Fault Tree analysis including component dependencies

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The inability of commonly used risk assessment methodologies to model component dependency is often recognised to be a barrier towards more realistic modelling of systems complexity. This is a significant limitation associated with widely adopted techniques such as Fault and Event Trees (FT/ET), which rely on the often unjustified assumption of stochastic independence among systems' components. The research presented provides mathematical solutions able to tackle the two main challenges associated with the modelling of dependency in reliability and safety system analysis: on the one hand, the achievement of adequate flexibility for representing realistically a wide variety of system settings and interrelations; on the other, the computational feasibility of the methodology, still retaining the familiarity of commonly used approaches (i.e. FT/ET). The proposed methodology relies on the use of Binary Decision Diagrams as well as on the manipulation of joint probabilities according to Bayes' theorem. The technique is demonstrated using a simple numerical application for verification.

Keywords: Fault Trees, Safety Analysis, Component Dependency, Degradation, Markov Models, Petri Nets

1. Methodology

Binary Decision Diagrams (BDDs) offer a methodology for the analysis of FTs and the subsequent estimate of relevant reliability metrics, such as system failure probability, system failure intensity and component importance measures Drechsler and Becker (2013). The probability of system failure can be calculated as the sum of the probability associated to individual BDD paths, which are sets of events connecting the BDD top node to a terminal 1.Under the assumption of independence paths probabilities can be simply computed as the product of the probabilities of the individual events (working and failed) which they entail. Such procedure is not adequate in the presence of dependencies.

The proposed algorithm offers an alternative for the computation of BDDs with component dependencies. This is achieved through five step:

(1) Paths identification:events included in each possible path are recorded and to classified according to the dependency group reference.
(2) Independent path element probability computation:since the joint probability of independent events is equal to the product of the individual events probabilities, the contribution to the overall path probability of independent components for each path can be computed in parallel with the path identification in step 1.
(3) Dependent path probability computation: once the contribution associated with dependency group are known for each path, their probability can be calculated on the basis of the joint probability in input, and finally multiplied by the path probability since the group is independent from other dependency groups.
(4) System failure probability Once the probability of each BDD path has been computed, the failure probability of the system is calculated as:

$$Q_{system} = \sum_{i=1}^{m} q(P_i) \tag{1}$$

where $q(P_i)$ refers to the probability associated with the $i-th$ of the m BDD paths.
(5) Birnbaum's meausure of importance: For independent components, the value of the Birnbaum's measure of importance can be calcu-

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lated as:

$$G(X_j) = \sum_{i|X_j \in P_i} \frac{q(P_i)}{q(X_j)} - \sum_{k|\overline{X_j} \in P_k} \frac{q(P_k)}{q(\overline{X_j})} \quad (2)$$

where $q(X_j)$ and $q(\overline{X_j})$ refer respectively to the probability of failure and success of the component X_j.

2. Numerical Application

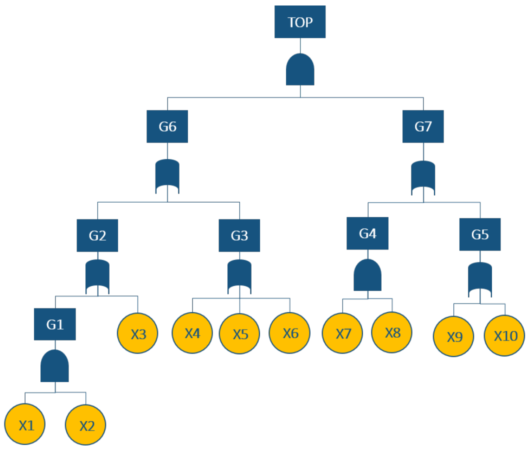

Fig. 1.: Fault Tree structure for the case-study analysed

In order to test the capabilities of the proposed methodology, a simple case study focusing on the fault tree structure shown in Fig.1 has been analysed. The case study includes three dependency groups. The FT was first converted to the BDD structure shown in Fig.2, where the three dependency groups are highligthed. The top event probability was computed according to the methodology discussed, resulting equal to $5.8600e^{02}$. The Birnbaum importance measured were also estimated according to Eq.2, resulting in the values shown in Table 1. Finally the failure intensity associated with the top event was computed from the Birnbaum measure of importance, resulting equal to $1.2440e^{-01}h^{-1}$.

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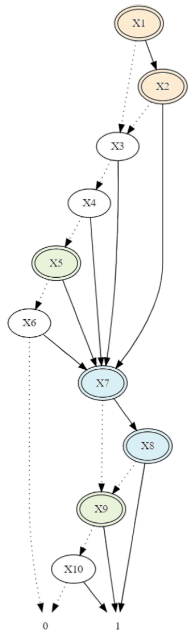

Fig. 2.: BDD structure analysed. Double ellipses refer to dependent components, while color shades have been assigned to different dependency groups

Table 1.: Components Birnbaum's Measure of Importance

Component	Birnbaum Measure
X1	$2.4390e^{-02}$
X2	$2.4360e^{-02}$
X3	$4.7694e^{-02}$
X4	$4.7987e^{-02}$
X5	$1.9498e^{-02}$
X6	$4.7695e^{-02}$
X7	$4.9035e^{-02}$
X8	$4.9035e^{-02}$
X9	$1.7898e^{-01}$
X10	$1.9384e^{-01}$

References

Drechsler, R. and B. Becker (2013). *Binary decision diagrams: theory and implementation*. Springer Science & Business Media.