

**Developing a sustainable and scalable
model for the professional learning of
mathematics teachers**

**The LeMaPS (Lessons for Mathematical
Problem-solving) Project**

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Acknowledgments and dedication

The project team would like to acknowledge the cooperation and hard work of the many teachers and students without whom the project could not have been possible. Also, there are many other organisations and individuals who have supported the project in many ways and continue on the journey that the project has started and which we have travelled together. It has been a privilege to work with them: we thank them all and wish them good luck with their professional learning in the future.

The team would also like to dedicate this summary of its work to the memory of Professor Malcolm Swan who unfortunately was diagnosed with a terminal illness in 2016. He sadly died at a far too early age in April 2017. His input to the work of the project was as always invaluable and he was, and remains, sorely missed. He was both a friend and influential colleague to all members of the team. Without his initial interest in problem-solving and mathematics education research this project would not have happened: it was to Malcolm that our now Japanese colleagues and friends reached out. We would like to take this opportunity to recognise his massive influence throughout every aspect of the work we report here.

Professor Geoff Wake
Dr. Colin Foster
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Executive Summary

The LeMaPS project, in the period 2014-16, worked with clusters of schools and wider networks in England to introduce Lesson Study processes with an emphasis on developing the teaching of mathematical problem-solving. Fundamental to the project was the intention to use a high-quality approach incorporating the essential principles of the Japanese model of Lesson Study. The work of the project was guided by research questions that focused on scalability and sustainability and how best to support the developing groups of teachers.

The project met its objectives of developing and supporting a network of high-quality lesson study groups working to a range of models. This was achieved despite considerable system turbulence caused by significant changes in school structures and governance taking place, including: transfer of responsibilities from local education authorities to individual schools, rapid expansion of academies, and a shift of much of initial teacher education (ITE) from universities to schools.

Also, during the period of the project, a network of 37 maths hubs was established across England, led by outstanding teaching schools. Funded nationally, the hubs are charged with supporting the professional development of teachers of mathematics of all phases in their locality. The project worked substantially with this emerging hub network, along with other networks, to develop Lesson Study for mathematical problem-solving.

Our main conclusions are:

- 1 High-quality lesson study is supported by a culture in which teachers aim to become expert practitioners by researching their classroom practice collaboratively within and across schools and drawing on research knowledge and outside expertise. This involves teachers in spending much more time than usual on the process of knowledge-informed planning, known as *Kyozai Kenku* in Japan. Live observation by all participants in research lessons and the contribution of an expert, known as *Koshi* in Japan, are also essential.
- 2 Lesson Study is often used as a term to describe different modes of teacher professional collaboration, including in its most reduced form simply observing a colleague's lesson. However, when Lesson Study is conducted with high quality, the level of learning can have a sustained impact on teachers' everyday practice, and their ability to relate to each other professionally.
- 3 Developing suitable models of high-quality lesson study at scale requires:
 - Teachers with a good understanding of the Lesson Study process, informed by multiple cycles of participation;
 - Champions, involved in co-ordination and expert roles, to lead the Lesson Study process;
 - Funded projects/training with structured guidance to develop champions with strong understanding and commitment.
- 4 Teachers involved in high-quality Lesson Study consider it an excellent way to learn to teach problem-solving and mathematics more widely.
- 5 High-quality Lesson Study is difficult to achieve for many practical reasons, which include a lack of commitment in some schools to bearing the cost (in terms of time as well as money). This situation may be exacerbated when school leadership experience of Lesson Study has been informed by reduced forms that can be ineffective.

- 6 School leaders can embrace and sustain Lesson Study in mathematics and also as a school-wide mechanism for continued professional development. This requires their understanding of how high-quality lesson study, when aided by external experts, raises the level of learning and level of collaboration among teachers.
- 7 Tools that provide detailed insight into all aspects of the process are helpful in supporting and developing teachers' implementation of Lesson Study. Such tools must relate clearly to the teaching and learning of the subject.

Recommendations for policy and practice

At a system level the Department for Education working closely with the National Centre for Excellence in Teaching Mathematics and drawing on expert advice from the Royal Society's Advisory committee on Mathematics Education should seek to ensure:

- 1 acknowledgement, celebration and reward of expertise in teachers' classroom practice in subject teaching.
- 2 development of a culture in the teaching profession in which teachers at all stages of their career expect to work with colleagues in and across schools to *inquire* into and develop expertise in teaching their subject.
- 3 Universities are expected, and resourced, to contribute to supporting research-informed development of the teaching profession and subject expertise.
- 4 Lesson Study is used in collaborative research partnerships that inform future iterations of curriculum design (as in Japan).
- 5 Initial Teacher Education initiates new entrants to the profession into Lesson Study practices in schools that work with high-quality principles.
- 6 Professional associations and others are supported to contribute to the initiation, development and sustaining of networks of expertise and experts in Lesson Study and subject teaching Details of the network should be communicated widely.
- 7 There is strategic development of a network of acknowledged expertise and experts in the lesson study process *and* research-informed subject expertise in teaching and learning.
- 8 Academics, publishers and others are resourced to support the use of high-quality lesson study in pursuit of expert teaching through the publication of appropriate materials.
- 9 Schools, professional associations and others are supported in sharing what they learn from their lesson study research lessons. This can be achieved by promoting open-house lesson study events and publishing and distributing outcomes from research lessons developed around particular themes.

At a school level:

- 10 School policy and management should value and reward the development of expertise in subject teaching.
- 11 Schools should support a culture of inquiry into what constitutes subject teaching expertise and focus this by establishing specific research themes within the school for periods of time.
- 12 Schools should devote time and energy to the process of Lesson Study.
- 13 Schools should collaborate with each other to ensure cross-fertilisation of ideas.
- 14 Schools should draw on expertise from outside of Lesson Study groups to ensure

that groups can benefit from research and professional knowledge and state-of-the-art practice to stimulate future work of the group.

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0 Preface

In the Foreword to the 2013 Royal Society's Advisory Committee on Mathematics Education (ACME) Report *Empowering teachers: success for learners* that surveyed the landscape of professional development in relation to mathematics teaching, Professor Stephen Sparks, Chair of the Advisory Committee on Mathematics Education (ACME) wrote, "Secondly, our review confirmed that excellent professional development is sustained and specific. Teachers will need access to good quality professional development throughout their careers in order to appreciate the changing uses of mathematics and the latest understanding of student learning."

This is achieved in Japan through Lesson Study. This involves teachers in researching their practice in a systematic and sustained part of their working lives. It is very much part of being a teacher: being a professional (Fujii, 2016; Lewis, 2016; Lewis and Murata, 2006).

Lesson study as a practice meets all of the requirements of what research tells us makes for effective professional learning for mathematics teachers. That is, that professional learning is:

- **Experiential:** stimulating & drawing on teachers' experiences.
- **Sustained:** cycles of planning, predicting, enactment & reflection.
- **Grounded:** practical, well-resourced; related to context & culture.
- **Safe:** teachers able to speak their minds, permission to take risks.
- **Collaborative:** involving networks of teachers & administrators.
- **Informed:** by outside expertise and research.
- **Provocative:** involving both pressure and support.
- **Focused:** attentive to the development of the mathematics itself

(Guskey, 2002; Joubert and Sutherland, 2009; Villegas-Reimers, 2003)

This report gives an insight into the work and outcomes of the project "*Developing a sustainable and scalable model for the professional learning of mathematics teachers*" that drew on Lesson Study as a model to work with teachers to develop teaching for mathematical problem-solving. Thus, the project tackled two areas of importance for teachers as they work to meet the challenges of introducing a new curriculum and changes to assessment at GCSE: it addressed needs in relation to improving teaching and learning whilst also seeking to support teachers develop new models of professional identity.

Lesson Study has been increasingly seen around the world as a model of professional learning that has the potential to make a substantial difference to mathematics teaching and learning (Takahashi and McDougal, 2015). However, for many reasons it cannot be transplanted from Japan directly into another country's education system. Much adaptation is necessary for all manner of societal and cultural reasons.

The project team, based at the University of Nottingham, were very much aware of these at the outset of the project having worked with colleagues from Tokyo Gakugei University both in England and in Japan. This report, therefore, gives some insights into what it is possible to adapt from the Japanese model in English schools. The report is a summative account of the work undertaken, but by its very nature with an aim of considering developing Lesson Study for mathematical problem-solving the journey is not complete, in many ways it has just begun.

1 Introduction, Context and Objectives of the Project

The Project *Developing a sustainable and scalable model for the professional learning of mathematics teachers* is more widely known as the LeMaPS (Lessons for Mathematical Problem-solving) Project. In the period 2014-16 it has sought the possibility of developing new and sustainable models of partnerships that support professional learning in secondary school mathematics. There were two distinctive aspects of the project:

- the Japanese model of Lesson Study informed the central mode of professional learning;
- it concerned problem-solving in mathematics. That is, the process of tackling extended, unstructured problems that require students to model situations with mathematics, make reasoned assumptions, construct chains of reasoning and interpret solutions in context.

Lesson Study

Lesson Study is a concept of professional learning that is focused on practitioner enquiry into teaching, learning and classroom practice. It has become increasingly widely known and adapted for use across geographical and cultural boundaries since the publication of Stigler and Hiebert's book *The Teaching Gap* (1999) that explored what makes for good teaching drawing on evidence from around the world. This identified the model of Lesson Study that emanates from Japan as having significant impact on teaching practices. However, in its adaptation to different cultural settings it is often transformed in ways that may limit its effectiveness because key elements often become watered down or disappear altogether. The project sought to ensure fidelity to key features of the original Japanese model by working closely with researchers of the IMPULS (International Math-teacher Professionalization Using Lesson Study) project based at Tokyo Gakugei University in Japan. A pilot study had been funded by the Bowland Charitable Trust and had taken place during the academic year 2012-13. It investigated the potential of Japanese Lesson Study to support the professional learning of mathematics teachers and had involved two groups of teachers (across schools) and HEI representatives in the collaborative design, delivery and study of "research lessons" based on Bowland mathematics materials (www.bowlandmaths.org.uk/).

“*Lesson Study makes it possible to attain the vision enshrined in the quote from Professor John Hattie: The greatest influence in student progression in learning is having highly expert, inspired and passionate teachers and school leaders working together to maximise the effect of their teaching.*”

Head Teacher who adopted Lesson Study across his school.

The main outcomes of the pilot were:

- i. a small network of expertise in initiating and facilitating lesson study as a process, by HEI and other participants;
- ii. increased expertise in, and understanding of, problem-solving and mathematics teaching more widely by participating teachers
- iii. an interest from all schools to continue (with others keen to join) and embed lesson study in their normal professional development activities.

Problem-solving

The PISA series of international comparative studies has raised the profile of problem-solving in mathematics and this is reflected in recent changes to the National Curriculum (DfE, 2013a) and GCSE specifications (DfE, 2013b), where 20-30% of assessment will in future be expected to have problem-solving as its focus.

Both the pilot and the main LeMaPS study focused on the development of process skills for mathematical problem-solving. This has not been prominent in reported uses of lesson study around the world, including in Japan where the tendency is to use problem-solving to focus on concept development. Indeed, it was because of the acknowledged expertise in problem-solving in the University of Nottingham's Centre for Research in Mathematics Education that the IMPULS colleagues had developed a relationship with the group. Currently in Japan, Lesson Study groups are being used to inform a forthcoming curriculum change that will incorporate mathematical problem-solving in the form of modelling.

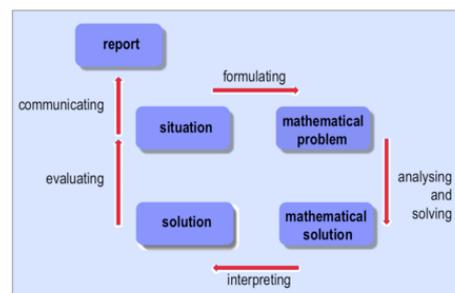
In the LeMaPS project, research lessons for mathematical problem-solving were at the centre of the lesson study cycle. The focus was on students developing mathematical problem-solving skills rather than building specific mathematical content knowledge. The pilot project identified that developing lessons focused on the learning of the key processes in problem-solving is demanding and is much less well understood than developing lessons focused on concept development (Foster, Wake, & Swan, 2014).

The LeMaPS project sought proof-of-concept of new and sustainable models of partnerships that support professional learning in secondary school mathematics for problem-solving. Thus, the project sought to initiate two major changes in the professional development of teachers of mathematics: the introduction of high-quality lesson study *and* the development of lessons in which the progressive improvement in students' problem-solving processes is the focus. Features of the project that ensured participation by teachers included:

- an increased focus on problem-solving in the curriculum,
- that it was informed by expertise in Lesson Study principles by the IMPULS group in Japan
- recognised expertise in problem-solving of the project team.

“ Every single person that I know who has been involved with it has always talked about how impressed they are and how they feel it has changed the way they teach. They also talk of how much they've learned from the deep discussions [they have had] of student learning”

Teacher coordinator of a Lesson Study network.



For teachers, therefore, the project had the potential of multiple benefits focused on perceived areas of need in the professional development.

The project was carried out a time of considerable change in systems of school structures and governance. This provided major challenges for the work of the project that had been proposed at a time when Local Authorities were responsible for the support of schools in their local area and with which the project team had established networks and intended to work. However, during the period of the project, following the Government's white paper of 2010, *The Importance of Teaching* (DfE, 2010), and their objective of moving to a school-led self-improving system, the responsibilities for schools in England transferred from 153 locally elected local education authorities to individual schools. This has resulted in schools, particularly Secondary Schools, becoming academies funded directly by central government, with many being part of multi-academy trusts and around 1200 organisations now responsible for schools. These changes that were underway and substantial at the time of the project led to infrastructure turmoil, with schools developing new and often competitive relationships with each other. The lesson study model the project explored required collaboration between schools and this was therefore consequently not always easy to achieve.

“*Knowing some of the things we know now... it will cause me - it will prompt me - to start to teach differently and to plan differently.*”

Classroom teacher.

A further significant change during the period of the project was the introduction of the network of maths hubs. Just prior to the commencement of the project the government announced that it was to fund over 30 maths hubs (now settling at a total of 37), geographically spread, and led by outstanding teaching schools. These are funded by the Department for Education through the National Centre for Excellence in Teaching Mathematics (NCETM). They have the aim of bringing together mathematics education professionals from “neighbouring schools, colleges, universities, CPD providers, maths experts and employers ... to spread excellent practice even more widely, for the benefit of all pupils and students.”¹

Thus the project took place at a time when schools were undergoing transformation in terms of their relationships with other schools, and during the initiation of a new network of local maths hubs that were charged with leading professional development of mathematics teachers nationally. These changes led to periods of uncertainty about how to support new and emerging networks of teachers engaged in professional development, not least because of changes in funding arrangements for such activities.

These changes provided major challenges for the implementation of the project in the way it had initially been conceptualised. However, they also provided new opportunities, particularly as the new and emerging networks provided the potential for the project to be influential at their inception by mere virtue of the project being active at this time.

Objectives

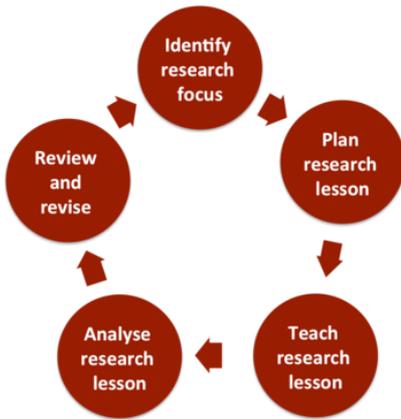
The objectives of the project were framed around two research questions:

RQ1. How can models of professional learning for secondary school teachers, based on lesson study, be developed and sustained within current and changing systems and structures of school governance and funding mechanisms?

RQ2. What supporting tools would help collaborative partnerships to implement lesson study for mathematical problem-solving in effective ways that are both sustainable and scalable?

Fundamental to the work of the project team was therefore a duality of work in research and development. In the next section we describe detail of how this proceeded.

Finally, an important aim of the project was not in fact be a 'project' in the conventional sense. There is much evidence (Adelman and Taylor, 2003) that funded projects in education make an impact whilst they are funded, particularly because they often provide funding for participants. Once this is withdrawn and participants are no longer working on the project the potential for continued impact is inevitably weakened. For this reason, and since the aim of LeMaPS was to develop a sustainable and scalable model of professional learning, it was decided at the outset to minimise for participants the prominence of their involvement in a project and rather to emphasise their involvement in developing their professional practice as teachers focussed on their development of teaching expertise. Consequently, participants in the project were not funded by the project (although many, as we shall report, successfully sought and developed their own means of funding time to accommodate their professional learning).



““ When you really deeply consider a problem - and planning that problem - and consider the pupils' responses, anticipating their responses - it helps you later plan any lesson- every lesson - not just the research lesson but every lesson. ””

Teacher coordinator of a Lesson Study network.

““ The process of Kyozaï Kenkyū challenged me to understand precisely how students would approach the chosen task so that, rather than just anticipating how students would respond, the lesson was designed to draw out the responses that would really deepen conceptual understanding. ””

Teacher

The LeMaPS Lesson Study Approach

Lesson study involves a community of teachers, drawing on research and professional knowledge, working with an outside expert. This community works in a sustained way through cycles of inquiry that focus on developing teaching practices that best support student learning and understanding problem-solving. Each cycle involves five steps.

1. The group begins by identifying a **research focus**. In the LeMaPS project, the focus was problem-solving, so research questions related to the teaching of problem-solving processes, rather than content. For example, “How can we better enable students to select and use mathematical representations when problem-solving?”
2. A **detailed lesson plan** is then produced by a small planning team from the wider lesson study group. Ideally this will include some input from the outside expert. The intention of the plan is to provide a lesson which will enable the group to answer the research question. At the heart of the plan is
 - the careful anticipation of how students will respond to the task and how the teacher might respond in turn;
 - an outline of what pupil progress in problem-solving may look like in the particular aspect of problem-solving being investigated.
 The plan also anticipates what the teacher will do at vital moments in the lesson to progress learning and help students overcome their difficulties.
3. The **research lesson** is taught by one of the planning team. This lesson is observed carefully by all members of the lesson study group, including teachers from other schools in the cluster, the outside expert and student teachers.
4. The research lesson is then analysed in the **post-lesson discussion** involving the teacher and all the observers. The outside expert is expected to make a particularly significant contribution to the post-lesson discussion by providing insights informed by research and in-depth knowledge of mathematical problem-solving.
5. Sometimes this leads to the collaborative development of a revised version of the lesson plan and a **summary of what has been learned**. The lesson plan and outcomes are disseminated around the group. This learning is then taken account of when the cycle begins again, perhaps with a new research question.

The Lesson Study cycle requires substantial time and effort and typically a group may hold one lesson per term.

The LeMaPS project sought to develop Lesson Study Groups and networks nationally.

The project team drew on a number of theoretical ideas to inform the different aspects of their work. Etienne Wenger's work on communities of practice (1998) was one of these. Wenger defines a community of practice as a group of people who share a passion for something that they know how to do and who interact regularly to learn how to do it better. Consequently, a teacher might be considered in their professional life to be working in a community of practice focused quite broadly on teaching. It may also be helpful to consider them participating in a number of different 'sub-communities' such as those that focus more narrowly on aspects of subject curriculum (e.g. mathematics teaching), pastoral care (e.g. form tutor) or leadership as well as within communities beyond their school (e.g. professional associations). From this perspective the aim of LeMaPS was to investigate how to initiate, develop and sustain Lesson Study communities of practice.

Wenger suggests that in developing communities of practice attention needs to be paid to elements of the Domain, Community, and the Practice. In terms of the Domain this requires defining clearly the area of shared inquiry and key issues, in terms of community understanding and being sensitive to the relationships among members and the sense of belonging and in terms of the practice (in the LeMaPS project Lesson Study), paying attention to the body of knowledge and providing supporting methods, stories, cases, tools, documents. The latter aspects of Practice are supported by the LeMaPS toolkit, as are the former to some extent. However, key individuals, "brokers" in Wenger's terms, are also essential. These key networkers need to have a vision of what is to be achieved and have the wherewithal to facilitate the development of a new community in the context in which they work. A key aspect of the work of the project team, therefore, was to develop this connected network of brokers at a national level as champions of Lesson Study in addition to supporting the development of local groups.

2 Methodology: The course of the project

In general terms the research methodology of the project can be considered as a hybrid of both design research that informed the design of tools to support the Lesson Study group and case study research that focused on understanding what works in building effective models of Lesson Study groups working with high quality.

In this section of the report we outline some of the challenges and opportunities that occurred during the timeframe of the project and their implications together with how the project responded and adapted the detail of the methodology to the changing environment.

Challenges and opportunities

As was outlined in section 1 the organisational structures of schools and local networks provided both challenges and opportunities for the project.

The Changes in relation to school governance that occurred during the period of the project are perhaps most starkly illustrated by considering the methodology outlined in the project proposal and contrasting this with the actuality of operationalising the work of the project. The proposal was to work with Local Authorities which at that time were responsible for the governance of most local schools. As a result of this, local mechanisms of support and collegiality were expected to be used to help develop sustainable models of Lesson Study. However, coinciding with the start of the project the government increasingly devolved funding for schools from Local Authorities directly to schools in pursuit of its Academisation programme. During the period of the project the typical groups that existed to support teachers' professional development ceased to exist within Local Authorities and their advisory teachers became scarce if not non-existent. and took up a leadership post in a school in a different county.

“ The importance of planning for me has been a really big impact of lesson study and I think that it has influenced every area of my practice. The more deeply you plan the better the outcomes will be for you and your pupils. ”

Teacher coordinator of a Lesson Study network.

On the other hand, the development of the maths hub network provided a significant opportunity for the project. However, this was also not without its challenges, particularly because of the timing of the development with the initiation of the network lying squarely in the timeframe of the project. The school taking the lead role in each hub was announced on 1 July 2014, six months into implementation of the LeMaPS project. Their work started at the beginning of the 2014 autumn term, but as might be expected it took well into 2015 for their work to start to develop new networks of collaboration and have impact on the system. Thus, the opportunity the maths hub network provided for the LeMaPS project was badly timed in terms of the agreed funded period of the project. The project team is grateful to the Nuffield Foundation for supporting its work for a year beyond the initially agreed time-frame.

Through until 2016 there were no straightforward mechanisms in place that supported developmental projects such as LeMaPS to connect with the hubs at a national level, as the Department for Education (DfE) had identified its own priority projects¹ that the hubs were to engage with. However, the hub leadership at a national level became increasingly aware that many of the hubs' work groups², whilst perhaps not being focused on lesson study, were referring to lesson study in some form as being a mode of working within various initiatives including those within the National Priority Projects, such as the Mathematical Reasoning Project.

Hub leadership at a national level recognised the potential for cross-hub collaboration on a number of issues, particularly informed by the case of LeMaPS, and developed a number of cross-hub collaborative projects. Lesson Study informed by the LeMaPS approach was instigated as one of these projects. During 2016 the project worked closely with these Hub Leads responsible for the development of cross-hub collaboration in lesson study. The LeMaPS project contributed substantially to the four workshops (two in each of two locations - East Midlands East and North West¹) that were held for key participants from approximately half of the Maths Hubs. LeMaPS was central to the planning and in providing materials from the toolkit to ensure that both high quality lesson study and problem-solving approaches were central to the experience of participants. These participants have now been encouraged to initiate and develop their own collaborative lesson study communities.

In summary, the demise of existing support networks focused on local Authorities and introduction of the maths hubs brought to the project:

- a) **Challenges** of (i) badly aligned timescales of development and (ii) the lack of support for national developments that are initiated and led by organisations such as universities and not supported as part of the government's (DfE's) agenda.
- b) **Opportunities** to work directly with emerging communities of mathematics teachers engaged in hub-supported cross-school collaboration in pursuit of professional learning relating directly to classroom practice.

Building Lesson Study groups and networks

Developing a national network of experts and expertise in Lesson Study Practices for Problem-solving was a priority of the project. This was fundamentally and primarily supported by our connection with Tokyo Gaku University's project IMPULS (International Math-teacher Professionalization Using Lesson Study). Over the period of the project a formal memorandum of understanding was exchanged between the universities of Tokyo Gaku and Nottingham. This was of major significance in terms of supporting the development in England of a

“ It makes for very effective professional development. ”

Consultant working with Head Teachers and Heads of Maths to establish Lesson Study clusters..

¹ National Priority Projects are projects that the hubs have to undertake on behalf of the DfE (for example, The Shanghai Teacher Exchange Programme).

² Work Groups are the programmes the individual hubs develop to run professional development events local to the hub.

“ Lesson study has been how we've made progress with problem-solving. In my team back at my school there are some of us who have been involved with lesson study for problem-solving and there are other people who haven't. It's been very, very clear to me how much further along in understanding and confidence in doing problem-solving lessons those of us who have been involved with lesson study have got when compared to those who haven't - it's been very striking. ”

Head of Maths implementing Lesson Study across maths departments in a Multi-Academy Trust

network of Lesson Study champions at a national level. Over the course of the project 28 educators (6 in mid-2014, 11 in mid-2015 and 11 in mid-2016) with different roles (teachers, advisers, Higher Education Initial Teacher Educators, Maths Leads in the growing number of multi-academy Trusts and so on) in the UK system attended the IMPULS annual lesson study immersion programme. Each cohort was prepared by a one-day conference prior to their visit and in the case of participants in Summer 2015 a two-day summer school on mathematical problem-solving was held in Nottingham. All participants have also been invited to, and most if not all have attended, a number of other conferences/workshops organised by the LeMaPS team, as well as calling upon the team, and each other, to be involved in different aspects of their local Lesson Study activity.

The network of maths hubs, whilst helpful in providing access to, and allowing us to work with, key players regionally/locally was not in its initial stages helpful to the project at a national level This was understandable, as the remit of each hub was to work at a geographically local level and in initial stages of their work each had to develop new models of working and establishing their own networks and relationships. In the first instance the LeMaPS project developed relationships with the two most local hubs and a hub in the East of England and supported their work in establishing a LeMaPS model of Lesson Study. This was useful preparation for when the Hub leadership at a national level recognised the need to have a structure which allowed for cross-hub collaboration. Consequently, a number of cross-hub collaborative projects were developed with funding to support their collaboration. Lesson Study informed by the LeMaPS approach was instigated as one of these projects. This development in 2015-16 was very helpful in allowing for both sustainability and scalability. Four workshops (two in each of two locations) were held for key participants from approximately half of the 35 Maths Hubs. LeMaPS was central to the planning and provided materials from the toolkit to ensure that both high quality lesson study and problem-solving approaches were central to the experience of participants. These participants have been encouraged to initiate and develop their own collaborative lesson study communities.

The Leads of this Maths Hub development in effect took over the organising of a national development project in Lesson Study, based on the LeMaPS approach, with reach across the Maths Hubs. This is work similar to that carried out by the project team of LeMaPS, but in this case ownership is firmly within the national structure of professional learning for mathematics teachers (the Maths Hubs). In this sense, both important features of scalability and sustainability have been ensured (at least during the lifetime of the hubs).

Further to this development is the ongoing work of the Maths Hubs on part in lesson study. This requirement has been partly instigated because of the effectiveness of the LeMaPS approach that has been seen in other aspects of the work of the Hubs. This approach has been

essential in National Priority Projects undertaken by the Hubs that are developing approaches to teaching Multiplicative Reasoning and Mathematical Reasoning (web ref). Importantly the Maths Hubs at both national and local levels have funded teachers to work with LeMaPS, including funding teachers to visit Japan and facilitate lesson study communities on their return.

Ultimately the development of local Lesson Study groups is the *raison d'être* of the LeMaPS project. As outlined in Section 1 of this report, the context in which the project worked deviated considerably from that expected at the time of proposing the project. The project team sought to work within the new and emerging systems and structures of school governance and directed efforts in working with both hub-supported Lesson Study groups and those emergent within other networks and communities. The models that have been developed are varied in the source of their instigation, support and expertise. They include Lesson Study groups working within:

- A Multi-Academy Trust (x 3)
- A project developed and funded by the London Challenge and facilitated by a LA Advisory Teacher
- A project initiated by a Consultant with funding from the LA that used to employ them
- A university-supported city-wide intervention
- A Hub-based work group
- A University ITE course

Brief details of these different models are given in the next section of this report as illustrative of the variety of possibilities that can be offered to post-project newcomers. They show different models of collaborative lesson research advocated by the project and we suggest practical ways in which they might facilitate their development.

Dissemination

Dissemination was an important aspect of the project, not, as is often the case with funded projects, to report outcomes, but rather to support the scaling of sustainable networks. Dissemination activity was instigated not only under the auspices of the project, but also through other events, often as part of the work of professional associations. This included presentations that drew on the tools of the developing toolkit to a wide range of national and professional associations.

The project also instigated a number of well attended conferences/seminars/ workshops of its own held both in Nottingham but also in London and Cambridge. A range of articles for professional audiences were developed as part of the project and the Nottingham team presented their research at a number of national and international academic conferences.

Details of this dissemination work can be found as an appendix to this report.

3 Outcomes

“ One of the outcomes is the conversations we’ve hadand as a team for the last three or four weeks are starting to happen more naturally.”

Maths teacher and Assistant Principal

The project met its objectives of developing and supporting a network of high-quality lesson study groups working to a range of models. This was achieved despite considerable system turbulence caused by significant changes in school structures and governance taking place, including: transfer of responsibilities from local education authorities to individual schools, rapid expansion of academies, and a shift of much of initial teacher education (ITE) from universities to schools.

Also, during the period of the project, a network of 37 maths hubs was established across England, led by outstanding teaching schools. Funded nationally, the hubs are charged with supporting the professional development of teachers of mathematics of all phases in their locality. The project worked substantially with this emerging hub network, along with other networks, to develop Lesson Study for mathematical problem-solving.

By the end of the project, there emerged a critical mass of maths education leaders who believe that high quality lesson study is key to many of the issues faced by the profession. They believe that lesson study will help lead to dramatic improvement in mathematics education. They also see that it will help create a collaborative and reflective ethos both within and across schools, and make teaching a more rewarding profession.

The final conference of the project brought together some of these education leaders who include teachers, Senior leadership Teams and schools and Multi-Academy Trusts, hub leads, consultants and university staff (some of whose expertise go well beyond problem-solving or maths secondary education). They reported that the project, through the various aspects of its activity had initiated lesson study groups at a number of different levels including within schools and Multi-Academy-Trusts, within Initial Teacher Education programmes, and in a modified form as part of the priority project work of the Hubs. In the latter case the lesson study principles have been used to form the work of “teacher research groups” investigating teaching and learning towards learning that supports problem-solving with multiplicative reasoning, mathematical reasoning and the more complex problems that are emerging to assess problem-solving at GCSE. ensures that the work that was initiated

Some of the emerging community are coming together, to continue to pursue the work initiated in Lesson Study by the project, under a flagship organisation ‘Collaborative Lesson Research’. This aims to continue the work started by the project, to disseminate high quality lesson study practices more widely not only for mathematical problem-solving, but more generally for mathematics and beyond.

Here we give, as illustrative, brief details of a number of different models of lesson study groups/networks that the project has generated

and supported. There are others that work to other variations of the collaborative model of lesson study that LeMaPS promoted and the toolkit gives access to some of these and collaborative-lesson-research.uk intends to develop further the sharing of these different models.

Models of lesson study groups/networks that have been developed and supported during the LeMaPS project.

Model 1

Multi-Academy Trust (collaboration between academies)

Uses lesson study within structures of collaboration across the Trust, e.g. in PD programmes that are developed in an inter-Trust “market”. All teachers are obliged during a year to participate in one of these programmes. Maths teachers have the option of opting into the programme.

Key contextual dimension:

There is a desire from leadership to promote inter-Trust collaboration and develop a culture of teacher inquiry.

Key Actors:

Head of Maths is a member of Senior Leadership and is an Enthusiast for problem-solving (relating to new National Curriculum and assessment demands). Mathematics Teacher supported this work and enthusiasm at an organisational level and in implementation.

Outcome:

Across the Trust the LeMaPS teachers established a lesson study programme focussed on lesson study for problem-solving that teachers from across 12 schools participate.

Model 2

Consultant (with ex-LA links)

Developed Lesson Study groups to meet local priorities in mathematics education. Worked with local Senior Managers in Schools to draw on some LA funding to facilitate start-up and dissemination events.

Key contextual dimension:

Local Authority has a residual ‘responsibility’ to support local schools and has designated remaining small pockets of funding to support the teaching and learning of mathematics

Key Actor(s):

Consultant who was previously mathematics advisor to the LA and with strong networks with maths teachers (including Heads of Departments) across schools and also school leadership. Has expertise in applying for funding to support work in schools.

Outcome:

Established programme of work led by consultant that maintained a local authority focus of collaboration throughout the timeframe of the project. This was celebrated at an annual lesson study conference instigated and organised by the consultant during the timeframe of the project.

Model 3**LA Advisory Teacher**

Developed Lesson Study to support LA schools with grant captured from external source (London Challenge Fund) with support from the LeMaPS project team. Lesson Study model followed very closely Japanese model and outcomes evaluated as condition of funding.

Key contextual dimension:

One of few remaining LAs with central funding to support LA schools quite widely. (Low achieving LA)

Key Actor(s):

Energetic LA Advisory Teacher who is well-connected within LA with mathematics teachers and also nationally active in National Association of Mathematics Advisers.

Outcome:

This project continued during the timeframe of the project and was reported at the London Challenge Fund annual conference. Work initiated by this project has been substantially maintained by the schools involved.

Model 4**University Initial Teacher Education (PGCE)**

Developed Lesson Study to support ITE students' learning in relation to lesson planning, pedagogy and didactics. Students worked collaboratively in small groups across schools and with their mentors. Programme culminated at the end of the year with a whole group (50 students) lesson study event.

Key contextual dimension:

Developed from on-going research within the mathematics education research group and ITE professional values of 'teachers as learners'.

Key Actor(s):

Lead university tutor working with research colleagues who have been using lesson study as a method of collaborative research with local schools. Good connections with local maths hub lead involved in this research provides a growing culture of lesson study in the region.

Outcome:

This introduction to lesson study continues as part of the University's ITE/PGCE route.

Model 5**Hub-based**

Working directly to develop Lesson Study as a mode of professional learning for mathematical problem-solving working directly with the LeMaPS project

Key contextual dimension: (professional cultures)

Hub lead school is a prominent teaching school in the region and has a strong commitment to 'teaching as learning'. Strong connections with the project team university.

Key Actor(s):

Work initiated by young teacher who had been part pilot study. Worked with head of Maths also Maths Hub lead.

Outcome:

Lesson Study developed within a local school cluster and scaled up substantially and now working at a national level.

LeMaPS Toolkit

The project developed a toolkit to support lesson study groups in a flexible way. This provides tools that are related to the process of lesson study – in the context of mathematical problem-solving. Of particular importance in this toolkit, available at www.lemaps.org are the video sequences that provide insight into high quality Lesson Study in practice.

Teachers that the project worked with, report that the toolkit is used by them when working with colleagues to help them engage in in the different phases of their lesson study. For example, all groups indicate that an issue that they have to confront at an early stage of their lesson study work is that of identifying and refining their research question. The toolkit provides advice in relation to this and a video segment that shows a group of teachers working towards identifying a research question as part of their lesson for mathematical problem-solving. In this way the toolkit can help exemplify, and provide insights into, the reality of how the lesson study process can be ‘made real’

The Diagram that follows outlines the structure of the toolkit.

LeMaPS Toolkit overview

Why is Lesson Study for Problem-solving needed?

Lesson Study for Professional Development

Lesson Study for Problem-solving

Lesson Study for enhancing the curriculum

How can I create and sustain a lesson study group?

Organising a lesson study community

Running a workshop on lesson study

How can I plan for a research lesson?

Overview

Planning Meeting 1

Identifying the research focus

Selecting a suitable problem-solving task

Planning Meeting 2

Working on the task and anticipating student responses

Recognising progress

Planning Meeting 3

Completing the lesson plan

What happens on the day of the research lesson?

Organising the research lesson

The pre-lesson briefing

Conducting and observing the lesson

Planning and leading the post-lesson discussion

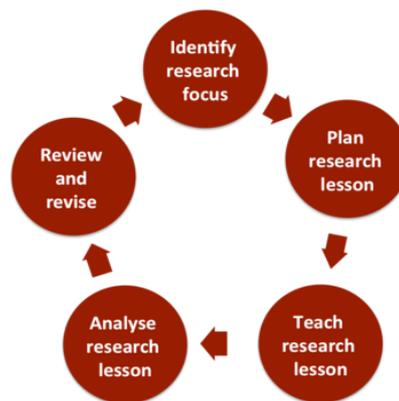
The role of the outside expert

What happens after the research lesson?

Reflecting on the research lesson and revising and disseminating the lesson plan

What can I learn from what others have experienced?

Case studies and lessons learned



4 Conclusions

There is a recognition that while problem-solving remains a critical area of focus for mathematics teachers, it does not provide a wide enough platform for schools and teachers to experiment with and become experts in lesson study practices. It is vital to persuade school leadership to support the practice. There is more chance of success in this if lesson study has a broader base and is established for professional development for all teachers.

In order to achieve this, the best way forward may be to use lesson study in mathematical problem-solving in a strategic way to influence both mathematics education and more generally other subject leaders. It remains important to focus on problem-solving for these reasons:

- It is an area within mathematics, which comes closest to other subjects – many of the tools developed for problem-solving can be applied more easily in other subject areas.
- This is an area that is of growing importance in the mathematics curriculum and in assessment at GCSE (from 2017 onwards <https://www.gov.uk/government/publications/gcse-changes-a-summary/summary-of-changes-to-gcses-from-2015>)with a known need for professional development (see for example <https://nrich.maths.org/pd>)

Further to this we note that both lesson study and mathematical problem-solving are of continuing interest internationally in education research and mathematics education research respectively. For example, the World Alliance of Lesson Study organisation is receiving increasing interest from the education research community and the work of LeMaPS and the Collaborative Lesson Research group has been well-received in this forum as well as at other international research conferences such as the annual conferences of the International Group for the Psychology of Mathematics Education (IGPME) and the International American Educational Research Association (AERA).

The project benefitted from opportunities to consider a wide range of different models of lesson study implementation and details of these were found by interviewing many participants across all roles including those leading lesson study in a variety of circumstances. School leaders as well as those supporting lesson study more generally, for example, in the maths hubs and NCETM were also consulted. Within the project timeframe we also had the opportunity to interview lesson study participants and leaders in Japan including those working at a prefecture (regional) level, in subject associations, university research groups, in schools etc. Our analysis of the different contexts led us our *main conclusions*:

- 1 High-quality lesson study is supported by a culture in which teachers aim to become expert practitioners by researching their classroom practice collaboratively within and across schools and drawing on research knowledge and outside

expertise. This involves teachers in spending much more time than usual on the process of knowledge-informed planning, known as *Kyozai Kenku* in Japan. Live observation by all participants in research lessons and the contribution of an expert, known as *Koshi* in Japan, are also essential.

- 2 Lesson Study is often used as a term to describe different modes of teacher professional collaboration, including in its most reduced form simply observing a colleague's lesson. However, when Lesson Study is conducted with high quality, the level of learning can have a sustained impact on teachers' everyday practice, and their ability to relate to each other professionally.
- 3 Developing suitable models of high-quality lesson study at scale requires:
 - Teachers with a good understanding of the Lesson Study process, informed by multiple cycles of participation;
 - Champions, involved in co-ordination and expert roles, to lead the Lesson Study process;
 - Funded projects/training with structured guidance to develop champions with strong understanding and commitment.
- 4 Teachers involved in high-quality Lesson Study consider it an excellent way to learn to teach problem-solving and mathematics more widely.
- 5 High-quality Lesson Study is difficult to achieve for many practical reasons, which include a lack of commitment in some schools to bearing the cost (in terms of time as well as money). This situation may be exacerbated when school leadership experience of Lesson Study has been informed by reduced forms that can be ineffective.
- 6 School leaders can embrace and sustain Lesson Study in mathematics and also as a school-wide mechanism for continued professional development. This requires their understanding of how high-quality lesson study, when aided by external experts, raises the level of learning and level of collaboration among teachers.
- 7 Tools that provide detailed insight into all aspects of the process are helpful in supporting and developing teachers' implementation of Lesson Study. Such tools must relate clearly to the teaching and learning of the subject.

5 Recommendations

High quality lesson study has the potential to develop the professional knowledge base of teachers in relation to the different aspects of what is necessary to develop facility in mathematics: 1) Mathematical concepts; (2) Mathematical principles including properties, rules, formulas and theorems; (3) Mathematical skills including operations by following certain procedures and steps, construction of figures, and data processing; and (4) Mathematical thinking methods underlying mathematical contents and skills. The LeMaPS project tackled a specific aspect from the latter of these categories, that of problem-solving. This is an area of the curriculum that was at the time of the project gaining in importance in the mathematics curriculum and it was found that Lesson Study supported teachers in their development of knowledge of how to improve their teaching of problem-solving. To ensure that lesson study might be used more widely to support such growth in teachers' knowledge for teaching we make recommendations that might inform policy and practice at both a system and school level. These focus in general in developing a system in which increased knowledge of teachers in relation to effective teaching is valued and this is achieved by teachers engaging in reflexive practice in collaborative groups that seek answers to professional questions.

Recommendations for policy and practice

At a system level:

- 1 Policies that underpin career structures for teachers should acknowledge, celebrate and reward expertise in teachers' classroom practice in subject teaching rather than in the main setting expectations that career progression should focus on management.
- 2 It is important to develop a culture in the teaching profession in which teachers at all stages of their career expect to work with colleagues in and across schools to *inquire* into and develop expertise in teaching their subject. Where possible organisations such as NCETM, subject associations, school alliances and so on should work strategically, and where possible, in collaboration, to support the development of such a culture.
- 3 Universities and other emergent organisations such as 'Research schools' should be expected and resourced to contribute to supporting research-informed development of the teaching profession and subject expertise.
- 4 Lesson Study should be used in collaborative research partnerships that inform future iterations of curriculum design (as in Japan).
- 5 Where it isn't already the case, Initial Teacher Education should initiate students into Lesson Study practices *in schools that work with high-quality principles*.
- 6 Professional associations and others should contribute to the

initiation, development and sustaining of networks of expertise and experts in Lesson Study and subject teaching. Details of the network should be communicated widely.

- 7 There should be strategic development of a network of acknowledged expertise and experts in the lesson study process *and* research-informed subject expertise in teaching and learning.
- 8 Academics, publishers and others should support the use of high-quality lesson study in pursuit of expert teaching through the publication of appropriate materials.
- 9 Schools, professional associations and others should be encouraged to share what they learn from their lesson study research lessons. This can be achieved by promoting open-house lesson study events and publishing and distributing outcomes from research lessons developed around particular themes.

At a school level:

- 10 School policies and managers should explicitly recognise, value and reward the development of expertise in subject teaching rather than only operating models of career progression that have expectations that value management outside of the classroom.
- 11 Schools should support a culture of inquiry into what constitutes subject teaching expertise and focus this by establishing specific research themes within the school for periods of time.
- 12 Schools should devote time and energy to the process of Lesson Study.
- 13 Schools should collaborate with each other to ensure cross-fertilisation of ideas.
- 14 Schools should draw on expertise from outside of school-based Lesson Study, such as expertise, for example, from mathematics education researchers, lead practitioners working with the NCETM, maths leads working across Multi Academy Trusts and so on, to ensure that groups can benefit from research and professional knowledge and state-of-the-art practice to stimulate future work of the group

6 Next steps – moving beyond proof of concept

Our conclusion is that the model of lesson study suitable for scaling up and sustaining for the UK is the high-quality version, because of the powerful impact it can have on individual teachers and the community of teachers. The impact is felt by individual teachers, as they change their everyday practice, but also as they learn to communicate with each other professionally. Lesser versions of lesson study may be conducted once or twice, even for several years, but will generate less and less impact over time. The high-quality version is our proof of concept model.

The project has identified what the key elements are for such a model of lesson study. The project has also produced essential tools that clarify these key elements for those interested in conducting lesson study in mathematical problem-solving.

Several of these 'key elements', however, require creation of professional capacity 'en masse' that does not exist in the UK today. For example, it is essential to have an army of 'external advisors' who can raise the level of discussion in post lesson discussions through their final commentary in post-lesson discussions. At the moment, there are a handful of people who can do so for mathematical problem-solving, by the virtue of their past experience of working directly with IMPULS, but even they are not ready to 'train' others in replicating what they do. It is essential that we go further in developing processes to build the needed capacity systematically and eventually without the recourse to colleagues in Japan. This requires 'development work' well beyond proof of concept.

Fortunately, the project has generated a critical mass of maths education leaders who believe that high quality lesson study is key to many of the issues faced by the profession. They believe that lesson study will help lead to dramatic improvement in mathematics education. They also see that it will help create collaborative and reflective ethos both within and across schools and networks. This has the potential to make teaching a more rewarding profession.

These people, who include teachers, SLTs, hub leads, consultants and university staff (some of whose expertise go well beyond problem-solving or maths secondary education), are now coming together under a flagship 'Collaborative Lesson Research.org' to continue the work started by the project. They are ready to undertake further development work to disseminate high quality lesson study practices more widely for mathematical problem-solving and mathematics more widely and through that to influence SLTs and other subjects.

7 Update

The LeMaPS project ran its course with a number of interesting and successful outcomes as reported here. Perhaps the most substantive of these was the development of a small but determined community of leaders in lesson study convinced by the effectiveness and potential for professional transformation that might be afforded by high quality lesson study of the form central to the project. This group has been working to establish and strategically grow their community. Most of this activity has been under the auspices of the group 'Collaborative Lesson Research' (www.collaborative-lesson-research.uk).

This organisation is establishing charity status and developing a business plan that will support its future activities. To date such activity has involved supporting a number of day conferences hosted at the Institute of Education, University College London, and at Sheffield Hallam University. The group has also continued to work with colleagues from the IMPULS project to hold a number of workshops looking in detail at important aspects of lesson study such as supporting *kyozai kenkyu* (detailed lesson planning), developing curriculum knowledge expertise. These have been held in Manchester and London in 2017 (one week), 2018 (two days) and there are arrangements to have a repeat week in December 2019. Some of the activity undertaken and support of lesson study groups has been achieved through the collaboration between the project team at the University of Nottingham and the academic group IMPULS at Tokyo Gakugei University. Central to this was the six-month sabbatical of Professor Keiichi Nishimura hosted by the University of Nottingham in the period April 2016 – September 2016. During this time a substantial amount of work was supported across the lesson study networks as well as academic collaboration between the academics in Japan and the UK. This resulted in a symposium, *Understanding Lesson Study from Different Theoretical Perspectives*, being held at the American Educational Research Association Annual Meeting in 2018.

Further to this, the Nottingham team, with Dr Colin Foster now at the University of Loughborough, was awarded an ESRC Japan Connections Grant (ES/S014292/1), Exploring socially distributed professional knowledge for coherent curriculum design. This project will run from January 2019 – March 2020, using lesson study to explore how lesson study and mathematics teaching in mathematics in Japan is supported by didactical tools that are integrated throughout Japanese text books for students, teacher guides and research guides that support lesson study research lessons.

The LeMaPS team, and more widely the lesson study community that it has generated, now focused around the Collaborative Lesson Research group continues on its journey

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Appendix.

Dissemination.

During the lifetime of the project presentations were made to the following national and professional organisations:

- The Association of Teachers of Mathematics
- Mathematical Association
- The Specialist Schools and Academies Trust
- The Core Maths Support Programme
- The National Association of Mathematics Advisors
- The Maths Hubs Forum
- World ORT

The project held a number of well-attended conferences/seminars/workshops of its own including:

What can we learn from Japan about professional development for mathematics teachers? (King's College, December 2014)

Japanese Lesson Study: Critical role of external experts (UCL Institute of education, September 2016)

Teaching as a learning profession: policy issues (University of Cambridge, September 2016)

Developing High Quality Lesson Study in the UK: Lessons from the past and shaping the future (University of Nottingham, November 2016)

Another aspect of dissemination related to the work of the project was the development of a number of articles written by a number of maths educators closely connected with lesson study groups working on the project including:

Blinko, J. (2015). What is a book-look looking for? *Mathematics Teaching*, 246, 12-16.

Lewis, M. (2015). Assessment: Beyond right and wrong. *Mathematics Teaching*, 249, 21-23.

Archer, R. (2016). Lesson Study, a trip to Japan. *Mathematics Teaching*, 250, 36-40.

Simmons, M. (2016). The role of 'koshi' in UK lesson study. *Mathematics Teaching*, 250, 41-43.

Seleznyov, S. (2016). Lesson study in the UK: possibilities and challenges. *Equals*, 21(1), 5-10.

The project team also used the research of the project to develop theoretical understanding of lesson study and presented at a number of national and international conferences including:

- British Society for Research in Learning Mathematics (BSRLM)
- The annual meeting of the International Group for the Psychology of Mathematics Education (PME)
- Congress of European Research in Mathematics Education (CERME)
- The annual meeting of the American Educational research Association (AERA)
- World Alliance of Lesson Study (WALS)

Work, to date, resulted in the following publications:

- Foster, C., Wake, G., & Swan, M. (2014). Mathematical knowledge for teaching problem-solving: Lessons from lesson study. In S. Oesterle, C. Nicol, P. Liljedahl, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36*, Vol. 3, pp. 97–104. Vancouver, Canada: PME.
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Further to this the team collaborated with colleagues at Stellenbosch University, South Africa to gain a British Academy Grant to develop a lesson study model. The project *Towards a sustainable model of professional practice for mathematics student teachers* successfully developed and implemented a new model of professional practice for

student teachers at Stellenbosch University which could serve as example in initial teacher education more widely in South Africa. More details can be found at

<https://lessonstudystellenboschuniversity.wikispaces.com/>