Extension of Common Measures of Importance for Dynamic and Dependent Tree Theory (D^2T^2)

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Extended Abstract

Traditional Fault Tree Analysis (FTA), known as Kinetic Tree Theory (KTT), was derived by Vesely [1] in the 1970s to model and analyse engineering systems. The tree structure provides a clear visual representation of the causes of system failure in terms of component and software failures and human errors. FTA has 2 stages, qualitative analysis which involves identifying the necessary and sufficient causes of system failure, i.e., the minimal cut sets and quantitative analysis which involves calculating the system unavailability, system failure frequency and measures of importance.

When assessing a system, its performance is dependent upon that of its components. Certain components or minimal cut sets will play a more significant role in causing or contributing to system failure than others. The contribution that a component or a minimal cut set makes to system failure is known as its importance. The concept of importance was first introduced by Birnbaum in 1969 [2], since this time, numerous measures of importance have been developed to assess the different roles that a component failures or minimal cut sets can play in the deterioration of the system state. Measures of importance can be categorised as either deterministic or probabilistic and assign a value between 0 and 1 to each component or minimal cut set, with 1 signifying the highest level of contribution. Deterministic measures of importance are also known as structural measures of importance, and they assess the importance of a component or minimal cut set without taking account of the reliability of the component(s). Probabilistic measures of importance take component failure probabilities and intensities into account and are, therefore, far more useful than deterministic measures in practical reliability problems.

Importance analysis enables engineers to rank the contribution each component or minimal cut set makes to system failure. In this way, weaknesses within the system can be identified and resources can be used most efficiently to improve system reliability. This paper will focus on two measures, Birnbaum's Measure of Component Importance, and the Criticality Measure of Component Importance.

Birnbaum's measure of importance is denoted by $G_i(q)$, and defined as the probability that component i is critical to systems failure, i.e., the system is in a working state such that the failure of component i causes it to fail. An expression for this measure is given in equation 1:

$$G_i(q) = Q_{sys}(1_i, q) - Q_{sys}(0_i, q) \tag{1}$$

Where, Q_{sys} is the system unavailability function and $Q_{sys}(1_i, q)$ is the probability that the system fails with component i failed and $Q_{sys}(0_i, q)$ is the probability that the system fails with component i working. An alternative expression for this measure is given in equation 2:

$$G_i(q) = \frac{\delta Q_{sys}(t)}{\delta q_i(t)} \tag{2}$$

The criticality measure of importance is defined as the probability that component i is critical to the system and has failed, weighted by the system unavailability at time t. An expression for this measure for systems involving only independent basic events is given in equation (3)

$$I_{C_i} = \frac{G_i(q) \cdot q_i(t)}{Q_{sys}(t)} \tag{3}$$

Both of these measures can be efficiently calculated during FTA, however, there are a few major limitations of FTA. Specifically, the necessary assumptions of constant component failure and repair rates and independence of component failure and repair rates. Neither assumption is appropriate for modern engineering systems. In 2023 Andrews and Tolo [3] published the D^2T^2 methodology designed specifically to address these limitations in the most efficient manner. The methodology retains the tree structure and combines the use of BDDs, Markov Methods and SPN models to analyse engineering systems involving non-constant failure and repair rates and component dependencies as efficiently as possible. The methodology ensures that no matter how far apart the dependent events are in the tree structure, the dependency model only features these components. As such, the dependency models are minimal, maximising efficiency.

The D^2T^2 methodology is a multi-layer methodology which culminates in a final BDD for the top gate of the Fault Tree. A variety of forms can feed into this BDD, and each form can have a variety of inputs too. Figure 1 illustrates the possible inputs for each element of a system.

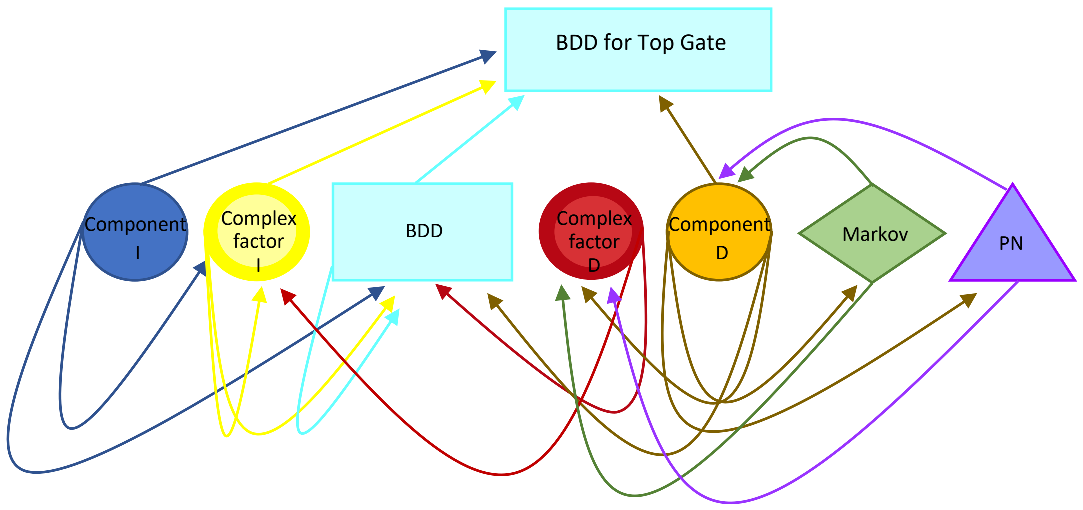
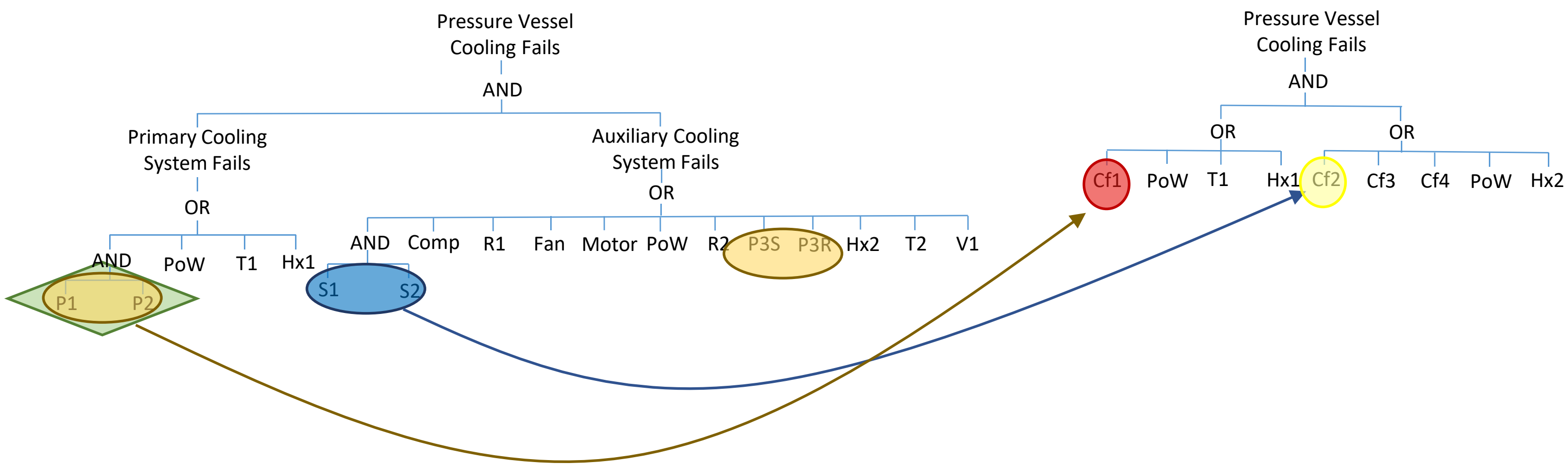


Figure 1: Illustration of possible inputs for a system analysed using the D^2T^2 methodology

Consider the Pressure Vessel Cooling System case study introduced previously by Andrews and Tolo, the Fault Tree for this system is shown in figure 2.



At present, the methodology enables the calculation of both the system unavailability and the system failure frequency. However, in order to obtain the full range of output, the methodology needs to be extended to calculate measures of importance.

This paper suggests an extension that enables the calculation of the following key measures of importance:

- Birnbaum's measure of importance
- The Criticality Measure of Importance

References

1. Vesely W.E., (1970), A Time Dependent Methodology for Fault Tree Evaluation, Nuclear Engineering and Design, Vol 13, pp337-360.
2. Andrews, J.D, Tolo, S., (2023), Dynamic and Dependent Tree Theory (D^2T^2): A Framework for the Analysis of Fault Trees with Dependent Basic Events,