Testing for Unit Roots in the Presence of a Possible Break in Trend and Non-Stationary Volatility*

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Abstract

In this paper we analyse the impact of non-stationary volatility on the recently developed unit root tests which allow for a possible break in trend occurring at an unknown point in the sample, considered in Harris, Harvey, Leybourne and Taylor (2008) [HHLT]. HHLT's analysis hinges on a novel break fraction estimator which, where a break in trend occurs, is consistent for the true break fraction at rate $O_p(T^{-1})$. Unlike other available estimators, however, when there is no trend break HHLT's estimator converges to zero at rate $O_p(T^-1/2)$. In their analysis HHLT assume the shocks to follow a linear process driven by IID innovations. Our first contribution is to show that HHLT's break fraction estimator retains the same consistency properties as demonstrated by HHLT for the IID case when the innovations display non-stationary behaviour of a quite general form, including, for example, the case of a single break in the volatility of the innovations which may or may not occur at the same time as a break in trend. However, as we subsequently demonstrate, the limiting null distribution of HHLT's proposed unit root statistic based around this estimator is no longer pivotal in the presence of non-stationary volatility. Associated Monte Carlo is presented to quantify the impact of various models of non-stationary volatility on the both the asymptotic and finite sample behaviour on the test. A solution to the identified inference problem is then provided by considering wild bootstrap-based implementations of the HHLT test, using the trend break estimator from the original sample data. The proposed bootstrap method does not require the practitioner to specify a parametric model for volatility. The bootstrap is shown to perform very well in practice across a range of non-stationary volatility models.

Keywords: Unit root test; quasi difference de-trending; trend break; non-stationary volatility; wild bootstrap.

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