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Improved Methods for System Reliability Modelling

Professor John Andrews

SPAA & Dependability@Siemens 2023
Nuremberg 8-9 November 2023

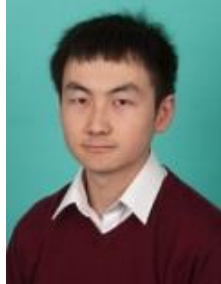


Academic Staff

- Prof John Andrews
- Dr Rasa Remenyte-
Prescott
- Dr Luis Neves
- Dr Darren Prescott
- Dr Silvia Tolo
- Dr Derek Yan

NxGen Project Manager

- Kate Sanderson



Started in 2009 with my appointment to a research chair in Infrastructure Asset Management supported by Network Rail and the Royal Academy of Engineering.



Research Activities

Modelling to support the prevention of system failures and the mitigation of their consequences

- Risk and Reliability Engineering
- Asset Management
- Resilience Engineering

Industrial Sectors

- Railways
- Nuclear
- Fuel Cell
- Oil & Gas
- Aerospace
- Military
- Manufacturing
- Healthcare



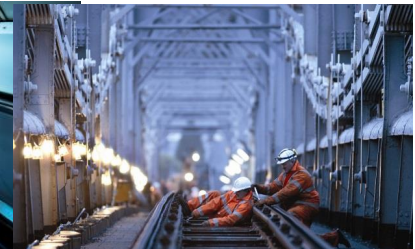
HS2



Lloyd's Register
Foundation



Lloyd's
Register

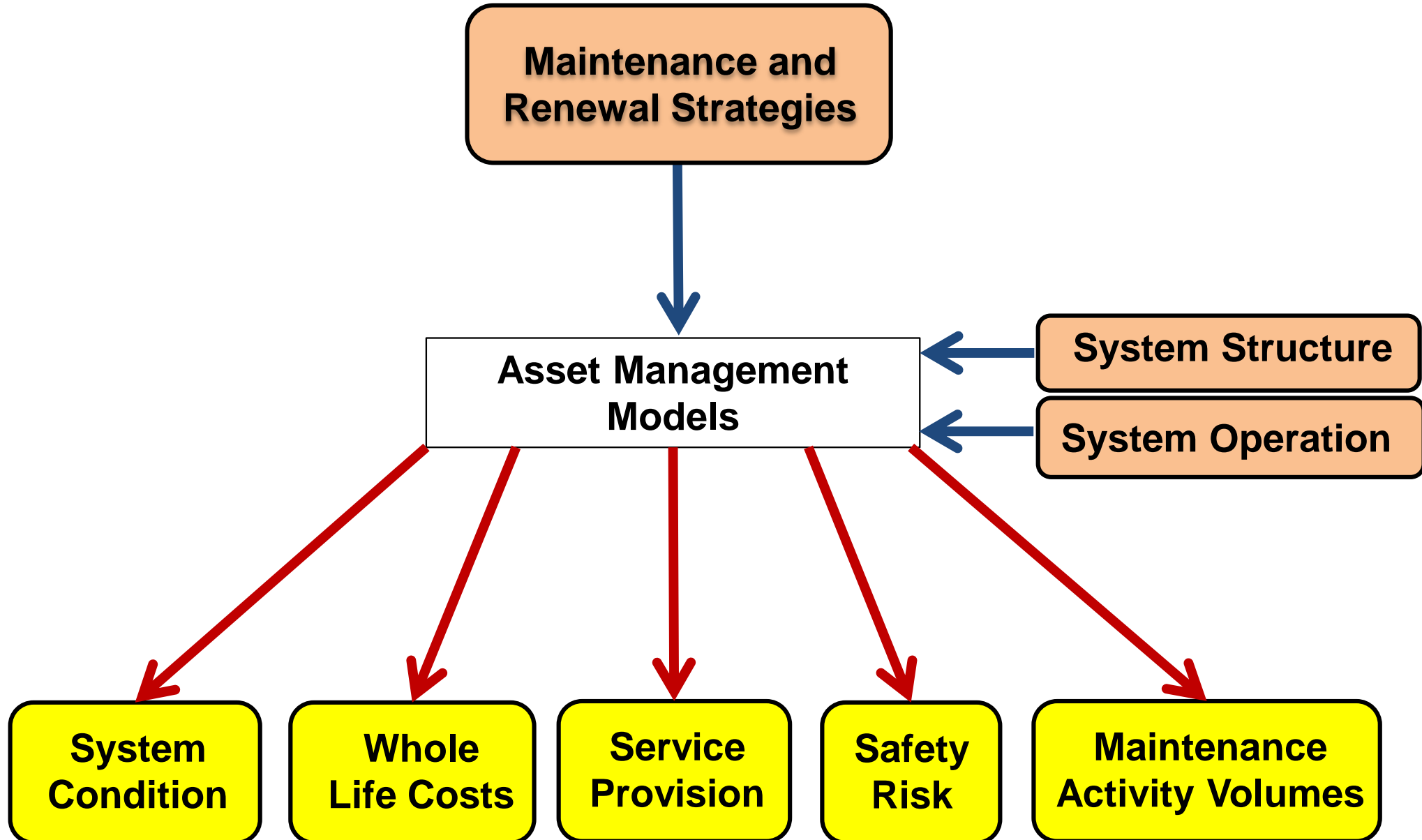




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Railway Infrastructure Asset Management





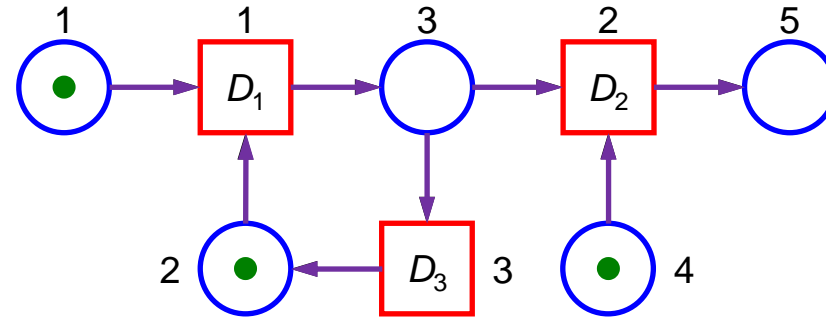
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Modelling Method

Petri Nets

Petri Net Basics and Definitions



i

Places, p_i

- Marked with tokens

j

Transitions, t_j

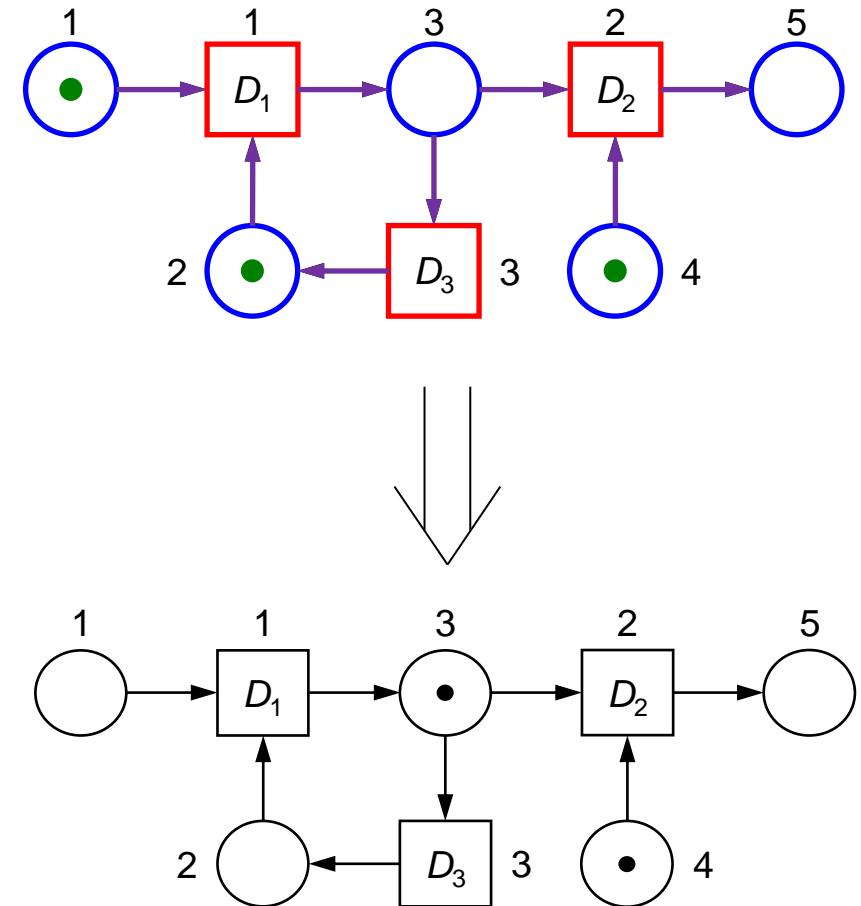
- Time delay D_j determines token movement.
- Type:
 - immediate if $D_j = 0$
 - timed if $D_j \neq 0$

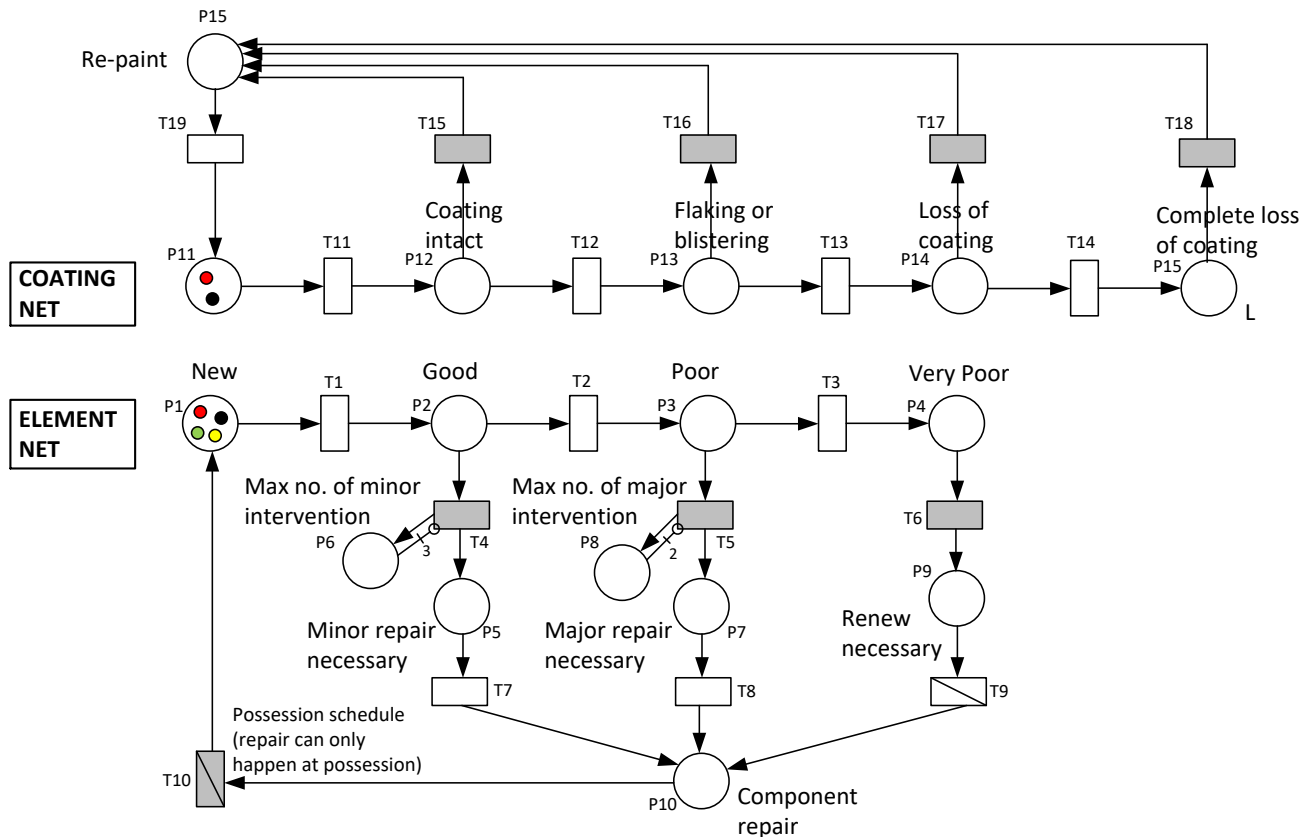
Edges

- From place to transition or transition to place.

- Movement of tokens governed by the firing rule...

- If all input places of a transition are marked by at least one token then this transition is called **enabled**.
- After a delay $D \geq 0$ the transition **fires**. The firing removes one token from each of its input places and adds one token to each of its output places.





Features

- Any distribution of times to transition
- Capable of modelling very complex maintenance strategies
- Concise structure
- Solution by Monte Carlo simulation
- Produces distributions of durations and no of incidences of different states
- Easy to modularise and link module models to form system model



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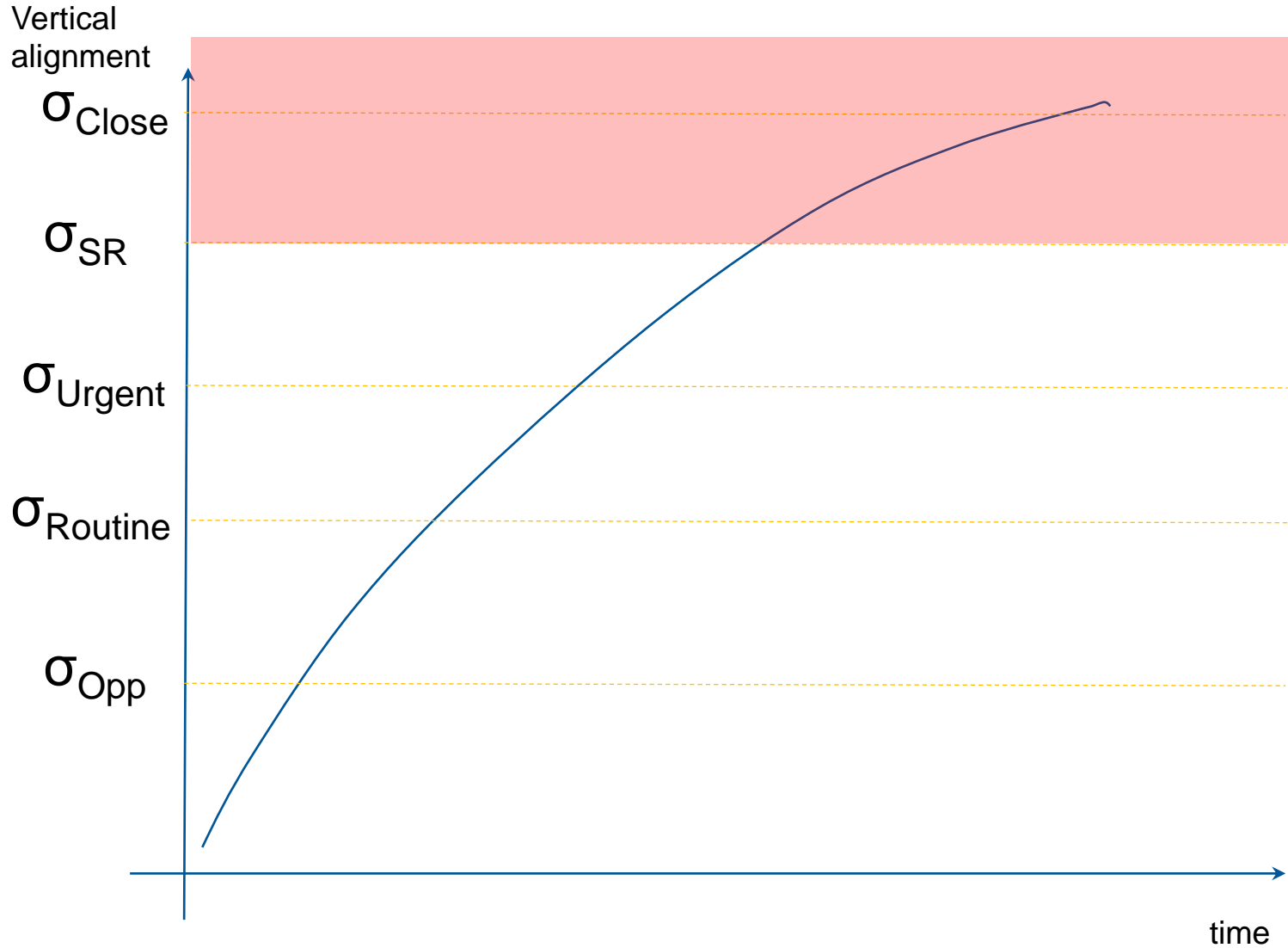


Case Study

Maintaining Railway Track Geometry

Vertical alignment of 200m sections

Vertical Alignment Degradation (200 m section)



LC

Line Closure /
emergency maintenance

SR

Speed Restriction /
emergency
maintenance

UR

Urgent Maintenance

PO

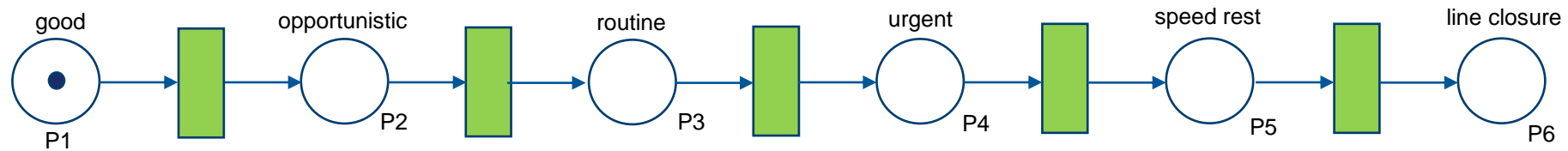
Poor Condition /
Routine Maintenance

OP

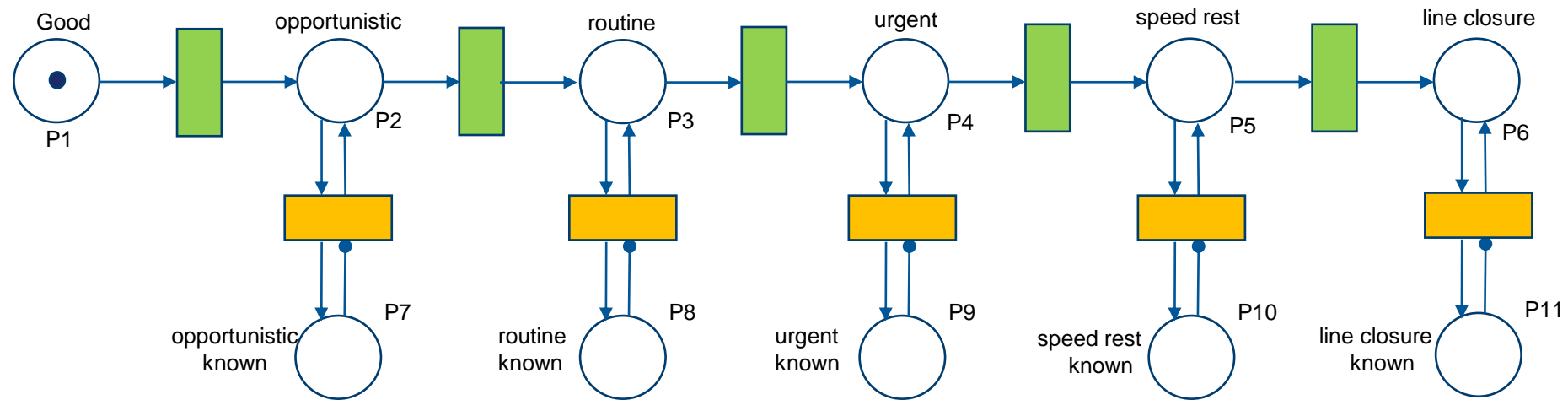
Opportunistic
Maintenance

GD

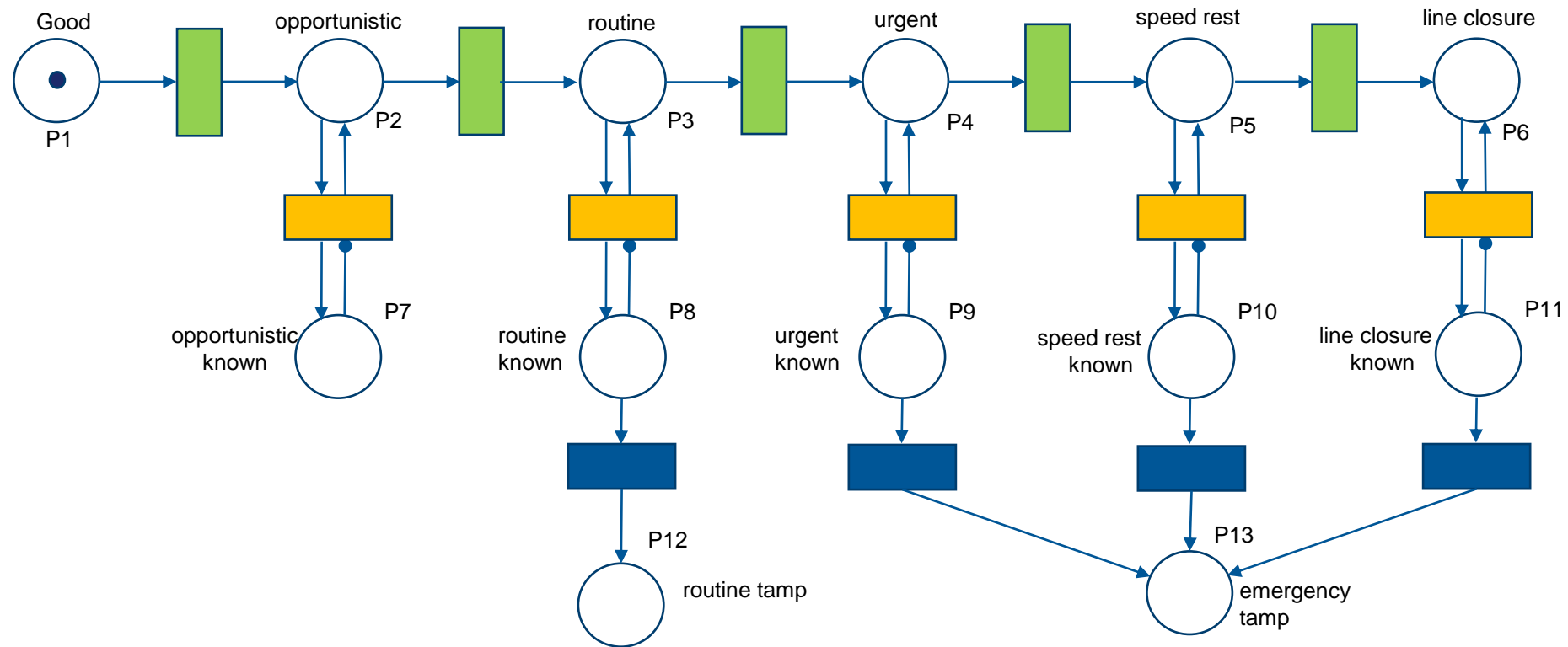
Good Condition



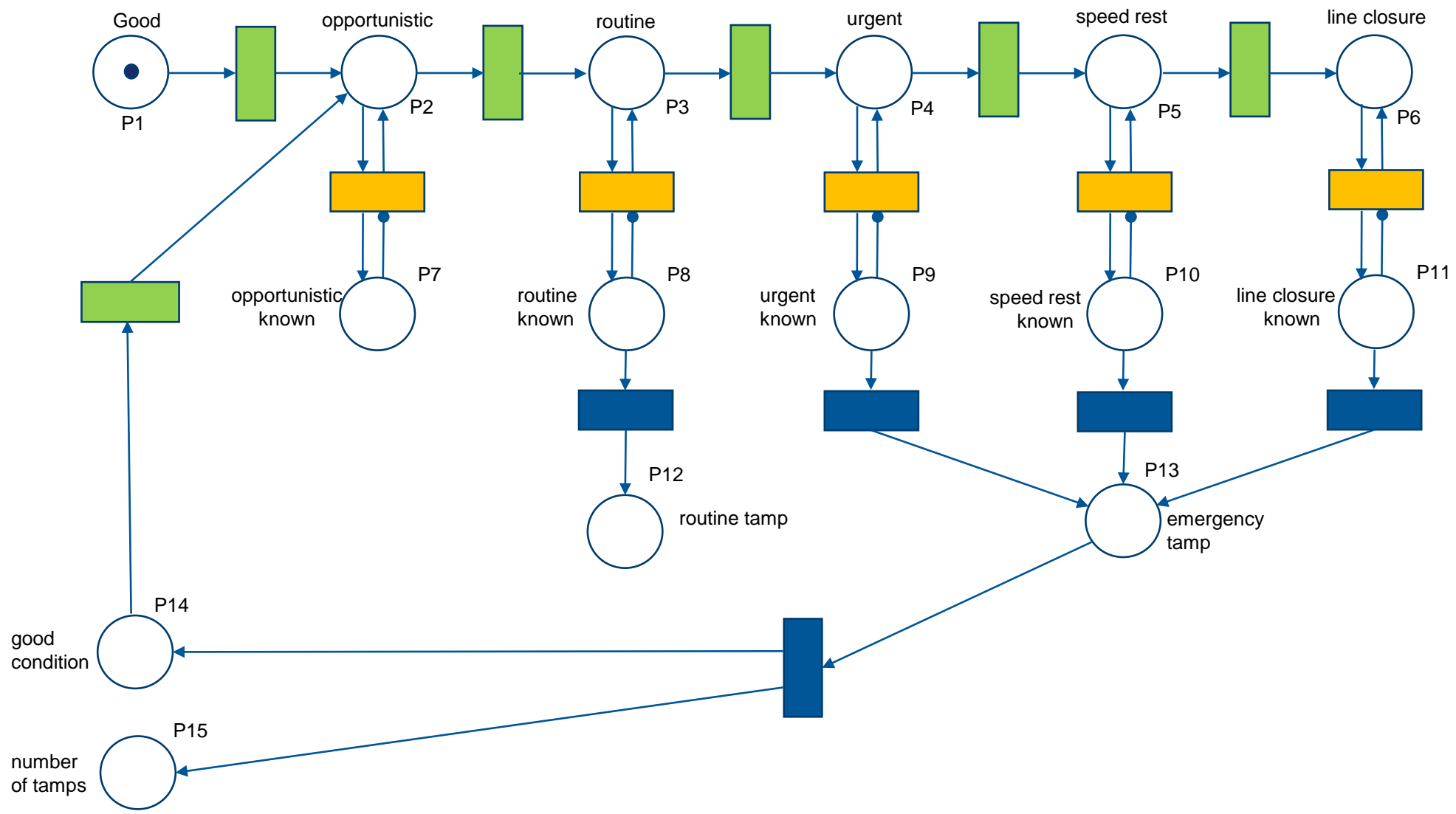
Degradation



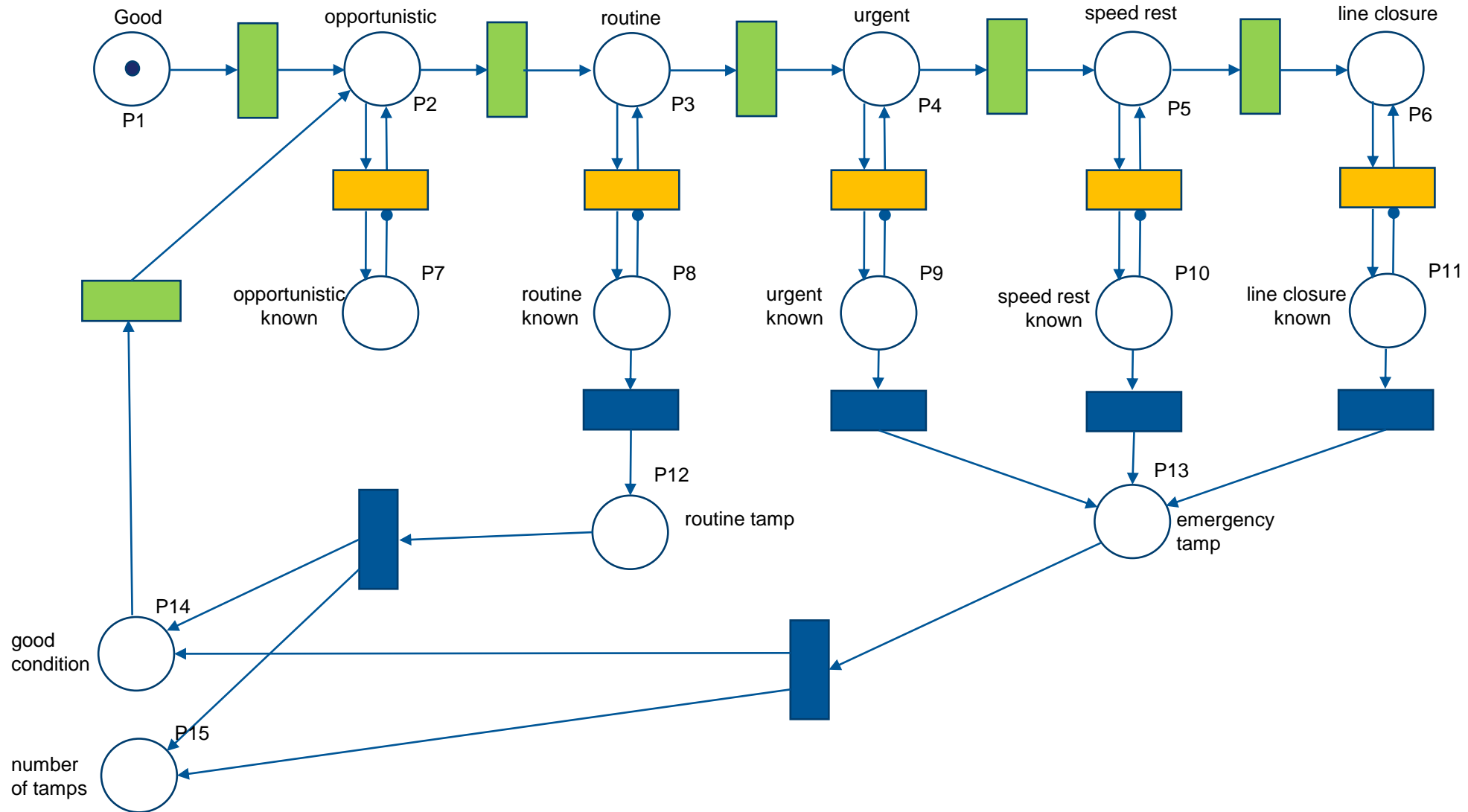
Inspection



Repair Options



Emergency Repair



Degradation time distributions account for the variation of all track sections along a route.

Routine Repair



Model results – Asset Condition Performance

Condition	Condition Known?	Min Value	Average Value	Max Value	Comment
Good		92.66%	95.2%	97.31%	
Opportunistic		0.27%	0.42%	0.59%	
Routine		2.58%	3.11%	5.72%	
Urgent		1.12%	1.16%	1.18%	
Speed Restriction needed	Known	0.0%	0.005 %	0.018 %	Service disruption
	Unknown	0.0%	0.043 %	0.056 %	Potential safety issue
Line Closure needed	Known	0.0%	0.005 %	0.018 %	Service disruption
	Unknown	0.0%	0.057 %	0.07 %	Potential safety issue

Event	Number		
	Min	Average	Max
Track Inspections	391	391	391
Routine Intervention (tamp)	0.0	3.7	12.5
Emergency Intervention (tamp)	0.0	2.58	3.11
Speed Restriction	0.0	0.2	2.3
Line Closure	0.0	0.028	1.57



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Resilience Engineering



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Track Buckling

Hot Weather



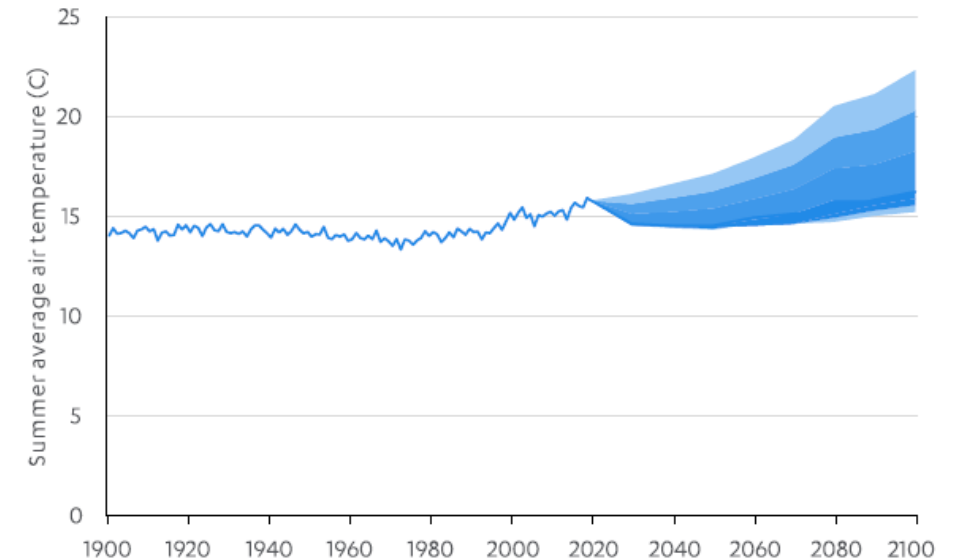
Effects of Climate Change – period of sustained high temperature

- Expansion in the rails means that tamping risks causing them to buckle.
- No tamping - causes a drift towards a poorer condition.
- Track can be in any state at the start of the heatwave.

Questions

- How many days of high temperature before the risk of a safety incident or a service disruption becomes unacceptable?
- How is maintenance best performed prior to a period of high temperature to ensure geometry resilience?
- How long after the high temperature period to clear the backlog of work?

Figure 4.1 Predicted average summer temperatures in the UK (1900 – 2100)* 125

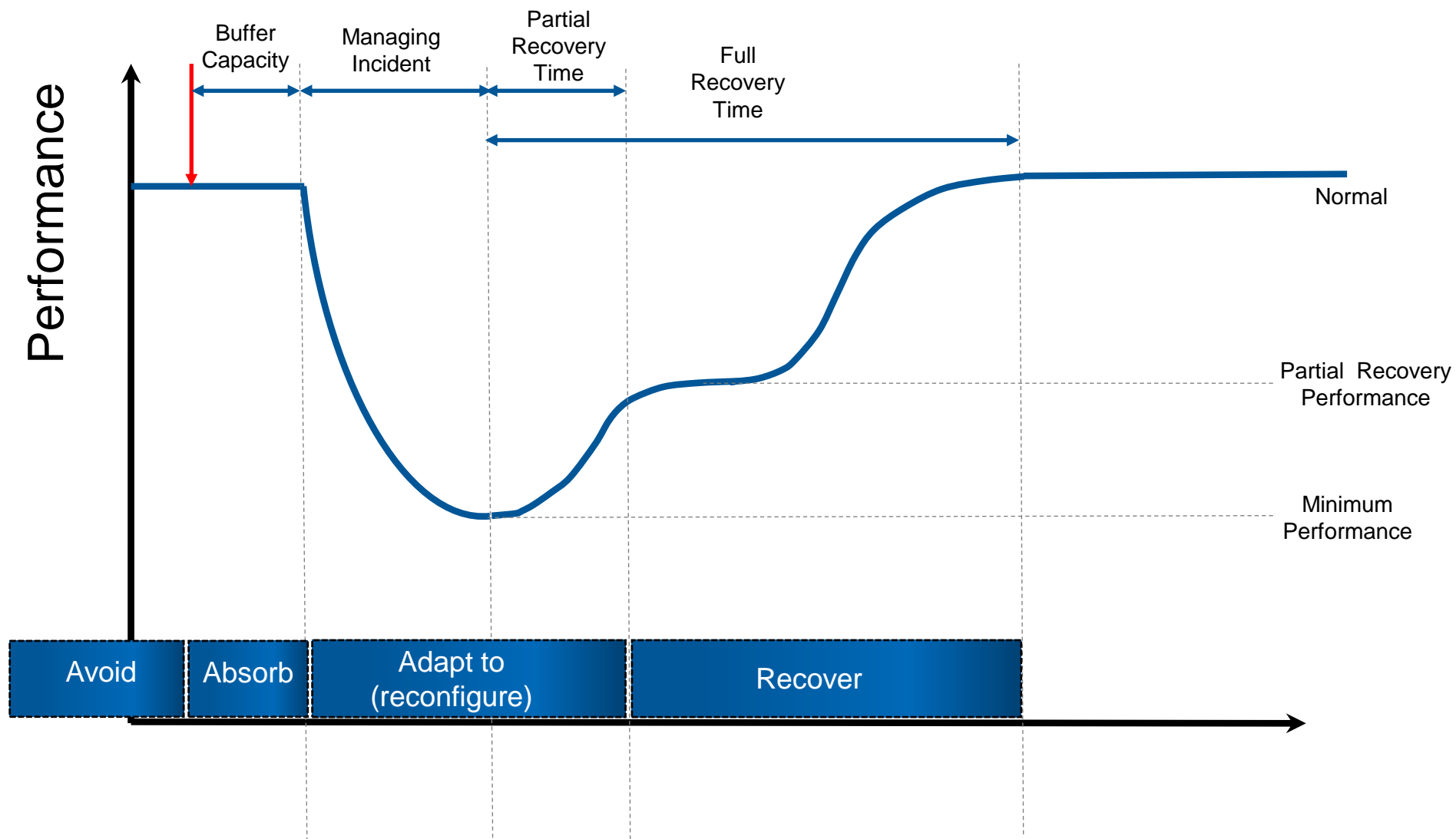


* Range of projected values based on the minimum and maximum of all UKCP18 temperature scenarios, at the 5th and 95th percentile. Source: UKCP18

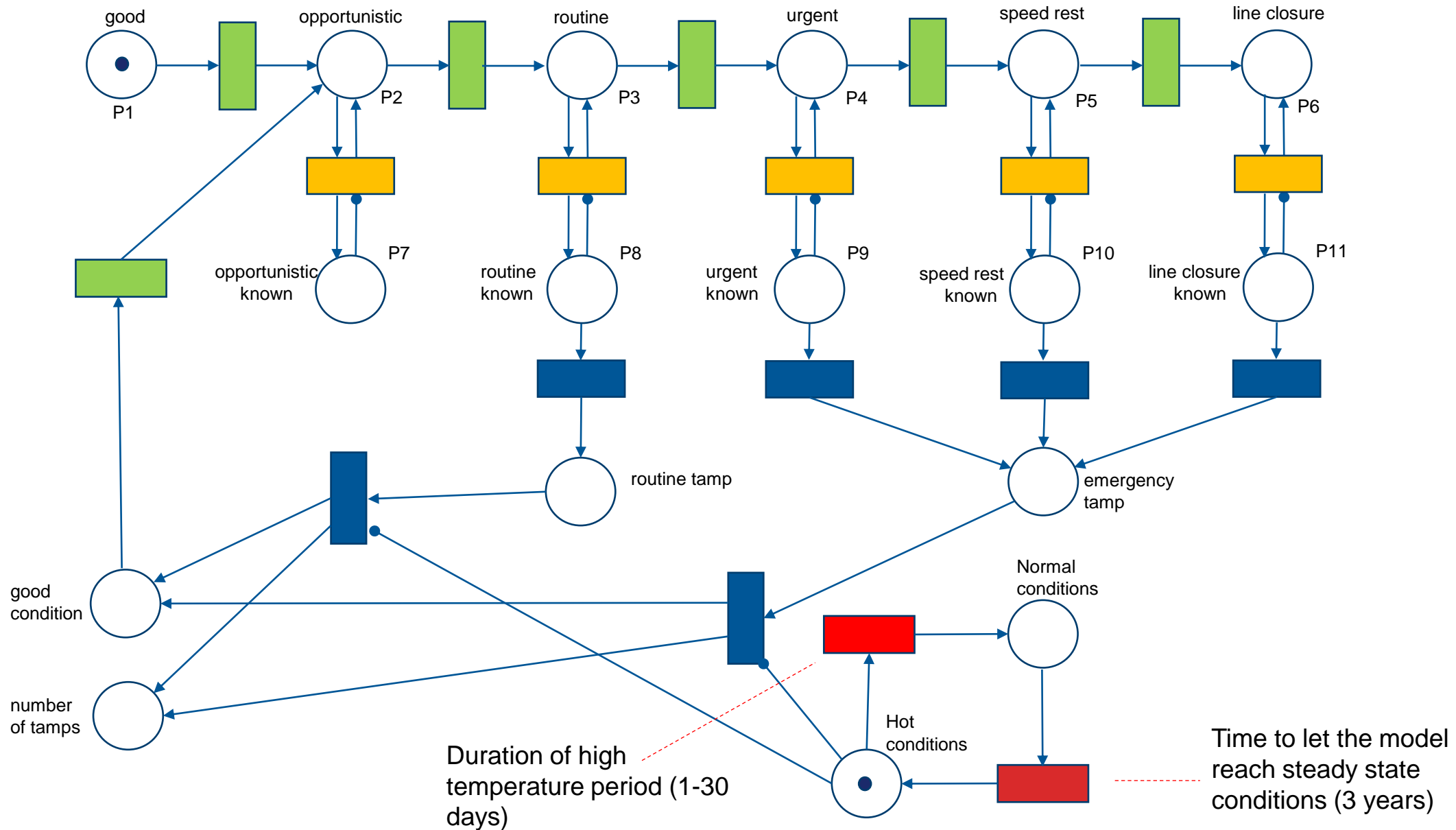
Anticipate, React, Recover, Resilient Infrastructure Systems, National Infrastructure Commission, May 2020



Engineering System Resilience



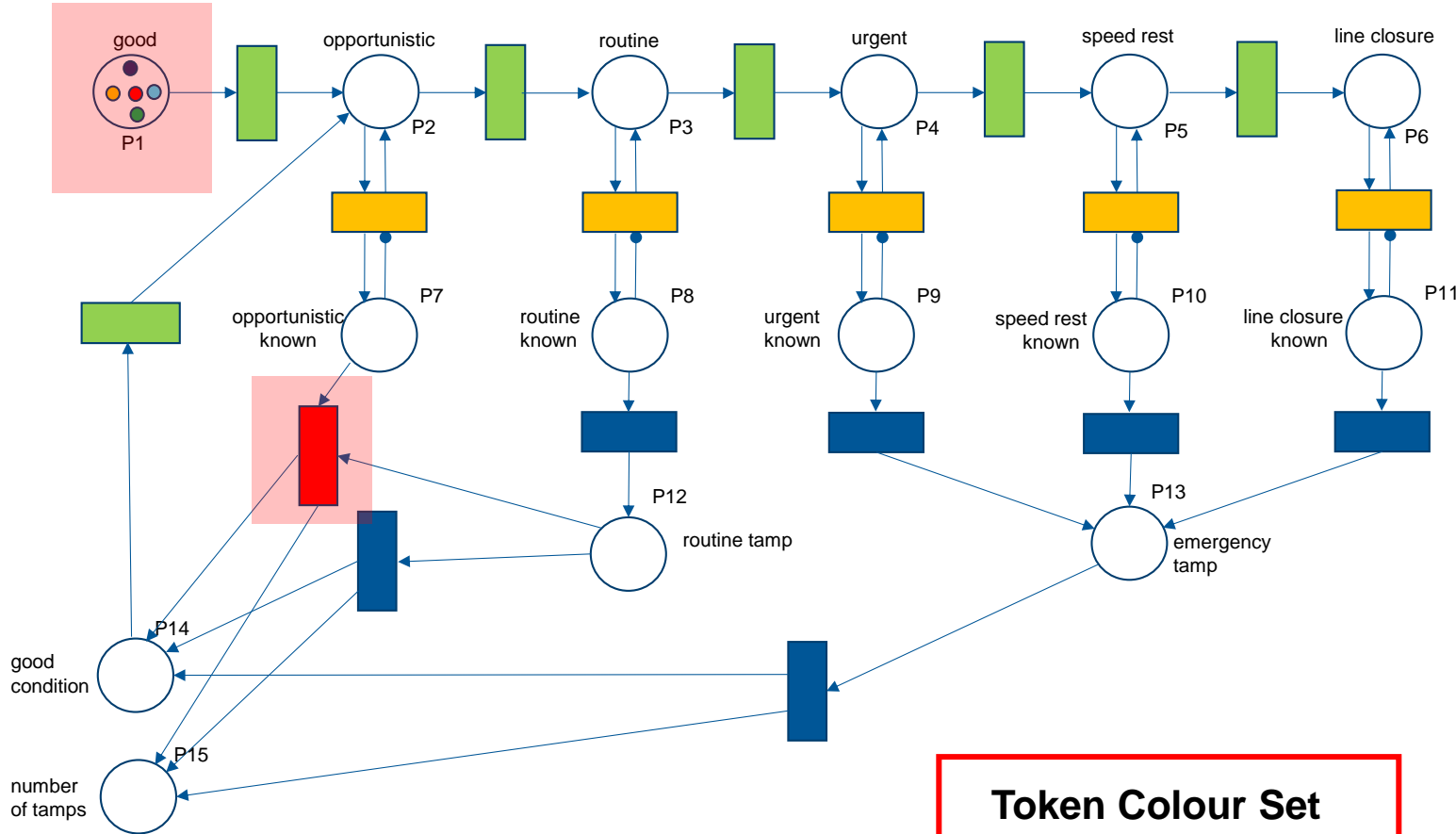
Modified Model – weather module



1280 track sections

Days into heatwave	Expected Number at full capacity	Expected number with speed restrictions	Expected number of line closures	Tamping backlog at end of heatwave	
				Routine	Urgent
0	1279.89	0.11	0.00	0.35	0.0
5	1279.23	0.77	0.00	2.56	2.22
10	1279.15	0.85	0.00	4.77	4.43
15	1278.89	1.11	0.00	7.26	4.45
20	1278.55	1.45	0.00	9.13	4.48
25	1278.46	1.54	0.00	11.61	4.51
30	1278.12	1.88	0.00	13.74	4.53

System / Route Model – Coloured Petri Nets



Token Colour Set

- Section ID
- Location
- Tamping history
- Time stamp

- Coloured tokens represent each section
 - localised transition parameters
 - transition times stored within the token
- Transition constantly receptive to firing.



- Simple example has been used to present the capabilities of Petri Net modelling approaches to support decisions on Railway Infrastructure Resilience Modelling
- The models are incredibly flexible and capable of:
 - mimicking the maintenance processes and strategies carried out no matter how complex.
 - applicable to a broad range of applications – such as climate change.
 - extension to include different failure modes:
 - twist, horizontal alignment, cyclic top, gauge
 - rail grinding and welding
 - other forms of maintenance – stone blowing / ballast cleaning
- Can be extended to include different asset types to produce a system or a route model – allowing a system level decision process

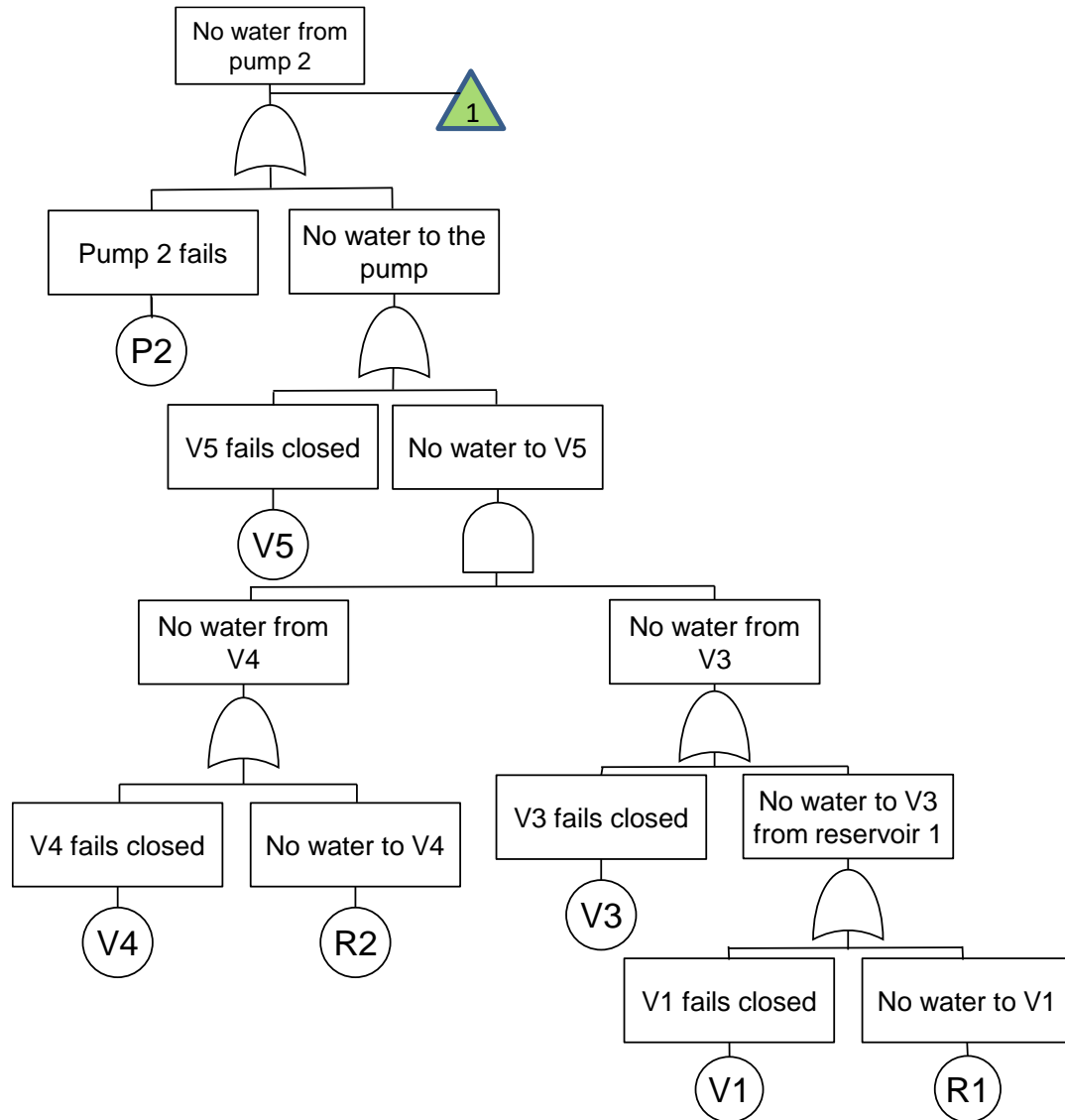


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Quantified Risk Assessment

Probabilistic Safety Assessment



Component failure models

- Limited maintenance process detail

- No Repair: $Q(t) = F(t) = 1 - e^{-\lambda t}$

- Revealed: $Q(t) = \frac{\lambda}{\lambda + \nu} (1 - e^{-(\lambda + \nu)t})$

- Unrevealed: $Q_{AV} = \lambda \left(\frac{\theta}{2} + \tau \right)$

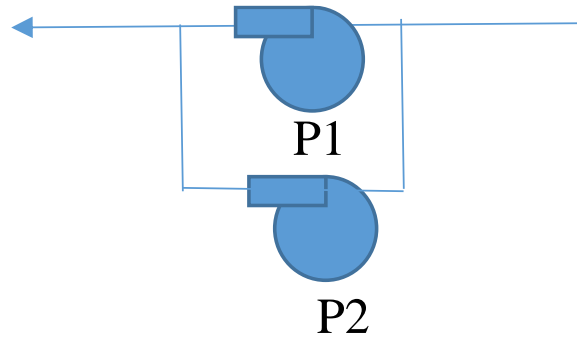
- Snap-shot in time

PROJECT AIMS

- Incorporate:
 - non-constant failure rates
 - dependent events
 - dynamic features
 - highly complex maintenance strategies



Standby Systems



Standby System

- Pump P1 operational.
- When P1 fails P2 takes over the duty

Hot Standby

Both pumps are operational but the fluid is just driven by P1. On failure of P1, the fluid now passes through P2

**P1 & P2
Independent**

Warm Standby

Pump P2 is not operational in standby. It becomes operational when P1 fails. It can fail in standby but with a lower rate than when operational.

P1 & P2 Dependent

Cold Standby

Pump P2 is not operational in standby. It becomes operational when P1 fails. It cannot fail in standby.

P1 & P2 Dependent



Dependency Examples

Type	Description	Example
Secondary Failure	When one component fails it increases the load on a second component which then experiences an increased failure rate	Two pumps both operational and sharing the load. Each pump has the capability to deliver the full demand should the other pump fail
Opportunistic Maintenance	<p>A component fails which causes a system shutdown or the requires specialist equipment for the repair.</p> <p>The opportunity is taken to do work on a second component which has not failed but is in a degraded state</p>	<p>Components on a circuit board.</p> <p>Components in a sub-sea production module</p>
Common Cause	When one characteristic (eg materials, manufacturing, location, operation, installation maintenance) causes the degraded performance in several components	<p>Incorrect maintenance done on several identical sensors</p> <p>Impact breaks the circuit on cables routed in the same way to different redundant channels</p>
Queueing	Failed components all needing the same maintenance resource are queued. Then repaired in priority order	Limited number of maintenance teams, equipment or spares

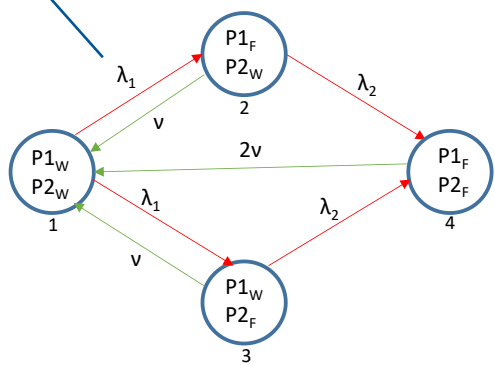
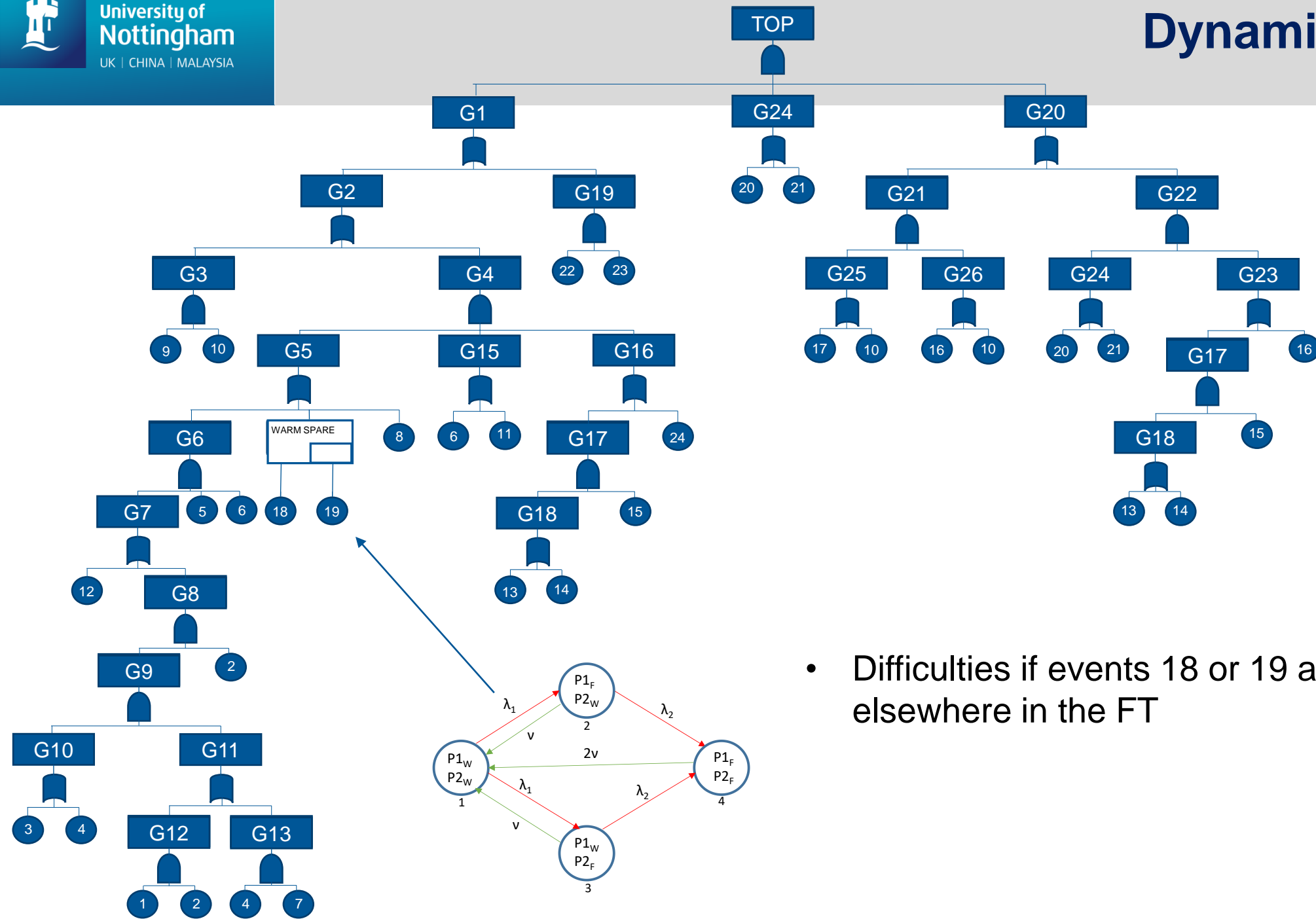


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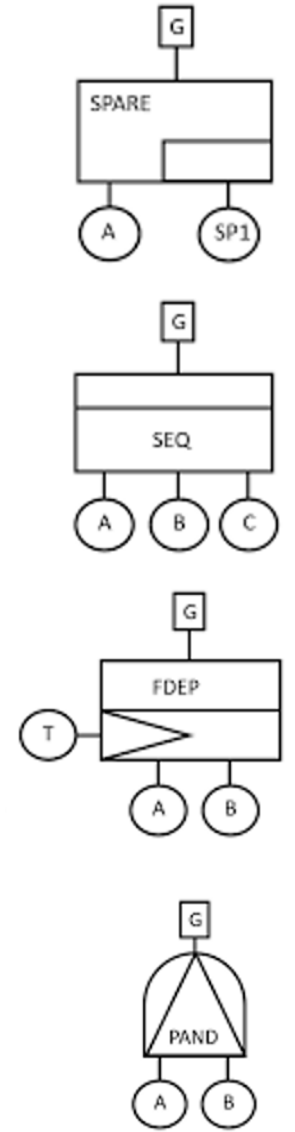
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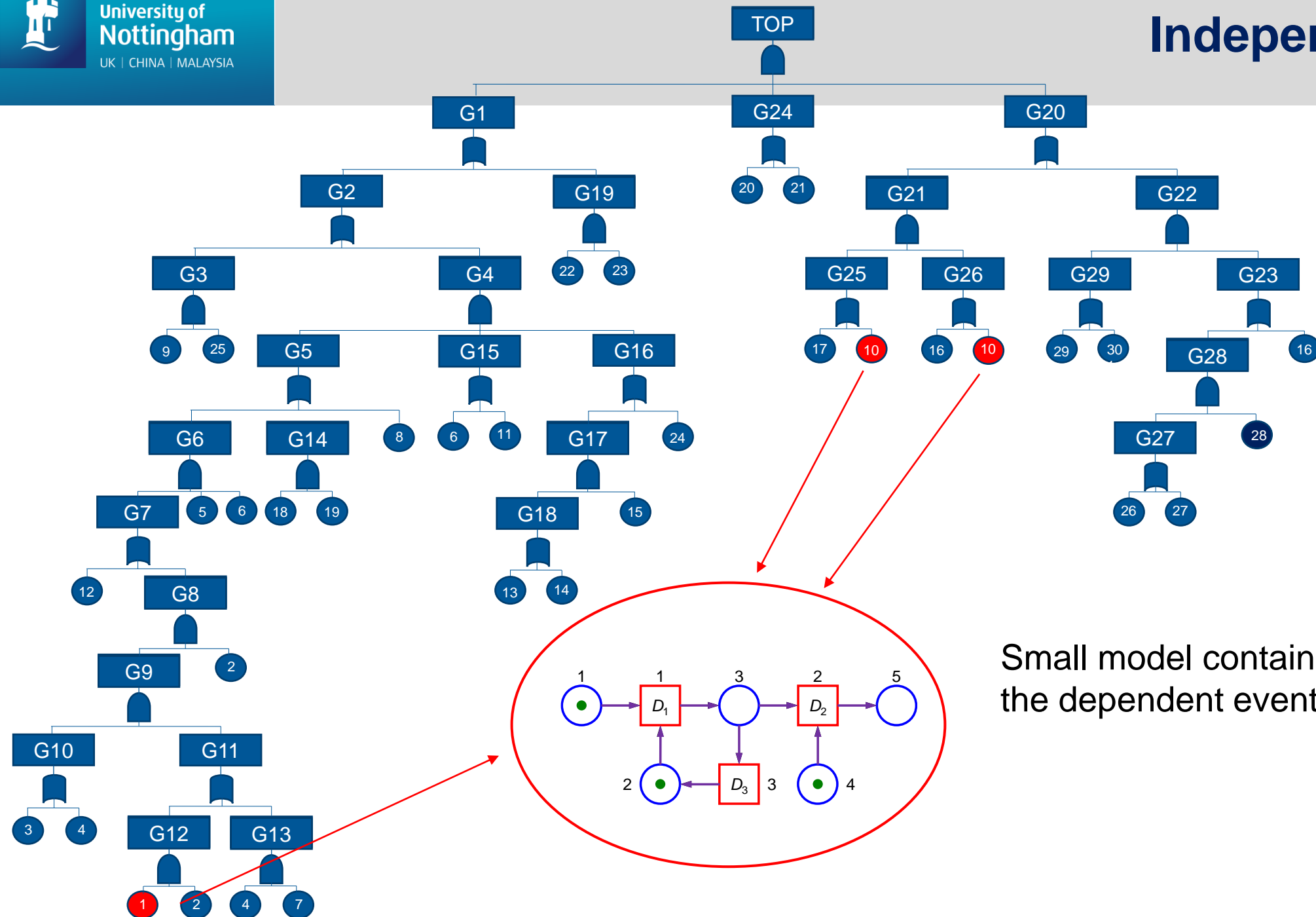
Dynamic & Dependent Tree Theory (D²T²)

A Fault Tree Analysis Framework



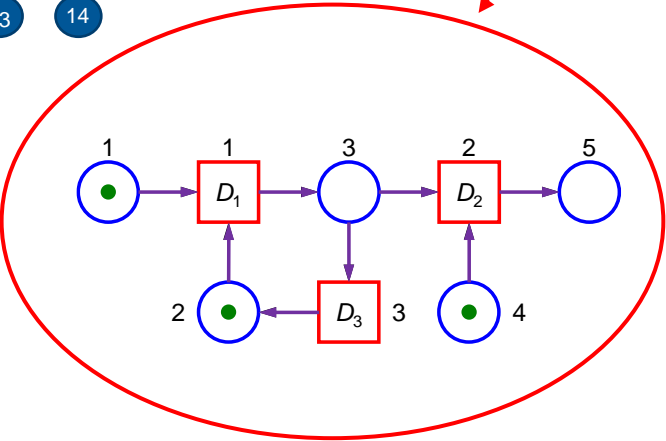
- Difficulties if events 18 or 19 appear elsewhere in the FT





Maintenance dependency's can affect events which are not geographically close in the FT structure

Small model containing only the dependent events





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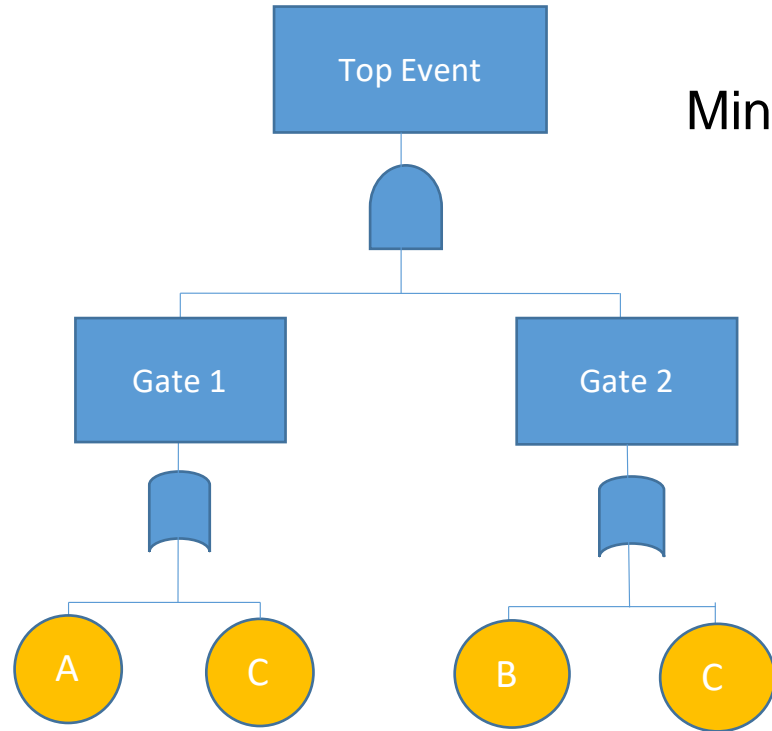
Integration of Fundamental Quantification Methodologies

Fault Tree Analysis => Binary Decision Diagrams (BDD)

Petri Nets

Markov Methods

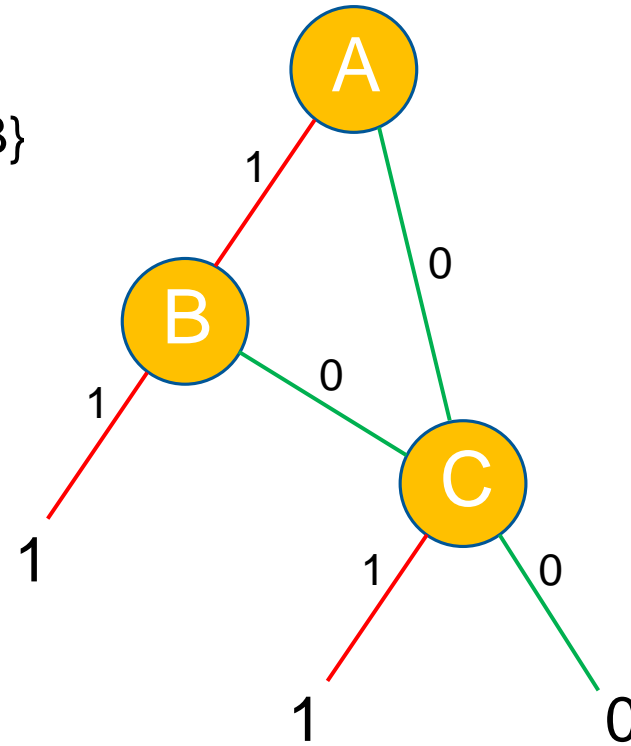
Binary Decision Diagrams – Top Event Probability



Min Cut Sets: {C}, {A, B}



ORDERING $A < B < C$



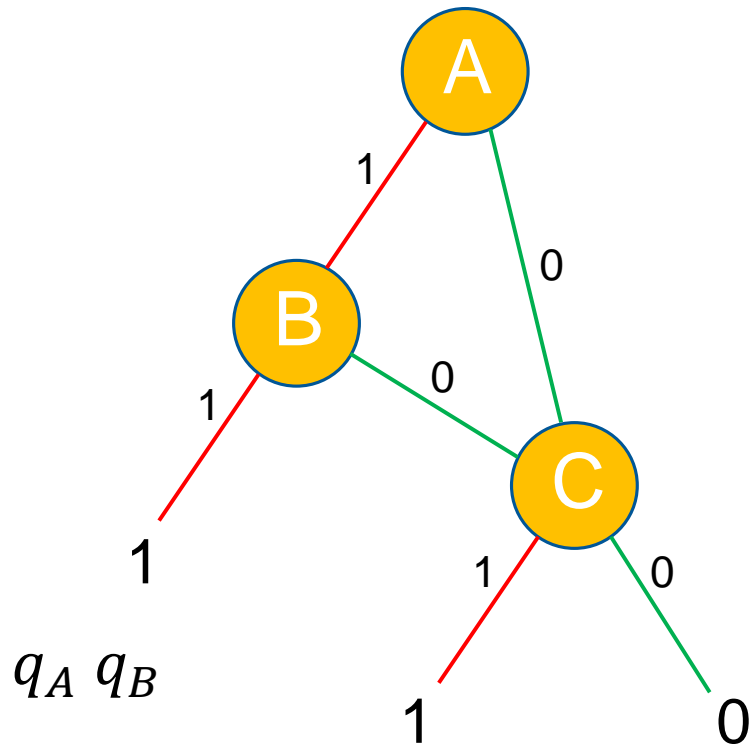
+ OR
· AND

$$TOP = A.B + C$$

$$TOP = A.B + A.\bar{B}.C + \bar{A}.C$$

$$Q_{SYS} = q_A q_B + q_C - q_A q_B q_C$$

Binary Decision Diagrams – Top Event Probability



$$q_A q_B$$

$$q_A(1 - q_B)q_C$$

$$+(1 - q_A) q_C$$

$$Q_{SYS} = q_A q_B + q_A(1 - q_B)q_C + (1 - q_A) q_C$$

$$= q_A q_B + q_C - q_A q_B q_C$$

- Exact
 - Fast
 - Efficient
- } No need to derive the Min Cut Sets as an intermediate step

***** Disjoint paths to failure *****



Dependencies

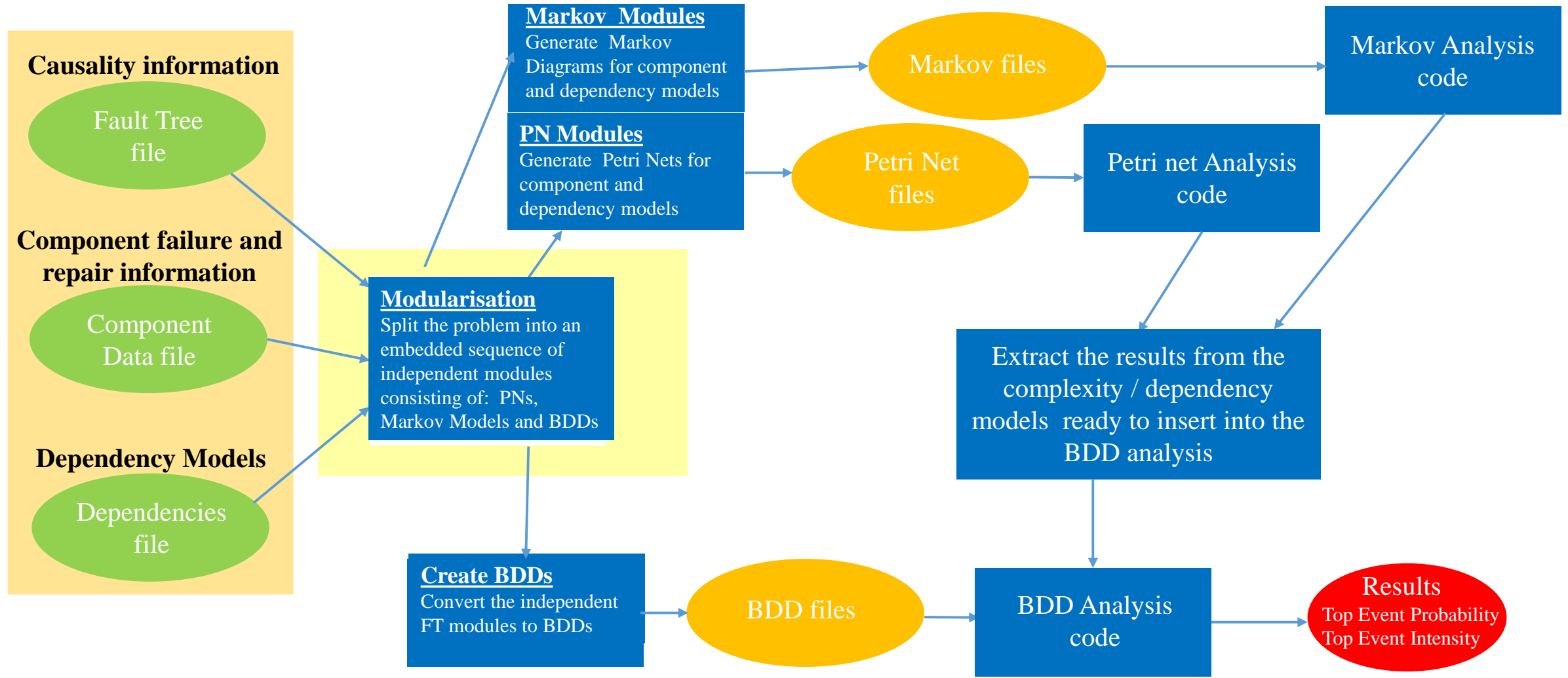
- Model the dependencies and complexities using Petri Nets or Markov models
 - Always use the *simplest dependency model*

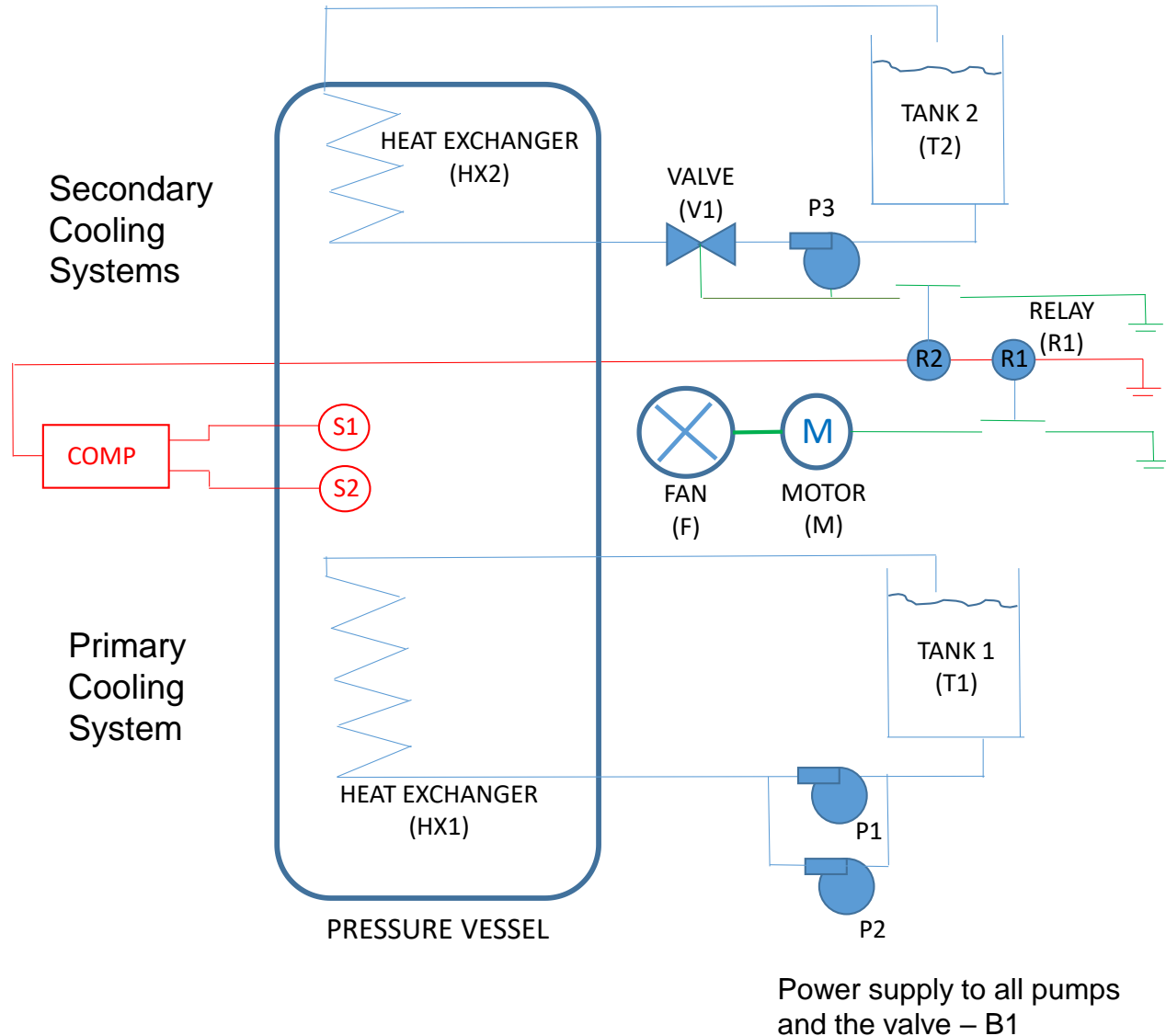
Binary Decision Diagrams

- Dependencies are just required to be considered on each path
- Path numbers can be very high so every effort needs to be made to *minimise the size of the BDD*
 - minimise the fault tree size using an effective modularisation
 - effective variable ordering



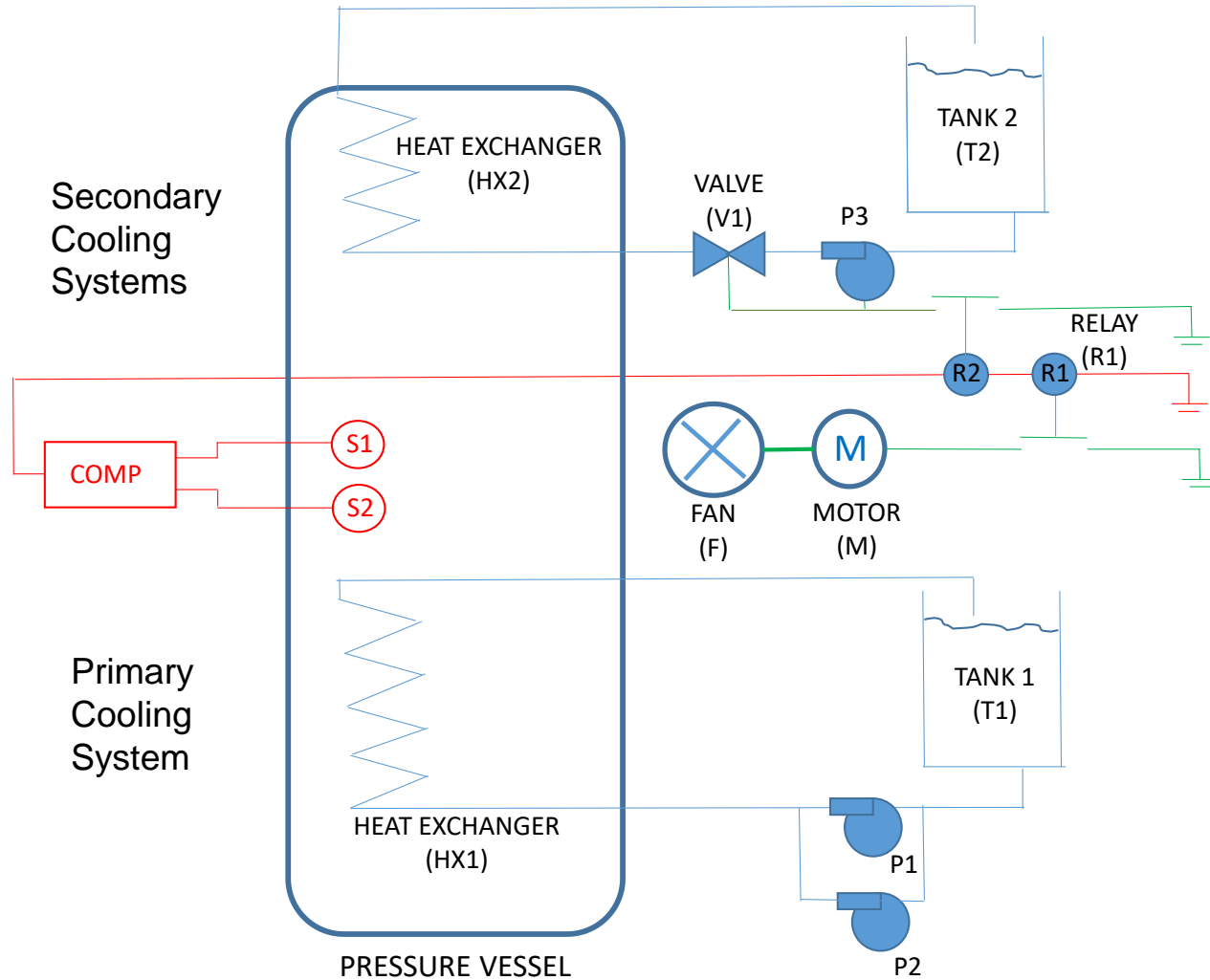
Basic Structure of the Code





Sub-Systems

- **Primary Cooling Water System**
 - Tank (T1), Pumps (P1,P2), Heat Exchanger (Hx1), Power Supply (B1)
- **Detection System**
 - Sensors (S1,S2), Computer (Comp)
- **Secondary Cooling Water System**
 - Tank(T2), Pump (P3), Heat Exchanger (Hx2), Valve (V1), Relay (R2), Power Supply (B1)
- **Secondary Cooling Fan System**
 - Fan (F), Motor (M), Relay (R1)



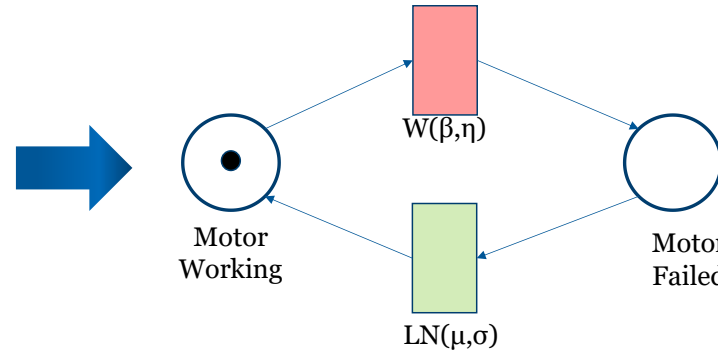
Power supply to all pumps and the valve – B1

Complex Features

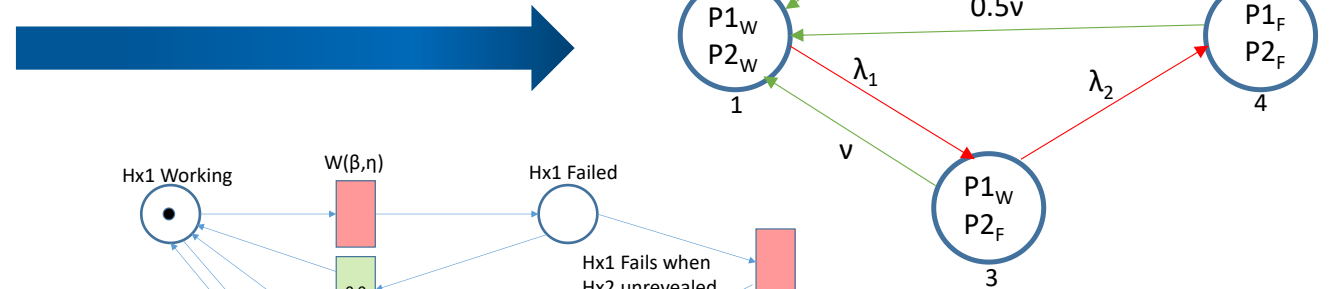
- **Non-constant failure / repair rates**
 - Motor M - Weibull failure time distribution and a lognormal repair time distribution
- **Dependencies**
 - Pumps P1 & P2 – if one fails it puts increased load (and increases the failure rate) of the other
 - Heat Exchangers Hx1 & Hx2 - when one needs replacement – needs specialist equipment and both are replaced
 - Pump P3 - two events P3S and P3R are clearly dependent

Complexity and Dependency Models

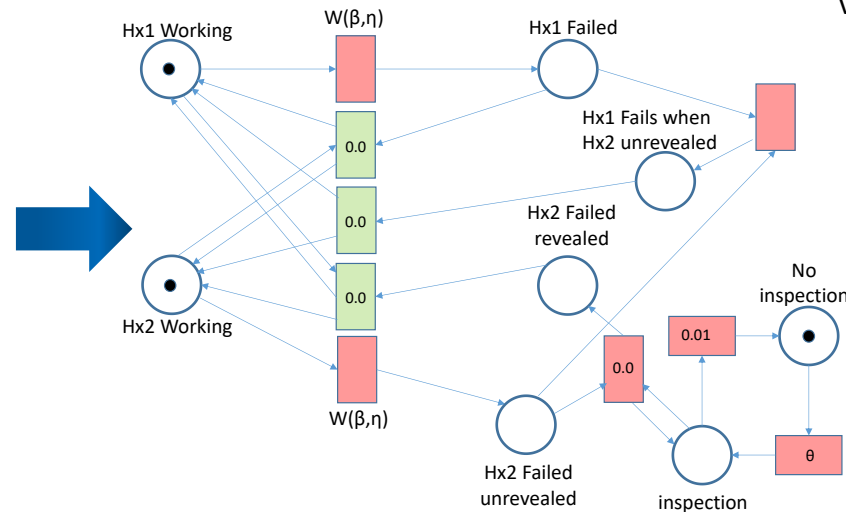
- **Non-constant failure / repair rates**
 - Motor M - Weibull failure time distribution and a lognormal repair time distribution



- **Dependencies**
 - Pumps P1 & P2 – if one fails it puts increased load (and increases the failure rate) of the other



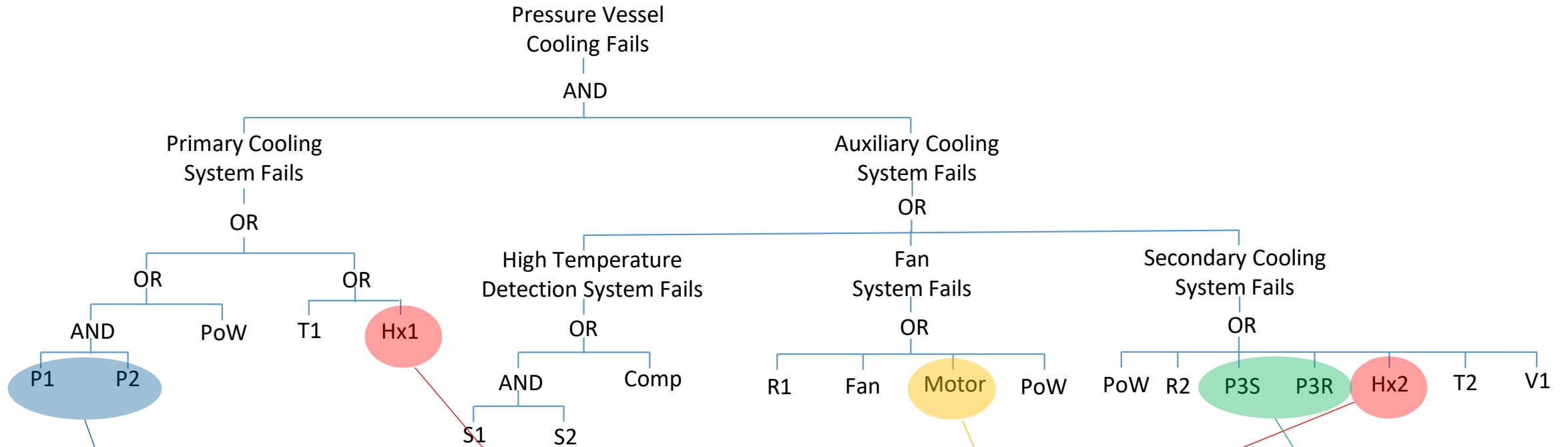
- Heat Exchangers Hx1 & Hx2 - when one needs replacement – needs specialist equipment and both are replaced



- Pump P3 - two events P3S and P3R are clearly dependent

$$\begin{aligned}
 q_{P3} &= q_{P3S} + (1.0 - q_{P3S})\lambda_{P3R}t_{period} \\
 &= 0.05 + 0.095 \times 10^{-4} \times 30 \\
 &= 0.05285
 \end{aligned}$$

Fault Tree Structure and Dependent Events



Pumps P1 & P2 – if one fails it puts increased load (and increases the failure rate) of the other

Heat Exchangers Hx1 & Hx2 - when one needs replacement – needs specialist equipment and both are replaced

Non-constant failure / repair rates

Pump P3 - two events P3S and P3R are clearly dependent

- **Contraction**

Subsequent gates of the same type are contracted into a single gate

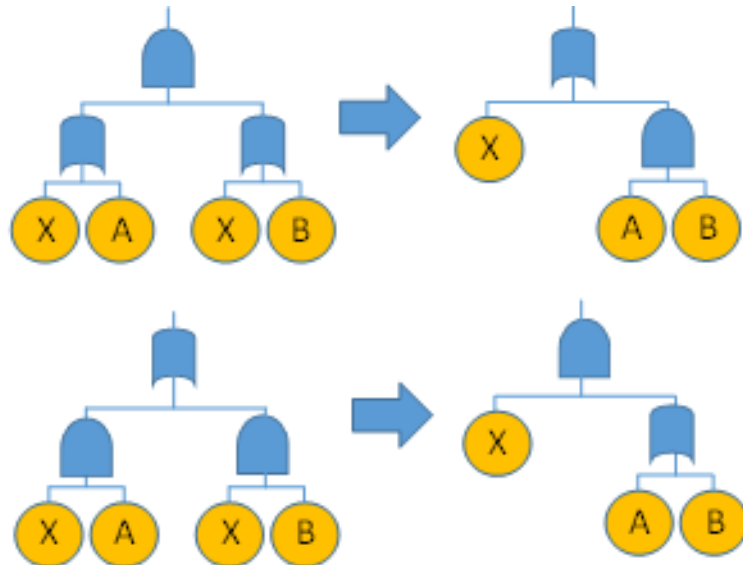
- **Factorisation**

Extracts factors expressed as groups of events that always occur together in the same gate type. The factors can be any number of events if they satisfy the following:

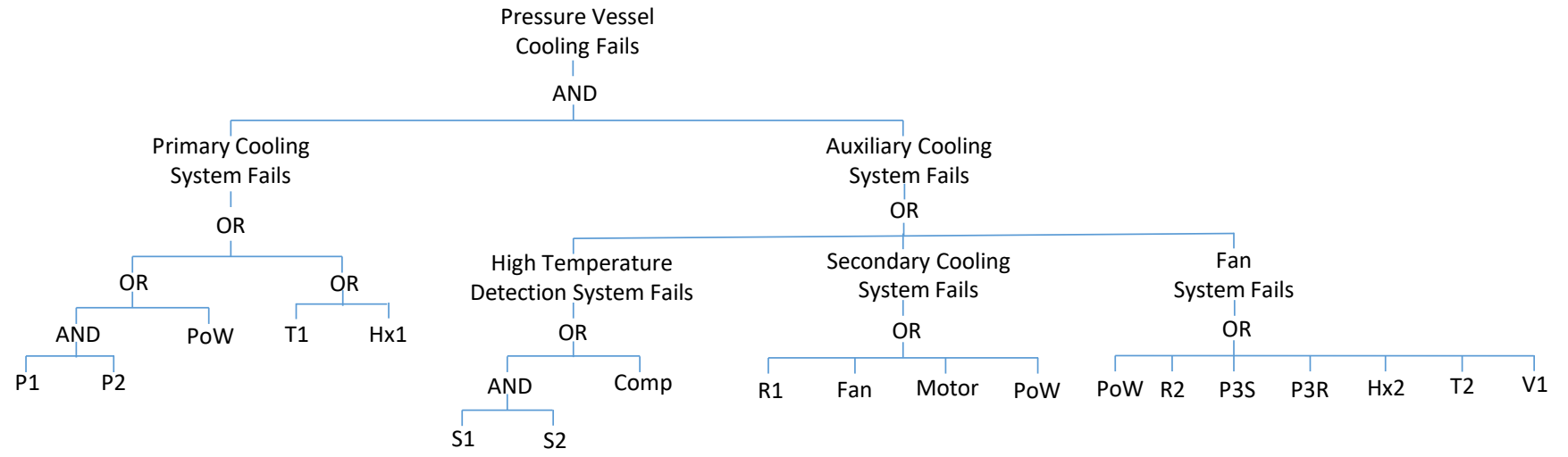
- All events in the group are independent and initiators
- All events in the group are independent and enablers.
- All events in the group feature a dependency and contain all events in the same dependency group.

- **Extraction**

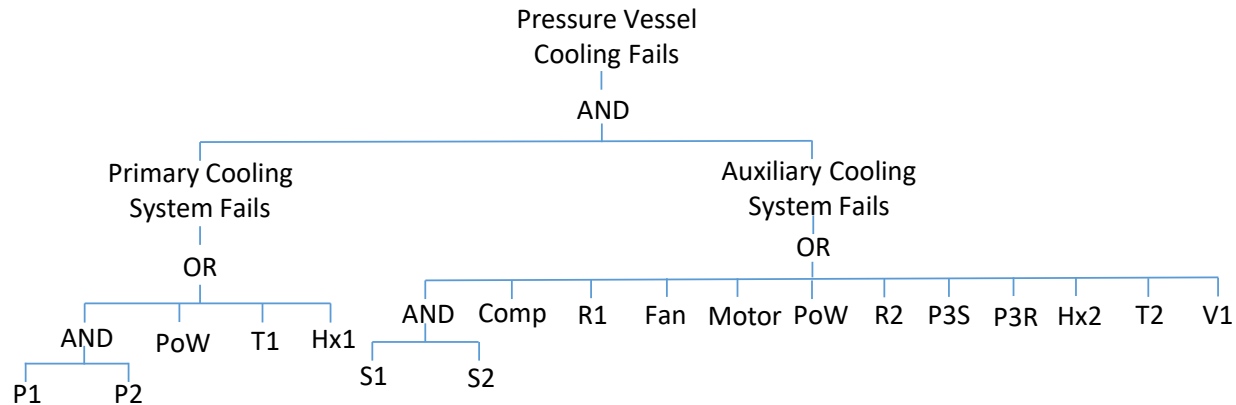
Restructure:



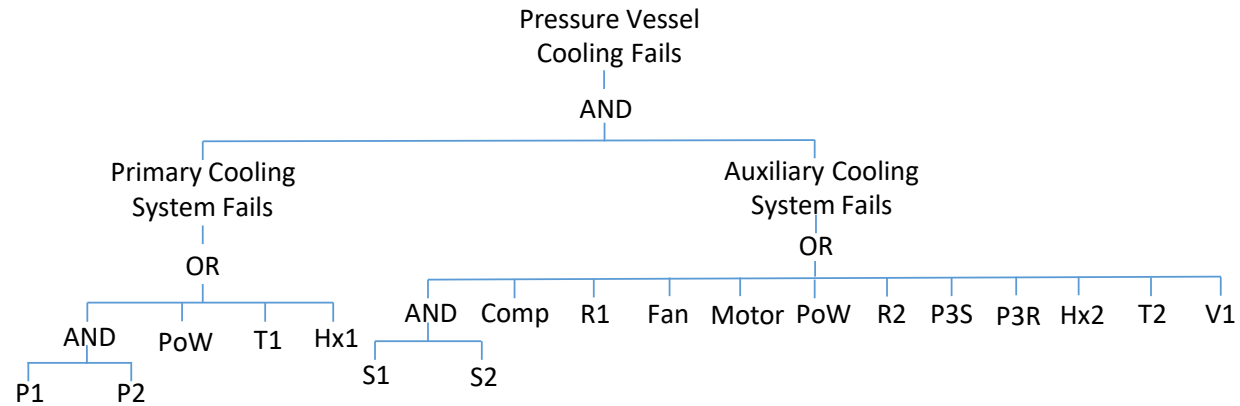
Modularisation (1)



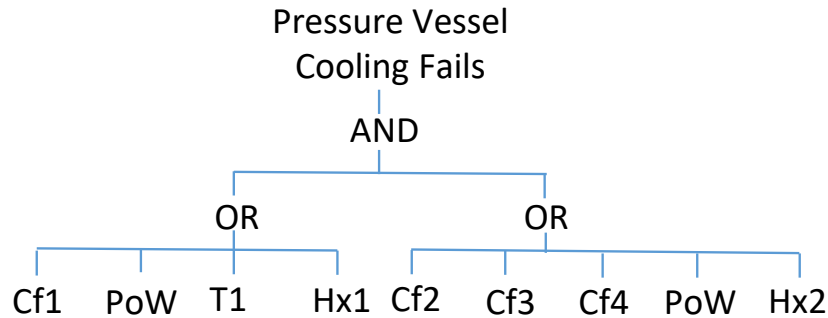
Contraction 1



Modularisation (2)



Factorise 1



$$Cf_1 = P1.P2$$

(dependency group D1 – initiators)

$$Cf_2 = S1.S2$$

(independent enablers)

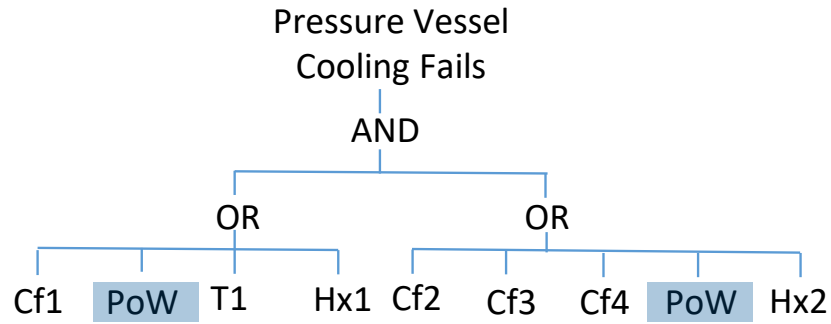
$$Cf_3 = Comp + R1 + Fan + Motor + R2 + T2 + V1$$

(independent enablers)

$$Cf_4 = P3S + P3R$$

(dependency group D3 – enablers)

Modularisation (3)



$$Cf_1 = P1.P2$$

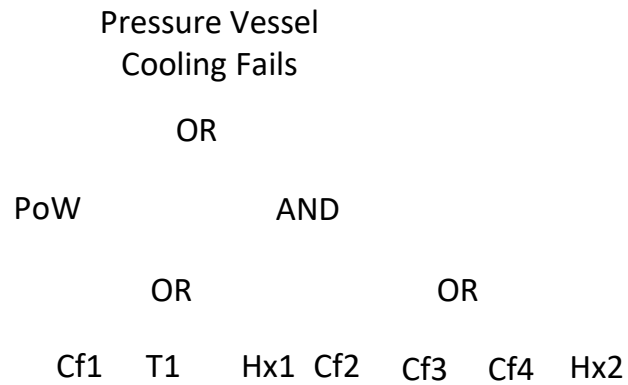
$$Cf_2 = S1.S2$$

$$Cf_3 = Comp + R1 + Fan + Motor + R2 + T2 + V1$$

$$Cf_4 = P3S + P3R$$

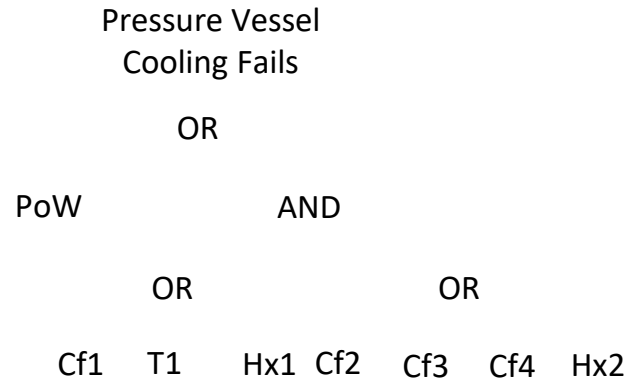


Extract 1



Contraction 2 -- No change

Modularisation (4)



$$Cf_1 = P1.P2$$

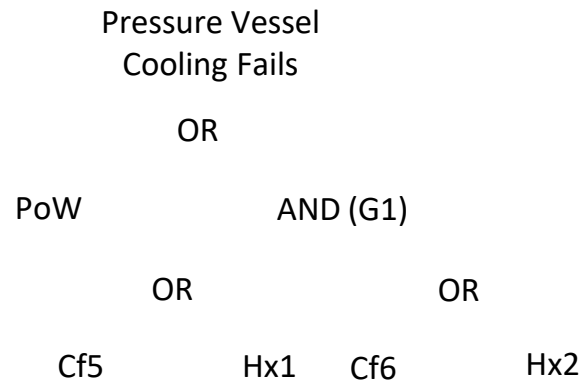
$$Cf_2 = S1.S2$$

$$Cf_3 = Comp + R1 + Fan + Motor + R2 + T2 + V1$$

$$Cf_4 = P3S + P3R$$



Factorise 2

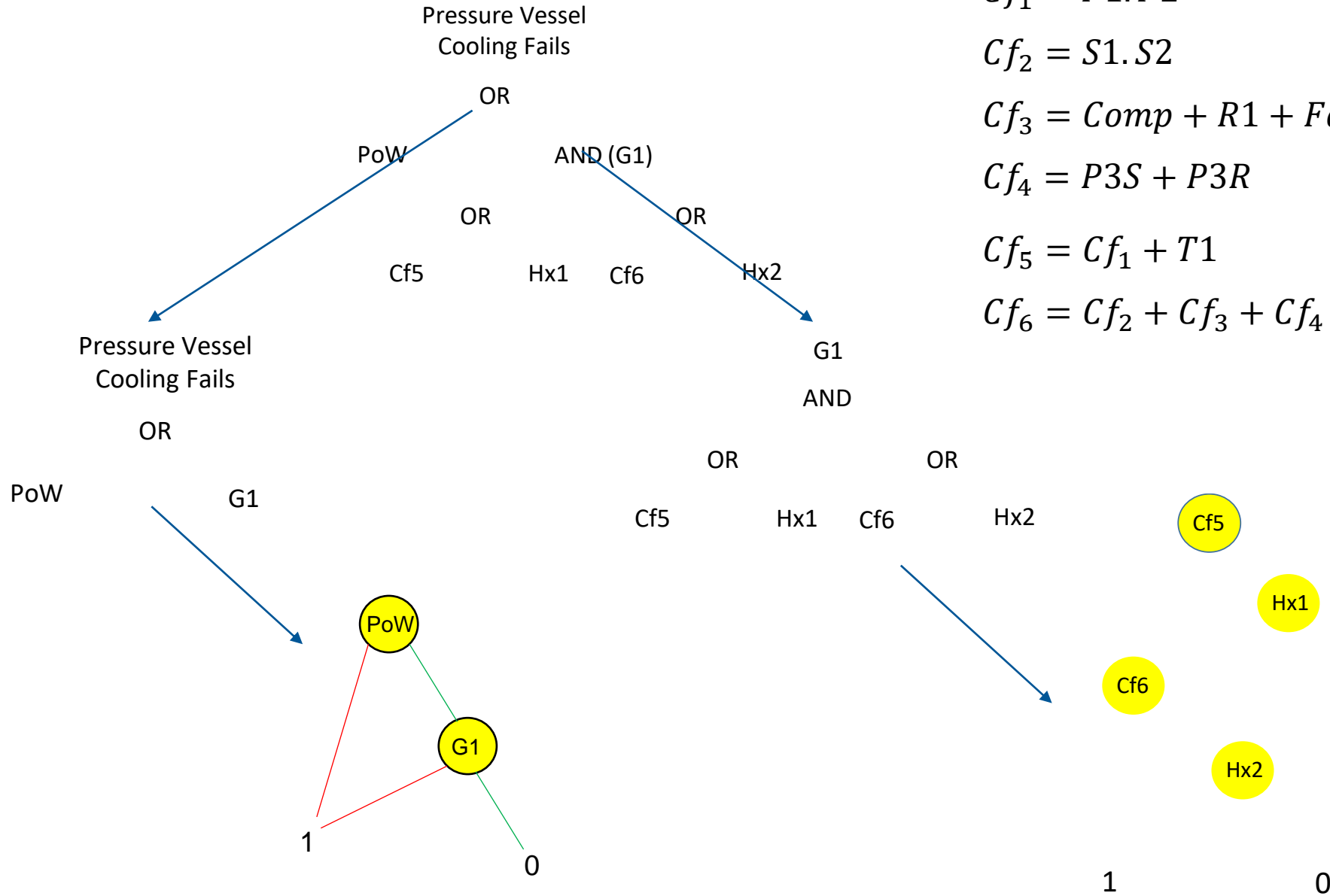


$$Cf_5 = Cf_1 + T1$$

$$Cf_6 = Cf_2 + Cf_3 + Cf_4$$

Simplest possible Faunet representation

Modularisation (5) - Rauzy & Dutuit



$$Cf_1 = P1.P2$$

$$Cf_2 = S1.S2$$

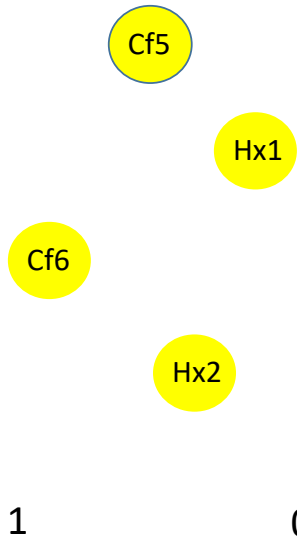
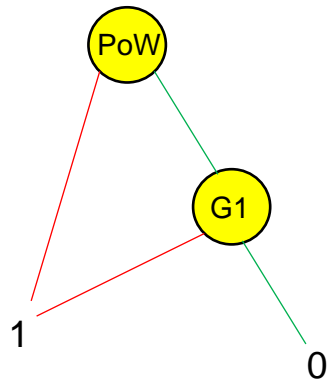
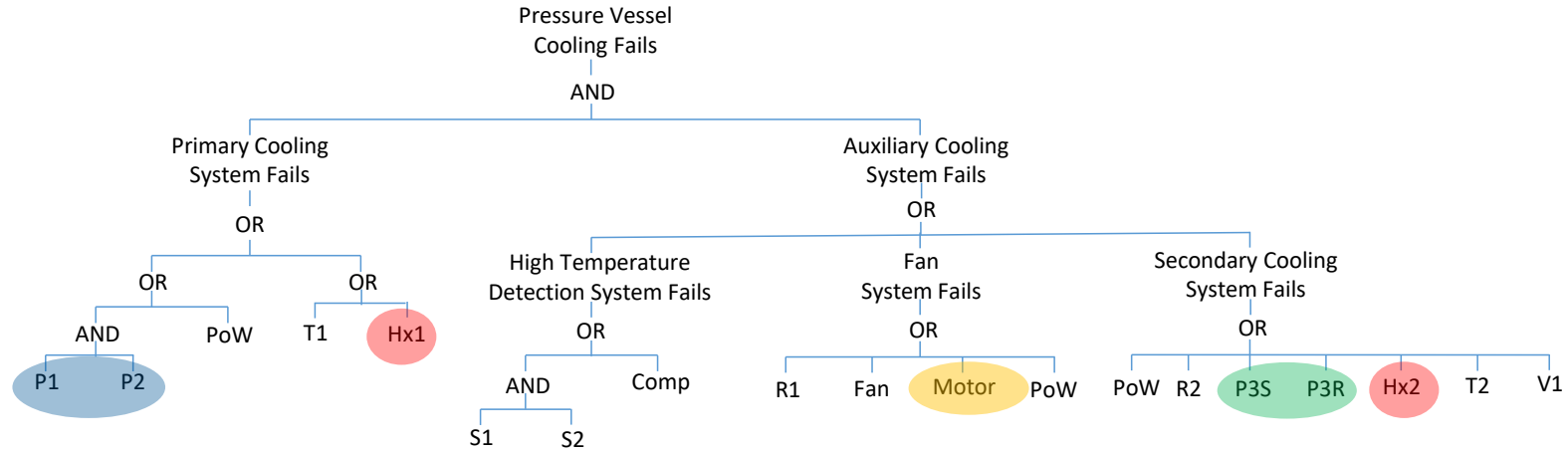
$$Cf_3 = Comp + R1 + Fan + Motor + R2 + T2 + V1$$

$$Cf_4 = P3S + P3R$$

$$Cf_5 = Cf_1 + T1$$

$$Cf_6 = Cf_2 + Cf_3 + Cf_4$$

Modularisation



$$Cf_1 = P1.P2$$

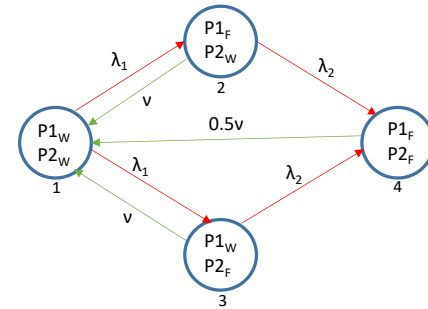
$$Cf_2 = S1.S2$$

$$Cf_3 = Comp + R1 + Fan + Motor + R2 + T2 + V1$$

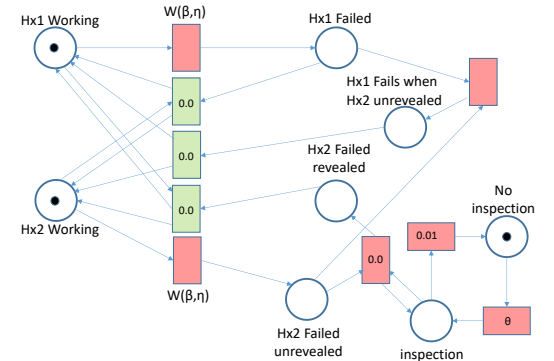
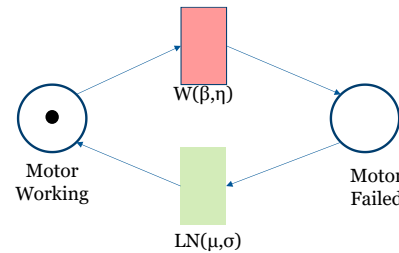
$$Cf_4 = P3S + P3R$$

$$Cf_5 = Cf_1 + T1$$

$$Cf_6 = Cf_2 + Cf_3 + Cf_4$$

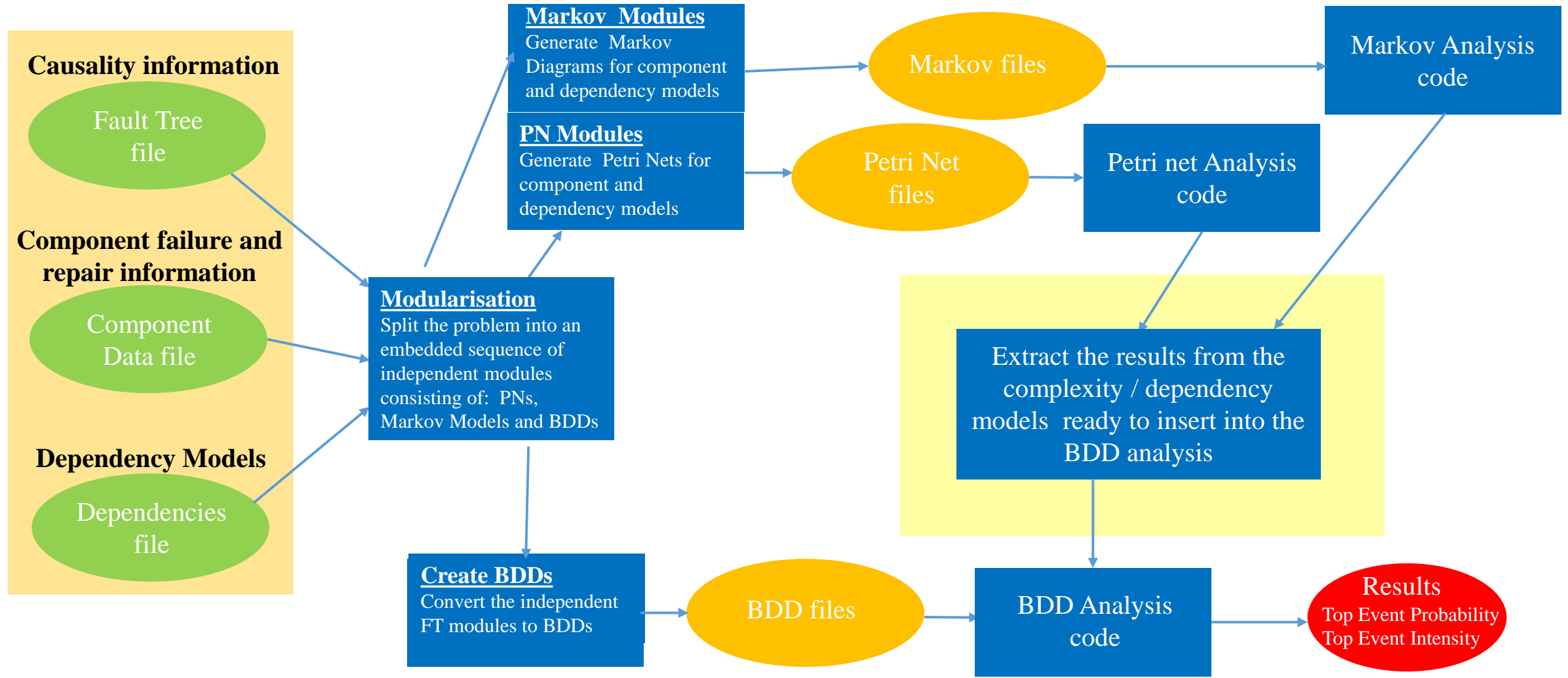


$$q_{cf3} = q_{P3S} + (1.0 - q_{P3S})\lambda_{P3R}t_{period}$$





Basic Structure of the Code



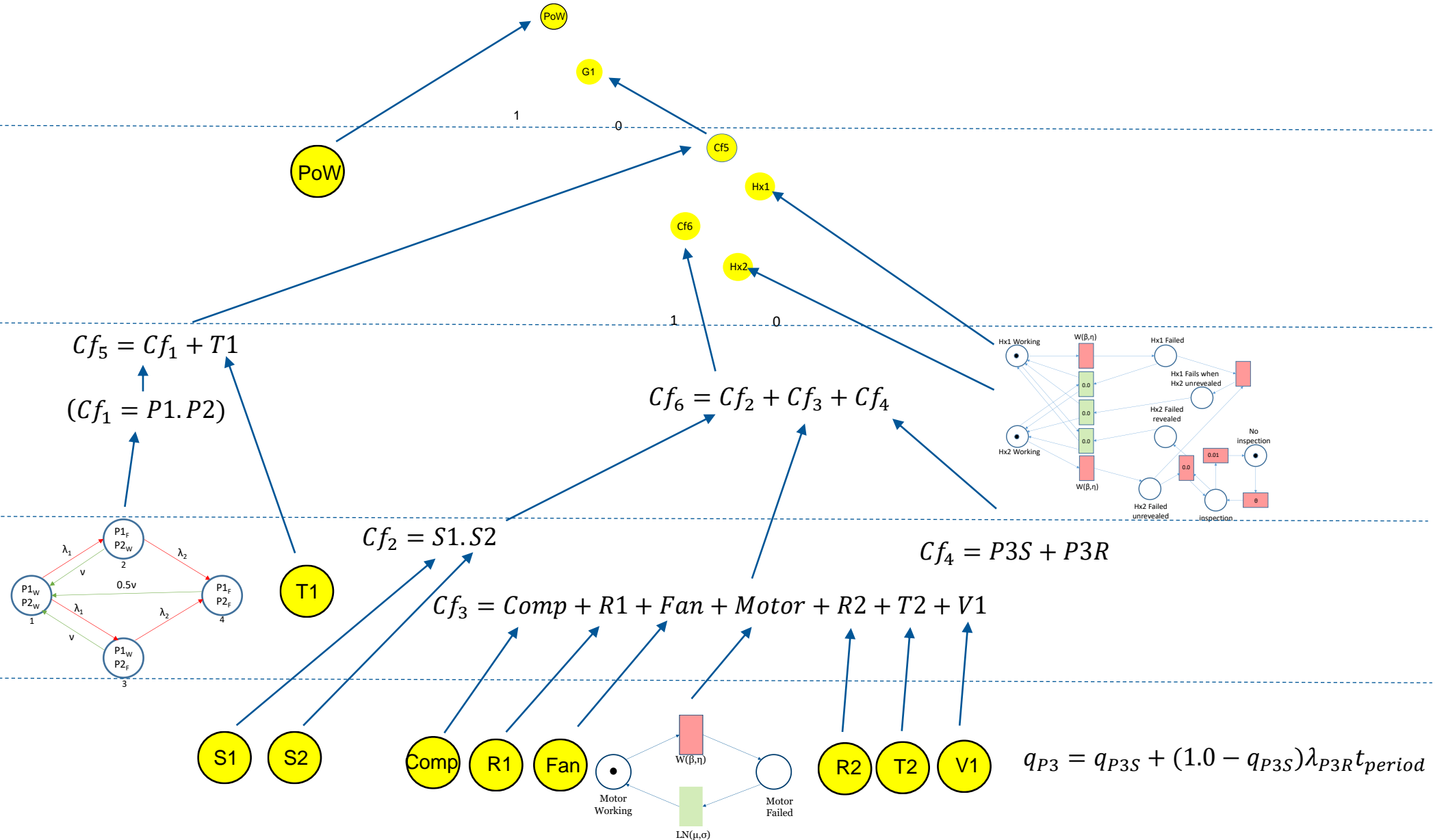
Level 0

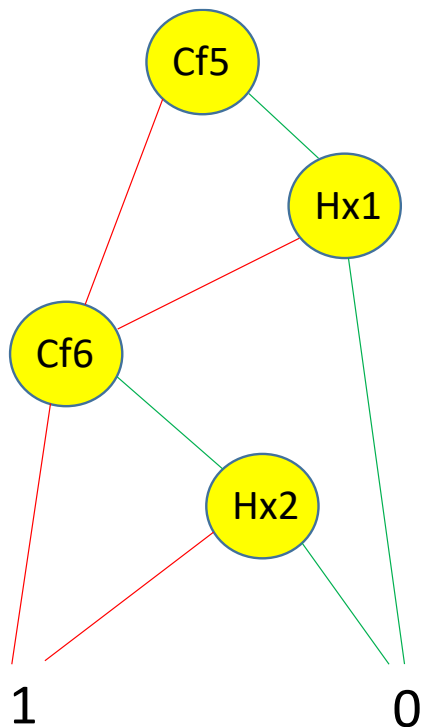
Level 1

Level 2

Level 3

Level 4





j	$path_j$	$lpath_j$	$Dpath_j^1$
1	$Cf5_1, Cf6_1$	$Cf5_1, Cf6_1$	
2	$Cf5_1, Cf6_0, Hx2_1$	$Cf5_1, Cf6_0$	$Hx2_1$
3	$Cf5_0, Hx1_1, Cf6_1$	$Cf5_0, Cf6_1$	$Hx1_1$
4	$Cf5_0, Hx1_1, Cf6_0, Hx2_1$	$Cf5_0, Cf6_0$	$Hx1_1, Hx2_1$

$$Q_{G1} = \sum_{j=0}^{npath} \left[P(lpath_j) \cdot \prod_{k=1}^{ndep} P(Dpath_j^k) \right]$$

$$Q_{path1} = P(Cf5_1) \cdot P(Cf6_1) = 0.0010830$$

$$Q_{path2} = P(Cf5_1) \cdot (1 - P(Cf6_1)) \cdot P(Hx2_1) = 8.8052957 \times 10^{-6}$$

$$Q_{path3} = (1 - P(Cf5_1)) \cdot P(Cf6_1) \cdot P(Hx1_1) = 0.0$$

$$Q_{path4} = (1 - P(Cf5_1)) \cdot (1 - P(Cf6_1)) \cdot P(Hx1_1, Hx2_1) = 0.0$$

$$Q_{G1} = 0.00109175$$



- Top Event Frequency Calculations
- Qualitative FTA – remains unchanged
- Importance measures
- Large FTA calculations
- Event Tree Analysis



- Dynamic and Dependent Tree Theory, D²T², enables the evaluation of fault trees which are not limited by the restrictions which apply to conventional fault trees solved by Kinetic Tree Theory.
- Retains the familiar and popular fault tree causality structure.
- Utilises BDDs, Petri Nets and Markov Models.
- The Petri net and Markov models dedicated to solve the complexities and dependencies are minimal in size.
- Modularisation of the fault tree minimises the size of the BDD utilised in the system evaluation (and therefore the number of paths).



Thank you for listening – any questions ?

Professor John Andrews
Faculty of Engineering
University of Nottingham

john.andrews@nottingham.ac.uk