	46	<0.001	0.054
CON10	-0.787	80.304	72	0.235	0.018
CON11	-0.708	67.422	64	0.361	0.012
CON12	-0.842	37.011	55	0.970	<0.001
				Average variance extracted	0.537

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 50 Model parameters and item fit statistics of the 1-factor model of perceived value of learning mathematics and confidence in mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
VAL1	0.881	39.496	38	0.403	0.011
VAL2	0.717	42.015	41	0.427	0.008
VAL3	0.725	44.020	45	0.513	<0.001
VAL4	0.796	58.927	45	0.080	0.030
VAL5	0.720	56.342	50	0.250	0.019
VAL6	0.659	57.409	52	0.282	0.017
VAL7	0.656	57.665	46	0.116	0.027
CON1	0.853	59.407	56	0.353	0.013
CON2	0.863	80.769	70	0.178	0.021
CON3	0.843	70.118	50	0.032	0.034
CON4	0.879	66.170	71	0.640	<0.001
CON5	0.901	71.372	56	0.081	0.028
CON6	0.920	58.312	67	0.767	<0.001
CON7	-0.856	61.915	51	0.141	0.025
CON8	-0.866	88.707	74	0.117	0.024
CON9	-0.896	100.256	70	0.010	0.035
CON10	-0.801	81.860	75	0.275	0.016
CON11	-0.705	66.756	67	0.485	<0.001
CON12	-0.842	34.016	56	0.991	<0.001
				Average variance extracted of	
				VAL	0.548
				CON	0.729
				Correlation between VAL and CON	0.547

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 51 Model parameters and item fit statistics of the 2-factor model of perceived value of learning mathematics and confidence in mathematics.

model	C2	df	p-value	SRMR	RMSEA	CFI
1-factor	1378.954	152	<0.001	0.113	0.153	0.904
2-factor	523.380	151	<0.001	0.065	0.085	0.971

C2 is a limited-information goodness-of-fit test statistic for ordinal IRT, an alternative to χ^2 (Cai & Monroe, 2014).

Table 52 Goodness-of-fit indices of the models of perceived value of learning mathematics and confidence in mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
VAL1	0.735	74.258	60	0.102	0.026
VAL2	0.688	63.760	58	0.281	0.017
VAL3	0.715	57.198	60	0.579	<0.001
VAL4	0.656	73.696	71	0.390	0.010
VAL5	0.653	77.322	84	0.684	<0.001
VAL6	0.618	63.065	83	0.949	<0.001
VAL7	0.603	75.449	76	0.496	<0.001
ENJ1	0.597	89.945	72	0.075	0.026
ENJ2	0.910	65.172	48	0.050	0.032
ENJ3	0.597	70.002	73	0.578	<0.001
ENJ4	0.787	58.826	66	0.722	<0.001
ENJ5	0.826	59.656	64	0.631	<0.001
ENJ6	0.528	91.982	81	0.190	0.019
ENJ7	0.593	70.470	86	0.887	<0.001
				Average variance extracted	0.471

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 53 Model parameters and item fit statistics of the 1-factor model of perceived value of learning mathematics and enjoyment of mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
VAL1	0.864	73.949	54	0.037	0.032
VAL2	0.732	62.255	56	0.263	0.018
VAL3	0.742	62.134	65	0.578	<0.001
VAL4	0.773	75.354	65	0.178	0.021
VAL5	0.712	74.598	82	0.707	<0.001
VAL6	0.661	62.886	81	0.932	<0.001
VAL7	0.663	75.471	74	0.431	0.007
ENJ1	0.577	91.547	73	0.070	0.027
ENJ2	0.959	61.685	47	0.074	0.030
ENJ3	0.615	68.669	73	0.622	<0.001
ENJ4	0.804	61.449	67	0.668	<0.001
ENJ5	0.867	60.594	65	0.632	<0.001
ENJ6	0.530	91.775	82	0.216	0.018
ENJ7	0.614	75.810	83	0.699	<0.001
				Average variance extracted of	
				VAL	0.545
				ENJ	0.527
				Correlation between VAL and ENJ	0.762

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 54 Model parameters and item fit statistics of the 2-factor model of perceived value of learning mathematics and enjoyment of mathematics.

model	C2	df	p-value	SRMR	RMSEA	CFI
1-factor	535.672	77	<0.001	0.087	0.129	0.909
2-factor	296.812	76	<0.001	0.066	0.090	0.956

C2 is a limited-information goodness-of-fit test statistic for ordinal IRT, an alternative to χ^2 (Cai & Monroe, 2014).

Table 55 Goodness-of-fit indices of the models of perceived value of learning mathematics and enjoyment of mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
VAL1	-0.314	113.130	67	<0.001	0.044
VAL2	-0.239	90.807	68	0.034	0.030
VAL3	-0.331	98.345	75	0.037	0.029
VAL4	-0.317	93.566	70	0.032	0.031
VAL5	-0.399	96.518	76	0.056	0.027
VAL6	-0.268	125.527	82	0.001	0.038
VAL7	-0.197	68.927	65	0.346	0.013
ANX1	0.856	87.888	49	0.001	0.047
ANX2	0.775	115.441	51	<0.001	0.059
ANX3	0.902	133.198	43	<0.001	0.076
ANX4	0.874	181.168	48	<0.001	0.088
ANX5	0.858	109.719	49	<0.001	0.059
ANX6	0.787	81.633	51	0.004	0.041
				Average variance extracted	0.377

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 56 Model parameters and item fit statistics of the 1-factor model of perceived value of learning mathematics and anxiety about mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
VAL1	0.891	71.410	61	0.170	0.022
VAL2	0.716	54.092	60	0.690	<0.001
VAL3	0.712	62.781	64	0.520	<0.001
VAL4	0.800	62.573	63	0.491	<0.001
VAL5	0.713	75.812	70	0.297	0.015
VAL6	0.649	86.695	69	0.074	0.027
VAL7	0.662	60.098	63	0.580	<0.001
ANX1	0.865	66.783	68	0.519	<0.001
ANX2	0.767	73.185	73	0.472	0.003
ANX3	0.891	68.541	63	0.295	0.016
ANX4	0.852	79.519	57	0.026	0.033
ANX5	0.883	37.920	33	0.255	0.020
ANX6	0.827	41.724	32	0.117	0.029
				Average variance extracted of	
				VAL	0.546
				ANX	0.720
				Correlation between VAL and ANX	-0.240

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 57 Model parameters and item fit statistics of the 2-factor model of perceived value of learning mathematics and anxiety about mathematics.

model	C2	df	p-value	SRMR	RMSEA	CFI
1-factor	1740.937	65	<0.001	0.214	0.267	0.579
2-factor	294.464	64	<0.001	0.076	0.100	0.942

C2 is a limited-information goodness-of-fit test statistic for ordinal IRT, an alternative to χ^2 (Cai & Monroe, 2014).

Table 58 Goodness-of-fit indices of the models of perceived value of learning mathematics and anxiety about mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
CON1	0.855	50.008	49	0.433	0.008
CON2	0.833	130.632	57	<0.001	0.061
CON3	0.845	44.116	43	0.424	0.009
CON4	0.860	102.525	52	<0.001	0.053
CON5	0.891	67.818	49	0.039	0.034
CON6	0.925	71.942	51	0.028	0.035
CON7	-0.845	49.910	39	0.113	0.029
CON8	-0.835	76.784	65	0.150	0.023
CON9	-0.874	65.829	39	0.113	0.029
CON10	-0.783	73.417	58	0.083	0.028
CON11	-0.729	58.865	75	0.915	<0.001
CON12	-0.857	58.414	53	0.283	0.017
ENJ1	0.335	70.011	60	0.177	0.022
ENJ2	0.742	62.968	55	0.215	0.021
ENJ3	0.491	70.945	64	0.257	0.018
ENJ4	0.592	61.256	60	0.431	0.008
ENJ5	0.649	64.554	61	0.354	0.013
ENJ6	0.523	72.997	66	0.259	0.018
ENJ7	0.650	62.939	60	0.373	0.012
				Average variance extracted	0.577

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 59 Model parameters and item fit statistics of the 1-factor model of confidence in mathematics and enjoyment of mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
CON1	0.853	64.053	57	0.243	0.019
CON2	0.856	81.693	72	0.204	0.020
CON3	0.842	49.583	51	0.530	<0.001
CON4	0.875	63.885	70	0.683	<0.001
CON5	0.899	49.593	56	0.714	<0.001
CON6	0.923	55.640	64	0.762	<0.001
CON7	-0.856	39.832	34	0.227	0.022
CON8	-0.861	74.695	71	0.359	0.012
CON9	-0.892	73.981	67	0.261	0.017
CON10	-0.801	83.369	75	0.238	0.018
CON11	-0.716	66.169	80	0.867	<0.001
CON12	-0.848	70.791	68	0.385	0.011
ENJ1	0.542	56.987	54	0.365	0.013
ENJ2	0.959	59.106	46	0.093	0.029
ENJ3	0.622	59.661	60	0.488	<0.001
ENJ4	0.798	47.405	50	0.578	<0.001
ENJ5	0.864	75.510	51	0.014	0.037
ENJ6	0.545	73.280	66	0.252	0.018
ENJ7	0.641	64.585	59	0.288	0.017
				Average variance extracted of	
				CON	0.728
				ENJ	0.527
				Correlation between CON and ENJ	0.710

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 60 Model parameters and item fit statistics of the 2-factor model of confidence in mathematics and enjoyment of mathematics.

model	C2	df	p-value	SRMR	RMSEA	CFI
1-factor	1539.774	152	<0.001	0.096	0.163	0.902
2-factor	826.993	151	<0.001	0.083	0.114	0.952

C2 is a limited-information goodness-of-fit test statistic for ordinal IRT, an alternative to χ^2 (Cai & Monroe, 2014).

Table 61 Goodness-of-fit indices of the models of confidence in mathematics and enjoyment of mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
CON1	-0.827	87.373	63	0.023	0.034
CON2	-0.837	59.221	46	0.091	0.029
CON3	-0.800	84.808	50	0.002	0.045
CON4	-0.842	136.078	73	<0.001	0.050
CON5	-0.879	90.504	60	0.007	0.039
CON6	-0.895	85.865	66	0.051	0.030
CON7	0.869	99.924	51	<0.001	0.053
CON8	0.866	155.710	54	<0.001	0.074
CON9	0.904	99.390	49	<0.001	0.055
CON10	0.838	94.437	57	0.001	0.044
CON11	0.715	72.297	57	0.083	0.028
CON12	0.836	91.613	51	<0.001	0.048
ANX1	0.764	71.216	60	0.152	0.023
ANX2	0.654	79.153	67	0.147	0.023
ANX3	0.772	64.091	53	0.141	0.025
ANX4	0.809	55.307	56	0.501	<0.001
ANX5	0.708	107.528	63	<0.001	0.045
ANX6	0.606	75.818	70	0.296	0.016
				Average variance extracted	0.649

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 62 Model parameters and item fit statistics of the 1-factor model of confidence in mathematics and anxiety about mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
CON1	-0.846	62.843	64	0.517	<0.001
CON2	-0.859	68.154	59	0.194	0.021
CON3	-0.833	65.000	55	0.168	0.023
CON4	-0.871	121.651	76	0.001	0.042
CON5	-0.900	93.532	63	0.008	0.038
CON6	-0.917	82.811	69	0.123	0.024
CON7	0.866	89.476	69	0.049	0.029
CON8	0.872	98.824	63	0.003	0.041
CON9	0.903	69.027	69	0.476	0.001
CON10	0.818	74.822	72	0.387	0.011
CON11	0.709	61.960	66	0.618	<0.001
CON12	0.844	79.725	60	0.045	0.031
ANX1	0.875	54.633	52	0.385	0.012
ANX2	0.774	84.608	57	0.010	0.038
ANX3	0.899	70.753	48	0.018	0.037
ANX4	0.869	51.144	49	0.389	0.011
ANX5	0.872	106.555	51	<0.001	0.056
ANX6	0.808	59.779	56	0.340	0.014
				Average variance extracted of	
				CON	0.731
				ANX	0.724
				Correlation between CON and ANX	0.766

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 63 Model parameters and item fit statistics of the 2-factor model of confidence in mathematics and anxiety about mathematics.

model	C2	df	p-value	SRMR	RMSEA	CFI
1-factor	1555.026	135	<0.001	0.091	0.175	0.908
2-factor	723.244	134	<0.001	0.068	0.113	0.962

C2 is a limited-information goodness-of-fit test statistic for ordinal IRT, an alternative to χ^2 (Cai & Monroe, 2014).

Table 64 Goodness-of-fit indices of the models of confidence in mathematics and anxiety about mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
ENJ1	-0.200	88.503	68	0.048	0.029
ENJ2	-0.555	153.147	100	0.001	0.039
ENJ3	-0.442	97.658	83	0.130	0.022
ENJ4	-0.466	149.267	87	<0.001	0.045
ENJ5	-0.460	128.791	84	0.001	0.039
ENJ6	-0.452	126.154	91	0.009	0.033
ENJ7	-0.590	122.322	89	0.011	0.032
ANX1	0.847	116.502	59	<0.001	0.052
ANX2	0.764	131.118	63	<0.001	0.055
ANX3	0.895	135.914	60	<0.001	0.059
ANX4	0.873	158.018	59	<0.001	0.068
ANX5	0.845	118.712	59	<0.001	0.053
ANX6	0.781	127.513	62	<0.001	0.054
				Average variance extracted	0.440

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 65 Model parameters and item fit statistics of the 1-factor model of enjoyment of mathematics and anxiety about mathematics.

	Factor Loading	S_X2	df.S_X2	*p*.S_X2	RMSEA.S_X2
ENJ1	0.547	73.398	65	0.222	0.019
ENJ2	0.959	86.896	82	0.335	0.013
ENJ3	0.625	78.170	79	0.505	<0.001
ENJ4	0.802	90.443	77	0.140	0.022
ENJ5	0.863	87.682	84	0.370	0.011
ENJ6	0.534	94.877	78	0.094	0.025
ENJ7	0.629	90.030	80	0.208	0.019
ANX1	0.866	70.685	77	0.681	<0.001
ANX2	0.768	72.721	82	0.758	<0.001
ANX3	0.893	72.750	73	0.486	<0.001
ANX4	0.856	51.432	59	0.748	<0.001
ANX5	0.883	86.703	77	0.211	0.019
ANX6	0.826	106.458	78	0.018	0.032
				Average variance extracted of	
				ENJ	0.525
				ANX	0.722
				Correlation between ENJ and ANX	-0.444

S_X2 is signed chi-squared test, an item fit index for ordinal IRT (Kang & Chen, 2008).

Table 66 Model parameters and item fit statistics of the 2-factor model of enjoyment of mathematics and anxiety about mathematics.

model	C2	df	p-value	SRMR	RMSEA	CFI
1-factor	1598.800	65	<0.001	0.175	0.257	0.666
2-factor	398.232	63	<0.001	0.092	0.121	0.927

C2 is a limited-information goodness-of-fit test statistic for ordinal IRT, an alternative to χ^2 (Cai & Monroe, 2014).

Table 67 Goodness-of-fit indices of the models of enjoyment of mathematics and anxiety about mathematics.

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