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The first and last word in debates: Plaintive plaintiffs

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The First and Last Word in Debates: Plaintive Plaintiffs*

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Abstract

Is it better to have the first or the last word in two-round debates like common law trials? If litigants always share available witnesses then they never prefer to present first (to lead), and may prefer to follow; and they never prefer to choose the order after observing the available witnesses than to always follow. Litigants can prefer to lead if they have different available witnesses because the outcomes when some litigant leads coincide with the outcomes in an alternative game where that litigant follows, but commits to its reaction function before observing its available witnesses. Leading is therefore a commitment device.

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1. Introduction

1.1. The order of presentation

According to conventional wisdom, advocates who compete to persuade a listener should try to present the first or the last argument in a debate: the first speaker anchors a listener's interpretation of subsequent arguments; whereas the final argument is influential if listeners only remember the last word.

Cognitive limitations are doubtless important in some debates, but the order of presentation can be significant for strategic reasons. According to David Axelrod, Obama's 2012 campaign manager, the decision to start running attack ads early contributed to the campaign's success.[1] This effect might be due to anchoring; but we think that seizing the agenda is more about steering the subsequent debate. On the other hand, Obama's tactics in the second Presidential debate in 2012 suggest that having the last word may be advantageous. Romney's claim that 47% of voters pay no income tax had been leaked before the debate.[2] Romney had presumably prepared a response; but Obama's success in the debate turned on his decision not to raise the issue till his concluding remarks.[3] This effect might be due to voters' short-term memory; but most viewers likely knew about Romney's claim before the debate itself. Finally, FOMC chairs have selected alternative orders: Greenspan chose to speak and to vote first at FOMC meetings, whereas Bernanke spoke last.[4] We explore the strategic consequences of the first and the last word in debates by analyzing a game-theoretic model in which the listener is not cognitively limited.

In multi-round debates, such as those between Presidential candidates, a given speaker could present first and last; and the middle word might be most important. By contrast, she has either the first or the last word in two-round debates with alternating speakers. We focus for simplicity on the latter case, as instanced by common law trials, where the defendant conventionally presents its evidence second. Common law trials are an attractive application of two-round debates because the extensive form and payoffs are each commonly known and well defined; the role of experienced attorneys and judges suggests that equilibrium plausibly predicts choices; and because psychologists have used mock juries to assess the effect of varying the order. We ask when the conventional order serves the interests of either litigant or of the judge/jury?

We address this question by comparing play across two games. Each game is played by a defendant (D), a plaintiff/prosecution (P), and a judge/jury (J); the games only differ according to the identity of the first litigant to present evidence (the *leader*). Nature starts each game by choosing the facts at issue (the *state*), including whether the defendant is factually guilty, and each litigant's available witnesses (its *witness set*): where a witness is a strict subset of states that contains the realized state, as in Milgrom (1981). The leader

[1] In Axelrod's words: "We defined the race and Governor Romney before the conventions, and he was digging out of that hole for the remaining two months."

[2] Mother Jones: "SECRET VIDEO: Romney tells millionaire donors what he REALLY thinks of Obama voters", Sept. 17, 2012.

[3] Slate: "When candidates attack: Obama won Tuesday night's slugfest, but Romney will live to fight another day", Oct. 17, 2012.

[4] See, for example, Blinder (2009).

observes its own (and possibly its rival's) witness set, and chooses which witness(es) to present: its *evidence*. After observing the leader's evidence, the follower recalls some or all of those witnesses and/or presents a (possibly empty) selection from its own witness set. In other words, only the leader needs to present new evidence: so our model captures the burden of proof in common law trials. J ends the game by acquitting or convicting after observing the evidence presented, but not the state or the witness sets. D always wants to be acquitted; P always wants a conviction; and J wants to avoid miscarriages of justice. We characterize each game via its perfect Bayesian equilibria, and refer to the ensuing payoff vectors as outcomes.

These games typically have several outcomes; so we say that a player prefers an order of presentation if it expects (before the state and witness sets are realized) to be better off at every equilibrium of the game with that order than at every equilibrium of the game with the other order. This criterion for preferring an order is, of course, partial; but, in light of their conflicting preferences over verdicts, D prefers an order if and only if P prefers the other order. We use our criterion to explore how differences between the litigants' realized witness sets determine whether litigants prefer to lead or to follow and J's preference over the order.

It is useful for our purposes to separate out a natural benchmark case in which, at each realized state, Nature assigns the same available witnesses to both litigants: a case which we call *discovery*. We first prove that litigants cannot prefer to lead in discovery games. This result is intuitive: the follower is allowed to present any evidence which the leader has suppressed, and can therefore not be worse off in equilibrium than when J observes all available evidence. As D and P have conflicting interests, the leader cannot be better off in equilibrium than when all available evidence is presented; so litigants cannot prefer to lead. However, litigants could be better off following, as we demonstrate by example. In this example, the game in which P leads only has a separating equilibrium, while the other game also has equilibria with a wrongful conviction. Flexibility is valuable here because the wrongful conviction occurs when J observes the same evidence at a pair of witness sets: the defendant is factually innocent at one of these witness sets, while P has direct proof of factual guilt at the other witness set.[5] If P leads then it must present any available direct proof in equilibrium, else J would acquit after D's best response; so J observes different evidence at different witness sets, and acquits at one of them. By contrast, J observes the same evidence (viz. no direct proof) and convicts at both witness sets in an equilibrium when D leads: D is deterred from deviating by P's implicit threat to directly prove factual guilt when available. Comparing across equilibrium correspondences, P prefers to follow; so D also prefers to follow. This argument may help to explain Obama's successful debating tactic: raising Romney's claim at the end of the debate obviated the need to demonstrate that he could counter Romney's defense. We also show that J can only prefer an order in discovery games if litigants prefer to follow.

Call games in which the two litigants' witness sets differ at some realizations: *non-discovery games*. The model has too many moving parts to provide results for non-discovery games as general as those for discovery games. In particular, we cannot provide general sufficient conditions for litigants to prefer to present first (reversing the result for

[5] J must convict after observing such evidence in any equilibrium.

discovery games). We address this problem by providing three reasons, each plausible in the context of trials, why litigants might prefer to lead in non-discovery games, each illustrated by example. We then characterize the common feature of these examples which results in a preference to lead:

First, the fact that some witness is available to a given litigant may inform J about the state. A leader who presents a witness available to both litigants prevents the follower from proving availability because the follower is allowed to recall that witness, even if it was not in its witness set; so litigants might prefer to lead. Second, as the leader alone must meet a burden of proof, pooling across witness set pairs is only possible if a witness is available to the leader at different witness set pairs; so the identity of the leader can matter.[6] The third reason applies to games in which each litigant is uncertain of its rival's available witnesses. Prima facie, resolution of this uncertainty seems to reinforce the advantage of following; but, as we show, this intuition turns out to be wrong. In this example, P only ever has a single witness, but D does not know who that is, and might therefore regret the evidence it presents if it leads. However, this is only part of the picture: D's best response at one witness set pair as follower adversely affects the verdict which J then reaches in equilibrium at another witness set pair. The example is constructed such that the latter effect dominates ex ante.

In fact, all of the examples share this feature. Specifically, the flexibility gained by responding on a case-by-case basis (viz. after observing its available witnesses) may be ex ante disadvantageous because some best responses at one witness set pair may impose negative externalities at other witness set pairs. The follower might therefore ex ante gain by committing to its strategy; and leading, like commitment, may obviate the follower's cross-witness set coordination problem. We formalize this intuition by analyzing a game in which the follower publicly commits to its strategy before observing its witness set. We show that, in each of the three examples, the outcomes of this commitment game coincide with the outcomes of the game in which a litigant leads. In other words, litigants prefer to lead because equilibrium play then replicates the effect of prior commitment. More generally, leadership could alleviate commitment problems.

In commitment games, the follower resolves its coordination problem by ensuring that J observes different evidence at different witness set pairs. A given litigant (say, D) could alternatively secure separation if it could decide on the order after observing its witness set, as in Italian criminal trials, where the defendant may ask to lead. We end the paper by characterizing equilibrium play in such *ex post order* games, and then using our criterion to ask whether the option to choose the order can ex ante advantage a player: that is, whether a player could prefer to play the ex post order game over a litigant always (viz. at every witness set pair) leading or always following. We focus on games in which litigants always share the same witness set: recall that, in such discovery games, litigants cannot prefer to lead, but may prefer to follow. The novel feature of ex post order games is that the choice of order may itself signal the realized witness set. Nevertheless, our results carry over in the following sense: D can neither prefer to play the ex post order game to always following nor prefer always leading to playing the ex post order game. Furthermore, D

[6] We provide a related example to demonstrate that J alone may prefer an order: again contrary to discovery games.

could prefer always following to playing the ex post order game, and could prefer playing the ex post order game to always leading. Finally, J cannot prefer to play the ex post order game over a litigant always leading, and may prefer playing one of the discovery games over playing the ex post order game. These results suggest some reasons why defendants are not allowed to choose the order in common law trials.

We survey the related literature in the next subsection, present our model of trials in Section 2, and analyze play in discovery games in Section 3. Section 4 considers games which are not played under discovery, and interprets the results therein by analyzing commitment games. Section 5 characterizes play in ex post order games. We summarize our results in Section 6, and then assess how they depend on features of the model which do not apply to other debates. We collect most proofs in an Appendix.

1.2. Related literature

We relate our model and results, in sequence, to the game-theoretic, the legal, and the psychology literatures.

Our focus on preferences over the order recalls a well-known IO literature. In particular, leadership is advantageous when firms' choices are strategic substitutes. The most fundamental differences are that we treat J's response as binary and endogenous; so there is no obvious analog to strategic substitutability. However, this literature teaches the valuable lesson that flexibility could be disadvantageous in strategic settings.

Our model is part of a literature on persuasion games, and is particularly related to papers which study sequential debates in which litigants cannot directly prove the state:

The literature on persuasion games started with Milgrom (1981), who shows that a single litigant with state-independent preferences over the verdict separates if every state can be directly proved. We allow litigants to directly prove the state as a special case; but each game would then only have separating equilibria, and the order of presentation would not matter.

Our model is related most closely to Lipman and Seppi (1995), who study sequential debates in which litigants share a common witness set (our discovery benchmark), and can present any or all of the available witnesses: a condition which we also impose, and which they dub Full Reports (aka normality). In contrast to our model, litigants can observe the state, and J is not restricted to two possible verdicts. Perhaps more crucially, Lipman and Seppi also allow litigants to present messages from a rich cheap talk language:[7] an assumption which they exploit to construct an equilibrium which separates across states. By contrast, we exclude (cheap talk) speeches in order to focus on trials: cf. Section 2.1 below. We prove that discovery games have an equilibrium which separates across witness sets (rather than states), and demonstrate that non-discovery games need not have such separating equilibria.

Glazer and Rubinstein (2001) study a debate in which available witnesses are defined such that litigants never share the same witness set, and Full Reports also fails. In further contrast to our model, J chooses the order of presentation, and commits to the (binary)

[7]We refer to a witness as *cheap talk* if it is available to both litigants, irrespective of Nature's choice.

evidence-dependent verdict it reaches. In Glazer and Rubinstein's example, J chooses sequential over simultaneous presentation; and the outcome of the optimal mechanism can also be realized in an equilibrium of a game in which J cannot commit. This equilibrium could violate a condition they call Debate Consistency: prescribing J to reach different verdicts when the leader and the follower swap the evidence they present. Our model differs in various respects exactly because we focus on common law trials: we do not allow players to commit; and, more importantly, we assume Full Reports because litigants are allowed to present as much (relevant) evidence as they like. This assumption has substantive effects in our model: we show in the Conclusion that litigants could prefer to present first in discovery games if Full Reports fails. Nevertheless, there are some parallels: no player can prefer litigants who always share the same available witnesses to present simultaneously than to present in sequence; and separating equilibria of a given discovery game may fail Debate Consistency on the path.

Chen and Olszewski (2014) study a debate with a fixed order of presentation, two equiprobable states and a continuum of verdicts. Litigants privately observe up to two signals which are variously correlated with the state (and are therefore ordered by strength), and commit ex ante to their strategies in sequence: where a strategy specifies which signal realization to present when two signals are available (so Full Reports fails). In contrast to Chen and Olszewski, we explore preferences over orders of presentation rather than how witness strength affects litigant strategies; and we study commitment games in order to support our claim that leadership may solve the *follower's* commitment problem.[8]

Various other papers on debates are more tangentially related. In particular in Shin (1994), litigants simultaneously select from their privately observed witness sets, and J can reach a finite number of verdicts. Shin interprets the weight that the jury places on the evidence presented by each litigant in equilibrium as a burden of proof. We interpret the burden of proof as a condition which the leader's presentation must satisfy, else J would end the trial and find for the follower.

Ottaviani and Sorensen (2001) study the implications of changing the order in which litigants present cheap talk messages. In contrast to our model, the 'state' also includes litigant competence, which they cannot observe; and litigant payoffs have a private (career concern) component.[9] The ensuing herding effects play no role in our model.

Our model is also related to dynamic Bayesian persuasion games, where litigants choose a signal in sequence, possibly after observing the realization of their predecessors' signals. Litigants choose from the same set of signals: an assumption analogous to our model of discovery games. In Li and Norman (forthcoming), there is a unique (subgame perfect) outcome for generic preferences. Discovery games also have (perfect Bayesian) equilibria; but there may be several equilibrium outcomes. The key difference, in this respect, is that J could infer something about the realized witness set pair (and thereby the state) when a litigant deviates in our model, but not in Bayesian persuasion games.

The order in which litigants present at trial is neither constitutionally determined

[8] In Chen and Olszewski, the equilibrium under commitment is also an equilibrium of the game *with the same order* without commitment.

[9] Hahn (2011) extends this model by allowing litigants to present witnesses.

nor fixed by statute. Lawyers have typically explained the conventional order in non-consequentialist terms: defendants do not need to justify themselves before an accusation has been made.[10] From a consequentialist standpoint: Kozinski (2015) claims that leading might anchor beliefs; whereas Damaska (1973) argues that any such advantage "pales to insignificance when assessed against the disadvantage of having to argue before knowing how the prosecution's case will develop." (fn 48, p.529)[11] This argument seems to conflate two apparently complementary arguments: following allows a litigant to tailor its presentation to the evidence presented by its rival (flexibility); and also to respond to witnesses who are unexpectedly available to its rival. The first effect could apply in situations where litigants share the same available witnesses; the second effect requires asymmetric information. We demonstrate that these two effects can work in opposite directions. Indeed, we show that a litigant who is uncertain of its rival's witness set may prefer to lead.

While the effect of trial order has hitherto not been addressed game theoretically, a literature in psychology (starting with Lund (1925)) asks how the order of presentation affects verdicts? Mock jury trials (starting with Walker et al (1972)) fix the evidence that each litigant presents and varies the order in which this evidence is presented - which nullifies the strategic effects which we address.[12] The underlying theory is rooted in individual psychology, referring to the workings of imperfect memory (whether primacy or recency effects predominate) and to priming or anchoring effects. Indeed, the theory seems to apply equally to the order in which the litigants present and the order in which a given litigant presents its witnesses; our analysis, by contrast, only concerns the former case.

We conjecture that memory effects are less important when juries can deliberate (cf. Ellsworth (1989)) or when attorneys make closing statements than in mock jury trials. Indeed, if memory effects were important in real trials then one might expect significant differences between juries which could and couldn't discuss the evidence during the trial. Evidence from Arizona (which allowed discussion during civil trials) suggests no significant effects: cf. Hannaford et al (2000) and Diamond et al (2003). We also conjecture that opening statements reduce anchoring effects.

2. Model

We present our main model in this section. We describe the sequence in common law trials in Section 2.1, define the games in Section 2.2, and explain our criterion for preference over orders of presentation in Section 2.3.

[10] A defendant's right to be presumed innocent in criminal trials may be grounded in respect for persons: cf. Roberts and Zuckerman (2010) Ch 6.3.

[11] Similarly, Williams (1963) suggests that the common law order may benefit defendants:

"The best reason for the English arrangement is that it enables the defending counsel to see how the prosecution case develops before deciding whether to put his client in the witness box." (p82)

[12] Evidence from these experiments is mixed: for example, Kerstholt and Jackson (1998) report that earlier presentation can be beneficial or detrimental to the defendant, depending on the background information given to subjects. See also Costabile and Klein (2005) and Engel et al (2017).

2.1. Common law trials

Prior to a common law trial, litigants may be required to disclose available witnesses to each other.

Common law trials with a single defendant (D) and a single plaintiff/prosecutor (P) typically contain the following phases:

- P and then D make opening statements, which must be announcements of the evidence to be presented.
- P calls witnesses, who are cross-examined by D. (Speeches are not allowed.)
- D can present a motion to end the trial and acquit, on the grounds that P has not met its burden of proof.
- If the motion is dismissed then D calls witnesses, who are cross-examined by P. (Speeches are again not allowed.)
- P may be allowed, in unusual circumstances, to call witnesses to rebut surprise claims made by D's witnesses.
- P and then D make closing statements, which remind the judge/jury (J) of the evidence, and can suggest interpretations thereof.
- J reaches a verdict.

In the next subsection, we present a model which captures most of these features.

2.2. Games

We model debates as games in which litigants present evidence after observing their available witnesses. Accordingly, we start by defining available witnesses and evidence in terms of more primitive notions.

We suppose that Nature randomly selects state $\mathbf{s}$, representing the facts at issue, from some finite support $\mathbf{S}$. The defendant is *factually guilty* in states $\mathbf{s} \in \mathbf{G}$, and is *factually innocent* in the other states.

Conditional on the realized state, Nature selects the witnesses available to each litigant. We treat a witness (w) as a nonempty, strict subset of $\mathbf{S}$. A realized witness is reliable, in the sense that it contains the realized state. Accordingly, we can interpret witness w as either testimony or physical/documentary proof that the realized state $\mathbf{s}$ is an element of w. In particular, we say that witness w *directly proves* factual guilt if w only contains states in $\mathbf{G}$.[13] As each witness is reliable, witnesses w and w' can only both be available if the realized state is in $w \cap w'$; so we can extend our notion of direct proof to combinations of witnesses. The supposition that w is a strict subset means that it is informative, excluding some states (and is therefore not cheap talk).

[13] w directly proves factual innocence if it does not contain any states in $\mathbf{G}$.

Litigant l's *witness set*, denoted W_l, consists of each of l's available witnesses. We refer to the union of all available witnesses at W_l as the *full report* at W_l, and to the two litigants' witness sets as the *witness set pair*. The notation hitherto introduced illustrates a convention: we use subscripts to denote litigants and superscripts for other purposes.

The games which we analyze are played by two litigants, the defendant (D) and the plaintiff (P), and a judge/jury (J). In each game, one of the litigants is designated as the *leader* (litigant L), and the other litigant as the *follower* (litigant F): each designation defining an *order of presentation*.

Given either order, the game proceeds in four rounds:

<u>Round 0</u> (Nature)

Nature chooses state $\mathbf{s}$ with probability $p(\mathbf{s})$ from support $\mathbf{S}$. (We will refer to $p(.)$ as the *prior distribution*.) Conditional on realized state $\mathbf{s}$, Nature chooses witness set pair W_D, W_P with probability $\pi(W_D, W_P|\mathbf{s})$ from support $\mathbf{W}$, whose generic element is denoted W. We suppose that W_l is nonempty (each litigant l has an available witness) at every $W \in \mathbf{W}$.

<u>Round 1</u> (Leader)

We allow for two cases: either the leader only observes its own witness set or it observes both witness sets. The leader does not observe the state in either case. The leader then chooses which nonempty subset of its available witnesses to call. We denote this collection of witnesses as e_L, and refer to it as the *leader's evidence*. Nonemptiness means that the leader must present *some* evidence (which cannot be cheap talk).

<u>Round 2</u> (Follower)

The follower observes e_L, its available witnesses and possibly the leader's available witnesses (but not the state), and then decides either to *pass* (that is, to call no witness) or to call any combination of witness(es) who have either been called by the leader or are available to the follower.[14] We write the follower's choice as e_F, and refer to it as the *follower's evidence*. Note that the follower's evidence, in our sense, need not contain any witnesses, whereas the leader cannot pass.

We call the two litigants' choices: the *evidence pair*, which we denote $\{e_L, e_F\}$: by convention, an evidence pair is written with the leader's choice first, whereas a witness set pair is written with D's witness set first. It will prove useful to write $e_L \cup e_F$ for the union of all witnesses presented by the two litigants.

<u>Round 3</u> (J)

After observing the evidence pair, J ends the game by reaching a verdict (v), deciding whether to acquit (α) or convict (γ).

A strategy for the leader lists the evidence it presents at each witness set W_L (possibly conditional on W_F); a strategy for the follower lists the evidence it presents at each witness set W_F after observing evidence $e_L \in W_L$; and a strategy for J is a selection $v \in \{\alpha, \gamma\}$ after observing each possible evidence pair $\{e_L, e_F\}$.

We suppose that the litigants only care about the verdict, and have conflicting preferences thereon. Specifically, irrespective of the state and of the witness set pair: D earns 1 if J acquits and 0 otherwise, while P earns 1 if J convicts and 0 otherwise. We say that

[14] Our implicit assumption that each litigant can call any combination of its available witnesses is called *Full Reports* in the literature.

D's [resp. P's] *favored verdict* is acquittal [resp. conviction], and write litigant l's favored verdict as v_l.

J seeks to avoid convicting a factually innocent or acquitting a factually guilty defendant. Thus, in contrast to litigants, J's preference over verdicts is state-dependent. Specifically, J loses $1-d$ if it acquits at any state $\mathbf{s} \in \mathbf{G}$, loses d if it convicts at any state in $\mathbf{S} \backslash \mathbf{G}$, and makes no loss otherwise, where $d \in (0,1)$ equals the standard of proof: that is, the posterior belief at which J is indifferent between acquittal and conviction.[15] Given these asssumptions, J strictly prefers to acquit at witness set pair W when

$$(1-d) \sum_{\mathbf{s} \in \mathbf{G}} \pi^{-1}(\mathbf{s}|W) < d \sum_{\mathbf{s} \notin \mathbf{G}} \pi^{-1}(\mathbf{s}|W)$$

where $\pi^{-1}(\mathbf{s}|W)$ is the posterior probability of state $\mathbf{s}$ when the witness set pair is W. We write $W \in \mathbf{W}^{\alpha}$ if this inequality holds at W and $W \in \mathbf{W}^{\gamma}$ if the reverse inequality holds at that witness set pair. We suppose that J is not indifferent at any witness set pair in $\mathbf{W}$.

Partitioning $\mathbf{W}$ according to J's ideal verdict allows us to associate each possible evidence pair $\{e_L, e_F\}$ with the verdicts that J might then reach. We say that $\{e_L, e_F\}$ *induces* conviction [resp. acquittal] if the combination of witnesses in e_L and e_F directly proves that the realized state is in [resp. is not in] $\mathbf{G}$.

The leader forms beliefs about the state and the follower's witness set after observing its own and possibly the follower's witness sets; the follower forms beliefs about the state and the leader's witness set after observing e_L, its own and possibly the leader's witness sets; and J forms beliefs about the state and the witness set pair after observing the evidence pair. However, only the leader's belief about the follower's witness set and J's beliefs about the witness set pair are strategically relevant; so we will ignore the other beliefs.

The strategy sets, payoffs and beliefs detailed above define a game, which we denote $\Gamma_{L,F}$. If D [resp. P] is the leader then we write the game as $\Gamma_{D,P}$ [resp. $\Gamma_{P,D}$].

Each strategy combination in a game determines the payoffs that a player earns at every witness set pair, and therefore its $\#\mathbf{W}$-length payoff vector. $\Gamma_{D,P}$ and $\Gamma_{P,D}$ have the same set of payoff vectors. Nature's distribution of choices, $p(.)$ and $\{\pi(.|\mathbf{s}\}_{\mathbf{s} \in \mathbf{S}}$ determines each player's expected (across witness set pairs) payoff at a given strategy combination. To simplify exposition, we assume that no player earns the same payoff at any pair of strategy combinations which prescribe a different verdict at some witness set pair. This assumption imposes generic conditions on the distribution of Nature's choices. (We will refer to this assumption below as the *genericity condition*.)

We solve each game by characterizing those pure strategy perfect Bayesian equilibria at which J reaches verdict v after observing any evidence pair which induces v (viz. off as well as on the path): a condition which Lipman and Seppi (1995) call *feasibility*.[16]

[15] The standard of proof is reasonable doubt in criminal trials, and the balance of probabilities in civil trials.

[16] See Fudenberg and Tirole (1991) pp331-333 for a formal definition of a PBE. Their Condition B(iii) precludes J from drawing inferences about the state after observing unexpected evidence pairs.

We refer to such strategy combinations and beliefs as *equilibria*, and say that an equilibrium is *separating* if it prescribes different evidence pairs at different witness set pairs. We refer to the payoff vector reached on an equilibrium path as an *outcome*, writing $\omega_{L,F}$ for the set of outcomes in $\Gamma_{L,F}$, aka the *outcome correspondence*. J acquits at witness set W if and only if $W \in \mathbf{W}^{\alpha}$ in a *separating outcome* (which could be prescribed by a non-separating equilibrium).

If an equilibrium of a game prescribes acquittal [resp. conviction] at a witness set pair W that is not in $\mathbf{W}^{\alpha}$ [resp. not in $\mathbf{W}^{\gamma}$] then we say that it prescribes a *wrongful acquittal* [resp. a *wrongful conviction*] at W. In other words, we abuse common language by defining miscarriages of justice in terms of witness set pairs rather than states.

We end this subsection by explaining two of our modelling assumptions.

We treat the leader and the follower asymmetrically. Our supposition that the leader must present some evidence represents its *burden of proof*. This assumption is motivated by the common law rule that J must acquit if the prosecution has not presented sufficiently persuasive evidence, a rule which does not apply to the defendant.[17]

As common law trials occur in public, the defendant can recall a prosecution witness - as our model allows. On the other hand, the prosecution can only present rebuttal evidence if one of the defendant's witnesses has given surprise testimony. Our assumption that J reaches a verdict in Round 3 may be justified on the understanding that interrogation and cross-examination reveal all that a witness knows.

2.3. Preferences over orders of presentation

Our main results will explore the conditions under which a player prefers to lead or to follow. We interpret this as a question about the player's preferences over outcome correspondences. Each game typically has multiple outcomes, as illustrated by the following example:

Example 1 *There are three states:* $\mathbf{S} = \{\mathbf{m}, \mathbf{o}, \mathbf{mo}\}$*, where* $\mathbf{mo}$ *is the only factually guilty state. There are two witnesses:*

$$w^1 = \{\mathbf{m}, \mathbf{mo}\} \text{ and } w^2 = \{\mathbf{o}, \mathbf{mo}\}.$$

and three witness sets:

$$W^1 = \{w^1\},\ W^2 = \{w^2\} \text{ and } W^{12} = \{w^1, w^2\}.$$

The conditional distribution of witness set pairs is

$$\pi(W^1, W^1|\mathbf{m}) = \pi(W^2, W^2|\mathbf{o}) = \pi(W^{12}, W^{12}|\mathbf{mo}) = 1.$$

The prior distribution satisfies

$$\frac{p(\mathbf{mo})}{p(\mathbf{mo}) + p(\mathbf{m})} < d < \frac{p(\mathbf{mo})}{p(\mathbf{mo}) + p(\mathbf{o})}.$$

[17] See Spier (2007) Section 3.2 for further discussion.

Example 1 can be interpreted as follows. D is commonly known to have motive and/or opportunity to commit the crime, but J only wants to convict if it believes that D has both. There are two factually innocent states: in one ($\mathbf{s} = \mathbf{m}$), D only has motive; in the other ($\mathbf{s} = \mathbf{o}$), D only has opportunity. Each witness is available in a factually innocent and in the factually guilty state. In state $\mathbf{m}$, litigants can prove and only prove motive by presenting w^1; in state $\mathbf{o}$, litigants can prove and only prove opportunity by presenting w^2. In the factually guilty state, each litigant could prove motive and opportunity by presenting both witnesses or could prove either motive or opportunity by presenting one witness.

Before proceeding, we use Example 1 to further illustrate some concepts and notation: $\mathbf{W}^\alpha$ consists of the witness set pairs W^1, W^1 and W^2, W^2; W^{12}, W^{12} is the only witness set pair in $\mathbf{W}^\gamma$. The union of w^1 and w^2 represents the evidence that J observes if the litigants jointly call both witnesses; is only available at witness set pair W^{12}, W^{12}; and is the full report at W^{12}. J observes evidence pair $\{w^1, w^2\}$ after the leader has presented w^1 and the follower has presented w^2. This evidence pair is available at witness set pairs W^1, W^1 and W^{12}, W^{12}, but not at W^2, W^2. Any evidence pair with this combination directly proves factual guilt (viz. that the state is $\mathbf{mo} = w^1 \cap w^2$) and therefore induces conviction. Evidence pair $\{w^1, w^1\}$ is available at W^1 and at W^{12}, so it does not induce a verdict.

Proposition 1 below will imply that both games (viz. with each litigant as leader) have a separating equilibrium, in which J acquits unless the witness set pair is W^{12}, W^{12}. Both games also have a common, non-separating outcome. In both games, this outcome is supported by an equilibrium which prescribes the leader to present w^1 at W^1, W^1 and to present w^2 otherwise, the follower to pass on the path, and J to acquit after and only after observing either $\{w^1, pass\}$ or $\{w^1, w^1\}$.[18] Off the path, the equilibrium in $\Gamma_{D,P}$ prescribes P to present both witnesses after any deviation by D at W^{12}, W^{12}; whereas the equilibrium in $\Gamma_{P,D}$ prescribes D to always pass. The separating and the non-separating outcomes only differ at W^2, W^2, where the non-separating equilibrium prescribes a wrongful conviction. If we selected a separating equilibrium in $\Gamma_{D,P}$ and a non-separating equilibrium in $\Gamma_{P,D}$ then each litigant would prefer to lead; and each litigant would prefer to follow if we reversed the selection.

This selection problem applies much more generally. Moreover, refinements based on forward induction do not reduce the multiplicity of outcomes because each litigant has the same preference ordering over verdicts at every witness set pair.[19] Accordingly, we now provide a criterion for preference over multiple outcomes which does not rely on selection arguments.

First, some preliminaries. The outcome of a game specifies each player's payoff at each witness set pair, and its expected payoff (across witness set pairs). We say that a player strictly prefers one outcome over another if its expected payoff is higher. Now two

[18] The lower bound on d implies that J acquits at W^1, W^1; the upper bound implies that J otherwise convicts.

[19] Hart et al's (2017) truth-leaning refinement would require each litigant to present its full report unless it strictly prefers not to do so. The supposition that litigants are inclined to tell the whole truth is surely inappropriate to trials or indeed to Presidential debates.

outcomes of a game can only differ if they are supported by equilibria which prescribe different verdicts at some witness set pair. The genericity condition therefore implies that each player has a strict preference over any pair of distinct outcomes.

Consider the outcome correspondences of the two games: $\omega_{D,P}$ and $\omega_{P,D}$. We say that *player Q prefers that D leads* (or prefers playing $\Gamma_{D,P}$ to playing $\Gamma_{P,D}$) if $\omega_{D,P}$ and $\omega_{P,D}$ are nonempty and player Q

Condition 1 Weakly prefers every outcome in $\omega_{D,P}$ over every outcome in $\omega_{P,D}$; and

Condition 2 Strictly prefers some outcome in $\omega_{D,P}$ over some outcome in $\omega_{P,D}$.

Analogous conditions define a preference for P to lead. As litigants have opposing preferences over the verdict, P prefers to lead [resp. follow] if and only if D prefers to lead [resp. follow].

Conditions 1 and 2 define a partial ordering over games. In particular, no player can prefer an order if both games share two or more distinct outcomes (as in Example 1). Consequently, our criterion is hard to satisfy: which cuts in favor of our negative results, and against our positive results. We could, alternatively, work with natural conditions which are yet harder to satisfy: requiring players to prefer one outcome over another at every witness set pair, rather than in expectation. All of our results below would also satisfy this tighter condition.

3. Discovery games

Discovery rules require litigants to share their available witnesses, and are enforced by means such as depositions, interrogatories and motions to inspect and copy documents. Discovery rules were introduced in federal civil trials in 1938 to improve fact-finding and to prevent litigants from introducing surprise evidence at trial (cf. Subrin (1998)).[20] In this section, we study preferences over the order of presentation when discovery rules apply to both litigants. Our model allows us to address this case by imposing conditions on $\mathbf{W}$: the support of witness set pairs. Specifically, we will describe games in which $W_D = W_P$ for every $W_D, W_P \in \mathbf{W}$ as *discovery games*.[21] This condition, which is satisfied in Example 1 above, implies that the leader knows the follower's witness set. We can simplify the notation in discovery games by writing W as the common witness set. We will focus on discovery games throughout this section.

Our main results in this section will establish that litigants cannot prefer to lead, but may prefer to follow in discovery games. We start by providing a result of independent interest, which we will exploit below.

Proposition 1 *Every discovery game has a separating equilibrium.*

We prove Proposition 1 by construction in Section 1 of the Appendix.

[20] Bone (2012) provides further details, and reviews the law and economics literature. Litigants in civil trials in civil law systems are typically not required to disclose.

[21] There are two possible interpretations: either litigants only observe their own available witnesses but know that witness sets are perfectly correlated across litigants; or Nature initially endows each litigant with different witnesses, but the discovery process allows each litigant to call any witness initially endowed to either of the litigants.

According to the construction, J reaches verdict v after observing any evidence pair $\{e_L, e_F\}$ whose union, $e_L \cup e_F$, is the full report at $W \in \mathbf{W}^v$; and the leader presents the full report at every witness set because the follower responds to any leader evidence by presenting the full report unless it can persuade J to reach its favored verdict.

The equilibrium strategy combination prescribes J to reach a verdict which only depends on the evidence pair via its union, and therefore satisfies Glazer and Rubinstein's (2001) notion of Debate Consistency, both on and off the path. However, it is easy to confirm that equilibria of discovery games may fail Debate Consistency on the path, as we illustrate in Section 1 of the Appendix.

Proposition 1 implies that each discovery game has an outcome. We can therefore use the criterion introduced in Section 2.3 to consider preferences over the order of presentation. If each discovery game only had a separating outcome then the order could not matter. Our main result in this section asserts that players may prefer an order:

Theorem 1 *In discovery games:*

a) *Litigants cannot prefer to lead;*

b) *Litigants can prefer to follow; and*

c) *J can prefer an order, but can only do so if the litigants prefer to follow.*

Theorem 1 implies that the conventional order is optimal from defendants' point of view in discovery games, without appealing to principles like a presumption of innocence.

Proof We prove Theorem 1a) in Section 1 of the Appendix. The proof relies on

Lemma *If $\Gamma_{L,F}$ has an equilibrium which prescribes verdict the leader's favored verdict at witness sets $\mathbf{W}^*$ then $\Gamma_{F,L}$ has an equilibrium which prescribes that verdict at every witness set in $\mathbf{W}^*$.*

Consider any equilibrium (say, X) of $\Gamma_{L,F}$. We prove Lemma by using X to construct the following strategy combination (Y) in $\Gamma_{F,L}$. At every witness set $W \in \mathbf{W}^*$, Y prescribes the leader (now litigant F) to present the same evidence as X prescribed litigant L to present at W; and prescribes litigant F to present the full report at every other witness set (where X prescribes the follower's favored verdict). This strategy combination and associated beliefs form an equilibrium. We then prove that Lemma implies a), exploiting the transitivity of preferences over outcomes. This part of the proof does not rely on $\Gamma_{L,F}$ and $\Gamma_{F,L}$ being discovery games: a property which we will exploit again in Section 4.3.

b) We prove this part via an example in which one game only has a separating outcome, while the other game also has another outcome:

Example 2 *There are four states: $\mathbf{S} = \{\mathbf{i}^1, \mathbf{i}^2, \mathbf{i}^3, \mathbf{g}\}$, and the defendant is only factually guilty in state $\mathbf{g}$. There are three witnesses:*

$$w^1 = \{\mathbf{i}^1, \mathbf{i}^2, \mathbf{g}\},\ w^2 = \{\mathbf{i}^2, \mathbf{i}^3, \mathbf{g}\} \text{ and } w^3 = \{\mathbf{g}\}$$

and four witness sets:

$$W^1 = \{w^1\},\ W^2 = \{w^2\},\ W^{12} = \{w^1, w^2\} \text{ and } W^{123} = \{w^1, w^2, w^3\}.$$

In every state, the two litigants share the same witness set. The conditional distribution of this witness set is

$$\pi(W^1|\mathbf{i}^1) = \pi(W^{12}|\mathbf{i}^2) = \pi(W^2|\mathbf{i}^3) = \pi(W^{123}|\mathbf{g}) = 1.$$

The prior distribution satisfies

$$\max\{\frac{p(\mathbf{g})}{p(\mathbf{g}) + p(\mathbf{i}^2)}, \frac{p(\mathbf{g})}{p(\mathbf{g}) + p(\mathbf{i}^3)}\} < d < \frac{p(\mathbf{g})}{p(\mathbf{g}) + p(\mathbf{i}^1)}.$$

The conditions on priors imply that J observes the same evidence pair at W^1 and at W^{123} and convicts in any equilibrium of either game with a non-separating outcome. J cannot observe the same evidence pair at W^2 and at W^{123} or at W^{12} and at W^{123} in an equilibrium of either game because J would then acquit; and P could then profitably deviate to presenting w^3 at W^{123}, as it directly proves factual guilt.

Claim 1 *If Example 2 then every outcome of $\Gamma_{P,D}$ is separating.*

Proof Suppose, per contra, that J observes the same evidence pair at W^1 and at W^{123} in an equilibrium. J must then acquit at W^{12} and convict at W^{123}. As $W^{12} \subset W^{123}$, P could then profitably deviate to presenting w^1 at W^{12}.■

Claim 2 *If Example 2 then $\Gamma_{D,P}$ has an equilibrium with a wrongful conviction at W^1, and has no other non-separating outcomes.*

Proof Consider the following strategy combination and beliefs:

- D presents w^1 at W^{123}, and presents the full report at every other witness set;
- P presents the full report in response to D's evidence at every witness set other than W^{123}, where it presents w^1 in response to w^1, and otherwise presents the full report;
- J believes that the realized witness set is
  - W^{123} and convicts if either litigant presents evidence which contains w^3;
  - W^1 or W^{123} and convicts after observing $\{w^1, w^1\}$; and
  - In $\mathbf{W}^{\alpha}$ and acquits after observing any other evidence pair $\{e_D, e_P\}$.

D cannot profitably deviate at W^{123} because P would then secure conviction by presenting w^3. J's beliefs satisfy Bayes rule and feasibility. The strategy combination and beliefs therefore form an equilibrium.

The conditions on priors preclude any other non-separating outcome.■

Proposition 1 and Claim 1 imply that $\omega_{P,D}$ consists of a separating outcome alone; while Proposition 1 and Claim 2 imply that $\omega_{D,P}$ consists of a separating outcome and an outcome with a wrongful conviction at W^1. Consequently, both litigants must prefer to follow, proving part **b)**.

c) J strictly prefers the separating outcome of any game over any other outcome. Example 2 illustrates a discovery game in which $\Gamma_{P,D}$ only has a separating outcome, whereas $\Gamma_{D,P}$ also has another outcome; so J prefers D to lead.

Suppose that J prefers litigant L to lead, and write the two games as $\Gamma_{L,F}$ and $\Gamma_{F,L}$. Proposition 1 implies that both games have a separating outcome, which J prefers over any other outcome; so $\Gamma_{F,L}$ alone must have a non-separating equilibrium (say, X) which prescribes miscarriages of justice at some witness set(s). X cannot prescribe J to wrongfully reach verdict v_F at any witness set, else Lemma would imply that $\Gamma_{L,F}$ also has a non-separating outcome. Hence, X must prescribe v_L at every witness set where there is a miscarriage of justice. Litigants must then prefer to follow.■

The criterion which we introduced in Section 2.3 compares outcome correspondences, and therefore seems rather unwieldy. Theorem 1 provides a strikingly clean description of preferences over the order without fully characterizing these correspondences.

Presenting second allows the follower to condition the evidence it presents on the leader's choice at each witness set. Example 2 illustrates why such flexibility may be advantageous in discovery games. The full report at W^{123} induces conviction; so J must convict at W^{123} in both games. However, the evidence which the leader presents at W^{123} determines J's verdict at W^1. In the non-separating equilibrium of $\Gamma_{D,P}$, D presents w^1 at W^{123} because P threatens to directly prove factual guilt otherwise; so P also secures a conviction at W^1. P cannot make such a threat if it leads: it must then directly prove factual guilt at W^{123}, and J must acquit at W^{12} in $\Gamma_{P,D}$.

The converse of Theorem 1c) is false: litigants would prefer to follow if both games had non-separating equilibria, and all miscarriages of justice in $\Gamma_{P,D}$ [resp. $\Gamma_{D,P}$] were wrongful acquittals [resp. convictions];[22] but J would then not prefer either order.

Theorem 1 also holds in some natural variants on our model. In Section 2.3, we required a litigant who prefers one outcome over another to be better in expectation, rather than at every witness set pair. Theorem 1a) would obviously hold if we adopted the stronger criterion for preference between outcomes. More interestingly, inspection of the proofs above reveals that the other parts of Theorem 1 also generalize. Theorem 1 would also hold in another variant on the game where one or more litigants observe the state. Finally, an argument akin to that used to prove Lemma establishes that, if litigants share available witnesses, then J cannot prefer litigants to present simultaneously, and that litigants cannot prefer presenting simultaneously to always following.

We end this section with an Observation, which will be useful in Section 5:

Observation Consider the following mirror image of Example 2.

Example 2' *There are four states:* $\mathbf{S} = \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{g}^3, \mathbf{i}\}$, *and the defendant is only factually innocent in state* $\mathbf{i}$. *There are three witnesses:*

$$w^4 = \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{i}\},\ w^5 = \{\mathbf{g}^2, \mathbf{g}^3, \mathbf{i}\} \text{ and } w^6 = \{\mathbf{i}\}$$

[22] Example A3 in the Appendix illustrates this possibility.

and four witness sets:

$$W^4 = \{w^4\},\ W^5 = \{w^5\},\ W^{45} = \{w^4, w^5\} \text{ and } W^{456} = \{w^4, w^5, w^6\}.$$

In every state, the two litigants share the same witness set. The conditional distribution of this witness set is

$$\pi(W^4|\mathbf{g}^1) = \pi(W^{45}|\mathbf{g}^2) = \pi(W^5|\mathbf{g}^3) = \pi(W^{456}|\mathbf{i}) = 1.$$

The prior distribution satisfies

$$\frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1) + p(\mathbf{i})} < d < \min\{\frac{p(\mathbf{g}^2)}{p(\mathbf{g}^2) + p(\mathbf{i})}, \frac{p(\mathbf{g}^3)}{p(\mathbf{g}^3) + p(\mathbf{i})}\}.$$

Arguments which directly mirror those in Claims 1 and 2 imply that, if Example 2', then $\Gamma_{P,D}$ has an equilibrium with a wrongful acquittal at W^4 while $\Gamma_{D,P}$ only has separating outcomes.■

4. Non-discovery games

In the last section, we demonstrated that litigants cannot prefer to lead, but may prefer to follow, and that J can only prefer an order if litigants prefer to follow in discovery games. In this section, we demonstrate that these results may fail in games which are not played under discovery. In Sections 4.1 and 4.2, we show that litigants can prefer to lead because of the burden of proof or because a leader alone can prove availability or because witness sets are privately observed. In Section 4.3, we define commitment games, and show that leadership without commitment generates the same outcomes as follower commitment in the examples used in Sections 4.1 and 4.2: leadership is, in effect, an optimal commitment device for the follower. Section 4.4 proves by example that J alone might prefer an order, and collects the main results in this section.

4.1. No-subpoena games

According to our model, a litigant can only call a witness who is in its own witness set or who has already been called by its rival. In the benchmark case of discovery games, litigants always have the same available witnesses; so the leader both knows and can call any witness available to its rival. In this subsection, we consider cases in which Nature may assign different available witnesses to the two litigants, but reveals the witness set pair to both litigants. We refer to such cases as *no-subpoena games*.

No-subpoena games are realistic extensions of discovery games for two reasons. First, discovery rules apply to both litigants in civil trials, but predominantly to the prosecution in criminal trials.[23] If discovery rules only apply to one litigant (say, P) then W_D must

[23] Failure of the prosecution to provide the defendant in a criminal trial with material evidence violates due process (*Brady* v. *Maryland* 373 US 83 (1963)), though these rights were restricted in *Kyles* v. *Whitley* (1995). Under Federal Rules of Criminal Procedure 16, a defense request for discovery triggers

contain W_P. A no-subpoena game would then be played if P knew W_D but could not subpoena witnesses in $W_D \backslash W_P$. This situation is exemplified by the 5^{th} Amendment right not to testify in criminal trials: P may know that D would confess under cross-examination, but is not allowed to subpoena the defendant. Second, discovery rules are sometimes difficult to enforce because a litigant may not be able to adequately describe the requested witness: for example, a litigant may be sure that its rival has a favorable witness in its files, but does not know which file to search. In these circumstances, a no-subpoena game would again be played; but, in this case, one litigant's witness set need not contain its rival's witness set.

We divide this subsection into two parts, each illustrating a reason why litigants may prefer to lead in no-subpoena games.

4.1.1. The burden of proof

In this part, we provide an example of a no-subpoena game in which litigants prefer to lead because they then bear the burden of proof. Phrased this way, the result seems surprising; but it turns on the fact that the burden may restrict the outcome correspondence.

Example 3 *There are four states:* $\mathbf{S} = \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{i}^1, \mathbf{i}^2\}$, *where the defendant is factually guilty in states* $\mathbf{g}^1$ *and* $\mathbf{g}^2$. *There are four witnesses:*

$$w^1 = \{\mathbf{i}^1, \mathbf{i}^2\},\ w^2 = \{\mathbf{g}^1, \mathbf{i}^2\},\ w^3 = \{\mathbf{g}^1, \mathbf{i}^1, \mathbf{i}^2\} \text{ and } w^4 = \{\mathbf{g}^2\}$$

and five witness sets:

$$W^1 = \{w^1\},\ W^2 = \{w^2\},\ W^3 = \{w^3\},\ W^{13} = \{w^1, w^3\} \text{ and } W^4 = \{w^4\}.$$

The conditional distribution of witness set pairs satisfies[24]

$$\pi(W^3, W^1|\mathbf{i}^1) = \pi(W^3, W^2|\mathbf{g}^1) = \pi(W^{13}, W^2|\mathbf{i}^2) = \pi(W^4, W^4|\mathbf{g}^2) = 1.$$

The prior distribution satisfies

$$\frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1) + p(\mathbf{i}^1)} < d < \frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1) + p(\mathbf{i}^2)}.$$

Each litigant observes the witness set pair.

Claim 3 *If Example 3 then litigants prefer to lead, and* J *prefers* P *to lead.*

a reciprocal obligation to give notice of evidence and witnesses to be called. Litigants in civil cases must disclose evidence, even if it is not requested by the opposing side. (The rules are detailed in Federal Rules of Civil Procedure (2010) Title V and in the 1998 Civil Procedure Rules for England.) Counsel for criminal defendants in civil law systems typically have pre-trial access to the investigative file: cf. Caianiello (2019).

[24] Recall our convention of writing witness set pairs as W_D, W_P.

We prove Claim 3 in Section 2 of the Appendix. Neither game can have an equilibrium which prescribes a wrongful conviction at W^{13}, W^2 because D could profitably deviate to presenting w^1, which directly proves factual innocence. We first construct a separating equilibrium in $\Gamma_{D,P}$. The lower bound on d allows us to construct another equilibrium, which prescribes J to acquit after observing $\{w^3, pass\}$ at W^3, W^1 and at W^3, W^2, and therefore wrongfully acquit at W^3, W^1. We then construct a separating equilibrium in $\Gamma_{P,D}$. This outcome ia unique because J cannot observe the same evidence pair at W^3, W^1 and at W^3, W^2 when P leads: an effect of the burden of proof. In sum, $\omega_{D,P}$ contains $\omega_{P,D}$. As the extra outcome has a wrongful acquittal, litigants prefer to lead, and J prefers P to lead.

Example 3 illustrates how the burden of proof can explain a litigant preference to lead in no-subpoena games. Specifically, the burden requires the leader not to pass; so pooling at W^3, W^1 and W^3, W^2 is only possible if the leader has a common witness at those witness set pairs. This condition is satisfied in $\Gamma_{D,P}$, where J can observe $\{w^3, pass\}$ at both witness set pairs, but not in $\Gamma_{P,D}$. This motive is impossible in discovery games, where the order of presentation does not affect whether J can observe the same evidence pair at different witness sets.[25]

4.1.2. Proving availability

Our model allows the follower to recall any witness that the leader has presented. In this part, we exploit the idea that the identity of a litigant who presents evidence may determine what that evidence directly proves. Specifically, we provide an example of a no-subpoena game with two key features: first, J would reach verdict v_l if it knew that witness w was available to litigant l; second, l can only prove that $w \in W_l$ if it is the first litigant to present w. Conversely, the other litigant could prevent l from proving that $w \in W_l$ by presenting w first.

Example 4 *There are three states:* $\mathbf{S} = \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{i}\}$*; the defendant is only factually innocent in state* $\mathbf{i}$*. There are three witnesses:*

$$w^1 = \{\mathbf{g}^1, \mathbf{i}\},\ w^2 = \{\mathbf{g}^2\} \text{ and } w^3 = \{\mathbf{i}\}$$

and three witness sets:

$$W^1 = \{w^1\},\ W^2 = \{w^2\} \text{ and } W^3 = \{w^3\}.$$

The conditional distribution of witness set pairs satisfies

$$\pi(W^1, W^1|\mathbf{g}^1) = \pi(W^2, W^2|\mathbf{g}^2) = \pi(W^1, W^3|\mathbf{i}) = 1.$$

The prior distribution satisfies $\frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1)+p(\mathbf{i})} < d$*. Each litigant observes the witness set pair.*

[25] The burden of proof could, of course, cut the other way. An earlier version of this paper contains a related example in which the burden of proof creates a motive for litigants to prefer to follow.

Claim 4 *If Example 4 then litigants prefer to lead, and J prefers P to lead.*

We prove Claim 4 in Section 2 of the Appendix. P alone can directly prove factual guilt at W^1, W^1 by presenting w^1; so every equilibrium of $\Gamma_{P,D}$ must be separating. $\Gamma_{D,P}$ has a separating equilibrium as well as an equilibrium which prescribes D to present w^1 at W^1, W^1, thereby preventing P from proving that $w^1 \in W_P$, and resulting in J wrongfully acquitting. Consequently, litigants prefer to lead, while J prefers P to lead. This motive does not turn on the burden of proof: for allowing the leader to pass would not change the outcome correspondence on either order. Furthermore, the motive is impossible in discovery games, where J knows that a witness is available to one litigant if and only if it is available to both litigants.[26]

4.2. Incomplete information games

Thus far, we have considered no-subpoena games in which each litigant knows the witnesses available to its rival. In this subsection, we consider games in which at least one litigant is uncertain of its rival's available witnesses. We provide a variant on Example 1 (in Section 2.3), which illustrates how a litigant may prefer to lead in such games.

Example 5 *There are three states:* $\mathbf{S} = \{\mathbf{m}, \mathbf{o}, \mathbf{mo}\}$, *where* $\mathbf{mo}$ *is the only factually guilty state. There are two witnesses:*

$$w^1 = \{\mathbf{m}, \mathbf{mo}\} \text{ and } w^2 = \{\mathbf{o}, \mathbf{mo}\}$$

and three witness sets:

$$W^1 = \{w^1\},\ W^2 = \{w^2\} \text{ and } W^{12} = \{w^1, w^2\}.$$

The conditional distribution of witness set pairs satisfies

$$\begin{aligned} \pi(W^1, W^1|\mathbf{m}) &= \pi(W^2, W^2|\mathbf{o}) = 1; \\ \pi(W^{12}, W^1|\mathbf{mo}) &= 1 - \pi(W^{12}, W^2|\mathbf{mo}) \equiv \pi \in (\frac{1}{2}, 1), \end{aligned}$$

The prior distribution satisfies

$$\frac{p(\mathbf{mo})\pi}{p(\mathbf{mo})\pi + p(\mathbf{m})} < d < \frac{p(\mathbf{mo})(1-\pi)}{p(\mathbf{mo})(1-\pi) + p(\mathbf{o})}.$$

Each litigant only observes its own witness set.

The interpretation of **m** and **o** as motive and opportunity in Example 1 carries over to Example 5.

Claim 5 *If Example 5 then litigants prefer to lead, and J prefers D to lead.*

[26] Proving availability could, of course, cut the other way. An earlier version of this paper contains a related example in which litigants prefer to follow because of the follower's inability to prove availability of a witness.

We prove Claim 5 in Section 2 of the Appendix. In $\Gamma_{D,P}$, D must present the same evidence at witness set pairs W^{12}, W^1 and W^{12}, W^2, which is w^1 because $\pi > 1/2$; so the lower bound on d implies that J wrongfully acquits at W^{12}, W^1. Furthermore, J observes different evidence pairs at W^2, W^2 and at W^{12}, W^2, and therefore (rightfully) acquits at W^2, W^2. By contrast, D always passes in an equilibrium of $\Gamma_{P,D}$. It cannot present w^1 at W^{12}, W^2 in any equilibrium because J would then have to convict at W^{12}, W^2 and acquit at W^2, W^2; and D could then profitably deviate to passing at W^{12}, W^2. In sum, $\Gamma_{D,P}$ has a unique outcome whose sole miscarriage of justice is a wrongful acquittal at W^{12}, W^1, whereas $\Gamma_{P,D}$ has a unique outcome with a wrongful acquittal at W^{12}, W^1 and a wrongful conviction at W^2, W^2. Consequently, litigants prefer to lead, and J prefers D to lead.

This result is rather striking in light of (our reading of) a claim by Damaska (1973) which we quoted in Section 1.2: that incomplete information provides an additional reason for preferring to follow because the follower can then not be surprised by the leader's available witnesses. Learning that P has witness w^2 available at W^{12}, W^2 does not allow D to secure an acquittal; but its equilibrium response at W^{12}, W^2 harms D at W^2, W^2.[27]

Proposition 1 states that discovery games have a separating outcome, as do non-discovery games satisfying Examples 3 and 4. Example 5 illustrates a non-discovery game which lacks a separating outcome.

We argued above that Example 3 illustrates the burden of proof reason why litigants may prefer to lead in a no-subpoena game. Example 5 shares this property: w^1 is available at witness sets W^1 and W^{12}; and $\Gamma_{D,P}$ has an equilibrium in which D presents w^1 at witness set pairs W^1, W^1, W^{12}, W^1 and W^{12}, W^2. However, this property is not sufficient for litigants to present the same evidence pairs at W^{12}, W^1 and at W^1, W^1 or at W^{12}, W^2 and at W^2, W^2.

4.3. Commitment

We have provided three examples of non-discovery games in which litigants prefer to lead. In this subsection, we will argue that these examples share a common feature: the follower's ability to respond on a case-by-case basis may impose negative externalities across witness set pairs, which would be resolved if the follower could commit ex ante to its strategy. We will argue that, in these examples, games in which the follower can commit have the same outcome correspondence as games in which the order of presentation is reversed. In other words, presenting first acts like a commitment device.

The follower may, ex ante, gain from commitment because the evidence it presents at a given witness set pair affects its payoff at other witness set pairs via J's inference. Consider, for example, $\Gamma_{D,P}$'s non-separating equilibrium in Example 4, where P passes at W^1, W^1 and at W^1, W^3, and J acquits at both witness set pairs. P passes at W^1, W^3 because J would also acquit were P to present w^3; but this ex post optimal choice implies that J observes the same evidence pair at the two witness set pairs, and therefore acquits. If P could ex ante commit (and J could observe the commitment) then it would choose to respond differently to w^1 at W^1, W^1 and at W^1, W^3; and J would then convict at W^1, W^1.

[27] Incomplete information could, of course, cut the other way. An earlier version of this paper contains a related example in which incomplete information creates a motive for litigants to prefer to follow.

Example 4 has two special features. First, P's commitment would ensure that J observes different evidence pairs at the two witness set pairs, irrespective of D's choice at W^1, W^1 and at W^1, W^{12}. In other words, the commitment would work irrespective of whether it is observed by D. Second, P's commitment would work, irrespective of whether it had already observed (and therefore signals) its witness set. Neither of these features need hold in settings more general than Example 4. We formalize the potential advantages of commitment by defining a game in which the follower moves before observing its witness set, and in which both of the other players observe the follower's commitment.

Take some game $\Gamma_{L,F}$. The commitment game $\Gamma^c_{L,F}$ has the following time line. F starts play by choosing its strategy: that is, the evidence it presents in response to L's evidence at each follower witness set. Nature then selects the state and the witness set pair; L presents evidence, having observed its own witness set and F's strategy; F implements its chosen strategy; and J reaches a verdict after observing F's strategy and the evidence pair. Players earn the same payoff at any terminal node in $\Gamma^c_{L,F}$ as at the equivalent terminal node in $\Gamma_{L,F}$; but F's maximand is now its ex ante expected payoff across realized witness set pairs. We analyze commitment games using the solution concept described in Section 2.2, which implies that J cannot learn about the realized witness set pair from any follower deviation, and that J must reach the verdict induced by an evidence pair, irrespective of whether F has deviated.[28] We denote the outcome correspondence of $\Gamma^c_{L,F}$ by $\omega^c_{L,F}$. The assumption that J observes F's strategy limits the punishment that J can impose on a deviating follower, and therefore limits $\omega^c_{L,F}$.

We now demonstrate that $\omega^c_{L,F} = \omega_{F,L}$ in each of Examples 3-5 for *some* designation of leader $L \in \{D, P\}$.

Claim 3C *If Example 3 then $\Gamma^c_{D,P}$ and $\Gamma_{P,D}$ have the same outcomes, and the outcome correspondence in $\Gamma_{P,D}$ contains the unique outcome in $\Gamma^c_{D,P}$.*

Recall that $\Gamma_{P,D}$ only has a separating outcome, whereas $\Gamma_{D,P}$ has a separating equilibrium and an equilibrium with a wrongful acquittal at W^3, W^2. We prove Claim 3C (in Section 2 of the Appendix) by constructing separating equilibria in both commitment games. P prevents a wrongful acquittal at W^3, W^2 in $\Gamma^c_{D,P}$ by committing to present different evidence in response to w^1 at W^3, W^1 and at W^3, W^2. The burden of proof precludes any outcome with a wrongful acquittal in $\Gamma^c_{P,D}$, just as in $\Gamma_{P,D}$.

Claim 4C *If Example 4 then $\Gamma^c_{D,P}$ and $\Gamma_{P,D}$ have the same outcomes, and the outcome correspondence in $\Gamma_{P,D}$ contains the unique outcome in $\Gamma^c_{D,P}$.*

Recall that $\Gamma_{P,D}$ only has a separating outcome, whereas $\Gamma_{D,P}$ has separating equilibria and equilibria with a wrongful acquittal at W^1, W^1. We prove Claim 4C (in Section 2 of the Appendix) by constructing separating equilibria in both commitment games. In $\Gamma^c_{D,P}$, as in $\Gamma_{D,P}$, D presenting w^1 at W^1, W^1 prevents P from proving availability of w^1. However, P prevents a wrongful acquittal at W^1, W^1 in $\Gamma^c_{D,P}$ by committing to different responses to w^1 at W^1, W^1 and at W^1, W^3.

[28] J's beliefs are only defined after observing evidence pairs consistent with D's commitment.

Claim 5C *If Example 5 then $\Gamma^c_{D,P}$ and $\Gamma_{P,D}$ have the same outcomes, and the outcome correspondences in $\Gamma_{P,D}$ and in $\Gamma^c_{D,P}$ are disjoint.*

Recall that $\Gamma_{P,D}$ has a unique equilibrium with a wrongful conviction at W^2, W^2 and a wrongful acquittal at W^{12}, W^1; whereas the only miscarriage of justice in the unique outcome of $\Gamma_{D,P}$ is a wrongful acquittal at W^{12}, W^1. We prove Claim 5C (in Section 2 of the Appendix) by constructing an equilibrium of each commitment game which prescribes a wrongful acquittal at W^{12}, W^1 and no other miscarriages of justice. D avoids the wrongful conviction at W^2, W^2 by committing to different responses to w^2 at W^2, W^2 and at W^{12}, W^2; as a result, J acquits at W^2, W^2. P cannot secure a wrongful conviction at W^{12}, W^2 in $\Gamma^c_{D,P}$ because $\pi > 1/2$ implies that D presents w^1 at W^{12}, W^2 and at W^{12}, W^1, but presents w^2 at W^2, W^2.

In sum, for Examples 3-5, the outcome correspondence of a commitment game coincides with the outcome correspondence of the game with the other order for *an* assignment of roles: viz. some litigant $L \in \{D, P\}$. In this sense, leading is a commitment device for a litigant; so *both* litigants prefer to lead in each example because of their conflicting interests. In this sense, advantageous commitment explains a preference to lead. More generally, Claims 3C-5C suggest that a litigant might prefer to lead because the ensuing outcome *alleviates* its commitment problem qua follower.

Our analysis of these examples suggests that the results could be generalized in several directions. We now note some limits to this suggestion.

In the equilibria which we construct to prove that leadership is a commitment device in Examples 3-5, the evidence which the follower commits to present depends on the follower's witness set, but not on the leader's evidence. We use this feature to prove uniqueness. Neither property generalizes to all commitment games.

In Examples 3-5, commitment is valuable for some litigant F in the sense that it is strictly better off committing qua follower; and $\omega^c_{L,F} = \omega_{F,L}$ (commitment replicates leadership). However, valuable commitment does not imply that $\omega^c_{L,F} = \omega_{F,L}$, as Example A1 in Section 2 of the Appendix illustrates. Absent commitment, both games have a unique and common outcome with a wrongful acquittal. By contrast, $\Gamma^c_{D,P}$ has a unique, separating outcome. Commitment is therefore valuable to P, who prefers to commit as follower than to lead (with or without commitment). Indeed, the separating outcome is not in either $\omega_{D,P}$ or $\omega_{P,D}$.

One might also wonder whether a preference to lead implies that $\omega^c_{L,F} = \omega_{F,L}$? Proposition A1, in Section 2 of the Appendix, exploits an example of a no-subpoena game (Example A2) to disprove this conjecture. In this example, $\omega_{D,P}$ consists of an outcome with a wrongful acquittal and an outcome with a wrongful conviction, while $\omega_{P,D}$ consists of the outcome with the wrongful conviction; so litigants prefer to lead.[29] However, both commitment games have a separating outcome; so litigants prefer to lead, but each commitment game has a different outcome correspondence to either of the non-discovery games.

[29] The absence of a pooling equilibrium could be interpreted as a burden of proof effect, as in Example 3.

Commitment may also be valuable when litigants always share available witnesses (and play a *commitment discovery game*): for example, $\omega^c_{P,D}$ consists of the separating outcome alone in Example 1, whereas $\omega_{P,D}$ also contains an outcome with a wrongful conviction. More generally, Proposition A2 in Section 2 of the Appendix asserts that every commitment discovery game has a separating equilibrium; and that $\omega^c_{L,F}$ cannot contain an outcome which is worse for F than the separating outcome. The argument for the first claim is constructive; the argument for the second part involves constructing a profitable deviation for the follower to presenting the full report at every witness set, irrespective of the leader's evidence. J must then expect to observe different evidence pairs at different witness sets, and must therefore reach separating verdicts. In further contrast to Examples 3-5, $\omega^c_{L,F}$ may contain several outcomes: for example, $\omega^c_{D,P}$ contains both of the outcomes in $\omega_{D,P}$ in Example 2.

Proposition A2 leaves open the possibility that litigants prefer to lead because $\omega^c_{L,F}$ is strictly contained in $\omega_{L,F}$, consists of the separating outcome, and coincides with $\omega_{F,L}$. However, this is impossible in discovery games because all such games have a separating equilibrium (Proposition 1) and $\omega_{F,L}$ contains an outcome which is worse than separation for the follower whenever $\omega_{L,F}$ does (Lemma).

4.4. J's preferences

Theorem 1c) asserts that J can only prefer an order if litigants prefer to follow in discovery games; and the same property holds in Examples 3-5. In this subsection, we demonstrate by example that it does not carry over to all non-discovery games. We then collect this section's main results in Theorem 2.

Example 6 *There are five states:* $\mathbf{S} = \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{g}^3, \mathbf{i}^1, \mathbf{i}^2\}$*; the defendant is only factually innocent in states* $\mathbf{i}^1$ *and* $\mathbf{i}^2$*. There are four witnesses:*

$$w^1 = \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{i}^1\},\ w^2 = \{\mathbf{g}^1, \mathbf{g}^3, \mathbf{i}^1, \mathbf{i}^2\},\ w^3 = \{\mathbf{g}^2, \mathbf{i}^2\} \text{ and } w^4 = \{\mathbf{g}^3\}$$

and five witness sets:

$$W^1 = \{w^1\},\ W^2 = \{w^2\},\ W^{12} = \{w^1, w^2\},\ W^3 = \{w^3\},\ W^4 = \{w^4\} \text{ and } W^{24} = \{w^2, w^4\}.$$

The conditional distribution of witness set pairs satisfies

$$\pi(W^1, W^{12}|\mathbf{g}^1) = \pi(W^1, W^3|\mathbf{g}^2) = \pi(W^2, W^{12}|\mathbf{i}^1) = \pi(W^3, W^2|\mathbf{i}^2) = \pi(W^{24}, W^4|\mathbf{g}^3) = 1.$$

The prior distribution satisfies

$$\max\{\frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1) + p(\mathbf{i}^2)}, \frac{p(\mathbf{g}^3)}{p(\mathbf{g}^3) + p(\mathbf{i}^1)}\} < d < \frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1) + p(\mathbf{i}^1)}.$$

Each litigant observes the witness set pair.

Claim 6 *If Example 6 then J prefers D to lead, but litigants do not prefer an order.*

We prove Claim 6 in Section 2 of the Appendix. The burden of proof assumption implies that every outcome in $\Gamma_{D,P}$ is separating; and it is easy to construct such an equilibrium. It is also easy to construct a separating equilibrium in $\Gamma_{P,D}$. We then demonstrate that $\Gamma_{P,D}$ also has an equilibrium which prescribes a wrongful conviction at W^2, W^{12}, and an equilibrium which prescribes a wrongful acquittal at W^2, W^{12}. J then prefers D to lead. However, litigants do not have a preferred order because each litigant ranks one of the non-separating outcomes of $\Gamma_{P,D}$ over the separating outcome, and the latter outcome over the other non-separating outcome of $\Gamma_{P,D}$.

We end this section by summarizing its main results in

Theorem 2 *Litigants may prefer to lead and J alone may prefer an order in non-discovery games.*

5. Ex post order games

Miscarriages of justice occur in equilibrium when litigant l cannot persuade J to reach her favored verdict at some witness set pair $W \in \mathbf{W}^{v_l}$ (that is, where v_l is the rightful verdict). In the last section, we demonstrated that a litigant might resolve this coordination problem by leading. In this section, we consider an alternative means of separation: some litigant (say, D) chooses whether to lead after observing its witness set; so the chosen order is itself a signal. Specifically, we define and characterize outcomes in what we call an ex post order game. We use the results to ask whether D could *ex ante* prefer playing the ex post order game to playing one in which the order is fixed (as in our previous models), or conversely. We will focus on games in which litigants are commonly known to share a witness set; so the comparators are the two discovery games, as analyzed in Section 3. We will show that, in such games, offering D the option to choose an order cannot be better for D than always following, and may leave it worse off; and that D cannot prefer to always lead, and may prefer to play the ex post order game.

Ex post order games are of analytical interest; but they are also relevant in Italian criminal trials, where the defendant may ask to lead: defendants typically arrive at trial having observed their available witnesses, and are not repeat players.[30] They are also relevant in some non-judicial contexts, such as committees, whose chair may decide the order in which members speak on a case-by-case basis.

The game which we analyze in this section starts with Nature choosing the state and witness set, revealing the witness set to both litigants; D then chooses an order (say, L, F), and the game is then played out according to the rules specified in Section 2.2 for $\Gamma_{L,F}$ (after Nature's move). We denote such games by Γ, and refer to them as *ex post order games*. We analyze Γ using the solution concept introduced in Section 2.2, and apply the criterion for preferring an order in Section 2.3 to preferences over playing ex post order and discovery games.

Our first result asserts that

[30] In *United States* v *Mezzanatto* (1995), the Supreme Court ruled that procedural rights at a criminal trial can be (voluntarily) waived by a defendant.

Proposition 2 *Every outcome in a discovery game is an outcome in the ex post order game.*

We prove Proposition 2 (in Section 3 of the Appendix) by taking any equilibrium of a discovery game with any order L, F and constructing an equilibrium of Γ with the same outcome, in which D chooses order L, F at every witness set. According to the construction, J punishes D for choosing the other order (F, L) by only subsequently acquitting at an evidence set if presenting the full report induces acquittal: that is, J draws the most skeptical possible inference from the evidence pair. J must acquit at such a witness set in any discovery game; so the equilibrium prescribes acquittal at this witness set if D has chosen prescribed order L, F.

Proposition 2 implies that D chooses to lead at every witness set in an equilibrium of Γ, even though it never prefers to play $\Gamma_{D,P}$ over $\Gamma_{P,D}$ (cf. Theorem 1a)). Furthermore, Propositions 1 and 2 jointly imply that Γ has a separating outcome.

In Section 3 of the Appendix, we use an example (Example A3) to demonstrate that ex post order games may have outcomes which are not outcomes in either discovery game: cf. Claim A3. Example A3 has eight states, bolting the four states in Example 2 to the four states in its mirror image, Example 2'. $\mathbf{W}$ is the union of the witness sets in Example 2 (say, $\mathbf{W}^2$) and the witness sets in Example 2' (say, $\mathbf{W}^{2'}$): so $\mathbf{W}^2$ and $\mathbf{W}^{2'}$ are disjoint.

Claims 1 and 2 and Observation (all in Section 3) imply that, if Example A3, then $\Gamma_{P,D}$ has an equilibrium which prescribes a wrongful conviction at W^1 and no other miscarriage of justice; and that $\Gamma_{D,P}$ has an equilibrium which prescribes a wrongful acquittal at W^4 and no other miscarriage of justice. In particular, no outcome of a discovery game has a wrongful acquittal and a wrongful conviction. However, we can construct an equilibrium (say, X) of the ex post order game with this property:

This equilibrium prescribes D to choose order P, D at every witness set in $\mathbf{W}^2$, and order P, D at every witness set in $\mathbf{W}^{2'}$. If D has chosen the prescribed order at a witness set in $\mathbf{W}^2$ then X prescribes play in an equilibrium of a discovery game satisfying Example 2 with a wrongful conviction; and if D has chosen the prescribed order at a witness set in $\mathbf{W}^{2'}$ then X prescribes play in an equilibrium of a discovery game satisfying Example 2' with a wrongful acquittal. D cannot profitably deviate to the other order at witness sets in $\mathbf{W}^2$ because X prescribes J to always convict in the continuation of play; and D cannot profitably deviate to the other order at witness sets in $\mathbf{W}^{2'}$ because X prescribes play in a separating equilibrium in Example 2' in the continuation. We formalize these arguments in Claim A6, which we prove in Section 3 of the Appendix.

We now apply the criteria for preferring to lead or to follow, which we introduced in Section 2.3, to compare playing an ex post order game versus playing each of the two discovery games. In Example A3, D cannot prefer (to play) Γ to playing either discovery game because each discovery game has a non-separating outcome; so Proposition 2 implies that Γ shares two outcomes with each discovery game, precluding a preferred game. The same argument precludes D preferring to play either discovery game over playing Γ.

Theorem 3 states that this property is not generally true. We state Theorem 3 in terms of D's preferences; P obviously has the opposite preferences.

Theorem 3 *Suppose that litigants always share the same available witnesses.*

a) *D cannot prefer playing ex post order game Γ to playing $\Gamma_{P,D}$ or playing $\Gamma_{D,P}$ to playing Γ;*

b) *D may prefer playing $\Gamma_{P,D}$ to playing Γ and may prefer playing Γ to playing $\Gamma_{D,P}$;*

c) *J cannot prefer playing Γ to playing either discovery game, but may prefer playing a discovery game to playing Γ.*

The interesting feature of Theorem 3, which we prove in Section 3 of the Appendix, is that D could be made worse off by being given the option to choose an order. We demonstrate that D cannot prefer playing the ex post order game, using the observation that D cannot prefer to play Γ if it shares at least two outcomes with $\Gamma_{P,D}$. Propositions 1 and 2 then imply that D can only prefer to play Γ if $\Gamma_{P,D}$ only has separating equilibria, whereas Γ also has an equilibrium which prescribes wrongful acquittals at some witness sets. If these conditions are satisfied then we can construct an equilibrium of $\Gamma_{P,D}$ which prescribes some wrongful acquittals, contrary to the implications of the supposition that D prefers to play Γ. On the other hand, Example 2 illustrates a situation in which D is strictly better off always following than being allowed to choose the order of presentation ex post. A mirror image argument establishes that D cannot prefer playing $\Gamma_{D,P}$ to playing Γ; and Example 2' illustrates a situation in which D prefers playing the ex post order game to always leading. Finally, Proposition 2 immediately implies that J cannot prefer to play the ex post order game; and Example 2 illustrates a situation in which J prefers to play a discovery game.

The arguments used in this section rely on our supposition that litigants always share available witnesses, as in discovery games. It is instructive to adapt the examples of non-discovery games in Section 4 to define ex post order games. In each example, the outcomes of the associated ex post order games are exactly $\omega_{D,P} \cup \omega_{P,D}$. In Examples 3 and 4, $\omega_{P,D} \subset \omega_{D,P}$; so D cannot prefer $\Gamma_{D,P}$ to Γ, but prefers Γ to $\Gamma_{P,D}$. However in Example 5, $\omega_{D,P}$ and $\omega_{P,D}$ are disjoint, and D prefers to lead; so D prefers $\Gamma_{D,P}$ to Γ and Γ to $\Gamma_{P,D}$.

6. Conclusion

We have studied players' preferences over the order of presentation in a model that tries to capture essential features of common law trials. Our results indicate the importance of distinguishing between discovery and non-discovery games. In the former case, litigants cannot prefer to lead, but may prefer to follow; in the latter case, litigants might prefer to lead. These results suggest that the objectives of procedure (whatever those are) might better be served by sometimes changing the existing order. Any such change would doubtless require further procedural reforms: for example, requiring that the indictment be accompanied by a more detailed summary of P's case to satisfy the 6th Amendment requirement that D be "informed of the nature and cause of the accusation" in criminal cases. However, we believe that changing the order is practically possible, in part because civil law criminal cases usually start with interrogation of the defendant (cf. Damaska (1973) p528). On the other hand, we have also shown that a litigant would prefer to always follow than to choose an order ex post when litigants always share their available witnesses.

We have obtained our results in a model where the leader bears the burden of proof; litigants can present any combination of available witnesses (Full Reports); but each litigant has a single opportunity to present new witnesses. All of these assumptions are natural when modelling common law trials, but they might fail in other debates: Full Reports might fail because the listener has a limited attention span; whereas speakers are allowed to alternately present evidence or to pass over any fixed number of rounds in other debates. We will now discuss the robustness of our results on discovery games when these assumptions fail.

Our results on discovery games would still hold if the leader were allowed to pass. To see this, note that litigants cannot prefer to lead because the proof of Lemma does not turn on a burden of proof, and the rest of the proof only depends on outcomes. On the other hand, Example 2 would still illustrate a preference to follow because the conditions on priors imply that J must observe the same evidence pair at W^1 and at W^{123} in any non-separating equilibrium, irrespective of the burden. Indeed, the only results in the paper which depend on the burden of proof assumption are in Section 4.1.1, where we show that it may explain a preference to lead; and we noted in that section that the other examples do not depend on the burden.

Discovery games which fail Full Reports (e.g. because each litigant can only present one witness) may lack a separating equilibrium. To see this, consider Example 1 (from Section 2.3). If D leads then the game has a separating equilibrium because P could induce conviction by presenting whichever witness D did not present at W^{12}; but if P leads then the game would not have a separating equilibrium because D would then always pass at W^{12}. However, both of these games have an equilibrium in which J observes $\{w^2, pass\}$ (and then convicts) at W^2 and at W^{12}. Consequently, litigants prefer to lead in this variant on discovery games.

There are many alternative stopping rules: for example, litigants may alternately present evidence or pass over any fixed number of rounds: a set-up akin to Presidential debates. If litigants share available witnesses then the game has an equilibrium in which evidence is only presented in the last two rounds - which may shed light on Obama's success in the 2012 debate with Romney. Another possibility is that litigants alternate in presenting evidence (starting in Round 1) until a litigant passes, at which point J reaches a verdict and the game ends. Proposition 1 and Lemma hold in this set-up; so no litigant can prefer to present in odd-numbered rounds. However, litigants cannot prefer to present in even-numbered rounds. To see this recall that, if Example 2 holds then $\Gamma_{D,P}$ has an equilibrium in which J observes $\{w^1, pass\}$ and convicts at W^1 and at W^{123}. It is easy to confirm that this is also an outcome with the alternative stopping rule if D presents in odd-numbered rounds. We also argued that every outcome of $\Gamma_{P,D}$ must be separating because P could profitably deviate to presenting w^3 at W^{123} unless J convicts after observing $\{w^1, w^2\}$ - in which case, P could profitably deviate to presenting w^1 at W^{12}. This argument fails with the alternative stopping rule because P does not forego the opportunity to directly prove guilt at W^{123} by presenting w^1 in Round 1. Specifically, there is an equilibrium in which J acquits after observing $\{w^1, w^2, pass\}$ and convicts after observing $\{w^1, w^2, w^3, pass\}$.[31] P can no longer profitably deviate to presenting w^1 at W^{12} in Round 1, while D cannot profitably deviate from passing to presenting w^3 at W^{123}. This argument implies that the same non-separating outcome exists, irrespective of the order of presentation. We draw the following lesson. In our model, the leader commits to the evidence which it decides to present; with the alternative stopping rule, a litigant who does not call some witness in one round can do so later if its rival has not passed. This argument relies on the supposition that litigants are sure to alternate: if the follower might end the debate after two rounds then litigants can prefer to follow.

We have hitherto used our model to consider preferences over the order of presentation. However, the model can also be used to address other questions, such as whether discovery improves trial outcomes: that is, whether J always prefers enforcement of discovery rules (for a fixed order of presentation)? The answer, surprisingly, is 'no'.[32] To see this, consider the no-subpoena game, Example 3 in Section 4.1.1. The proof of Claim 3 establishes that, if P leads, then the game has a unique, separating outcome in which J acquits if and only if the defendant is factually innocent. Now consider a variant on this game in which litigants must share their available witnesses at every witness set pair. The conditional distribution of witness sets in this discovery game then satisfies

$$\pi(W^{13}|\mathbf{i}^1) = \pi(W^{23}|\mathbf{g}^1) = \pi(W^{123}|\mathbf{i}^2) = \pi(W^4|\mathbf{g}^2) = 1,$$

where W^{13} and W^4 are defined in Example 3, W^{23} is the pair $\{w^2, w^3\}$, and W^{123} is the triple $\{w^1, w^2, w^3\}$. The lower bound on d in Example 3 implies that the game in which P leads has a non-separating equilibrium which prescribes P to present w^3 at W^{13} and W^{23}, and to present w^1 and w^4 respectively at witness sets W^{123} and W^4; D to pass after every history; and J to acquit after observing both evidence pair $\{w^3, pass\}$ (on the path) and $\{w^2, pass\}$ (off the path). The latter verdict is rationalized by J's inference that the realized witness set is W^{123}. Proposition 1 implies that the discovery and non-discovery games both have a separating equilibrium; and the argument above implies that the discovery game alone has another outcome. Using the criterion introduced in Section 2.3: J prefers to play the non-discovery game.

[31] $\{w^1, w^2, pass\}$ means that P presents w^1 in Round 1 and passes in Round 3; $\{w^1, w^2, w^3, pass\}$ should be read analogously.

[32] Cooter and Rubinfeld (1994) and Hay (1994) show how discovery can improve trial outcomes. See also Bone (2012).

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APPENDIX

We divide this Appendix into three parts: the first contains results on discovery games, the second contains results on non-discovery games, and the third contains results on ex post order games.

1. DISCOVERY GAMES

Proposition 1 *Every discovery game has a separating equilibrium.*

Proof
Consider the following construction:

- The leader presents the full report at every witness set;
- At each witness set W: the follower responds to any evidence e_L by presenting the full report unless $W \in \mathbf{W}^{v_L}$ and there is $W' \in \mathbf{W}^{v_F}$ and $e_F \in W$ such that $e_L \cup e_F$ is the full report at W', in which case the follower presents e_F;
- After observing $\{e_L, e_F\}$, J believes that the realized witness set is:
  - W and reaches verdict v if $e_L e_F$ is the full report at $W \in \mathbf{W}^v$;
  - In $\mathbf{W}^v$ and reaches verdict v if $\{e_L, e_F\}$ induces verdict v; and
  - In $\mathbf{W}^{v_L}$ and reaches verdict v_L otherwise.

This strategy combination implies that J's beliefs are feasible and satisfy Bayes rule. J best responds both to the evidence presented on the path and after the leader has deviated, as well as to any evidence pair which induces a verdict. Consequently, J cannot profitably deviate. For any e_L, the strategy combination prescribes J to reach the leader's favored verdict unless the follower presents some evidence e_F such that $e_L \cup e_F$ is the full report at some witness set in $\mathbf{W}^{v_F}$. Consequently, the follower cannot profitably deviate. Given J's prescribed strategy, the leader cannot profitably deviate if $W \in \mathbf{W}^{v_L}$; and, irrespective of e_L, the strategy combination prescribes J to reach the follower's favored verdict otherwise. Consequently, the leader cannot profitably deviate.■

Debate Consistency Consider the following example:

There are three states: $\mathbf{S} = \{\mathbf{g}^1, \mathbf{i}^1, \mathbf{i}^2\}$*, where the defendant is factually innocent in states* $\mathbf{i}^1$ *and* $\mathbf{i}^2$*. There are three witnesses:* $w^1 = \{g, i^1\}$, $w^2 = \{g, i^2\}$ and $w^3 = \{i^1, i^2\}$ *and two witness sets:* $W^{12} = \{w^1, w^2\}$ and $W^{123} = \{w^1, w^2, w^3\}$*. The conditional distribution of witness set pairs satisfies* $\pi(W^{12}|g) = \pi(W^{123}|i^1) = \pi(W^{123}|i^2) = 1$.

$\Gamma_{P,D}$ has a separating equilibrium which prescribes P to present w^1 at W^{12}, where D responds with w^2 and J convicts; and prescribes P to present w^2 at W^{12}, where D responds with w^1 and J acquits. J convicts after observing every evidence pair available at W^{12} except $\{w^2, w^1\}$, and acquits after a litigant has presented witness w^3. Hence, D cannot profitably deviate at W^{12}. On the other hand, P presents w^2 at W^{123} because it knows that D could induce acquittal by presenting w^3, and can therefore not profitably deviate. This equilibrium fails Debate Consistency on the path because J convicts after observing $\{w^1, w^2\}$ and acquits after observing $\{w^2, w^1\}$.

Theorem 1 *In discovery games:*

a) *Litigants cannot prefer to lead.*

The proof relies on

Lemma *If $\Gamma_{L,F}$ has an equilibrium which prescribes the leader's favored verdict at witness sets $\mathbf{W}^*$ then $\Gamma_{F,L}$ has an equilibrium which prescribes that verdict at every witness set in $\mathbf{W}^*$.*

Proof We focus on games in which D is the leader (viz. $\Gamma_{D,P}$) for expositional convenience: so the leader's favored verdict is acquittal. Let a given equilibrium of $\Gamma_{D,P}$ (say, X) partition $\mathbf{W}$ into the witness sets where it prescribes acquittal ($\mathbf{W}^A$) and its complement ($\mathbf{W}^C$), and prescribes each litigant l to present evidence $e_l^X(W)$ on the path at each witness set W. The full report at any $W \in \mathbf{W}^A$ [resp. $W' \in \mathbf{W}^C$] cannot induce conviction [resp. acquittal], else P [resp. D] could profitably deviate to presenting the full report.

Consider the following strategy combination (say, Y) and beliefs in $\Gamma_{P,D}$:

- Y prescribes P to present $e_D^X(W)$ at every $W \in \mathbf{W}^A$, and to present the full report at any other witness set;
- At any witness set $W \in \mathbf{W}^A$: Y prescribes D to respond to $e_D^X(W)$ by presenting $e_P^X(W)$, and to respond to any other evidence by presenting the full report at W;
- After P has presented e_P at a witness set $W \in \mathbf{W}^C$, Y prescribes D to present the full report unless $W \in \mathbf{W}^\gamma$ and
  - $e_P = e_D^X(W')$ for some witness set $W' \in \mathbf{W}^A$, and W contains $e_P^X(W')$. Y then prescribes D to respond to e_P by presenting $e_P^X(W')$; or
  - There is $W' \in \mathbf{W}^A \cup \mathbf{W}^\alpha$ and $W' \subset W$. Y then prescribes D to respond to e_P by presenting the full report at W'.
- After observing evidence pair $\{e_P, e_D\}$, J believes that the realized witness set is in $\mathbf{W}^A \cup \mathbf{W}^\alpha$ and acquits if
  - $e_P = e_D^X(W)$ and $e_D = e_P^X(W)$ for some $W \in \mathbf{W}^A$; or
  - $\{e_P, e_D\}$ induces acquittal; or
  - e_P and/or e_D is the full report at some witness set in $\mathbf{W}^A \cup \mathbf{W}^\alpha$.

  J believes that the realized witness set is in $\mathbf{W}^\gamma$ and convicts after observing any other evidence pair.

In sum, Y prescribes J to observe $\{e_D^X(W), e_P^X(W)\}$ at every $W \in \mathbf{W}^A$, and to observe both litigants presenting the full report at every $W' \in \mathbf{W}^C$. As X is an equilibrium, Y cannot prescribe J to observe the same evidence pair on the path at any $W \in \mathbf{W}^A$ and any $W' \in \mathbf{W}^C$.

Given the strategies that Y prescribes for litigants, J's beliefs are feasible and satisfy Bayes rule, and J cannot profitably deviate. To see this, note that Y prescribes J to acquit after observing evidence which induces acquittal; or is played on the path at witness sets in $\mathbf{W}^A$ (so J holds the same posterior beliefs after observing $\{e_D^X(W), e_P^X(W)\}$ in both games); or if a litigant presents the full report at some witness set in $\mathbf{W}^A$ or $\mathbf{W}^\alpha$. None of this evidence can induce conviction, else P could profitably deviate from X by presenting the full report at some $W \in \mathbf{W}^A$. Finally, Y never prescribes J to convict after observing evidence which induces acquittal, else D could profitably deviate from X by presenting the full report at some witness set in $\mathbf{W}^C$.

We now argue that litigants cannot profitably deviate at any witness set:

Consider any witness set $W \in \mathbf{W}^A$. Y prescribes P to present $e_D^X(W)$ and J to acquit after observing either $\{e_D^X(W), e_P^X(W)\}$ or D presenting the full report at W. D can therefore secure acquittal, irrespective of the evidence that P presents at W; so no litigant can profitably deviate from Y's prescription at W. On the other hand, if P follows Y's prescription by presenting the full report at any $W' \in \mathbf{W}^C$ then Y prescribes J to convict, irrespective of the evidence that D might present at W'. Consequently, P can secure conviction at W', and neither litigant can profitably deviate, on the path, at any witness set in $\mathbf{W}^C$. Off the path, Y prescribes D to present evidence at W which results in an acquittal after P has deviated, whenever such evidence is available.

These arguments imply that Y is an equilibrium in $\Gamma_{P,D}$ which prescribes acquittal at every witness set in $\mathbf{W}^A$. An equivalent argument establishes Lemma when P is the leader.■

a) follows from Lemma. To see this suppose, per contra, that D prefers to lead. Condition 2 in Section 2.3 then implies that D strictly prefers an outcome in $\Gamma_{D,P}$ (say, x) over an outcome in $\Gamma_{P,D}$ (say, y). Lemma then implies that D weakly prefers another outcome in $\Gamma_{P,D}$ (say, y') over x, and that D weakly prefers y over another outcome in $\Gamma_{D,P}$ (say, x'). Transitivity then implies that D prefers y' over x'; so Condition 1 would fail, contrary to the initial supposition. An analogous argument precludes P from preferring to lead.■

2. NON-DISCOVERY GAMES

Claim 3 *If Example 3 then litigants prefer to lead, and J prefers P to lead.*

Proof In Example 3, w^1 and w^4 respectively directly prove factual innocence and guilt; so J must acquit [resp. convict] in equilibrium after observing either litigant present w^1 [resp. w^4].

The following strategy combination and beliefs form a (separating) equilibrium in $\Gamma_{D,P}$:

- D presents w^3 at W^3, W^1 and at W^3, W^2; presents w^1 at W^{13}, W^2; and presents w^4 at W^4, W^4;

- Irrespective of e_D: P presents the full report at each of its witness sets; and
- J believes that the realized witness set pair is
  - W^3, W^1 and acquits after observing $\{w^3, w^1\}$ or $\{w^3, pass\}$;
  - W^3, W^2 and convicts after observing $\{w^3, w^2\}$;
  - W^{13}, W^2 and acquits if D has presented evidence containing w^1;
  - W^4, W^4 and convicts after a litigant has presented w^4.

This equilibrium prescribes J to acquit at W^3, W^1 and at W^{13}, W^2 and to convict otherwise.

The lower bound on d implies that $\Gamma_{D,P}$ also has a non-separating equilibrium in which

- D presents w^3 at W^3, W^1 and at W^3, W^2; presents w^1 at W^{13}, W^2; and presents w^4 at W^4, W^4;
- Irrespective of e_D: P passes at W^3, W^1 and at W^3, W^2; and presents the full report at W^{13}, W^2 and at W^4, W^4; and
- J believes that the realized witness set pair is
  - W^{13}, W^2 and acquits after observing $\{w^3, w^2\}$ or D present evidence containing w^1;
  - W^3, W^1 or W^3, W^2 and acquits after observing $\{w^3, pass\}$;
  - W^3, W^1 and acquits after observing $\{w^3, w^1\}$;
  - W^4, W^4 and convicts if a litigant has presented w^4.

This equilibrium prescribes J to convict at W^4, W^4, and otherwise to acquit.

J cannot observe the same evidence pair at W^3, W^2 and at W^{13}, W^2 alone in any equilibrium because the upper bound on d implies that J would then convict; and D could then profitably deviate to presenting w^1, inducing acquittal, at W^{13}, W^2. $\Gamma_{D,P}$ can therefore have no outcomes other than those detailed above.

The following strategy combination and beliefs form a (separating) equilibrium in $\Gamma_{P,D}$:

- P presents w^1 at W^3, W^1; presents w^2 at W^3, W^2 and at W^{13}, W^2; and presents w^4 at W^4, W^4;
- D presents the full report at each of its witness sets, irrespective of e_P; and
- J believes that the realized witness set pair is
  - W^3, W^1 and acquits after observing $\{w^1, w^3\}$ or $\{w^1, pass\}$;
  - W^3, W^2 and convicts after observing $\{w^2, w^3\}$ or $\{w^2, pass\}$;
  - W^{13}, W^2 and acquits if D has presented evidence containing w^1;

– W^4, W^4 and convicts if a litigant has presented w^4.

J cannot observe the same evidence pair at W^3, W^1 and at W^{13}, W^2 because of the burden of proof. J can also not observe the same evidence pair at W^3, W^2 and at W^{13}, W^2 in equilibrium because J would then have to convict; and D could then profitably deviate to presenting $w^1 w^3$.

In sum, $\Gamma_{D,P}$ has a separating equilibrium and another equilibrium with a wrongful acquittal at W^3, W^2; while $\Gamma_{P,D}$ only has a separating equilibrium. Consequently, each litigant prefers to lead, while J prefers P to lead.■

Claim 4 *If Example 4 then litigants prefer to lead, and J prefers P to lead.*

Proof Witnesses w^2 and w^3 respectively directly prove factual guilt and innocence; so a litigant who presents w^2 [resp. w^3] induces conviction [resp. acquittal]. On the other hand, P alone induces conviction by presenting w^1 first.

The following strategy combination and beliefs form a (separating) equilibrium in $\Gamma_{P,D}$ which prescribes

- P to present w^1 at W^1, W^1, w^2 at W^2, W^2, and w^3 at W^1, W^3;
- D to present the full report at each of its witness sets, irrespective of e_P; and
- J to believe that the realized witness set pair is
  - W^1, W^1 and convict if P has presented w^1;
  - W^2, W^2 and convict if P has presented w^2;
  - W^1, W^3 and convict if P has presented w^3.

This equilibrium prescribes J to acquit at W^1, W^3, and otherwise to convict. Every equilibrium of $\Gamma_{P,D}$ must clearly be separating.

$\Gamma_{D,P}$ has a separating equilibrium which prescribes

- D to present the full report at each of its witness sets;
- P to present the full report at each of its witness sets, irrespective of e_D; and
- J to believe that the realized witness set pair is
  - W^2, W^2 and convict if a litigant has presented w^2;
  - W^1, W^1 and convict after observing $\{w^1, w^1\}$;
  - W^1, W^3 and acquit after observing P present w^3 and $\{w^1, pass\}$.

In light of the lower bound on d, it also has a non-separating equilibrium which prescribes

- D to present w^1 at W^1, W^1 and at W^1, W^3; and to present w^2 at W^2, W^2;

- P to pass at W^1, W^1 and at W^1, W^3; and to present w^2 at W^2, W^2; and
- J to believe that the realized witness set pair is
  - W^2, W^2 and convict if a litigant has presented w^2;
  - W^1, W^3 and acquit if P has presented w^3; and
  - W^1, W^1 or W^1, W^3 and acquit otherwise.

This equilibrium prescribes J to convict at W^2, W^2, and otherwise to acquit.

In light of the wrongful acquittal at W^1, W^1, litigants prefer to lead, while J prefers P to lead.■

Claim 5 *If Example 5 then litigants prefer to lead, and J prefers D to lead.*

Proof As $\pi > 1/2$, the following strategy combination and J's beliefs form an equilibrium in $\Gamma_{D,P}$ (omitting the litigants' beliefs):

- D presents w^1 when its witness set is W^1 or W^{12}, and otherwise presents w^2;
- Irrespective of e_D: P presents the full report at each of its witness sets; and
- J believes that the realized witness set pair is
  - W^1, W^1 or W^{12}, W^1 and acquits after observing $\{w^1, w^1\}$ and $\{w^1, pass\}$;
  - W^2, W^2 and acquits after observing $\{w^2, w^2\}$ and $\{w^2, pass\}$;
  - W^{12}, W^1 or W^{12}, W^2 and convicts if D has presented w^1w^2;
  - W^{12}, W^1 and convicts after observing $\{w^2, w^1\}$; and
  - W^{12}, W^2 and convicts after observing $\{w^1, w^2\}$.

This equilibrium prescribes J to convict at W^{12}, W^2, and otherwise to acquit.

The game has a unique outcome: for D cannot present w^1w^2 in equilibrium when its witness set is W^{12} because J would then have to acquit after observing $\{w^1, pass\}$, $\{w^1, w^1\}$, $\{w^2, pass\}$ and $\{w^2, w^2\}$; so $\pi \in (0, 1)$ implies that D could profitably deviate to presenting w^1 when its witness set is W^{12}. Furthermore, D cannot present w^2 when its witness set is W^{12} because J would then have to acquit after observing $\{w^1, pass\}$ and $\{w^1, w^1\}$; so $\pi > 1/2$ implies that D could profitably deviate to presenting w^1.

The following strategy combination and J's beliefs form an equilibrium in $\Gamma_{P,D}$:

- P presents the full report at each of its witness sets;
- D passes, irrespective of e_P; and
- J believes that the realized witness set pair is
  - W^1, W^1 or W^{12}, W^1 and acquits after observing $\{w^1, w^1\}$ and $\{w^1, pass\}$;

– W^{12}, W^1 or W^{12}, W^2 and convicts after observing $\{w^2, w^2\}$ and $\{w^2, pass\}$;
– W^{12}, W^1 or W^{12}, W^2 and convicts after observing an evidence pair whose combination is $w^1 w^2$.

This equilibrium prescribes J to convict at W^2, W^2 and at W^{12}, W^2, and otherwise to acquit.

The game has a unique outcome because J must observe the same evidence pair and acquit at W^1, W^1 and at W^{12}, W^1: it must acquit after observing the same evidence pair; and if it observed different evidence pairs then it must acquit at W^1, W^1, and D could then profitably deviate at W^{12}, W^1 to replicating its prescribed response to w^1 at W^1, W^1. Furthermore, J must observe the same evidence pair and convict at W^2, W^2 and at W^{12}, W^2: it must convict after observing the same evidence pair; and if it observed different evidence pairs then it must acquit at W^2, W^2, and D could then profitably deviate at W^{12}, W^2 to replicating its prescribed response to w^2 at W^2, W^2.

Comparing outcomes in the two games: each litigant prefers to lead and J prefers D to lead because J reaches the same verdict in both games at every witness set pair other than W^2, W^2, where J only acquits in $\Gamma_{D,P}$.■

Claim 3C *If Example 3 then* $\Gamma^c_{D,P}$ *and* $\Gamma_{P,D}$ *have the same outcomes, and the outcome correspondence in* $\Gamma_{P,D}$ *contains the unique outcome in* $\Gamma^c_{D,P}$.

Proof Consider the following strategy combination and beliefs in $\Gamma^c_{P,D}$:

- D commits to present the full report at each of its witness sets;
- Irrespective of D's strategy: P presents the full report at each of its witness sets; and
- Irrespective of D's strategy, J believes that the realized witness set pair is
  – W^3, W^1 and acquits if P has presented w^1;
  – W^3, W^2 and convicts after observing $\{w^2, w^3\}$ and $\{w^2, pass\}$;
  – W^{13}, W^2 and acquits after observing $\{w^2, w^1 w^3\}$ and $\{w^2, w^1\}$;
  – W^4, W^4 and convicts if a litigant has presented w^4.

This strategy combination prescribes no miscarriages of justice. J's beliefs satisfy Bayes rule, and are feasible; so J cannot profitably deviate. P cannot deviate; D cannot profitably deviate, given the other players' strategies, because J would convict after observing $\{w^2, pass\}$. The strategy combination and beliefs therefore form a separating equilibrium; and, as P's witness sets are singletons, $\Gamma^c_{P,D}$ only has a separating outcome.

Consider the following strategy combination and beliefs in $\Gamma^c_{D,P}$:

- P commits to present the full report at each of its witness sets;
- Irrespective of P's strategy: D presents the full report at each of its witness sets; and

- Irrespective of P's strategy, J believes that the realized witness set pair is
  - W^3, W^1 and acquits after observing $\{w^3, w^1\}$ and $\{w^3, pass\}$;
  - W^3, W^2 and convicts after observing $\{w^3, w^2\}$;
  - W^{13}, W^2 and acquits if D has presented evidence containing w^1;
  - W^4, W^4 and convicts if a litigant has presented w^4.

This strategy combination prescribes no miscarriages of justice. J's beliefs satisfy Bayes rule, and are feasible; so J cannot profitably deviate. D cannot profitably deviate because J acquits at W^{13}, W^2; and P cannot profitably deviate because J acquits after observing $\{w^3, pass\}$ and $\{w^1 w^3, pass\}$. This outcome is unique because D can induce acquittal by presenting the full report at W^{13}, W^2.

Thus, the outcome correspondence in $\Gamma^c_{D,P}$ coincides with the outcome correspondence in $\Gamma_{P,D}$; but the outcome correspondence in $\Gamma^c_{P,D}$ is contained in the outcome correspondence in $\Gamma_{D,P}$.■

Claim 4C *If Example 4 then $\Gamma^c_{D,P}$ and $\Gamma_{P,D}$ have the same outcomes, and the outcome correspondence in $\Gamma_{P,D}$ contains the unique outcome in $\Gamma^c_{D,P}$.*

Proof Consider the following strategy combination and beliefs in $\Gamma^c_{P,D}$:

- D commits to present the full report at each of its witness sets;
- P presents the full report at each of its witness sets; and
- Irrespective of P's choice: J believes that the realized witness set pair is
  - W^1, W^1 and convicts if P has presented w^1;
  - W^1, W^3 and acquits if P has presented w^3;
  - W^2, W^2 and convicts if a litigant has presented w^2.

This strategy combination prescribes no miscarriages of justice. J's beliefs satisfy Bayes rule, and are feasible; so J cannot profitably deviate. P cannot deviate; D cannot profitably deviate as J can infer the witness set pair from P's evidence. Consequently, $\Gamma^c_{P,D}$ has a separating outcome which is, of course, unique.

Consider the following strategy combination and beliefs in $\Gamma^c_{D,P}$:

- P commits to present the full report at each of its witness sets;
- D presents the full report at each of its witness sets; and
- Irrespective of P's strategy: J believes that the realized witness set pair is
  - W^1, W^1 and convicts if P has presented w^1;
  - W^1, W^3 and acquits after observing $\{w^1, w^3\}$ and $\{w^1, pass\}$;

– W^2, W^2 and convicts if a litigant has presented w^2.

This strategy combination prescribes no miscarriages of justice. J's beliefs satisfy Bayes rule, and are feasible; so J cannot profitably deviate. D cannot deviate. P cannot profitably deviate because J would acquit after observing the same evidence pair at W^1, W^1 and at W^1, W^3. The strategy combination and beliefs therefore form a separating equilibrium in $\Gamma^c_{D,P}$; and every outcome is separating.

Thus, the outcome correspondence in $\Gamma^c_{D,P}$ coincides with the outcome correspondence in $\Gamma_{P,D}$; but the outcome correspondence in $\Gamma^c_{P,D}$ is contained in the outcome correspondence in $\Gamma_{D,P}$.■

Claim 5C *If Example 5 then $\Gamma^c_{D,P}$ and $\Gamma_{P,D}$ have the same outcomes, and the outcome correspondences in $\Gamma_{P,D}$ and in $\Gamma^c_{D,P}$ are disjoint.*

Proof Consider the following strategy combination and beliefs in $\Gamma^c_{P,D}$:

- D commits to present w^1 at W^1, W^1 and at W^{12}, W^1, to present w^2 at W^2, W^2, and to pass at W^{12}, W^2;
- Irrespective of D's strategy: P presents the full report at each of its witness sets; and
- Irrespective of D's strategy: J believes that the realized witness set pair is
  - W^1, W^1 and acquits after observing $\{w^1, pass\}$ and $\{w^1, w^1\}$;
  - W^{12}, W^1 and convicts after observing $\{w^1, w^1w^2\}$ and $\{w^1, w^2\}$.

  If D presents $e^1_D = w^2$ or *pass* in response to w^2 at W^2, W^2 and $e^2_D \neq e^1_D$ in response to w^2 at W^{12}, W^2 then J believes that the realized witness set pair is
  - W^2, W^2 and acquits after observing $\{w^2, e^1_D\}$;
  - W^{12}, W^2 and convicts after observing $\{w^2, e^2_D\}$.

  If D presents e_D in response to w^2 at W^2, W^2 and at W^{12}, W^2 then J believes that the realized witness set pair is W^2, W^2 or W^{12}, W^2 and convicts after observing $\{w^2, e_D\}$.

This strategy combination prescribes a wrongful acquittal at W^{12}, W^1. J's beliefs satisfy Bayes rule, and are feasible; so J cannot profitably deviate. P cannot deviate. D cannot profitably deviate because J would convict after observing $\{w^2, e_D\}$ if D presented e_D in response to w^2 at W^2, W^2 and at W^{12}, W^2. Consequently, the strategy combination and beliefs form an equilibrium in $\Gamma^c_{P,D}$. There are no other outcomes because P's witness sets are all singletons.

Consider strategy combinations and beliefs in $\Gamma^c_{D,P}$ which prescribe:

- P to commit to present the full report at each of its witness sets;;

- If P has not deviated then D presents w^2 when its witness set is W^2, and otherwise presents w^1; and
- If P has not deviated then J believes that the realized witness set pair is
  - W^1, W^1 or W^{12}, W^1 and acquits after observing $\{w^1, w^1\}$;
  - W^2, W^2 and acquits after observing $\{w^2, w^2\}$;
  - W^{12}, W^2 and convicts after observing $\{w^1w^2, w^1\}$, $\{w^1w^2, w^2\}$ and $\{w^2, w^1\}$.

This strategy combination prescribes a wrongful acquittal at W^{12}, W^1. J's beliefs satisfy Bayes rule, and are feasible; so J cannot profitably deviate. Given D's strategy, P cannot profitably deviate. Given P's strategy, D cannot profitably deviate to presenting w^2 or w^1w^2 when its witness set is W^{12}. Consequently, the strategy combination and beliefs form part of an equilibrium in $\Gamma^c_{P,D}$. It is easy to confirm that $\Gamma^c_{P,D}$ has no other outcomes.

Thus, the outcome correspondence in $\Gamma^c_{P,D}$ coincides with the outcome correspondence in $\Gamma_{D,P}$, but the outcome correspondences in $\Gamma^c_{D,P}$ and in $\Gamma_{P,D}$ differ.■

Example A1 *There are three states:* $\mathbf{S} = \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{i}\}$, *where* $\mathbf{i}$ *is the only factually innocent state. There are three witnesses:*

$$w^1 = \{\mathbf{g}^1, \mathbf{i}\},\ w^2 = \{\mathbf{i}\} \text{ and } w^3 = \{\mathbf{g}^2\}$$

and three witness sets:

$$W^1 = \{w^1\},\ W^{12} = \{w^1, w^2\} \text{ and } W^3 = \{w^3\}.$$

The conditional distribution of witness set pairs satisfies

$$\pi(W^1, W^1|\mathbf{g}^1) = \pi(W^3, W^3|\mathbf{g}^2) = \pi(W^1, W^{12}|\mathbf{i}) = 1.$$

The prior distribution satisfies $\frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1)+p(\mathbf{i})} < d$. *Each litigant observes the witness set pair.*

Witnesses w^2 and w^3 respectively directly prove factual innocence and guilt; so, in each game, J must acquit at W^1, W^{12} if w^2 is presented, and convict at W^3, W^3 if w^3 is presented.

J must reach the same verdict at W^1, W^1 and at W^1, W^{12} in any equilibrium of $\Gamma_{D,P}$ because it would otherwise have to convict at W^1, W^1; and P could then profitably deviate to passing at W^1, W^{12}. In fact, every equilibrium prescribes

- D to present the full report at each of its witness sets;
- P to make the same choice at W^1, W^1 and at W^1, W^{12}; and
- J to acquit if and only if D has presented w^1.

$\Gamma_{P,D}$ cannot have a separating equilibrium for an analogous reason: J would then have to convict at W^1, W^1 alone; so P could profitably deviate to presenting w^1 at W^1, W^{12}. The only outcome of $\Gamma_{P,D}$ can be supported by an equilibrium which prescribes

- P to present the full report at each of its witness sets;
- D to pass at each of its witness sets, irrespective of P's strategy; and
- J to convict if and only if P has presented w^3.

In sum, both games share a unique outcome, in which J wrongfully acquits at W^1, W^1. P's problem in $\Gamma_{D,P}$ is that it could profitably deviate at W^1, W^{12} (to replicate its prescribed response at W^1, W^1) from any putative equilibrium which prescribed conviction at W^1, W^1.

$\Gamma^c_{D,P}$ has a unique, separating outcome, supported by an equilibrium in which:

- P commits to pass at W^1, W^1 and otherwise to present the full report;
- D presents the full report at each of its witness sets; and
- J convicts unless P's evidence includes w^2.

Commitment is therefore valuable to P, who prefers to commit as follower than to lead (with or without commitment). Indeed, the separating outcome is not in either $\omega_{D,P}$ or $\omega_{P,D}$. However, commitment is not valuable to D: $\Gamma^c_{P,D}$ has the same outcome as $\Gamma_{P,D}$.

Example A2 *There are four states:* $\mathbf{S} = \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{i}^1, \mathbf{i}^2\}$*, where the defendant is factually guilty in states* $\mathbf{g}^1$ *and* $\mathbf{g}^2$*. There are four witnesses:*

$$w^1 = \{\mathbf{g}^1, \mathbf{i}^2\},\ w^2 = \{\mathbf{g}^1, \mathbf{i}^1\},\ w^3 = \{\mathbf{g}^1, \mathbf{i}^1, \mathbf{i}^2\} \text{ and } w^4 = \{g^2\}$$

and five witness sets:

$$W^1 = \{w^1\},\ W^2 = \{w^2\},\ W^3 = \{w^3\},\ W^{13} = \{w^1, w^3\} \text{ and } W^4 = \{w^4\}$$

The conditional distribution of witness set pairs satisfies

$$\pi(W^3, W^2|\mathbf{i}^1) = \pi(W^{13}, W^2|\mathbf{g}^1) = \pi(W^{13}, W^1|\mathbf{i}^2) = \pi(W^4, W^4|\mathbf{g}^2) = 1.$$

The prior distribution satisfies

$$\frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1) + p(\mathbf{i}^2)} < d < \frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1) + p(\mathbf{i}^1)}.$$

Each litigant observes the witness set pair.

Proposition A1 *If Example A2 then*

a) *Litigants prefer to lead;*

b) *The outcome correspondence of* $\Gamma^c_{L,F}$ *differs from the outcome correspondence of* $\Gamma_{F,L}$ *for any assignment of leader:* $L \in \{D, P\}$.

Proof Note that either litigant presenting w^4 induces conviction. We simplify exposition by ignoring play at W^4, W^4 because J always convicts at that witness set pair in equilibrium, irrespective of whether the follower can commit.

a) We start by characterizing $\Gamma_{D,P}$, which has an equilibrium which prescribes

- D to present w^3 at W^3, W^2 and at W^{13}, W^2 and w^1 at W^{13}, W^1;
- Irrespective of e_D: P to present w^2 at W^3, W^2 and at W^{13}, W^2, and to pass at W^{13}, W^1; and
- J to be believe that the realized witness set pair is
  - W^{13}, W^1 and acquit after observing $\{w^1, pass\}$, $\{w^1, w^1\}$, $\{w^3, w^1\}$ and $\{w^1w^3, w^1\}$;
  - W^{13}, W^2 and convict after observing $\{w^1, w^2\}$, $\{w^1w^3, w^2\}$, $\{w^1w^3, pass\}$ and $\{w^3, pass\}$;
  - W^{13}, W^2 or W^3, W^2 and convict after observing $\{w^3, w^2\}$.

This strategy combination and beliefs forms an equilibrium which prescribes a wrongful conviction at W^3, W^2. $\Gamma_{D,P}$ has another equilibrium, which prescribes

- D to present w^3 at every witness set pair;
- P to pass unless it can respond to w^1 or w^1w^3 by presenting w^2; and
- J to be believe that the realized witness set pair is
  - W^{13}, W^1 and acquit after observing $\{w^1, pass\}$, $\{w^3, pass\}$, $\{w^3, w^1\}$, $\{w^1w^3, pass\}$ and $\{w^1w^3, w^1\}$;
  - W^{13}, W^2 and convict after observing $\{w^1, w^2\}$ and $\{w^1w^3, w^2\}$;
  - W^3, W^2 and acquit after observing $\{w^3, w^2\}$;
  - W^3, W^2 or W^{13}, W^2 or W^3, W^2 and acquit after observing $\{w^3, pass\}$.

This strategy combination and beliefs forms an equilibrium which prescribes a wrongful acquittal at W^3, W^2.

$\Gamma_{D,P}$ cannot have a separating equilibrium because J would then have to acquit at W^3, W^2 and convict at W^{13}, W^2; and D could then profitably deviate to presenting w^3 at W^{13}, W^2.

$\Gamma_{P,D}$ has an equilibrium which prescribes

- P to present the full report at each of its witness sets;
- D to pass, irrespective of e_P; and

- J to be believe that the realized witness set pair is
  - W^{13}, W^1 and acquit if P has presented w^1;
  - W^{13}, W^2 and convict if P has presented w^2 and D has not passed;
  - W^3, W^2 or W^{13}, W^2 and convict after observing $\{w^2, pass\}$.

This strategy combination and beliefs forms an equilibrium which prescribes a wrongful conviction at W^3, W^2. This is the only outcome of $\Gamma_{P,D}$. In particular, $\Gamma_{P,D}$ cannot have a separating equilibrium because J would then have to always convict at W^{13}, W^2 and must therefore always convict at W^3, W^2.

In sum, $\omega_{P,D} \subset \omega_{D,P}$ and the outcome in $\omega_{D,P} \backslash \omega_{P,D}$ has a single wrongful acquittal; so litigants prefer to lead.

b) Consider the following strategy combination and beliefs in $\Gamma^c_{P,D}$:

- D passes at W^3, W^2 and at W^{13}, W^1, and presents w^1 at W^{13}, W^2;
- Irrespective of D's strategy: P presents the full report at every witness set pair;
- J believes that the realized witness set pair is W^3, W^1 if P has presented w^1.

  If D commits to presenting $e^1_D = w^3$ or *pass* at W^3, W^2 and $e^2_D \neq e^1_D$ at W^{13}, W^2 then J believes that the realized witness set pair is:
  - W^3, W^2 and acquits after observing $\{w^2, e^1_D\}$; and
  - W^{13}, W^2 and convicts after observing $\{w^2, e^2_D\}$.

  If D presents e_D at both W^3, W^2 and W^{13}, W^2 then J believes that the realized witness set pair is W^3, W^2 or W^{13}, W^2 and convicts after observing $\{w^2, e_D\}$.

This strategy combination prescribes no miscarriages of justice. P cannot deviate. J's beliefs satisfy Bayes rule, and are feasible; so J cannot profitably deviate. D cannot profitably deviate because of J's response.

In sum, $\Gamma^c_{P,D}$ has a separating outcome, unlike $\Gamma_{D,P}$.

$\Gamma^c_{D,P}$ also has a separating outcome, supported by an equilibrium which prescribes, inter alia:

- Irrespective of e_D: P to pass at W^3, W^2 and at W^{13}, W^1 and to present w^2 at W^{13}, W^2;
- Irrespective of P's strategy: D to present the full report at every witness set pair;
- If P has not deviated then J believes that the realized witness set pair is
  - W^3, W^2 and acquits after observing $\{w^3, pass\}$;
  - W^{13}, W^1 and acquits after observing $\{w^1 w^3, pass\}$;
  - W^{13}, W^2 and convicts if P has presented w^2.

Given D's strategy, P cannot profitably deviate to *pass* in response to w^1w^3 at W^{13}, W^1 and at W^{13}, W^2. Given P's strategy and J's ensuing beliefs, D cannot profitably deviate at W^3, W^2 or at W^{13}, W^2 (because J always convicts) or at W^{13}, W^1 (because J acquits).

This strategy combination forms part of an equilibrium because J learns the realized witness set pair from D's evidence, irrespective of P's strategy, and because P's strategy prevents D from securing an acquittal at W^{13}, W^2.

In sum, the outcome correspondence of $\Gamma^c_{D,P}$ also contains an outcome which is not in the outcome correspondence of $\Gamma_{P,D}$.■

Proposition A2

a) *Every commitment discovery game has a separating equilibrium;*

b) *The outcome correspondence of a commitment discovery game only contains outcomes which the follower weakly prefers over the separating outcome.*

Proof

a) Consider the following strategy combination and beliefs in $\Gamma^c_{L,F}$:

- The follower commits to presenting the full report after the leader has presented e_L at witness set W, unless $W \in \mathbf{W}^{v_L}$ and there is $e_F \in W$ such that $e_L \cup e_F$ is the full report at some witness set $W' \in \mathbf{W}^{v_F}$ or $e_L \cup e_F$ induces v_F, in which case the follower commits to respond by presenting some such e_F;
- If the follower has not deviated from its prescribed strategy then the leader presents the full report at every witness set. If the follower has deviated and would present $e_F(e; W)$ at some witness set $W \in \mathbf{W}^{v_F}$ then the leader presents the full report unless there is $e_L \in W$ such that $e_L \cup e_F(e_L; W)$ is the full report at some $W \in \mathbf{W}^{v_L}$ or induces v_L, in which case the leader presents e_L;
- J's beliefs about the realized witness set are determined by the follower's commitment. If J observes any $\{e_L, e_F\}$ which induces verdict v_l then it believes that realized witness set is in $\mathbf{W}^{v_l}$ and is consistent with the follower's commitment, and reaches verdict v_l. Let $\{e_L, e_F\}$ not induce a verdict.
  - If the follower has not deviated then J believes that the realized witness set is in $\mathbf{W}^v$ and reaches verdict v if and only if litigant l's evidence (say, e_l) is the full report at witness set $W \in \mathbf{W}^v$ and contains its rival's evidence;
  - If the follower has deviated then J believes that the realized witness set is in $\mathbf{W}^{v_L}$ and reaches verdict v_L unless the follower is only committed to presenting e_F at some witness set(s) in $\mathbf{W}^{v_F}$.

J cannot profitably deviate either after observing an evidence pair which is prescribed by F's observed and L's prescribed strategies at some witness set W or if the evidence pair induces a verdict or if the follower only presents e_F at witness set(s) in $\mathbf{W}^{v_F}$. The evidence pair does not otherwise induce a verdict; so J cannot profitably deviate from its prescribed strategy.

The leader's strategy ensures that J reaches verdict v_L at every witness set $W \in \mathbf{W}^{v_L}$. If $W \in \mathbf{W}^{v_F}$ and the follower has not deviated then the leader cannot prevent J from reaching v_F by presenting any evidence contained in the full report at W; and the leader's prescribed strategy secures verdict v_F whenever possible (given W and J's strategy). Consequently, the leader cannot profitably deviate at any witness set or after any follower commitment.

If the follower presents the full report at every witness set then the strategy combination prescribes no miscarriages of justice. There is no follower strategy which secures verdict v_F at any witness set in $\mathbf{W}^{v_L}$; so the follower cannot profitably deviate.

b) Suppose, per contra, that $\Gamma^c_{L,F}$ has an equilibrium whose outcome is worse for the follower than the separating outcome. If F were to deviate to presenting the full report at every witness set, irrespective of the evidence which the leader presents, then J would observe a different evidence pair at each witness set, and would therefore have to reach verdict v at every witness set $W \in \mathbf{W}^v$. The deviation would therefore be profitable.■

Claim 6 *If Example 6 then J prefers D to lead, but litigants do not prefer an order.*

Proof If D [resp. P] presents w^3 then the ensuing evidence pair induces acquittal [resp. conviction]. As J must convict after observing evidence w^4, $p(\mathbf{g}^3)/[p(\mathbf{g}^3)+p(\mathbf{i}^1)] < d$ implies that J reaches different verdicts at W^2, W^{12} and at W^{24}, W^4 in any equilibrium of either game. Accordingly, we henceforth simplify exposition by suppressing reference to W^{24}, W^4.

The burden of proof assumption implies that every outcome in $\Gamma_{D,P}$ is separating; and it is easy to confirm that there is a separating equilibrium which prescribes D to present the full report at each of its witness sets, and P to present the full report, irrespective of D's choice.

$\Gamma_{P,D}$ has a separating equilibrium which prescribes P to present the full report at each of its witness sets, D to present the full report, irrespective of e_P, and J to convict unless D has presented w^3 or the evidence pair is $\{w^1 \cup w^2, w^2\}$.

$\Gamma_{P,D}$ also has an equilibrium which prescribes a wrongful conviction, and an equilibrium which prescribes a wrongful acquittal. Specifically, consider the following strategy combination and beliefs:

- P presents w^2 at W^1, W^{12}, at W^2, W^{12} and at W^3, W^2, and presents w^3 at W^1, W^3;
- Irrespective of e_P: D presents w^3 at W^3, W^2, and otherwise passes; and
- J believes that the realized witness set pair is
  - W^1, W^{12} or W^2, W^{12} and convicts after observing $\{w^2, pass\}$;
  - W^1, W^3 and convicts if P has presented w^3;
  - W^3, W^2 and acquits after observing $\{w^2, w^3\}$;
  - W^2, W^{12} and acquits after observing $\{w^1 \cup w^2, w^2\}$;

– W^1, W^{12} and convicts otherwise.

It is easy to confirm that no player can profitably deviate. This equilibrium prescribes a wrongful conviction at W^2, W^{12}.

Now consider the following strategy combination and beliefs:

- P presents w^1 at W^2, W^{12}, presents w^2 at W^1, W^{12} and at W^3, W^2, and presents w^3 at W^1, W^3;
- D passes, irrespective of e_P; and
- J believes that the realized witness set pair is
  - W^1, W^{12} or W^3, W^2 and acquits after observing $\{w^2, pass\}$;
  - W^1, W^{12} and convicts after observing $\{w^1, w^1\}$, $\{w^2, w^1\}$ and $\{w^1 \cup w^2, w^1\}$;
  - W^1, W^3 and convicts if P has presented w^3;
  - W^3, W^2 and acquits after observing $\{w^2, w^3\}$;
  - W^2, W^{12} and acquits otherwise.

It is easy to confirm that no player can profitably deviate. This equilibrium prescribes a wrongful acquittal at W^2, W^{12}.

In sum, $\Gamma_{D,P}$ only has a separating outcome, whereas $\Gamma_{P,D}$ has a separating equilibrium and equilibria with miscarriages of justice; so J prefers D to lead. Each litigant ranks one of the non-separating outcomes of $\Gamma_{P,D}$ over the separating outcome, and the latter outcome over the other non-separating outcome of $\Gamma_{P,D}$; so neither litigant prefers an order of presentation.■

3. EX POST ORDER GAMES

Proposition 2 *Every outcome in a discovery game is an outcome in the ex post order game.*

Proof Let X denote an equilibrium of a discovery game with a given order, which we denote $\Gamma_{L,F}$. We will argue that Γ has an equilibrium in which D chooses order L, F at every witness set, and X is then played. J does not update its beliefs about the realized witness set after observing D's choice of order on the path of this putative equilibrium; so, by definition of X, no player can profitably deviate once D has chosen order L, F.

Suppose that D deviates to choosing order F, L at some witness set W, and consider the following strategy combination (say, Y) in the continuation:

- The leader (now litigant F) presents the full report at W;

- If D is the follower (after the deviation) then it presents the full report at W unless there is $e_D \in W \cup pass$ such that the evidence pair induces acquittal, in which case L presents e_D;

  If P is the follower then it presents the full report at W unless presenting the full report induces acquittal and presenting e_D does not induce acquittal, in which case L passes;

- After observing $\{e_F, e_L\}$, J believes that the realized witness set is in
  - $\mathbf{W}^\alpha$ and acquits if the evidence pair induces acquittal; and
  - $\mathbf{W}^\gamma$ and convicts otherwise.

J does not update its beliefs about the realized witness set after observing the unexpected order (F, L). After subsequently observing some evidence pair $\{e_F, e_L\}$, J's beliefs are feasible and satisfy Bayes rule; so J cannot profitably deviate. The unexpected follower (L) cannot profitably deviate, given e_F and J's prescribed strategy, because the evidence it presents causes J to reach L's favored verdict whenever possible. The unexpected leader (F) cannot profitably deviate at W because, according to Y's prescriptions, J only reaches v_F after F has presented e_F if J reaches v_F after F has presented the full report at W.

We now turn to D's choice of an order. As X is an equilibrium of $\Gamma_{L,F}$, it must prescribe acquittal at every witness set whose full report induces acquittal. The specified strategy combination implies that J would acquit at those witness sets if D deviated to order F, L, and would otherwise convict. Consequently, D cannot profitably deviate to choosing order F, L at any witness set.■

Example A3 *There are eight states:* $S = \{\mathbf{g}, \mathbf{g}^1, \mathbf{g}^2, \mathbf{g}^3, \mathbf{i}, \mathbf{i}^1, \mathbf{i}^2, \mathbf{i}^3\}$, *where D is factually guilty in states* $\mathbf{g}, \mathbf{g}^1, \mathbf{g}^2$ *and* $\mathbf{g}^3$. *There are six witnesses:*

$$\begin{aligned} w^1 &= \{\mathbf{i}^1, \mathbf{i}^2, \mathbf{g}\},\ w^2 = \{\mathbf{i}^2, \mathbf{i}^3, \mathbf{g}\},\ w^3 = \{\mathbf{g}\}, \\ w^4 &= \{\mathbf{g}^1, \mathbf{g}^2, \mathbf{i}\},\ w^5 = \{\mathbf{g}^2, \mathbf{g}^3, \mathbf{i}\} \text{ and } w^6 = \{\mathbf{i}\} \end{aligned}$$

and eight witness sets:

$$\begin{aligned} W^1 &= \{w^1\},\ W^2 = \{w^2\},\ W^{12} = \{w^1, w^2\},\ W^{123} = \{w^1, w^2, w^3\}, \\ W^4 &= \{w^4\},\ W^5 = \{w^5\},\ W^{45} = \{w^4, w^5\},\ W^{456} = \{w^4, w^5, w^6\} \end{aligned}$$

whose conditional distribution satisfies

$$\begin{aligned} \pi(W^1|\mathbf{i}^1) &= \pi(W^{12}|\mathbf{i}^2) = \pi(W^2|\mathbf{i}^3) = \pi(W^{456}|\mathbf{i}) = \pi(W^{123}|\mathbf{g}) \\ &= \pi(W^4|\mathbf{g}^1) = \pi(W^{45}|\mathbf{g}^2) = \pi(W^5|\mathbf{g}^3) = 1. \end{aligned}$$

The prior distribution satisfies

$$\max\{\frac{p(\mathbf{g})}{p(\mathbf{g}) + p(\mathbf{i}^2)}, \frac{p(\mathbf{g})}{p(\mathbf{g}) + p(\mathbf{i}^3)}, \frac{p(\mathbf{g}^1)}{p(\mathbf{g}^1) + p(\mathbf{i})}\} < d$$

$$< \min\{\frac{p(\mathbf{g}^2)}{p(\mathbf{g}^2) + p(\mathbf{i})}, \frac{p(\mathbf{g}^3)}{p(\mathbf{g}^3) + p(\mathbf{i})}, \frac{p(\mathbf{g})}{p(\mathbf{g}) + p(\mathbf{i}^1)}\}.$$

Claim A6 *If Example A3 then some outcome of the ex post order game is not an outcome of either discovery game.*

Proof Example A3 is just the combination of Example 2 and its mirror image, Example 2'. In particular, witnesses w^1, w^2 and w^3 are never available at a witness set where w^4, w^5 or w^6 are available, and conversely.

Claims 1 and 2 (in Section 3) imply that, if Example 2, then $\Gamma_{D,P}$ has an equilibrium with a wrongful conviction at W^1 while $\Gamma_{P,D}$ only has separating equilibria. Observation (in Section 3) notes that, if Example 2', then $\Gamma_{P,D}$ has an equilibrium with a wrongful acquittal at W^4 while $\Gamma_{D,P}$ only has separating equilibria. Consequently, if Example A3 and P leads then we can construct an equilibrium which prescribes a wrongful conviction at W^1 and no other miscarriage of justice; and if D leads then we can construct an equilibrium which prescribes a wrongful acquittal at W^4 and no other miscarriage of justice. It is easy to see that neither discovery game has another non-separating outcome.

We can exploit these arguments to construct an equilibrium of Γ (say, X) which prescribes a a wrongful conviction at W^1 and a wrongful acquittal at W^4. Specifically, partition $\mathbf{W}$ into $\mathbf{W}_{D,P} = \{W^1, W^2, W^{12}, W^3\}$ (the witness sets in Example 2) and its complement in $\mathbf{W}$, denoted $\mathbf{W}_{P,D}$. X prescribes D to choose order D, P at every witness set in and only in $\mathbf{W}_{D,P}$. As the union of witness sets in $\mathbf{W}_{D,P}$ is disjoint from the union of witness sets in $\mathbf{W}_{P,D}$, J can infer any deviation from the prescribed order at some witness set from the evidence pair. X can therefore prescribe the play and beliefs that constitute a non-separating equilibrium of $\Gamma_{D,P}$ at every witness set in $\mathbf{W}_{D,P}$ (cf. the proof of Claim 2) and prescribe the play and beliefs that constitute a non-separating equilibrium of $\Gamma_{P,D}$ at every witness set in $\mathbf{W}_{P,D}$. Once D has chosen the prescribed order, J's beliefs satisfy Bayes rule and feasibility, and no player can profitably deviate.

Now suppose that D deviates to order P, D at a witness set in $\mathbf{W}_{D,P}$. In Example 2, W^{123} contains all of the other witness sets, and presenting its full report induces conviction. Accordingly, let X prescribe both litigants to present the full report at every witness set and J to infer that the realized witness set is W^{123} and convict after observing any evidence pair which follows D's deviation to order P, D. Let X also prescribe the play that constitutes a separating equilibrium of $\Gamma_{D,P}$ at every witness set in $\mathbf{W}_{P,D}$ after D deviates to order D, P at a witness set in $\mathbf{W}_{P,D}$. Given the play prescribed thereafter, D cannot profitably deviate to another order because X does not prescribe acquittal after D has deviated to the other order at any witness set where it prescribes conviction on the path.

In sum, X is an equilibrium in Γ whose outcome is disjoint from $\omega_{P,D} \cup \omega_{D,P}$.■

Theorem 3 *Suppose that litigants always share the same available witnesses.*

a) *D cannot prefer playing ex post order game Γ to playing $\Gamma_{P,D}$ or playing $\Gamma_{D,P}$ to playing Γ;*

b) *D may prefer playing $\Gamma_{P,D}$ to playing Γ and may prefer playing Γ to playing $\Gamma_{D,P}$;*

c) *J cannot prefer playing Γ to playing either discovery game, but may prefer playing a discovery game to playing Γ.*

Proof

a) We start by showing that D cannot prefer playing Γ over playing $\Gamma_{P,D}$. Suppose otherwise. If $\Gamma_{P,D}$ has a non-separating outcome then D cannot prefer to play Γ because Propositions 1 and 2 imply that both games share two outcomes; and D can then not prefer Γ to $\Gamma_{P,D}$. Accordingly, suppose that $\Gamma_{P,D}$ only has separating equilibria; so Γ must have an equilibrium (say, X) which prescribes J to acquit after observing the same order and evidence pair at every witness set in each of a collection of subsets of $\mathbf{W}$, say $\{\mathbf{V}^n\}$, at least one of which contains a witness set in $\mathbf{W}^\gamma$(so X prescribes a wrongful acquittal). We write $e_D(e_P; W)$ for the evidence which X prescribes D to present in response to e_P at W. We also write $\mathbf{V}$ for the union of collections $\mathbf{V}^n$, $\mathbf{V}^-$ for $\mathbf{W}\backslash\mathbf{V}$, and $\overline{\mathbf{V}}^n$ for the (nonempty) intersection of witness sets in $\mathbf{V}^n$. As X is an equilibrium, there is no witness set $W \in \mathbf{V}$ such that presenting the full report induces conviction, else P could profitably deviate from X to presenting the full report at W. Consequently, for every evidence pair $\{e_L, e_F\}$ that J could observe at any $W \in \mathbf{V}$, there is a witness set $W^* \in \mathbf{W}^\alpha$.

We will prove that D cannot prefer to play Γ by arguing that $\Gamma_{P,D}$ then has an equilibrium (call it Y) which prescribes acquittal at every witness set in $\mathbf{V}$, and which prescribes verdict v at every witness set in $\mathbf{W}^v \cap \mathbf{V}^-$:

At each $W \in \mathbf{V}^n$, Y prescribes

- P to present some $e_P^n \in \overline{\mathbf{V}}^n$;
- D to respond to e_P by presenting some $e_D(e_P; W) \in \overline{\mathbf{V}}^n$.

  At each $W \in \mathbf{V}^-$, Y prescribes
- P to present the full report;
- D to respond to e_P by presenting the full report unless $W \in \mathbf{W}^\gamma$ and
  - e_P is the full report at some witness set $W' \in \mathbf{V}^- \cap \mathbf{W}^\alpha$, in which case D presents e_P; or
  - e_P is contained in some $W' \in \mathbf{V}^n$ and W contains $e_D(e_P; W')$, in which case D presents $e_D(e_P; W')$; and
- After observing any $\{e_P, e_D\}$ such that $e_P \cup e_D$ is contained in some witness set in $\mathbf{V}$, J to believe that the realized witness set is
  - In $\overline{\mathbf{V}}^n$ and acquit after observing $\{e_P^n, e_D(e_P^n; W)\}$;
  - W^* otherwise.

  After observing any $\{e_P, e_D\}$ such that $e_P \cup e_D$ is not contained in any witness set in $\mathbf{V}$, J to believe that the realized witness set is
  - Some witness set in $\mathbf{W}^v$ and reach verdict v if the evidence pair induces v;
  - $W \in \mathbf{W}^\gamma$ and convict if e_P is the full report at W and $e_D\backslash e_P$ is empty;

– Some witness set in $\mathbf{W}^{\alpha}$ which contains $e_P \cup e_D$ and acquit otherwise.

J's prescribed beliefs are feasible. We now argue that Y never prescribes J to infer two distinct realized witness sets after observing a given evidence pair. This property fails if and only if there is a witness set $W \in \mathbf{V}^n$ which contains the full report at some $W' \in \mathbf{V}^- \cap \mathbf{W}^{\gamma}$. This is impossible if X prescribes the same order at W and at W' because $W' \subset W$, X prescribes conviction at W', and X is an equilibrium. It is also impossible if X prescribes different orders at W and at W' because $W' \subset W$ implies that D could then profitably deviate from X by choosing the other order at W'. In sum, J's beliefs are mutually consistent. They satisfy Bayes rule because Y prescribes the same belief after observing the evidence pair prescribed at each collection $\mathbf{V}^n$ as X prescribes after observing the evidence pair it prescribes at that collection. Finally, J cannot profitably deviate, conditional on its prescribed beliefs.

It is easy to confirm that neither litigant can profitably deviate from Y's prescription; so Y forms an equilibrium in $\Gamma_{P,D}$. Y has a non-separating outcome because, by construction, X prescribes wrongful acquittal at some witness set.

In sum, whenever Γ has an equilibrium which prescribes wrongful acquittal at some witness set, $\Gamma_{P,D}$ has a non-separating outcome. Proposition 2 then implies that the games share two outcomes; so D cannot prefer Γ over $\Gamma_{P,D}$.

A mirror image argument precludes D from preferring to always lead over playing the ex post order game. Specifically, if D preferred to play $\Gamma_{D,P}$ then Proposition 2 implies that $\Gamma_{D,P}$ only has a separating outcome, and Γ must have an equilibrium which prescribes conviction at collections of witness sets, including a wrongful conviction. We can then use the same approach as above to construct an equilibrium of $\Gamma_{D,P}$ which prescribes a wrongful conviction, contrary to the initial supposition that D prefers to always lead.

b) Example 2 illustrates a situation in which D prefers to always follow than to play the ex post order game. Recall that $\Gamma_{P,D}$ then only has a separating outcome, and $\Gamma_{D,P}$ also has an outcome with a wrongful conviction. Proposition 2 implies that Γ shares $\Gamma_{D,P}$'s non-separating outcome; and the condition on priors implies that every other outcome of Γ is separating. Hence, D prefers playing $\Gamma_{P,D}$ to playing Γ in this example. On the other hand, Example 2' illustrates a situation in which D prefers playing Γ to playing $\Gamma_{D,P}$ for mirror image reasons.

c) Propositions 1 and 2 immediately imply the first claim. Example 2 illustrates a situation in which J prefers playing a discovery game to playing Γ.■