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CONTEXTUALISATION

Introduction/ Background

Being literate with Mathematics and being an effective problem solver involves having an understanding of, and fluency with, a range of mathematical content and the ability to use that mathematics to solve real-world problems[1]. This principle is embedded as a key purpose for both GCSE (end of Key Stage 4) and Functional Skills Mathematics qualifications:

> GCSE: "A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject."[2]
>
> Functional Skills: "A key aim for Functional Skills mathematics specifications is that they enable the student to gain confidence and fluency in, and a positive attitude towards, mathematics. Students will convey their confidence in using mathematics when they can demonstrate a sound grasp of mathematical knowledge and skills … and apply it to solve mathematical problems."[3]

'Contextualisation' – or engagement with mathematical ideas and structures in real-world situations – is a key element of mathematical courses that aim to prepare students for effective and successful engagement in daily life, workplace settings, and broader social, economic, political and environmental activities.[4] Clearly, such a capacity to successfully engage with contextualised mathematics is essential for many Further Education (FE) students, not least because for many of them their study programmes and their aspirations are vocationally focussed.

Contextualisation of mathematics may also serve to motivate students: it has the potential to highlight the practical relevance and usefulness of mathematics ensuring a sense of utility and purpose[5]. Contextualisation supports students in forming a relationship with mathematics that focuses on its use-value rather than merely for its exchange value as a gatekeeper subject[6].

Many students entering the FE sector have negative attitudes to mathematics, often influenced by unsuccessful prior learning experiences at school (see briefing paper *Motivating and engaging learners*[7]). Engaging with mathematics both in general real-world situations and in situations directly relevant to specific vocational pathways have demonstrated some success in improving students' learning experiences in mathematics[8, 9]. However, the weight of evidence suggests that context in itself is not the main factor that influences performance in examinations and that other factors such as content, question style, and word count are better determinants of difficulty[10].

Contextualisation has the potential to encourage the use of technologies relevant to real-world practices (see briefing paper *Technology and Data*[11]) and facilitates engagement with the types of concrete resources and problem-solving experiences facilitated in Mastery pedagogy (see briefing paper *Mastery*)[12]. Furthermore, studies examining approaches to learning mathematics in which learners develop mathematical thinking in ways that draw on initial understanding of a realistic or realisable situation have been shown to improve students' attitudes and beliefs about the value of mathematics, their capacity to engage with multiple representations and methods, and their problem-solving skills[13, 14].

However, contextualisation is not a simple process and research points to the lack of resemblance between classroom mathematics and workplace practices. This highlights the difficultly that is inherent in the transfer of mathematical knowledge developed in formal academic (school) settings to other contexts[15]. The fundamental difference between the nature of the role of mathematics in the different settings of school and the workplace is key here. In school, mathematics is the object of study; by contrast, in the workplace it is ensuring successful outcomes that is central and mathematics if it is used is merely a tool to supports this. Strategies that have been adopted to help with this in mathematics curriculum design have included developing mathematics through tasks that have a high degree of realism and authenticity. In some cases this may be supported by the use of carefully constructed boundary objects for supporting learning across different situations, and a focus on the development of competencies that facilitate the construction and deconstruction of mathematical meaning in varied sites of application[16].

Contextualisation also presents specific challenges for lecturers and students. Developing approaches to teaching and learning mathematics that draw on context and useful resources requires a detailed understanding of both mathematical content and of appropriate real-world situations in which the content can be applied in ways that support learning of mathematics and problem-solving competence. This may add further challenge for FE lecturers, some of whom may have relatively low levels of confidence with mathematics content and pedagogical knowledge.[17] For students, the use of contexts may add a further level of complexity to a problem by requiring effective literacy, comprehension and interpretation skills. Contextualised problems may consequently take students longer to complete adding additional challenge for many in examinations. Students, therefore, need to be taught how to engage efficiently and successfully with contextualised problems, adding a further demand of teachers.

Key Issues

The use of contexts in the teaching and learning of mathematics gives rise to a number of questions. These relate broadly to the categories of context, learning process, and metacognition:

Contexts

- What counts as an effective context to support the learning of mathematics? Are some contexts more effective than others? [18, 19]
- What types of contexts may be used to best effect? Do they have to be authentic? (What counts as authentic?) Or, can they be modified to foreground specific mathematical principles?[20, 21, 22]
- What impact does using different types of contexts have on mathematical learning and experiences?[23, 24]

Learning Process

- What is the most effective way to structure teaching and learning activities when the key aim is the development of mathematical knowledge (with context giving meaning and facilitating access to the mathematics)?[25]
- What is the most effective way to structure teaching and learning activities when the key aim is to engage in real-world problems (with mathematics used as a mediating tool)? [26]
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The questions above point to the need to distinguish between approaches that have a different end-goal in mind:

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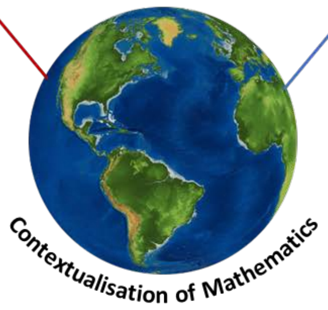
Approach 1 – Mathematics is foregrounded

- Contexts are used to give meaning and access to mathematical contents.
- The contexts are a means to an end – a tool for accessing and giving meaning to abstract mathematics.
- The ultimate goal for the learning process is a deeper understanding of a mathematical concept that can be applied to other situations.



Approach 2 – Context is foregrounded

- Mathematics is used and applied to make sense of contexts.
- The mathematics is a means to an end – a tool for making sense of a situation or for modelling a situation.
- The ultimate goal for the learning process is to complete a task in the context or a deeper understanding of the context.

Both approaches have relevance in the FE context, with Approach 1 arguably better suited to GCSE preparations and Approach 2 to performing tasks and solving problems in vocational contexts. However, we need to be clear about this distinction and explicit with students about their rationale for using particular contexts and the 'rules of the game' for engaging with those contexts. Failure to do this leaves students uncertain about whether it is the mathematics or the context that provides the key focus of learning. Equally, those tasked with developing resources to support teaching and learning practices in FE need to be clear and explicit about the 'direction of travel' and whether context is being used to support mathematical learning or mathematics is being used to support understanding of a context.

Metacognition

- What strategies can be used to support students' comprehension and interpretation skills for dealing with contextualised mathematics problems?[27, 28]

With the above in mind, the following four broad needs are identified for consideration:

1 *Clarity about types and uses of contexts, and principles that characterise effective use of contexts in mathematics learning.*

2 *Guidance on how to structure an effective learning sequence that involves using contexts to enhance mathematical understanding. This approach will be prioritised when preparing students for GCSE.*

3 *Guidance on how to structure an effective learning sequence that involves the use of mathematics as a mediating tool to engage with real-world problems in vocational contexts. This approach will be prioritised when preparing students for problem solving in Functional Skills and (pre-) vocational courses such as T Levels.*

4 *Development of specific strategies to support students' comprehension and interpretation skills, and to develop their metacognitive skills for solving contextual mathematics problems.*

Suggestions to support each issue are discussed in more detail below.

Issue 1 - Types and uses of contexts

When used appropriately, contexts provide meaning to abstract mathematical content and provide students with an experience of the use-value of mathematics for the world outside of the classroom. However, when contexts are used that bear little resemble to the students' own, or easily imagined, real-life experience, this has the opposite effect of convincing students that the only way to make mathematics appear meaningful is to create unrealistic scenarios to demonstrate its usefulness.

The MEI (Mathematics, Education and Industry) (2017) *Guide to developing contextualised teaching and learning resources* is a particularly useful document for supporting the development of contextualised resources for use in post-16 settings. This guide provides suggestions, for example, on:

- different uses of contexts (e.g. consolidation, learning new skills, developing problem-solving skills)
- advice on what makes a good context and how this may differ in an examination preparation course versus a vocational course focused on workplace preparation
- examples of suitable contexts for use specifically in post-16 vocational settings

It includes a checklist of things to consider when creating a contextualised resource (see Appendix A).

A possible useful addition to this resource is consideration of different context types[29] and how these can be used to support students' learning in different vocational courses:

- Context free - foreground formal and abstract mathematical structures *[Content agenda]*
- Authentic contexts - bear a close resemblance to real-life, include the use of genuine artefacts (e.g. newspaper articles; bills; construction plans), and, where possible, take account of all possible variables that may come into play in the scenario. *[Context agenda]*
- 'Cleaned' or modified contexts - a simplified version of an authentic context, deliberately cleaned or modified to either make the context easier to access or to foreground a particular mathematical concept or a particular problem *[Context and content agenda]*
- Parables - situations that involves an unnamed person, organisation or entity, deliberately constructed to accentuate a mathematical feature or concept. The anonymity of the characters gives the problem situation universal applicability (e.g. 'A games show contestant has to choose one box from a row of four …'. Importantly, the situation is still one in which students can place themselves, but students know that the main aim is to do or learn mathematics rather than understand the context.
  [Content agenda]
- Contrived or artificial contexts - invented contexts designed to fit a particular mathematical point, irrespective of how appropriate or reflective these are to real life. *[Content agenda]*

Context free, cleaned and *fictitious* contexts are all relevant to GCSE-preparation courses, primarily since these contexts types foreground mathematical contents and methods. *Authentic* and *cleaned* contexts are more relevant to optional vocational programmes, since these context types foreground understanding and engagement with the context. In general, *contrived contexts* should be avoided since these trivialise the contexts and the link between the mathematics and the real-world.

<u>*Suggestion*</u>: A formal classification of context types used in resources developed to support post-16 programmes would ensure that students are adequately exposed to different context types and are given opportunities to develop the skills needed to engage with a variety of problem-solving experiences. (This could be achieved by using a system of tagging if resources are available electronically).

Several additional issues worth considering around the effective use of contexts in post-16 vocational programmes are:

- Degree of Realism: Given the spectrum of context types available and the different agendas associated with each, there should be a clear motivation and rationale for the degree of realism used in a contextual task – and this must be explicitly communicated to the students. Students must know the expected outcome of the learning process (mathematical knowledge or production output) and the range of acceptable tools and knowledge that they are able to bring to bear on the problem-solving process (e.g. whether or not informal everyday strategies are allowed). In short, students must be made aware of the 'rules of the game' and whether the game is mainly about mathematics or mainly about the context, or both (this is particularly important as students move from mathematics specific courses to (pre-)vocational programmes).

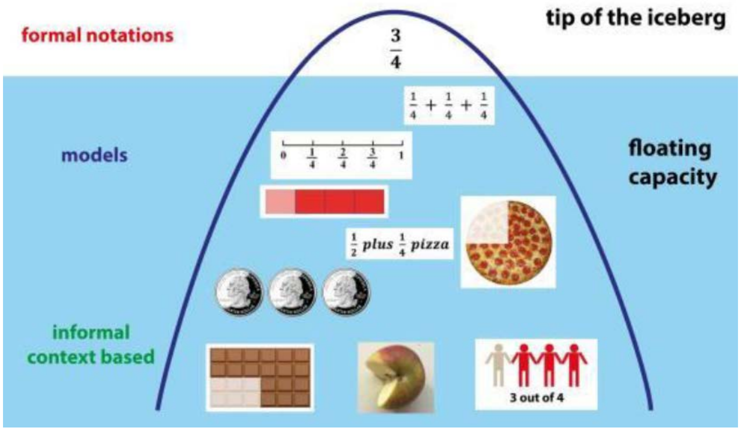
- Relevance and familiarity: Students respond more positively to contexts that they are familiar with and those that have direct relevance to their current learning and lived experiences . Less familiar contexts are more demanding for students and increase demand on comprehension skills.
- Language: Careful consideration must be given to the quantity of language interpretation and comprehension required to engage successfully with contextual scenarios and problems. Language used can impact on the difficulty of a problem scenario, can lead to misinterpretations, and can detract from any mathematical practices under scrutiny. Issue 4 below offers some suggestions to support students' interpretation and comprehension skills.

Issue 2 - Structuring effective learning sequences by using contexts to enhance mathematical understanding

The *Realistic Mathematics Education* (RME) approach provides a comprehensive, research-informed teaching strategy for the use of contextualisation to support the learning of mathematical content and the development of problem-solving and reasoning skills. This approach has demonstrated particular impact in the post-16 GCSE-resit context[30], signalling the relevance and suitability of this approach to the work of the CfEM.

The aim of RME is to support a slow move towards formal mathematical procedures and systems through students' active engagement with contexts and representations ('model building') that encourages visualisation of mathematical processes and structures, and the connections between these[31, 32]. We drw attention to the following key elements of RME:

1. *Contexts*: The use of 'realisable' contexts to support the exploration of mathematical content. 'Realisable' refers to situations that are meaningful to students and that they can imagine and relate to. The contexts do not have to present authentic representations of real-world practice and can, instead, be carefully developed or 'cleaned' to ensure access to specific mathematical principles.
2. *Models*: Models are crucial and serve to bridge the gap between informal understanding linked to contexts and more formal systems and principles. Students need to be taught how to construct models, how to use the same models to represent different scenarios, how to use different models to represent the same scenario, and how to build connections between models as they move towards more formal procedures. This use of models recognises the 'level principle' of mathematical understanding as students pass through various levels of understanding starting with context-related methods and solutions, then moving to more general shortcuts and schemas, and finally to understanding how all of these concepts and strategies are related[33]. *Progressive formalisation of models* is key as the developed models change from context-dependent models to general models with wider applicability across a range of problems and content domains.



(Hough, Solomon, Dickinson and Gough, 2017[34] – adapted from Webb, Boswinkel and Dekker, 2008)[35]

There are two different ways of working with models in a RME approach:

- ‘Horizontal mathematisation’ involves using a model to solve a problem – *the process of ‘doing’*[36] *mathematics*
- ‘Vertical mathematisation’ involves using the model to develop a more formal understanding of the mathematical structure itself by reorganising, finding shortcuts and recognising the wider applicability of the model to other scenarios – *the process of ‘becoming’ someone who uses and communicates mathematically with others*[37]*.*

Both forms of mathematisation are necessary to support the development of students’ problem-solving skills and their deeper understanding of formal mathematical procedures.

3. *Intertwinement principle*: Problems span across content domains and across topics within content domains so that students are expected to draw on a wide selection of models, strategies and content to explore mathematical concepts and solve problems in relation to those concepts.

4. *Active learning*: Students are active participants in the learning process and directly engage in problem-solving processes. Teachers facilitate whole class discussions and collaborative group work providing opportunities to share and challenge models, strategies, and thinking. Teachers use careful questioning to support and direct students’ thinking.

This approach is fully consistent with Mastery pedagogy[38] and with current UK-specific guidance on principles of effective mathematics practice[39], and provides a proven framework to support resource development, teaching practice and professional development opportunities for post-16 GCSE and GCSE-resit preparation.

Issue 3 - Structuring an effective learning sequence to engage with real-world problems in vocational contexts

As highlighted previously, vocational activities focused on workplace practices are motivated by the production of specific workplace outcomes, commonly associated with the successful completion of a task or the production of an object. This shift away from mathematics as the object of study towards mathematics as a tool for supporting the production process requires a different approach for resource development to support students’ successful engagement in these activities. Specifically, there needs to be greater emphasis on specific understanding of the relevant vocational contexts, the tools and models commonly used in those contexts (including prevalent forms of communication and representation), and the ways in which formal mathematics can support successful participation in these contexts.

Following this approach, MEI provides a useful document that indicates how specific content and assessment objectives link to vocational courses and contexts[40]:

Strand	Ref	Content	Construction and the built environment	Health, social care and childcare	Business, administration and entrepreneurship
Number	N1	Order positive and negative numbers, decimals and fractions.	Ordering heights and costs of buildings, soil bearing capacities or U values of different materials. Comparing and ordering components for construction - for example, imperial measures are commonly used for materials such as wood, nails and screws, and could be ordered in increasing size.	Child development - age ranges and expected development. Temperatures (e.g. safe room temperature, body temperature, bath water, etc.). Cost of care plans. Reading and understanding ‘use by’ dates on food and medicines and deciding what order to use them in.	Order performance figures for different regions, or sales people, from highest to lowest. Sorting figures or files in date order. Comparing quotes from several different sub-contractors.

This ‘context grid’, together with the supplied exemplar resources, is an invaluable resource for thinking about how specific mathematical content can be applied in particular vocational contexts.

However, a potential limitation of the resource is the lack of direct focus on *general mathematical competences*[41] (GMCs) that facilitate successful engagement with mathematical content across a range of contextualised activities. Earlier work around these GMCs has been developed to provide a framework for mathematics for the new T Levels. It is likely that development work towards establishing the GMCs will proceed in parallel to the work of the CfEM. The ten GMCs being considered are:

- Measuring with precision
- Estimating, calculating and error spotting
- Working with proportion
- Using rules and formulae
- Processing data
- Understanding data and risk
- Interpreting and representing with mathematical diagrams
- Communicating using mathematics
- Costing a project
- Optimising work processes

These broad competency categories provide a potential structure for the design of resources that consider the ways in which students and workers might bring together both content and competencies when engaging with mathematics in different contexts. Each GMC has relevance in a range of vocational, workplace and daily-life scenarios and embodies engagement with a cross-section of mathematical contents.

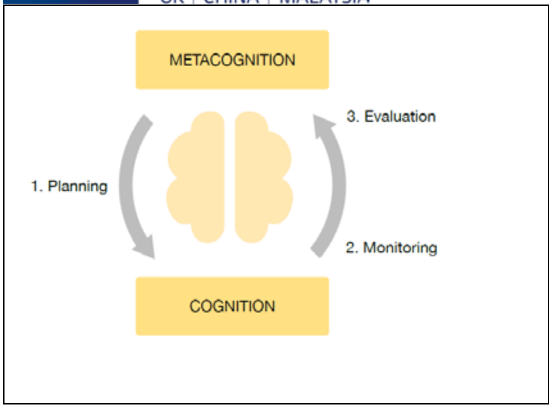
We suggest that as the work of the CfEM and T-level implementation proceeds these competency categories be used as a framework to structure the development of resources to support vocational students' engagement with mathematical content during their vocational programmes. This framework should demonstrate examples of how each GMC can be enacted in different vocational contexts and the potential learning of mathematical content.

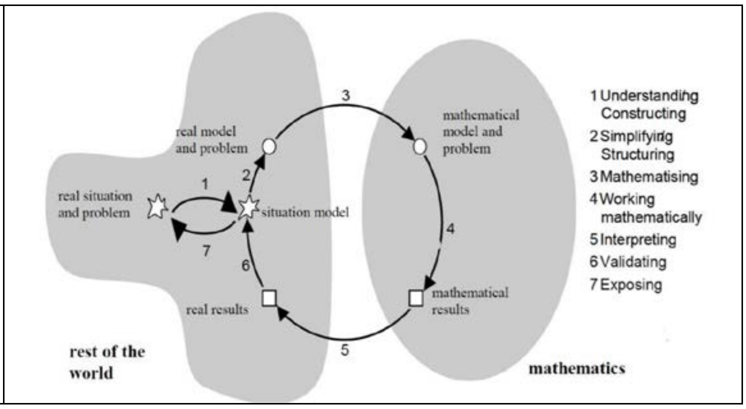
Issue 4 – Strategies to develop students' comprehension, interpretation and metacognitive skills to support contextual problem-solving

Effective problem-solving in contextual scenarios involves a blending of engagement with (a) mathematical content, (b) contextual elements and (c) competencies[42]. Students need to be shown, explicitly, how to engage in problem-solving activities, including how to interpret and filter relevant information, access relevant mathematical content, work systematically, communicate thinking, and reason, justify and critique their own and others' thinking and models. Students need to be equipped with 'metacognitive' strategies to deconstruct, monitor and direct their own learning, and with 'self-regulation' strategies so that they can identify and gauge strengths and weaknesses in their learning activity and act so as to improve their learning experience[43].

A recommended strategy in this regard is the use of comprehensive worked examples, deliberately developed to model specific elements of problem-solving practice, activity or thought process. These worked examples must serve to exemplify the general self-regulation cycle of 'planning, monitoring, evaluating' and a maths-specific problem solving process:

General self-regulation cycle for completing tasks[44]	Maths-specific problem-solving process[45]





The worked examples can demonstrate a range of problem-solving processes or skills, including for example:

- Strategies for interpreting, filtering and recording relevant contextual and mathematical information needed for a problem-solving experience
- Systematic and logical working through the cycle of problem-solving processes
- Structured working and layout
- Strategies for effective model building
- Use of multiple representations to investigate a problem
- Effective and varied forms for communicating an answer
- Critical analysis of a model developed by someone else
- Justifying a solution
- Deconstructing the thought process used to solve a problem
- Validating a solution and the efficiency of the method used.

Summary

Contextualisation

Contextualisation of mathematics that relates mathematical structure to real-world situations should be a key element of mathematical courses that aim to prepare students for effective and successful engagement in daily life, workplace settings, and broader social, economic, political and environmental activities. Capacity to successfully engage with such mathematics should be considered essential to FE students as they become young adults with many having vocational concerns and aspirations. A contextualised approach to mathematics is consequently likely to motivate students as they realise its practical relevance and usefulness to their other studies.

Our aim will be to develop approaches initially that seek to:

- Structure effective learning that draws on context(s) to enhance mathematical understanding (the Realistic Mathematics Education (RME) approach)
- Support students' comprehension, interpretation and metacognitive skills to support contextual problem-solving.

Further to this, a more long-term approach will develop a resource bank that will support mathematics learning that connects with contexts from the wide range of post-16 vocational programmes available to students. This work will connect with developments of mathematics in courses other than GCSE resits, particularly the mathematics in T-Levels.

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Appendices

MEI checklist[46] of questions to consider when developing contextualised resources

Is the task…	Explanation	Check
Authentic and realistic	Is it a genuine example? Can learners see how the task relates to real life?	
Meaningful	Does it draw on the learners' own experience?	
Purposeful	Can learners see why they are using the maths?	
Effective	Are learners learning from the task?	
Motivating	Do learners want to tackle the problem?	
Relevant	Can learners see where they could use the maths in their vocational area or personal interests?	
Challenging	Do all learners feel they have had to work to solve the task?	
Accessible	Can all learners tackle the problem when they see it or are they overwhelmed?	
Inclusive	Can all genders, ethnicities and cultures relate to it?	
Empowering	Does the task help learners develop confidence and independence?	
Differentiated	Can it meet the needs of different learners?	
Encouraging	Do all learners feel they have achieved something?	
Inspiring	Do learners want to carry on and do more maths?	
Holistic	Could it develop other skills without compromising the learning of maths?	

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[18] For a discussion of different context types and relevance to mathematics learning, see:
Chris du Feu, "Naming and Shaming," *Mathematics in School*, *30*, No. 3 (2001): 2–8.

[19] For an expanded categorisation of du Feu's context taxonomy in relation to a vocational type mathematics qualification see:
Rajan Debba, *An Exploration of the Strategies used by Grade 12 Mathematical Literacy Learners when Answering Mathematical Literacy Examination Questions Based on a Variety of Real-Life Contexts,* Master of Education (South Africa: University of KwaZulu-Natal, 2011. Available online at:
http://researchspace.ukzn.ac.za/jspui/bitstream/10413/5814/1/Debba_Rajan_2011.pdf .

[20] Both the Realistic Mathematics pedagogic approach and the OECD PISA assessment frameworks rationalise the use of 'realistic' and 'realisable', but not necessarily authentic, contexts in mathematics teaching, learning and assessment at a school level. See:

- Maria Van den Heuvel-Panhuizen, "The Didactical Use of Models in Realistic Mathematics Education: An Example from a Longitudinal Trajectory on Percentage," *Educational Studies in Mathematics*, 54, No. 1 (2003): 9–35.
- OECD (2013).

[21] By contrast, the studies below present a specific case for the use of contexts with a high degree of authenticity (rather than simply realistic contexts) in mathematics-related work with *adults*:

- Lynda Ginsburg, Myrna Manly and Mary Jane Schmitt, *The Components of Numeracy* (Harvard, US: National Centre for the Study of Adult Learning and Literacy, 2006).
- T. Scott Murray, Yvan Clermont and Marilyn Binkley, *International Adult Literacy Survey (IALS) - Measuring Adult Literacy and Life Skills: New Frameworks for Assessment* (Canada: Statistics Canada, 2005).

[22] Palm (below) provides a framework for classifying the degree of authenticity of a context by considering the dimensions of 'fidelity' and 'comprehensiveness':
T. Palm, "Theory of Authentic Task Situations," in *Words and Worlds: Modelling Verbal Descriptions of Situations, eds.* Lieven Verschaffel, Brian Greer, Wim Van Dooren & Swapna Mukhopadhyay (Rotterdam: Sense Publishers, 2009): 3-19.

[23] For a discussion of students' experiences of engaging with contextualised mathematics tasks in GCSE assessments, see:
Jackie Greatorex, "Context in Mathematics Examination Questions," *Research Matters*, 17 (2014): 18-23.

[24] The research below explores the impact of the 'nearness' and 'farness' of contexts from students lived experiences on their engagement with contextualised mathematics tasks:

Godfrey Sethole, Busi Goba, Jill Adler and Renuka Vithal, "Fine-Tuning a Language Of Description For Mathematics Items Which Incorporate The Everyday," in *Mathematics Classrooms in Twelve Countries: The Insider's Perspective*, ed. David Clark, Christine Keitel, Yoshinori Shimizu (Rotterdam: Sense, 2006): 117-130.

[25] The Realistic Mathematics Education pedagogic approach provides an effective structure for employing context to support mathematical learning. This approach will be discussed in more detail in the section below.
See, for example:

- Maria Van den Heuvel-Panhuizen and Paul Drijvers (2014).
- Adrian Treffers, "RME in the Netherlands 1980–1990," *RME in primary school*, ed. in Len Streefland (Utrecht: Freudenthal Institute, 1991).

[26] Geoff Wake (2014) provides curriculum design principles that "facilitate the competencies and subjective needs of students as that become workers (and, more generally, citizens) with the capacity to develop mathematical understandings of, and facility with, new practices with which they might engage." (p.84)

[27] Education Endowment Foundation (EEF), *Improving Mathematics in Key Stages 2 and 3: Guidance Report* (London, UK: EEF, 2017).

[28] Education Endowment Foundation (EEF), *Metacognition and Self-Regulated Learning: Guidance Report* (London, UK: EEF, 2018).

[29] Chris du Feu (2001); Rajan Debba (2011).

[30] Sue Hough, Yvette Solomon, Paul Dickinson and Steve Gough, *Investigating the Impact of a Realistic Mathematics Education Approach on Achievement and Attitudes in Post-16 GCSE resit classes* (Manchester Metropolitan University: Faculty of Education, 2017).

[31] Maria Van den Heuvel-Panhuizen (2003).

[32] Sue Hough, Yvette Solomon, Paul Dickinson and Steve Gough, *Investigating the Impact of a Realistic Mathematics Education Approach on Achievement and Attitudes in Post-16 GCSE resit classes* (Manchester Metropolitan University: Faculty of Education).

[33] Maria Van den Heuvel-Panhuizen and Paul Drijvers (2014).

[34] Sue Hough, Yvette Solomon, Paul Dickinson and Steve Gough (2017).

[35] David Webb, Nina Boswinkel and Truss Dekker, "Beneath the Tip of the Iceberg: Using Representations to Support Student Understanding," *Mathematics Teaching in the Middle School*, *14*, no. 2 (2008): 110-113.

[36] Geoff Wake (2014)

[37] Geoff Wake (2014)

[38] Briefing paper - *Mastery*

[39] EEF (2017).

[40] Mathematics, Education and Industry (MEI), *A toolkit for contextualizing post-16 GCSE Mathematics* (Towbridge, UK: MEI, 2017). http://mei.org.uk/contextualisation-toolkit

[41] J. S. Williams, Geoffrey Wake and A. Jervis, "General Mathematical Competence in Vocational Education," in *Mathematics Education for the 21st Century*, eds. Celia Hoyles, C. Morgan and G Woodhouse (London: Falmer Press, 1999)

[42] Geoff Wake (2014).

[43] EEF (2018), 8-9.

[44] EEF (2018), 10.

[45] Werner Blum and D Leiß, "How Do Students and Teachers Deal with Modeling Problems?" in *Mathematical Modeling (ICTMA12): Education, Engineering and Economics,* eds. Christopher Haines, Peter Galbraith, Werner Blum and Sanowar Khan (Chichester: Horwood Publishing, 2006): 222 – 231.

[46] Mathematics, Education and Industry (MEI), *Contextualising post-16 GCSE Mathematics: a toolkit A guide to developing contextualised teaching and learning resources* (Towbridge, UK: MEI, 2017): 6. http://mei.org.uk/contextualisation-toolkit.