Cosmic Acceleration and Modified Gravity

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Outline

- Falsifiability of $\Lambda$CDM and Smooth Dark Energy
  Distance-Redshift vs Structure Growth

- Modified gravity
  Formal equivalence of dark energy and modified gravity
  Nonlinear screening mechanism to return GR locally
  Chameleon and Vainshtein signatures

- Toy model examples: $f(R)$, DGP, massive gravity

- Collaborators on the Market
  Alexander Belikov  Pierre Gratia
  Amol Upadhye  Ignacy Sawicki
  Mark Wyman
Cosmic Acceleration

- Geometric measures of distance redshift from SN, CMB, BAO

Standard(izable) Candle Supernovae Luminosity v Flux

Standard Ruler Sound Horizon v CMB, BAO angular and redshift separation
Mercury or Pluto?

- General relativity says Gravity = Geometry

- And Geometry = Matter-Energy

- Could the missing energy required by acceleration be an incomplete description of how matter determines geometry?
Two Potentials

- Line Element

\[ ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2 \]

- **Newtonian** dynamical potential \( \Psi \)

- **Space curvature** potential \( \Phi \)

- As in the parameterized **post Newtonian approach**, cosmological tests of the \( \Phi/\Psi \)

- **Space curvature** per unit dynamical mass

- Given parameterized **metric**, matter falls on **geodesics**
Dynamical vs Lensing Mass

- Newtonian potential: $\Psi = \delta g_{00} / 2g_{00}$ which non-relativistic particles feel

- Space curvature: $\Phi = \delta g_{ii} / 2g_{ii}$ which also deflects photons

- Tests of space curvature per unit dynamical mass are the least model dependent
Dynamical vs Lensing Mass

• Newtonian potential: $\Psi = \frac{\delta g_{00}}{2g_{00}}$ which non-relativistic particles feel

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• Tests of space curvature per unit dynamical mass are the least model dependent, but one suffices cosmologically combined with distance
Modified Gravity = Dark Energy?

- **Solar system** tests of gravity are informed by our knowledge of the local stress energy content.
- With **no other constraint** on the stress energy of dark energy other than conservation, modified gravity is **formally equivalent** to dark energy.

\[
F(g_{\mu\nu}) + G_{\mu\nu} = 8\pi G T^{M}_{\mu\nu} \quad - F(g_{\mu\nu}) = 8\pi G T^{DE}_{\mu\nu} \\
G_{\mu\nu} = 8\pi G [T^{M}_{\mu\nu} + T^{DE}_{\mu\nu}]
\]

and the **Bianchi identity** guarantees \( \nabla^{\mu} T^{DE}_{\mu\nu} = 0 \)

- **Distinguishing** between dark energy and modified gravity requires **closure relations** that relate components of stress energy tensor.
- **For matter components**, closure relations take the form of **equations of state** relating density, pressure and anisotropic stress.
**Smooth Dark Energy**

- **Scalar field** dark energy has $\delta p = \delta \rho$ (in constant field gauge) – relativistic sound speed, **no anisotropic stress**

  - **Jeans stability** implies that its energy density is **spatially smooth** compared with the **matter** below the sound horizon

  $$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2$$

  $$\nabla^2 \Phi \propto \text{matter density fluctuation}$$

- **Anisotropic stress** changes the amount of **space curvature** per unit dynamical mass: negligible for both matter and smooth dark energy

  $$\nabla^2(\Phi + \Psi) \propto \text{anisotropic stress} \approx 0$$

in contrast to **modified gravity** or force-law models
Falsifiability of Smooth Dark Energy

- With the smoothness assumption, dark energy only affects gravitational growth of structure through changing the expansion rate.
- Hence geometric measurements of the expansion rate predict the growth of structure:
  - Hubble Constant
  - Supernovae
  - Baryon Acoustic Oscillations
- Growth of structure measurements can therefore falsify the whole smooth dark energy paradigm:
  - Cluster, Void Abundance
  - Weak Lensing
  - Velocity Field (Redshift Space Distortion)
Falsifying $\Lambda$CDM

- CMB determination of **matter density** controls all determinations in the **deceleration** (matter dominated) epoch
- **Planck:** $\Omega_m h^2 = 0.1426 \pm 0.0025 \rightarrow 1.7\%$
- **Distance** to recombination $D_*$ determined to $\frac{1}{4} 1.7\% \approx 0.43\%$  
  ($\Lambda$CDM result 0.46%)  
  [ $-0.11 \Delta w - 0.48 \Delta \ln h - 0.15 \Delta \ln \Omega_m - 1.4 \Delta \ln \Omega_{\text{tot}} = 0$ ]  
- **Expansion rate** during any redshift in the deceleration epoch determined to $\frac{1}{2} 1.7\%$
- **Distance to any redshift** in the deceleration epoch determined as

$$D(z) = D_* - \int_z^{z_*} \frac{dz}{H(z)}$$

- **Volumes** determined by a combination $dV = D_A^2 d\Omega d\tau / H(z)$
- **Structure** also determined by growth of fluctuations from $z_*$
Value of Local Measurements

- With high redshifts fixed, the largest deviations from the dark energy appear at low redshift $z \sim 0$

- By the Friedmann equation $H^2 \propto \rho$ and difference between $H(z)$ extrapolated from the CMB $H_0 = 38$ and 67 is entirely due to the dark energy density in a flat universe.

- With the dark energy density fixed by $H_0$, the deviation from the CMB observed $D_*$ from the $\Lambda$CDM prediction measures the equation of state (or evolution of the dark energy density)

$$p_{\text{DE}} = w \rho_{\text{DE}}$$

- Likewise current amplitude of structure, e.g. local cluster abundance, tests the smooth dark energy paradigm.
$H_0$ is Undervalued

- Flat constant $w$ dark energy model
- Determination of Hubble constant gives $w$ to comparable precision
- At $w=-1$, Planck predicts $h=0.673\pm0.012$

For evolving $w$, equal precision on average or pivot $w$, equally useful for testing a cosmological constant
$H_0$ is for Hints, Naught

- **Actual distance ladder measurements** prefer larger value

- ...but **BAO inference** prefers the low value $68.4 \pm 1$
Pinning the Past

- **Fixed distance to recombination** $D_A(z \sim 1100)$
- **Fixed initial fluctuation** $G(z \sim 1100)$
- **Constant** $w = w_{DE}$; (with free functions null deviations at $z=0$ possible but contrived)

$\Delta D_A/D_A \quad \Delta H^{-1}/H^{-1} \quad \Delta G/G \quad \Delta w_{DE} = 1/3$

Hu (2004)
Falsifying Quintessence

- Dark energy slows growth of structure in highly predictive way
- Deviation significantly $>2\%$ rules out $\Lambda$ with or without curvature
- Excess $>2\%$ rules out quintessence with or without curvature and early dark energy (as does $>2\%$ excess in $H_0$)
Dynamical Tests of Acceleration

- Dark energy slows growth of structure in highly predictive way

Mortonson, Hu, Huterer (2009)

Cosmological Constant

Quintessence

MS1054
z=0.826
Growth and Clusters

- Growth measurements vs Planck predictions

• Statistically discrepant at the $\sim 3\sigma$ level
Void Abundance

- Voids present interesting means to test gravity since they are the least screened
- Devising and quantifying statistics still lags halos Li, Koyama, Zhao (2012)

Jennings, Li, Hu (2013)
Falsify in Favor of What?
Dynamical vs Lensing Mass

- Newtonian potential: $\Psi = \frac{\delta g_{00}}{2g_{00}}$ which non-relativistic particles feel

- Space curvature: $\Phi = \frac{\delta g_{ii}}{2g_{ii}}$ which also deflects photons

- Tests of space curvature per unit dynamical mass are the least model dependent, but one suffices cosmologically combined with distance
Parameterized Post-Friedmann Approach(es)

- Parameterize **cosmic acceleration** sector, or whole **dark sector**, e.g. 
  Hu (1998), with conserved effective stress tensor

- Equivalent to assigning **equations of state** for fluctuations

- Balance **simplicity/efficiency** with **generality**

- Linear regime: **covariantly** describe **horizon** and quasistatic **Newtonian** limits

  - **Anisotropic stress** (slip) and **effective density** (Newton constant)
    Caldwell et al (1997); Hu & Sawicki (1997); Amendola et al (1997); ...

  - **General stress tensor** Baker et al (2012); EFT Bloomfeld talk; EOS Pearson talk but 

- Non-linear regime: **screening mechanisms** - Chameleon, 
  symmetron, Vainstein Hu & Sawicki (1997); Li & Hu (2011); Brax et al (2012)
Nonlinearly Screened DOFs

- Modifications of gravity will introduce new propagating degrees of freedom (Weinberg)

- These DOFs mediate fifth forces and may lead to ghost and tachyon instabilities

- Even attempts to modify gravity on cosmological scales (IR) will have consequences for small scales (e.g. vDVZ discontinuity)

- Fifth forces are highly constrained in the solar system and lab

- Must be screened by a nonlinear mechanism in the presence of matter source: chameleon, symmetron, Vainshtein...

- Realization in models: $f(R)$, DGP, galileon, massive gravity

- $f(R)$, DGP examples solved from horizon scales through to dark matter halo scales with $N$-body simulations
Cast of $f(R)$ Characters

- $R$: Ricci scalar or “curvature”
- $f(R)$: modified action (Starobinsky 1980; Nojiri & Odintsov 2003; Carroll et al 2004)
  \[ S = \int d^4x \sqrt{-g} \left[ \frac{R + f(R)}{16\pi G} + \mathcal{L}_m \right] \]
- $f_R \equiv df/dR$: additional propagating scalar degree of freedom (metric variation)
- $f_{RR} \equiv d^2f/dR^2$: Compton wavelength of $f_R$ squared, inverse mass squared
- $B$: Compton wavelength of $f_R$ squared in units of the Hubble length
  \[ B \equiv \frac{f_{RR}}{1 + f_R} \frac{R'}{H} \frac{H}{H'} \]
- $' \equiv d/d\ln a$: scale factor as time coordinate
Form of $f(R)$ Models

- Transition from zero to constant across an adjustable curvature scale
- Slope $n$ controls the rapidity of transition, field amplitude $f_{R0}$ position
- Background curvature stops declining during acceleration epoch and thereafter behaves like cosmological constant

Hu & Sawicki (2007)
Three Regimes

- Fully worked $f(R)$ example show 3 regimes
- **Superhorizon** regime: constant comoving curvature, $g(a)$
- **Linear** regime - closure $\leftrightarrow$ “smooth” dark energy density:

  \[
  k^2(p - \psi)/2 = 4\pi G a^2 \Delta \rho \\
  (\Phi + \Psi)/(\Phi - \Psi) = g(a, k)
  \]

  In principle $G(a)$ but conformal invariance: deviations order $f_R$

- **Non-linear** regime, scalar $f_R$:

  \[
  \nabla^2(p - \psi)/2 = -4\pi G a^2 \Delta \rho \\
  \nabla^2 \psi = 4\pi G a^2 \Delta \rho + \frac{1}{2}\nabla^2 f_R
  \]

  with non-linearity in the field equation

  \[
  \nabla^2 f_R = g_{\text{lin}}(a) a^2 (8\pi G \Delta \rho - N[f_R])
  \]
Non-Linear Chameleon

• For $f(R)$ the field equation

$$\nabla^2 f_R \approx \frac{1}{3}(\delta R(f_R) - 8\pi G\delta \rho)$$

is the non-linear equation that returns general relativity

• High curvature implies short Compton wavelength and suppressed deviations but requires a change in the field from the background value $\delta R(f_R)$

• Change in field is generated by density perturbations just like gravitational potential so that the chameleon appears only if

$$\Delta f_R \leq \frac{2}{3} \Phi,$$

else required field gradients too large despite $\delta R = 8\pi G\delta \rho$ being the local minimum of effective potential
Non-Linear Dynamics

- Supplement that with the **modified Poisson equation**

\[ \nabla^2 \Psi = \frac{16\pi G}{3} \delta \rho - \frac{1}{6} \delta R(f_R) \]

- Matter evolution given metric unchanged: usual **motion of matter** in a gravitational potential \( \Psi \)

- Prescription for \( N \)-body code

- **Particle Mesh** (PM) for the Poisson equation

- Field equation is a non-linear Poisson equation: **relaxation** method for \( f_R \)

- **Initial conditions** set to GR at high redshift
Environment Dependent Force

- Chameleon suppresses extra force (scalar field) in high density, deep potential regions
Environment Dependent Force

- For large background field, gradients in the scalar prevent the chameleon from appearing

Oyaizu, Lima, Hu (2008) [AMR high resolution: Zhao, Li, Koyama]
Cluster Abundance

- Enhanced abundance of rare dark matter halos (clusters) with extra force

\[ |f_{R0}| = 10^{-4} \]

\[ \text{Full simulation} \]

\[ \text{No chameleon} \]

\[ \text{Spherical collapse} \]

Lima, Schmidt, Oyaizu, Hu (2008)
Cluster $f(R)$ Constraints

- Clusters provide best current cosmological constraints on $f(R)$ models
- Spherical collapse rescaling to place constraints on full range of inverse power law models of index $n$

Sun, Stars, Galaxies

- Solar system is chameleon dressed by our galaxy
- Rotation curve $v/c \sim 10^{-3}$, $\Phi \sim 10^{-6} \sim |\Delta f_R|$ limits cosmological field
- In dwarf galaxies this can reach a factor of a few lower yielding environmental differences between stellar objects of varying potential
  
  Jain, Vikram, Sakstein (2012); Davis, Lim, Sakstein, Shaw (2011)

| $|f_R| = 0.5 \times 10^{-6}$ | $|f_R| = 1.0 \times 10^{-6}$ | $|f_R| = 2.0 \times 10^{-6}$ |
|-----------------------------|-----------------------------|-----------------------------|
| $\rho$                      | $\rho$                      | $\rho$                      |

$\frac{R}{\kappa^2}$ (g cm$^{-3}$)

$\rho$

$|f_R| = 0.5 \times 10^{-6}$

$|f_R| = 1.0 \times 10^{-6}$

$|f_R| = 2.0 \times 10^{-6}$

Hu & Sawicki (2007)
Solar System & Lab

- **Strictly valid for solar system / lab or are beyond effective theory?**
- **If former, solar system \( f(R) \) tests of more powerful by at least 10** (Hu & Sawicki 2009; exosolar tests: Jain et al., Davis et al.)
- **Laboratory tests: within factor of 2 of ruling out all gravitational strength chameleon models** \[ m < 0.0073 (\xi \rho/10 \text{g cm}^3)^{1/3} \text{eV} \]

Already exceeded the vacuum scale (1000 km) and earth (1 cm) of Vainshtein models (Nicolis & Rattazzi 2004)

Upadhye, Hu, Khoury (2012)
Chameleon Pile-Up

- Chameleon threshold at intermediate masses \( (10^{13} \, h^{-1} \, M_{\odot}) \)
- Mergers from smaller masses continues, to higher masses stops
- Pile up of halos at threshold
Chameleon Mass Function

- Simple **single parameter** extension covers **variety of models**
- Basis of a halo model based **post Friedmann parameterization** of chameleon

\[
\Delta n_M/n_M = f_{R0} = 10^{-6}
\]

Li & Hu (2011); Lombriser et al (2013); Kopp et al (2013)
- Connect to linear regime with interpolation of HaloFit

Li & Hu (2011)
Motion: Environment & Object

- Self-field of a "test mass" can saturate an external field (for $f(R)$ in the gradient, for DGP in the second derivatives)

Hui, Nicolis, Stubbs (2009)
Jain & Vanderplas (2011)
Zhao, Li, Koyama (2011)
DGP Braneworld Acceleration

- Braneworld acceleration (Dvali, Gabadadze & Porrati 2000)

\[ S = \int d^5x \sqrt{-g} \left[ \frac{(5) \mathcal{R}}{2\kappa^2} + \delta(\chi) \left( \frac{(4) \mathcal{R}}{2\mu^2} + \mathcal{L}_m \right) \right] \]

with crossover scale \( r_c = \kappa^2 / 2\mu^2 \)

- Influence of bulk through Weyl tensor anisotropy - solve master equation in bulk (Deffayet 2001)

- Matter still minimally coupled and conserved

- Exhibits the 3 regimes of modified gravity

- Weyl tensor anisotropy dominated conserved curvature regime \( r > r_c \) (Sawicki, Song, Hu 2006; Cardoso et al 2007)

- Brane bending scalar tensor regime \( r_\ast < r < r_c \) (Lue, Soccimarro, Starkman 2004; Koyama & Maartens 2006)

- Nonlinear General Relativistic regime \( r < r_\ast = (r_c^2r_g)^{1/3} \) where \( r_g = 2GM \) (Dvali 2006)
Nonlinear Interaction

Nonlinearity in field equation recovers linear theory if $N[\phi] \to 0$

$$\nabla^2 \phi = g_{\text{lin}}(a)a^2 \left(8\pi G \Delta \rho - N[\phi]\right)$$

- For $f(R)$, $\phi = f_R$ and

$$N[\phi] = \delta R(\phi)$$

a nonlinear function of the field

Linked to gravitational potential

- For DGP, $\phi$ is the brane-bending mode and

$$N[\phi] = \frac{r_c^2}{a^4} \left[ (\nabla^2 \phi)^2 - (\nabla_i \nabla_j \phi)^2 \right]$$

a nonlinear function of second derivatives of the field

Linked to density fluctuation - Galileon invariance - no self-shielding of external forces
Vainshtein Suppression

• Modification to gravitational potential saturates at the Vainshtein radius $\sim (G M r_c^2)^{1/3}$
DGP N-Body

- **DGP nonlinear derivative interaction solved by relaxation revealing the Vainshtein mechanism**

Newtonian Potential  Brane Bending Mode

Schmidt (2009); Chan & Scoccimarro (2009); Li, Zhao, Koyama (2013)
Weak Vainshtein Screening

- Screening occurs when objects are separated by a Vainshtein radius
- Vainshtein radius depends on mass $m^{1/3}$
- Halos in compensated voids experience acceleration toward the center proportional to $m$

Belikov & Hu (2012)
Strong Vainshtein Screening

- Objects separated by much less than Vainshtein radius
- Screened acceleration also mass dependent due to nonlinearity
- Universal precession rate is not universal: corrections scale as \((M_B/M_A)^{3/5}\)

Hiramatsu, Hu, Koyama, Schmidt (2012)
• Calculation of the metric ratio $g = \Phi + \Psi / \Phi - \Psi$ requires solving for the propagation of metric fluctuations into the bulk

• Encapsulated in the off brane gradient which closes the system (e.g. normal branch $g = -1/(2Hr_C + 1)$ until deep in de Sitter)

DGP CMB Large-Angle Excess

- Extra dimension modify gravity on large scales
- 4D universe bending into extra dimension alters gravitational redshifts in cosmic microwave background

Massive Gravity

- DGP model motivated re-examination of massive gravity models
- Nonlinearly complete Fierz-Pauli action: Vainshtein strong coupling (restoring vDVZ continuity), no Boulware Deser ghost, effective theory out to $\Lambda_3$ Arkani-Hamed, Georgi, Schwartz (2003)
- Massive gravity action [de Rham, Gabadadze, Tolley et al, Hassan & Rosen, ... (2010-2012)]

$$S = \frac{M_p}{2} \int d^4 x \sqrt{-g} \left[ R - \frac{m^2}{4} \sum_{n=0}^{4} \beta_n S_n (\sqrt{g^{-1}\eta}) \right]$$

where $\eta$ is a fiducial (Minkowski) metric

- Diffeomorphism invariance can be restored by introducing Stückelberg fields (aka vierbeins of fiducial metric)

$$g^{-1}\eta \rightarrow g^{-1}f = g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}$$

which carry transformation from unitary to arbitrary gauge
Self Acceleration

- Graviton mass \( \sim H_0 \) provides self-acceleration
- Generalizing results de Rham et al, Koyama et al, Mukohyama et al… for any isotropic matter a **cosmological constant stress-energy is an exact solution** Gratia, Hu, Wyman (2012); Volkov (2012)

\[
\rho_m = -p_m = \frac{m^2 M_p^2}{2} P_0
\]

where \( P_0 \) constant given \( \alpha_n \)
- Cosmic **acceleration** if \( m \sim H_0 \), remains constant for arbitrarily large radial matter perturbations
- St"uckelberg fields are inhomogeneous in isotropic coordinates d’Amico et al (2011) - flat fiducial metric is not Minkowski in FRW coordinates
- **Stress-energy** depends only on spatial St"uckelberg fields, leaving a set of solutions that differ in \( \phi_0 \) or the choice of **unitary time**
Self Acceleration

• Self-accelerating solution approached from arbitrary initial conditions? classically and quantum-mechanically stable?

• Field fluctuations again decouple with spatial St¨uckelberg field obeying first order closed equation

• Stable to radial field perturbations Wyman, Hu, Gratia (2012)

\[ \frac{\delta p}{\delta \rho} = \frac{a \ddot{a}}{3 \dot{a}^2} \]

e.g. de Sitter \( \frac{\delta p}{\delta \rho} = 1/3 \) - but eos generally anisotropic

• St¨uckelberg dynamics determined by unitary time: special cases with no dynamics, no stress energy perturbations Gumrukcuoglu et al

• Stability to anisotropic perturbations and higher order terms in action? Koyama et al; de Felice et al; d’Amico; Khosravi et al

• Effective theory to 1000km in vacuum, on earth 1cm or 1km? Burrage, Kaloper, Padilla (2012)
Singularities

- Massive gravity is bimetric theory, second metric dynamical or not
- Offers new opportunities for singularities - coordinate singularities in GR can become physical, removing in one not both
- Some static black hole solutions unphysical (reachable by dynamics?) Gruzinov & Mirbabayi (2011); Deffayet & Jacobson (2011); Nieuwenhuizen (2011); Volkov (2013) if metrics are simultaneously diagonal
- Simple example: determinant singularity dynamically generated in a recollapsing open universe Gratia, Hu, Wyman (2013)
  Coordinates where fiducial metric is flat has $\tilde{t} \propto a$ - transformation singular at $\dot{a} = 0$
  Singularity in $g^{-1}f$ is coordinate invariant
  Non-dynamical $f$ theory undefined here, non-positive definite solution continuous
- Promoting the second metric to dynamical changes the nature of singularities, including det=0 example even in limit that approximately fixed by large $M_{\text{pl}}$ Gratia, Hu, Rosen, Wyman (in prep)
Summary

- Formal equivalence between dark energy and modified gravity
- Practical inequivalence of smooth dark energy and extra propagating scalar fifth force
- Smooth dark energy (e.g. quintessence) highly falsifiable: consistency tests may indicate modified gravity before detailed parameterized tests in linear regime
- Specific modified gravity models highly falsifiable (falsified!)
- Paradigmatic lessons: nonlinearity in field equations screen deviations
  - Chameleon/symmetron: deep potential well
  - Vainshtein: high local density
- Characteristic signatures of different screening mechanisms
- $N$-body studies maturing