Symmetry-improved 2PI approach to the Goldstone-boson IR problem of the SM effective potential

Daniele Teresi

daniele.teresi@ulb.ac.be

Service de Physique Théorique, Université Libre de Bruxelles, Belgium

based on

A. Pilaftsis and D. Teresi, arXiv:1511.05347 [hep-ph].

and

A. Pilaftsis and D. Teresi, J. Phys. Conf. Ser. 631 (2015) 1, 012008. arXiv:1502.07986 [hep-ph].

5TH UK-QFT meeting, Nottingham, 15 Jan 2016

Introduction

Resummations needed in many areas of QFT, e.g.

- equilibrium and non-equilibrium thermal QFT
- IR divergences, extrapolations to high energies? (this talk)

Cornwall-Jackiw-Tomboulis 2PI effective action [Cornwall, Jackiw, Tomboulis, 1974]

- Implicitly resums infinite sets of diagrams
- Thermal masses, finite width effects, effective potential, ...

Truncations of CJT do not encode symmetries properly
[Baym, Grinstein, 1977; Amelino-Camelia, 1997]

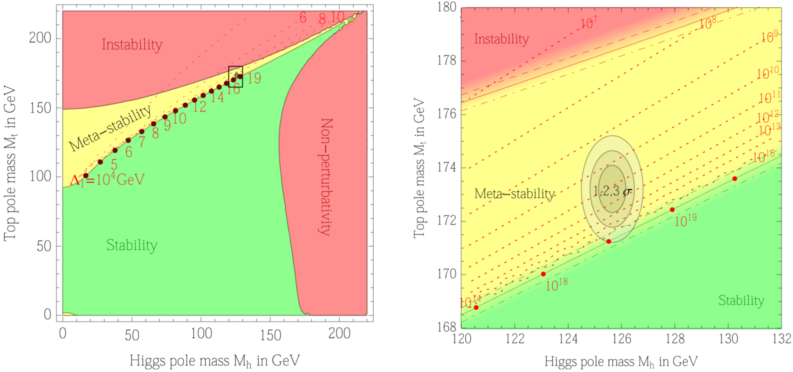
Standard Ward Identities not satisfied $\Longrightarrow$ massive Goldstone bosons

Symmetry-improved CJT effective action

Pilaftsis and Teresi, Nucl. Phys. B 874 (2013) 2, 594. arXiv:1305.3221 [hep-ph]

Further developments:
[Mao, 2013; Brown, Whittingham, 2015; Pilaftsis, Teresi, 2015; Garbrecht, Millington 2015]

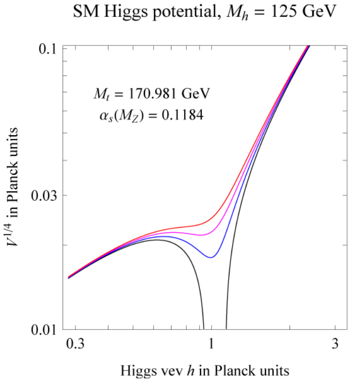
Metastability of the SM at NNLO

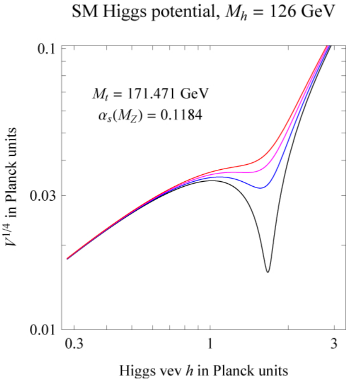


[Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, 2013]

Uncertainties of the SM potential at NNLO

Higgs Potential versus Variations in Top Mass by 0.1 MeV

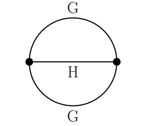




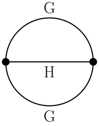
[Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia, 2012]

The Goldstone-boson catastrophe in the Standard Model [Martin, 2013]

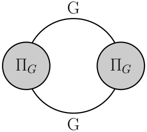
Contributions to the SM effective potential:



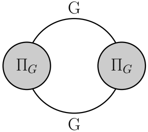
$\sim \quad m_G^2 \log m_G^2$ ✓

$\frac{d}{d\phi}$ 

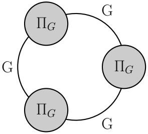
$\sim \quad \log m_G^2$ ✗



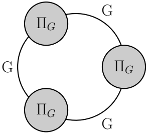
$\sim \quad \log m_G^2$ ✗

$\frac{d}{d\phi}$ 

$\sim \quad \frac{1}{m_G^2}$ ✗



$\sim \quad \frac{1}{m_G^2}$ ✗

$\frac{d}{d\phi}$ 

$\sim \quad \left(\frac{1}{m_G^2}\right)^2$ ✗

$$m_G^2 = \lambda\phi^2 - m^2 \qquad M_G^2 = m_G^2 + \Pi_G^{(1)}|_{k=0} + \ldots = 0 \quad \text{at } \phi = v$$

Partial-resummation approach

Why do we care?

- conceptually: $V_{\rm eff}$ should be well-defined for all ϕ
  unphysical instability at $\phi = v$
- quantitatively: IR div. at $\phi \neq v$ can have a large impact at $\phi = v$, and therefore to the extrapolated high-energy $V_{\rm eff}$

Partial resummation [Martin, 2014; Elias-Miro, Espinosa, Kostandin, 2014]

- approximately resum ring diagrams:
  $$V^{(1)} \to \frac{3}{4(16\pi^2)}\left(m_G^2 + \Pi_G(0)\right)^2\left[\log\left(\frac{m_G^2+\Pi_G(0)}{\mu^2}\right) - \frac{3}{2}\right]$$
- $\frac{d}{d\phi}V^{(1)}$ still divergent, because so is $\frac{d}{d\,m_G^2}\Pi_G(0)$
  $$\Longrightarrow \quad \Pi_G(0) \to \Pi_g \equiv \Pi_G(0) - \frac{3\lambda}{(16\pi^2)}\,m_G^2\left(\log(m_G^2/\mu^2) - 1\right)$$
- subtract double-counted diagrams

Partial-resummation approach

Why do we care?

- conceptually: V_{eff} should be well-defined for all ϕ
  unphysical instability at $\phi = v$
- quantitatively: IR div. at $\phi \neq v$ can have a large impact at $\phi = v$, and therefore to the extrapolated high-energy V_{eff}

Partial resummation [Martin, 2014; Elias-Miro, Espinosa, Kostandin, 2014]

- approximately resum ring diagrams:

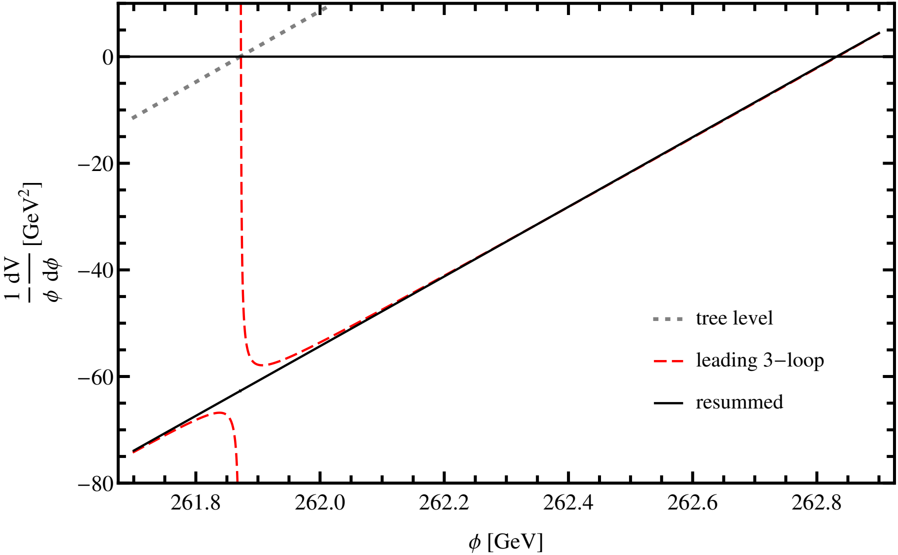
$$V^{(1)} \to \frac{3}{4(16\pi^2)}\left(m_G^2 + \Pi_G(0)\right)^2 \left[\log\left(\frac{m_G^2 + \Pi_G(0)}{\mu^2}\right) - \frac{3}{2}\right]$$

- $\dfrac{d}{d\phi}V^{(1)}$ still divergent, because so is $\dfrac{d}{d\,m_G^2}\Pi_G(0)$

$$\Longrightarrow \quad \Pi_G(0) \to \Pi_g \equiv \Pi_G(0) - \frac{3\lambda}{(16\pi^2)}\, m_G^2\left(\log(m_G^2/\mu^2) - 1\right)$$

- subtract double-counted diagrams

Partial-resummation approach: results



2PI CJT effective action

Generalize the standard 1PI Effective Action:

- local and bi-local sources $J(x)$, $K(x,y)$:
$$Z[J,K] \;=\; \int \mathcal{D}\phi \, e^{i(S[\phi] + J_x\phi_x + \frac{1}{2}K_{xy}\phi_x\phi_y)} \;=\; e^{iW[J,K]}$$
- $\frac{\delta W[J,K]}{\delta J_x} = \langle \widehat{\phi}_x \rangle \equiv \phi_x \qquad 2\frac{\delta W[J,K]}{\delta K_{xy}} = \langle \widehat{\phi}_x \widehat{\phi}_y \rangle \equiv \phi_x\phi_y + i\Delta_{xy}$
- double Legendre transform:
$$\Gamma[\phi,\Delta] = W[J,K] - J_x\phi_x - \tfrac{1}{2}K_{xy}(i\Delta_{xy} + \phi_x\phi_y)$$

Self-consistent Equations of Motion

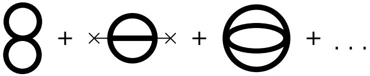
$$\frac{\delta\Gamma[\phi,\Delta]}{\delta\phi_x} = -J_x - K_{xy}\,\phi_y \qquad\qquad \frac{\delta\Gamma[\phi,\Delta]}{\delta\Delta_{xy}} = -\frac{i}{2}\,K_{xy}$$

physical solution: extremum of $\Gamma[\phi,\Delta]$

2PI effective action - explicit form

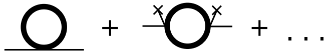
$$\Gamma[\phi,\Delta] = S[\phi] + \frac{i}{2}\,\mathrm{Tr}\ln\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,\Delta{\Delta^{0}}^{-1} - i\,\Gamma^{(2)}_{2\mathrm{PI}}[\phi,\Delta]$$

$\Gamma^{(2)}_{2\mathrm{PI}}[\phi,\Delta] =$

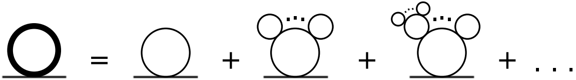


Equations of Motion

- $\dfrac{\delta\Gamma}{\delta\phi} = 0$
- $\dfrac{\delta\Gamma}{\delta\Delta} = 0 \;\rightarrow$ SDE: $\Delta^{-1} = {\Delta^{0}}^{-1} +$

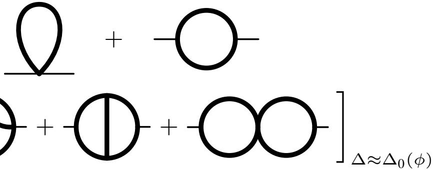


Hartree-Fock:



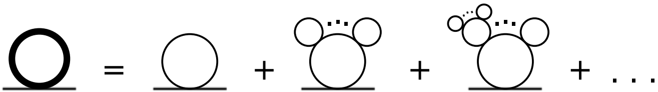
Symmetry-improved 2PI approach

- More complete resummation:
  - first-principle approach
  - it takes into account the momentum-dependence of self-energy insertions
  - more topologies
  - no ad-hoc subtraction of $m_G^2 \log m_G^2$ contributions in Π_G
- $-\dfrac{1}{\phi}\dfrac{d\widetilde{V}_{\rm eff}}{d\phi} \equiv \Delta_G^{-1}(\phi)\big|_{k=0} \supset$

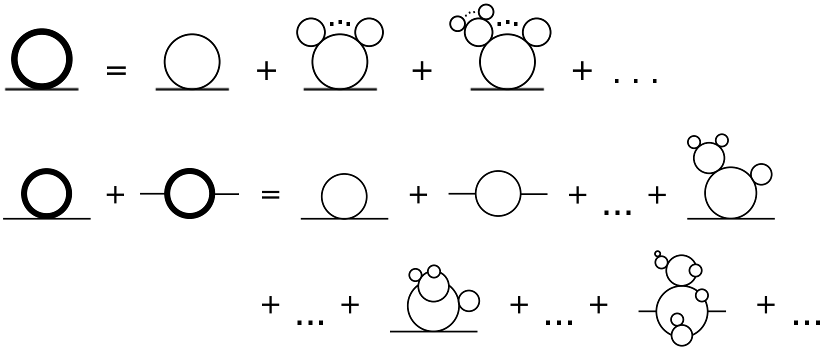


- No IR divergences: the would-be divergent self-energies are 2PR
- Correct threshold properties:
  - at the minimum Goldstone bosons really massless inside loops
  - IR divergences really resummed, not hidden in truncation artifacts
  - no unphysical instabilities

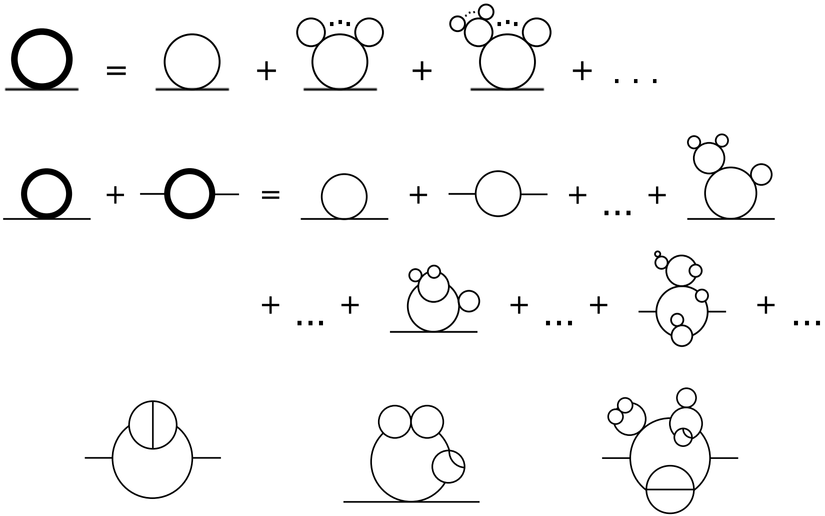
2PI resummation



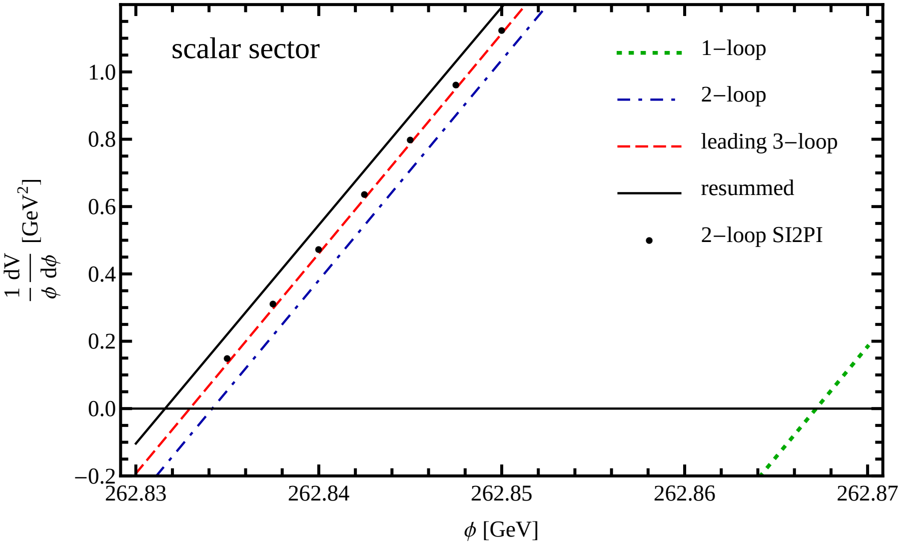
2PI resummation



2PI resummation



Symmetry-improved 2PI: scalar sector



Global $SU(2)_L \times U(1)_Y$ model

Global $SU(2)_L \times U(1)_Y$ model with:

- Higgs doublet $\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(\phi + H + i\,G^0) \end{pmatrix}$
- 3rd-generation quark doublet $Q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$
- top-quark singlet t_R
- top-quark Yukawa Lagrangian $-\mathcal{L}_{\mathrm{Y}} \;=\; h_t\,\varepsilon^{ab}\,\overline{Q}_{L,a}\Phi_b^\dagger\,t_R \;+\; \mathrm{H.c.}$

Include chiral fermions semi-perturbatively by performing

double-Legendre-transform w.r.t. Φ, single-Legendre transform w.r.t. Q_L, t_R:

Global $SU(2)_L \times U(1)_Y$ model

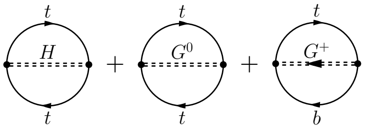
Global $SU(2)_L \times U(1)_Y$ model with:

- Higgs doublet $\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(\phi + H + i\,G^0) \end{pmatrix}$
- 3rd-generation quark doublet $Q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$
- top-quark singlet t_R
- top-quark Yukawa Lagrangian $-\mathcal{L}_{\rm Y} \;=\; h_t\,\varepsilon^{ab}\,\overline{Q}_{L,a}\Phi^\dagger_b\,t_R \;+\; \text{H.c.}$

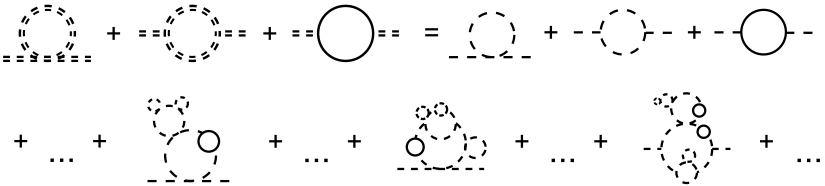
Include chiral fermions semi-perturbatively by performing double-Legendre-transform w.r.t. Φ, single-Legendre transform w.r.t. Q_L, t_R:

$$\Gamma^{(2)}[\phi, \Delta^H, \Delta^G, \Delta^+] \;=\; \Gamma^{(2)}_{\rm scalar}[\phi, \Delta^H, \Delta^G, \Delta^+] \;+\; 3\,i\;\mathrm{Tr}\ln S^{\alpha\,(0)}[\phi]$$

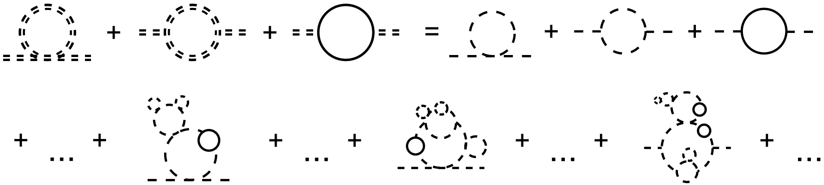
$$-\,i\,\Big\{ \quad + \quad + \quad \Big\}$$

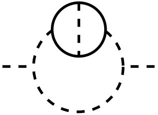


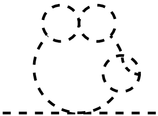
2PI resummation including fermions semi-perturbatively

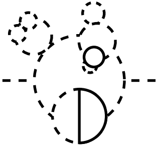


2PI resummation including fermions semi-perturbatively









Equations of motion:

$$\Delta^{-1,H}(k) = (1+\delta Z_1)\,k^2 + (3\lambda + \delta\lambda_1^A + 2\delta\lambda_1^B)\,\phi^2 - (m^2 + \delta m_1^2)$$
$$+ (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\,\mathcal{T}_H + (\lambda + \delta\lambda_2^A)\,\mathcal{T}_G + 2\,(\lambda + \delta\lambda_2^A)\,\mathcal{T}_+$$
$$- 18\lambda^2\phi^2\,\mathcal{I}_{HH}(k) - 2\lambda^2\phi^2\,\mathcal{I}_{GG}(k) - 4\lambda^2\phi^2\,\mathcal{I}_{++}(k) + \boldsymbol{\Sigma}_H(k)$$

$$\Delta^{-1,G}(k) = (1+\delta Z_1)\,k^2 + (\lambda + \delta\lambda_1^A)\,\phi^2 - (m^2 + \delta m_1^2) + (\lambda + \delta\lambda_2^A)\,\mathcal{T}_H$$
$$+ (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\,\mathcal{T}_G + 2\,(\lambda + \delta\lambda_2^A)\,\mathcal{T}_+ - 4\lambda^2\phi^2\,\mathcal{I}_{HG}(k) + \boldsymbol{\Sigma}_G(k)$$
$$\Delta^{-1,+}(k) = (1+\delta Z_1)\,k^2 + (\lambda + \delta\lambda_1^A + \boldsymbol{\delta\lambda_1^{cb}})\,\phi^2 - (m^2 + \delta m_1^2) + (\lambda + \delta\lambda_2^A)\,\mathcal{T}_H$$
$$+ (\lambda + \delta\lambda_2^A)\,\mathcal{T}_G + 2\,(2\lambda + \delta\lambda_2^A + \delta\lambda_2^B)\,\mathcal{T}_+ - 4\lambda^2\phi^2\,\mathcal{I}_{H+}(k) + \boldsymbol{\Sigma}_+(k)$$

Additional counterterm needed

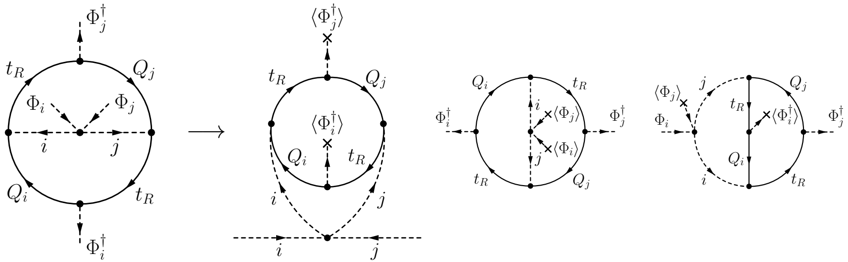
custodially-breaking counterterm $\delta\lambda_1^{\rm cb}$

Custodially-violating artifacts

- At zero momentum, $\Gamma_{1\text{PI}}$ depends only on $\Phi^\dagger\Phi = \frac{1}{2}(\phi^2 + H^2 + G^0 G^0) + G^- G^+$
- As in custodial $SU(2)_L \times SU(2)_R \sim O(4)$
- $SU(2)_L \times SU(2)_R$ is not a symmetry of the theory, hardly broken by Yukawa h_t
- In the 1PI formalism, zero-momentum custodially-violating terms must cancel:
- In 1-loop 2PI EoMs, only first self-energy is generated $\rightarrow$ custudially-violating spurious terms
- Quantitatively tiny effects

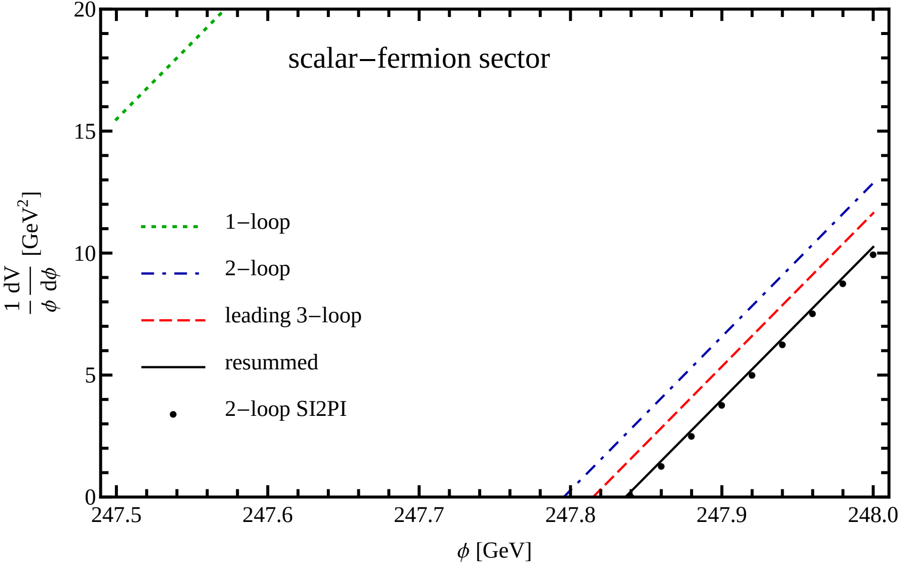
Custodially-violating artifacts

- At zero momentum, Γ_{1PI} depends only on $\Phi^\dagger\Phi = \frac{1}{2}(\phi^2 + H^2 + G^0G^0) + G^-G^+$
- As in custodial $SU(2)_L \times SU(2)_R \sim O(4)$
- $SU(2)_L \times SU(2)_R$ is not a symmetry of the theory, hardly broken by Yukawa h_t
- In the 1PI formalism, zero-momentum custodially-violating terms must cancel:

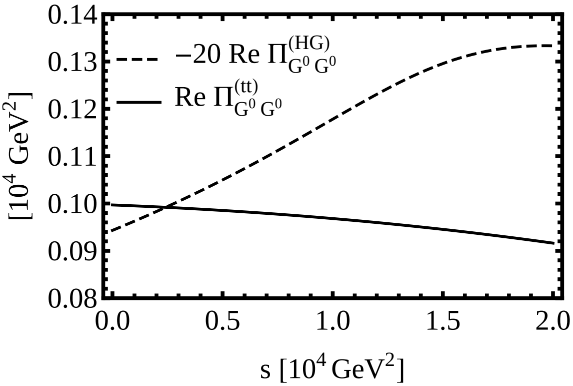


- In 1-loop 2PI EoMs, only first self-energy is generated $\rightarrow$ custudially-violating spurious terms
- Quantitatively tiny effects

Symmetry-improved 2PI: scalar-fermion sector

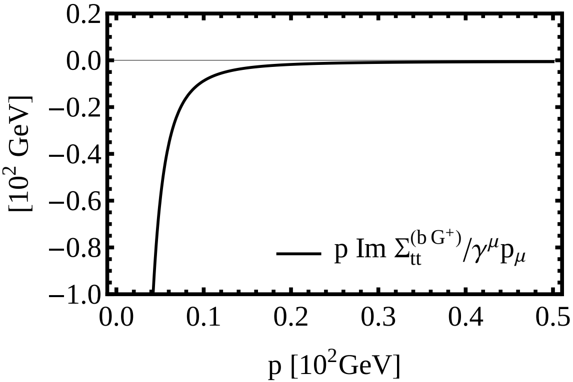


Why is the agreement better now?



The end of the story?

We have treated the fermion propagators perturbatively, but they get significantly dressed in the IR:



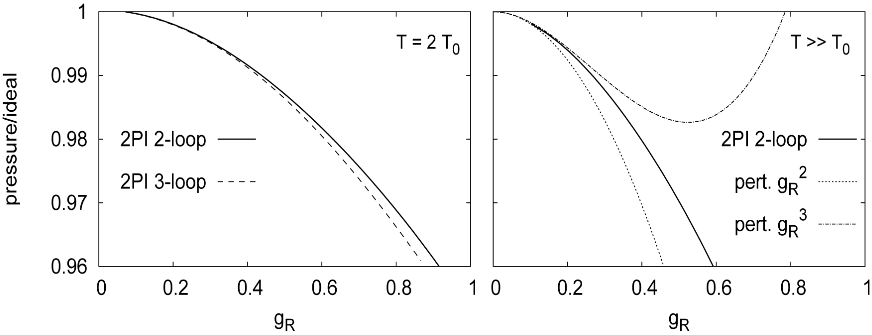
Warning: Can the fully-2PI inclusion of fermions alter our conclusions?

Conclusions

- Resummations needed in many areas of QFT
- The CJT formalism is a powerful, *first-principle*, theoretical tool to perform *automatically* “fractal” resummations
- To address models with global symmetries:
  Symmetry-improved 2PI [Pilaftsis and Teresi, 1305.3221]
- Good field-theoretical properties $\Longrightarrow$ appropriate to study the IR divergences of the SM effective potential due to Goldstone bosons
  [Pilaftsis and Teresi, 1511.05347, 1502.07986]
  - Inclusion of chiral fermions in a semi-perturbative way
  - Renormalization taking into account spurious custodially-violating effects
  - Existing approximate method in the literature only partially accurate
    - quite good for fermion loops (top loop important in SM)
    - inaccurate for scalar loops (stop loop important in MSSM, ...)
- Future directions: fully-2PI inclusion of chiral fermions, high-energy behaviour of the effective potential, gauge symmetries, ...

Backup slides

CJT significantly improves convergence



[Berges, Borsanyi, Reinosa, Serreau, 2005]

Goldstone theorem

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi^i)^2 + \frac{m^2}{2}(\varphi^i)^2 - \frac{\lambda}{4}[(\varphi^i)^2]^2 \qquad \phi^1 = v + H,\ \phi^2 = G$$

Standard 1PI Ward Identities

$$\Gamma[\mathcal{O}\phi] = \Gamma[\phi] \quad \Longrightarrow \quad \frac{\delta\Gamma}{\delta\phi^i_x} T^a_{ij}\phi^j_x = 0$$

Goldstone theorem: $v \int_x \frac{\delta^2\Gamma}{\delta G_x \delta G_y} = 0$ at the extremum of $\Gamma[\phi]$

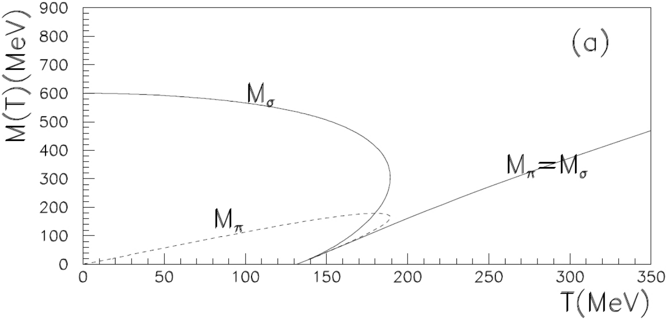
2PI Ward Identity

$$\Gamma[\mathcal{O}\phi, \mathcal{O}\Delta\mathcal{O}^T] = \Gamma[\phi, \Delta]$$

$$\Longrightarrow \quad v \int_x \frac{\delta^2\Gamma}{\delta G_x \delta G_y} + \frac{\delta^2\Gamma}{\delta G_y \delta\Delta^{GH}_{xz}}(\Delta^H_{xz} - \Delta^G_{xz}) = 0$$

No Goldstone theorem for the truncated 2PI effective action

Massive Goldstone bosons



[Petropoulos, 1998]

Symmetry-improved equations of motion

Constraint: $v \int_x \Delta^{G\,-1}_{xy} = 0$

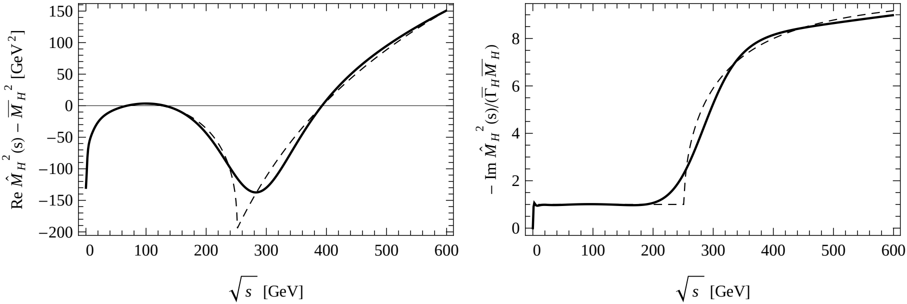
Introduce Lagrange multiplier L:

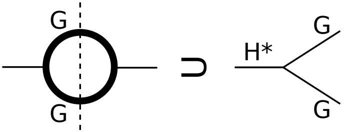
$$\frac{\partial \Gamma_{\rm tr}[v, \Delta]}{\partial v} = L\, m^2$$
$$\frac{\delta \Gamma_{\rm tr}[v, \Delta]}{\delta \Delta_i(k)} = 0$$
$$v\, \Delta_G^{-1}\big|_{k=0} = 0$$

A reducible singularity has been IR regulated

Effectively $\frac{\delta \Gamma_{\rm tr}[\phi, \Delta]}{\delta \phi} = 0$ has been replaced by $\underbrace{\frac{\delta \Gamma_{\rm tr}[\phi]}{\delta \phi}}_{= v\, \Delta_G^{-1}|_{k=0}} = 0$

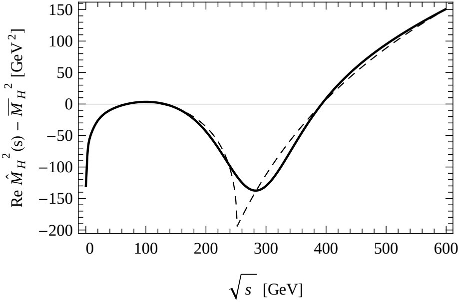
Higgs self-energy in symmetry-improved 2PI

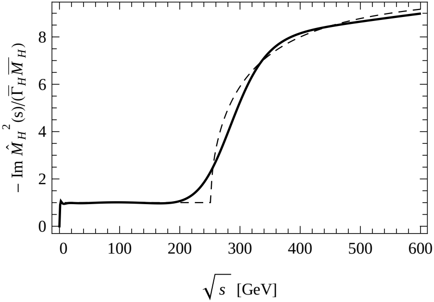


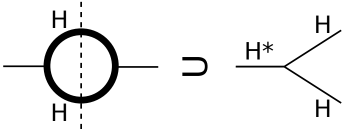


for $\sqrt{s} > 0$

Higgs self-energy in symmetry-improved 2PI

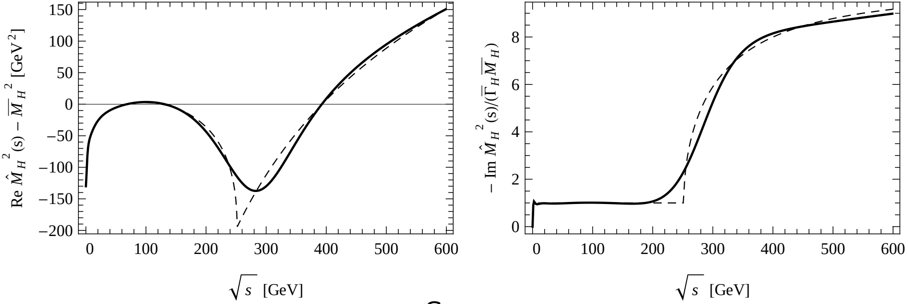


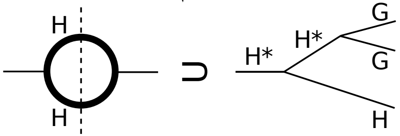




for $\sqrt{s} > 2\,\overline{M}_H$

Higgs self-energy in symmetry-improved 2PI





also for $\sqrt{s} \lesssim 2\,\overline{M}_H$