Timescape Cosmology:
Modifying the Geometry of the Universe

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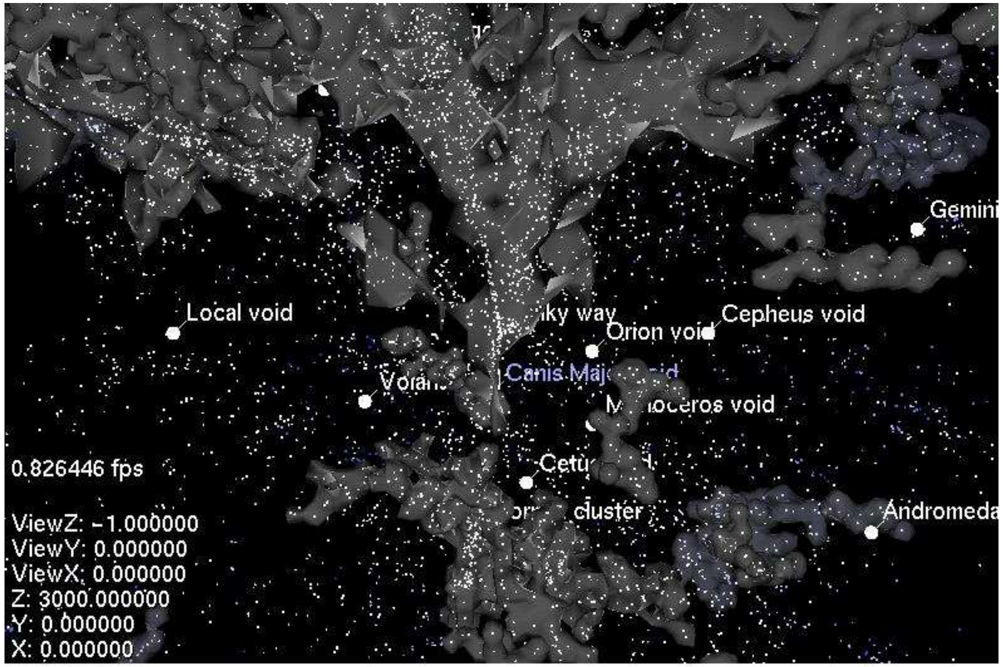


DLW: **Class. Quan. Grav. 28 (2011) 164006**

New J. Phys. 9 (2007) 377

Phys. Rev. Lett. 99 (2007) 251101

Phys. Rev. D78 (2008) 084032

Phys. Rev. D80 (2009) 123512

B.M. Leith, S.C.C. Ng & DLW:

ApJ 672 (2008) L91

P.R. Smale & DLW, **MNRAS 413 (2011) 367**

P.R. Smale, **MNRAS 418 (2011) 2779**

DLW, P.R. Smale, T. Mattsson & R. Watkins, **arXiv:1201.5371**

J.A.G. Duley, M.A. Nazer & DLW: **arXiv:1306.3208**

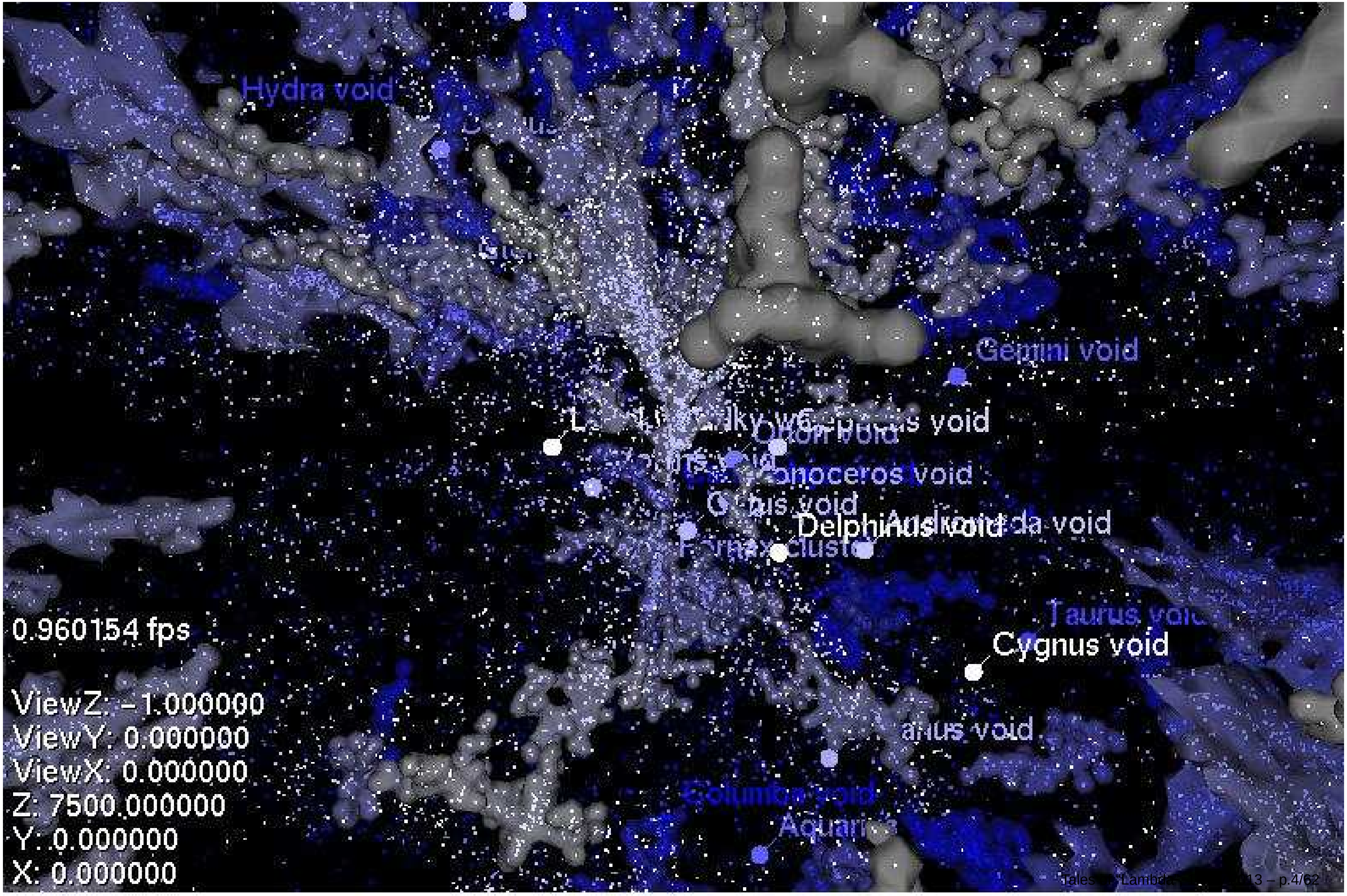
What is "dark energy"?

- Usual explanation: a homogeneous isotropic form of "stuff" which violates the strong energy condition. (Locally pressure $P = w\rho c^2$, $w < -\frac{1}{3}$.)
  Best-fit close to cosmological constant, Λ, $w = -1$.
- *Cosmic coincidence*: Why now? Why $\Omega_{\Lambda 0} \sim 2\Omega_{M0}$, so that a universe which has been decelerating for much of its history began accelerating only at $z \sim 0.7$?
- Onset of acceleration coincides also with the nonlinear growth of large structures. Are we oversimplifying the geometry?
- My answer, Timescape scenario: *Dark energy is a misidentification of gradients in quasilocal kinetic energy of expansion of space.*

From smooth to lumpy

- Universe was very smooth at time of last scattering; fluctuations in the fluid were tiny ($\delta\rho/\rho \sim 10^{-5}$ in photons and baryons; $\sim 10^{-4}$,10^{-3} in non–baryonic dark matter).
- FLRW approximation very good early on.
- Universe is very lumpy or inhomogeneous today.
- Recent surveys estimate that 40–50% of the volume of the universe is contained in voids of diameter $30h^{-1}$ Mpc. [Hubble constant $H_0 = 100h$ km/s/Mpc] (Hoyle & Vogeley, ApJ 566 (2002) 641; 607 (2004) 751)
- Add some larger voids, and many smaller minivoids, and the universe is *void–dominated* at present epoch.
- Clusters of galaxies are strung in filaments and bubbles around these voids.

6df: voids & bubble walls (A. Fairall, UCT)



Fitting problem (Ellis 1984)

- On what scale are Einstein's field equations (EFEs) valid?

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Scale on which matter fields are coarse–grained to produce the energy–momentum tensor on r.h.s. not prescribed
- general relativity only well tested for isolated systems – e.g., solar system or binary pulsars – for which $T_{\mu\nu} = 0$
- Usual approach: FLRW + Newtonian-style potentials evolved into nonlinear regime by N-body simulations
- Other approaches: cut and paste exact solutions, e.g., Einstein-Straus vacuole (1946) $\rightarrow$ Swiss cheese models; LTB vacuoles $\rightarrow$ meatball models

Layers of coarse-graining in cosmology

1. Atomic, molecular, ionic or nuclear particles coarse-grained as fluid in early universe, voids, stars etc
2. Collapsed objects – stars, black holes coarse-grained as isolated objects;
3. Stellar systems coarse-grained as dust particles within galaxies;
4. Galaxies coarse-grained as dust particles within clusters;
5. Clusters of galaxies as bound systems within expanding walls and filaments;
6. Voids, walls and filaments combined as regions of different densities in a smoothed out expanding cosmological fluid.

Dilemma of gravitational energy…

- In GR spacetime carries *energy* & *angular momentum*

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- On account of the strong equivalence principle, $T_{\mu\nu}$ contains localizable energy–momentum only
- Kinetic energy and energy associated with spatial curvature are in $G_{\mu\nu}$: variations are "quasilocal"!
- Newtonian version, $T - U = -V$, of Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3}$$

where $T = \frac{1}{2}m\dot{a}^2x^2$, $U = -\frac{1}{2}kmc^2x^2$, $V = -\frac{4}{3}\pi G\rho a^2x^2m$; $\mathbf{r} = a(t)\mathbf{x}$.

What is a cosmological particle (dust)?

- In FLRW one takes observers "comoving with the dust"
- Traditionally galaxies were regarded as dust. However,
  - Neither galaxies nor galaxy clusters are homogeneously distributed today
  - Dust particles should have (on average) invariant masses over the timescale of the problem
- Must coarse-grain over expanding fluid elements larger than the largest typical structures
- ASIDE: Taking galaxies as dust leads to flawed argument against backreaction (Peebles 0910.5142)

$$\Phi_{\rm Newton}(\text{galaxy}) \sim v_{\rm gal}^2/c^2 \sim 10^{-6}$$

ΛCDM self-consistent; but galaxies, clusters do not justify FLRW background

Largest typical structures

Survey	Void diameter	Density contrast
PSCz	$(29.8 \pm 3.5)h^{-1}$Mpc	$\delta_\rho = -0.92 \pm 0.03$
UZC	$(29.2 \pm 2.7)h^{-1}$Mpc	$\delta_\rho = -0.96 \pm 0.01$
2dF NGP	$(29.8 \pm 5.3)h^{-1}$Mpc	$\delta_\rho = -0.94 \pm 0.02$
2dF SGP	$(31.2 \pm 5.3)h^{-1}$Mpc	$\delta_\rho = -0.94 \pm 0.02$

Dominant void statistics in the Point Source Catalogue Survey (PSCz), the Updated Zwicky Catalogue (UZC), and the 2 degree Field Survey (2dF) North Galactic Pole (NGP) and South Galactic Pole (SGP), (Hoyle and Vogeley 2002,2004). More recent results of Pan et al. (2011) using SDSS Data Release 7 similar.

- Particle size should be a few times greater than largest typical structures (voids with $\delta_\rho \equiv (\rho - \bar{\rho})/\bar{\rho}$ near -1)
- Coarse grain dust “particles” – fluid elements – at Scale of Statistical Homogeneity (SSH) $\sim 100/h$ Mpc

Coarse-graining at SSH

- In timescape model we will coarse-grain “dust” at SSH
- Scale at which fluid cell properties from cell to cell remain similar *on average* throughout evolution of universe
- Notion of “comoving with dust” will require clarification
- Variance of expansion etc relates more to internal degrees of freedom of fluid particle than differences between particles
- Coarse-graining over internal gravitational degrees of freedom means that we no longer deal with a single global geometry: *description of geometry is statistical*

Averaging and backreaction

- In general $\langle G^{\mu}{}_{\nu}(g_{\alpha\beta})\rangle \neq G^{\mu}{}_{\nu}(\langle g_{\alpha\beta}\rangle)$

Three approaches

1. Perturbative schemes about a given background geometry;
2. Spacetime averages (e.g., Zalaletdinov);
3. Spatial averages on hypersurfaces based on a $1+3$ foliation (e.g., Buchert).

- Perturbative schemes deal with *weak backreaction*
- Approaches 2 and 3 can be fully nonlinear giving *strong backreaction*
- No obvious way to average tensors on a manifold, so extra assumptions or structure needed

Buchert averaging

- Average scalar quantities only on domain in spatial hypersurface $\mathcal{D} \in \Sigma_t$; e.g.,

$$\langle \mathcal{R} \rangle \equiv \left(\int_{\mathcal{D}} \mathrm{d}^3x \sqrt{{}^3 g} \mathcal{R}(t, \mathbf{x}) \right) / \mathcal{V}(t)$$

where $\mathcal{V}(t) = \int_{\mathcal{D}} \mathrm{d}^3x \sqrt{{}^3 g}$, $\quad {}^3 g \equiv \det({}^3 g_{ij}) = -\det({}^4 g_{\mu\nu})$.

- Now $\sqrt{{}^3 g}\, \theta = \sqrt{-{}^4 g}\, \nabla_\mu U^\mu = \partial_\mu (\sqrt{-{}^4 g}\, U^\mu) = \partial_t (\sqrt{{}^3 g})$, so

$$\langle \theta \rangle = \left(\partial_t \mathcal{V} \right) / \mathcal{V}$$

- Generally for any scalar Ψ, get commutation rule

$$\partial_t \langle \Psi \rangle - \langle \partial_t \Psi \rangle = \langle \Psi \theta \rangle - \langle \theta \rangle \langle \Psi \rangle = \langle \Psi\, \delta\theta \rangle = \langle \theta\, \delta\Psi \rangle = \langle \delta\Psi\, \delta\theta \rangle$$

where $\delta\Psi \equiv \Psi - \langle \Psi \rangle$, $\delta\theta \equiv \theta - \langle \theta \rangle$.

Buchert-Ehlers-Carfora-Piotrkowska -Russ-Soffel-Kasai-Börner equations

For irrotational dust cosmologies, with energy density, $\rho(t,\mathbf{x})$, expansion scalar, $\theta(t,\mathbf{x})$, and shear scalar, $\sigma(t,\mathbf{x})$, where $\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}$, **defining** $3\dot{\bar{a}}/\bar{a} \equiv \langle\theta\rangle$, we find average cosmic evolution described by exact Buchert equations

$$\begin{aligned}
&(1) & 3\frac{\dot{\bar{a}}^2}{\bar{a}^2} &= 8\pi G\langle\rho\rangle - \tfrac{1}{2}\langle\mathcal{R}\rangle - \tfrac{1}{2}\mathcal{Q} \\
&(2) & 3\frac{\ddot{\bar{a}}}{\bar{a}} &= -4\pi G\langle\rho\rangle + \mathcal{Q} \\
&(3) & \partial_t\langle\rho\rangle &+ 3\frac{\dot{\bar{a}}}{\bar{a}}\langle\rho\rangle = 0 \\
&(4) & \partial_t\left(\bar{a}^6\mathcal{Q}\right) &+ \bar{a}^4\partial_t\left(\bar{a}^2\langle\mathcal{R}\rangle\right) = 0 \\
& & \mathcal{Q} &\equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle
\end{aligned}$$

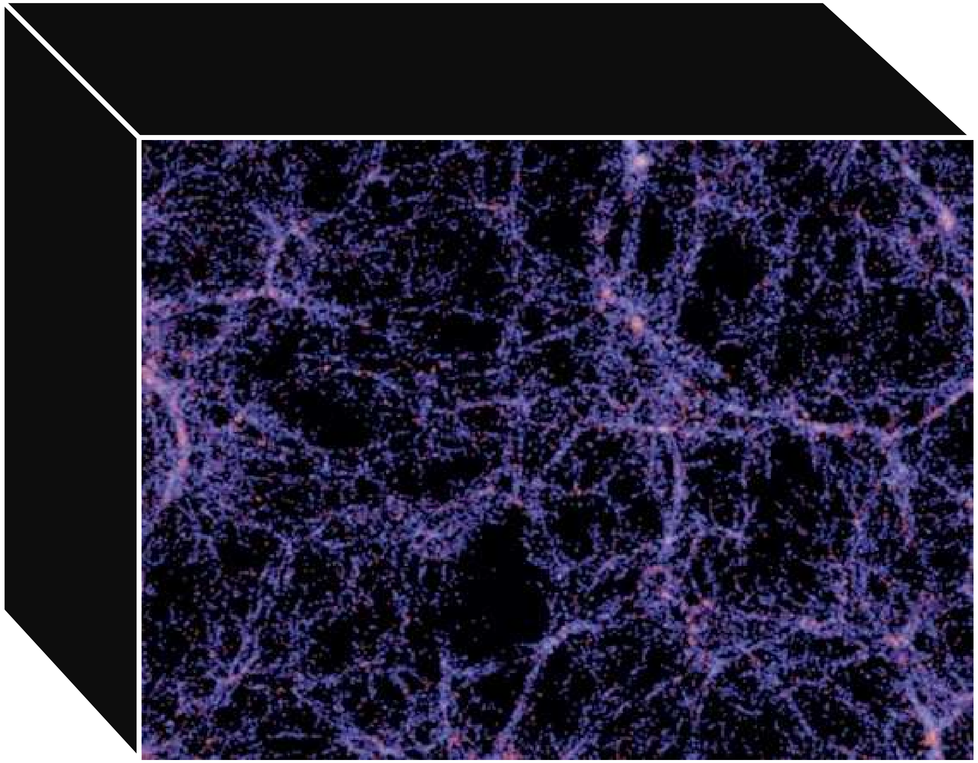
Backreaction in Buchert averaging

- *Kinematic backreaction* term can also be written

$$\mathcal{Q} = \frac{2}{3}\langle(\delta\theta)^2\rangle - 2\langle\sigma^2\rangle$$

  i.e., combines variance of expansion, and shear.
- Eq. (6) is required to ensure (3) is an integral of (4).
- Buchert equations look deceptively like Friedmann equations, but deal with *statistical* quantities
- The extent to which the back–reaction, $\mathcal{Q}$, can lead to apparent cosmic acceleration or not has been the subject of much debate (e.g., Ishibashi & Wald 2006):
  - How do statistical quantities relate to observables?
  - What about the time slicing?
  - How big is $\mathcal{Q}$ given reasonable initial conditions?

Within a statistically average cell

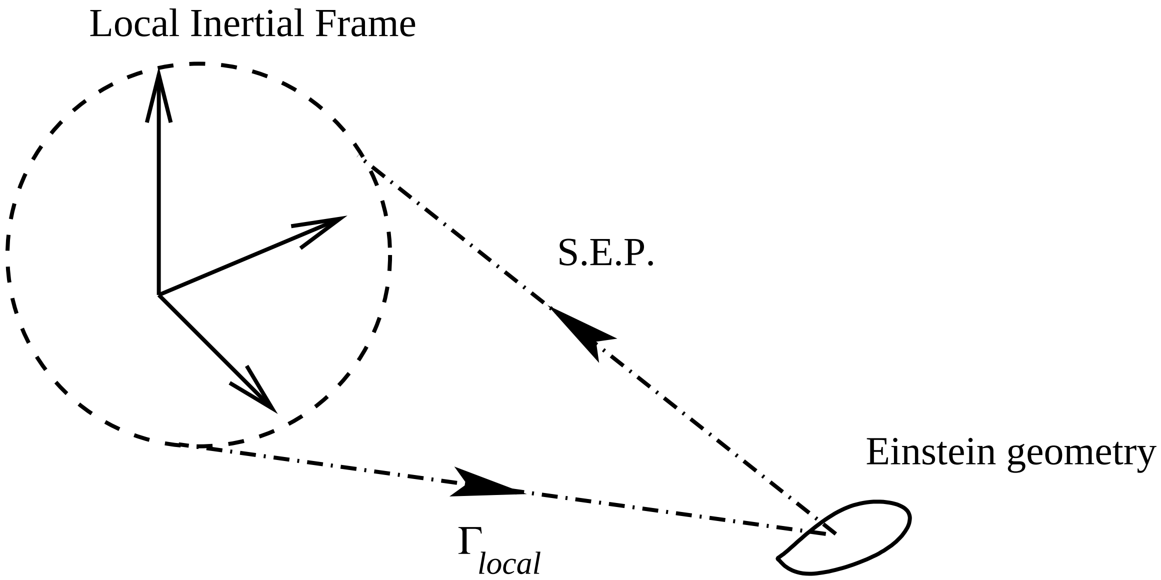


- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim -1$.
- GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rulers & clocks and volume average ones

The Copernican principle

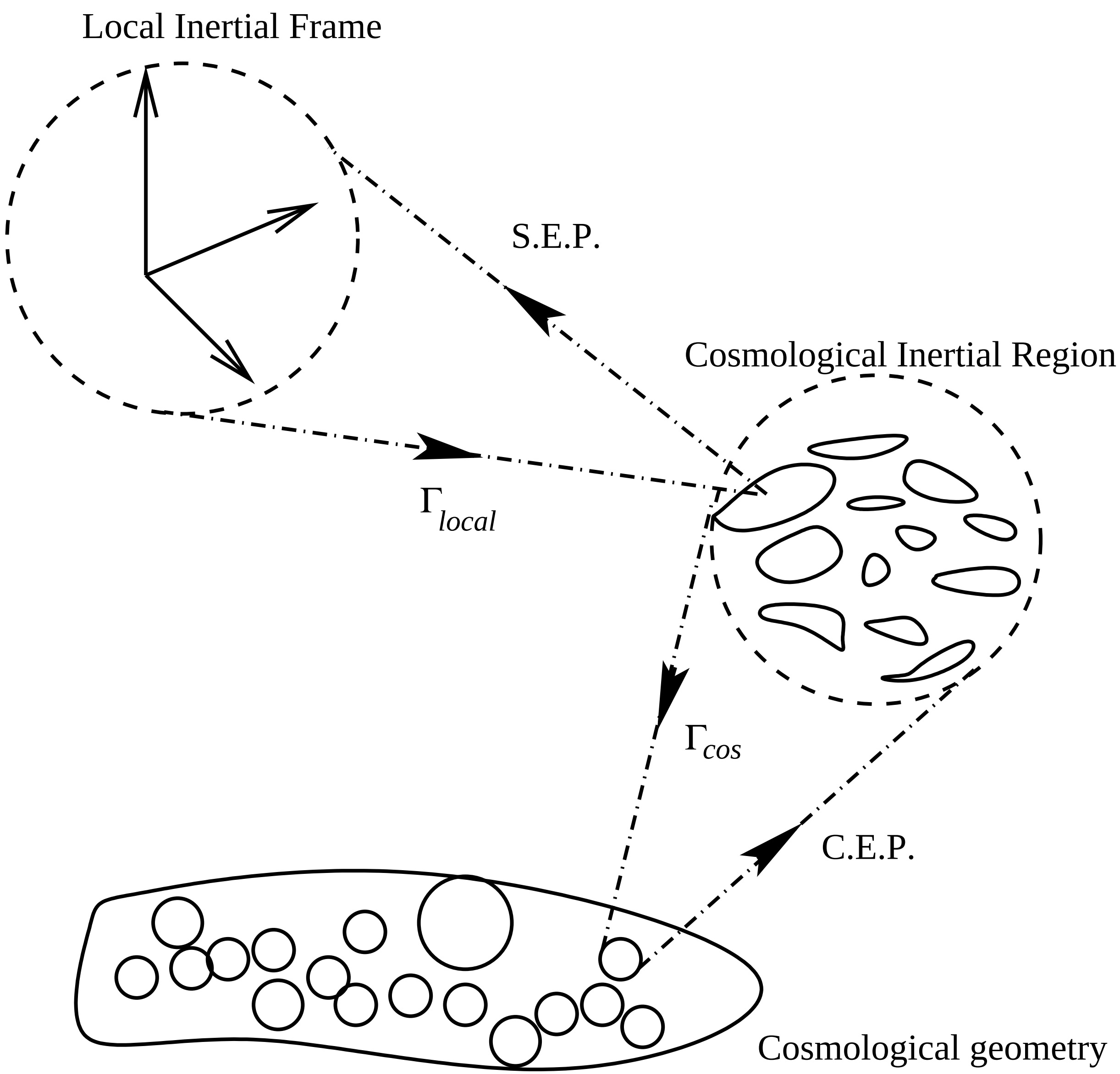
- Retain Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume–average environment (void)

Back to first principles. . .



- Need to address Mach's principle: *"Local inertial frames are determined through the distributions of energy and momentum in the universe by some weighted average of the apparent motions"*
- Need to separate non-propagating d.o.f., in particular regional density, from propagating modes: shape d.o.f.

Back to first principles. . .



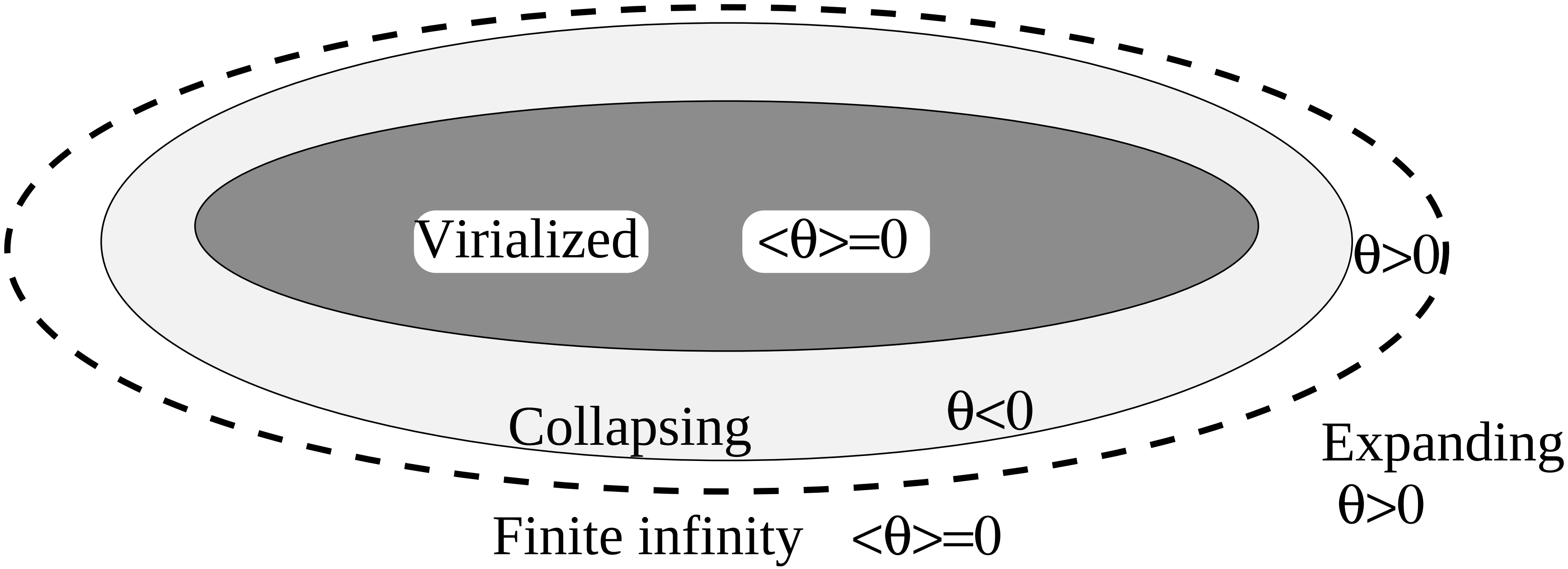
Cosmological Equivalence Principle

- *In cosmological averages it is always possible to choose a suitably defined spacetime region, the cosmological inertial region, in which average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation*,

$$\mathrm{d}s^2_{\mathrm{CIF}} = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right],$$

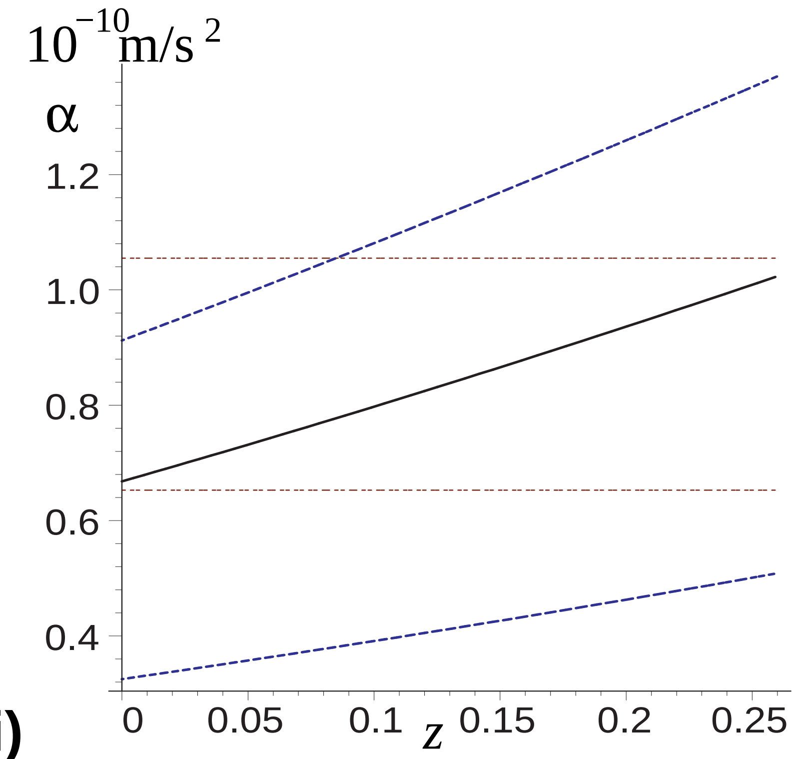
- Defines Cosmological Inertial Frame (CIF)
- Accounts for regional average effect of density in terms of frames for which the state of rest in an expanding space is indistinguishable from decelerating expansion of particles moving in a static space

Finite infinity

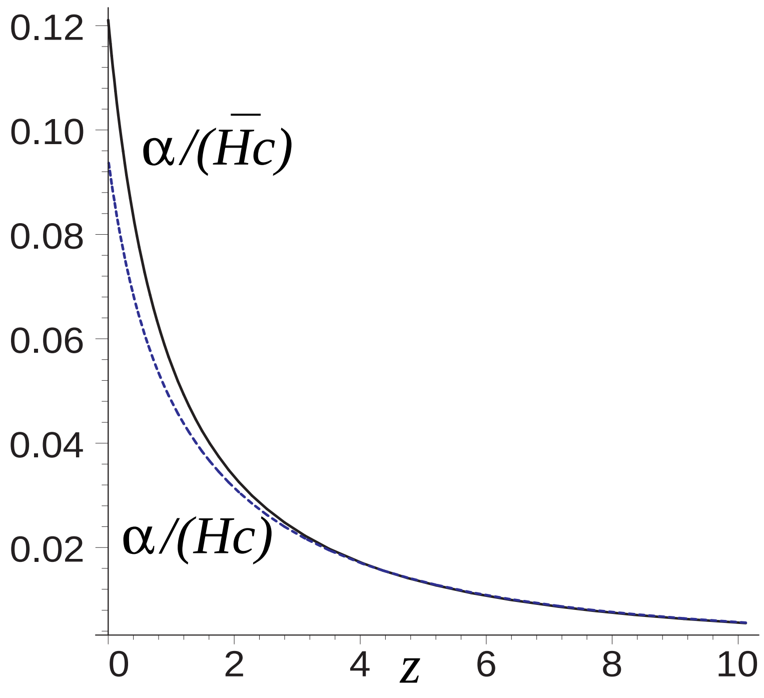


- Define *finite infinity*, “*fi*” as boundary to *connected* region within which *average expansion* vanishes $\langle\theta\rangle = 0$ and expansion is positive outside.
- Shape of *fi* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.

Relative deceleration scale



(i)



(ii)

By cosmological equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude $\alpha = H_0 c\bar{\gamma}\dot{\bar{\gamma}}/(\sqrt{\bar{\gamma}^2 - 1})$ beyond which *weak field cosmological general relativity* will be changed from Newtonian expectations: **(i)** as absolute scale nearby; **(ii)** divided by Hubble parameter to large z.

- Relative volume deceleration of expanding regions of different local density/curvature, leads cumulatively to canonical clocks differing by $\mathrm{d}t = \bar{\gamma}_\mathrm{w}\,\mathrm{d}\tau_\mathrm{w}$

Two/three scale model

- Split spatial volume $\mathcal{V}=\mathcal{V}_i\bar{a}^3$ as disjoint union of negatively curved void fraction with scale factor $a_{\rm v}$ and spatially flat “wall” fraction with scale factor $a_{\rm w}$.

$$\begin{aligned}\bar{a}^3 &= f_{\rm wi}a_{\rm w}{}^3+f_{\rm vi}a_{\rm v}{}^3\equiv\bar{a}^3(f_{\rm w}+f_{\rm v})\\ f_{\rm w} &\equiv f_{\rm wi}a_{\rm w}{}^3/\bar{a}^3, \qquad f_{\rm v}\equiv f_{\rm vi}a_{\rm v}{}^3/\bar{a}^3\end{aligned}$$

- $f_{\rm vi}=1-f_{\rm wi}$ is the fraction of present epoch horizon volume which was in uncompensated underdense perturbations at last scattering.

$$\bar{H}(t)=\frac{\dot{\bar{a}}}{\bar{a}}=f_{\rm w}H_{\rm w}+f_{\rm v}H_{\rm v}; \qquad H_{\rm w}\equiv\frac{1}{a_{\rm w}}\frac{{\rm d}a_{\rm w}}{{\rm d}t}, \quad H_{\rm v}\equiv\frac{1}{a_{\rm v}}\frac{{\rm d}a_{\rm v}}{{\rm d}t}$$

- Here t is the Buchert time parameter, considered as a collective coordinate of dust cell coarse-grained at SSH.

Phenomenological lapse functions

- According to Buchert average variance of θ will include internal variance of $H_{\rm w}$ relative to $H_{\rm v}$.
  Note $h_r \equiv H_{\rm w}/H_{\rm v} < 1$.
- Buchert time, t, is measured at the *volume average* position: locations where the local Ricci curvature scalar is the same as horizon volume average
- In timescape model, rates of wall and void centre observers who measure an isotropic CMB are fixed by the uniform quasilocal Hubble flow condition, i.e.,

$$\frac{1}{\bar a}\frac{{\rm d}\bar a}{{\rm d}t} = \frac{1}{a_{\rm w}}\frac{{\rm d}a_{\rm w}}{{\rm d}\tau_{\rm w}} = \frac{1}{a_{\rm v}}\frac{{\rm d}a_{\rm v}}{{\rm d}\tau_{\rm v}}; \qquad \text{or} \qquad \bar H(t) = \bar\gamma_{\rm w} H_{\rm w} = \bar\gamma_{\rm v} H_{\rm v}$$

where $\bar\gamma_{\rm v} = \frac{{\rm d}t}{{\rm d}\tau_{\rm v}}$, $\bar\gamma_{\rm w} = \frac{{\rm d}t}{{\rm d}\tau_{\rm w}} = 1 + (1-h_r) f_v / h_r$, are *phenomenological lapse functions* (NOT ADM lapse).

Other ingredients

- $\langle \mathcal{R} \rangle = k_v / a_{\rm v}{}^3 = k_{\rm v} f_{\rm vi}{}^{2/3} f_{\rm v}{}^{1/3} / \bar{a}^3$ since $k_{\rm w} = 0$
- Assume that average shear in SSH cell vanishes; more precisely neglect $\mathcal{Q}$ *within* voids and walls *separately*

$$\langle \delta\theta^2 \rangle_{\rm w} = \tfrac{3}{4} \langle \sigma^2 \rangle_{\rm w} \qquad \langle \delta\theta^2 \rangle_{\rm v} = \tfrac{3}{4} \langle \sigma^2 \rangle_{\rm v}$$

  Justification: for spherical voids expect $\langle \sigma^2 \rangle = \langle \omega^2 \rangle = 0$; for walls expect $\langle \sigma^2 \rangle$ and $\langle \omega^2 \rangle$ largely self-canceling.

- Only remaining backreaction is variance of relative volume expansion of walls and voids

$$\mathcal{Q} = 6 f_{\rm v} (1 - f_{\rm v}) \left(H_{\rm v} - H_{\rm w} \right)^2 = \frac{2 \dot{f}_{\rm v}{}^2}{3 f_{\rm v} (1 - f_{\rm v})}$$

- Solutions known for:
  - dust (DLW 2007);
  - dust + Λ (Viaggiu, 2012), taking $\bar{\gamma}_{\rm w} = \bar{\gamma}_{\rm v} = 1$;
  - dust + radiation (Duley, Nazer + DLW, 1306.3208)

Bare cosmological parameters

- Buchert equations for volume averaged observer, with $f_{\rm v}(t) = f_{\rm vi} {a_{\rm v}}^3/\bar{a}^3$ (void volume fraction) and $k_{\rm v} < 0$

$$\bar{\Omega}_M + \bar{\Omega}_R + \bar{\Omega}_k + \bar{\Omega}_{\mathcal{Q}} = 1,$$

$$\bar{a}^{-6}\partial_t \left(\bar{\Omega}_{\mathcal{Q}} \bar{H}^2 \bar{a}^6\right) + \bar{a}^{-2}\partial_t \left(\bar{\Omega}_k \bar{H}^2 \bar{a}^2\right) = 0\,.$$

where the *bare parameters* are

$$\bar{\Omega}_M = \frac{8\pi G \bar{\rho}_{M0} \bar{a}_0^3}{3\bar{H}^2 \bar{a}^3}, \qquad \bar{\Omega}_R = \frac{8\pi G \bar{\rho}_{R0} \bar{a}_0^4}{3\bar{H}^2 \bar{a}^4},$$

$$\bar{\Omega}_k = \frac{-k_{\rm v} {f_{\rm vi}}^{2/3} {f_{\rm v}}^{1/3}}{\bar{a}^2 \bar{H}^2}, \qquad \bar{\Omega}_{\mathcal{Q}} = \frac{-{\dot{f}_{\rm v}}^{\,2}}{9 f_{\rm v}(1-f_{\rm v})\bar{H}^2}\,.$$

Tracker solution

- PRL 99 (2007) 251101:

$$\bar{a} = \frac{\bar{a}_0 (3\bar{H}_0 t)^{2/3}}{2+f_{v0}} \left[3f_{v0}\bar{H}_0 t + (1-f_{v0})(2+f_{v0}) \right]^{1/3}$$

$$f_v = \frac{3f_{v0}\bar{H}_0 t}{3f_{v0}\bar{H}_0 t + (1-f_{v0})(2+f_{v0})},$$

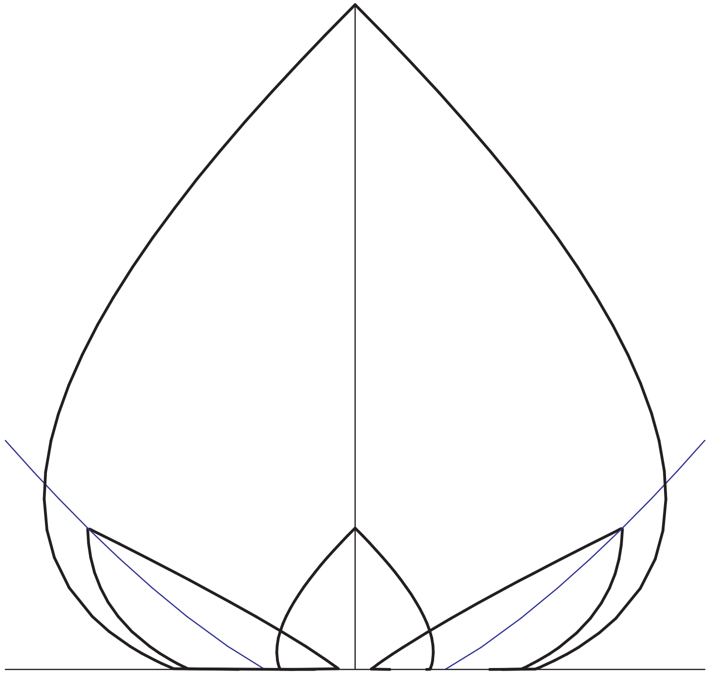
- Other parameters (drop subscript w on $\bar{\gamma}_w$):

$$\bar{\gamma} = 1 + \tfrac{1}{2} f_v = \tfrac{3}{2}\bar{H}t$$

$$\bar{\Omega}_M = \frac{4(1-f_v)}{(2+f_v)^2}; \quad \bar{\Omega}_k = \frac{9f_v}{(2+f_v)^2}; \quad \bar{\Omega}_{\mathcal{Q}} = \frac{-f_v(1-f_v)}{(2+f_v)^2}$$

$$\tau_w = \tfrac{2}{3}t + \frac{2(1-f_{v0})(2+f_{v0})}{27f_{v0}\bar{H}_0} \ln\left(1 + \frac{9f_{v0}\bar{H}_0 t}{2(1-f_{v0})(2+f_{v0})}\right)$$

Past light cone average



- Interpret solution of Buchert equations by radial null cone average

$$ds^2 = -dt^2 + \bar{a}^2(t)\,d\bar{\eta}^2 + A(\bar{\eta},t)\,d\Omega^2,$$

where $\int_0^{\bar{\eta}_\mathcal{H}} d\bar{\eta}\,A(\bar{\eta},t) = \bar{a}^2(t)\mathcal{V}_{\rm i}(\bar{\eta}_\mathcal{H})/(4\pi)$.

- LTB metric but NOT an LTB solution

Physical interpretation

- Conformally match radial null geodesics of spherical Buchert geometry to those of finite infinity geometry with uniform local Hubble flow condition $\mathrm{d}t = \bar{a}\,\mathrm{d}\bar{\eta}$ and $\mathrm{d}\tau_{\mathrm{w}} = a_{\mathrm{w}}\mathrm{d}\eta_{\mathrm{w}}$. But $\mathrm{d}t = \bar{\gamma}\mathrm{d}\tau_{\mathrm{w}}$ and $a_{\mathrm{w}} = {f_{\mathrm{wi}}}^{-1/3}(1-f_{\mathrm{v}})\,\bar{a}$. Hence *on radial null geodesics*

$$\mathrm{d}\eta_{\mathrm{w}} = \frac{{f_{\mathrm{wi}}}^{1/3}\mathrm{d}\bar{\eta}}{\bar{\gamma}\,(1-f_{\mathrm{v}})^{1/3}}$$

Define η_{w} by integral of above on radial null-geodesics.

- Extend spatially flat wall geometry to dressed geometry

$$\mathrm{d}s^2 = -\mathrm{d}\tau_{\mathrm{w}}^2 + a^2(\tau_{\mathrm{w}})\left[\mathrm{d}\bar{\eta}^2 + r_{\mathrm{w}}^2(\bar{\eta},\tau_{\mathrm{w}})\,\mathrm{d}\Omega^2\right]$$

where $r_{\mathrm{w}} \equiv \bar{\gamma}\,(1-f_{\mathrm{v}})^{1/3}\,{f_{\mathrm{wi}}}^{-1/3}\eta_{\mathrm{w}}(\bar{\eta},\tau_{\mathrm{w}})$, $a = \bar{a}/\bar{\gamma}$.

Dressed cosmological parameters

- N.B. The extension is NOT an isometry

$$\text{N.B.}\qquad \mathrm{d}s^2_{fi} = -\mathrm{d}\tau_{\mathrm{w}}^2 + {a_{\mathrm{w}}}^2(\tau_{\mathrm{w}})\left[\mathrm{d}\eta_{\mathrm{w}}^2 + \eta_{\mathrm{w}}^2\mathrm{d}\Omega^2\right]$$
$$\rightarrow \mathrm{d}s^2 = -\mathrm{d}\tau_{\mathrm{w}}^2 + a^2\left[\mathrm{d}\bar\eta^2 + r_{\mathrm{w}}^2(\bar\eta,\tau_{\mathrm{w}})\,\mathrm{d}\Omega^2\right]$$

- Extended metric is an effective “spherical Buchert geometry” adapted to wall rulers and clocks.
- Since $\mathrm{d}\bar\eta = \mathrm{d}t/\bar a = \bar\gamma\,\mathrm{d}\tau_{\mathrm{w}}/\bar a = \mathrm{d}\tau_{\mathrm{w}}/a$, this leads to *dressed parameters* which do not sum to 1, e.g.,

$$\Omega_M = \bar\gamma^3\bar\Omega_M\,.$$

- Dressed average Hubble parameter

$$H = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tau_{\mathrm{w}}} = \frac{1}{\bar a}\frac{\mathrm{d}\bar a}{\mathrm{d}\tau_{\mathrm{w}}} - \frac{1}{\bar\gamma}\frac{\mathrm{d}\bar\gamma}{\mathrm{d}\tau_{\mathrm{w}}}$$

Dressed cosmological parameters

- H is greater than wall Hubble rate; smaller than void Hubble rate measured by wall (or any one set of) clocks

$$\bar{H}(t) = \frac{1}{\bar{a}}\frac{\mathrm{d}\bar{a}}{\mathrm{d}t} = \frac{1}{a_{\mathrm{v}}}\frac{\mathrm{d}a_{\mathrm{v}}}{\mathrm{d}\tau_{\mathrm{v}}} = \frac{1}{a_{\mathrm{w}}}\frac{\mathrm{d}a_{\mathrm{w}}}{\mathrm{d}\tau_{\mathrm{w}}} < H < \frac{1}{a_{\mathrm{v}}}\frac{\mathrm{d}a_{\mathrm{v}}}{\mathrm{d}\tau_{\mathrm{w}}}$$

- For tracker solution $H = (4{f_{\mathrm{v}}}^2 + f_{\mathrm{v}} + 4)/6t$
- Dressed average deceleration parameter

$$q = \frac{-1}{H^2 a^2}\frac{\mathrm{d}^2 a}{\mathrm{d}\tau_{\mathrm{w}}^2}$$

Can have $q < 0$ even though $\bar{q} = \frac{-1}{\bar{H}^2 \bar{a}^2}\frac{\mathrm{d}^2\bar{a}}{\mathrm{d}t^2} > 0$; difference of clocks important.

Redshift, luminosity distance

- Cosmological redshift (last term tracker solution)

$$z+1=\frac{a}{a_0}=\frac{\bar{a}_0\bar{\gamma}}{\bar{a}\bar{\gamma}_0}=\frac{(2+f_{\rm v}){f_{\rm v}}^{1/3}}{3{f_{\rm v0}}^{1/3}\bar{H}_0 t}=\frac{2^{4/3}t^{1/3}(t+b)}{{f_{\rm v0}}^{1/3}\bar{H}_0 t(2t+3b)^{4/3}}\,,$$

where $b=2(1-f_{\rm v0})(2+f_{\rm v0})/[9f_{\rm v0}\bar{H}_0]$

- Dressed luminosity distance relation $d_L=(1+z)D$ where the *effective comoving distance* to a redshift z is $D=a_0 r_{\rm w}$, with

$$r_{\rm w}=\bar{\gamma}\,(1-f_{\rm v})^{1/3}\int_t^{t_0}\frac{{\rm d}t'}{\bar{\gamma}(t')(1-f_{\rm v}(t'))^{1/3}\bar{a}(t')}\,.$$

Redshift, luminosity distance

- Perform integral for tracker solution

$$D_A = \frac{D}{1+z} = \frac{d_L}{(1+z)^2} = (t)^{\frac{2}{3}} \int_t^{t_0} \frac{2\mathrm{d}t'}{(2+f_{\mathrm{v}}(t'))(t')^{2/3}}$$
$$= t^{2/3}(\mathcal{F}(t_0) - \mathcal{F}(t))$$

where

$$\mathcal{F}(t) = 2t^{1/3} + \frac{b^{1/3}}{6} \ln\left(\frac{(t^{1/3}+b^{1/3})^2}{t^{2/3} - b^{1/3}t^{1/3} + b^{2/3}} \right) + \frac{b^{1/3}}{\sqrt{3}} \tan^{-1}\left(\frac{2t^{1/3} - b^{1/3}}{\sqrt{3}\, b^{1/3}} \right).$$

t given implicitly in terms of z by previous relation

Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2\left(1 - f_{\mathrm{v}}\right)^2}{(2 + f_{\mathrm{v}})^2}\,.$$

  As $t \to \infty$, $f_{\mathrm{v}} \to 1$ and $\bar{q} \to 0^+$.

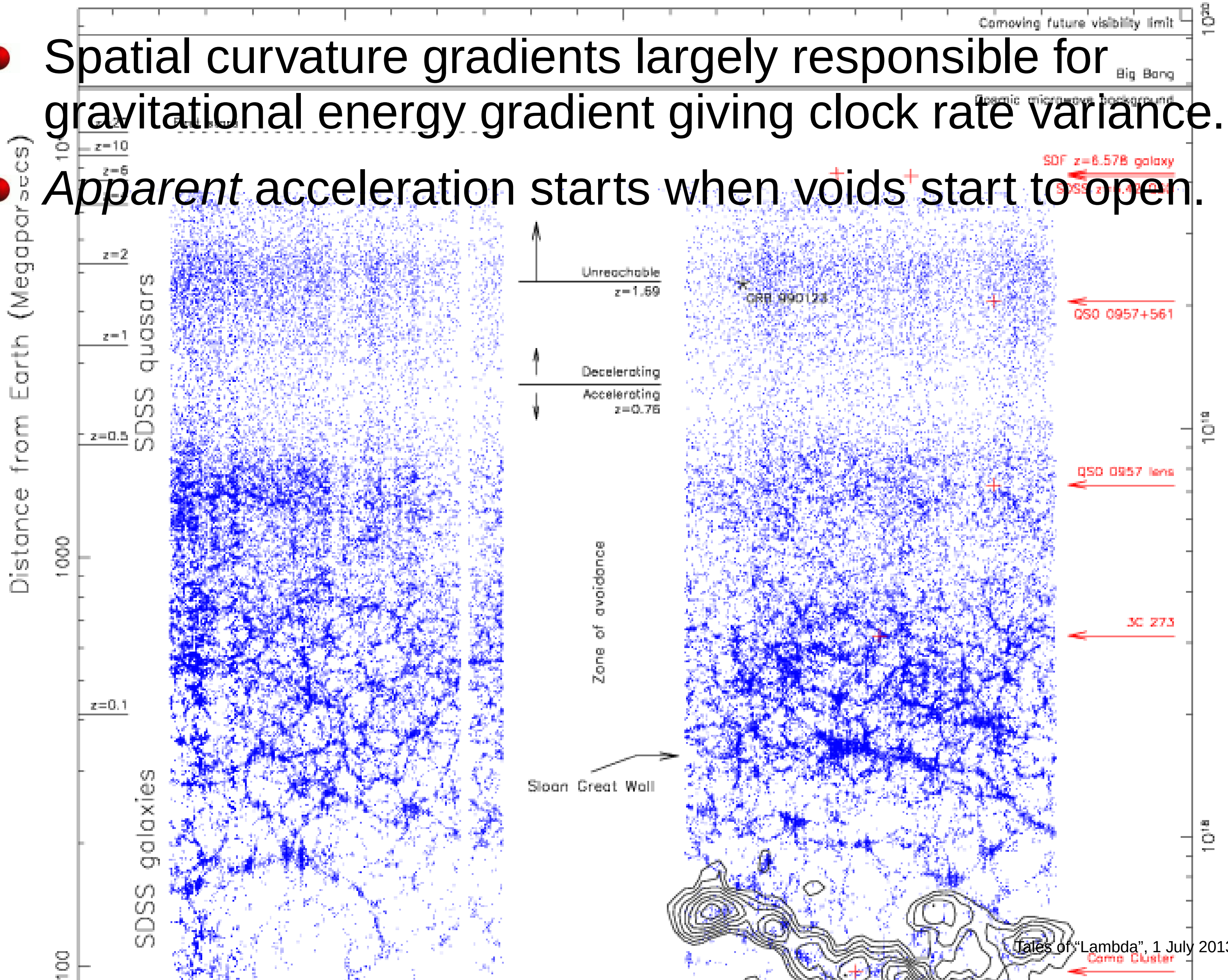
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-\left(1 - f_{\mathrm{v}}\right)\left(8{f_{\mathrm{v}}}^3 + 39{f_{\mathrm{v}}}^2 - 12f_{\mathrm{v}} - 8\right)}{\left(4 + f_{\mathrm{v}} + 4{f_{\mathrm{v}}}^2\right)^2}\,,$$

  Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small f_{v}; changes sign when $f_{\mathrm{v}} = 0.58670773\ldots$, and approaches $q \to 0^-$ at late times.

Cosmic coincidence problem solved

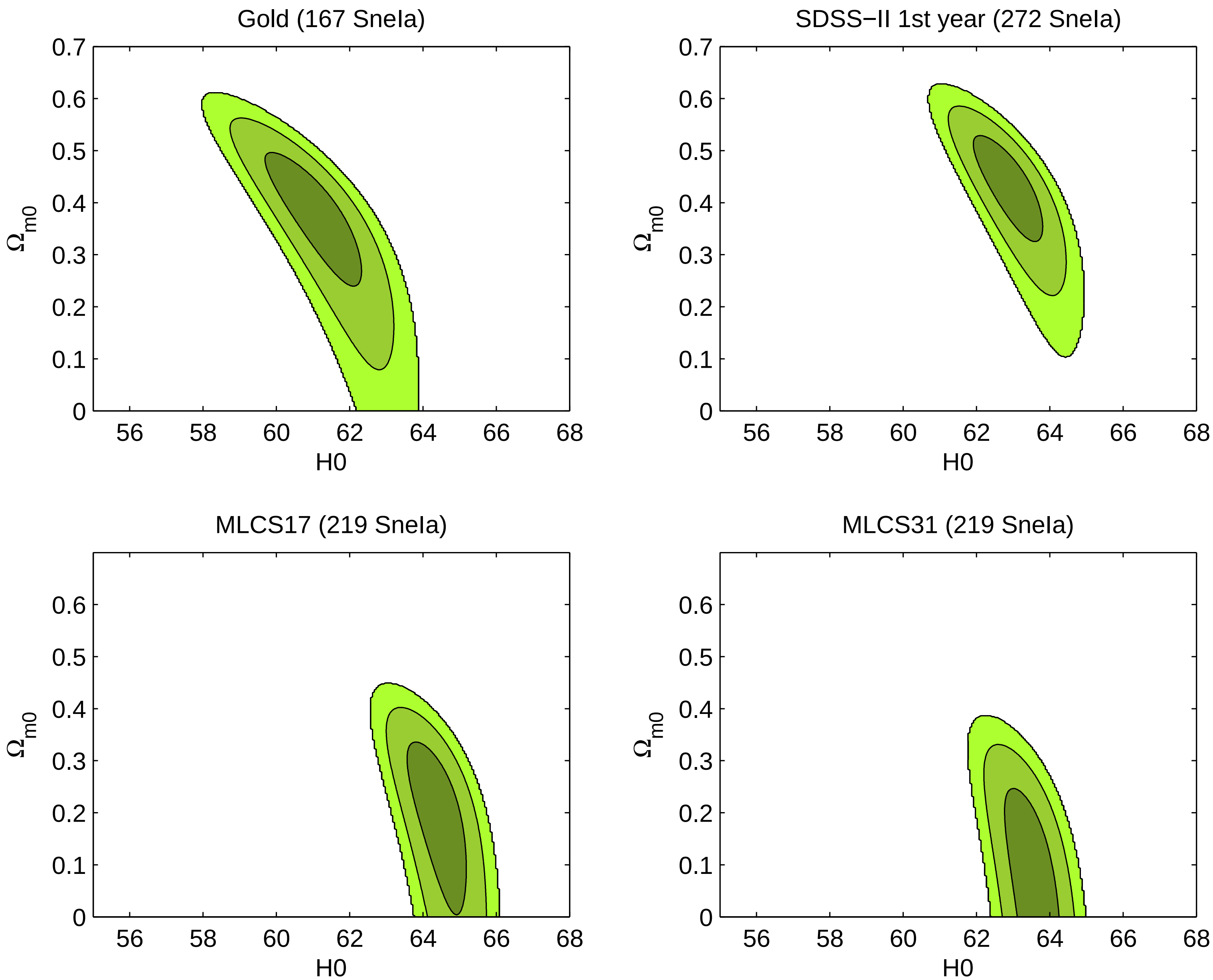
- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- *Apparent* acceleration starts when voids start to open.



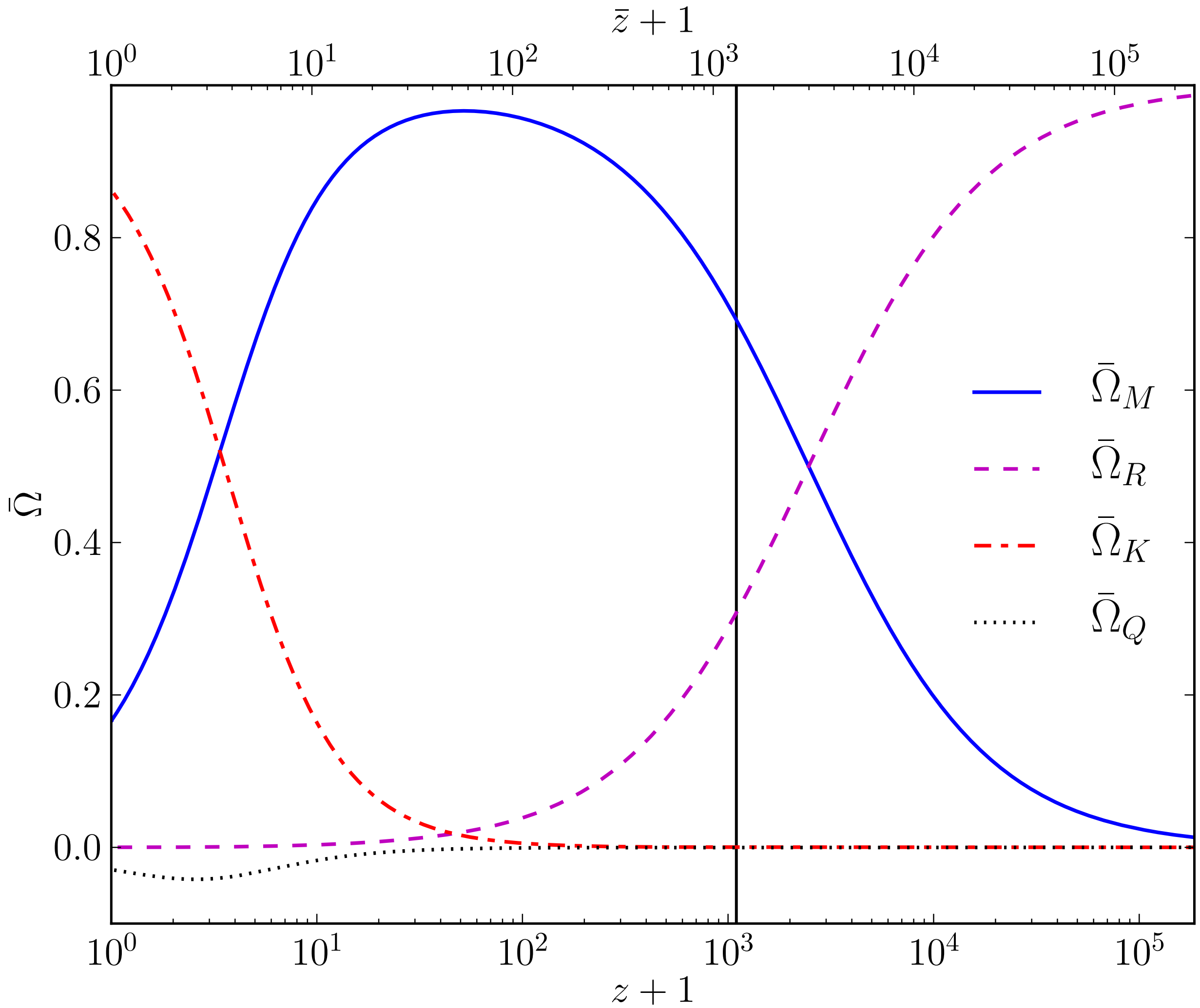
Smale + DLW, MNRAS 413 (2011) 367

- SALT/SALTII fits (Constitution,SALT2,Union2) favour ΛCDM over TS: $\ln B_{\mathrm{TS:\Lambda CDM}} = -1.06, -1.55, -3.46$
- MLCS2k2 (fits MLCS17,MLCS31,SDSS-II) favour TS over ΛCDM: $\ln B_{\mathrm{TS:\Lambda CDM}} = 1.37, 1.55, 0.53$
- Different MLCS fitters give different best-fit parameters; e.g. with cut at statistical homogeneity scale, for MLCS31 (Hicken et al 2009) $\Omega_{M0} = 0.12^{+0.12}_{-0.11}$; MLCS17 (Hicken et al 2009) $\Omega_{M0} = 0.19^{+0.14}_{-0.18}$; SDSS-II (Kessler et al 2009) $\Omega_{M0} = 0.42^{+0.10}_{-0.10}$
- Supernovae systematics (reddening/extinction, intrinsic colour variations) must be understood to distinguish models
- Foregrounds, and inclusion of SneIa below SSH an important issue

Supernovae systematics



Bare cosmological parameters



J.A.G. Duley, M.A. Nazer & DLW, arXiv:1306.3208:

full numerical solution with matter, radiation

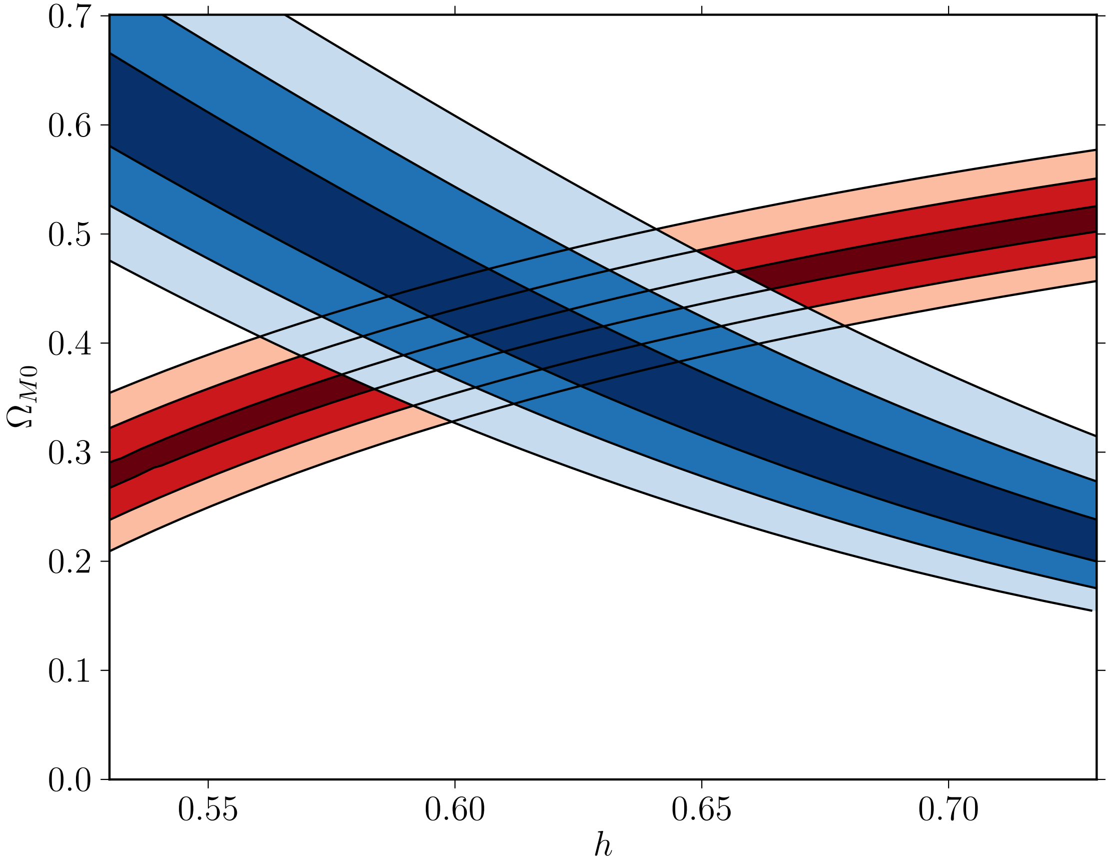
CMB – calibration of sound horizon

- Physics at last–scattering same as matter + radiation FLRW model. What is changed is relative calibration of parameters.
- Proper distance to comoving scale of the sound horizon at any epoch for volume–average observer [$\bar{x} = \bar{a}/\bar{a}_0$, so $\bar{x}_{\rm dec} = \bar{\gamma}_0^{-1}(1+z_{\rm dec})^{-1}$]

$$\bar{D}_s = \frac{\bar{a}(t)}{\bar{a}_0}\frac{c}{\sqrt{3}}\int_0^{\bar{x}_{\rm dec}} \frac{{\rm d}\bar{x}}{\bar{x}^2\bar{H}\sqrt{1+0.75\,\bar{x}\,\bar{\Omega}_{B0}/\bar{\Omega}_{\gamma 0}}}\,,$$

- For wall observer $D_s(\tau) = \bar{\gamma}^{-1} D_s$
- Determine epoch of photon decoupling, and baryon drag epoch directly from Peebles equation etc with numerical solution

CMB constraints from Planck



Parameters within the (Ω_{M0},H_0) plane which fit the angular scale of the sound horizon $\theta_* = 0.0104139$ (blue), and its comoving scale at the baryon drag epoch as compared to Planck value $98.88\, h^{-1}$Mpc (red) to within 2%, 4% and 6%, with photon-baryon ratio $\eta_{B\gamma} = 4.6$–5.6×10^{-10} within 2σ of all observed light element abundances (including lithium-7).

Parameters using Planck constraints

- Dressed Hubble constant $H_0 = 61.7 \pm 3.0$ km/s/Mpc
- Bare Hubble constant $H_{w0} = \bar{H}_0 = 50.1 \pm 1.7$ km/s/Mpc
- Local max Hubble constant $H_{v0} = 75.2^{+2.0}_{-2.6}$ km/s/Mpc
- Present void fraction $f_{v0} = 0.695^{+0.041}_{-0.051}$
- Bare matter density parameter $\bar{\Omega}_{M0} = 0.167^{+0.036}_{-0.037}$
- Bare baryon density parameter $\bar{\Omega}_{B0} = 0.030^{+0.007}_{-0.005}$
- Dressed matter density parameter $\Omega_{M0} = 0.41^{+0.06}_{-0.05}$
- Dressed baryon density parameter $\Omega_{B0} = 0.074^{+0.013}_{-0.011}$
- Age of universe (galaxy/wall) $\tau_{w0} = 14.2 \pm 0.5$ Gyr
- Age of universe (volume-average) $t_0 = 17.5 \pm 0.6$ Gyr
- Apparent acceleration onset $z_{acc} = 0.46^{+0.26}_{-0.25}$

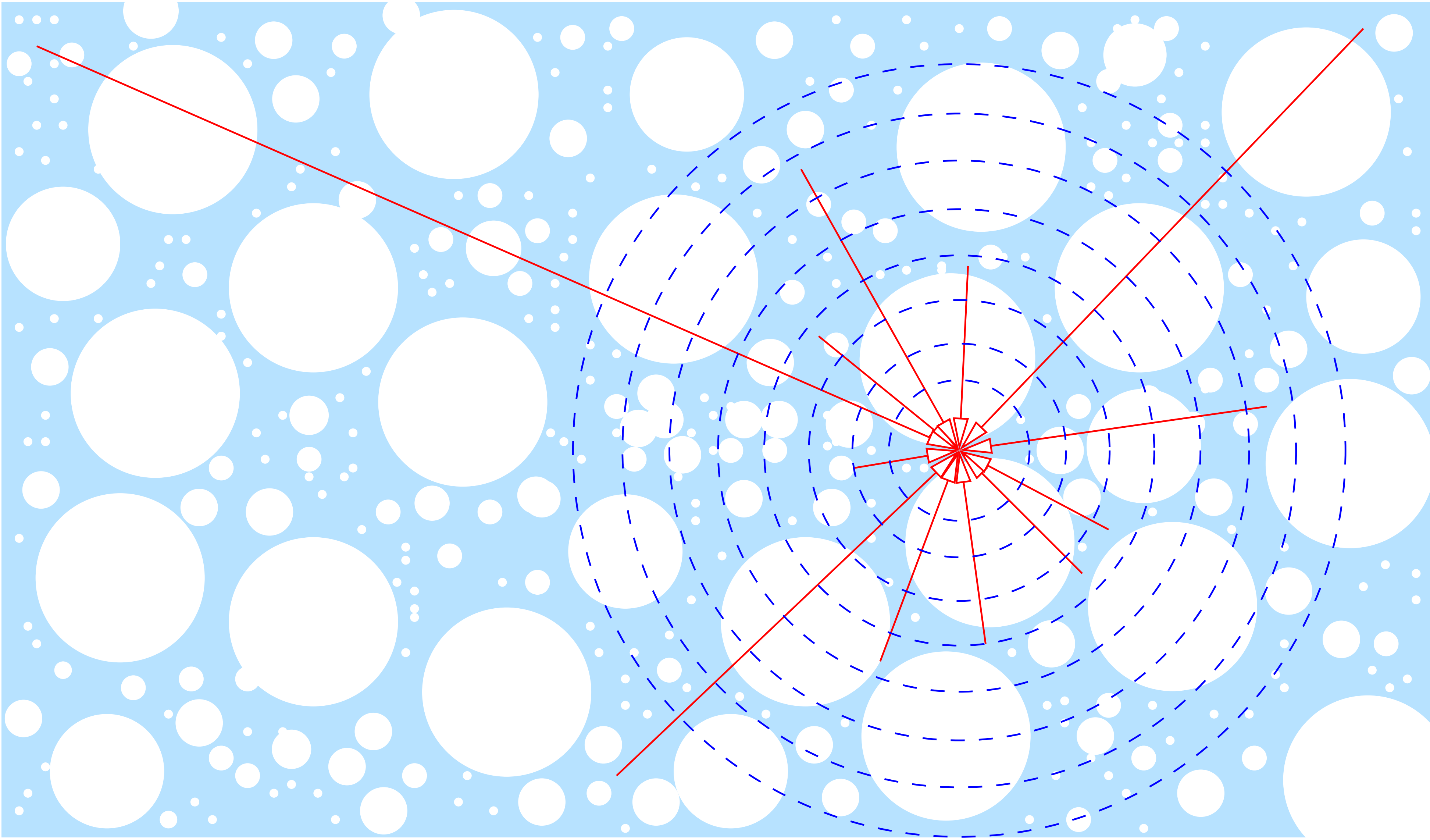
Other tests versus ΛCDM

See Phys. Rev. D80 (2009) 123512 for tests of average observational quantities

- Phenomenological equivalents of $w(z)$, *Om*(z) statistic etc
- Alcock–Paczyński test
- redshift–time drift (Sandage–Loeb) test
- Clarkson, Bassett and Lu homogeneity test

However, potentially most interesting results are from variance of Hubble flow below scale of statistical homogeneity

Apparent Hubble flow variance



Peculiar velocity formalism

- Standard framework, FLRW + Newtonian perturbations, assumes peculiar velocity field

$$v_{\rm pec} = cz - H_0 r$$

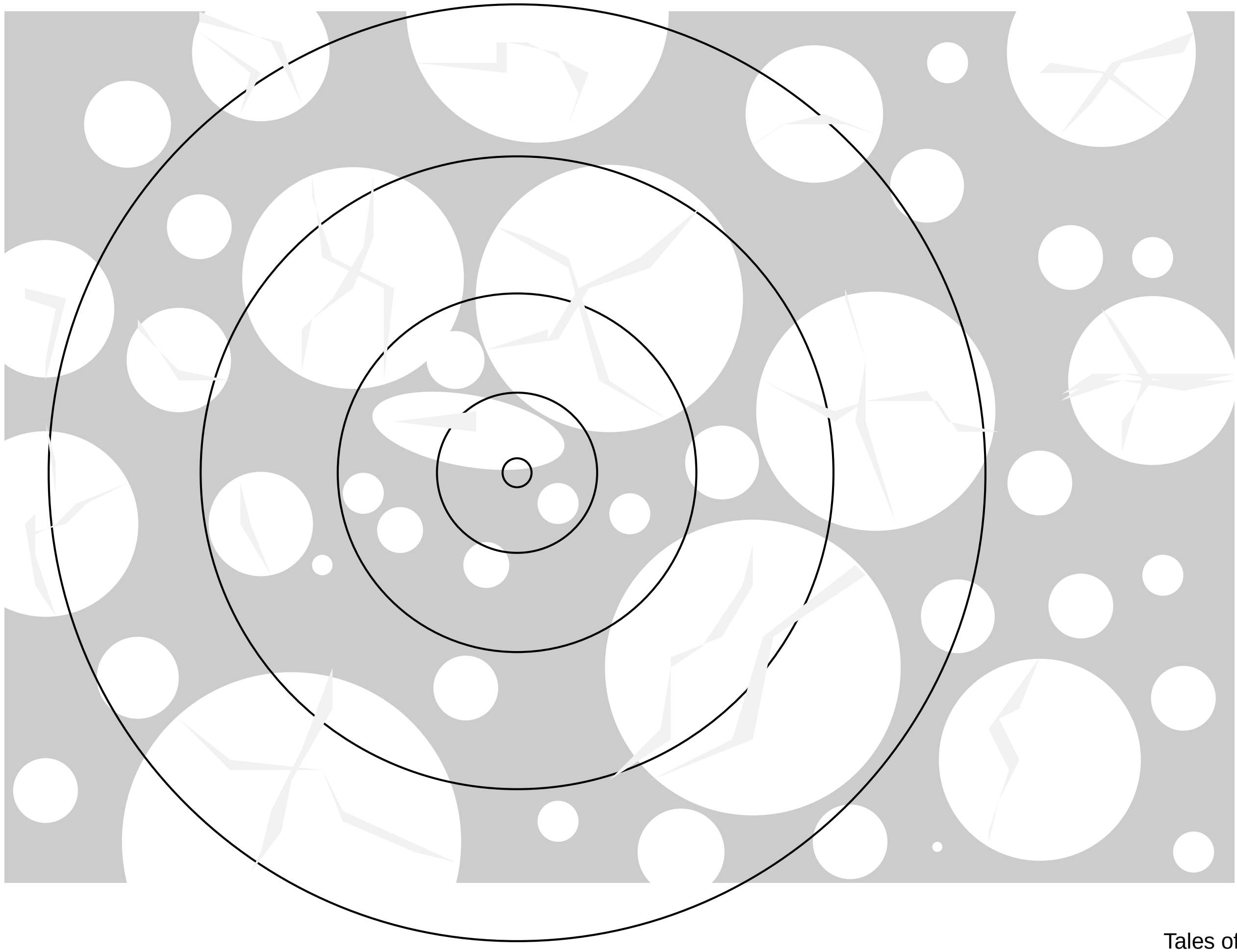
generated by

$$\mathbf{v}(\mathbf{r}) = \frac{H_0 \Omega_{M0}^{0.55}}{4\pi} \int \mathrm{d}^3\mathbf{r}'\, \delta_m(\mathbf{r}')\, \frac{(\mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|^3}$$

- After 3 decades of work, despite contradictory claims, the $\mathbf{v}(\mathbf{r})$ does not to converge to LG velocity w.r.t. CMB
- Agreement on direction, not amplitude or scale (Lavaux et al 2010; Bilicki et al 2011; . . .)
- Suggestions of bulk flows inconsistent with ΛCDM (Watkins, Feldman, Hudson 2009. . .)

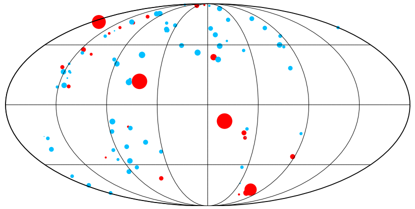
Spherical averages

- Determine variation in Hubble flow by determining best-fit linear Hubble law in spherical shells

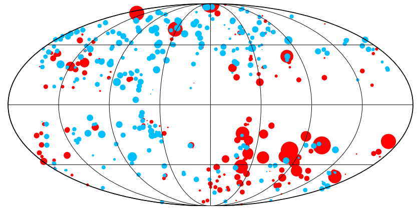


Analysis of COMPOSITE sample

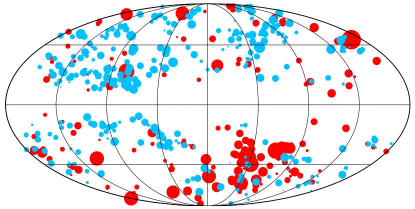
- Use COMPOSITE sample: Watkins, Feldman & Hudson 2009, 2010, with 4,534 galaxy redshifts and distances, includes most large surveys to 2009
- Distance methods: Tully Fisher, fundamental plane, surface brightness fluctuation; 103 supernovae distances.
- average in *independent spherical shells*
- Compute H_s in $12.5\,h^{-1}$Mpc shells; combine 3 shells $> 112.5\,h^{-1}$Mpc
- Use data beyond $156.25\,h^{-1}$Mpc as check on H_0 normalisation – COMPOSITE sample is normalized to $100\,h$ km/s/Mpc



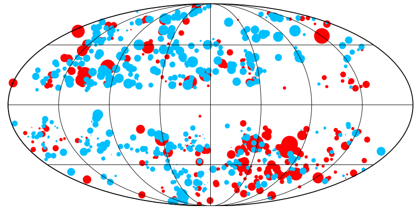
(a) 1: $0 - 12.5\ h^{-1}$ Mpc $N = 92$.



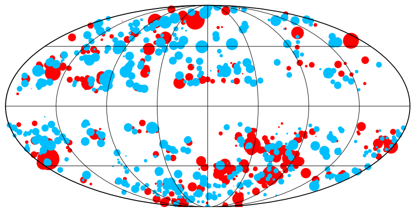
(b) 2: $12.5 - 25\ h^{-1}$ Mpc $N = 505$.



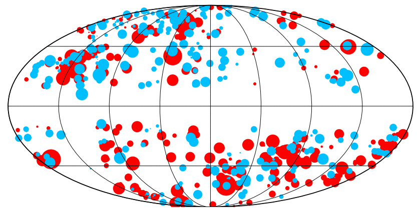
(c) 3: $25 - 37.5\ h^{-1}$ Mpc $N = 514$.



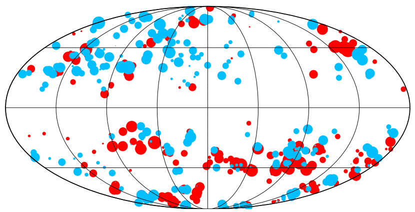
(d) 4: $37.5 - 50\ h^{-1}$ Mpc $N = 731$.



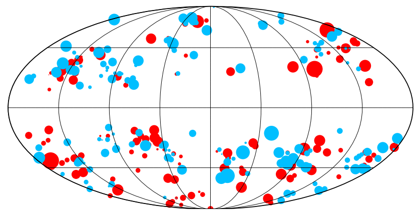
(e) 5: $50 - 62.5\ h^{-1}$ Mpc $N = 819$.



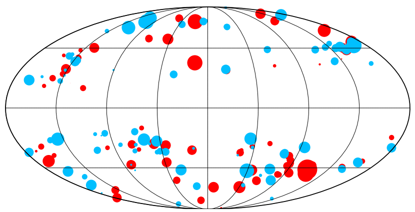
(f) 6: $62.5 - 75\ h^{-1}$ Mpc $N = 562$.



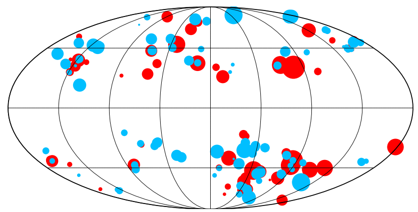
(g) 7: $75 - 87.5\ h^{-1}$ Mpc $N = 414$.



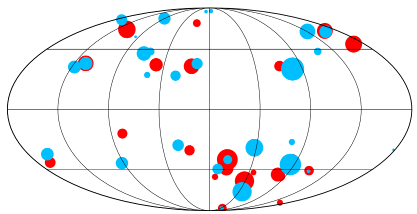
(h) 8: $87.5 - 100\ h^{-1}$ Mpc $N = 304$.



(i) 9: $100 - 112.5\ h^{-1}$ Mpc $N = 222$.

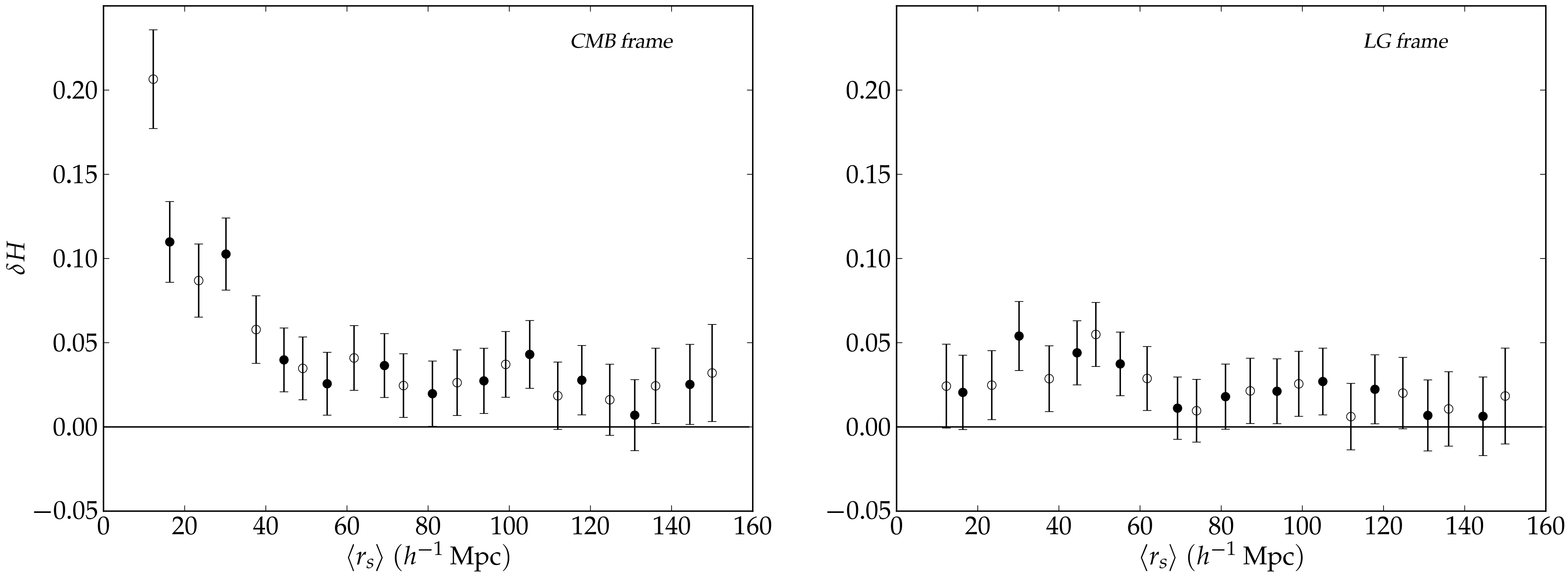


(j) 10: $112.5 - 156.25\ h^{-1}$ Mpc $N = 280$.



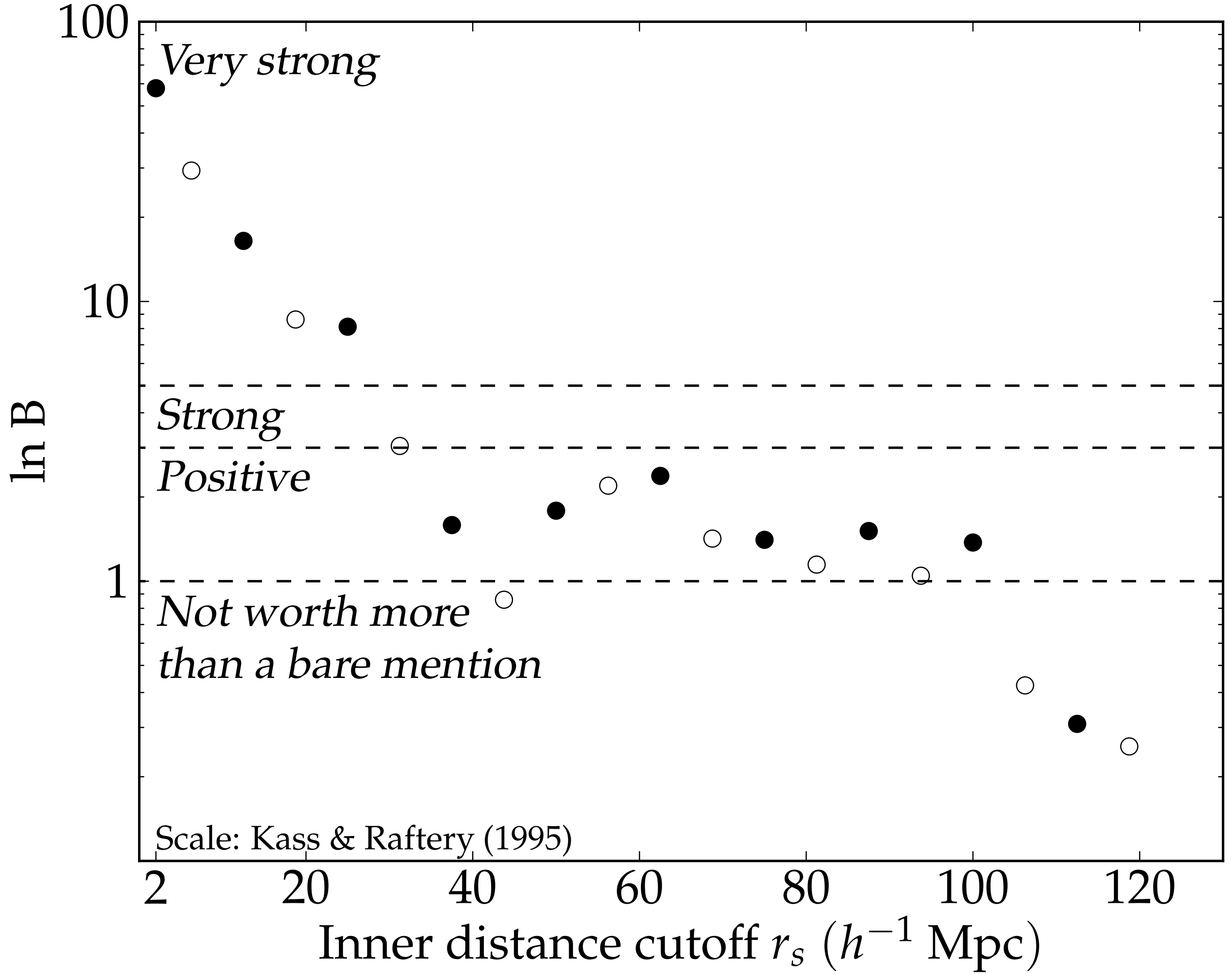
(k) 11: $156.25 - 417.4\ h^{-1}$ Mpc $N = 91$.

Radial variance $\delta H_s = (H_s - H_0)/H_0$



- Two choices of shell boundaries (closed and open circles); for each choice data points uncorrelated
- Analyse linear Hubble relation in rest frame of CMB; Local Group (LG); Local Sheet (LS). LS result very close to LG result.

Bayesian comparison of uniformity



- Hubble flow more uniform in LG frame than CMB frame with very strong evidence

Boosts and spurious monopole variance

- H_s determined by linear regression in each shell

$$H_s = \left(\sum_{i=1}^{N_s} \frac{(cz_i)^2}{\sigma_i^2}\right)\left(\sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2}\right)^{-1},$$

- Under boost $cz_i \to cz_i' = cz_i + v\cos\phi_i$ for uniformly distributed data, linear terms cancel on opposite sides of sky

$$\begin{aligned} H_s' - H_s &\sim \left(\sum_{i=1}^{N_s} \frac{(v\cos\phi_i)^2}{\sigma_i^2}\right)\left(\sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2}\right)^{-1} \\ &= \frac{\langle (v\cos\phi_i)^2\rangle}{\langle cz_i r_i\rangle} \sim \frac{v^2}{2H_0\langle r_i^2\rangle} \end{aligned}$$

Angular variance

Two approaches; fit

1. McClure and Dyer (2007) method – can look at higher multipoles

$$H_\alpha = \frac{\sum_{i=1}^{N} W_{i\,\alpha}\, cz_i\, r_i^{-1}}{\sum_{j=1}^{N} W_{j\,\alpha}}$$

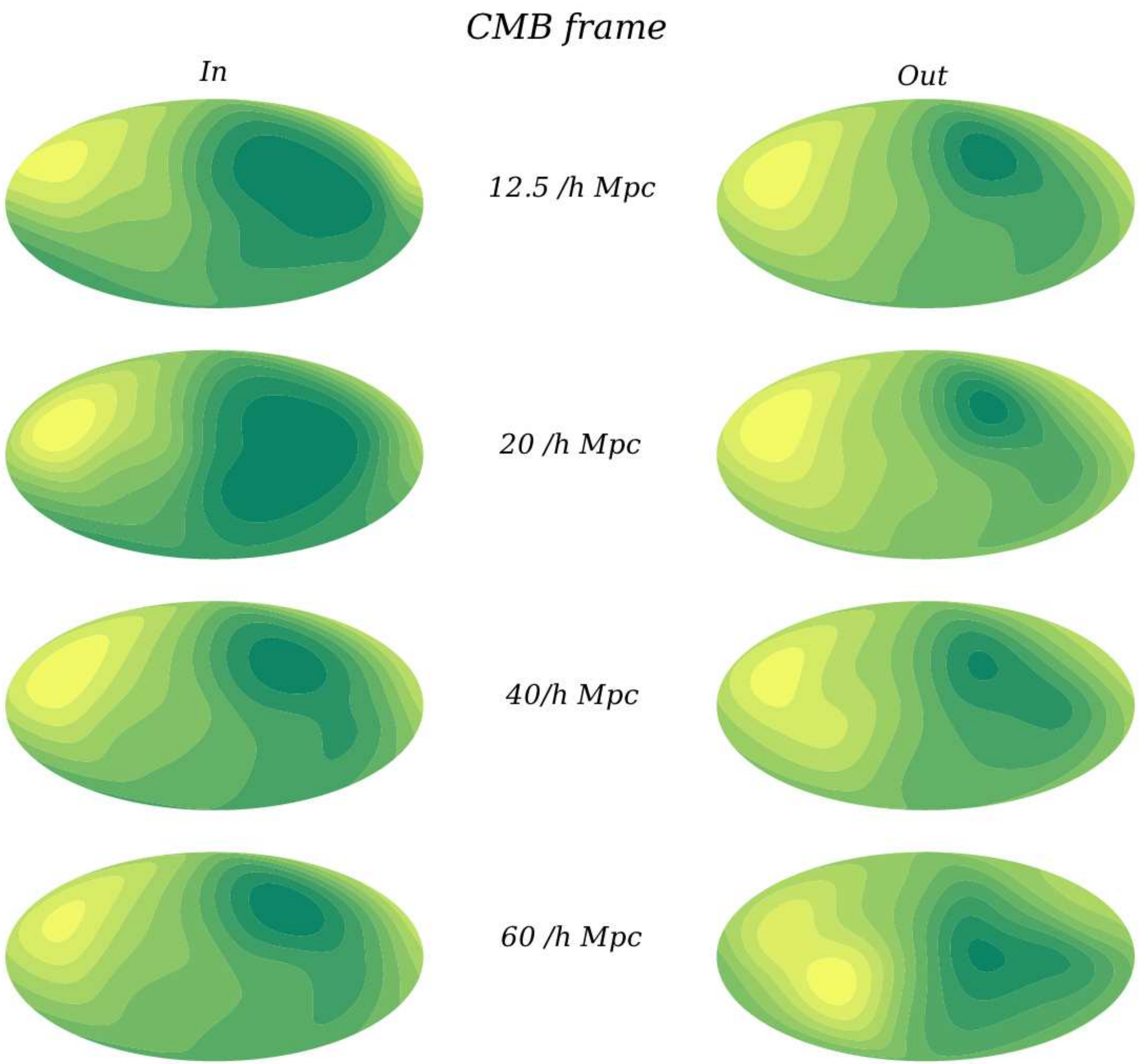
   where with $\cos\theta_i = \vec{r}_{\rm grid}\cdot\vec{r_i}$, $\sigma_\theta = 25^\circ$ (typically)

$$W_{i\,\alpha} = \frac{1}{\sqrt{2\pi}\sigma_\theta}\exp\left(\frac{-\theta_i^2}{2\sigma_\theta^2}\right)$$

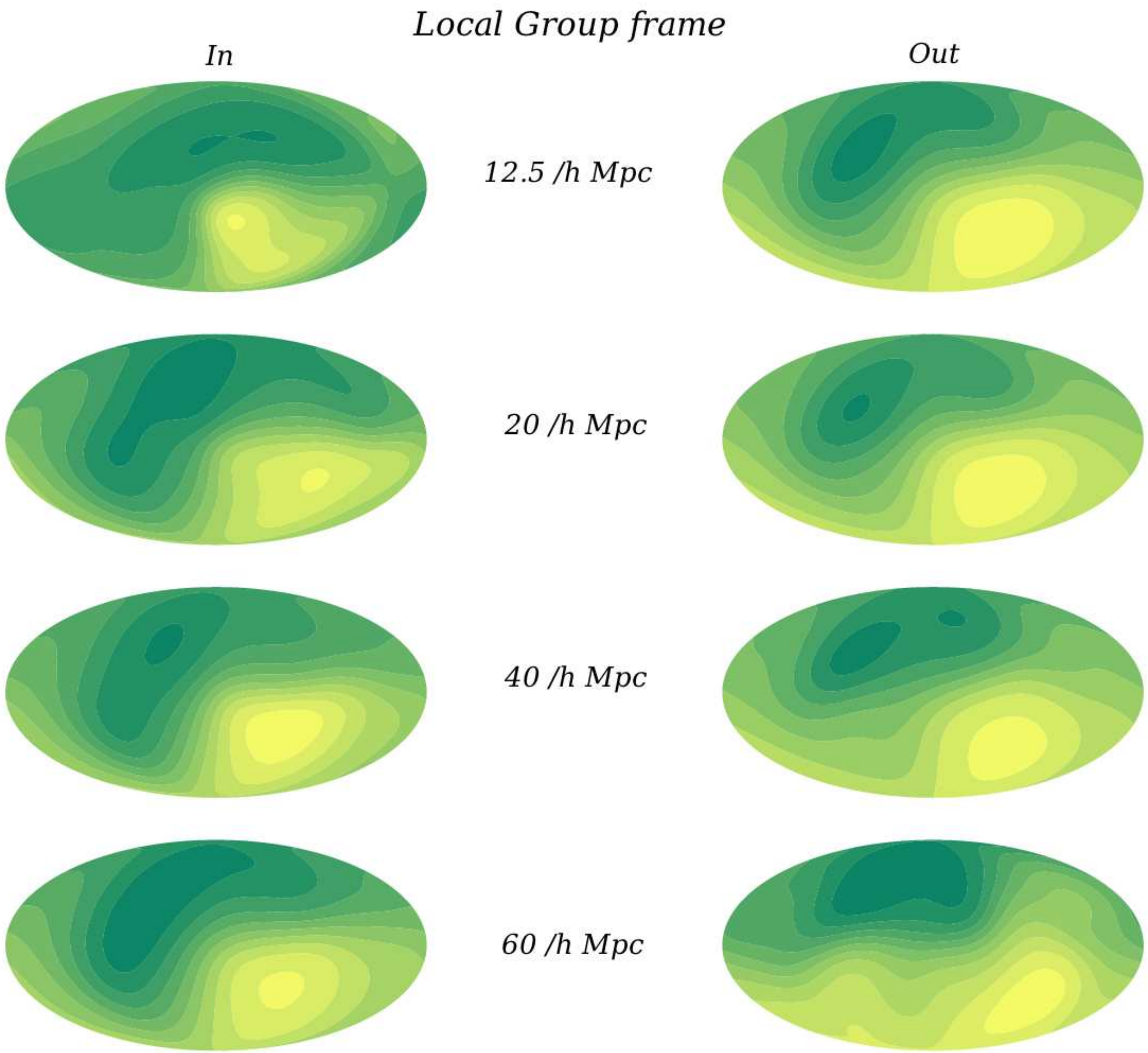
2. Simple dipole

$$\frac{cz}{r} = H_0 + b\cos\phi$$

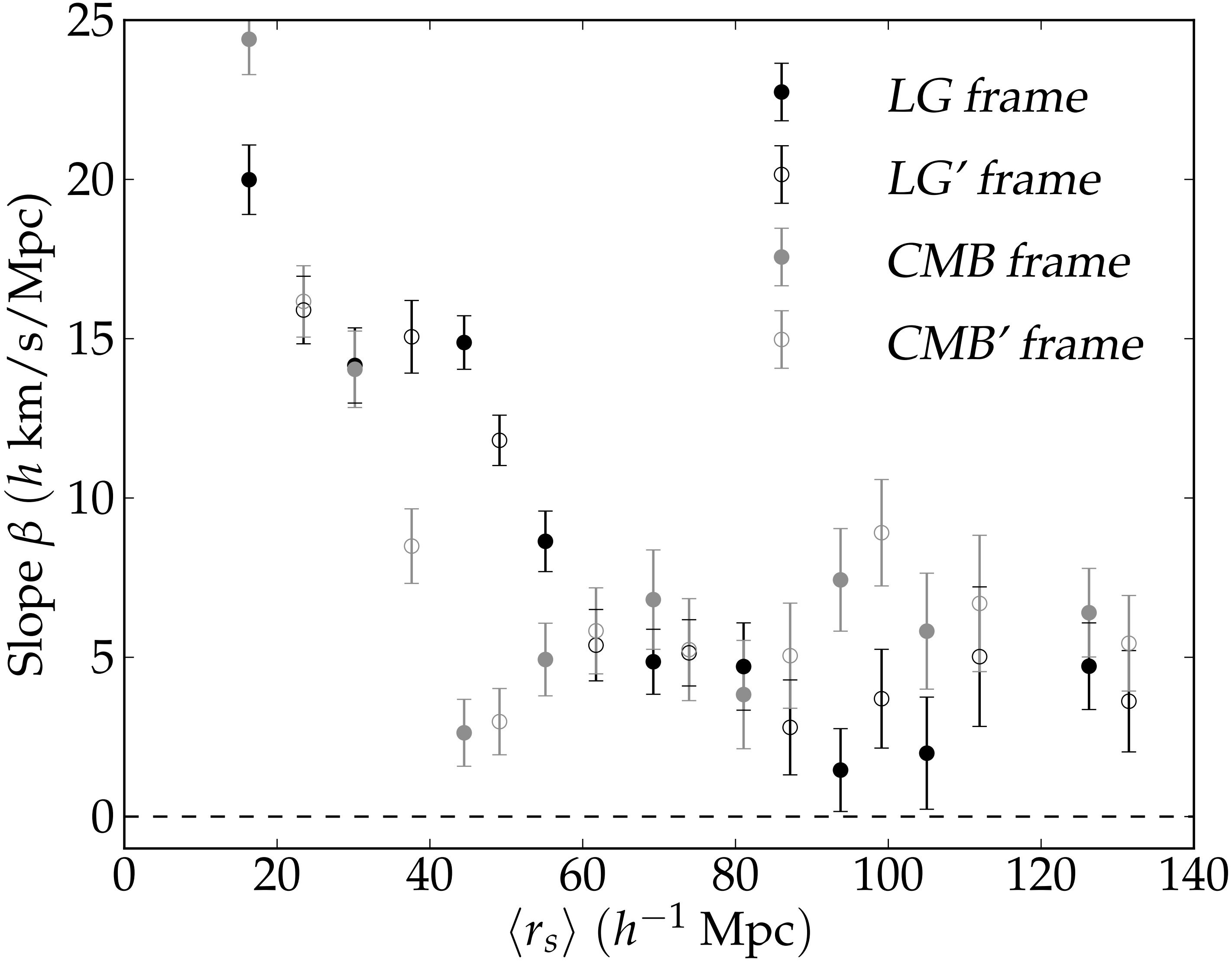
Hubble variance: CMB frame



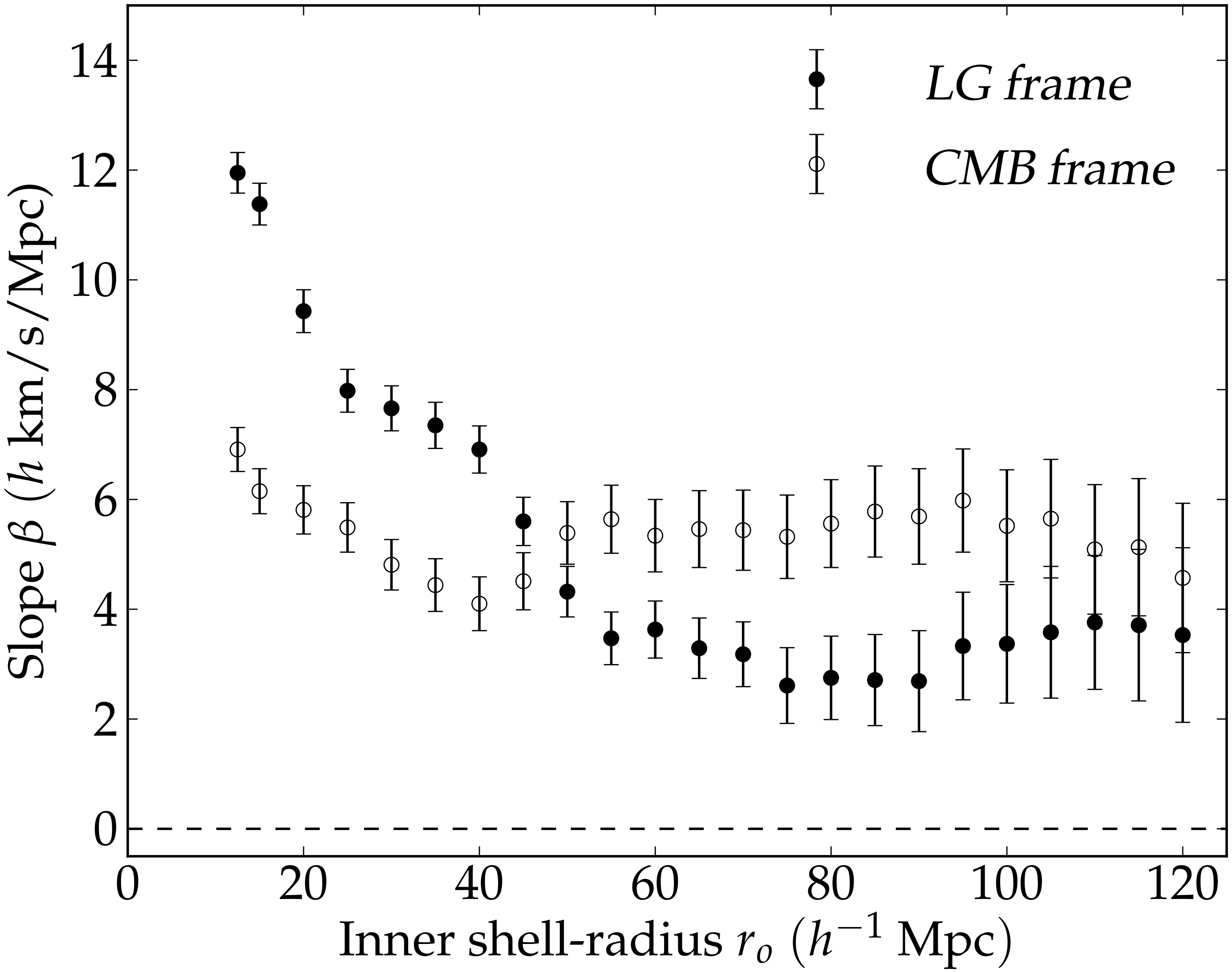
Hubble variance: LG frame



Value of β in $\frac{cz}{r} = H_0 + \beta \cos \phi$

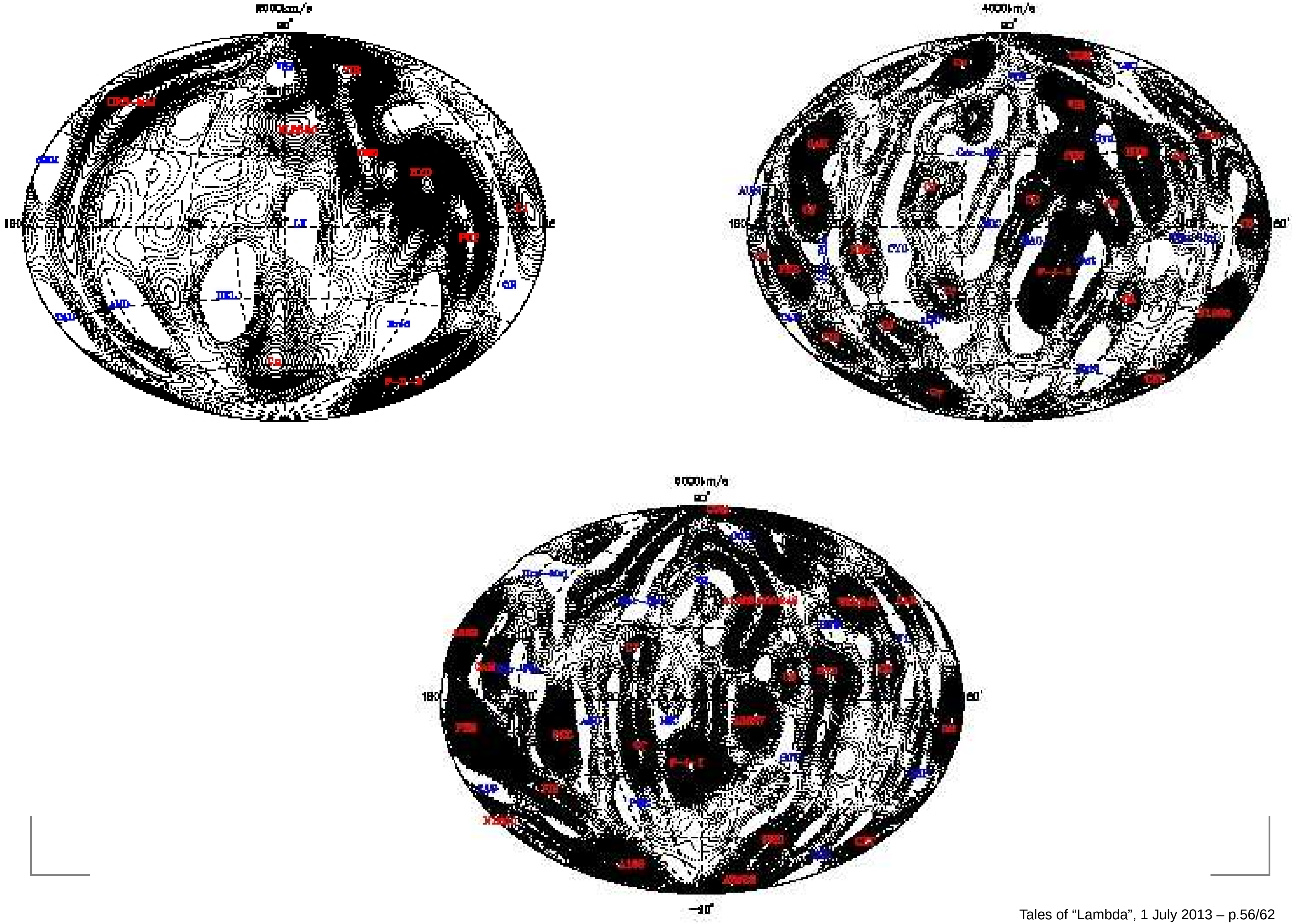


Value of β in $\frac{cz}{r} = H_0 + \beta \cos \phi$

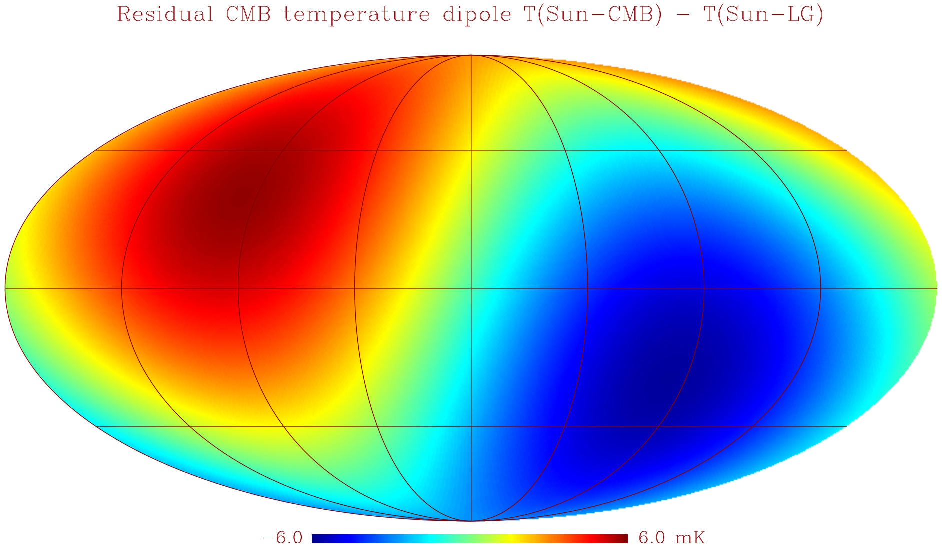


Result: arXiv:1201.5371

- CMB dipole usually interpreted as result of a boost w.r.t. *cosmic rest frame*, composed of our motion w.r.t. barycentre of Local Group plus a motion of the Local Group of 635 km s^{-1} towards ? Great Attractor? Shapley Concentration ? ??
- But Shapley Supercluster, is at $\gtrsim 138\,h^{-1}$Mpc > Scale of Statistical Homogeneity
- We find Hubble flow is significantly more uniform in rest frame of LG rather than standard "rest frame of CMB"
- Suggests LG is not moving at 635 km s^{-1}; but $\exists$ 0.5% foreground anisotropy in distance-redshift relation from foreground density gradient on $\lesssim 65\,h^{-1}$Mpc scales



Correlation with residual CMB dipole



- Digitize skymaps with HEALPIX, compute

$$\rho_{HT} = \frac{\sqrt{N_p}\,\sum_\alpha \bar{\sigma}_\alpha^{-2}(H_\alpha - \bar{H})(T_\alpha - \bar{T})}{\sqrt{\left[\sum_\alpha \bar{\sigma}_\alpha^{-2}\right]\left[\sum_\alpha \bar{\sigma}_\alpha^{-2}(H_\alpha - \bar{H})^2\right]\left[\sum_\alpha (T_\alpha - \bar{T})^2\right]}}$$

- $\rho_{HT} = -0.92$, (almost unchanged for $15^\circ < \sigma_\theta < 40^\circ$)
- Alternatively, t-test on raw data: null hypothesis that maps uncorrelated is rejected at 24.4σ.

Redshift-distance anisotropy

- As long as $T \propto 1/a$, where $a_0/a = 1 + z$ *for some appropriate average*, not necessarily FLRW, then small change, δz, in the redshift of the surface of photon decoupling – due to foreground structures – will induce a CMB temperature increment $T = T_0 + \delta T$, with

$$\frac{\delta T}{T_0} = \frac{-\delta z}{1 + z_{\rm dec}}$$

- With $z_{\rm dec} = 1089$, $\delta T = \pm(5.77 \pm 0.36)$ mK represents an increment $\delta z = \mp(2.31 \pm 0.15)$ to last scattering
- **Proposal**: rather than originating in a LG boost the ± 5.77 mK dipole is due to a small anisotropy in the distance-redshift relation on scales $\lesssim 65\, h^{-1}$Mpc.

Redshift-distance anisotropy

- For spatially flat ΛCDM

$$D = \frac{c}{H_0} \int_1^{1+z_{\rm dec}} \frac{{\rm d}x}{\sqrt{\Omega_{\Lambda 0} + \Omega_{M0} x^3 + \Omega_{R0} x^4}}$$

For standard values $\Omega_{R0} = 4.15 h^{-2} \times 10^{-5}$, $h = 0.72$

- $\Omega_{M0} = 0.25$, find $\delta D = \mp(0.33 \pm 0.02)\, h^{-1}$Mpc;
- $\Omega_{M0} = 0.30$, find $\delta D = \mp(0.32 \pm 0.02)\, h^{-1}$Mpc;
- timescape model similar.
- Assuming that the redshift-distance relation anisotropy is due to foreground structures within $65\, h^{-1}$Mpc then $\pm 0.35\, h^{-1}$Mpc represents a ± 0.5% effect

Questions, consequences. . .

- Ray tracing of CMB sky seen by off-centre observer in LTB void gives $|a_{10}| \gg |a_{20}| \gg |a_{30}|$ (Alnes and Amarzguioui 2006). When applied to realistic parameters for our setup the effective "peculiar velocity" of 635 km s^{-1} is matched with $a_{20}/a_{10} \lesssim$ 1%. Ray-tracing studies are in progress (K. Bolejko)
- Strong evidence for a non-kinematic dipole in radio galaxy data: Rubart and Schwarz, arXiv:1301.5559
- Evidence for Doppler boosting of CMB sky seen at small angles in Planck data, but changes significantly when large angle multipoles included: arXiv:1303.5087
- Clearly will a significant non-kinematic component to the CMB dipole will impact large angle anomalies

Next steps: Modified Geometry

- Characterization of statistical geometry, quasilocal kinetic energy
- Quasilocal conservation laws (Epp, Mann & McGrath 2012) formalism relevant for application to bounding spheres of finite infinity regions
- Shape Dynamics (Gomes, Gryb and Koslowski 2011,2012,. . .) – a CMC (Constant Mean extrinsic Curvature) formulation of gravity with 3d conformal invariance – might be adapted for statistical geometry
- Ultimately potential links to quantum cosmology, Jacobson hydrodynamic description, holographic cosmology etc

Conclusion

- Apparent cosmic acceleration can be understood purely within general relativity; by (i) treating geometry of universe more realistically; (ii) understanding fundamental aspects of general relativity which have not been fully explored – *quasi–local gravitational energy*, of *gradients* in kinetic energy of expansion etc.
- "Timescape" model gives good fit to major independent tests of ΛCDM with new perspectives on many puzzles – e.g., primordial lithium abundance anomaly; local/global differences in H_0; large angle CMB anomalies
- Many tests can be done to distinguish from ΛCDM. Must be careful not to assume Friedmann equation in any data reduction.