Constraining the effective action with external sources

based on work (1509.07847) in collaboration with Björn Garbrecht, Technische Universität München

Peter Millington, University of Nottingham

UK-QFT V; Friday, 15th January, 2016; University of Nottingham

Outline

- Motivation
- **Method of external sources**
  [Garbrecht, PM, 1509.07847]
- Example applications:
  - **Cornwall-Jackiw-Tomboulis 2PI** effective action
    [Cornwall, Jackiw, Tomboulis, PRD 10 (1974) 2428]
  - **Coppens-Verschelde 2PPI** effective action
    [Coppens, Verschelde, PLB 287 (1992) 133; 295 (1992) 83; Z. Phys. C 57 (1993) 349; 58 (1993) 319]
  - **Pilaftsis-Teresi symmetry-improved** effective action
    [**see talk by Daniele Teresi**; Pilaftsis, Teresi, NPB 874 (2013) 594; JPCS 631 (2015) 012008; 1511.05347]
- Concluding remarks

ab initio motivation

- How should we calculate the effective action when the **extremal classical** and **quantum paths** are non-perturbatively far away from one-another?

- For instance, what happens when the **classical path** corresponds to a **stable configuration**, but the **quantum path** does **not**?

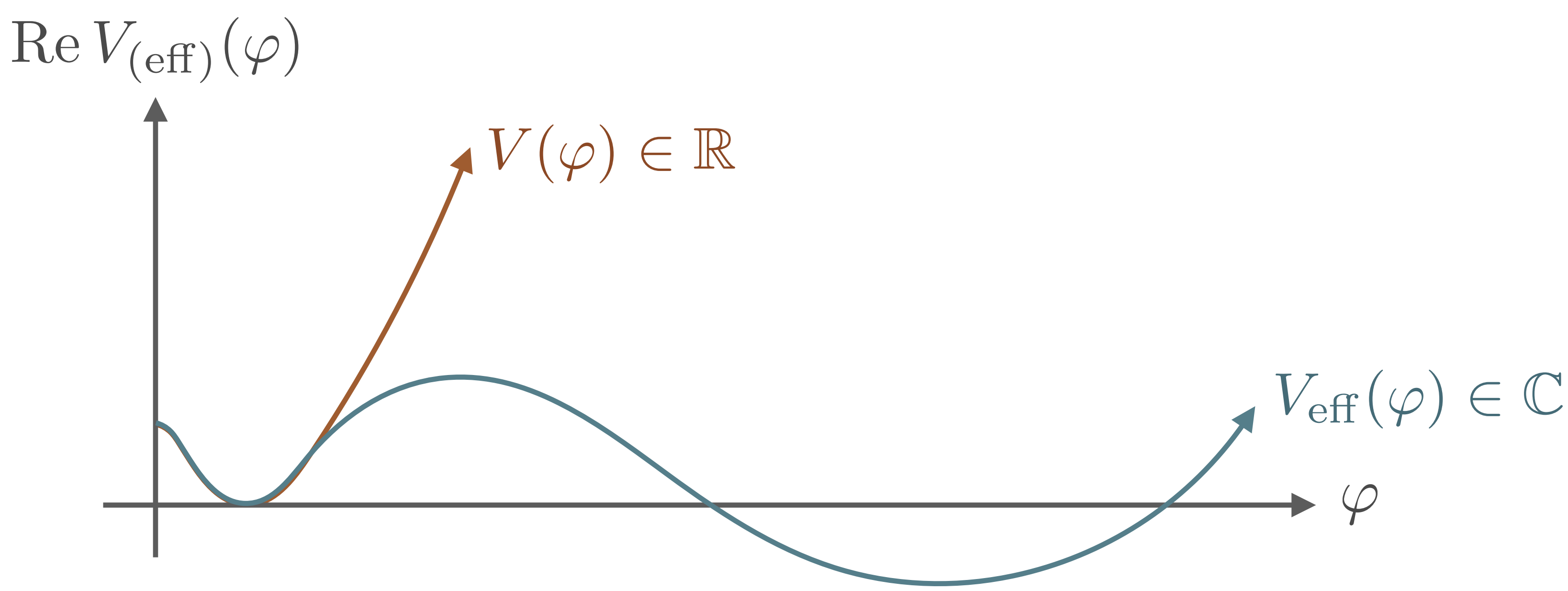
- This is the case for the **decay of meta-stable vacua** when a lower, global minimum emerges by virtue of radiative corrections.
  [Garbrecht, PM, PRD 92 (2015) 125022; cf. Weinberg, PRD 47 (1993) 4614;
  cf. talks by Carlos Tamarit and Stephen Stopyra]

- Can we perform the **saddle-point approximation** of the the **path integral** itself along the **quantum path** directly?
  [Garbrecht, PM, PRD 91 (2015) 105021 for the case of the 1PI effective action;
  cf. talk by Dimitri Skliros; Skliros, 1510.02549; Ellis, Mavromatos, Skliros 1512.02604.]

ab initio motivation

- This is relevant to determining the stability of the **electroweak vacuum** of the SM.
  [Cabibbo, Maiani, Parisi, Petronzio, NPB 158 (1979) 295; Sher, Phys. Rept. 179 (1989) 273; PLB 317 (1993) 159; Isidori, Ridolfi, Strumia, NPB 609 (2001) 387; **see also talk by Stephen Stopyra**.]

- State-of-the-art calculations are **indicative** of **metastability**.
  [Elias-Miró, Espinosa, Giudice, Isidori, Riotto, Strumia, PLB 709 (2012) 22; Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia, JHEP 1208 (2012) 098; Alekhin, Djouadi; Moch, PLB 716 (2012) 214; Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, JHEP 1312 (2013) 089; Bednyakov, Kniehl, Pikelner, Veretin, 1507.08833; Di Luzio, Isidori, Ridolfi, 1509.05028; **see also talk by Stephen Stopyra**.]



- Can we interpret the **imaginary-part** of the **perturbatively-calculated** effective potential in terms of the **decay rate** of an initially homogeneous false-vacuum state?
  [Weinberg, Wu, PRD 36 (1987) 2474; **cf. talk by Jean Alexandre.**]

ex post motivation

- Can we move easily between **different realisations** of the effective action (for there are many more than we will mention in this talk)?
  [We have seen two so far in the **talks by Dimitri Skliros** and **Daniele Teresi**.]

- How can we ensure that **truncations** of the effective action respect the **symmetries** that the full effective action does?
  [for an introduction to this problem in QED: Reinosa and J. Serreau, JHEP 0711 (2007) 09;
  for an introduction to this problem for global symmetries: **talk by Daniele Teresi** as well as
  Pilaftsis, Teresi, NPB 874 (2013) 594; JPCS 631 (2015) 012008; 1511.05347 and references therein.**]**

  In other words, how can we ensure that our truncations respect the **Ward(-Takahashi) identities** in the case of global (local, Abelian) symmetries or the **Slavnov-Taylor identities** for local, non-Abelian symmetries?

- How can we ensure **gauge independence** of quantities that depend on the value of the effective action away from the vacuum?
  [**cf. talk by Carlos Tamarit;** Plascencia, Tamarit, 1510.07613**.**]

Method of external sources

- We consider the Euclidean **two-point** effective action (for convenience) — the results, however, are general:

$$\Gamma[\phi,\Delta] = -\hbar\ln Z[\mathcal{J},\mathcal{K}] + \mathcal{J}_x[\phi,\Delta] + \frac{1}{2}\mathcal{K}_{xy}[\phi,\Delta]\big(\phi_x\phi_y + \hbar\Delta_{xy}\big)$$

$$Z[\mathcal{J},\mathcal{K}] = \int[\mathrm{d}\Phi]\exp\bigg[-\frac{1}{\hbar}\bigg(S[\Phi] - \mathcal{J}_x[\phi,\Delta]\Phi_x - \frac{1}{2}\mathcal{K}_{xy}[\phi,\Delta]\Phi_x\Phi_y\bigg)\bigg]$$

where the one- and two-point functions ϕ and Δ are (by definition) **independent** of one-another.

- For concreteness, we consider a **real scalar field** with **quartic self-interactions**:

$$\mathcal{L}_x = \frac{1}{2}\big(\partial_\mu\Phi_x\big)^2 + \frac{1}{2}m^2\Phi_x^2 + \frac{\lambda}{4!}\Phi_x^4$$

- Can we **force** the system along the **extremal quantum path** (or indeed any other path) by choosing suitable **non-vanishing external sources** and, at the same time, obtain a **self-consistent** realisation of the effective action?

- Note that this contrasts the standard realisations of the effective action, where the **physical vacuum limit** is obtained for **vanishing external sources** $\mathcal{J}$ and $\mathcal{K}$.

Why is this not off-the-wall crazy?

1. For **macroscopic systems**, the physical limit is obtained for **non-vanishing external sources**. These encode information about the statistical ensemble from the density matrix.
   [Jordan, PRD 33 (1986) 444; Calzetta, Hu, PRD 35 (1987) 495; 37 (1988) 2878;
   see also PM, Pilaftsis, PRD 88 (2013) 085009; Berges, AIP Conf. Proc. 739 (2005) 3-62.]

2. The two-point effective action is a double **Legendre transform**, with **conjugate** variables

$$(\mathcal{J}_x, \mathcal{K}_{xy}) \longleftrightarrow (\phi_x, \Delta_{xy})$$

   In the same way that the Legendre transform from **Lagrangian** to **Hamiltonian dynamics** sets the (generalised) velocity to be a function of the conjugate momentum, i.e.

$$\dot{q} \equiv \dot{q}(p)$$

   that of the effective action sets

$$\mathcal{J}_x \equiv \mathcal{J}_x[\phi, \Delta] \qquad\qquad \mathcal{K}_{xy} \equiv \mathcal{K}_{xy}[\phi, \Delta]$$

   i.e. non-trivial functionals of ϕ and Δ, such that they remain **independent** of one-another.

$\Rightarrow$ ϕ and Δ **cannot** be the **physical** one- and two-point functions, $\varphi \equiv \varphi[\phi, \Delta]$ and $\mathcal{G} \equiv \mathcal{G}[\phi, \Delta]$ say, which **do** and **must depend** on one-another.

Evaluating the effective action

- **Saddle-point evaluation** of the **path integral** gives the **stationarity condition**

$$\left.\frac{\delta S[\Phi]}{\delta \Phi_x}\right|_{\Phi=\varphi} - \mathcal{J}_x[\phi,\Delta] - \mathcal{K}_{xy}[\phi,\Delta]\varphi_y = 0$$

- By definition of the **Legendre transform**

$$\frac{\delta \Gamma[\phi,\Delta]}{\delta \phi_x} = \mathcal{J}_x[\phi,\Delta] + \mathcal{K}_{xy}[\phi,\Delta]\phi_y \qquad \frac{\delta \Gamma[\phi,\Delta]}{\delta \Delta_{xy}} = \frac{\hbar}{2}\mathcal{K}_{xy}[\phi,\Delta]$$

giving the **quantum equation of motion**

$$\left.\frac{\delta \Gamma[\phi,\Delta]}{\delta \phi_x}\right|_{\varphi,\mathcal{G}} = \frac{\delta S[\varphi]}{\delta \varphi_x} + \hbar\left.\frac{\delta \Gamma_1[\phi,\Delta]}{\delta \phi_x}\right|_{\varphi,\mathcal{G}} + \mathcal{O}(\hbar^2) = 0$$

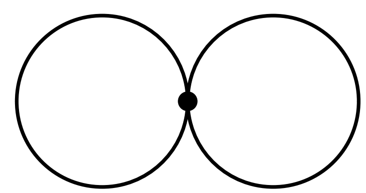
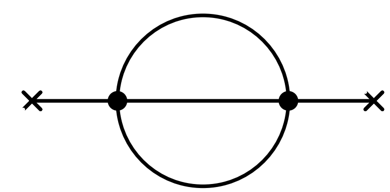
- Combining the two gives the **consistency relation**

$$\frac{\delta S[\varphi]}{\delta \varphi_x} = \mathcal{J}_x[\phi,\Delta] + \mathcal{K}_{xy}[\phi,\Delta]\varphi_y = -\hbar\left.\frac{\delta \Gamma_1[\phi,\Delta]}{\delta \phi_x}\right|_{\varphi,\mathcal{G}} - \mathcal{O}(\hbar^2)$$

Evaluating the effective action

- Eliminate ϕ, Δ, $\mathcal{J}$ and $\mathcal{K}$ in favour of the **physical configurations** φ and $\mathcal{G}$ by expanding around $\phi - \varphi = \mathcal{O}(\hbar)$ to obtain the effective action

$$\Gamma[\varphi, \mathcal{G}] = S[\varphi] + \frac{\hbar}{2}\mathrm{tr}\big[\ln\big(\mathcal{G}^{-1} * G_0\big) + G^{-1} * \mathcal{G} - 1\big] + \hbar^2\Gamma_2[\varphi, \mathcal{G}] + \mathcal{O}(\hbar^3)$$

$$\hbar^2\Gamma_2[\varphi, \mathcal{G}] = -$$  $$-$$ 

where all 1PR and 2PR diagrams have cancelled, as we would expect.

- The **physical two-point function** is defined via

$$\mathcal{G}^{-1}_{xy}[\phi, \Delta] = G^{-1}_{xy}(\varphi) - \mathcal{K}_{xy}[\phi, \Delta] \qquad\qquad G^{-1}_{xy}(\varphi) \equiv \frac{\delta^2 S[\Phi]}{\delta\Phi_x \delta\Phi_y}\bigg|_{\varphi}$$

- This is in contrast to the standard evaluation, where one instead eliminates $\mathcal{J}$, $\mathcal{K}$ and the **classical configuration** φ^{cl} in favour of ϕ and Δ by expanding around $\phi - \varphi^{\mathrm{cl}} = \mathcal{O}(\hbar)$
[see e.g. Carrington, EPJC 35 (2004) 383-392]

- The result looks exactly like the standard effective action, except that the **path integral** is **evaluated** along **some quantum path** of the system.

So what?

- We have **two external sources**, but the **consistency relation** imposes only **one constraint**.

- What is the **other constraint**? Well … whatever we want from the following non-exhaustive list:

1. Setting the **two-point source** (evaluated at the **unphysical configurations**) to **zero** trivially gives the **Jackiw 1PI** effective action.
   [Jackiw, PRD 9 (1974) 1686.]

2. Choosing the **two-point source** to **vanish** when evaluated at the **physical configurations** will give the **CJT 2PI** effective action.

3. Choosing a **local two-point source** will give the **CV 2PPI** effective action.

4. Inspired by the **PT symmetry-improved** effective action, choosing the **Ward identities** to constrain the two-point source will allow us to **preserve symmetries** in truncations of the effective action.

5. For **statistical systems**, we may take any of the above with an additive contribution to the sources that encodes information about the **statistical ensemble.**

CJT 2PI effective action

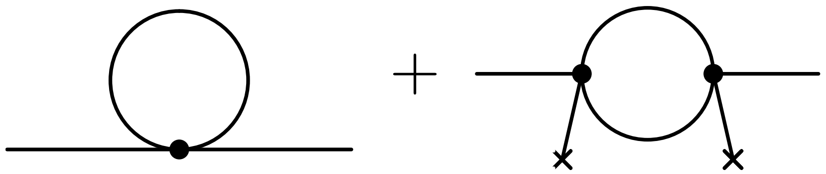
- In order to recover the CJT 2PI effective action, we require the **physical one-** and **two-point functions** to correspond to the **extremal quantum path**.
  [Cornwall, Jackiw, Tomboulis, PRD 10 (1974) 2428]

- This requires that

$$\mathcal{J}_x[\varphi, \mathcal{G}] = 0 \qquad\qquad \mathcal{K}_{xy}[\varphi, \mathcal{G}] = 0$$

- Choosing

$$-\mathcal{K}_{xy}[\phi, \Delta] = 2\hbar \left. \frac{\delta \Gamma_2[\phi, \Delta]}{\delta \Delta_{xy}} \right|_{\varphi, \mathcal{G}} =$$



we obtain the standard **Schwinger-Dyson equation** for the **physical two-point function** and the CJT 2PI effective action.

- By expanding around $\phi - \varphi = \mathcal{O}(\hbar)$, we can show that the sources do indeed vanish when evaluated at the physical one- and two-point functions.

- But the kernel of the Gaussian part of the path integral is now the **dressed two-point function**, i.e. the **saddle-point evaluation** of the path integral is evaluated along the **extremal quantum path** directly.

So what?

- Consider the **decay of a false vacuum state** to a lower, **radiatively-generated** minimum.
  [Garbrecht, PM, PRD 91 (2015) 125022; cf. Weinberg, PRD 47 (1993) 4614.]

- The saddle-point evaluation around …

  - the **classical extremum** contains the following Gaussian integral:

$$Z[0] = e^{-S[\varphi^{\rm cl}]/\hbar} \int [\mathrm{d}\hat{\Phi}] e^{-\hat{\Phi}_x G^{-1}_{xy}(\varphi^{\rm cl})\hat{\Phi}_y}$$

  - the **quantum extremum** contains a different Gaussian integral:

$$Z[\mathcal{J},\mathcal{K}] = e^{-S[\varphi]/\hbar} \int [\mathrm{d}\hat{\Phi}] e^{-\hat{\Phi}_x \{G^{-1}_{xy}(\varphi) - \mathcal{K}_{xy}[\phi,\Delta]\}\hat{\Phi}_y} = e^{-S[\varphi]/\hbar} \int [\mathrm{d}\hat{\Phi}] e^{-\hat{\Phi}_x \mathcal{G}^{-1}_{xy}[\phi,\Delta]\hat{\Phi}_y}$$

- The **dressed two-point function** sees the **instability**; the **tree-level function** does **not**.

- The **spectrum** of the **tree-level inverse two-point function** is **positive definite**; the **spectrum** of the **dressed two-point function** contains one **negative** and four **zero eigenvalues**, corresponding to dilatations and translational invariance of the nucleated critical bubble.
  [see e.g. Callan, Coleman, PRD 16 (1977) 1762; see also Langer, Annals Phys. 41 (1967) 108 [Annals Phys. 281 (2000) 941.]

- The analytic structure of the two functional integrals is different. (This is what we mean by "**non-perturbatively far away**.")

CV 2PPI effective action

- The CV 2PPI effective action resums all **point self-energy insertions** into the **dressed two-point function** and is defined as
[Coppens, Verschelde, PLB 287 (1992) 133; 295 (1992) 83; Z. Phys. C 57 (1993) 349; 58 (1993) 319]

$$\Gamma^{\rm 2PPI}[\phi,\Delta] = -\hbar \ln Z[\mathcal{J},\mathcal{K}] + \mathcal{J}_x[\phi,\Delta] + \frac{1}{2}\mathcal{K}_x[\phi,\Delta]\big(\phi_x^2 + \hbar\Delta_{xx}\big)$$

- In the standard approach, this is recast in the form

$$\Gamma^{\rm 2PPI}[\phi,\Delta] = S[\phi] + \hbar\,\Gamma_1^{\rm 2PPI}[\phi, M^2(\phi,\Delta)] + \hbar^2\,\Gamma_2^{\rm 2PPI}[\phi, M^2(\phi,\Delta)] - \frac{\lambda}{8}\hbar^2\Delta_{xx}^2$$

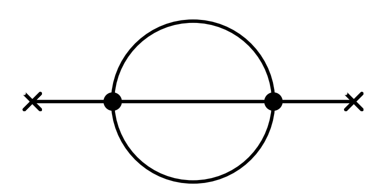
where Δ has a squared mass given by

$$M^2(\phi,\Delta) = m^2 + \frac{\lambda}{2}(\phi_x^2 + \hbar\,\Delta_{xx})$$

and

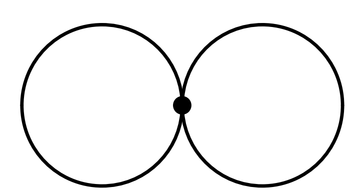
$$\hbar\,\Gamma_1^{\rm 2PPI}[\phi, M^2(\phi,\Delta)] = \frac{\hbar}{2}{\rm tr}\ln\Delta^{-1} * G_0$$

$$\hbar^2\,\Gamma_2^{\rm 2PPI}[\phi, M^2(\phi,\Delta)] = -$$



- Notice that that we have **artificially** isolated a term

$$-\frac{\lambda}{8}\hbar^2\Delta_{xx}^2 =$$

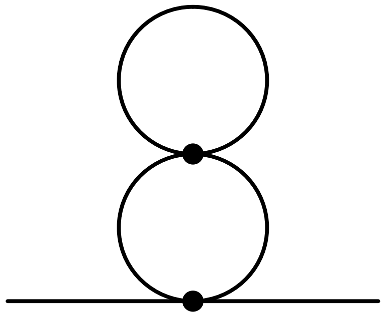


CV 2PPI effective action

- The equation of motion for the **dressed two-point function** (at coincidence) is given by

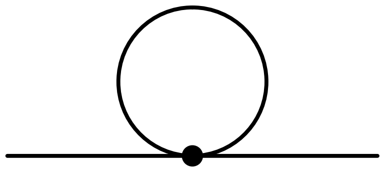
$$\Delta_{xx} = 2\frac{\delta\Gamma_1^{\rm 2PPI}[\phi, M^2(\phi,\Delta)]}{\delta M^2(\phi,\Delta)} + 2\hbar\frac{\delta\Gamma_2^{\rm 2PPI}[\phi, M^2(\phi,\Delta)]}{\delta M^2(\phi,\Delta)}$$

- Note that we have had to neglect **artificially** the **double-bubble diagram**. This is to prevent **double-counting** of



already counted in the resummation.

- Applying the **method of external sources**, we simply choose a **local two-point source**

$$\mathcal{K}_{xy}[\phi,\Delta] = -\,2\hbar\frac{\delta\Gamma_1[\varphi,\mathcal{G}]}{\delta\varphi_x^2}\delta^{(4)}(x-y) = \quad$$  $$+\mathcal{O}(\hbar^2)$$

applied to the two-point effective action with **no need to worry about double-counting**. That's it!

Symmetry preservation

- Consider a **globally O(2)-invariant model** with **SSB**:

$$\mathcal{L}_x = \frac{1}{2}\left(\partial_\mu \Phi_x^i\right)^2 + \frac{1}{2}m^2\left(\Phi_x^i\right)^2 + \frac{\lambda}{4}\left(\Phi_x^i\right)^2\left(\Phi_x^j\right)^2 \qquad i = 1,2 \qquad m^2 < 0$$

- The **Ward identities** read

$$\frac{\delta\Gamma[\phi,\Delta]}{\delta\phi_x^i} T_{ij}^a \phi_x^j + \frac{\delta\Gamma[\phi,\Delta]}{\delta\Delta_{xy}^{ij}}\left(T_{ik}^a \Delta_{xy}^{kj} + T_{jl}^a \Delta_{xy}^{il}\right) = 0$$

which can be used (in the spirit of the **PT symmetry-improved** effective action) to constrain the **two-point source**

$$\frac{\delta\Gamma[\phi,\Delta]}{\delta\phi_x^i} T_{ij}^a \phi_x^j + \frac{\hbar}{2}\mathcal{K}_{xy}^{ij}[\phi,\Delta]\left(T_{ik}^a \Delta_{xy}^{kj} + T_{jl}^a \Delta_{xy}^{il}\right) = 0$$

- If we expand the first term around the **extremal one-point function**, we can show that it vanishes, and hence the **two-point source** must satisfy

$$\mathcal{K}_{xy}^{ij}[\phi,\Delta]\left(T_{ik}^a \Delta_{xy}^{kj} + T_{jl}^a \Delta_{xy}^{il}\right) = 0$$

- But the second term also vanishes when evaluated at the extremal one- and two-point functions, so what have we learnt, if anything at all?

Hartree-Fock approximation

- Take only the **local self-energy corrections** to the **Higgs** and **Goldstone propagators**.

- It is known that the **Goldstone boson** will acquire a **pathological non-zero mass** in the **SSB phase** in the **Hartree-Fock approximation**.
  [Baym, Grinstein, PRD 15 (1977) 2897; Amelino-Camelia, PLB 407 (1997) 268; Petropoulos, JPG 25 (1999) 2225 ; Lenaghan, Rischke, JPG 26 (2000) 43.]

- Many authors have suggested solutions to this problem, including the elegant **PT symmetry-improved** effective action.
  [Petropoulos, JPG 25 (1999) 2225; Lenaghan, Rischke, JPG 26 (2000) 431; Baacke, Michalski, PRD 67 (2003) 085006; Ivanov, Riek, Knoll, PRD 71 (2005) 105016; Ivanov, Riek, van Hees, Knoll, PRD 72 (2005) 036008; Seel, Struber, Giacosa, Rischke, PRD 86 (2012) 125010; Grahl, Seel, Giacosa, Rischke, PRD 87 (2013) 096014; Markó, Reinosa, Szép, PRD 87 (2013) 105001; Nemoto, Naito, Oka, EPJA 9 (2000) 245; van Hees, Knoll, PRD 66 (2002) 025028;
  see talk by Daniele Teresi; Pilaftsis, Teresi, NPB 874 (2013) 594; JPCS 631 (2015) 012008; 1511.05347]

- The **Ward identities** and **consistency relation** tell us that the choice of two-point sources that preserves Goldstone's theorem is

$$\mathcal{K}^{GG}[\phi,\Delta]=\mathcal{K}^{HH}[\phi,\Delta]=-2\hbar\frac{\delta\Gamma_2^{(\mathrm{HF})}[\phi,\Delta]}{\delta\Delta_{xy}^{HH}}\bigg|_{\varphi,\mathcal{G}}$$

  i.e. we should take the **local Higgs self-energy** as the correction to the Goldstone propagators **not** the **local Goldstone self-energy**.

Hartree-Fock Approximation

- In the **SSB phase**, the leading correction to the **Higgs vev** is given by
$$\delta\varphi_x^H = -\lambda v \mathcal{G}_{xy}^{HH}\big(3\mathcal{G}_{yy}^{HH} + \mathcal{G}_{yy}^{GG}\big) = \frac{\lambda v}{2m^2}\big(3\mathcal{G}_{xx}^{HH} + \mathcal{G}_{xx}^{GG}\big) + \mathcal{O}(\hbar)$$
- The correction to the **Goldstone propagator** is given by
$$\mathcal{G}_{xy}^{-1,GG} \supset 2\hbar\lambda v \delta\varphi_x^H \delta^{(4)}(x-y) - \mathcal{K}_{xy}^{GG}[\phi,\Delta]$$
The constraint from the **Ward identities** tell us
$$\mathcal{K}_{xy}^{GG} = -2\hbar \frac{\delta\Gamma_2^{(\mathrm{HF})}[\phi,\Delta]}{\delta\Delta_{xy}^{HH}}\bigg|_{\varphi,\mathcal{G}} = -\hbar\lambda\big(3\mathcal{G}_{xx}^{HH} + \mathcal{G}_{xy}^{GG}\big)\delta^{(4)}(x-y)$$
instead of the **standard result**
$$\mathcal{K}_{xy}^{GG} = -2\hbar \frac{\delta\Gamma_2^{(\mathrm{HF})}[\phi,\Delta]}{\delta\Delta_{xy}^{GG}}\bigg|_{\varphi,\mathcal{G}} = -\hbar\lambda\big(\mathcal{G}_{xx}^{HH} + 3\mathcal{G}_{xy}^{GG}\big)\delta^{(4)}(x-y)$$
- Since the squared mass is negative, the order $\hbar$ corrections **cancel algebraically**, and we find that the Goldstone propagator
$$\mathcal{G}_{xy}^{-1,GG} = -\delta^{(4)}(x-y)\partial_x^2$$
which is clearly **massless**.
- Had we used the **standard result**, we would instead have found
$$\mathcal{G}_{xy}^{-1,GG} = \delta^{(4)}(x-y)\big[-\partial_x^2 - 2\hbar\lambda\big(\mathcal{G}_{xx}^{HH} - \mathcal{G}_{xx}^{GG}\big)\big]$$

Hartree-Fock approximation

- The **algebraic cancellation** of the **loop corrections** to the **Goldstone boson** in the **SSB phase** still holds in the presence of the **thermal corrections**

$$\mathcal{G}^{HH}_{xx}\big|_{\text{therm}} \approx \mathcal{G}^{GG}_{xx}\big|_{\text{therm}} \approx \frac{T^2}{12}$$

  with the **thermal Higgs mass** given by

$$m_H^2 = -2m^2 - \frac{8\lambda T^2}{12}$$

- The **mass gap equations** for the **Higgs** and **Goldstone bosons** read

$$m_H^2 = 3\lambda v_{\text{HF}}^2 + m^2 + \lambda(3\mathcal{G}^{HH}_{xx} + \mathcal{G}^{GG}_{xx})$$

$$m_G^2 = \lambda v_{\text{HF}}^2 + m^2 + \lambda(3\mathcal{G}^{HH}_{xx} + \mathcal{G}^{GG}_{xx})$$

- It follows algebraically that the **dressed Higgs vev** is

$$v_{\text{HF}}^2 = \frac{m_H^2 - m_G^2}{2\lambda}$$

- Thus, at the **critical temperature**, $T_c = \sqrt{3}v$, when both the **Higgs** and **Goldstone masses vanish**, the **vev** also **vanishes**, giving the correct **second-order phase transition**.

- These results are in complete agreement with those found using the **PT symmetry-improved** effective action.

Concluding remarks

- We have presented a novel approach to the effective action, which allows a wide range of results to be obtained within a single unique framework.

- We have illustrated …

  - how one may obtain the usual **CJT 2PI effective action**, with the advantage that the path integral itself is evaluated along the **extremal quantum path** directly;

  - how one may obtain the **CV 2PPI effective action**, without having to worry about **double-counting** diagrams;

  - how **Goldstone's theorem** can be preserved in the **Hartree-Fock approximation** of a globally O(2)-invariant model, in the spirit of the **PT symmetry-improved effective action**.

- Next steps: local, Abelian and non-Abelian symmetries; gauge dependence; constraint effective potential; coarse-graining?