Models of coupled dark matter to dark energy

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The Dark Universe. What is dark energy? What is dark matter?

$\Lambda$CDM fits the data extremely well, but suffers from two fundamental problems: fine-tuning & coincidence. Alternatives: e.g. quintessence, modified gravity

DM and DE might be coupled. This can solve the coincidence problem.

Plethora of coupled models in the literature → the form of the coupling is chosen phenomenologically at the level of the field equations.

What is the most general phenomenological model we can construct?

We present three distinct classes (Types) of coupled DE models, introducing the coupling at the level of the action.
Theories of coupled DM/DE

- Consider Dark Energy (DE) coupled to Cold Dark Matter (c) [e.g. Kodama & Sasaki '84, Ma & Bertschinger '95]
- $T^{(c)}$ and $T^{(DE)}$ are not separately conserved:

$$\nabla_\mu T^{(c)}_{\mu \nu} = -\nabla_\mu T^{(DE)}_{\mu \nu} = J_\nu$$

- Various forms of coupling have been considered. Examples:
  - $J_\nu = C\rho_c \nabla_\nu \phi$ [Amendola '00]
  - $J_\nu = \Gamma \rho_c u^{(c)}_\nu$ [Valiviita, Majerotto, Maartens '08]
- FRW background with $\bar{J}_0$ ($\bar{J}_i = 0$ because of isotropy)

$$\dot{\rho}_c + 3H\rho_c = -\bar{J}_0$$

$$\dot{\rho}_{DE} + 3H\rho_{DE} = \bar{J}_0$$

- In previous examples: $\bar{J}_0 = C\bar{\rho}_c \dot{\phi}$, $\bar{J}_0 = a\Gamma \dot{\rho}_c$
- Many models in the literature, and their cosmological implications (CMB, P(k), Supernovae, growth, non-linear perturbations, N-body simulations, instabilities...)
- In order to make further progress, we need to construct general models.
The fluid pull-back formalism is a description of relativistic fluids at the level of the action [see review by Andersson & Comer '08 and references therein].

The action for GR coupled to an adiabatic fluid is taken to be

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}R - \int d^4x \sqrt{-g}f(n), \]

where \( n \) is the fluid/particle number density.

\( f(n) \) is (in principle) an arbitrary function, whose form determines the equation of state and speed of sound of the fluid.

For pressureless matter \( f = f_0 n \).

Vary the action \( S \) to get field and fluid equations.
We want to construct a model where the adiabatic fluid is explicitly coupled to a DE field $\phi$

Invariants: $n, Y = \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$, $Z = u^\mu \nabla_\mu \phi$

The Lagrangian has the form

$$L = L(n, Y, Z, \phi).$$

GR+quintessence+fluid is

$$L = Y + V(\phi) + f(n)$$

k-essence is

$$L = F(Y, \phi) + f(n)$$

Our total (general) action is

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} L(n, Y, Z, \phi)$$

We can now consider different classes of theories
Type 1

- Type-1 models are classified via

\[ L(n, Y, Z, \phi) = F(Y, \phi) + f(n, \phi) \]

with \( f(n) = g(n) e^{\alpha(\phi)} \)

- Notation: \( \phi_\mu \equiv \nabla_\mu \phi \), \( f_n \equiv df/dn \), \( F_Y \equiv \partial F/\partial Y \) etc.

- These models describe a K-essence scalar field coupled to matter. If \( F = Y + V(\phi) \), we describe coupled quintessence models.

- Field energy-momentum tensor \( T^\phi_{\mu\nu} = F_Y \phi_\mu \phi_\nu - F g_{\mu\nu} \)

- Fluid energy-momentum tensor with \( \rho = f \) and \( P = n f_n - f \)

- The evolution of the fluid energy density \( \rho \) is given by

\[ u^\mu \nabla_\mu \rho + (\rho + P) \nabla_\mu u^\mu = Z \rho \alpha \phi \]

- Coupling current \( J_\mu = -\rho \alpha \phi \phi_\mu \)
Type 2

- Type-2 models are classified via

\[ L(n, Y, Z, \phi) = F(Y, \phi) + f(n, Z) \]

- Field energy-momentum tensor same as Type-1
- Fluid energy-momentum tensor with \( \rho = f - Z f_Z \) and \( P = n f_n - f \)
- CDM has \( P = 0 \) which means \( f = nh(Z) \)
- We parameterize \( h(Z) \) in terms of an integral of a new function \( \beta(Z) \) (this simplifies the equations)
- The evolution of the fluid energy density \( \rho \) is given by

\[ u^\mu \nabla_\mu \rho + \rho \nabla_\mu u^\mu = -Z \nabla_\mu (\rho \beta u^\mu) \]

- Coupling current \( J_\mu = \nabla_\nu (\rho \beta u^\nu) \phi_\mu \)
Type 3

- Type-3 models are classified via

\[ L(n, Y, Z, \phi) = F(Y, Z, \phi) + f(n) \]

- Field energy-momentum tensor

\[ T^\phi_{\mu\nu} = F_{Y\phi} \phi_{\mu} \phi_{\nu} - F g_{\mu\nu} - Z F_Z u_{\mu} u_{\nu} \]

- Fluid energy-momentum tensor same as Type-1

- Evolution of the fluid energy density \( \rho \) same as Type-1 (but different momentum transfer)

- Coupling current

\[ J_{\mu} = \nabla_{\nu} (F_Z u^{\nu}) \tilde{\phi}_\mu + F_Z D_{\mu} Z + Z F_Z u^{\nu} \nabla_{\nu} u_{\mu} \]

- Type 3 is very special \( \rightarrow \bar{J}_0 = 0 \): no coupling at the background field equations!

- Furthermore, the energy-conservation equation remains uncoupled even at the linear level

- Type-3 is a pure momentum-transfer theory
Cosmology: Type 1

For Type-1, coupled quintessence is described by $F = Y + V(\phi)$. CDM fluid means $f = n e^{\alpha(\phi)}$.

We investigate the background FRW space-time and linear perturbations (dots are derivatives wrt conformal time $\tau$)

$$\bar{\rho}_\phi = \frac{1}{2} \frac{\dot{\phi}^2}{a^2} + V(\phi), \quad \bar{P}_\phi = \frac{1}{2} \frac{\dot{\phi}^2}{a^2} - V(\phi)$$

- **Background Klein Gordon:**
  $$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2V_\phi = -a^2\bar{\rho}\alpha_\phi$$

- **Evolution of CDM density:**
  $$\dot{\bar{\rho}} + 3\bar{\rho}\mathcal{H} = \bar{\rho}\alpha_\phi \dot{\phi}$$
  with solution
  $$\bar{\rho} = \rho_0 a^{-3} e^{\alpha(\phi)}$$

- **Synchronous gauge:**
  $$ds^2 = -a^2 d\tau^2 + a^2 \left[ (1 + \frac{1}{3} h) \gamma_{ij} + D_{ij} \nu \right] dx^i dx^j$$

- **Perturbed Klein-Gordon:**
  $$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + \left( k^2 + a^2 V_{\phi\phi} \right) \phi + \frac{1}{2} \phi \ddot{h} = -a^2 \alpha_\phi \bar{\rho} \delta$$
The perturbed CDM equations are

\[ \dot{\delta} = -k^2 \theta - \frac{1}{2} \dot{h} + \alpha \phi \dot{\phi} \]

\[ \dot{\theta} = -H \theta - \alpha \phi \dot{\phi} \left[ \theta - \theta^{(\phi)} \right] \]

with \( \theta^{(\phi)} = \phi / \dot{\phi} \)

We can now study the effect of the coupling to the evolution of the background as well as to the CMB temperature and matter power spectra. We use a modified version of CAMB and compare with the uncoupled case.

We choose the 1EXP quintessence potential \( V(\phi) = V_0 e^{-\lambda \phi} \)

We choose \( \alpha(\phi) = \alpha_0 \phi \), with \( \alpha_0 \) a constant. This model has been studied extensively [e.g. Xia '09, Tarrant et al. '12]

We keep \( \lambda = 1.22 \) (fixed) and vary \( V_0 \) and \( \bar{\rho}_{c,0} \) so that each cosmology (coupled and uncoupled) evolves to the PLANCK cosmological parameters values [Ade et al. '13]
The CDM density is higher at early times for the couple case, in order to evolve to the same cosmological parameters today. The matter-radiation equality occurs earlier in the coupled case.
$P(k)$ affected on small scales. There is more dark matter at early times, matter-radiation equality earlier. Only small scale perturbations have time to enter the horizon and grow during radiation-dominated era. The turnover happens on smaller scales. The growth is enhanced, small scale power increases, larger $\sigma_8$. 
Small scale anisotropies decrease. The locations of the CMB peaks shift towards smaller scales. Large-scale anisotropies (ISW effect) decrease.
Cosmology: Type 2

- $\bar{\rho}^\phi$ and $\bar{P}^\phi$ same as Type 1
- Background KG:
  \[
  \left(1 - \frac{\bar{\rho}\beta Z}{1+Z\beta}\right)\left(\ddot{\phi} - \mathcal{H}\dot{\phi}\right) + 3\mathcal{H}\dot{\phi} + a^2 V_\phi = 0
  \]
- We choose a sub case with $\beta(Z) = \beta_0/Z$
- The background CDM equation is solved to give
  \[
  \bar{\rho}_c = \bar{\rho}_{c,0} a^{-3} \ddot{Z}^{\beta_0/(1-\beta_0)}
  \]
- Since $\ddot{Z} = -\dot{\phi}/a$, $\bar{\rho}_c$ depends on the time derivative $\dot{\phi}$ instead of $\phi$ itself which is a notable difference from the Type-1 case.
- We also derive the perturbed KG and perturbed CDM equations
Type 2: Evolution of $\Omega_{\text{cdm}}$

The figure shows the evolution of $\Omega_{\text{cdm}}$ as a function of scale factor $a$ for two different values of $\beta_0$: $\beta_0 = 0$ (solid line) and $\beta_0 = 1/11$ (dashed line). The plot indicates how the density parameter for cold dark matter changes over time, reflecting the expansion of the universe and the evolution of its density components.
Type 2: Matter power spectra

\[ P_0(k) \propto h^{-3} \text{Mpc}^{-3} \]

\[ k \text{ (h Mpc}^{-1}) \]

Graph showing the matter power spectrum \( P_0(k) \) for different values of \( \beta_0 \):
- \( \beta_0 = 0 \)
- \( \beta_0 = 1/11 \)
Type 2: CMB temperature spectra

\[ |(l+1)C_{\ell}/2\pi \mu \text{k}^2| \]

- \( \beta_0 = 0 \)
- \( \beta_0 = 1/11 \)

\[ \Delta C_{\ell} \]

\[ l \]

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Cosmology: Type 3

- We consider $F = Y + V(\phi) + \gamma(Z)$
- $\bar{\rho}^\phi = \frac{1}{2} \frac{\dot{\phi}^2}{a^2} + \frac{\dot{\phi}}{a} \gamma Z + \gamma + V$ and $\bar{P}^\phi = \frac{1}{2} \frac{\dot{\phi}^2}{a^2} - \gamma - V$
- Background KG: $(1 - \gamma ZZ)(\dddot{\phi} - \mathcal{H} \dot{\phi}) + 3a \mathcal{H} (\gamma Z - \bar{Z}) + a^2 V_\phi = 0$
- We choose a sub case with $\gamma(Z) = \gamma_0 Z^2$
- Type 3 is special $\rightarrow$ no coupling appears at the background level fluid equations
- We also derive the perturbed KG and perturbed CDM equations — note the $\delta$ evolution is the same as in the uncoupled case. This is a pure momentum-transfer coupling up-to linear order.
- This case has ghost/strong coupling problems for $\gamma_0 \geq 1/2$. Introducing the coupling at the level of the action helps identify pathologies/instabilities.
Hubble parameter $H(z)$

- **no coupling**
- **Type 1**, $\alpha_0 = 0.15$
- **Type 2**, $\beta_0 = \frac{1}{11}$
- **Type 3**, $\gamma_0 = 0.15$

Simon et al. (2005)

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Type 3: Matter power spectra

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Type 3: CMB temperature spectra

\[ \frac{\Delta C_l}{2 \pi \mu K^2} \]

\[ \gamma_0 = 0 \]
\[ \gamma_0 = 0.15 \]
Conclusions

▶ We have presented a novel method to construct general interacting dark energy models
▶ We have constructed 3 general classes of theories
▶ To study cosmological implications, we used a specific form for the coupling function and a specific potential. Observational signatures depend heavily on these choices.
▶ However, it is possible to parameterize the field equations by introducing all possible type of terms that can appear in a coupled theory using the PPF approach.
▶ This can pave the way for model-independent approach to test coupled theories, by reducing the large parameter space of free functions and coupling types to a small set of constants

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