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Abstract

Is it better to have the first or the last word in debates? The answer, in two-round debates like common law trials, depends on whether litigants share the available evidence. If so, then litigants never prefer to present first, but may prefer to present second. However, litigants may otherwise prefer to present first because doing so replicates the follower's ex ante optimal commitment.

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1. Introduction

According to conventional wisdom, advocates who compete to persuade a listener should try to present the first or the last argument in a debate: the first speaker anchors a listener's interpretation of subsequent arguments; whereas the final argument is influential if listeners only remember the last word.

Cognitive limitations are doubtless important in some debates, but the order of presentation can be significant for strategic reasons. According to David Axelrod, Obama's 2012 campaign manager, the decision to start running attack ads early contributed to the campaign's success.[1] This effect might be due to anchoring; but we think that seizing the agenda is more about steering the subsequent debate. On the other hand, Obama's tactics in the second Presidential debate in 2012 suggest that having the last word may be advantageous. Romney's claim that 47% of voters pay no income tax had been leaked before the debate.[2] Romney had presumably prepared a response; but Obama's success in the debate turned on his decision not to raise the issue till his concluding remarks.[3] This effect might be due to voters' short-term memory; but most viewers likely knew about Romney's claim before the debate itself. Finally, FOMC chairs have selected alternative orders: Greenspan chose to speak and to vote first at FOMC meetings, whereas Bernanke spoke last.[4] We explore the strategic consequences of the first and the last word in debates by analyzing a game-theoretic model in which the listener is not cognitively limited.

In multi-round debates, such as those between Presidential candidates, a given speaker could present first and last; and the middle word might be most important. By contrast, she has either the first or the last word in two-round debates with alternating speakers. We focus for simplicity on the latter case, as instanced by common law trials, where the defendant conventionally presents its evidence second. Common law trials are an attractive application of two-round debates because the extensive form and payoffs are each commonly known and well defined; the role of experienced attorneys and judges suggests that equilibrium plausibly predicts choices; and because psychologists have used mock juries to assess the effect of varying the order. We ask when the conventional order serves the interests of either litigant or of the judge/jury?[5]

We address this question by comparing equilibrium outcomes across two *fixed order* games, which only differ according to the identity of the first mover (or *leader*). Nature starts each game by choosing the facts at issue (the *state*), including whether the defendant is factually guilty, and then privately informs each litigant of its evidence set: the initial endowment of witnesses that the litigant can call.[6] The leader then presents any selection

[1] In Axelrod's words: "We defined the race and Governor Romney before the conventions, and he was digging out of that hole for the remaining two months."

[2] Mother Jones: "SECRET VIDEO: Romney tells millionaire donors what he REALLY thinks of Obama voters", Sept. 17, 2012.

[3] Slate: "When candidates attack: Obama won Tuesday night's slugfest, but Romney will live to fight another day", Oct. 17, 2012.

[4] See, for example, Blinder (2009).

[5] While defendants usually have the last word, a judge could vary the order of presentation if the resulting trial satisfied broader due process requirements.

[6] Formally, a witness is a strict subset of states that contains the realized state, as in Milgrom (1981).

from its evidence set in public; the follower responds by presenting witnesses that the leader called and/or a (possibly empty) subset of its available witnesses; and the judge/jury (henceforth J) then acquits or convicts after observing the evidence presented. Following procedure in common law trials, the leader has the burden of proof: it loses the case if it does not present any evidence. The defendant always wants to be acquitted; the plaintiff/prosecution always wants a conviction; and J wants to avoid miscarriages of justice. Fixed order games typically have several equilibrium outcomes; so we say that a player prefers an order of presentation if it expects (before the state and evidence sets are realized) to be better off at every equilibrium of one game than at every equilibrium of the other game.

Our main results concern the benchmark case in which both litigants are endowed with the same evidence: a case which we call *discovery*. We first prove that litigants cannot prefer to present first, and then demonstrate by example that litigants can prefer to present second: the intuition, roughly, is that the follower can respond flexibly to the evidence that the leader has presented: 'roughly' because our criterion involves a comparison of outcome correspondences. We generalize this example by providing necessary conditions for litigants to prefer to present second: conditions which turn on the witnesses available across different evidence sets. Finally, we show that J only prefers an order if litigants prefer to present second.

None of these results (fully) generalize to games where the two litigants may be endowed with different available evidence. We provide three reasons, illustrated by example, why litigants might then prefer to lead. First, the fact that some evidence is initially available to a given litigant may inform J about the state; and a litigant can only prove availability by presenting that evidence. A leader who presents a witness available to both litigants prevents the follower from proving availability; so litigants might prefer to lead. Second, as J does not observe which witnesses are available to each litigant, its verdict must depend on the evidence presented. As litigants always share available evidence in discovery games, litigants could present the same evidence, irrespective of the leader's identity. This is not generally true outside discovery games. As the leader must meet a burden of proof, pooling is only possible if some evidence is available *to the leader* at two evidence sets: so the identity of the leader may then matter. The third reason applies to games in which each litigant is uncertain of its rival's available evidence. Prima facie, resolution of this uncertainty seems to reinforce the advantage of presenting second; but, as we show, this intuition turns out to be wrong. In this example, the follower does not gain from flexibility at any of its information sets; but its choice at one evidence set pair adversely affects the verdict at another evidence set pair.

In fact, all of the examples share this feature. The flexibility gained by responding on a case-by-case basis (viz. after observing its available evidence) may be ex ante disadvantageous because best responses at one evidence set pair may impose negative externalities at other evidence set pairs. Loosely speaking, the follower might ex ante gain by committing to its strategy. We formalize this intuition by analyzing a game in which the follower commits to its strategy before observing its available evidence. We show that, in each of the three examples, the equilibrium outcomes of this commitment game coincide with the equilibrium outcomes of the game in which a litigant presents first. In other words, litigants prefer to present first because play then replicates the effect of prior commitment.

As we noted above, litigants only prefer to present second in discovery games if evidence sets intersect in particular ways. We use variants on the examples above to provide different reasons why litigants might prefer to present second when they are not known to share available witnesses. We also demonstrate that J alone may prefer an order in non-discovery games, contrary to our result for discovery games.

Finally, we adopt an ex post perspective to the choice of order, focusing on cases in which litigants share the available evidence. Specifically, we analyze a game in which some specified litigant first observes the available evidence and then chooses whether to present first. Play then proceeds according to the rules of the fixed order game described above. We call this a *variable order game*. We prove that every equilibrium outcome of a fixed order game is an equilibrium outcome of the associated variable order game, and that the specified litigant cannot prefer to play the variable order game to always presenting second. We then show by example that a litigant may prefer to always present second than to play the variable order game.

We survey the related literature in Section 2, present our model of trials in Section 3, and analyze play in discovery games in Section 4. Section 5 considers games which are not played under discovery, and interprets the results therein by analyzing commitment games. Section 6 characterizes play in variable order games, and assesses the advantages of an option to choose the order. We summarize our results in Section 7, and then consider their applicability to debates which lack some features of trials such as different stopping rules. We also show that a litigant may prefer to debate with or indeed delegate presentation to a rival than to alone present evidence. We collect longer proofs in an Appendix.

2. Related literature

Our model is part of a literature on persuasion games, and is particularly related to papers which study debates in which litigants cannot directly prove the state:

The literature on persuasion games started with Milgrom (1981), who shows that a single litigant with state-independent preferences over the verdict separates if every state can be directly proved. Milgrom and Roberts (1986) extend this canonical model in two relevant directions. First, suppose that J is uncertain of the evidence available to a single litigant in each state. J could then not draw skeptical inferences which penalize presentation of imprecise evidence, and might therefore not learn the available evidence at every state in any equilibrium. Milgrom and Roberts also study debates, in which litigants with conflicting preferences over the verdict simultaneously present evidence to a judge, who is modelled as an automaton. Competition between the litigants may then reveal the state, even if J cannot draw rational inferences. Our model combines these features by studying a debate in which a rational J is uncertain of the evidence available to the litigants.[7]

Lipman and Seppi (1995) study sequential debates in which litigants share available evidence (our discovery benchmark), and can present any or all of the available witnesses:

[7] We follow (most of) the subsequent literature by focusing on debates in which litigants present in sequence.

a condition which we also impose, and which they dub Full Reports (aka normality). In contrast to our model, litigants can observe the state, and J can reach any number of verdicts. Perhaps more crucially, Lipman and Seppi also allow litigants to present messages from a rich cheap talk language. By contrast, we exploit our focus on trials to exclude (cheap talk) speeches. Lipman and Seppi exploit the rich language to construct an equilibrium which separates across states; whereas, we prove that discovery games have an equilibrium which separates across evidence sets (rather than states), and demonstrate that non-discovery games need not have such separating equilibria. Aside from the modelling difference, this paper has a distinctive focus on the presentation order, for which Lipman and Seppi's model is inappropriate in the following sense. The equilibrium correspondence of a game in which speeches are allowed strictly contains the equilibrium correspondence of our (discovery) game because J can ignore all cheap talk messages. Players cannot prefer an order if the two games share two or more equilibrium outcomes; so allowing speeches would preclude any player preferring an order when litigants share available evidence.

In Glazer and Rubinstein (2001), a state is a quintuple of aspects, each of which is coded for one litigant (and against the other litigant). Each witness reports the set of states which share the coding for one aspect; and the evidence available to a litigant in some state is the set of witnesses who report an aspect coded for that litigant; so Full Reports fails. Models which fail Full Reports seem inappropriate to common law trials because litigants are allowed to present as much (relevant) evidence as they like . Glazer and Rubinstein show that a J who could commit to the verdict it reaches after observing any evidence would prefer the litigants to present in sequence than simultaneously, would prefer that two litigants present one witness each than that a single litigant presents two witnesses, and may commit to a strategy which violates Debate Consistency: the verdict depends not only on the evidence jointly presented but also on which litigant presented a given witness. Furthermore, the outcome of the optimal mechanism can also be realized in an equilibrium of a game in which J cannot commit. If Full Reports held then J could achieve its first best (separation) in an equilibrium of each fixed order discovery game: so the order would not matter. These separating equilibria need not satisfy Debate Consistency. On the other hand, we show that litigants could prefer to present first in fixed order discovery games if Full Reports fails.

Chen and Olszewski (2014) study a sequential debate with two equiprobable states and a continuum of verdicts. Litigants privately observe up to two signals which are correlated with the state, and commit ex ante to their strategies in sequence: where a strategy specifies which signal realization to present when two signals are available (so Full Reports fails). Chen and Olszewski show that the equilibrium under commitment is also an equilibrium of the related game in which neither litigant can commit. We demonstrate that non-discovery games in which the follower alone may commit to its strategy may have the same equilibrium outcome correspondence as a game without commitment in which the order of presentation is reversed.[8]

Various other papers on debates are more tangentially related. In particular in Shin (1994), litigants simultaneously present their privately observed available evidence, and J can reach a finite number of verdicts. Shin interprets the weight that the jury places on

[8]In contrast to Chen and Olszewski, we treat commitment games as an expositional device.

the evidence presented by each litigant in equilibrium as a burden of proof. By contrast, the burden of proof is an exogenously given stopping rule here.

Ottaviani and Sorensen (2001) consider the implications of changing the order in which litigants present cheap talk messages (rather than evidence). In contrast to our model, the 'state' also includes litigant competence, which they cannot observe; and litigant payoffs have a private (career concern) component. The ensuing herding effects play no role in our model.[9]

The order in which litigants present at trial is neither constitutionally determined nor fixed by statute. Lawyers have typically explained the conventional order in non-consequentialist terms: defendants do not need to justify themselves before an accusation has been made.[10] From a consequentialist standpoint: Kozinski (2015) claims that presenting first might anchor beliefs; whereas Damaska (1973) argues that any such advantage "pales to insignificance when assessed against the disadvantage of having to argue before knowing how the prosecution's case will develop." (fn 48, p.529)[11] This argument seems to conflate two apparently complementary arguments: presenting second allows a litigant to

- Tailor its presentation to the evidence presented by its rival when the available evidence is commonly known by litigants; and
- Respond to evidence which is unexpectedly available to its rival.

The first effect could apply in situations where litigants share the same available evidence; whereas the second effect requires asymmetric information. We demonstrate that these two effects can work in opposite directions. Indeed, we show that a litigant who is uncertain of its rival's available evidence may prefer to present first.

The order in which witnesses testify before a grand jury has been more malleable: for example, the officer suspected of unlawfully killing Michael Brown in Ferguson, Missouri testified first. Referring to that case, Cassell observed that "... it was to the defendant's disadvantage to testify early. He'd be locked into a set of statements, and the grand jurors might later find inconsistencies."[12] This effect coheres with our results on discovery games.

While the effect of trial order has hitherto not been addressed game theoretically, a literature in psychology (starting with Lund (1925)) asks how the order of presentation affects verdicts? Mock jury trials (starting with Walker et al (1972)) fix the evidence available to each litigant and vary the order in which this evidence is presented - which nullifies the strategic effects which we address.[13] The underlying theory is rooted in individual psychology, referring to the workings of imperfect memory (whether primacy or

[9] Hahn (2011) extends this model by allowing litigants to present evidence.

[10] A defendant's right to be presumed innocent in criminal trials may be grounded in respect for persons: cf. Roberts and Zuckerman (2010) Ch 6.3.

[11] Similarly, Williams (1963) suggests that the common law order may benefit defendants:

"The best reason for the English arrangement is that it enables the defending counsel to see how the prosecution case develops before deciding whether to put his client in the witness box." (p82)

[12] Quoted in "Raised Hands and the Doubts of a Grand Jury" *New York Times* (11/29/14 page A1)

[13] Evidence from these experiments is mixed: for example, Kerstholt and Jackson (1998) report that earlier presentation can be beneficial or detrimental to the defendant, depending on the background information given to subjects. See also Costable and Klein (2005).

recency effects predominate) and to priming or anchoring effects. Indeed, the theory seems to apply equally to the order in which the litigants present and the order in which a given litigant presents its evidence; our analysis, by contrast, only concerns the former case.

We conjecture that memory effects are less important when juries can deliberate (cf. Ellsworth (1989)) or when attorneys make closing statements than in mock jury trials.[14] Indeed, if memory effects were important in real trials then one might expect significant differences between juries which could and couldn't discuss the evidence during the trial. Evidence from Arizona (which allowed discussion during civil trials) suggests no significant effects: cf. Hannaford et al (2000) and Diamond et al (2003).

3. Model (Fixed order)

We present our main model in this section. We describe the sequence in common law trials in Section 3.1, present our model in Section 3.2, and explain our criterion for preference over orders of presentation in Section 3.3.

3.1. Common law trials

Prior to a common law trial, litigants may be required to disclose available evidence to each other.

Common law trials with a single defendant (D) and a single plaintiff/prosecutor (P) typically contain the following phases:

- P and then D make opening statements, which must be announcements of the evidence to be presented.
- P calls witnesses, who are cross-examined by D. (Speeches are not allowed.)
- D can present a motion to dismiss, on the grounds that P has not met its burden of proof.
- If the motion is dismissed then D calls witnesses, who are cross-examined by P. (Speeches are again not allowed.)
- P may be allowed, in unusual circumstances, to call witnesses to rebut surprise claims made by D's witnesses.
- P and then D make closing statements, which remind the judge/jury (J) of the evidence, and can suggest interpretations thereof.
- J reaches a verdict.

In the next subsection, we present a model which captures most of these features.

[14] Similarly, opening statements may reduce anchoring effects.

3.2. The games

We analyze two games, each of which is played by D, P and J; a generic litigant is indexed by l and its rival by m. In one game, D presents first; we then say that D is the *leader* and P is the *follower*; in the other game, the roles of leader and follower are reversed. The extensive form of each game only depends on the leader's identity; while payoffs do not depend on this order. Accordingly, it is also convenient to denote the leader by $l = 1$ and the follower by $l = 2$.

Nature starts the game by selecting the facts of the case, and the evidence available to each litigant. The litigants then present evidence in sequence to J, which either acquits or convicts.

Time line

The game proceeds in four rounds.

<u>Round 0</u> (Nature)

We describe the possible facts at issue (the *state*) as a countable set S with generic element s. S is partitioned into two events, I and G. If $s \in I$ then the defendant is said to be *factually innocent*; if $s \in G$ then the defendant is said to be *factually guilty*. Nature chooses state s with a probability denoted by p_s; we refer to $\{p_s\}_{s \in S}$ as the *prior distribution*.

We assume, for expositional convenience, that no active player observes the state. This assumption will turn out to be immaterial for our analysis, and has an interpretative advantage: prosecutors who know that the defendant is factually innocent are obliged not to prosecute.[15]

Conditional on the realized state, Nature also chooses the evidence available to each litigant l, aka its *evidence set*, which we denote by E_l, and assume to be nonempty. We refer to E_1, E_2 as an *evidence set pair*. Nature reveals E_l to litigant l, and possibly to litigant $m \neq l$.

Each evidence set consists of witnesses and combinations thereof. A witness w is a nonempty, strict subset of S. (Strictness means that w is not cheap talk.) We interpret witness w as either testimony or physical/documentary evidence which proves that the true state s is an element of w. Let W_s denote the set of witnesses in state s: i.e. the power set of S minus the empty set and S itself. We then have $E_l \subseteq W_s$.

We denote evidence which combines witnesses e and f, the *composition* of e and f, as ef. This evidence proves, inter alia, that $s \in e \cap f$. We extend this notation to cover cases in which f is not a combination of witnesses (in fact, when $f = pass$: see below). More generally, the composition of two collections of witnesses is their union. We will refer to the combination of all witnesses available at E_l as the *full report* at E_l; so the full report at two distinct evidence sets must differ. As a notational convention, we will describe an evidence set by the listing the witnesses available (rather than all combinations thereof). We suppose that E_l satisfies a condition which we call Full Reports: if $e \in E_l$ and $f \in E_l$ then $ef \in E_l$. (We will return to the implications of Full Reports in Section 7.4.)

Some additional terminology will prove useful below. If evidence e is only available to

[15] The assumption does not affect our existence results; and, in our examples, allowing J to infer the available evidence is equivalent to allowing J to infer the state.

litigant l in factually innocent states then we say that litigant l presenting e *directly proves* factual innocence. We extend this definition in the obvious fashion to factually guilty states. We will describe evidence which is only available to litigant l at a single evidence set pair as *unambiguous*, and will say that unambiguous evidence *induces a verdict*. (The motive for this terminology will soon become clear.)

We denote the probability with which Nature chooses evidence set pair E_1, E_2 in state s by $\pi_s(E_1, E_2)$. Formally, $\pi : S \to \Delta(W_s \times W_s)$ denotes the probability distribution over $E_1, E_2 \in W_s \times W_s$. This definition implies that the distribution of evidence set pairs does not depend on the order. Specifically, in each state s, the probability that D and P respectively have available evidence E_D and E_P is the same, whether D or P is the leader. We write Σ for the union of the support of π_s across states s: that is, the set of possible evidence set pairs.

Round 1 (Leader)

Given its realized evidence set, the leader decides which witness(es) in E_1 to call. We describe our assumption that the leader must present some evidence as the *burden of proof assumption*. This assumption is motivated by the rule that J must acquit if P has not presented sufficiently persuasive evidence.[16] Our version of the burden does not distinguish between more and less informative evidence, but is expositionally convenient. In fact, the only results which depend on the burden of proof assumption are in Section 5.1.1.[17]

Round 2 (Follower)

Suppose that the leader has presented $e_1 \in E_1$ and that the follower's realized evidence set is E_2. As evidence is presented in public, the follower can observe, and therefore recall the leader's witnesses. Conditional on e_1 and E_2, the follower decides whether to *pass* (that is, to present no evidence) or to present some evidence in $E_2 \cup e_1$. In other words, the follower is not subject to a burden of proof.

Round 3 (J)

Call the evidence presented on some path the *evidence pair*. If the leader presents $e_1 \in E_1$ and the follower responds by presenting $e_2 \in e_1 \cup E_2 \cup pass$ then we denote the evidence pair as e_1, e_2. (By contrast, $e_1 e_2$ denotes the composition of e_1 and e_2.) After observing the evidence pair, J ends the game by reaching a verdict: that is, deciding whether to acquit (α) or convict (γ).

In common law trials, the leader can only present rebuttal evidence if one of the follower's witnesses has given surprise testimony. Our assumption that J reaches a verdict after Round 2 may be justified on the understanding that interrogation and cross-examination reveal all that a witness knows.[18]

In sum, a strategy for the leader lists the evidence it presents at every evidence set E_1 which is realized with positive probability at some state; a strategy for the follower lists the evidence (if any) which it presents at each evidence set E_2 after observing any evidence $e_1 \in E_1$, where $E_1, E_2 \in \Sigma$; and a strategy for J is a selection $v \in \{\alpha, \gamma\}$ for every feasible evidence pair e_1, e_2.

[16] See Spier (2007) Section 3.2 for further discussion.

[17] We will explain why Theorem 1 does not depend on the assumption in the next section.

[18] We will return to the implications of other stopping rules in Section 7.2.

Payoffs

We suppose that the litigants only care about the verdict, and have conflicting preferences thereon: D [resp. P] earns 1 [resp. 0] whenever J acquits, and earns 0 [resp. 1] otherwise. We say that D's [resp. P's] *favored verdict* is acquittal [resp. conviction], and write litigant l's favored verdict as v_l.

J loses $1-d$ [resp. d] if it acquits at a factually guilty state [resp. convicts at a factually innocent state], and loses 0 otherwise, where $d \in (0,1)$ equals the standard of proof: that is, the posterior belief at which J is indifferent between acquittal and conviction.[19]

J's payoffs allow us to usefully partition Σ: the set of evidence set pairs E_1, E_2. Let Φ be any nonempty collection of evidence set pairs. We write $v(\Phi)$ for the verdict(s) which maximize J's payoff if it only learns that the evidence set pair is in Φ: that is, when J assigns a probability proportional to $\sum_{s \in S} p_s \pi_s(E_1, E_2)$ to each evidence set pair $E_1, E_2 \in \Phi$, and of 0 to every other evidence set pair. We assume that $v(\Phi)$ is unique for every $\Phi \subseteq \Sigma$. Accordingly, we write Σ_v as the evidence set pairs E_1, E_2 such that $v(E_1, E_2) = v$ for $v = \alpha, \gamma$.

Games

The strategy sets and payoffs detailed above define a *fixed order* game, which we denote $\Gamma^{1,2}$. If D [resp. P] is the leader then we denote the game as $\Gamma^{D,P}$ [resp. $\Gamma^{P,D}$]. It will also be useful to write $\Gamma^{l,m}$ for the fixed order game in which litigant l leads and litigant m follows.

Solution concept

We solve each game by characterizing those pure strategy perfect Bayesian equilibria (PBEs) at which

- J reaches verdict v whenever a litigant presents evidence which induces v (viz. off as well as on the path): a condition which Lipman and Seppi (1995) call 'feasibility'; and
- The follower never recalls a witness that the leader has called (no-recall).

We refer to such strategy combinations and beliefs as *equilibria*, and to the payoff vector reached on an equilibrium path as an *outcome*.[20] Our main results will compare outcomes in the two games, which motivates no-recall: any PBE in which the follower recalled some of the leader's witnesses after some history has the same outcome as another PBE in which the follower passes after that history.

We write $\omega^{D,P}$ [resp. $\omega^{P,D}$] for the set of outcomes in $\Gamma^{D,P}$ [resp. $\Gamma^{P,D}$]. We will denote strategy combinations by capital letters and outcomes by lower case letters.

We say that an equilibrium is *separating* if it prescribes different evidence pairs at different evidence set pairs (rather than in different states). By definition, J must then reach verdict $v(E_1, E_2)$ at every evidence set pair E_1, E_2. We also say that a (non-separating)

[19] The standard of proof is reasonable doubt in criminal trials, and the balance of probabilities in civil trials.

[20] We refer readers to Fudenberg and Tirole (1991) pp331-333 for a formal definition of a perfect Bayesian equilibrium.

equilibrium prescribes a *wrongful acquittal* at E_1, E_2 if it prescribes acquittal at that evidence set pair and $v(E_1, E_2) = \gamma$; we define *wrongful convictions* analogously. An equilibrium prescribes a *miscarriage of justice* at E_1, E_2 if it prescribes either a wrongful acquittal or a wrongful conviction at that evidence set pair. (Note that we define miscarriages of justice in terms of evidence set pairs rather than states.)

3.3. Preferences over orders of presentation

Our main results will explore the conditions under which a player prefers to lead (viz. to present first) or to follow. We interpret this as a question about the player's preferences over outcome correspondences: each game typically has multiple outcomes, as illustrated by the following example:[21]

Example 1 *There are three states: $S = \{m, o, mo\}$, where mo is the only factually guilty state. There are two witnesses:*

$$e = \{m, mo\} \text{ and } f = \{o, mo\}.$$

and three evidence sets:

$$E = \{e\},\ F = \{f\} \text{ and } EF = \{e, f\}.$$

In every state, the two litigants share the same evidence set (so we write $E_1 = E_2 \equiv E$). The conditional distribution of this evidence set is

$$\pi_m(E) = \pi_o(F) = \pi_{mo}(EF) = 1.$$

The prior distribution satisfies

$$\frac{p_{mo}}{p_{mo} + p_m} < d < \frac{p_{mo}}{p_{mo} + p_o}.$$

Example 1 can be interpreted as follows. D is commonly known to have motive and/or opportunity to commit the crime, but J only wants to convict if it believes that D has both. There are two factually innocent states: in one ($s = m$), D only has motive, in the other ($s = o$), D only has opportunity. Both witnesses are available in a factually innocent and in a factually guilty state. In state m, litigants can prove and only prove motive by presenting e; in state o, litigants can prove and only prove opportunity by presenting f. Finally, in the factually guilty state, each litigant could present both witnesses (ef: the full report at EF), thereby inducing conviction, or could prove either motive or opportunity.

Proposition 1 below will imply that both games have a separating outcome, in which J acquits unless the evidence set is EF. Both games have another equilibrium which prescribes: the leader to present e at E and to present f otherwise; the follower to pass on the path; P to present f at EF in response to D presenting e in $\Gamma^{D,P}$; D to always pass

[21] Recall our notational convention: we list the witnesses in an evidence set, rather than their combinations.

in $\Gamma^{P,D}$; and J to acquit after and only after observing $e, pass$. The two outcomes only differ at F, where the non-separating equilibrium alone prescribes a wrongful conviction. If we selected the separating equilibrium in $\Gamma^{D,P}$ and the other equilibrium in $\Gamma^{P,D}$ then each litigant would prefer to lead; and each litigant would prefer to follow if we reversed the selection.

This selection problem applies much more generally. Moreover, standard refinements do not reduce the multiplicity of outcomes because each litigant has the same preference ordering over verdicts at every evidence set pair. Accordingly, we now provide a criterion for preference over multiple outcomes which does not rely on selection arguments.

We start by defining the preferences of player $q \in \{D, P, J\}$ over payoff vectors x and y in a game with a fixed order. We write $x_q(E_1, E_2)$ and $y_q(E_1, E_2)$ as the payoff that equilibria X and Y respectively prescribe player q to earn at evidence set pair E_1, E_2. We say that player q strictly prefers x over y at E_1, E_2 if $x_q(E_1, E_2) > y_q(E_1, E_2)$, and that player q strictly prefers x over y if

$$\sum_{s \in S} \sum_{E_1, E_2} p_s \pi_s(E_1, E_2)[x_q(E_1, E_2) - y_q(E_1, E_2)] > 0,$$

where the left hand side is the expected difference in payoffs across states and evidence set pairs. We define weak preferences and indifference analogously. These orderings are clearly transitive: a feature which we will exploit below.

We now use these criteria for preferences over outcomes to define preferences over sets of outcomes. Specifically, let x^* and y^* be two nonempty collections of outcomes. We say that player *q prefers x^* over y^** if it weakly prefers every $x \in x^*$ over every $y \in y^*$, and strictly prefers some $x' \in x^*$ over some $y' \in y^*$. This criterion defines a player's preference ordering over outcomes in a given game. However, we can use the same conditions to define preferences over the set of outcomes in the two fixed order games by identifying x^* with $\omega^{l,m}$ and y^* with $\omega^{m,l}$. We then say that *player q prefers that litigant $l \neq m$ leads* if $\omega^{l,m}$ and $\omega^{m,l}$ are nonempty and player q

Condition 1 Weakly prefers every outcome in $\omega^{l,m}$ over every outcome in $\omega^{m,l}$; and

Condition 2 Strictly prefers some outcome in $\omega^{l,m}$ over some outcome in $\omega^{m,l}$.

As litigants have opposing preferences over the verdict, P prefers to lead [resp. follow] if and only if D prefers to lead [resp. follow].

We suppose that no player is indifferent between distinct outcomes of either game.[22] Conditions 1 and 2 then define a partial ordering over games. In particular, no player can prefer that a litigant leads if both games share two or more distinct outcomes (as in Example 1). Consequently, our criterion is hard to satisfy: which cuts in favor of our negative results, and against our positive results. We could, alternatively, work with natural conditions which are yet harder to satisfy: requiring player q to prefer x over y at every evidence set pair, rather than in expectation. All of our results below would also satisfy this tighter condition (as we will note below).

Conditions 1 and 2 define preferences over the two fixed order games. In Sections 5.3 and 6, we will define a pair of related games, and extend Conditions 1 and 2 in an obvious fashion to cover preferences between playing a related game and a fixed order game.

[22] This is a genericity condition on Nature's choices.

4. Discovery games

Discovery rules require litigants to share their available evidence, and are enforced by means such as depositions, interrogatories and motions to inspect and copy documents. Discovery rules were introduced in federal civil trials in 1938 to improve fact-finding and to prevent litigants from introducing surprise evidence at trial (cf. Subrin (1998)).[23] In this section, we study preferences over the order of presentation when discovery rules apply to both litigants. Our model allows us to address this case by imposing conditions on Σ: the support of evidence set pairs. Specifically, we will describe games in which $E_1 = E_2$ for every $E_1, E_2 \in \Sigma$ as *fixed order discovery games.*[24] We will focus on discovery games throughout this section.

We can simplify the notation in discovery games. If $E_1 = E_2 = E$ then we denote the evidence set pair as E. It will be useful to say that the follower's response (e_2) in response to the leader presenting e_1 *completes* the full report at E if the composition of e_1 and e_2 equals the full report at E.

Our main results in this section will establish that litigants cannot prefer to lead, but may prefer to follow in discovery games. We start by providing a result of independent interest, which we will exploit below:

Proposition 1 *Every fixed order discovery game has a separating equilibrium.*

We prove Proposition 1 by construction in the Appendix. According to the construction, J reaches verdict $v(E)$ after observing any evidence pair e_1, e_2 whose composition is the full report at E; the leader presents the full report at every evidence set; and the follower completes the full report at evidence set E unless a) J then reaches the leader's favored verdict, and b) the follower can complete the full report at an evidence set $F \subset E$ such that $v(F)$ is its favored verdict. The follower is motivated to complete the full report by J's credible threat to reach the leader's favored verdict after observing any evidence pair e_1, e_2 which is not the full report at any evidence set and whose composition is ambiguous.

Proposition 1 implies that each fixed order game has an outcome. We can therefore use the criterion introduced in Section 3.3 to consider preferences over the order of presentation. If each fixed order game only had a separating outcome then no player could prefer an order. Our main result in this section asserts that players may prefer an order:

Theorem 1 *In fixed order discovery games:*

a) *Litigants cannot prefer to lead;*

b) *Litigants can prefer to follow; and*

c) *J can prefer an order, but can only do so if the litigants prefer to follow.*

[23] Bone (2012) provides further details, and reviews the law and economics literature.

[24] There are two possible interpretations: either litigants only observe their own available evidence but know that evidence sets are perfectly correlated across litigants; or Nature initially endows each litigant with different evidence, but the discovery process allows each litigant to call any witness initially endowed to either of the litigants.

Theorem 1 implies that the conventional order is optimal from defendants' point of view in discovery games, without appealing to principles like a presumption of innocence. We prove Theorem 1a) in the Appendix. The proof relies on

Lemma *Let $\Gamma^{1,2}$ and $\Gamma^{2,1}$ be fixed order discovery games. If $\Gamma^{1,2}$ has an equilibrium which prescribes verdict v_1 (the leader's favored verdict) at evidence sets Φ then $\Gamma^{2,1}$ has an equilibrium which prescribes verdict v_1 at every evidence set in Φ.*

Consider any equilibrium (say, X) of $\Gamma^{1,2}$. We prove Lemma by using X to construct the following strategy combination (Y) in $\Gamma^{2,1}$: at every evidence set $E \in \Phi$, Y prescribes litigant 1 to present the same evidence as X prescribed litigant 1 to present at E; and prescribes litigant 2 to present and litigant 1 to complete the full report at every other evidence set $E \notin \Phi$. This strategy combination, and associated beliefs, forms an equilibrium. We then prove that Lemma implies a), exploiting the transitivity of preferences over outcomes. This part of the proof does not rely on $\Gamma^{1,2}$ and $\Gamma^{2,1}$ being discovery games: a property which we will exploit again in Section 7.3..

We now prove parts b) and c):

Proof

b) We prove this part with an example in which one game only has a separating outcome, while the other game also has another outcome:

Example 2 *There are four states: $S = \{g_1, g_2, g_3, i\}$, and the defendant is only factually innocent in state i. There are three witnesses:*

$$e = \{g_1, g_2, i\},\ f = \{g_2, g_3, i\} \text{ and } h = \{i\}$$

and four evidence sets:

$$E = \{e\},\ F = \{f\},\ EF = \{e, f\} \text{ and } EFH = \{e, f, h\},$$

whose conditional distribution satisfies

$$\pi_{g_1}(E) = \pi_{g_2}(EF) = \pi_{g_3}(F) = \pi_i(EFH) = 1.$$

The prior distribution satisfies

$$\frac{p_{g_1}}{p_{g_1} + p_i} < d < \min\{\frac{p_{g_2}}{p_{g_2} + p_i}, \frac{p_{g_3}}{p_{g_3} + p_i}\}.$$

The conditions on priors imply that, in any non-separating equilibrium, litigants must present $e, pass$ at E and at EFH, and J must wrongfully acquit at E. To see this, note that h directly proves factual innocence, so J must acquit at EFH in any equilibrium; but the conditions on priors imply that J would convict were litigants to present the same evidence pair at EFH as at EF and/or F.

Claim 1 *In Example 2, $\Gamma^{P,D}$ has an equilibrium with a wrongful acquittal at E, and has no other non-separating outcome.*

Proof Consider the following strategy combination and beliefs:

- P presents e at E and at EFH, and presents the full report at every other evidence set;
- D passes in response to P presenting e, and otherwise completes the full report;
- J believes that the realized evidence set is
  - EFH and acquits if either litigant presents evidence which contains h;
  - F and convicts if at least one litigant presents f, but neither litigant presents e or h;
  - EF and convicts if at least one litigant presents e and at least one litigant presents f, but neither litigant presents h;
  - E or EFH and acquits after observing $e, pass$ or e, e.

P cannot profitably deviate at EFH because D could secure acquittal by presenting h. The strategy combination and beliefs therefore form an equilibrium.

The conditions on priors preclude any other non-separating outcome.■

Claim 2 *In Example 2, every outcome of* $\Gamma^{D,P}$ *is separating.*

Proof Any other outcome would have to be supported by an equilibrium which prescribes D to present e and P to then pass at E and at EFH; and J to acquit after observing $e, pass$ and to convict at EF. J must also acquit after observing e, e, e, f and e, ef, else P could profitably deviate at EFH. This implies that D could profitably deviate to presenting e at EF.■

Proposition 1 and Claim 1 imply that $\omega^{P,D}$ consists of a separating outcome and an outcome with a wrongful acquittal at E; while Proposition 1 and Claim 2 imply that $\omega^{D,P}$ only consists of a separating outcome. Consequently, both litigants must prefer to follow.

c) Example 2 illustrates a fixed order game in which J prefers an order (D as leader).

Suppose that J prefers litigant $l \neq m$ to lead. J strictly prefers a separating outcome over any other outcome; and both games possess a separating outcome by Proposition 1. Consequently, J prefers $\Gamma^{l,m}$ over $\Gamma^{m,l}$ if and only if $\Gamma^{m,l}$ alone has a non-separating outcome. No non-separating equilibrium of $\Gamma^{m,l}$ can then prescribe J to wrongfully reach verdict v_m at any evidence set E, else Lemma would imply that $\Gamma^{l,m}$ also has a non-separating equilibrium, contrary to our supposition that J prefers an order. As every non-separating equilibrium of $\Gamma^{m,l}$ only prescribes J to wrongfully reach verdict v_l, litigants prefer to follow.■

The criterion which we introduced in Section 3.3 compares equilibrium correspondences, and therefore seems rather unwieldy. Theorem 1 provides a strikingly clean description of preferences over the order without fully characterizing these correspondences.

Lemma implies that litigants cannot prefer to lead at any evidence set; and inspection of Example 2 reveals that litigants strictly prefer to follow at E and are indifferent at the other evidence sets, rather than in expectation. Hence, Theorem 1 would also hold on the stricter criterion which we mentioned at the end of Section 3.3.

Theorem 1 does not rely on the burden of proof assumption in the sense that it would also hold in a variant on fixed order games where the leader is allowed to pass. Part a) would hold because the proof of Lemma does not turn on a burden of proof, and the rest of the proof only depends on outcomes. Part b) would hold because the conditions on priors in Example 2 imply that J must observe the same evidence pair at E and at EFH in any non-separating equilibrium, irrespective of the burden. Inspection of the proof of Theorem 1 also reveals it would also hold in another variant on fixed order games where one or more litigants observes the state.

Presenting second allows the follower to condition the evidence it presents on the leader's choice at each evidence set. Example 2 illustrates why such flexibility may be advantageous in discovery games. The full report at EFH induces acquittal; so J must acquit at EFH in both games. However, the evidence which the leader presents at EFH determines J's verdict at E. In the non-separating equilibrium of $\Gamma^{P,D}$, P presents e at EFH because it anticipates that D would otherwise complete the full report, thereby inducing acquittal. D cannot present e at EFH in $\Gamma^{D,P}$ as P would optimally respond by presenting f rather than passing because equilibrium play at EF requires J to convict after observing e, f. In sum, the follower's capacity to respond to the leader's evidence (aka flexibility) is necessary and sufficient for litigants to prefer to present second in Example 2.

If the full report at every evidence set were unambiguous then every equilibrium of both games would be separating, else a litigant could profitably deviate to presenting or completing the full report. Hence, litigants can only prefer to follow if the full report at some evidence set is ambiguous. Moreover, Conditions 1 and 2 require every miscarriage of justice in an equilibrium of $\Gamma^{D,P}$ [resp. $\Gamma^{P,D}$] to be a wrongful conviction [resp. acquittal] if litigants prefer to follow. We will now argue that the following condition is necessary for litigants to prefer to follow. But first some notation: define Φ_v as the collections of evidence sets such that $v(\Phi) = v$; and define $-v$ as verdict $\{\alpha, \gamma\}\backslash v$.

Interlocking Evidence Sets (IES)

- **a** Σ *can be partitioned into collections of evidence sets* $\{\Phi^i\}$*, such that all evidence sets in* Φ^i *have a nonempty intersection. Some* $\Phi^i \subseteq \Phi_v$ *intersects with* Σ_α *and with* Σ_γ*; and no evidence available at an evidence set which is in some* $\Phi^i \subseteq \Phi_v$ *induces verdict* $-v$.

- **b** *There is a verdict* v *and a non-singleton evidence set in some* $\Phi^i \in \Phi_v$ *which intersects with an evidence set in* Φ_{-v}.

IES generalizes Example 2. To see this, partition Σ into $\{E \cup EFH\}$, $\{F\}$ and $\{EF\}$. Evidence e is in E and EFH; $E \in \Sigma_\gamma$ and $EFH \in \Sigma_\alpha$; and ef is ambiguous. Consequently, part a) is satisfied. Furthermore, EF contains e and f and is in Σ_γ, while f is in both EF and EFH; so part b) is also satisfied.

Proposition 2 *Litigants only prefer to follow in fixed order discovery games if IES holds.*

We prove Proposition 2 in the Appendix. The conditions in part a) are necessary for a strategy combination to be part of a non-separating equilibrium, and therefore for litigants to prefer an order. Part b) provides *some* necessary conditions for litigants to prefer to follow. Suppose that $\Gamma^{1,2}$ has some non-separating equilibrium, say X. We can then use the strategy combination in that equilibrium to construct a strategy combination, say Y, in $\Gamma^{2,1}$ with the same payoff vector as X. If Y is an equilibrium in $\Gamma^{2,1}$ then litigants cannot prefer an order because the fixed order games would share two outcomes. We show that the conditions in part b) are necessary to exclude this possibility.

J is best off in a separating equilibrium, which exists in both games (by Proposition 1). Consequently, if J prefers some litigant to be the leader then that game only has separating equilibria. Litigants could prefer to follow without J preferring an order: for example, if $\Gamma^{1,2}$ had an equilibrium which prescribed verdict v_2 at an evidence set in Σ_{v_1} and $\Gamma^{2,1}$ had an equilibrium which prescribed verdict v_1 at an evidence set in Σ_{v_2}.

The proofs of Proposition 1 and of Theorem 1a) and 1b) all rely on constructions in which J's strategy satisfies a condition which Glazer and Rubinstein (2001) call Debate Consistency: the verdict which J reaches after observing any evidence pair e_1, e_2 only depends on e_1e_2 and not, in particular, on which litigant presents e_1. However, it is easy to construct fixed order discovery games with equilibria which fail Debate Consistency: for example, suppose that there are two evidence sets, $EF = \{e, f\}$ and $EFH = \{e, f, h\}$, where $v(EF) = \alpha \neq v(EFH)$. $\Gamma^{D,P}$ then has a separating equilibrium which prescribes J to observe e, f at EF and f, e at EFH: D cannot profitably deviate at EFH because any evidence which includes h induces conviction.

5. Non-discovery games

In the last section, we demonstrated that litigants cannot prefer to lead, but may prefer to follow in discovery games if IES is satisfied. In this section, we consider games which are not played under discovery. We provide examples in which litigants prefer to lead, as well as examples of non-discovery games in which litigants prefer to follow, for reasons which do not apply in discovery games. In Section 5.1, we consider games in which a litigant knows of, but cannot call witnesses available to its rival; in Section 5.2, we consider games in which available evidence is privately observed. In Section 5.3, we interpret results from previous parts in terms of commitment; and demonstrate that J alone could prefer an order in Section 5.4.

5.1. No-subpoena games

According to our model, a litigant can only call a witness who is in its own evidence set or who has already been called by its rival. In the benchmark case of discovery games, litigants are always endowed with the same available evidence; so the leader both knows and can call any witness available to its rival. In this subsection, we consider cases in which Nature may assign different available evidence to the two litigants, but reveals the evidence set pair to both litigants. We refer to such cases as *no-subpoena games.*

No-subpoena games are realistic extensions of discovery games for two reasons. First, discovery rules apply to both litigants in civil trials, but predominantly to the prosecution in criminal trials.[25] If discovery rules only apply to one litigant (say, P) then E_D must include E_P. A no-subpoena game would then be played if P knew E_D but could not subpoena witnesses in $E_D \backslash E_P$. This situation is exemplified by the 5^{th} Amendment right not to testify in criminal trials: P may know that D would confess if interrogated, but cannot call D as a witness. Second, discovery rules are sometimes difficult to enforce because a litigant may not be able to adequately describe the requested evidence: for example, one litigant may be sure that its rival has favorable evidence in its files, but does not know which file to search. In these circumstances a no-subpoena game would again be played; but, in this case, one litigant's evidence set need not contain its rival's evidence set.

We divide this subsection into two parts, each illustrating a reason why litigants may prefer to lead in no-subpoena games.

5.1.1. The burden of proof

In this part, we provide an example of a no-subpoena game in which litigants prefer to lead because of our assumption that the leader bears the burden of proof. We also amend this example to provide a related reason (beyond IES) why litigants may prefer to follow.

Litigants can only pool at evidence set pairs E_1, E_2 and F_1, F_2 if E_1 and E_2 contain a common witness (say, w), whereas the follower may pass or present w at E_2 and at F_2 in response to the leader presenting w, even if E_2 and F_2 do not contain a common witness. In the latter case, the burden of proof precludes J from observing the same evidence pair in $\Gamma^{2,1}$. Example 3 below illustrates how this argument could result in litigants preferring to lead.

We will abuse notation in this section by writing an evidence set pair as E_D, E_P.[26]

Example 3 *There are four states:* $S = \{g_1, g_2, i_1, i_2\}$*, where the defendant is factually guilty in states* g_1 *and* g_2*. There are three witnesses:*

$$e = \{g_1, i_1, i_2\},\ f = \{i_1, i_2\} \text{ and } h = \{g_1, g_2, i_2\}$$

and four evidence sets:

$$E = \{e\},\ F = \{f\},\ H = \{h\} \text{ and } EF = \{e, f\}.$$

The conditional distribution of evidence set pairs (with D's evidence set written first) satisfies

$$\pi_{g_1}(E, H) = \pi_{g_2}(H, H) = \pi_{i_1}(E, F) = \pi_{i_2}(EF, H) = 1.$$

[25] Failure of the prosecution to provide the defendant in a criminal trial with material evidence violates due process (*Brady* v. *Maryland* 373 US 83 (1963)), though these rights were restricted in *Kyles* v. *Whitley* (1995). Under Federal Rules of Criminal Procedure 16, a defense request for discovery triggers a reciprocal obligation to give notice of evidence and witnesses to be called. Litigants in civil cases must disclose evidence, even if it is not requested by the opposing side. (The rules are detailed in Federal Rules of Civil Procedure (2010) Title V and the 1998 Civil Procedure Rules for England.)

[26] Our notation hitherto has distinguished according to the order of presentation.

The prior distribution satisfies

$$\frac{p_{g_1}}{p_{g_1}+p_{i_1}} < d < \min\{\frac{p_{g_1}}{p_{g_1}+p_{i_2}}, \frac{p_{g_2}}{p_{g_2}+p_{i_2}}\}.$$

In Example 3, D presenting h directly proves factual guilt; P presenting h does not. Furthermore, D [resp. P] can only present f in state i_2 [resp. i_1].

$\Gamma^{D,P}$ has a separating equilibrium in which J observes e, h at E, H; $h, pass$ at H, H; e, f at E, F; and $ef, pass$ at EF, H. The lower bound on d implies that $\Gamma^{D,P}$ also has an equilibrium in which J acquits after observing $e, pass$ at E, H and at E, F; convicts after observing $h, pass$ at H, H; and acquits after observing $ef, pass$ at EF, H. On the other hand, the upper bound on d and the availability of f at EF preclude an equilibrium in which J observes the same evidence pair at E, H and at EF, H.

$\Gamma^{P,D}$ has a separating equilibrium in which J convicts after observing h, e at E, H and $h, pass$ at H, H; and acquits after observing $f, pass$ at E, F, and h, ef at EF, H. As F and H have an empty intersection, $\Gamma^{P,D}$ cannot have an equilibrium in which J observes the same evidence pair at E, F and at E, H. The upper bound on d and the availability of f at EF preclude an equilibrium in which J observes the same evidence pair at H, H and at EF, H.

In sum, $\Gamma^{D,P}$ has a separating equilibrium and another equilibrium with a wrongful acquittal at E, H; while $\Gamma^{P,D}$ only has a separating equilibrium. Consequently, litigants prefer to lead, while J prefers P to lead. Example 3 therefore illustrates how the burden of proof may explain why litigants can prefer to lead in no-subpoena games. Specifically, the burden requires the leader not to pass; so pooling at two evidence set pairs is only possible if the leader has a common witness at those evidence set pairs.

Litigants might also prefer to follow because of the way the burden of proof determines play in no-subpoena games. To see this, consider Example 3': a variant on Example 3.

Example 3' *There are three states:* $S = \{g_1, g_2, i\}$*, where* i *is the only factually innocent state. There are four witnesses:*

$$e = \{g_1, i\},\ f = \{i\},\ h = \{g_1\} \text{ and } k = \{g_2\}$$

and four evidence sets:

$$E = \{e\},\ EF = \{e, f\},\ H = \{h\} \text{ and } K = \{k\}.$$

The conditional distribution of evidence set pairs (with D's evidence set written first) satisfies $\pi_{g_1}(H, E) = \pi_{g_2}(K, K) = \pi_i(EF, EF) = 1$.

The prior distribution satisfies $\frac{p_{g_1}}{p_{g_1}+p_i} < d$.

EF and H have an empty intersection; so $\Gamma^{D,P}$ can only have a separating equilibrium, in which J only acquits at EF, EF. $\Gamma^{P,D}$ has a separating equilibrium, in which J convicts after observing $k, pass$ at K, K and either e, h or $e, pass$ at H, E; P presents ef and D

completes the full report at EF, EF because f directly proves factual innocence (so J acquits). $\Gamma^{P,D}$ also has an equilibrium in which P presents e at EF, EF and at H, E, to which D responds by passing, while J acquits after observing $e, pass$. P cannot profitably deviate at EF, EF because witness f directly proves factual innocence, and has no other evidence available at H, E. Comparing this and the separating equilibrium, we see that litigants prefer to follow.

Example 3' illustrates a reason why litigants want to follow which does not apply in discovery games in the following sense. Suppose that we create a discovery variant on Ex 3' in which $\pi_{g_1}(E) = 1$. Part b) of IES can then only be satisfied in the separating partition (viz. each cell is a singleton); but part a) would then fail because no cell in the partition would contain evidence sets in Σ_α and Σ_γ. Similarly, if $\pi_{g_1}(H) = 1$ then every partition is necessarily separating; and the same argument applies.

5.1.2. Proving availability

Our model allows the follower to recall any witness that the leader has presented. In this part, we exploit the idea that the identity of a litigant who presents evidence may determine what that evidence directly proves.[27] Specifically, we provide an example of a no-subpoena game with two key features: first, J would reach verdict v_l if it knew that witness w were available to litigant $l \neq m$; second, l can only prove that $l \in E_l$ if it is the first litigant to present w. If $w \in E_m$ at evidence set pairs where $w \in E_l$ then, as leader, m could prevent l from proving that $w \in E_l$ by presenting w;[28] so litigants prefer to lead. We then amend this example to provide a related reason (beyond IES) why litigants may prefer to follow.

Example 4 *There are five states: $S = \{g, i_1, i_2, i_3, i_4\}$, and the defendant is only factually guilty in state g. There are four witnesses:*

$$e = \{g, i_1, i_2, i_3\},\ f = \{g, i_1, i_2\} \quad h = \{g, i_1, i_3\} \text{ and } k = \{i_2, i_3, i_4\}$$

and four evidence sets:

$$EF = \{e, f\},\ EH = \{e, h\},\ FH = \{f, h\} \text{ and } K = \{k\}.$$

The conditional distribution of evidence set pairs (with D's evidence set written first) satisfies

$$\pi_g(EH, EF) = \pi_{i_1}(EF, FH) = \pi_i(K, K) = 1 : i = i_2, i_3, i_4.$$

The prior distribution satisfies $\frac{p_g}{p_g + p_{i_1}} < d$.

[27] Example 3 above also has this property, albeit with different consequences.

[28] In other words, litigant l has no other evidence which proves that $e \in E_l$. This condition mirrors the conventional supposition that l cannot prove its ignorance.

D can only present f or k [resp. h] in factually innocent [resp. guilty] states; whereas P can only present e [resp. h or k] in factually guilty [resp. innocent] states.

$\Gamma^{D,P}$ has a non-separating equilibrium in which J acquits after D has presented either e or k, however P responds. D presents k at K, K and otherwise presents e; P passes on the equilibrium path at every evidence set pair; and J always acquits. J cannot profitably deviate after observing e, f because this evidence pair is also available at EF, F, H, which is realized in a factually innocent state. P can therefore not profitably deviate to presenting f in response to e at EH, EF. In other words, by presenting e as leader, D prevents P from proving that $e \in E_P$, and therefore that $s \in G$. However, J convicts at EH, EF in every equilibrium of $\Gamma^{P,D}$ because P can directly prove the factually guilty state by presenting witness e.

On the other hand, both games have a separating equilibrium. In $\Gamma^{D,P}$, J convicts unless D presents f or k (which directly prove factual innocence); D and P respectively present the full report and pass at each of their evidence sets; while J convicts at EH, EF and otherwise acquits. In $\Gamma^{P,D}$, J convicts unless P presents e (which directly proves factual guilt); P and D respectively present the full report and pass at each of their evidence sets; while J convicts at EH, EF and otherwise acquits.

In sum, $\Gamma^{D,P}$ has a separating and a non-separating equilibrium, while $\Gamma^{P,D}$ only has a separating equilibrium. D is acquitted at K, K in all of these equilibria, but is only acquitted at EH, EF in the non-separating equilibrium. Consequently, litigants prefer to lead. In addition, J prefers P to lead.

The key to these conclusions is that when D as leader presents e at EH, EF, P cannot prove that it could also have presented e. This is, of course, impossible in discovery games, where J knows that a witness is available to one litigant if and only if it is available to both litigants. Moreover, the burden of proof reason for preferring to lead, which we instanced in the last subsection, does not apply here because e is available to D at the same evidence set pairs (EF, FH and EH, EF) where f is available to P. Indeed, litigants would also prefer to lead in a variant on the games where the leader can pass.

Example 4 has another interesting property: the litigants prefer to lead, even though the full report of each litigant's evidence set at every evidence set pair in Σ is unambiguous. By contrast, we noted in Section 3 that this condition precludes any player preferring an order in discovery games (because every equilibrium in each game is then separating).

Litigants might also prefer to follow because of the follower's inability to prove availability of a witness. To see this, consider Example 4': a variant on Example 4:

Example 4' *There are five states: $S = \{g, i_1, i_2, i_3, i_4\}$, and the defendant is only factually guilty in state g. There are four witnesses:*

$$e = \{g, i_1, i_2, i_3\},\ f = \{g, i_1, i_2\} \quad h = \{g, i_1, i_3\} \text{ and } k = \{i_2, i_3, i_4\}$$

and five evidence sets:

$$E = \{e\},\ F = \{f\},\ H = \{h\},\ K = \{k\} \text{ and } EF = \{e, f\}.$$

The conditional distribution of evidence set pairs (with D's evidence set written first) satisfies

$$\pi_g(E, F) = \pi_{i_1}(E, EF) = \pi_{i_2}(EF, F) = \pi_{i_3}(H, H) = \pi_{i_4}(K, K) = 1.$$

The prior distribution satisfies

$$\frac{p_g}{p_{i_1} + p_{i_2} + p_g} < d < \min\{\frac{p_g}{p_{i_1} + p_g}, \frac{p_g}{p_{i_2} + p_g}\}.$$

D would prove factual innocence by presenting either f or h, and each litigant would prove factual innocence by presenting k.

$\Gamma^{P,D}$ has an equilibrium in which D is always acquitted and, in particular, is wrongfully acquitted at E, F. This equilibrium prescribes J to acquit after observing every evidence set pair; P to present h at H, H, k at K, K and to otherwise present f; and D to pass in response to any evidence presented by P. J cannot profitably deviate because f, e could be presented in factually innocent state i_1; so P can also not profitably deviate at any of its evidence sets.

This is the only equilibrium. To see this, note that $\Gamma^{P,D}$ does not have a separating equilibrium because P would have to present f at both E, F and EF, F, and J would have to convict at E, F and acquit at EF, F. D could then profitably deviate at E, F to mimicking the evidence prescribed at EF, F. $\Gamma^{P,D}$ can also not have an equilibrium in which litigants present the same evidence pair at E, F and at E, EF (but not at EF, F) because J would have to acquit at EF, F after observing f, ef and convict after observing f, e and $f, pass$, so D could profitably deviate to presenting ef at E, F and at E, EF. Analogous arguments preclude $\Gamma^{P,D}$ having an equilibrium in which litigants present the same evidence pair at E, F and at EF, F (but not at E, EF).

$\Gamma^{D,P}$ has an equilibrium which features a wrongful conviction at E, EF. This equilibrium prescribes D to present the full report at each of its evidence sets; P to pass on the path at each of its evidence sets; and J to convict after observing $e, pass$ and to acquit after D has presented ef, h or k. D's evidence set is a singleton at E, F and E, EF; so it cannot profitably deviate. The full report at each of D's other evidence sets directly proves factual innocence; so J cannot profitably deviate. Consequently, P cannot profitably deviate after D has presented the full report at each of its evidence sets.

$\Gamma^{D,P}$ also has an equilibrium in which D is always acquitted. This equilibrium prescribes J to acquit after observing every evidence set pair; D to present h at H, H, k at K, K and otherwise to present e; and P to pass at each of its evidence sets. J cannot profitably deviate because e, f could be presented in factually innocent state i_1; so P can also not profitably deviate at any of its evidence sets.

Finally, $\Gamma^{D,P}$ cannot have a separating equilibrium, as it would require J to acquit at E, EF and convict at E, F. D must present e at both evidence set pairs; so P could profitably deviate at E, EF to mimicking the response to e prescribed at E, F.

In sum, D is always acquitted in every equilibrium of $\Gamma^{P,D}$, while $\Gamma^{D,P}$ has an equilibrium with a wrongful conviction at E, EF. Consequently, litigants prefer to follow in Example 4' precisely because the follower cannot prove availability. This motive is impossible in games where litigants share an evidence set in $\{E, F, EF, H, K\}$ (and the conditions on priors in Example 4' apply). Specifically, suppose per contra that litigants prefer to follow in such a discovery game: so a) $\Gamma^{P,D}$ alone has an equilibrium with a wrongful acquittal; or b) $\Gamma^{D,P}$ alone has an equilibrium with a wrongful conviction.

If $EF \notin \Sigma$ then every equilibrium of each game has to be separating (because $E \cap F$ is empty). Now suppose that $EF \in \Sigma$. In case a), J must acquit after observing any evidence pair whose composition is ef and after observing (say) $e, pass$; and both discovery games have an equilibrium in which J acquits after observing $e, pass$. In case b), J must convict after observing any evidence pair whose composition is ef and after observing (say) $e, pass$; and both discovery games have an equilibrium in which J convicts after observing $e, pass$. Consequently, litigants cannot prefer to follow in either case.

Example 4' has two other interesting properties:

J expects to lose $p_g(1-d)$ at the outcome where it always acquits, and expects to lose $p_{i_1}d$ at the outcome in $\Gamma^{D,P}$ with a wrongful conviction. The conditions on priors therefore implies that J prefers P to lead.

Example 4' also demonstrates that neither no-subpoena game need have a separating equilibrium, whereas both discovery games have a separating equilibrium (Proposition 1), as well as in Examples 3 and 3'.

5.2. Incomplete information games

In the last subsection, we noted that asymmetric or unenforceable discovery rules may result in the play of no-subpoena games where each litigant knows the evidence available to its rival. In this subsection, we drop the latter condition. Specifically, we consider games in which each litigant may be uncertain of its rival's available evidence. We will call such cases *incomplete information games.*

We start with a variant on Example 1 (in Section 3.3), which illustrates how a litigant may prefer to lead in such games.

Example 5 *There are three states: $S = \{m, o, mo\}$, where mo is the only factually guilty state. There are two witnesses:*

$$e = \{m, mo\} \text{ and } f = \{o, mo\}$$

and three evidence sets:

$$E = \{e\},\ F = \{f\} \text{ and } EF = \{e, f\}.$$

The conditional distribution of evidence set pairs (with D's evidence set written first) satisfies

$$\pi_m(E, E) = \pi_o(F, F) = 1;\ \pi_{mo}(EF, E) = 1 - \pi_{mo}(EF, F) \equiv \pi \in (\frac{1}{2}, 1).$$

The prior distribution satisfies

$$\frac{p_{mo}\pi}{p_{mo}\pi + p_m} < d < \frac{p_{mo}(1-\pi)}{p_{mo}(1-\pi) + p_o}.$$

The interpretation of m and o as motive and opportunity in Example 1 carries over to Example 5.

As $\pi > 1/2$, the following strategy combination is part of an equilibrium in $\Gamma^{D,P}$:

- D presents e when its evidence set is E or EF, and otherwise presents f;
- P completes the full report at all of its evidence sets;
- J acquits after and only after observing either $e, pass$ or $f, pass$.

This equilibrium is unique: for D cannot present ef when its evidence set is EF because J would then acquit after observing $e, pass$ and $f, pass$; so D could profitably deviate to presenting either e or f. Furthermore, D cannot present f when its evidence set is EF because J would then acquit after observing $e, pass$; so $\pi > 1/2$ implies that D could profitably deviate to presenting e.

The following strategy combination is part of an equilibrium in $\Gamma^{P,D}$:

- P presents e when its evidence set is E, and otherwise presents f;
- D always passes;
- J acquits after and only after observing $e, pass$.

In every equilibrium, D passes in response to P presenting e [resp. f], else J would then convict, and would acquit after observing $e, pass$ [resp. $f, pass$]; so D could profitably deviate to passing. Consequently, this game has a unique outcome.

Comparing outcomes in the two games: each litigant prefers to lead at every evidence set pair because J reaches the same verdict in both games at every evidence set pair other than F, F, where J only acquits in $\Gamma^{D,P}$.

This result is rather striking in light of (our reading of) a claim by Damaska (1973) which we quoted in Section 2: that incomplete information provides an additional reason for preferring to follow because the follower can then not be surprised by the leader's available evidence. Learning that P has witness f at EF, F does not allow D to secure an acquittal; but its equilibrium response at EF, F harms D at F, F.

A simple variant on Example 5 reveals that litigants may again prefer to follow in incomplete information games for reasons which do not apply in discovery games.

Example 5' *As Example 5, except that the prior distribution satisfies*

$$\max\{\frac{p_{mo}\pi}{p_{mo}\pi + p_m}, \frac{p_{mo}(1-\pi)}{p_{mo}(1-\pi) + p_o}\} < d.$$

Each game has a unique equilibrium. Litigant play corresponds to that in Example 5; and J reaches the same verdict as in Example 5 at each evidence set pair when D

leads. The difference concerns the equilibrium verdicts in $\Gamma^{P,D}$, where J now acquits at F, F and at EF, F; so J acquits at every evidence set pair in equilibrium. Litigants then prefer to follow, even though every miscarriage of justice is a wrongful acquittal in the only equilibrium of $\Gamma^{D,P}$.

The motive to prefer following in Example 5' does not apply in games with the same states ($S = \{m, o, mo\}$) and witnesses (e and f) where litigants always share the same evidence set. P can then induce conviction by presenting ef in both games; so the analysis of Example 1 implies that litigants cannot prefer an order. On the other hand, litigants cannot prefer an order if $\pi_{mo}(EF) = 0$.

We argued above that Example 3 [resp. 3'] illustrates the burden of proof reason why litigants may prefer to lead [resp. follow] in a no-subpoena game. Examples 5 and 5' share this property: e is available at evidence sets E and EF; and $\Gamma^{D,P}$ has an equilibrium in which D presents e at evidence set pairs E, E, EF, E and EF, F. However, this property is not sufficient for litigants to present the same evidence pairs at EF, E and at E, E or at EF, F and at F, F.

5.3. Commitment

We have provided three examples of non-discovery games in which litigants prefer to lead. In this subsection, we will argue that these examples share a common feature: the follower's ability to respond on a case-by-case basis may impose negative externalities across evidence set pairs which would be resolved if the follower could commit ex ante to its strategy. We will argue that, in these examples, games in which the follower can commit have the same outcomes as games in which the order of presentation is reversed. In other words, presenting first acts like a commitment device.[29] We will also provide more general characterizations of equilibrium play in these commitment games.

To get some intuition, suppose that Σ consists of evidence set pairs E and F which satisfy the following conditions: the leader's available evidence at E and at F is the same witness; all evidence available to the follower at F is available to the follower at E; and $v(E) = v(E \cup F) = v_1 \neq v(F)$. J can then not reach different verdicts at E and at F as the follower could then profitably deviate at E to mimicking the evidence prescribed at F; so J must reach the leader's favored verdict at both evidence set pairs. However, J would reach v_2 at F if, at E, the follower could commit to presenting some evidence that is not available at F.

We formalize these ideas by defining a commitment game. Specifically, take some fixed order game $\Gamma^{1,2}$. The commitment game $\Gamma^{1,2}_c$ has the following time line. The follower starts play by choosing its strategy: that is, the evidence it presents in response to the leader's presentation at each follower evidence set. Nature then selects the state and the evidence set pair; the leader presents evidence, having observed its own evidence set and the follower's strategy; the follower implements its chosen strategy; and J reaches a verdict after observing the follower's strategy and the evidence pair. Players have the same payoff

[29]The link between commitment and the order of moves dates back to Hamilton and Slutsky (1990); but the question we address differs from those in the literature.

functions as in $\Gamma^{1,2}$. We analyze commitment games using the solution concept described in Section 3.2, and denote the outcome correspondence by $\omega_c^{1,2}$.

We now show that $\omega_c^{1,2}$ coincides with $\omega^{2,1}$ for some leader $1 \in \{D, P\}$ in Examples 3-5:

Example 3

In this example, $\Gamma^{D,P}$ has a separating equilibrium and an equilibrium in which J wrongfully acquits at E, H. By contrast, $\Gamma_c^{D,P}$ has a unique equilibrium in which P commits to presenting f at E, F and passing at E, H; so J convicts at E, H and H, H, and acquits at E, F and EF, H, which is the separating outcome. This is the only equilibrium of $\Gamma_c^{D,P}$, as P could otherwise profitably deviate to its strategy in the separating equilibrium. On the other hand, $\Gamma^{P,D}$ only has a separating equilibrium. In sum, presenting first replicates the effect of committing ex ante.

Example 4

In this example, $\Gamma^{D,P}$ has a separating equilibrium and an equilibrium with a wrongful acquittal at EH, EF; but this cannot be an outcome in $\Gamma_c^{D,P}$ because P could then profitably deviate to presenting f in response to e at EF, EH and passing in response to e at EF, FH. If D presented e at EH, EF then J would observe e, f and would have to convict, as e, f could only be played at EH, EF after P's (observed) deviation; and if D presented h or eh then J would again convict. P's deviation would therefore be profitable. Conversely, $\Gamma_c^{D,P}$ has a separating equilibrium in which P commits to present f in response to e and otherwise to present h at EF, FH and to always pass at EH, EF. Every equilibrium in $\Gamma^{P,D}$ is separating. $\Gamma_c^{D,P}$ also has a separating equilibrium, and can have no other equilibrium because D cannot prevent P from directly proving factual guilt by presenting eh at EH, EF. Thus, $\omega^{P,D} = \omega_c^{D,P}$ and $\omega^{D,P} \subset \omega_c^{P,D}$ in this example: litigants prefer to lead because P, as leader, would directly prove factual guilt at EH, EF, as it could also do by committing as a follower.

Example 5

In this example, $\Gamma^{P,D}$ has a unique equilibrium in which D always passes, and is wrongfully convicted at F, F; but this cannot be an outcome in $\Gamma_c^{P,D}$ because D could then profitably deviate to presenting e at EF, F. J still convicts at EF, F, but now acquits at F, F. Indeed, D commits to presenting e at EF, F, and is therefore rightfully acquitted at F, F in the only equilibrium of $\Gamma_c^{P,D}$. $\Gamma^{D,P}$ has an equilibrium in which J rightfully acquits at F, F because, by presenting first, D is forced to present the same evidence at EF, E and at EF, F. This replicates the equilibrium of $\Gamma_c^{P,D}$, where D commits to presenting the same evidence at the two evidence set pairs. It is easy to confirm that every equilibrium of $\Gamma_c^{D,P}$ is also separating. Thus, $\omega^{D,P} = \omega_c^{P,D} \neq \omega^{P,D}$ in this example.

In sum, $\omega^{2,1}$ and $\omega_c^{1,2}$ coincide in Examples 3-5 for *an* assignment of roles: viz. some $1 \in \{D, P\}$. In Example 3, this is true for both assignments: each litigant as follower prefers to commit, and the equilibrium outcomes in each pair ($\Gamma_c^{D,P}$,$\Gamma^{P,D}$) and ($\Gamma_c^{P,D}$,$\Gamma^{D,P}$) are the same. However, commitment is not valuable for D in Example 4 or for P in Example 5.

The property that $\omega_c^{1,2} = \omega^{2,1}$ does not generalize to all no-subpoena games, as Example 6 below illustrates:

Example 6 *There are five states:* $S = \{g_1, g_2, i_1, i_2, i_3\}$*, where* g_1 *and* g_2 *are the only factually guilty states. There are three witnesses:*

$$e = \{g_1, g_2, i_1, i_2\},\ f = \{g_1, i_1, i_2, i_3\} \text{ and } h = \{g_1, g_2, i_2, i_3\}$$

and six evidence sets:

$$E = \{e\},\ F = \{f\},\ H = \{h\},\ EF = \{e, f\},\ EG = \{e, g\} \text{ and } FG = \{f, g\}.$$

The conditional distribution of evidence set pairs (with D's evidence set written first) satisfies

$$\pi_{g_1}(EG, EF) = \pi_{g_2}(H, E) = \pi_{i_1}(E, F) = \pi_{i_2}(E, FG) = \pi_{i_3}(F, F) = 1.$$

The prior distribution satisfies

$$\max\{\frac{p_{g_1}}{p_{g_1} + p_{i_2}}, \frac{p_{g_1}}{p_{g_1} + p_{i_3}}\} < d < \frac{p_{g_1}}{p_{g_1} + p_{i_1}}.$$

Extending the criterion introduced in Section 3.3 in an obvious fashion, we prove Proposition 3 in the Appendix:

Proposition 3 *If Example 6 and litigants observe the evidence set pair then P prefers to play the commitment game* $\Gamma_c^{D,P}$ *over playing either fixed order game:* $\Gamma^{D,P}$ *and* $\Gamma^{P,D}$*.*

Thus far, we have considered commitment games based on non-discovery games. Commitment could also be valuable in fixed order discovery games, such as Example 1. It is easy to confirm that the commitment game based on Example 1 has a separating equilibrium. Indeed, the commitment game based on any fixed order discovery game has a separating equilibrium. Despite these parallels to non-discovery games, Theorem 1a) states that litigants cannot prefer to lead. The difference turns on Lemma, which holds in all fixed order discovery games, but which fails in Examples 3-5.

5.4. J's preferences

Theorem 1c) asserts that J can only prefer an order if litigants prefer to follow in discovery games. In this subsection, we demonstrate by example that this result does not carry over to non-discovery games. We then collect this section's main results in Theorem 2.

Example 7 *There are four states:* $S = \{g_1, g_2, i_1, i_2\}$*; the defendant is only factually guilty in states* g_1 *and* g_2*. There are three witnesses:*

$$e = \{g_1, g_2, i_1\},\ f = \{g_1, i_1, i_2\} \text{ and } h = \{g_2, i_1\}$$

and four evidence sets:

$$E = \{e\},\ F = \{f\},\ EF = \{e, f\} \text{ and } H = \{h\}.$$

The conditional distribution of evidence set pairs (with D's evidence set written first) satisfies

$$\pi_{g_1}(E, EF) = \pi_{g_2}(E, H) = \pi_{i_1}(F, EF) = \pi_{i_2}(H, F) = 1.$$

The prior distribution satisfies

$$\frac{p_{g_1}}{p_{g_1} + p_{i_2}} < d < \frac{p_{g_1}}{p_{g_1} + p_{i_1}}.$$

The burden of proof assumption implies that every outcome in $\Gamma^{D,P}$ is separating.

$\Gamma^{P,D}$ does not have a separating equilibrium because P could secure a conviction at F, EF if J convicts at E, EF. However, the game has two pooling equilibria, one with a wrongful conviction at F, EF and the other with a wrongful acquittal at E, EF. On the one hand, D is wrongfully convicted at F, EF if J observes $f, pass$ at E, EF and at F, EF; f, h at H, F; and $h, pass$ at E, H. On the other hand, D is wrongfully acquitted at E, EF if J observes $f, pass$ at E, EF and at H, F; $e, pass$ at F, EF; and $h, pass$ at E, H. with a wrongful acquittal at E, EF.

In sum, $\Gamma^{D,P}$ only has a separating outcome, whereas $\Gamma^{P,D}$ has two non-separating equilibria: one with a wrongful conviction and another with a wrongful acquittal. Consequently, J prefers D to lead, while neither litigant can prefer an order.

We end this section by summarizing our main results in

Theorem 2 *In fixed order non-discovery games:*

a) *Litigants may prefer to lead;*

b) *J alone may prefer an order.*■

6. Variable order games

Thus far, we have considered when players can prefer an order of presentation before knowing their available evidence. This perspective is relevant to the design of procedural rules for future trials. In some non-judicial contexts, a 'litigant' might choose the presentation order before observing its available evidence: for example, the chair might choose a procedure to govern all future committee meetings.

Results in Section 5 suggest that the choice of order might optimally depend on the parameters of the case.[30] One possible way to generate this flexibility is to allow a given litigant to choose the order after observing its available evidence. In this section, we consider the order which a given litigant (say, D) would choose *after observing its available evidence*. This perspective is relevant in Italian criminal trials, where the defendant may ask to lead: defendants typically arrive at trial having observed their available evidence, and are not repeat players. It is also relevant in some non-judicial contexts, such as committees, whose chair may decide the order in which members speak on a case-by-case basis.

[30] 'Optimally' means: if the optimal order depends on player payoffs.

The game which we analyze in this section starts with Nature choosing the state and available evidence sets, revealing E_l to litigant l; D then chooses an order, and the ensuing game is then played out according to the rules specified in Section 3.2. We focus on games in which litigants share available evidence, as in the fixed order discovery games of Section 4. D then chooses the order, knowing the evidence available to both litigants. We denote such games by Γ, and refer to them as *variable order discovery games*. We analyze Γ using the solution concept introduced in Section 3. We will argue that $\omega^{D,P} \cup \omega^{P,D}$, the set of outcomes in the two games $\Gamma^{1,2}$, is contained in the outcome correspondence of Γ, but that D cannot ex ante prefer to play the variable order discovery game over playing $\Gamma^{P,D}$.

Our first result compares outcomes in variable order games with outcomes in the discovery games of Section 4.

Proposition 4 *Every outcome in a fixed order discovery game is an outcome in the variable order discovery game.*

We prove Proposition 4 in the Appendix by taking any outcome of $\Gamma^{1,2}$ and constructing an equilibrium of Γ with the same outcome.

Proposition 4 implies that D chooses each given order at every evidence set in an equilibrium of Γ, even though it never prefers to lead (cf. Theorem 1a)). In one of these equilibria, D is deterred from presenting second at any evidence set because J draws the most skeptical possible inference from order P, D and any evidence pair. Furthermore, Propositions 1 and 4 jointly imply that Γ has a separating equilibrium.

We now use Example 8 (below) to demonstrate that variable order discovery games may have outcomes which are not outcomes in either fixed order discovery game.

Example 8 *There are eight states:* $S = \{g_1, g_2, g_3, g_4, i_1, i_2, i_3, i_4\}$*, where* D *is factually guilty in states* $g_1 - g_4$*. There are six witnesses:*

$$\begin{aligned} e &= \{g_1, g_2, i_1\},\ f = \{g_2, g_3, i_1\},\ h = \{i_1\}, \\ k &= \{g_4, i_2, i_3\},\ n = \{g_4, i_3, i_4\} \text{ and } q = \{g_4\} \end{aligned}$$

and eight evidence sets:

$$\begin{aligned} E &= \{e\},\ F = \{f\},\ EF = \{e, f\},\ EFH = \{e, f, h\}, \\ K &= \{k\},\ KN = \{k, n\},\ KNQ = \{k, n, q\},\ N = \{n\} \end{aligned}$$

whose conditional distribution satisfies

$$\begin{aligned} \pi_{g_1}(E) &= \pi_{g_2}(EF) = \pi_{g_3}(F) = \pi_{g_4}(KNQ) = \pi_{i_1}(EFH) \\ &= \pi_{i_2}(K) = \pi_{i_3}(KN) = \pi_{i_4}(N) = 1. \end{aligned}$$

The prior distribution satisfies

$$\max\{\frac{p_{g_1}}{p_{g_1} + p_{i_1}}, \frac{p_{g_4}}{p_{g_4} + p_{i_3}}, \frac{p_{g_4}}{p_{g_4} + p_{i_4}}\} < d < \min\{\frac{p_{g_2}}{p_{g_2} + p_{i_1}}, \frac{p_{g_3}}{p_{g_3} + p_{i_1}}, \frac{p_{g_4}}{p_{g_4} + p_{i_2}}\}.$$

All evidence which includes witness h [resp. q] induces acquittal [resp. conviction] in Example 8; all other evidence is ambiguous.

Fixed order discovery game $\Gamma^{P,D}$ with the parameters in Example 8 has an equilibrium in which J observes $e, pass$ at EFH and at E and the full report at every other evidence set; so the only miscarriage of justice is a wrongful acquittal at E. J cannot wrongfully convict at K after observing $k, pass$ because P could then profitably deviate to presenting k at KN. (This argument is equivalent to that used in the proof of Claim 2.) The only other outcome of $\Gamma^{P,D}$ is separating. Similarly, the only non-separating outcome of fixed order discovery game $\Gamma^{D,P}$ has a wrongful conviction at K.

Now consider the variable order discovery game Γ with the parameters in Example 8. Proposition 4 implies that Γ has a separating outcome and two other outcomes, each with a single miscarriage of justice. We demonstrate in the Appendix that Γ also has a non-separating equilibrium in which J wrongfully acquits at E and wrongfully convicts at K. D chooses order D, P at some, but not all evidence sets in this equilibrium, whereas it chooses the same order at every evidence set in the equilibrium constructed to prove Proposition 4.

The condition on priors in Example 8 implies that Γ has no other outcome. We now extend our definition of preferences in Section 3.3 to playing a variable order game versus playing a fixed order game. D cannot prefer to play $\Gamma^{P,D}$ over playing Γ because both games have a separating equilibrium and an equilibrium with a wrongful acquittal at E. However, D prefers to play $\Gamma^{P,D}$ over playing Γ at evidence set K because it is acquitted in every equilibrium in the former case, and convicted in some equilibria otherwise.

Theorem 3 below states that this property is more general than Example 8.

Theorem 3

- **a)** *D cannot prefer playing the variable order discovery game (Γ) to playing $\Gamma^{P,D}$, where it always follows;*
- **b)** *D may prefer playing $\Gamma$to playing $\Gamma^{D,P}$;*
- **c)** *D may prefer playing $\Gamma^{P,D}$ to playing Γ.*

We use Proposition 4 to prove Theorem 3a) in the Appendix. Specifically, we note that Theorem 3 can only fail if $\Gamma^{P,D}$ only has separating equilibria, and Γ also has a non-separating equilibrium; and we then show how to construct an equilibrium of $\Gamma^{P,D}$ with the same, non-separating outcome. We prove part b) via Example 2 in Section 4), and prove part c) via a related example.[31]

7. Summary and extensions

We have studied litigants' preferences over the order of presentation in a model that tries to capture essential features of common law trials. Our results indicate the importance of distinguishing between fixed order discovery games and fixed order games in which litigants

[31] Example 2 is detailed in the proof of Theorem 1b) in Section 4.

may have different available evidence. In the former case, litigants cannot prefer to lead, but may prefer to follow; in the latter case, litigants might prefer to lead. These results suggest that the objectives of procedure (whatever those are) might better be served by sometimes changing the existing order. Any such change would doubtless require further procedural reforms: for example, requiring that the indictment be accompanied by a more detailed summary of P's case to satisfy the 6th Amendment requirement that D be "informed of the nature and cause of the accusation" in criminal cases. However, we believe that changing the order is practically possible, in part because civil law criminal cases usually start with interrogation of the defendant (cf. Damaska (1973) p528). On the other hand, we have also shown that a litigant would prefer to always follow than to choose an order ex post when litigants always share available evidence.

We suggested in the Introduction that our results may shed light on debates in which participants cannot lie (e.g. because there are fact-checkers), but which lack some of the procedural rules of common law trials. We end the paper by discussing the robustness of our results on fixed order discovery games to alternative rules. We first show that a litigant may be advantaged by the presence of a litigant with opposite preferences (Section 7.1). We then explore the implications of alternative stopping rules (Section 7.2), of simultaneous presentations (Section 7.3) and of Full Reports (Section 7.4).

7.1. Advantageous competition

We have focused throughout on two-litigant games; but a comparison with one-litigant games is instructive. Specifically, consider a game (say, Γ^D) in which a single litigant (D) presents evidence to J, which then reaches a verdict. D would not choose to present evidence which directly proves factual guilt in equilibrium; so it is not surprising that Γ^D may not have a separating equilibrium. Example 9 below illustrates how the availability of evidence which directly proves factual guilt may result in a wrongful *conviction*:

Example 9 *There are three states: $S = \{g, i_1, i_2\}$, and the defendant is only factually guilty in state g. There are three witnesses:*

$$e = \{g, i_1\},\ f = \{i_2\} \text{ and } h = \{g\}$$

and three evidence sets,

$$E = \{e\},\ F = \{f\} \text{ and } EH = \{e, h\},$$

whose conditional distribution satisfies

$$\pi_g(EH) = \pi_{i_1}(E) = \pi_{i_2}(F) = 1.$$

The prior distribution satisfies $\frac{p_g}{p_g + p_{i_1}} > d$.

D cannot present witness h in any equilibrium of Γ^D because J would then acquit after observing e; so D could profitably deviate to presenting e at EH. The condition on priors implies that J must convict at E and at EH in every equilibrium of Γ^D.[32]

[32] This punchline is robust to dropping the burden of proof requirement.

D's problem in Γ^D is a commitment failure. Specifically, consider the commitment game Γ_c^D in which D chooses the evidence it would present at each evidence set before observing Nature's choice. D would commit to presenting witness h at EH in every equilibrium of Γ^D; and J would then acquit at E.

Now consider the fixed order discovery game $\Gamma^{D,P}$ which satisfies the conditions in Example 9. This game has two outcomes. One equilibrium replicates the outcome of Γ^D: D presents e at E and at EH, to which P responds by passing. Proposition 1 implies that $\Gamma^{D,P}$ also has a separating equilibrium: for example, D presents and P completes the full report at every evidence set. J acquits at E in these equilibria. Applying our criteria: both D and J prefer to play $\Gamma^{D,P}$ over playing Γ^D.[33] Perhaps yet more strikingly, Γ^P has the same outcomes as $\Gamma^{D,P}$: so D would prefer to delegate presentation of evidence to P.

7.2. Stopping rules

We have obtained our results in a model where litigants each have a single opportunity to present new witnesses: a feature of common law trials. We explore equilibrium play in games with alternative stopping rules in this subsection.

First, consider a debate between two litigants who alternately present evidence or pass over any fixed number of rounds: a set-up akin to Presidential debates. If litigants share available evidence then the game has an equilibrium in which evidence is only presented in the last two rounds - which may shed light on Obama's success in the 2012 debate with Romney.

Alternatively, consider games which only differ from our model in the following respect: litigants alternate in presenting evidence (starting in Round 1) until a litigant passes, at which point J reaches a verdict and the game ends. Proposition 1 and Lemma hold in discovery games; so no litigant can prefer to present in odd-numbered rounds. However, litigants cannot prefer to present in even-numbered rounds. To see this, recall our analysis of Example 2, which we used to prove that litigants in our model can prefer to follow:

We argued in Section 4 that our version of Example 2 has an equilibrium in which J observes $e, pass$ and acquits at E and at EFH if P leads. It is easy to confirm that this is also an outcome with the alternative stopping rule if P presents in odd-numbered rounds. We also argued that every equilibrium must be separating in our model if D leads because D could profitably deviate to presenting h at EFH unless J acquits after observing e, f - in which case, D could profitably deviate to presenting e at EF. This argument fails with the alternative stopping rule because D does not forego the opportunity to directly prove innocence at EFH by presenting e in Round 1. Specifically, there is an equilibrium in which J convicts after observing $e, f, pass$ and acquits after observing $e, f, h, pass$.[34] D can no longer profitably deviate to presenting e at EF in Round 1, while P cannot profitably deviate from passing to presenting f at EFH. This argument implies that the same non-separating outcome exists, irrespective of the order of presentation. We draw the following lesson. In our model, the leader commits to the evidence which it decides to

[33] D and J also prefer to play $\Gamma^{P,D}$ over playing Γ^D.

[34] $e, f, pass$ means that D presents e in Round 1 and passes in Round 3; $e, f, h, pass$ should be read analogously.

present; with the alternative stopping rule, a litigant who does not call some witness in one round can do so later if its rival has not passed. This argument relies on the supposition that litigants are sure to alternate. If the follower could end the debate after two rounds then litigants can prefer to follow.

7.3. Simultaneous presentations

Due process requires common law trials to be conducted in public; but other debates could be conducted simultaneously. In this subsection, we will argue that 'litigants' who share available evidence cannot prefer to present simultaneously over presenting second, and that they may prefer to follow.[35] The argument turns on proving a variant on Lemma. Specifically, let any equilibrium X of the simultaneous move game prescribe each litigant $l \in \{1,2\}$ to present $e_l(E)$ at each evidence set E, and J to reach verdict $v^X(e_1, e_2)$ after litigants 1 and 2 have respectively presented e_1 and e_2. Define $E_2(e)$ as $\{f \in E : v^\xi(e,f) = v_2\}$.

Consider the following strategy combination (say, Y) in $\Gamma^{1,2}$ which prescribes J to reach verdict $v^X(e_1, e_2)$ after observing any evidence pair e_1, e_2; and, at each each evidence set E: litigant 1 to present $e_1(E)$; and litigant 2's response to e to satisfy

$$e_2(e) = \begin{array}{cc} e_1(E) & \text{if } e = e_2(E) \\ \text{some } f \in E_2(e) & \text{if } e \neq e_2(E) \text{ and } E_2(e) \text{ is nonempty} \\ e_1(E) & \text{otherwise} \end{array}$$

As X is an equilibrium of the simultaneous move game, J cannot profitably deviate from Y's prescription at any evidence pair. If X prescribes verdict v_1 (1's favored verdict) at E then 2 cannot profitably deviate from Y at E by definition of $E_2(e)$. If X prescribes v_2 at E then $v^X[e, e_2(E)] = v_2$ for every $e \in E$, else 1 could profitably deviate from X at E. Hence, $E_2(e)$ is nonempty for every $e \in E \backslash e_1(E)$, and 1 cannot profitably deviate. In sum, Y specifies an equilibrium of $\Gamma^{1,2}$ with the same outcome as X. The arguments used to prove that Theorem 1a) follows from Lemma then imply that litigants cannot prefer to present simultaneously over playing $\Gamma^{1,2}$. It is easy to confirm that the simultaneous move game which satisfies Example 2 only has a separating equilibrium: that is, has the same outcome as $\Gamma^{D,P}$. The arguments used in the proof of Theorem 1b) then imply that litigants may prefer to follow over presenting simultaneously.

7.4. Full reports

We argued above that Full Reports (litigants can present any combination of available witnesses) is a natural assumption when modelling common law trials; but Full Reports might fail in other debates, e.g. because the listener has a limited attention span. Fixed order discovery games which fail Full Reports may lack a separating equilibrium: for example, if litigants could only present one witness and the conditions in Example 1 held

[35] To facilitate comparison, we suppose that both litigants must present some evidence in a simultaneous move game. Inspection of the argument below reveals that the same result applies if neither litigant had a burden of proof in either game.

(cf. Section 3.3) then $\Gamma^{D,P}$ would have a separating equilibrium because P could induce conviction by presenting whichever witness D did not present at EF. However, $\Gamma^{P,D}$ would not have a separating equilibrium because D would then always pass at EF. However, both of these games have an equilibrium in which J observes $f, pass$ (and then convicts) at F and at EF. Consequently, litigants prefer to lead in this variant on fixed order discovery games.

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APPENDIX

Proposition 1 *Every fixed order discovery game has a separating equilibrium.*

Proof Consider the following construction (recalling that v_l is the verdict favorable to l):

- The leader presents the full report at every evidence set;
- At each evidence set E: the follower responds to e_1 by completing the full report unless $v(E) = v_1$ and e_1 is also in another evidence set F such that $v(F) = v_2$ and $F \subset E$, in which case the follower completes the full report at F;
- After observing e_1, e_2 (where e_2 could be $pass$), J
  - believes that the realized evidence set is: E and reaches verdict $v(E)$ if e_1e_2 is the full report at E;
  - believes that the realized evidence set is in Σ_v and reaches verdict v if e_1e_2 induces verdict v; and
  - otherwise believes that the realized evidence set is in Σ_{v_1} and reaches verdict v_1.

This strategy combination implies that J best responds both to the evidence presented on the path and after the leader has deviated, as well as to any evidence pair which induces a verdict. Any other evidence pair is ambiguous; so J cannot profitably deviate.

If $v(E) = v_1$ then the leader cannot profitably deviate at any evidence set E because presenting the full report results in its favored verdict; and if $v(E) = v_2$ then the leader cannot profitably deviate because it anticipates that J will reach v_2 after the follower's response. The follower cannot profitably deviate because its response to the leader presenting some, but not all available witnesses at E results in J reaching its favored verdict whenever $v(F) = v_2$ for some $F \subset E$ which contains e_1.■

Theorem 1 *In fixed order discovery games:*

a) *Litigants cannot prefer to lead.*

The proof relies on

Lemma *Let $\Gamma^{1,2}$ and $\Gamma^{2,1}$ be fixed order discovery games. If $\Gamma^{1,2}$ has an equilibrium which prescribes verdict v_1 (the leader's favored verdict) at evidence sets Φ then $\Gamma^{2,1}$ has an equilibrium which prescribes verdict v_1 at every evidence set in Φ.*

Proof We focus on the case in which $1 = D$ for expositional convenience. There is then an equilibrium of $\Gamma^{D,P}$ (say, X) which partitions Σ into Φ and $\Sigma \backslash \Phi$, where X prescribes D and P to respectively present $e_D^X(E)$ and $e_P^X(E)$ and for J to reach verdict v after observing $e_D^X(E), e_P^X(E)$ at every $E \in \Phi$. The full report at $E \in \Phi$ [resp. $F \notin \Phi$] cannot induce conviction [resp. acquittal], else P [resp. D] could profitably deviate to completing the full report.

Consider the following strategy combination (say, Y) and beliefs in $\Gamma^{P,D}$:

- Y prescribes P to present $e_P = e_D^X(E)$ at every $E \in \Phi$, and to present the full report at any other evidence set;
- At any evidence set in Φ: Y prescribes D to respond to $e_P = e_D^X(E)$ by presenting $e_D = e_P^X(E)$, and otherwise to respond to e_P by completing the full report at E;
- At any evidence set $F \notin \Phi$: Y prescribes D to respond to e_P by completing the full report at F unless F contains some evidence e_D which does not complete the full report at F such that either
  - $e_P, e_D = e_D^X(E), e_P^X(E)$ for some $E \in \Phi$ or
  - e_D completes the full report at some evidence set in $\Phi \cup \Sigma_\alpha$.

  In both of these cases, Y prescribes D to present e_D in response to e_P;
- After observing e_P, e_D, J believes that the realized evidence set is in Σ_α if
  - $e_P, e_D = e_1^X(E), e_2^X(E)$ for some $E \in \Phi$ or
  - $e_P e_D$ induces α or
  - e_D completes the full report at some evidence set $E \in \Phi \cup \Sigma_\alpha$,

  and that the realized evidence set is in Σ_γ otherwise.

  Y prescribes J to acquit if and only if it believes that the realized evidence set is in Σ_α.

Y cannot prescribe the same evidence pair on the path at any $E \in \Phi$ and any $F \notin \Phi$. To see this, suppose per contra that X prescribes D to present the full report at F (say, F^*). X must then prescribe J to acquit after observing F^*, e_P for every $e_P \in E \cup pass$, else P could profitably deviate to passing at E. However, this implies that D could profitably deviate from X's prescription at F to presenting the full report because, by construction, X prescribes J to convict on the path at F.

Given the strategies that Y prescribes for litigants, J's beliefs are sequentially rational, and J cannot profitably deviate. To see this, note that Y only prescribes J to acquit after observing evidence which either induces α; or is played on the path at evidence sets in Σ_α^X (so J holds the same posterior beliefs after observing $e_1^X(E), e_2^X(E)$ in both games); or if the evidence completes the full report at some evidence set in Φ or in Σ_α. None of this evidence can induce conviction, else P could profitably deviate from X by presenting the full report at E. Finally, Y never prescribes J to convict after observing evidence which induces acquittal, else D could profitably deviate from X by presenting the full report at some evidence set in $\Sigma \backslash \Phi$.

We now argue that litigants cannot profitably deviate at any evidence set:

Consider an evidence set $E \in \Phi$. Y prescribes P to present $e_D^X(E)$ and J to acquit after observing either $e_D^X(E), e_P^X(E)$ or D completing the full report at E. D can therefore secure acquittal, irrespective of the evidence that P presents at E; so no litigant can profitably deviate from Y's prescription at E.

Now consider an evidence set $F \notin \Phi$. If $F \in \Sigma_\alpha$ then Y prescribes J to acquit whenever D completes the full report at F; so neither litigant can profitably deviate. If $F \in \Sigma_\gamma$ then P cannot profitably deviate from presenting the full report; and if P deviates to presenting e_P then D can profitably deviate from completing the full report if and only if there is e_D such that J acquits after observing e_P, e_D: to wit if the evidence pair is prescribed by Y or completes the full report at some $E \in \Phi$.

These arguments imply that Y is an equilibrium in $\Gamma^{P,D}$ which prescribes acquittal at every evidence set in Φ. An equivalent argument establishes Lemma when litigant 1 is P.■

a) follows from Lemma. To see this suppose, per contra, that D prefers to lead. Condition 2 in Section 3.3 then implies that $\Gamma^{D,P}$ has an equilibrium (say, X) which prescribes acquittal at some evidence set E, while $\Gamma^{P,D}$ has an equilibrium (say, Y) which prescribes conviction at E. Lemma then implies that $\Gamma^{P,D}$ has another equilibrium Y' which also prescribes acquittal at E, while $\Gamma^{D,P}$ has another equilibrium X' which prescribes conviction at E.

In sum, we have outcomes $x, x' \in \omega^{D,P}$ and $y, y' \in \omega^{P,D}$ such that

$$y'_D(E) \geq x_D(E) > y_D(E) \geq x'_D(E),$$

where $z_D(E)$ denotes the payoff for D that equilibrium Z prescribes at E: $z = x, y$. These inequalities imply that $y'_D(E) > x'_D(E)$, contrary to Condition 1. An analogous argument precludes P from preferring to lead.■

Proposition 2 *Litigants only prefer to follow in fixed order discovery games if IES holds.*

Proof Any equilibrium of $\Gamma^{1,2}$ partitions Σ into collections of evidence sets, say Φ^i, with a nonempty intersection; prescribes litigant l to present e^i_l at every evidence set in Φ^i; and prescribes J to reach verdict $v(\Phi^i)$ after observing the common evidence pair presented at each Φ^i. Proposition 1 implies that litigants can only prefer to follow if some equilibrium of $\Gamma^{1,2}$ (say, X) prescribes J to reach v_2 at some evidence set(s) in Σ_{v_1}. We write $e_l(E)$ for the evidence which X prescribes litigant $l \in \{1, 2\}$ to present on the path at evidence set E.

As X sometimes prescribes a miscarriage of justice, some evidence set in Σ_{v_2} must share a witness with an evidence set in Σ_{v_1}. Furthermore, no evidence which induces v_l can be available at any evidence set at which X prescribes v_m ($l \neq m$), else m could profitably deviate: so part a) of IES is necessary. We will show that, if X exists, then $\Gamma^{2,1}$ has an equilibrium with the same outcome unless part b) of IES is also satisfied.

We now use X to define a strategy combination Y for $\Gamma^{D,P}$ which prescribes the same verdicts at each evidence set as X. Specifically, Y prescribes

- Litigant 2 to present $e_1(E)$ at every evidence set E, and litigant 1 to respond to each $e \in E$ by presenting the same evidence as X prescribes 2 to present in response to e;
- J to reach the same verdict after observing litigants 2 and 1 presenting e and f respectively as X prescribed after observing 1 and 2 presenting e and f respectively.

Litigants cannot prefer an order if $\Gamma^{1,2}$ and $\Gamma^{2,1}$ share a non-separating outcome; so if litigants prefer to follow then some player must have a profitable deviation from Y at an evidence set. This player can clearly not be J.

Litigant 1 only has a profitable deviation at evidence set E in a collection $\Phi^i \in \Phi_{v_2}$ if E contains witnesses e and f and there is an evidence set F in a collection $\Phi^j \in \Phi_{v_1}$ such that $ef \in E \cap F$. Litigant 2 only has a profitable deviation at evidence set E in a collection $\Phi^i \in \Phi_{v_1}$ if E is not a singleton and there is an evidence set F in a collection $\Phi^j \in \Phi_{v_2}$ such that $e_1(E) \in E \cap F$.

In sum, $\Gamma^{1,2}$ and $\Gamma^{2,1}$ would share a non-separating outcome, precluding a preference to follow, unless IES held.■

Proposition 3 *If Example 6 and litigants observe the evidence set pair then P prefers to play the commitment game $\Gamma_c^{D,P}$ over playing either fixed order game: $\Gamma^{D,P}$ and $\Gamma^{P,D}$.*

Proof Note that D presenting h induces conviction; while P presenting h [resp. e] induces acquittal [resp. conviction].

$\Gamma^{D,P}$ cannot have an equilibrium in which J observes different evidence pairs at E, F and at EG, EF, as it would then have to acquit at the former and convict at latter evidence pair; so J would have to acquit after observing $e, pass$ and e, f, and D could profitably deviate to presenting e at EG, EF. $\Gamma^{D,P}$ can also not have an equilibrium in which J observes the same evidence pair at EG, EF and at E, F but a different evidence pair at E, FG: for J would then have to convict at EG, EF and at E, F and acquit at E, FG; and P could then profitably deviate at E, FG to mimicking the response to e prescribed at EG, EF and at E, F. However, it has equilibria in which J observes the same evidence pair at E, FG and at EG, EF, acquitting at these evidence pairs as well as at E, F. These equilibria prescribe a single miscarriage of justice: a wrongful acquittal at EG, EF.

P presenting e directly proves that the state is factually guilty; so J convicts at EG, EF and H, E in every equilibrium of $\Gamma^{P,D}$. J must observe $f, pass$ at F, F; so the condition on priors implies that J must acquit at $F.F$ in every equilibrium of $\Gamma^{P,D}$. The condition on priors also implies that J must acquit at E, FG in every equilibrium of $\Gamma^{P,D}$. Finally, J cannot observe the same evidence pair at EG, EF and at E, F but a different evidence pair at E, FG as J would convict in the former and acquit in the latter case. J can only convict at EG, EF and at E, F if it convicts after observing both $f, pass$ and f, e, else D could profitably deviate at those evidence pairs. P could then profitably deviate to presenting f at E, FG. In sum, every outcome of $\Gamma^{P,D}$ must be separating; and it is easy to confirm that $\Gamma^{P,D}$ has a separating equilibrium.

In sum, neither fixed order game has an equilibrium in which J convicts at E, F. By contrast, J convicts at E, F in every equilibrium of $\Gamma_c^{D,P}$. To see this, note that $\Gamma_c^{D,P}$ has an equilibrium in which

- P commits to pass in response to e at EG, EF and at E, F in response to f at F, F and to h at H, E, and commits to present f in response to e at E, FG;
- Given P's equilibrium commitment, D presents f at F, F, h at H, E, and e otherwise; and

- J convicts after observing $e, pass$ and $h, pass$, and acquits after observing $f, pass$.

The condition on priors implies that J cannot profitably deviate. D cannot deviate at E, F or at H, E; and, given P's commitment and J's strategy, it cannot profitably deviate at EG, EF because presenting evidence which contains h directly proves factual guilt. Finally, P cannot profitably deviate because J would not convict at E, FG or at F, F after any other commitment. The latter argument implies that the outcome is unique.

P weakly prefers to play $\Gamma_c^{D,P}$ over playing either fixed order game at every evidence set pair, and is better off playing $\Gamma_c^{D,P}$ at E, F; so P prefers to play the commitment game.■

Proposition 4 *Every outcome in a fixed order discovery game ($\Gamma^{D,P}$ or $\Gamma^{P,D}$) is an outcome in the variable order discovery game (Γ).*

Proof Let X denote an equilibrium of a fixed order discovery game with a given order, which we denote $\Gamma^{1,2}$. We will argue that Γ has an equilibrium in which D chooses order $1, 2$ at every evidence set, and X is then played.

By definition of X, no player can profitably deviate once D has chosen order $1, 2$: J's belief that this choice of an order does not inform J about the realized evidence set is sequentially rational because D chooses this order at every evidence set.

Suppose that D deviates to choosing order $2, 1$, and consider the following strategy combination in the continuation:

- The leader (now litigant 2) presents the full report at every evidence set;
- If D is the follower then it completes the full report; if P is the follower then it passes;
- J acquits after observing evidence pair e, f if ef induces acquittal, and otherwise convicts.

After observing the unexpected order $2, 1$ and some evidence pair e_1, e_2, J believes that the realized evidence set is in Σ_α if and only if $e_1 e_2$ induces acquittal. These beliefs satisfy feasibility (cf. Section 3.2). Given these beliefs, J can never profitably deviate after observing the unexpected order.

If P is the follower and D has presented evidence which induces acquittal then J would acquit, irrespective of the evidence which P presents; so P cannot profitably deviate from passing. P can also not profitably deviate if D has presented evidence which does not induce acquittal, as J then convicts. If D is the follower then it cannot profitably deviate because the full report induces acquittal if any evidence contained in the full report induces acquittal.

D as leader cannot profitably deviate: again because the full report induces acquittal if any evidence contained in the full report induces acquittal. P as leader cannot profitably deviate because it expects D to complete the full report.

We now turn to D's choice of an order. As X is an equilibrium of $\Gamma^{1,2}$, it must prescribe acquittal at every evidence set whose full report induces acquittal. The specified strategy combination implies that J would only acquit at those evidence sets if D deviated to order

2, 1. Consequently, D cannot profitably deviate to choosing order 2, 1 at any evidence set. The result follows from the definition of X, where J's beliefs after D chooses the expected order correspond to its beliefs in the equilibrium of $\Gamma^{1,2}$, and therefore satisfy Bayes rule.■

Example 8

We claim that Γ has an outcome with a wrongful acquittal at E and a wrongful conviction at N.

Partition Σ into $\Sigma^{D,P}$ and $\Sigma^{P,D}$, where

$$\Sigma^{P,D} = \{E, F, EF, EFH\} \text{ and } \Sigma^{D,P} = \{K, N, KN, KNQ\}$$

and consider the following strategy combination and beliefs:

- D chooses order D, P at every evidence set in $\Sigma^{D,P}$ and order P, D at every evidence set in $\Sigma^{P,D}$.

- Consider any evidence set in $\Sigma^{P,D}$. If D has chosen order P, D then P presents the full report at every evidence set except EFH, where it presents e. D completes the full report unless P has presented e, in which case D passes. If D has chosen order D, P then D presents the full report, and P always passes.

  Consider any evidence set in $\Sigma^{D,P}$. If D has chosen order D, P then D presents the full report at every evidence set except KNQ, where it presents n. P completes the full report unless D has presented n, in which case P passes. If D has chosen order P, D then P presents the full report, and D always completes the full report.

- If a litigant has presented evidence which contains h [resp. q] then J believes that the realized evidence set is EFH [resp. KNQ] and acquits [resp. convicts].

  If D has chosen order P, D then the table below describes the support of J's belief about the realized evidence set and its verdict after observing any evidence pair whose composition does not contain g and is in an evidence set in $\Sigma^{P,D}$:

Evidence pairs	J's belief	Verdict
$e, pass$	$EFH \cup E$	α
$f, pass$	F	γ
$ef, pass$ or e, f or f, e	EF	γ

- If J observes an evidence pair whose composition is in an evidence set in $\Sigma^{D,P}$ then J believes that the realized evidence set is KNQ and convicts.

  If D has chosen order D, P then the table below describes the support of J's belief about the realized evidence set and its verdict after observing any evidence pair whose composition is in $\Sigma^{D,P}$ but does not contain n:

Evidence pairs	J's belief	Verdict
$n, pass$	$KNQ \cup N$	γ
$k, pass$	K	α
$kn, pass$ or k, n or n, k	KN	α

If J observes an evidence pair whose composition is in an evidence set in $\Sigma^{P,D}$ but does not contain h then J believes that the realized evidence set is EF and convicts.

J's beliefs after observing any evidence pair satisfy Bayes rule on the path, and are feasible off the path; and J's verdict is sequentially rational. Moreover, J knows when D has deviated to choosing the extra-equilibrium order because evidence sets in $\Sigma^{P,D}$ and $\Sigma^{D,P}$ do not share any common witnesses. Given an order, neither litigant can profitably deviate; and D cannot profitably deviate to choosing an unexpected order.

These arguments imply that the strategy combination and beliefs above are an equilibrium of Γ, where J wrongfully acquits at E and wrongfully convicts at N. By contrast, the two discovery games each feature a single miscarriage of justice. Hence, ex post order games may have outcomes which are neither in $o^{P,D}$ nor in $o^{D,P}$.

Theorem 3

a) *D cannot prefer playing the variable order discovery game (Γ) to playing $\Gamma^{P,D}$, where it always follows;*

b) *D may prefer playing Γ to playing $\Gamma^{D,P}$;*

c) *D may prefer playing $\Gamma^{P,D}$ to playing Γ.*

Proof

a) If $\Gamma^{P,D}$ has a non-separating equilibrium then D cannot prefer to play Γ because Propositions 1 and 4 imply that both games have a separating equilibrium, and D cannot prefer playing Γ to playing $\Gamma^{P,D}$ if Γ and $\Gamma^{P,D}$ share two or more outcomes. Accordingly, suppose that $\Gamma^{P,D}$ only has separating equilibria.

D then prefers to play the variable order discovery game if and only if that game has an equilibrium (say, X) which prescribes a wrongful acquittal and no wrongful convictions. We will prove the result by using X to construct a non-separating equilibrium (say, Y) of $\Gamma^{P,D}$ with the same outcome as X.

X prescribes D to choose order D, P at some evidence sets (say, $\Sigma^{D,P}$), and to choose order P, D at the other evidence sets (say, $\Sigma^{P,D}$). Write $\Sigma_v^{l,m}$ for the evidence sets in $\Sigma^{l,m}$ at which X prescribes verdict v, and $e_l(E)$ for the evidence which X prescribes l to present at E: $l \neq m \in \{D, P\}$.

Y prescribes

- P to present $e_D(E)$ at every $E \in \Sigma_\alpha^{D,P}$, to present $e_P(E)$ at every $E \in \Sigma_\alpha^{P,D}$, and to present the full report at every other evidence set;

- D to respond to e_P by passing when X prescribes P to present e_P at some evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$, and otherwise to complete the full report;
- J to acquit after observing any evidence e, f such that ef is in some evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$ unless ef is the full report at some evidence set in $\Sigma_\gamma^{D,P} \cup \Sigma_\gamma^{P,D}$ or is not contained in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$.

Y is well-defined because the full report at any singleton evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$ cannot be the full report at any other evidence set; and because no singleton evidence set $F \in \Sigma_\gamma^{D,P} \cup \Sigma_\gamma^{P,D}$ can consist of either $e_D(E)$ for any $E \in \Sigma_\alpha^{D,P}$ or of $e_P(E)$ for any $E \in \Sigma_\alpha^{P,D}$ as, in each case, D could profitably deviate from X by choosing the other order at F.

J cannot profitably deviate because

- $\Sigma_\gamma^{D,P} \cup \Sigma_\gamma^{P,D} \subseteq \Sigma_\gamma$ as X does not prescribe any wrongful convictions;
- X prescribes llitigants to present the same evidence pair at the same collections of evidence sets as X unless $e_D(E) = e_P(F)$ for some $E \in \Sigma_\alpha^{D,P}$ and $F \in \Sigma_\alpha^{P,D}$, in which case J cannot improve on acquitting; and
- The full report at any evidence set in $[\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}] \cap \Sigma_\gamma$ must be ambiguous, else P could profitably deviate from X by presenting or completing the full report.

Y only prescribes J to convict after observing e, f if ef is the full report at some evidence set in Σ_γ or is not contained in any evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$. Hence, D cannot profitably deviate at any evidence set in $\Sigma_\gamma^{D,P} \cup \Sigma_\gamma^{P,D}$, and therefore at any evidence set.

If $E \in \Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$ then Y prescribes J to acquit after observing every e_P, e_D such that $e_P e_D$ is the full report at E; so P cannot profitably deviate at any evidence set in $\Sigma_\alpha^{D,P} \cup \Sigma_\alpha^{P,D}$, and therefore at any evidence set.

These arguments imply that Y is a non-separating equilibrium of $\Gamma^{P,D}$, contrary to the initial supposition that $\Gamma^{P,D}$ only has a separating outcome.

b) If the conditions in Example 2 hold then $\Gamma^{D,P}$ only has separating equilibria (by Claim 2), and the variable order game also an equilibrium with a wrongful acquittal (by Claim 1 and Propositions 1 and 4). As the variable order game has no other outcomes, D prefers to play the variable order game.

c) Consider games which satisfy the following conditions.[36] There are four states: $S = \{g, i_1, i_2, i_3\}$, and the defendant is only factually guilty in state g. There are three witnesses:

$$e = \{g\},\ f = \{g, i_2, i_3\} \text{ and } h = \{i_1, i_2\}$$

and four evidence sets:

$$E = \{e\},\ F = \{f\},\ FH = \{f, h\} \text{ and } EFH = \{e, f, h\},$$

whose conditional distribution satisfies

$$\pi_g(EFH) = \pi_{i_1}(E) = \pi_{i_2}(FH) = \pi_{i_3}(F) = 1.$$

[36] These conditions involve relabelling part of Example 8.

The prior distribution satisfies

$$\max\{\frac{p_g}{p_g + p_{i_2}}, \frac{p_g}{p_g + p_{i_3}}\} < d < \frac{p_g}{p_g + p_{i_1}}.$$

$\Gamma^{P,D}$ only has a separating equilibrium; $\Gamma^{D,P}$ also has an equilibrium with a wrongful conviction; and the equilibrium outcomes of Γ coincide with the outcomes of $\Gamma^{D,P}$. Consequently, D prefers to always follow over playing the variable order discovery game.■