Verification of the approximate Beckmann-Kirchhoff model on a series of sinusoidal profiles

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Abstract

Approximate scattering models have gained popularity over the past years due to their simplicity and fast computational speed to predict scattered light [1, 2]. In contrast to rigorous scattering models which use numerical techniques to solve Maxwell's equations, approximate models mostly neglect the effect of the polarisation of the incident light, multiple scattering, sharp edges, and other phenomena. Hence, such models should only be applied with slowly varying surfaces [1]. While many of the approximate models are restricted to surface features with relatively small height variations (typically less than half the wavelength of the incident light), the Backmann-Kirchhoff (BK) model can predict light scattering from surfaces with large height variations [2, 3], as long as the surfaces are "locally flat" with small curvatures. To obtain the most accurate result from the BK scattering model, well-distinguished validity conditions are essential. In the area of surface topography measurement, the BK model has been used in signal modelling [4] and measurement and correction of the 3D transfer function of coherence scanning interferometry [5], and 3D image formation in focus variation microscopy [6]. The BK model has been applied on certain structured surfaces/profiles, e.g, sinusoidal, rectangular, spherical [5, 7, 8], or even complex additive manufactured surfaces [9]. In this work, the scattered field from a series of sinusoidal profiles has been obtained using the BK model. The combination of sinusoidal profiles with different pitch and height values generates structured profiles within a wide range of radius of curvatures and slope angles. High slope angles and low radius of curvatures within a topography can cause the BK model to fail. The scattered field from a combination of sinusoidal profiles obtained by the BK model is compared to a rigorous boundary element method (BEM) (see Figure 1). The two-dimensional BEM is used to calculate the exact values of the angular distribution of the scattered field [10]. The BEM model solves linear partial differential equations along the boundaries and can be applied on any complex surface geometries. It is shown that the root-mean-square of the difference of the normalised scattered fields increases where the profile specifications (slope and curvature) are beyond the validity conditions of the BK model.

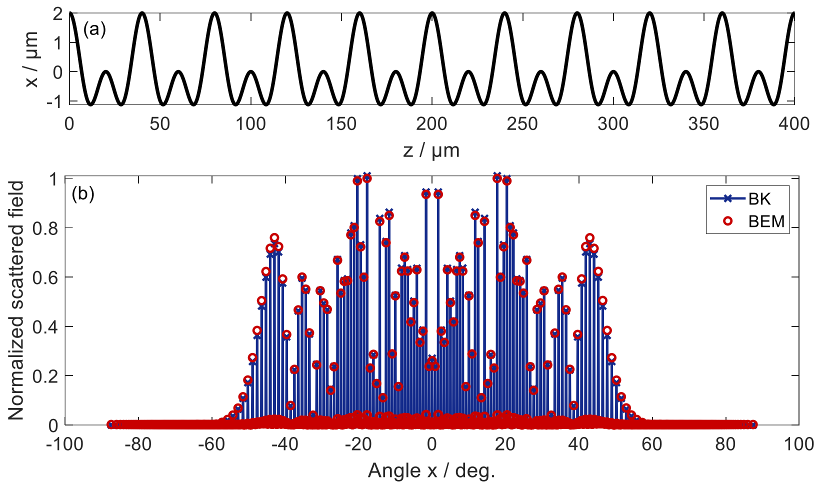

Figure 1: (a) an example of the combination of two sinusoidal profiles (20 µm and 40 µm pitches and 2 µm height), (b) angular distribution of the normalised scattered field obtained by the BK model (blue) and the BEM model (red).

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