Recent Progress in Effective Field Theory
Constructions of Dark Energy

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1. Theories of Dark Energy/Modified Gravity
2. Effective Field Theories
3. Covariant Approach
4. Perturbative Approach
Outline

1. Theories of Dark Energy/Modified Gravity
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The Presence of Dark Energy

\[ H^2 = \frac{8\pi G \rho}{3} + \frac{\Lambda}{3} - \frac{k}{a^2} \]

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda}{3} \]

The accelerated expansion of the universe \( \ddot{a}/a > 0 \) shows that either:

- There exists a Cosmological Constant \( \Lambda \)
- There exists exotic matter with \( P < -\rho/3 \)
- We have the theory wrong
There are Plenty of Fishes in the Sea

- Cosmological Constant
- Quintessence
- $k$-Essence
- Brans-Dicke theories
- Ghost Condensate
- Extra-dimensions *a la* ADD/RS/DGP/UED/ . . .
- $f(R)$ gravity, $f(T)$ gravity, . . .
- Gauss-Bonnet gravity
- Galileon models
- Massive gravity
- . . .
Towards a Generic Theory of Dark Energy?

In a low-energy four-dimensional limit, almost all theories of dark energy behave as GR + scalar field(s). Can we use this?

**Approaches**

- Battye and Pearson 2012 (fluid approach, perturbative EOMs)
- Parameterized Post-Friedmannian (Baker *et al.* 2012, multi-scalar perturbative EOMs)
- Pogosian, Silvestri and Buniy 2013 (generic perturbative EOMs)
- Covariant EFT (JB and Flanagan 2012, single scalar action)
Goals of Generalized Models

Predict the Background Evolution
- Expansion Rate $H(t)$
- Huge degeneracy in theory space

Predict Perturbative Phenomena
- Matter density perturbation $\delta$
- Matter velocity perturbation $v$
- Can lift degeneracy in background

\[
\frac{k^2}{a^2} \psi = -4\pi G \rho Q(a, k)(\delta + \frac{3aHv}{k})
\]

\[
\phi = R(a, k)\psi
\]

Aim: Construct theoretical priors on $Q$, $R$
Quantum Field Theory Overview

**Idea**
- Provide a low energy description of phenomena
- Ignore massive fields above the energy range of interest
- Anything that can happen, will happen
Effective Field Theories

Quantum Field Theory Overview

Implementation

- Specify fields, symmetries, cutoff
- Construct action with all possible operators
- Arrange operators in an appropriate expansion
- EFT provides rules for scaling of coefficients, and gives a handle on radiative corrections

\[ \mathcal{L} = \mathcal{L}_{\text{marginal, relevant}} + \sum \frac{c_k}{\Lambda^k} O_k, \quad c_k \sim 1 \]

- Fractional errors \( \propto (\sqrt{NE/\Lambda})^n \)
Challenges for Cosmological Effective Field Theory

- Cosmological models allow for large numbers of quanta in perturbative modes \( \sim m_P^2/H_0^2 \)
- Evolution of background solution leads to difficulties comparing operators
- Expect fields to move large distances over cosmological evolution

\[
\Delta \phi \sim m_P
\]
Two Classes of Effective Field Theory

**Background Independent**
- Works for all backgrounds, covariant expansion in $g_{\mu\nu}$, $\phi$, ... 
- $\mathcal{L} \sim R + (\nabla \phi)^2 + (\nabla \phi)^4 + \ldots$

**Background Dependent**
- Expand in deviations $\delta g_{\mu\nu}$, $\delta \phi$ from our cosmological background
- Covers a larger set of theories with finitely many terms
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Build on previous work by Weinberg (2008) who constructed a covariant EFT of Inflation.

<table>
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<tr>
<th>Similarities</th>
<th>Differences</th>
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<td>Describe accelerated expansion of the universe</td>
<td>Scales radically different</td>
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<td>Scalar field approach common to both</td>
<td>Introduction of matter</td>
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Overview of Approach

Construct a theory of Gravity + Scalar Field

- Identify a leading order action
- Identify perturbative corrections
- Apply selection rules to the theory
- Estimate scaling of operators
- Estimate regime of validity of theory
Covariant Approach

Action and Perturbations

Leading Order Action

\[ S_0 = \int d^4x \sqrt{-g} \left\{ \frac{m_p^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - U(\phi) \right\} + S_{\text{matter}} \left[ e^{\alpha(\phi)} g_{\mu\nu}, \{\psi\} \right] \]

- Use Einstein frame → canonical kinetic terms → allows comparison of operators

Perturb the Action

\[ \phi, g^{\mu\nu}, R_{\mu\nu\sigma\lambda}, \epsilon_{\mu\nu\sigma\lambda}, T_{\mu\nu}, \nabla_\mu, \Box \ldots \]
\[ \Delta S = \int d^4x \sqrt{-g} \left\{ a_1 (\nabla \phi)^4 + a_2 \Box \phi (\nabla \phi)^2 + a_3 (\Box \phi)^2 \right. \\
+ b_1 T^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi + b_2 T (\nabla \phi)^2 \\
+ b_3 T \Box \phi + b_4 R_{\mu \nu} T^{\mu \nu} + b_5 RT + b_6 T^{\mu \nu} \nabla_\mu \nabla_\nu + b_7 T \\
+ c_1 G^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi + c_2 R (\nabla \phi)^2 + c_3 R \Box \phi \\
+ d_1 R^2 + d_2 \left( R^{\mu \nu} R_{\mu \nu} - \frac{1}{3} R^2 \right) + d_4 \epsilon^{\mu \nu \lambda \rho} C_{\mu \nu} \alpha \beta C_{\lambda \rho \alpha \beta} \\
+ d_3 \left( R^2 - 4 R^{\mu \nu} R_{\mu \nu} + R_{\mu \nu \sigma \rho} R^{\mu \nu \sigma \rho} \right) \\
+ e_1 T^{\mu \nu} T_{\mu \nu} + e_2 T^2 + \ldots \right\} \\

S_{\text{matter}} \left[ e^{\alpha(\phi)} g^{\mu \nu} + e^{\alpha(\phi)} \left( \beta_1 \nabla_\mu \phi \nabla_\nu \phi + \beta_2 (\nabla \phi)^2 g^{\mu \nu} + \beta_3 \Box \phi g^{\mu \nu} \\
+ \beta_4 \nabla_\mu \nabla_\nu \phi + \beta_5 R_{\mu \nu} + \beta_6 R g_{\mu \nu} + \ldots \right), \{\psi\} \right] \]
Selection Rules

Choose Appropriate Operators

- Use a derivative expansion to fourth order
- Remove higher order derivatives in equations of motion ("reduce" the action)
- Impose the Weak Equivalence Principle
Resulting Theory

\[ S = \int d^4 x \sqrt{-g} \left\{ \frac{m_p^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - U(\phi) + a_1 (\nabla \phi)^4 \right. \]

\[ + b_2 T (\nabla \phi)^2 + c_1 G^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi \]

\[ + d_3 \left( R^2 - 4 R^{\mu \nu} R_{\mu \nu} + R_{\mu \nu \sigma \rho} R^{\mu \nu \sigma \rho} \right) \]

\[ + d_4 \epsilon^{\mu \nu \lambda \rho} C_{\mu \nu}^{\alpha \beta} C_{\lambda \rho \alpha \beta} \]

\[ + e_1 T^{\mu \nu} T_{\mu \nu} + e_2 T^2 + \ldots \right\} \]

\[ + S_m \left[ e^{\alpha(\phi)} g_{\mu \nu} \right] \]

- Coefficients are functions of \( \phi \)
- Parameter space is given by nine free functions
Scaling of Operators

- Naive scaling $\nabla^d \phi^n / M^{n+d-4}$ leads to heavily suppressed/finely tuned operators.
- Pseudo Nambu Goldstone Boson construction leads to a different scaling relationship with radiatively protected coefficients:
  \[
  \frac{\nabla^d \phi^n}{M^{d-2} m_P^{n-2}}
  \]
- Operators are effectively suppressed by $H^2 / M^2$.
- Remaining mass dimensions accounted for by factors of $m_P^2$.
- Coefficients are functions of $\phi / m_P$.
Regime of Validity

- Expansion breaks down
- Interesting regime
- Basic quintessence sufficient

\[ \frac{\delta \rho}{\rho} \]

\[ H_0 \]

\[ m_P \]
Regime of Validity

\[ \ln N \]

\[ \frac{m_p^2}{H_0^2} \]

Boundary of domain of validity of EFT

Background cosmology

\[ \delta \rho/\rho \sim M \]

\[ \delta \phi \sim m_p \]

\[ \delta \rho/\rho \sim 1 \]

\[ \delta \phi \sim 1 \]

\[ \sqrt{H_0 m_p} \]

\[ \sqrt{M m_p} \]
Effectiveness of this Approach

- Theory captures leading order dynamics within the regime of validity
- Unified treatment of cosmological background and perturbations
- Class of theories arises generically from a pNGB construction, which guarantees that quantum effects are under control

- By construction, nonlinear kinetic terms cannot give $O(1)$ contributions to dynamics
- Class of theories is indistinguishable from vanilla quintessence unless $M$ is near $H_0$
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Effective Field Theory of Inflation

- An EFT of perturbations has proved useful for inflation (Cheung et al. 2008) as well as quintessence (Creminelli et al. 2009)
- Background evolution must be specified
- Perturbative description more powerful
- Application to dark energy differs because of the presence of matter, and vastly different scales
General Parametrization of Single (Effective) Scalar Field Models

**Idea**

- Work in a gauge where scalar field perturbations are vanishing: \( \phi = \phi_0(t) \) (“unitary gauge”)
- Identify all objects invariant under the reduced symmetry
- Construct a general action from these objects
- Arrange action in a perturbative expansion
- Satisfy background equations (cancel tadpoles)
- Apply Effective Field Theory rules to select terms
- Restore perturbations
Matter Treatment

Decouple from Gravitational Model

- Assume Weak Equivalence Principle (WEP)
- Work in Jordan Frame
- Matter equations are independent of gravitational model
- Depend only on $\psi$, $\phi$ (Newtonian gauge)
Unitary Gauge

**Symmetries**

- Residual symmetry after gauge fixing is spatial diffeomorphism invariance $x^i = x^i(\tilde{x}^j, t)$
- Objects that transform appropriately are the following:
  - Functions of time: $f(t)$
  - Spatial metric: $h_{ij}$
  - Covariant spatial derivatives: $D_i$
  - 3D gravitational invariants: $R_{ijkl}, R_{ij}, R$
  - 3D volume form: $\epsilon^{ijk}$
  - Lapse: $N$
  - Time derivatives: $D_t = \partial_t - \mathcal{L}_N$
Combining the Objects into an Action

- Using equivalences, integration by parts, and demanding a single physical scalar degree of freedom enforces these objects are combined as

\[
S = \int d^3x \ dt \ N \sqrt{h} F \left[ t, \epsilon^{ijk}, h_{ij}, N, R_{ij}, D_i, K_{ij} \right]
\]

where \( K_{ij} = \frac{(D_t h_{ij})}{2N} \) is the extrinsic curvature tensor for surfaces of constant time.
Combining the Objects into an Action

- Trick: rearrange action as

\[
S = \int d^3x \ dt \ N \sqrt{h} F \left[ t, \epsilon^{ijk}, h_{ij}, N - N_0, R_{ij} - 2k h_{ij} / a(t)^2, D_i, K_{ij} + H(t) h_{ij} \right]
\]

\[
= \int d^3x \ dt \ N \sqrt{h} F \left[ t, \epsilon^{ijk}, h_{ij}, \delta g^{00}, \delta R_{ij}, D_i, \delta K_{ij} \right]
\]

- Relies upon the symmetry of FRW to construct perturbations
- Expand order by order in perturbations
- Typical to use \( g^{00} = -1/N^2 \) instead of \( N \)
Action in Unitary Gauge

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{m_P^2}{2} R \right\} + S_{\text{matter}} \]

- General Relativity
Action in Unitary Gauge

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{m_P^2}{2} R + \Lambda(t) - c(t) \delta g^{00} \right\} + S_{\text{matter}} \]

Quintessence
Action in Unitary Gauge

\[ S = \int d^4 x \sqrt{-g} \left\{ \frac{m_p^2}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} \right\} + S_{\text{matter}} \]

- Non-minimal coupling
- Background evolution entirely specified in terms of these three functions of time
- Equivalent to perturbative description of background independent approach
Action in Unitary Gauge

\[ S = \int d^4 x \sqrt{-g} \left\{ \frac{m_p^2}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 \right\} \\
+ S_{\text{matter}} \]

- k-essence
Perturbative Approach

Action in Unitary Gauge

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{m^2_P}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 ight. \\
\left. - \frac{\tilde{M}_1^3(t)}{2} \delta g^{00} \delta K_i^i \right\} + S_{\text{matter}} \]

- Galileon/Kinetic Braiding
Perturbative Approach

Action in Unitary Gauge

\[ S = \int d^4 x \sqrt{-g} \left\{ \frac{m_P^2}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 \right. \\
- \frac{\tilde{M}_1^3(t)}{2} \delta g^{00} \delta K_i^i - \frac{\tilde{M}_2^2(t)}{2} (\delta K_i^i)^2 - \frac{\tilde{M}_3^2(t)}{2} \delta K_j^i \delta K_i^j \\
+ \frac{\hat{M}_2^2(t)}{2} \delta g^{00} \delta R^{(3)} \right\} \\
+ S_{\text{matter}} \]

- Horndeski’s General Theory
Action in Unitary Gauge

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{m_P^2}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 \right. \\
- \frac{\tilde{M}_1^3(t)}{2} \delta g^{00} \delta K_i^i - \frac{\tilde{M}_2^2(t)}{2} (\delta K_i^i)^2 - \frac{\tilde{M}_3^2(t)}{2} \delta K_j^i \delta K_i^j \\
+ \frac{\hat{M}_2^2(t)}{2} \delta g^{00} \delta R^{(3)} + m_2^2(t) h^{ij} \partial_i g^{00} \partial_j g^{00} \left. \right\} \\
+ S_{\text{matter}} \\

- Ho\check{r}ava-Lifshitz Gravity
Use the Stückelberg trick to restore perturbations. Calculate:

- Effective stress-energy tensor, scalar equation of motion
- $Q(a, k), R(a, k)$
- Effective Newtonian constant
- Speed of sound of perturbations
- Stability

Expresses $Q(a, k), R(a, k)$ in terms of a few functions of time in the action

Stronger theoretical prior on theory space of modified gravity models
Benefits and Limitations of This Approach

Benefits

- Time dependence arises from a small number of coefficients in the action
- General parametrization of theory space
- Allows for model-independent constraints
- Based on an action construction: scalar, vector and tensor mode behavior all arise from the same coefficients

Limitations

- Agnostic as to background evolution
- Only applies to single (effective) scalar field models
- Requires $\phi_0(t)$ to be strictly monotonic in regime of interest
Example: Application to Horndeski

Most general single scalar field theory with 2nd Order EOMs

\[ S = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + S_{\text{matter}} \]

with

\[ \mathcal{L}_2 = K(\phi, X) \]
\[ \mathcal{L}_3 = -G_3(\phi, X)\Box \phi \]
\[ \mathcal{L}_4 = G_4(\phi, X) R + G_{4,X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) \right] \]
\[ \mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} (\nabla^\mu \nabla^\nu \phi) \]
\[ -\frac{1}{6} G_{5,X} \left[ (\Box \phi)^3 - 3(\Box \phi) (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) \right. \]
\[ + 2(\nabla^\mu \nabla_\alpha \phi) (\nabla^\alpha \nabla_\beta \phi) (\nabla_\beta \nabla_\mu \phi) \]

\[ X = -(\nabla \phi)^2 / 2 \]
Example: Application to Horndeski

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2} \delta g^{00} \delta K^i_i - \frac{\bar{M}_2^2(t)}{2} \left( \delta K^i_i \delta K^j_j + 2 \delta g^{00} \delta R^{(3)} \right) \right\} + S_{\text{matter}}[g_{\mu\nu}]
\]

- Six functions of time \( \Omega(t) \), \( \Lambda(t) \), \( c(t) \), \( M_2^4(t) \), \( \bar{M}_1^3(t) \) and \( \bar{M}_2^2(t) \) are related to free functions in general action
Example: Application to Horndeski

Observables

\[ G_{\text{eff}} = Q(a, k) = \frac{1}{4\pi} \frac{f_1 + f_2 \frac{a^2}{k^2}}{f_3 + f_4 \frac{a^2}{k^2}} \]
\[ R(a, k) = \frac{f_5 + f_6 \frac{a^2}{k^2}}{f_1 + f_2 \frac{a^2}{k^2}} \]

- In the quasistatic limit, \( f_i \) are written in terms of \( \Omega(t), \Lambda(t), c(t) \), \( \bar{M}_3^1(t) \) and \( \bar{M}_2^2(t) \)
- Using background equations of motion \( \rightarrow \) can eliminate \( c(t) \) and \( \Lambda(t) \) in favor of \( H(t) \)
- Reduced Horndeski perturbations to four functions of time only!
- Generalized Horndeski perturbation theory to curved FRW
Conclusions

- We have constructed two types of EFTs to describe dark energy/modified gravity.
- Covariant approach is very restricted in the models it can describe.
- Perturbative approach will hopefully yield a formalism to calculate stringent constraints on dynamical dark energy behavior.
- Effective field theory techniques are not as useful in cosmology as they are in particle physics.
Future Work

- Understand regime of validity in detail
- Relate to other formalisms and models
- Identify further theoretical constraints
- Investigate quantum phenomena
- Begin exploring parameter space
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