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The Extension of Commonly used Measures of Importance for Dynamic and Dependent Tree Theory (D^2T^2)

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Traditional Fault Tree Analysis (FTA), known as Kinetic Tree Theory (KTT), was derived by Vesely et al (1970) to model and analyse engineering systems. The tree structure provides a clear visual representation of the causes of system failure. FTA has two stages, qualitative and quantitative analysis. Part of the quantification process is to calculate measures of importance. FTA is limited by the necessary assumptions of constant component failure and repair rates and independence of component failure and repair rates. The D^2T^2 methodology overcomes these assumptions, but a new methodology to calculate measures of importance when such assumptions are not met is required. This paper proposes extensions to 5 common used measures of importance.

Keywords: Importance Measures, Fault Tree Analysis, Quantitative Analysis, Dependencies.

1. Introduction

The performance of a system is dependent upon that of its components and Minimal Cut Sets. Some components or minimal cut sets will play a more significant role in causing or contributing to system failure than others. The contribution that a component or a minimal cut set makes to system failure is its importance. Birnbaum first introduced the concept of importance in 1969. He developed what we now term Birnbaum's Measure of component importance which is denoted by $G_i(q)$, and defined as the probability that component i is critical to systems failure, i.e., the system is in a working state such that the failure of component i causes it to fail. An expression for this measure is given in Eq. (1):

$$G_i(q) = Q_{sys}(1_i, q) - Q_{sys}(0_i, q) \quad (1)$$

where, Q_{sys} is the system unavailability function and $Q_{sys}(1_i, q)$ is the probability that the system fails with component i failed and $Q_{sys}(0_i, q)$ is the probability that the system fails with component i working.

An alternative expression for this measure is given in Eq. (2):

$$G_i(q) = \frac{\partial Q_{sys}(t)}{\partial q_i(t)} \quad (2)$$

Since this time, numerous measures of importance have been developed to assess the role that a component failure plays in the deterioration of the system state. Each measure gives subtly different information. Measures of importance assign a value between 0 and 1 to each component, with 1 signifying the highest level of contribution.

Engineers can use importance analysis to rank the contribution each component or minimal cut set makes to system failure. In this way, weaknesses within the system can be identified and resources can be used most efficiently to improve system reliability. This paper will focus on five key measures, Birnbaum's Measure of importance defined in Eq. (1) and Eq. (2). The Criticality Measure of Importance, defined as the probability that component i is critical to the system and has failed, weighted by the system unavailability at time t. An expression for this measure for systems involving only independent basic events is given in Eq. (3).

$$I_{C_i} = \frac{G_i(q) \cdot q_i(t)}{Q_{sys}(t)} \quad (3)$$

The Risk Achievement Worth (RAW) which calculates the relative increase in the system unavailability when it is known that component i has failed. It can be calculated using Eq. (4).

$$I_i^{RAW} = \frac{Q_{sys}(1_i, t) - Q_{sys}(t)}{Q_{sys}(t)} \quad (4)$$

The Risk Reduction Worth (RRW) calculates the relative reduction in the system unavailability when it is known that component i is working. It is calculated using Eq. (5).

$$I_i^{RRW} = \frac{Q_{sys}(t) - Q_{sys}(0_i, t)}{Q_{sys}(t)} \quad (5)$$

The Fussell-Vesely measure of component importance is concerned with the contribution component failures make to system failure. It is calculated using Eq. (6).

$$I_{FV_i} = \frac{P(\bigcup_{\substack{k=1 \\ i \in C_K}}^{n_c} C_K)}{Q_{sys}(t)} \quad (6)$$

All of these measures can be efficiently calculated during FTA. However, for systems which experience dependencies between the component failures the D^2T^2 methodology can be employed which will require a new approach to calculating the importance measures.

2. Dynamic and Dependent Tree Theory

In Andrews Et al (2023). published the D^2T^2 methodology designed specifically to address limitations in the traditional method of fault tree analysis which restricted its ability to represent the performance of modern engineering systems. These limitations include the need for component failure and repair rates to be assumed constant, components failures to be independent and very limited processes employed to represent the asset management strategy. The methodology retains the tree structure and integrates Binary Decision Diagram (BDD), Markov and Stochastic Petri Net (SPN) methodologies to perform the analysis.

The D^2T^2 methodology is a multi-layer methodology which culminates in a final BDD for the top gate of the Fault Tree. A variety of sub-models can feed into this BDD, and each can have a variety of inputs too. Fig 1 illustrates the possible inputs for each element of a system.

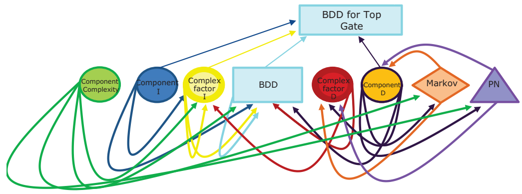

Fig 1: Illustration of possible inputs for a system analysed using the D^2T^2 methodology.

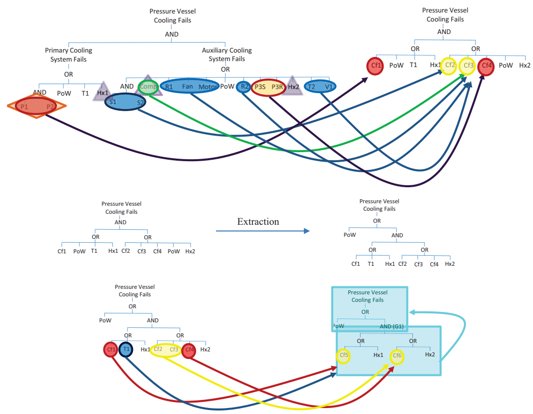

Fig. 2. D^2T^2 modularisation structure for the pressure vessel cooling system.

Fig. 2 illustrates the modularisation structure for the Pressure Vessel Cooling System case study introduced in Andrews et al (2023). This paper presents the algorithm to integrate the importance calculations into the D^2T^2 methodology.

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