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Centres for Excellence in Mathematics

MASTERY

Introduction

Mastery learning is an idea that can be traced back to 1971 and the educational psychologist Benjamin Bloom.[i] However, recent focus in England on the term has been brought about through the work of the NCETM[ii] and the national Maths Hubs programme.

The recent NCETM iteration of mastery has been heavily influenced by practices from Shanghai (in the form of a teacher exchange) and Singapore (in the form of textbooks). The NCETM have sought to emphasise 5 big ideas[iii] that they believe underpin mastery - coherence, representations, variation, mathematical thinking and fluency (see appendix 1). Strongly linked to the work of the NCETM two mastery textbook schemes have also been endorsed by the UK government – Maths No Problem![iv] (KS1 to KS2) and Power Maths[v] (KS1 to KS2). Outside of the government backed Maths Hubs initiative, the organisation Mathematics Mastery[vi] (linked to the Ark Academy chain) have also developed their own resources (running from KS1 to KS4).

Research on the impact of mastery includes a meta-analysis from the Education Endowment Foundation (EEF).[vii] They summarise that mastery has a moderate impact on student progress but note that there is a large amount of variation in results, definition and application. Two other recent reports based on trials in England are also worth noting. Firstly, the EEF have evaluated the Ark Academy Mathematics Mastery scheme stating that students make a small amount of additional progress.[viii] Secondly, Sheffield Hallam University have reported on the NCETM coordinated China-England Mathematics Teacher Exchange[ix]. Though care should be taken in understanding the results, a reasonable summary is that there was no evidence of positive effects for 11-year olds and a small positive effect for 7-year olds. Within the FE sector there have been some small-scale ETF funded trials of mastery approaches though nothing yet providing impact on attainment.

Within the Further Education (FE) sector a number of challenges are relevant to the development of mastery[x].

1. **Student motivation**
   There is a variety of student motivation depending on backgrounds, additional support needs, previous attainment and attendance.
2. **Resources**
   Schemes of work vary across settings and are seen by teachers as adaptable to allow personalisation and differentiation. This personalisation allows teachers to focus on specific areas of student weakness rather than aim for complete (re-)coverage.
3. **College structures**
   Colleges provide different structures for teaching maths within a timescale of only 8 months.
   - Centralised departments compared to dispersed departments.
   - Lessons ranging from 50 minutes four times per week compared to 3-hour sessions once per week.
   - Students of all attainment levels in one class compared to only students targeting a level 4 in the class.
4. **Teacher skills and beliefs**
   Recruiting qualified and specialist staff can be difficult in some areas of the country. Teachers value the use of mark schemes and exam style questions to illustrate to students where additional marks can be achieved.

Key Issues

As a framework to understand the issues faced in developing mastery in an FE context it will be considered under four categories[xi] – a mastery approach, a mastery curriculum, teaching for mastery and achieving mastery. In essence, a mastery approach should underpin all the work by shaping beliefs, a mastery curriculum should structure the mathematical content and shape resources, teaching for mastery should shape lesson design and achieving mastery should shape student understanding. All four of these areas should be addressed to achieve a successful and coherent mastery framework.

A **mastery approach** revolves around the belief that through hard work all students are capable of being successful at mathematics. This underpinning principle may be a challenge in the FE context as many students already lack belief and have low confidence-based on their GCSE results. Furthermore, the range of previous attainments in the class (potentially from U to a level 3) could provide a challenge for teachers.

A **mastery curriculum** must have the twin aims of creating a purposeful connected pathway through the curriculum alongside (where appropriate) steps in progressing understanding. It is the balance of these two aims that will be crucial in shaping mastery in an FE context. Many FE establishment have designed their own individual paths through the GCSE curriculum, see schemes of work as adaptable and may be reluctant to adopt one recommended path.

Teaching for mastery is a term that has come to describe a set of practices designed to keep all students progressing at the same rate. Combined with a coherent curriculum the belief here is that all students will progress at the same pace with less need to differentiate within lessons. However, as already mentioned, this is not a value shared by many FE settings. The NCETM have promoted the pedagogic principles of using variation and using representations in their 5 big ideas. In particular, the use of representations (such as the bar model) are likely to be well received in an FE context.[xii]

Achieving mastery describes that students know 'why', know 'that' and know 'how' rather than on simply memorising procedures. Linked to this the NCETM have promoted the utilisation of fluency and mathematical thinking in their 5 big ideas. In the FE context there should be a considered emphasis on developing exam style practice questions alongside exam style mark schemes.

Issue 1 – A mastery approach

> *Students (and their teachers) can have different beliefs about intellectual abilities. Some believe that intellectual abilities are basically fixed - … others believe that intellectual abilities can be cultivated and developed through application and instruction. They do not deny that people may differ in their current skill levels, but they believe that everyone can improve their underlying ability.*[xiii]

A mastery approach should be underpinned by the belief that through hard work all pupils are capable of being successful at mathematics. This belief is exemplified in the mastery work of the NCETM as "by working hard at mathematics they[students] can succeed".[xiv] Indeed research shows that for Asian students (both based in an Asian country or a Western country) a belief in hard work and success through effort are fundamental to success.[xv] Work on growth mindsets (from Carol Dweck and Jo Boaler[xvi] show a similar link between success and the belief that abilities and intelligence can be developed).

Some students in FE may need help to believe that they can succeed (and some teachers may need help in believing that their students can succeed). This message needs to be central to the development of mastery at FE without being reduced to gimmicky slogans. Students will begin to believe that they can succeed if resources are well designed with teachers able to bring out connections and key learning. Students need to be able to see that they can make sense of mathematics and this hinges on successful development of issues 2, 3 and 4. Such design and development will also improve teacher belief over time. FE teachers can be faced with a wide range of students including adult learners with limited confidence, EAL students with strong mathematical skills, students that have just fallen short of a level 4 and students that should arguably be studying the functional skills route. Resources and messages all need to be developed to help motivate students and encourage the belief that all can succeed.

Research could focus around comparing student confidence, attitudes and self-efficacy at the start of the GCSE FE programme to the end. Similarly, a focus could be placed on identifying a change in teacher attitudes over the year.

Issue 2

> *"Efforts focused only on low-level cognitive skills, attempted to break learning down into small segments, and insisted that students "master" each segment before being permitted to move on... Nowhere in Bloom's writing, however, can this kind of narrowness and rigidity be found. In fact, Bloom emphasised quite the opposite".*[xvii]

A mastery curriculum should create a connected and purposeful pathway through the curriculum **alongside** coherent steps in progressing understanding. (The equivalent principle in the NCETM's five big ideas is coherence and small steps). Both the connections and the steps must be considered to ensure that both teachers and students see maths at FE colleges as making sense and distinct from school-based maths.

A traditional mastery curriculum should provide a detailed and comprehensive breakdown of lessons. This structure may be helpful if topics are being introduced for the first time, to students with similar starting points and with sufficient teaching time. However, in FE colleges time is at a premium, students have varied starting points and they have already experienced the topics. Furthermore, in many FE settings schemes of work are seen as adaptable and an emphasis is placed on addressing areas of weakness rather than on aiming for complete coverage. Therefore, consideration needs to be given to:

- How to create resources suitable for a variety of settings.
- How topics should be grouped together.
- Which topics should be included (and which should be left out).
- The amount of guidance and flexibility given to teachers (particularly for any topics not included).
- How to equip teachers with a knowledge of alternative strategies and how they could draw these together.

The use of approved mastery textbooks at primary school level would indicate some consideration of a planned mastery curriculum but there is little evidence of one at the GCSE level in the UK context. Potential guidance may include ARK Mathematics Mastery, schemes from Shanghai, Japan or Singapore.

Research could focus around determining which topics should be included (potentially linking to the diagnostic questioning research).

Issue 3 – Teaching for Mastery

> *"Manipulatives and representations can be powerful tools for supporting pupils to engage with mathematical ideas. However, manipulatives and representations are just tools: how they are used is important".*[xviii]

Teaching for mastery is a term that has come to describe pedagogic practices that are utilised to keep all students progressing at the same rate. (In their 5 big ideas, the NCETM have drawn attention to the use of representation and structure along with variation). However, as already discussed many teachers in the FE context value personalisation and differentiation. So the intention of the pedagogic practices considered here is whether they enhance the motivation and understanding of all students.

The pedagogic practice that could provide a focus for research is the use of representation to enhance understanding. Whilst there are many possible representations that could be used, some research[xix] has shown that the bar model is effective in increasing students' ability to solve and understand word problems. Furthermore, the Education and Training Foundation (ETF) are currently funding a trial of mastery approaches, such as the bar model, in FE colleges.[xx]

General principles[xxi] on the use of representation includes:

- Ensure that there is a clear rationale.
- Help students (and teachers) to make sense of the link between the representation and the corresponding mathematical idea.
- Help students (and teachers) to understand the underlying mathematics (not just see the representation as a new procedure).

Alternative pedagogic principles to enhance understanding and motivation that could be the focus of research include:

- **The use of introductory problems**
  Introductory problems (for each topic) could be designed to motivate students and help students to understand the purpose of the mathematics they are studying. Setting a meaningful contextualised problem helps to create a rationale for more abstract parts of understanding. A recent DfE report[xxii] in effective post-16 practice states "aligning subjects with areas of interest such as a student's elective subject or contemporary topics can also encourage engagement."
- **The use of dialogic learning**
  Cultivating a classroom culture based around dialogic learning[xxiii] could encourage motivation and understanding. Part of this pedagogic approach is based around the recognition that students already have experience of the curriculum (along with a range of different methods). The teacher is not there to tell students a new method – but to synthesise understanding and help bring clarity.

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Issue 4 – Achieving Mastery

> *Intelligent practice is a term used to describe practice exercises that integrate the development of fluency with the deepening of conceptual understanding. Attention is drawn to the mathematical structures and relationships to assist in the deepening of conceptual understanding, whilst at the same time developing fluency through practice.*[xxiv]

Achieving mastery means that students know facts, can apply facts, are fluent in using procedures and have a depth of understanding that allows them to solve problems.[xxv] A traditional view of achieving mastery is that students achieve 80%[xxvi] in a test before proceeding to the next unit. Whilst this does not feel appropriate for an FE context it does highlight the need for carefully designed questions. (In terms of the NCETM's five big ideas these principles are picked up in the mentions of mathematical thinking and fluency).

Questions should be designed based around the following suggestions:

- **A focus on developing fluency and conceptual understanding**
  Fluency should be encouraged along with an emphasis on mathematical structures to deepen conceptual understanding.[xxvii] This should include small step progression of questions (in terms of both difficulty and concept) in the design of questions.
- **Provide exam style mark schemes**
  Teachers are aware of the importance of developing exam techniques in students and correspondingly spend large amounts of time in this area.[xxviii]
- **Include problem solving style questions**
  The new GCSE curriculum (first examined from June 2017) has an increased focus on problem solving.[xxix]

Hence, practice questions must be designed carefully and teachers equipped to take opportunities to bring out learning points.

The focus of research in this area could be on whether students develop an understanding of the steps involved in solving questions and an awareness of the allocation of marks.

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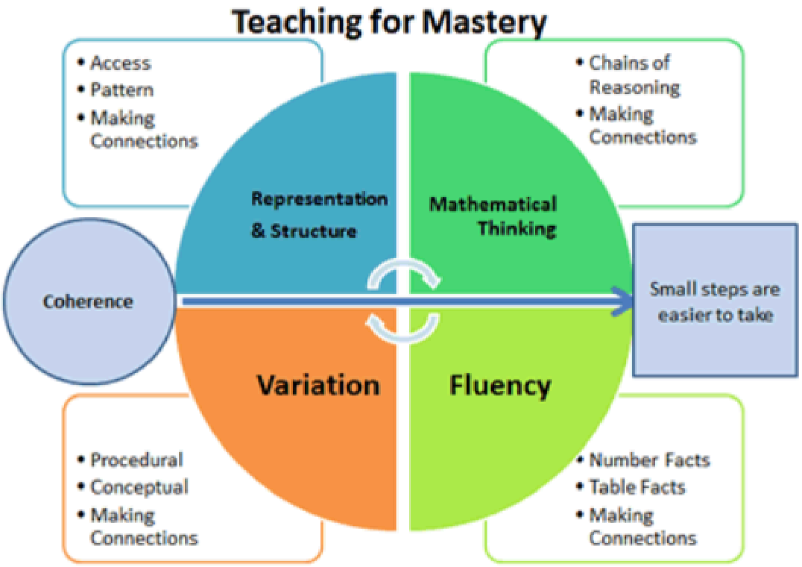
Appendix 1

Five Big Ideas in Teaching for Mastery

A central component in the NCETM/Maths Hubs programmes to develop Mastery Specialists has been discussion of Five Big Ideas, drawn from research evidence, underpinning teaching for mastery. This is the diagram used to help bind these ideas together:



(click to enlarge)

A true understanding of these ideas will probably come about only after discussion with other teachers and by exploring how the ideas are reflected in day-to-day maths teaching, but here's a flavour of what lies behind them:

Coherence
Connecting new ideas to concepts that have already been understood, and ensuring that, once understood and mastered, new ideas are used again in next steps of learning, all steps being small steps

Representation and Structure
Representations used in lessons expose the mathematical structure being taught, the aim being that students can do the maths without recourse to the representation

Mathematical Thinking
If taught ideas are to be understood deeply, they must not merely be passively received but must be worked on by the student: thought about, reasoned with and discussed with others

Fluency
Quick and efficient recall of facts and procedures and the flexibility to move between different contexts and representations of mathematics

Variation
Varying the way a concept is initially presented to students, by giving examples that display a concept as well as those that don't display it. Also, carefully varying practice questions so that mechanical repetition is avoided, and thinking is encouraged.

https://www.ncetm.org.uk/resources/50042

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