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Towards autonomous digital twins:  
uncertainty, data, computing, simulation, and ethics

ESRA Symposium

University of Strathclyde

Glasgow, UK

30<sup>th</sup> November – 1<sup>st</sup> December

2023

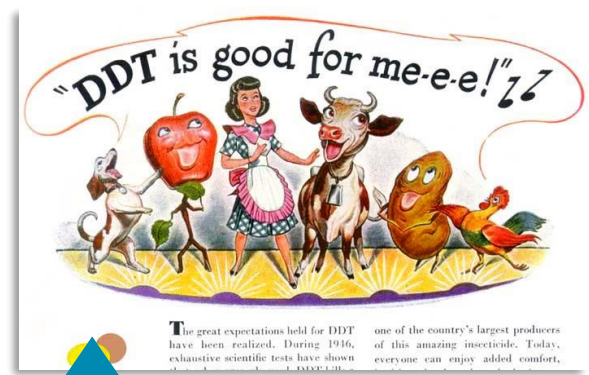
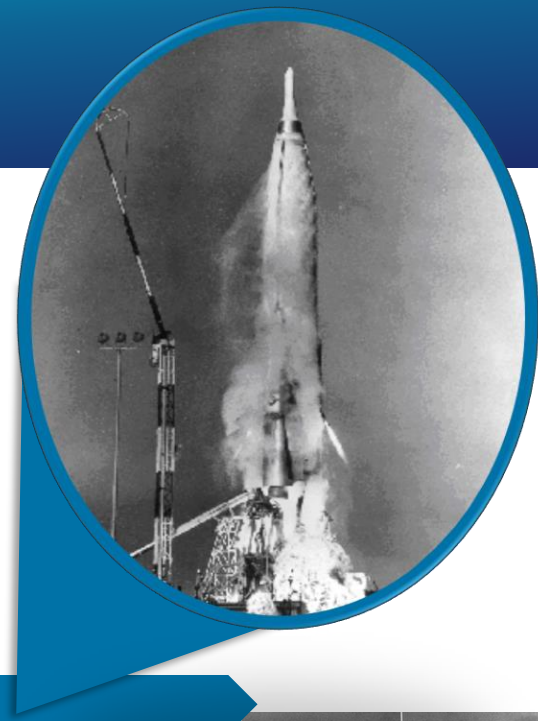
# Modelling dependencies in complex systems: Dynamic and Dependent Tree Theory (D<sup>2</sup>T<sup>2</sup>)

Dr Silvia Tolo



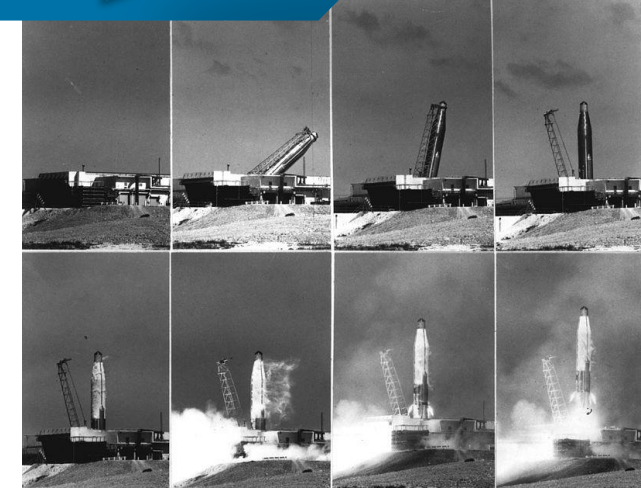
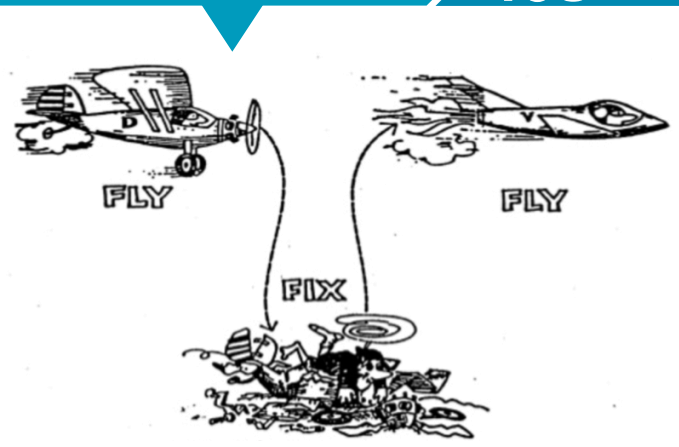
Foundation

# Why system safety?

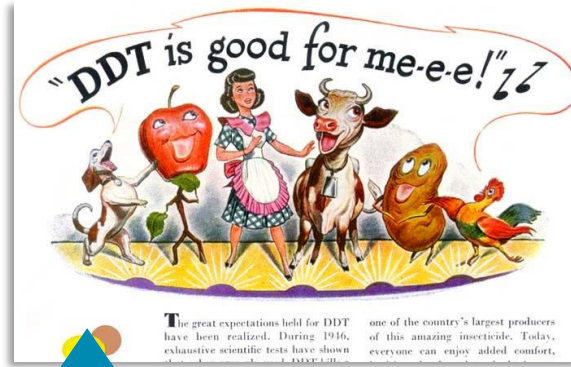
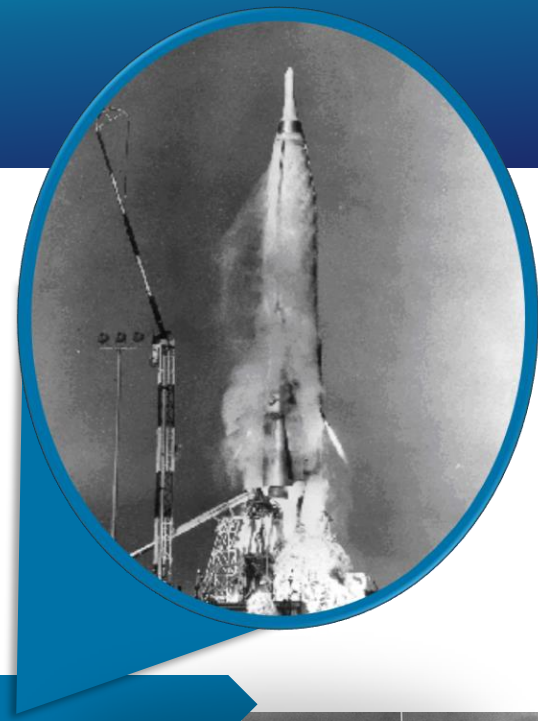


40s

50s

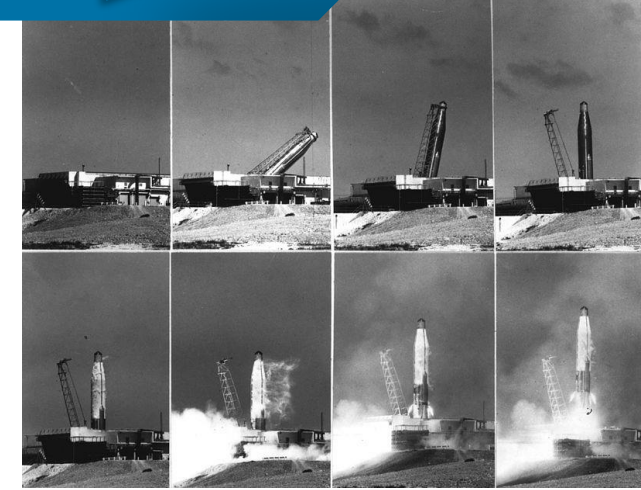
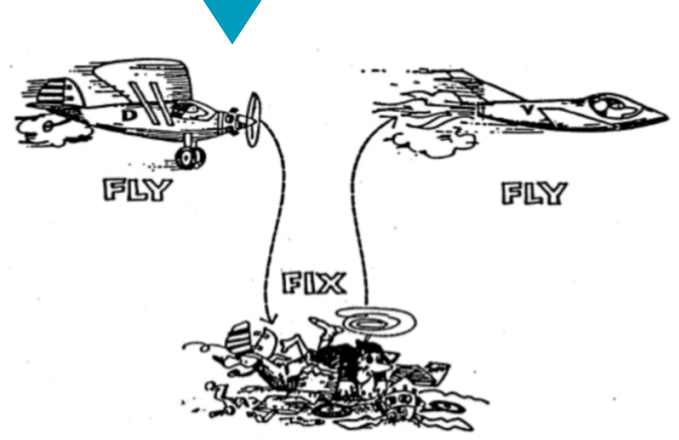


# Why system safety?



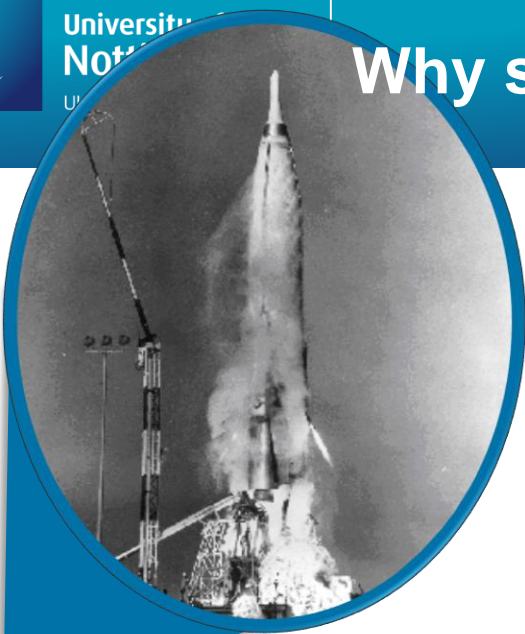
40s

50s

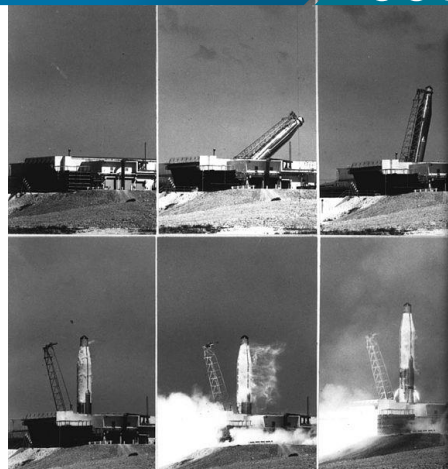




# Why system safety?



60s



**WEATHER**  
Forecast for Tucson generally fair, little change in temperatures.  
Yesterday: HIGH 74 LOW 56  
Year Ago: HIGH 85 LOW 59  
U.S. WEATHER BUREAU

**The Arizona Daily Star** FINAL  
★ An Independent Newspaper Printing The News Impartially ★  
TUCSON, ARIZONA, SATURDAY MORNING, JANUARY 28, 1967 622 1965 • THIRTY PAGES

**Apollo Training Craft Explodes**

**ASTRONAUTS DIE IN FIERY CAPSULE**

**U.S. Signs Outer Space N-Treaty**  
"Inspiring Moment" Hailed by Johnson  
By MAX FRANKS  
WASHINGTON — President Johnson today proclaimed a "historic moment" in which the United States, the Soviet Union and all other countries signed a treaty to limit military activities in outer space.

Johnson hailed it as an "inspiring moment" in the history of the human race and declared the treaty as a "first step toward bringing outer space free from the implications of war."

Similar ceremonies were held earlier yesterday in Moscow and London. But the treaty will not take effect until it is ratified by the U.S. and the Soviet Union, Britain and other countries.

**3 Spacemen Trained in Arizona**  
The three Apollo astronauts killed last night had trained at several scientific facilities here in Arizona.

Beginning with the Gemini 2 flight in 1965, astronauts had spent many hours at Kitt Peak, the observatory south-west of Tucson on the Pinal Mountains, at Grant Valley in Pinal and at the Grand Canyon.

Since its workers Arizona was selected for Astronauts shortly preceding the moon landing, volcanic regions, lava flows, minor craters and a geological phenomena like the Grand Canyon.

**Three-Man Crew Killed Instantly**  
CAPE KENNEDY, Fla., (AP) — The three Apollo 1 Astronauts were killed last night by a flash fire that trapped them aboard the huge spacecraft designed to take a man to the moon by 1970.

Locked behind sealed hatches and killed instantly just 218 feet above the ground were:

Air Force Col. Virgil (Gus) Grissom, a space pioneer and the first man to soar twice into the heavens; Air Force Col. Edward H. White II, first American to walk in space, and Navy Lt. Cmdr. Roger B. Chaffee, a rookie eagerly awaiting his first flight.

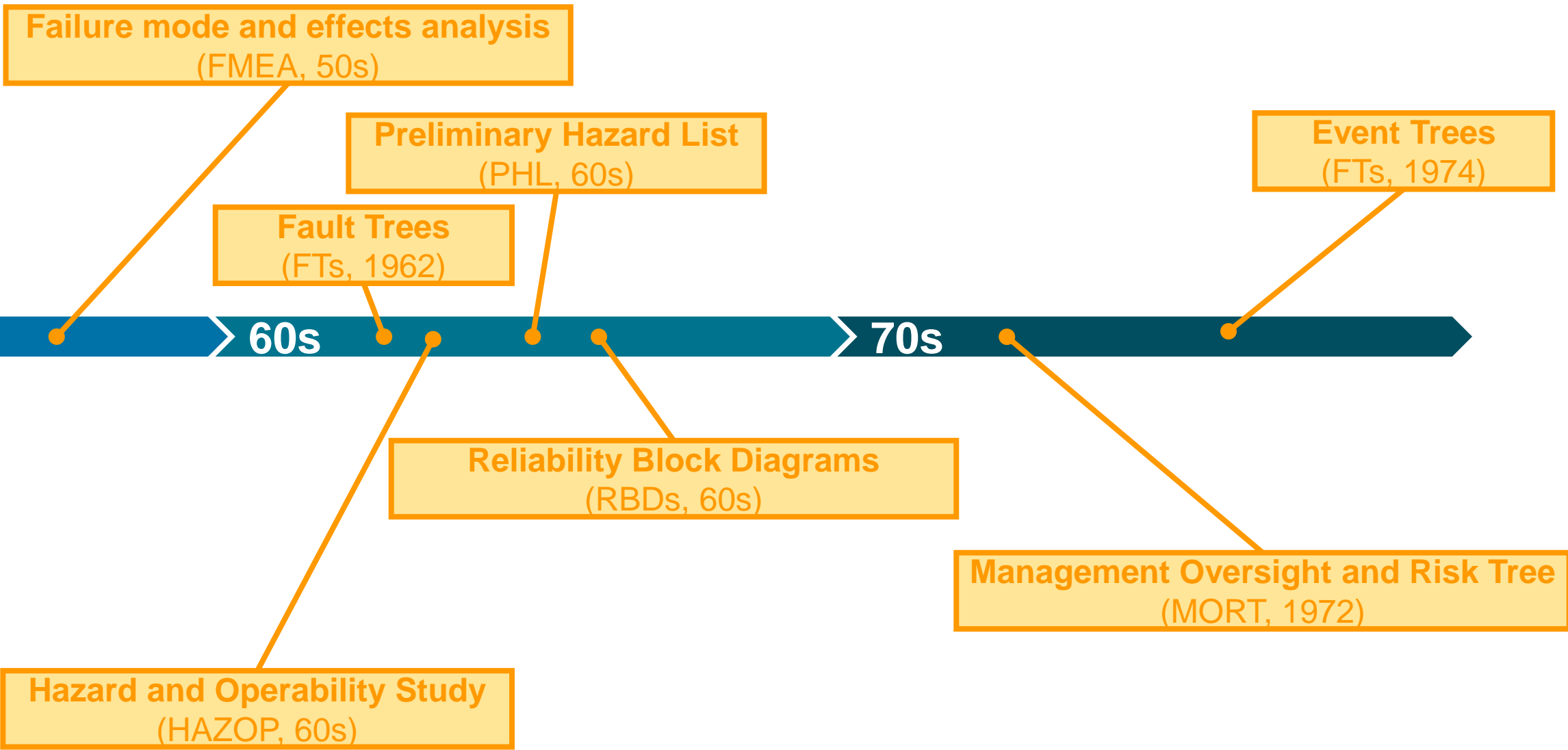
The three were hooked into a pure oxygen breathing system in their spacesuits and the oxygen fed the fire. Valves just moments trying to reverse the trapped men fell back one by one as they fought through dense, acid smoke toward the capsule.

Although the tragedy postponed indefinitely the Apollo 1's scheduled Feb. 21 blast off, space officials and

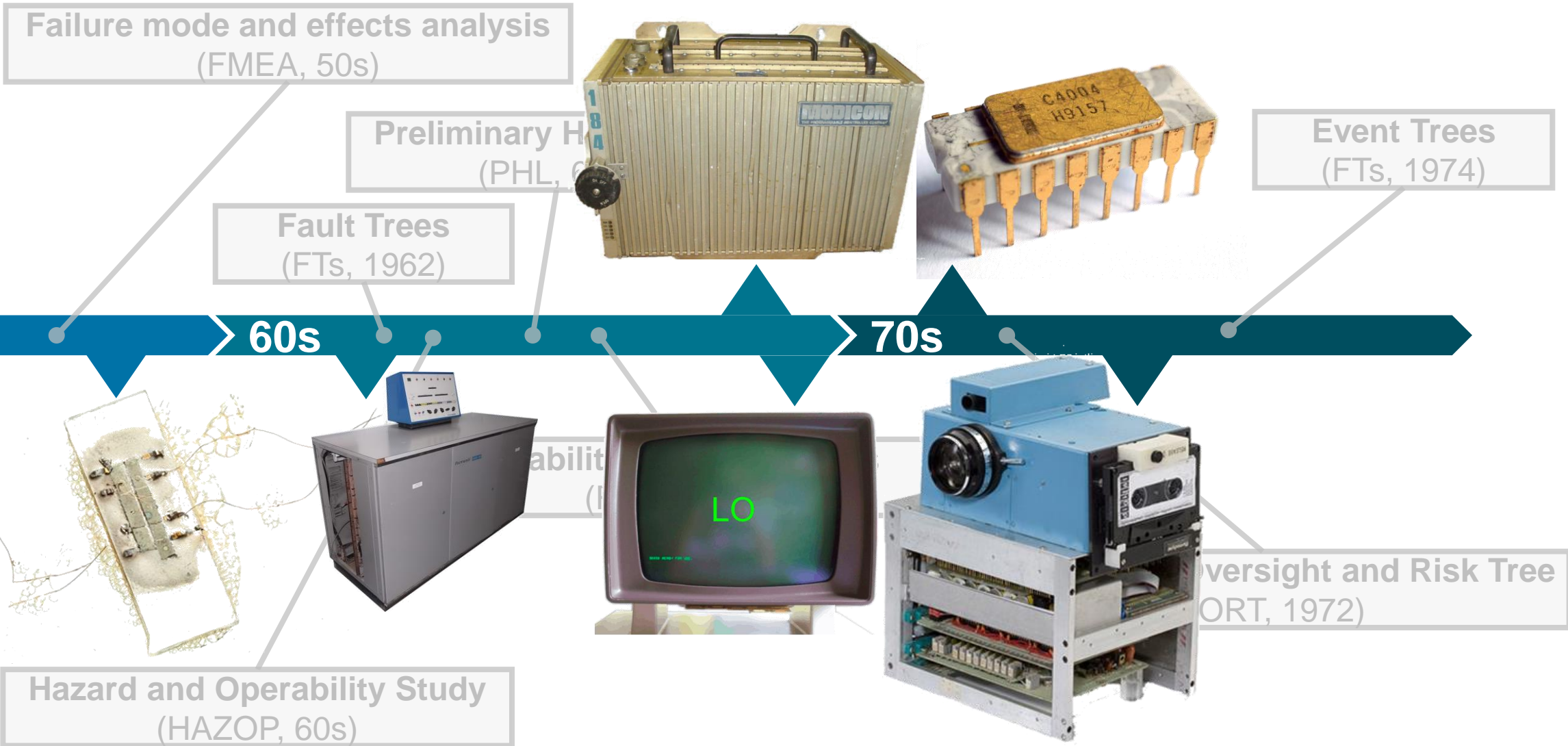
**Tragedy Will Hurt**



# Why system safety?



# Why system safety?





## LARGE-SCALE SYSTEMS

- High level of complexity
- Enormous number of components
- Low probability values
- Multiple stakeholders



EFFICIENCY ↑

## LARGE-SCALE SYSTEMS

- High level of complexity
- Enormous number of components
- Low probability values
- Multiple stakeholders





# Different systems...different tool?

TRADITIONAL  
TECHNIQUES

EFFICIENCY ↑



# Different systems...different tool?

TRADITIONAL  
TECHNIQUES

EFFICIENCY ↑

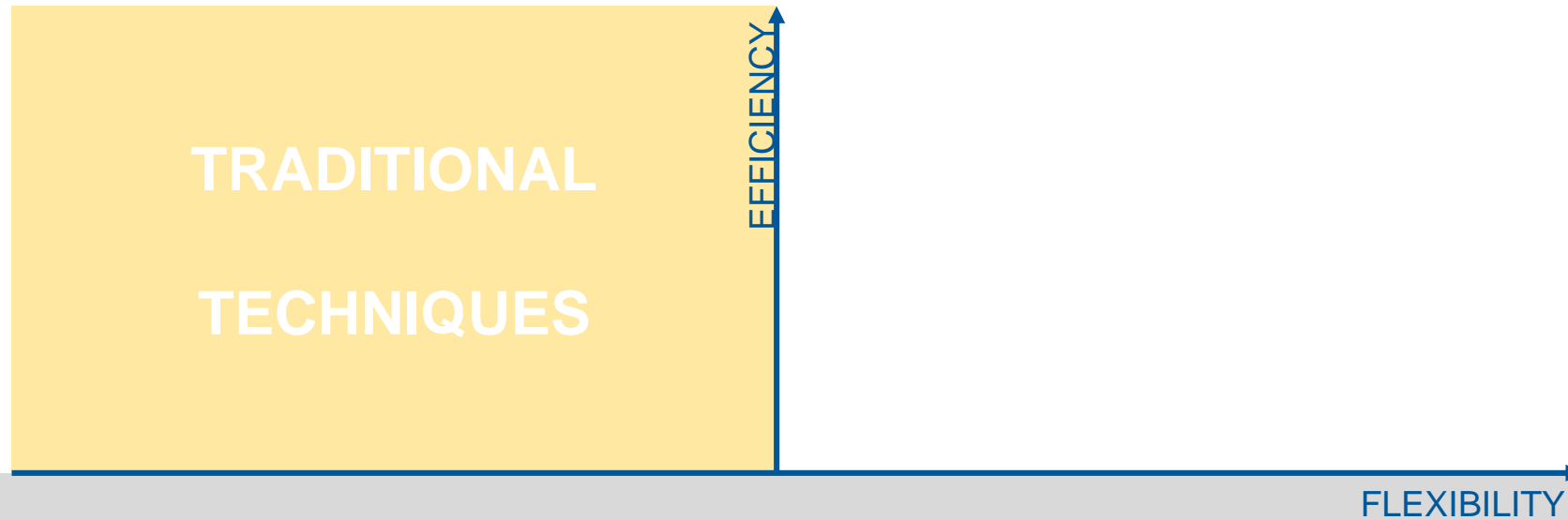
## HIGH LEVEL OF AUTOMATION AND CONTROL TECHNOLOGY

- systems un-negligibly dynamic
- human-technology interface
- increasingly complex maintenance strategies

Dependencies between  
failure events



# Different systems...different tool?

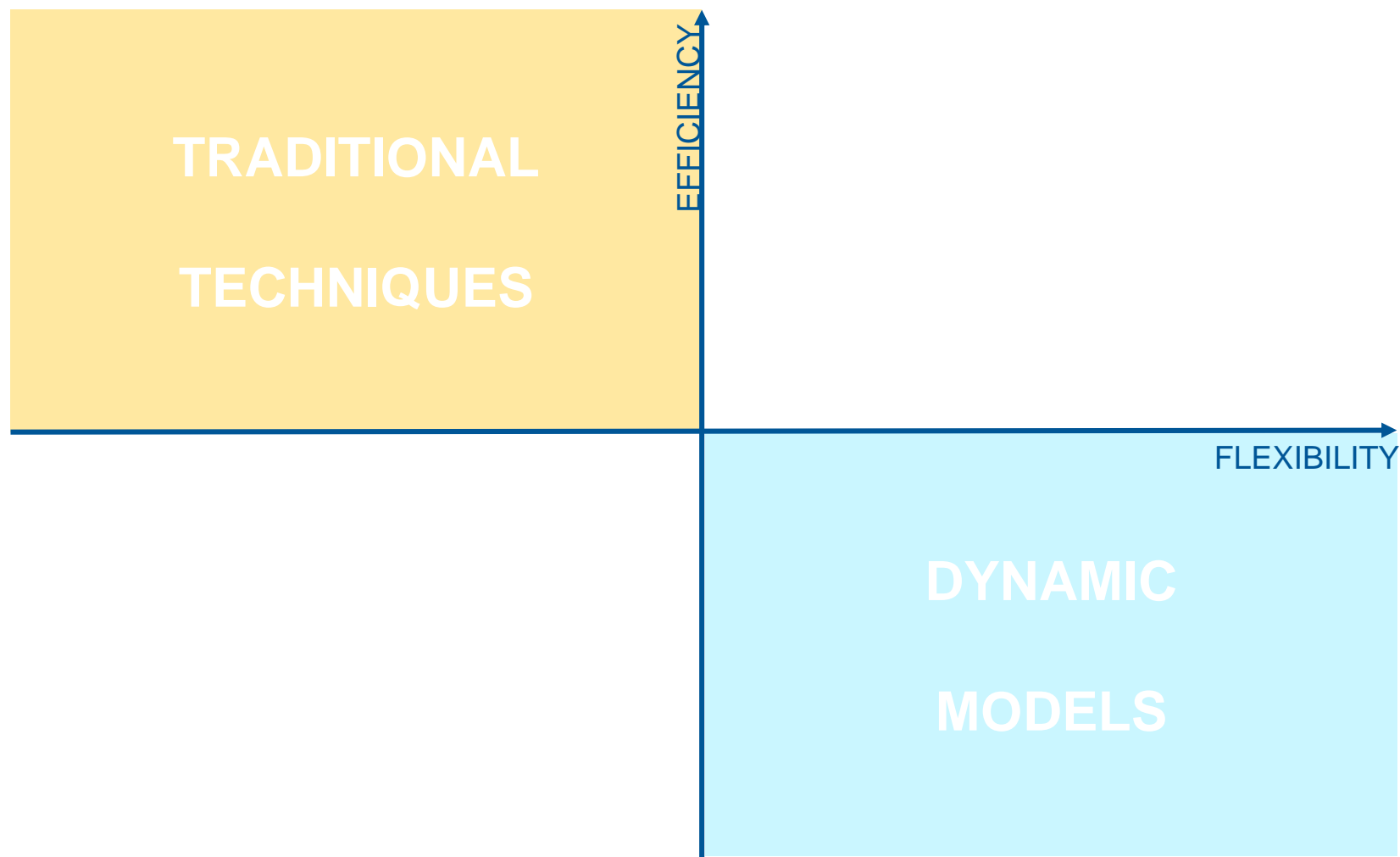


## HIGH LEVEL OF AUTOMATION AND CONTROL TECHNOLOGY

- systems un-negligibly dynamic
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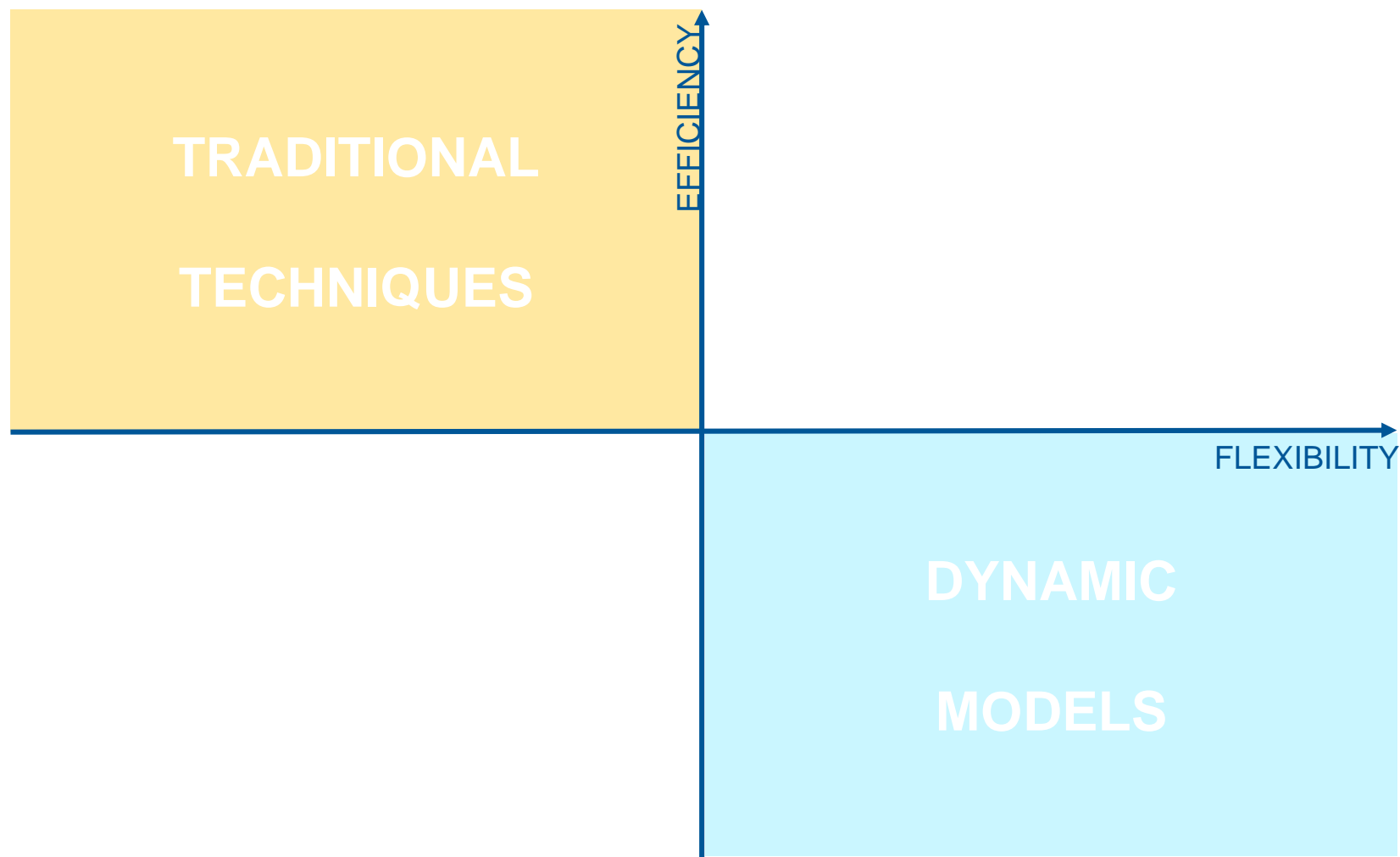


# Different systems...different tool?





# Different systems...different tool?





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**D<sup>2</sup>T<sup>2</sup>**

Dynamic and Dependent Tree Theory

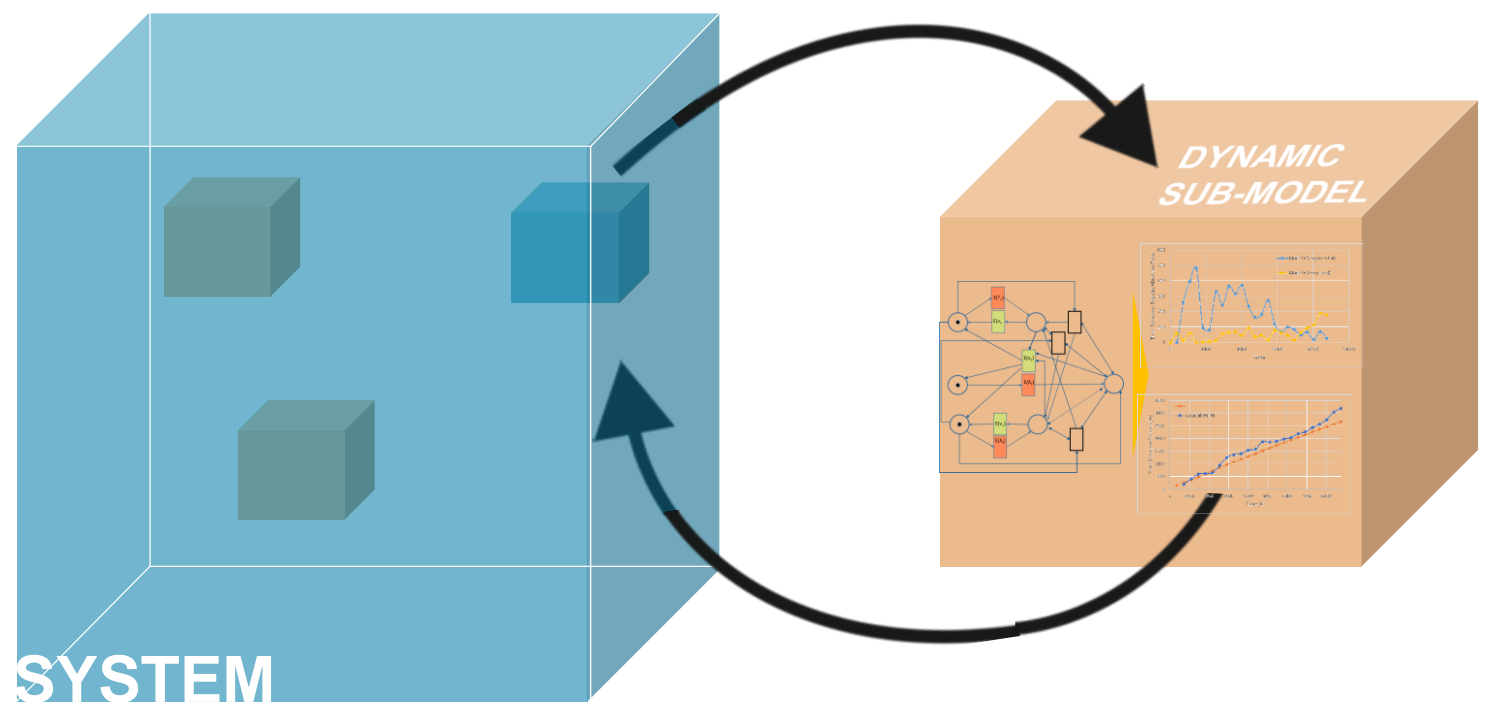


# D<sup>2</sup>T<sup>2</sup>: The Big Picture



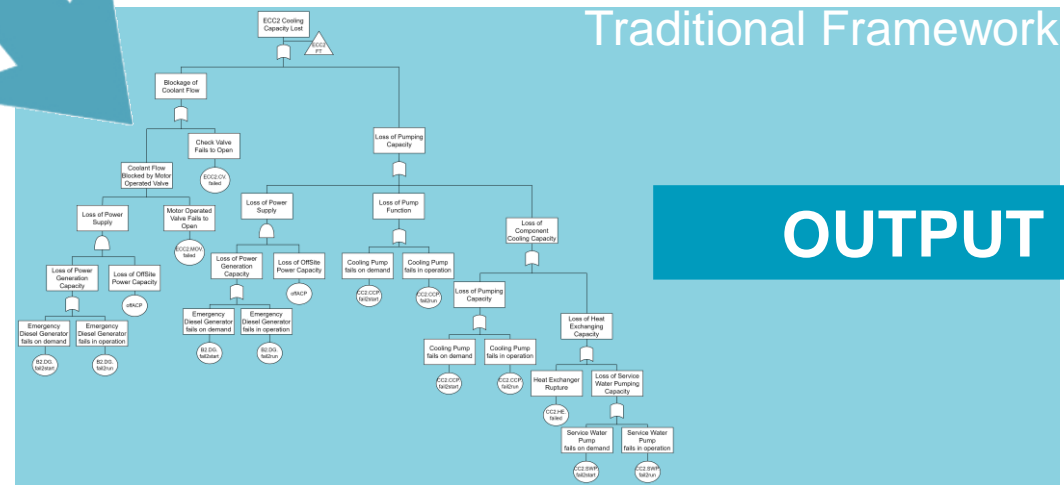
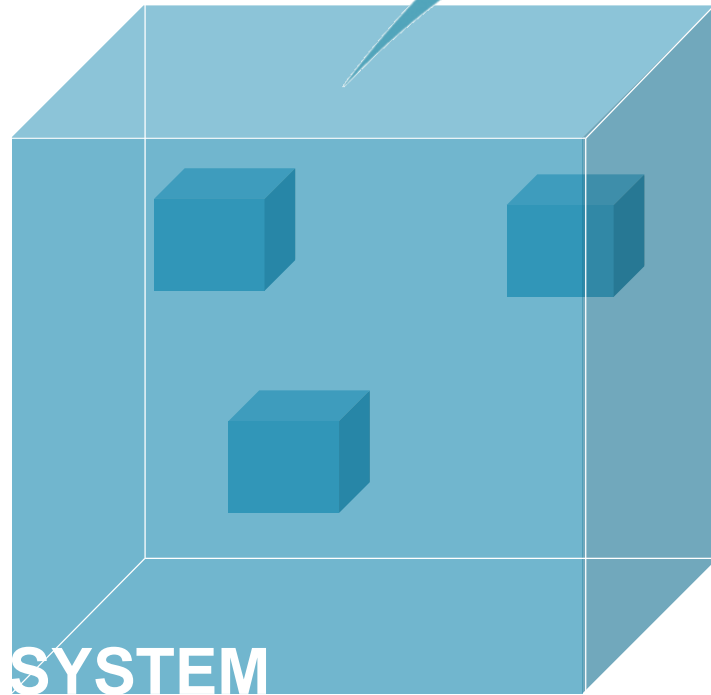


# D<sup>2</sup>T<sup>2</sup>: The Big Picture





# D<sup>2</sup>T<sup>2</sup>: The Big Picture



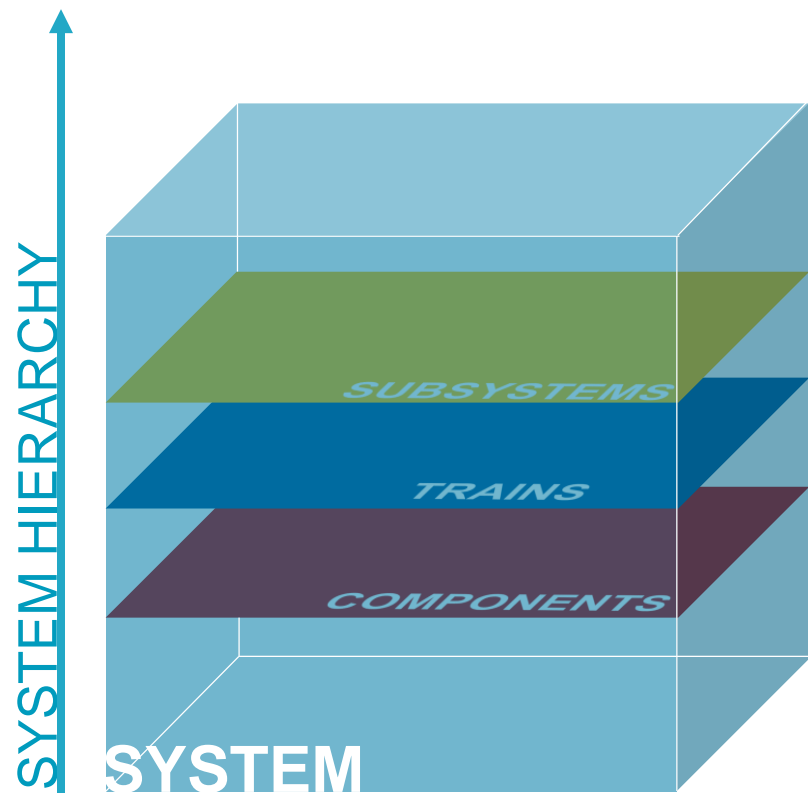
→ Tailored use of 'expensive' dynamic models

→ Preserves effectiveness of traditional techniques

→ Enhances modelling accuracy and flexibility

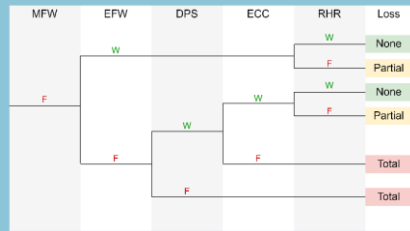
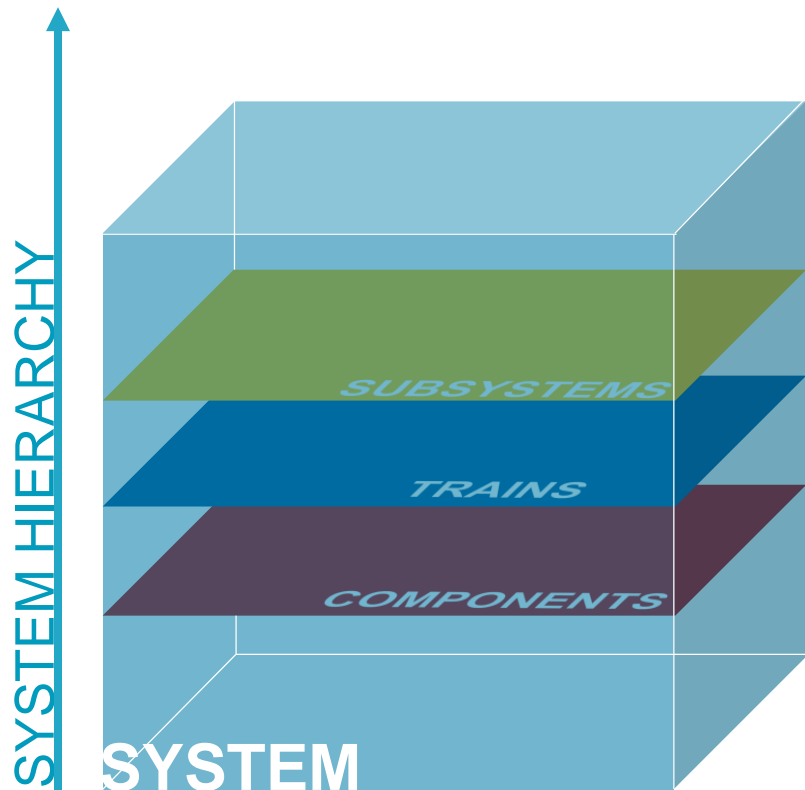


# Modelling Hierarchy





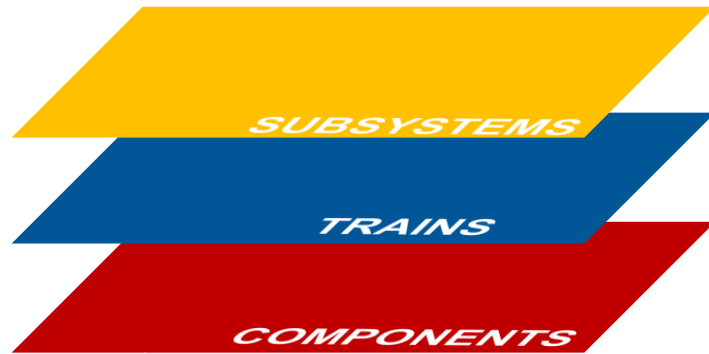
# Modelling Hierarchy



## Event Tree [ET]:

- Accident Sequence
- Subsystems interaction

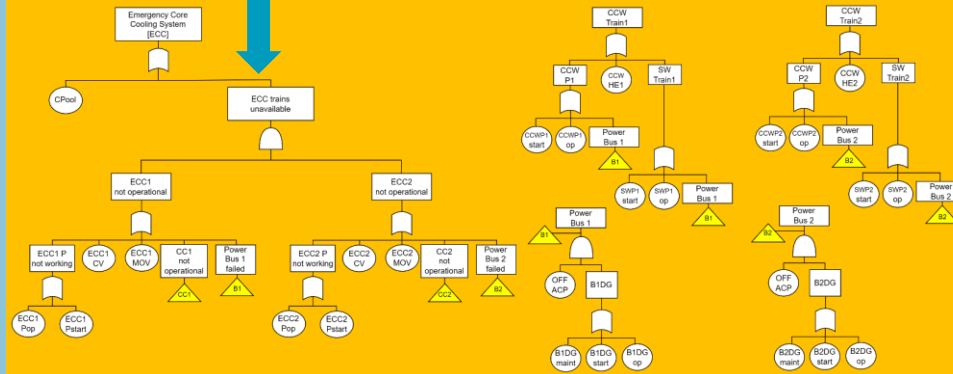
SYSTEM HIERARCHY



MFW	EFW	DPS	ECC	RHR	Loss
	W		W	W	None
			F	F	Partial
F			W	W	None
			F	F	Partial
	F				Total
		F			Total

Event Tree [ET]:

- Accident Sequence
- Subsystems interaction



Fault Tree [FT]:

- Sub-system failure

SYSTEM HIERARCHY



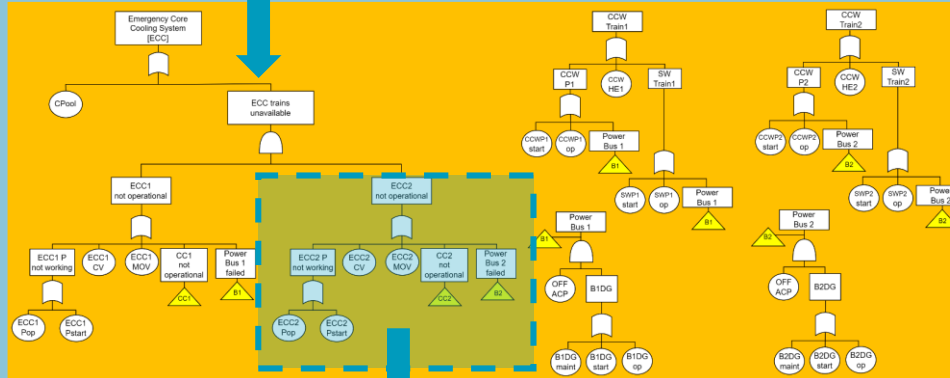
MFW	EFW	DPS	ECC	RHR	Loss
	W		W	W	None
			F	F	Partial
F			W	W	None
			F	F	Partial
	F				Total
		F			Total

## Event Tree [ET]:

- Accident Sequence
- Subsystems interaction

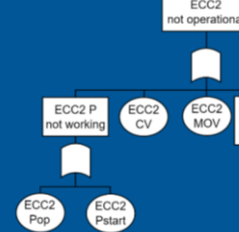
## Fault Tree [FT]:

- Sub-system failure



## Sub-Tree [FT]:

- Trains of identical components
- Redundancy



# Modelling Hierarchy

SYSTEM HIERARCHY



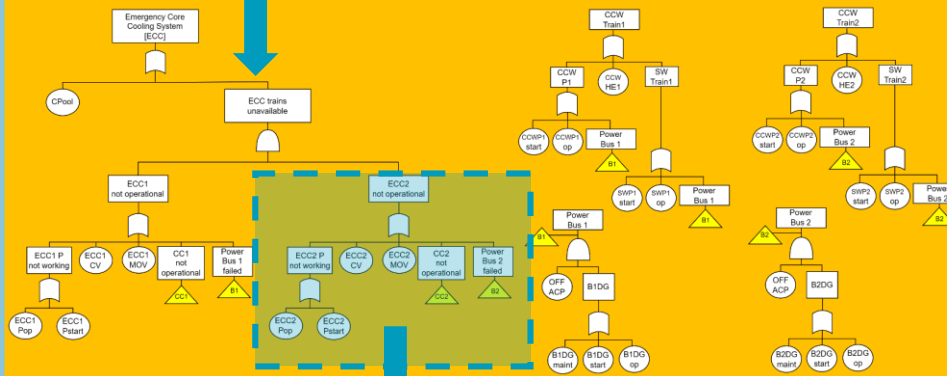
MFW	EFW	DPS	ECC	RHR	Loss
	W		W	W	None
			F	F	Partial
F			W	W	None
			F	F	Partial
	F				Total
		F			Total

## Event Tree [ET]:

- Accident Sequence
- Subsystems interaction

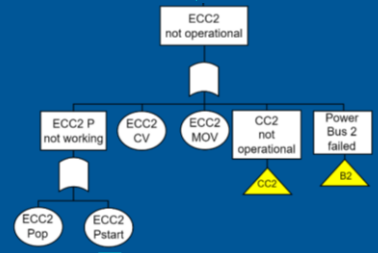
## Fault Tree [FT]:

- Sub-system failure



## Sub-Tree [FT]:

- Trains of identical components
- Redundancy



## Basic Event [BE]:

- Component Failure Mechanism
- Reliability Metrics





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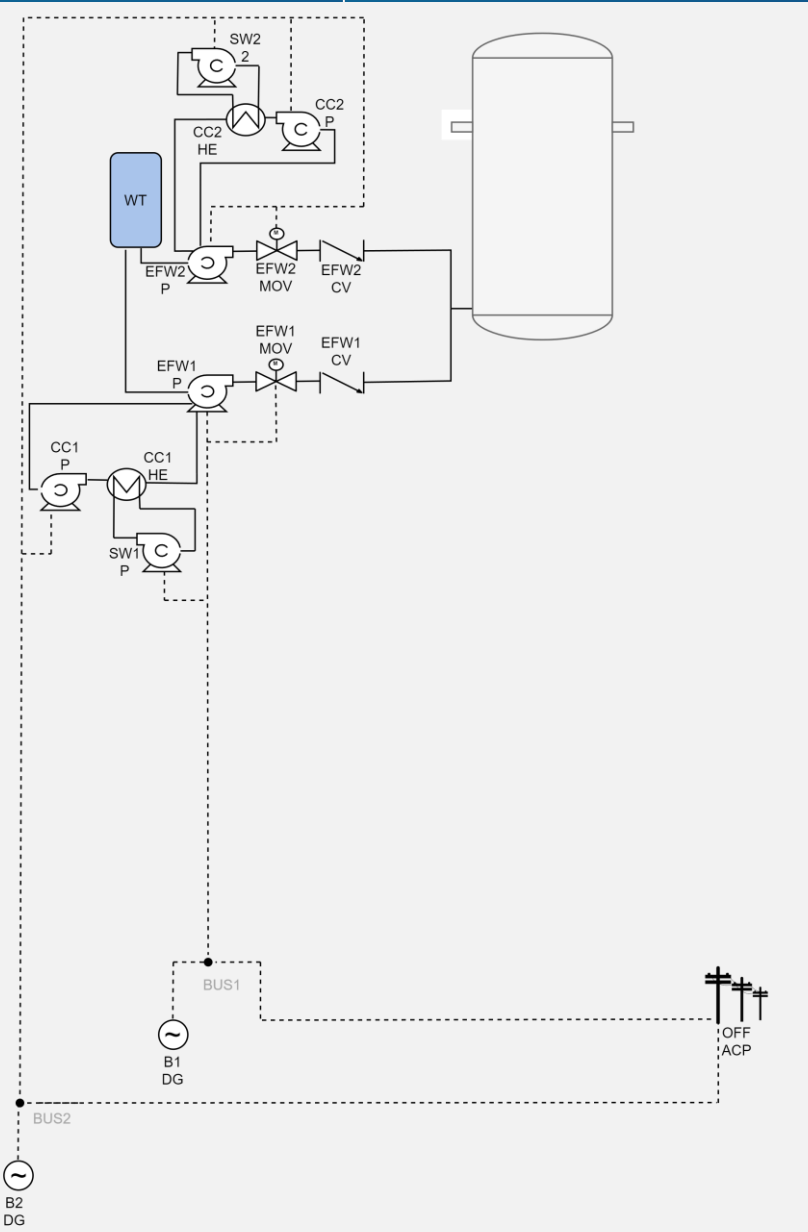
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# Hands on

D<sup>2</sup>T<sup>2</sup> application

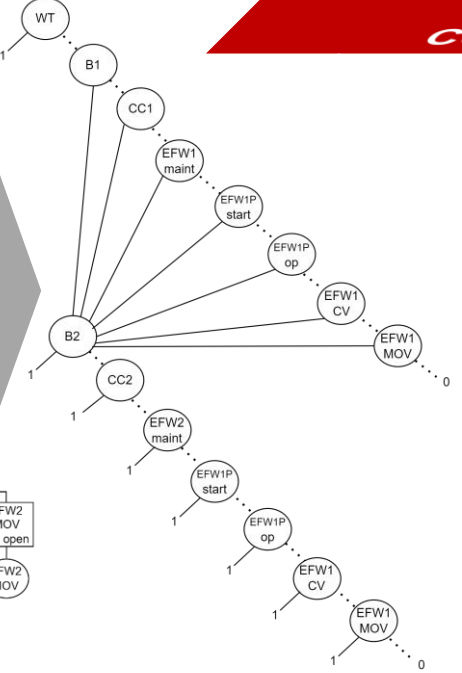
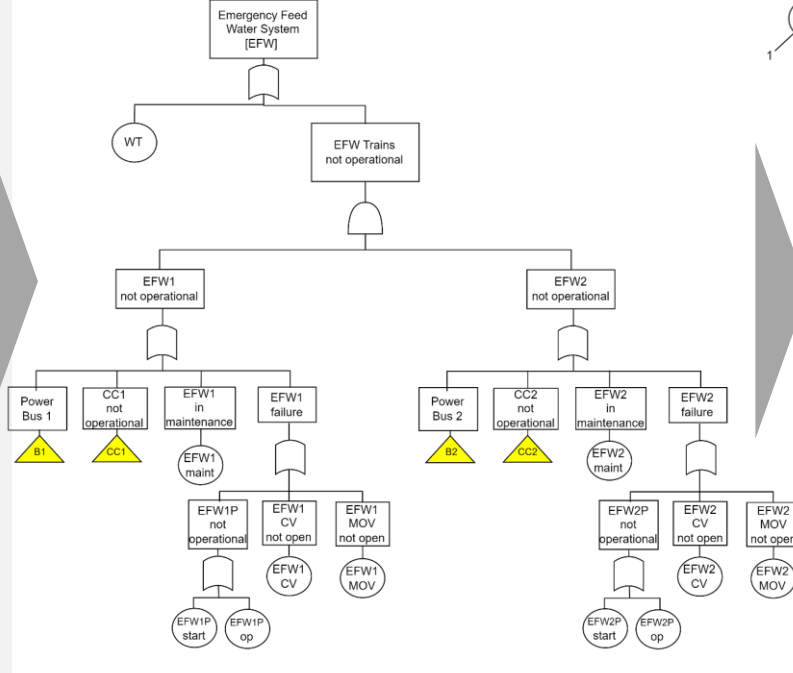
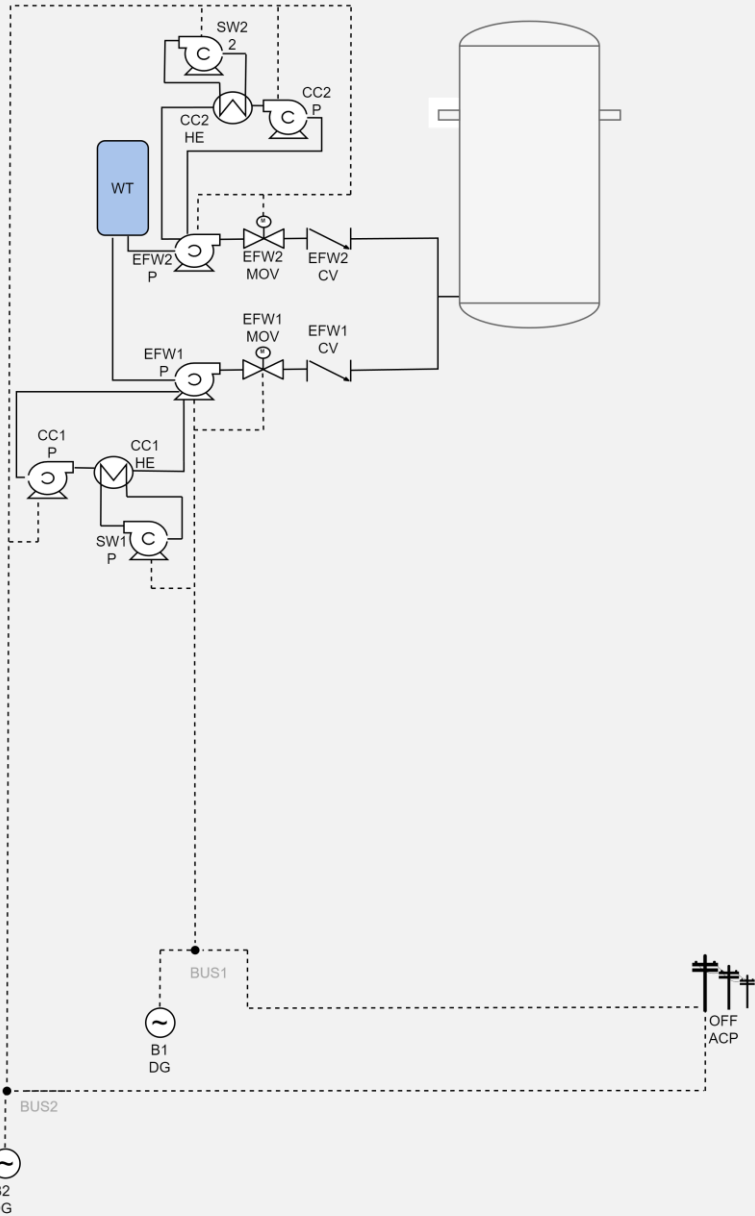


# D<sup>2</sup>T<sup>2</sup>: Components Dependency

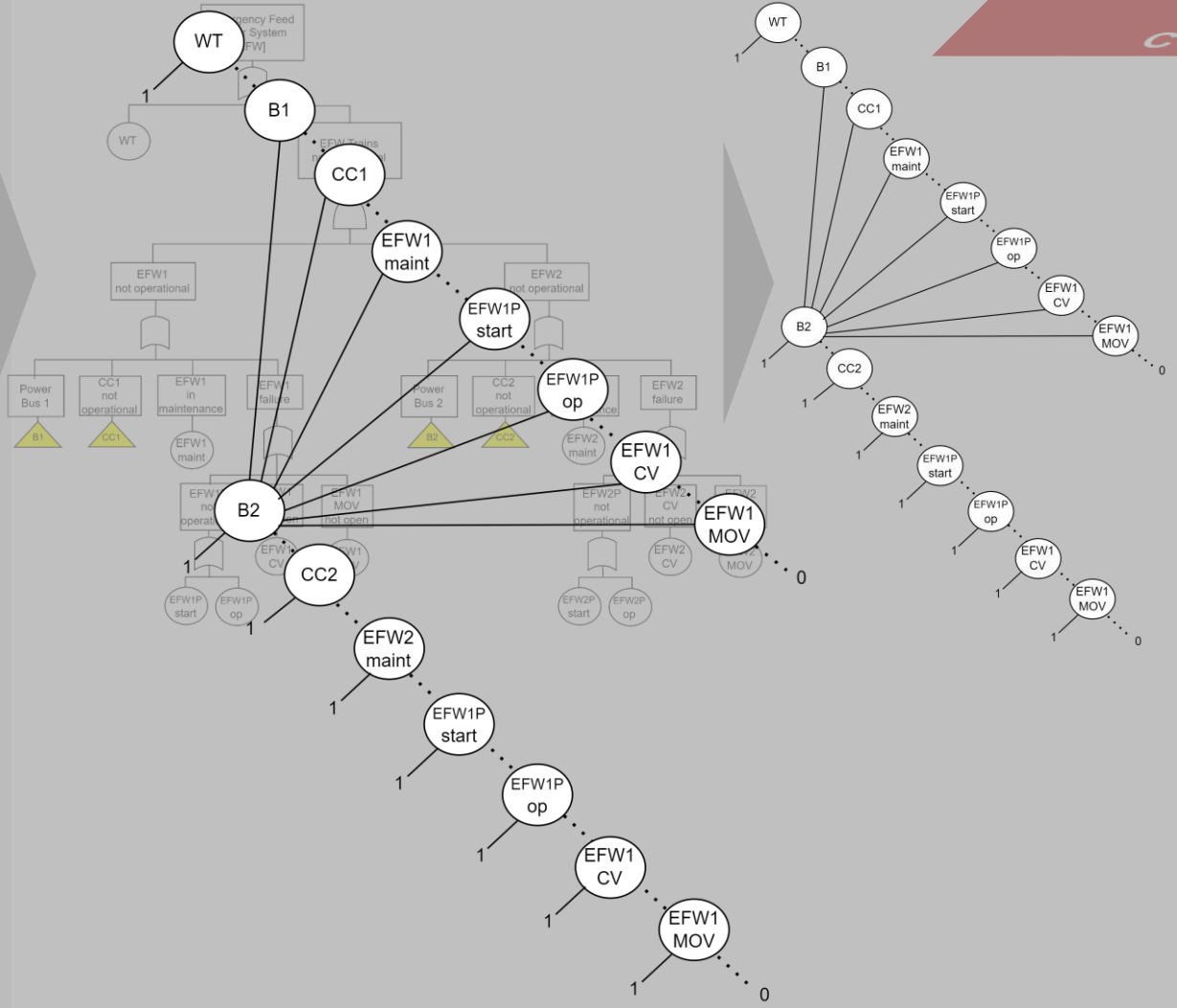
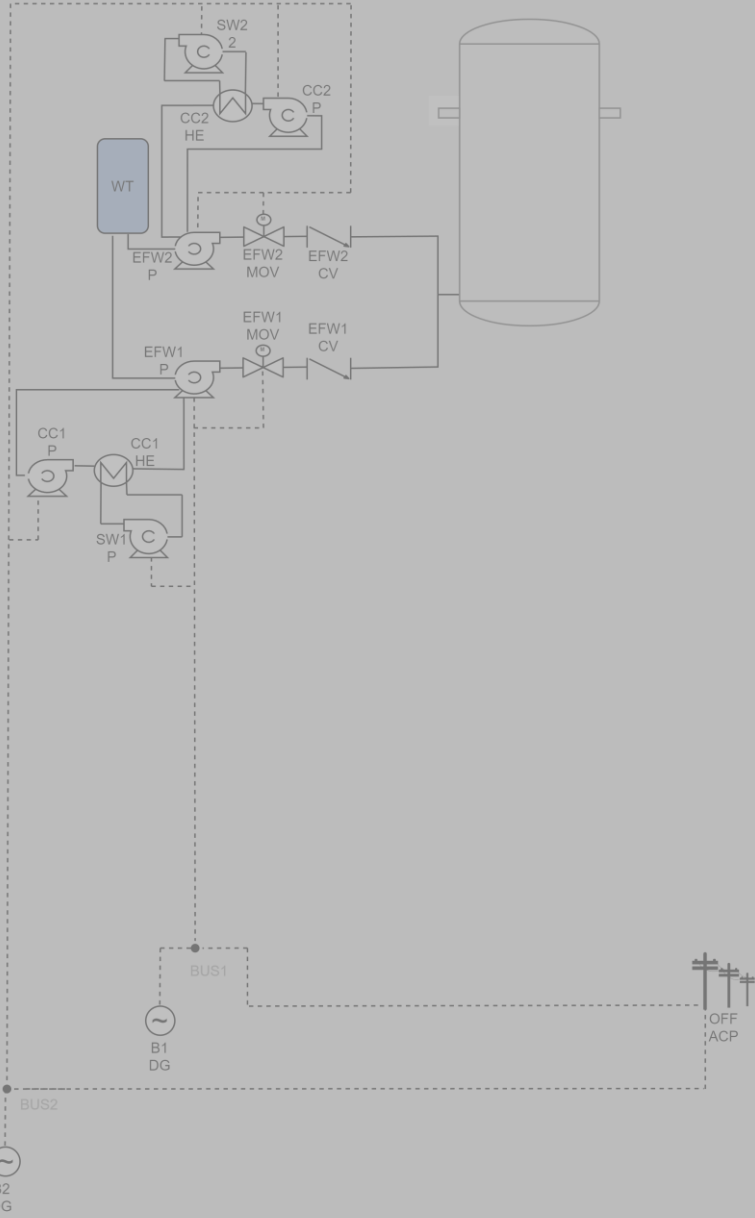




# D<sup>2</sup>T<sup>2</sup>: Components Dependency

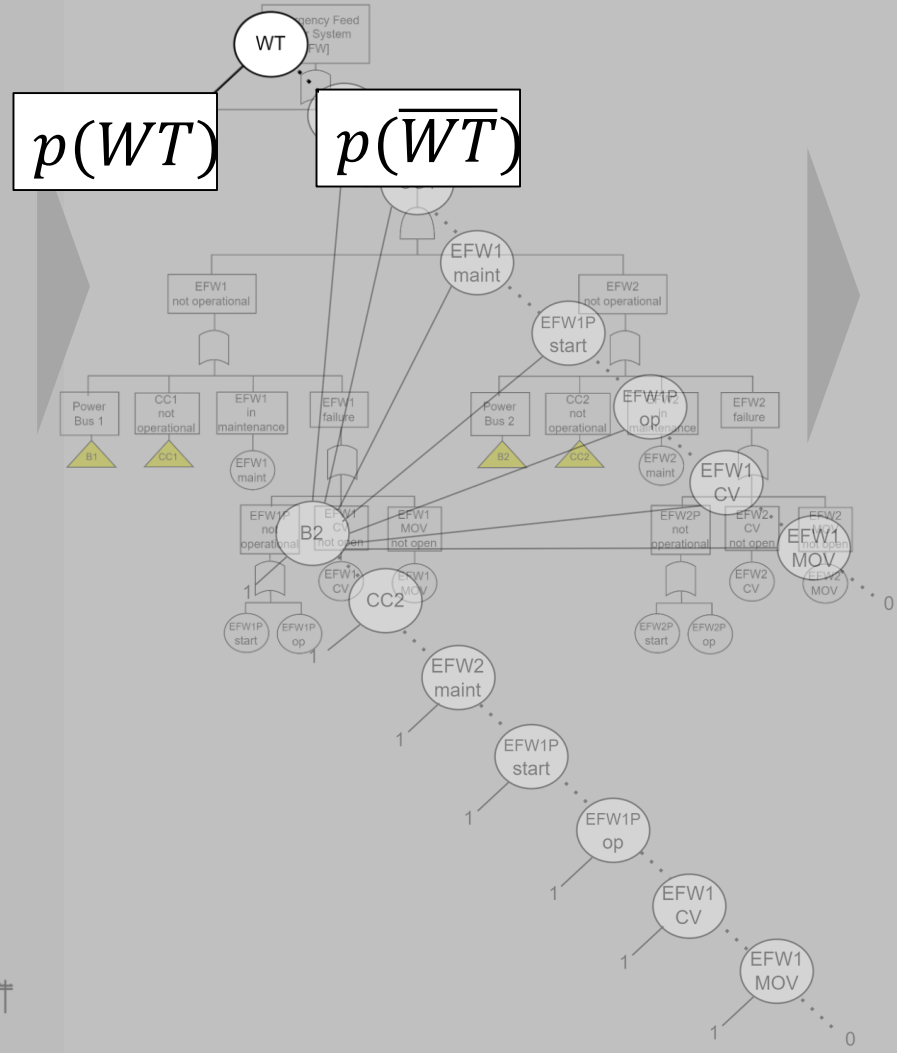
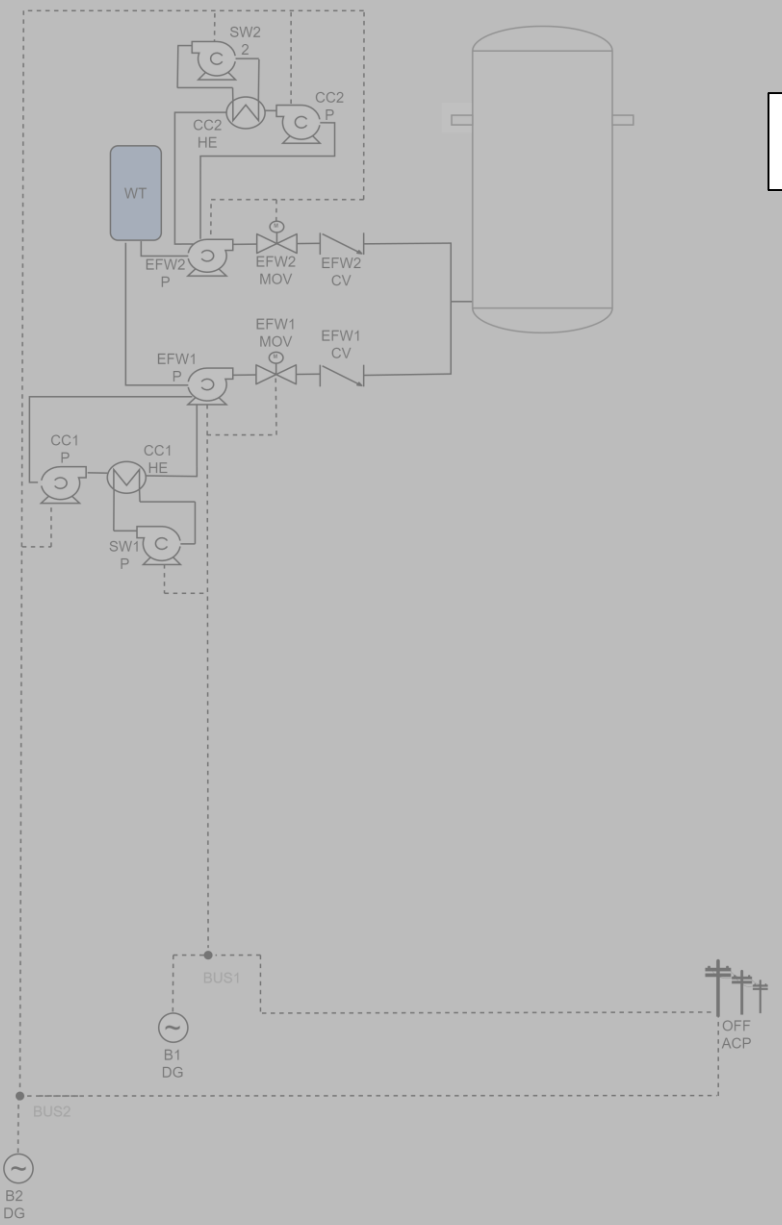


# D<sup>2</sup>T<sup>2</sup>: Components Dependency



COMPONENTS

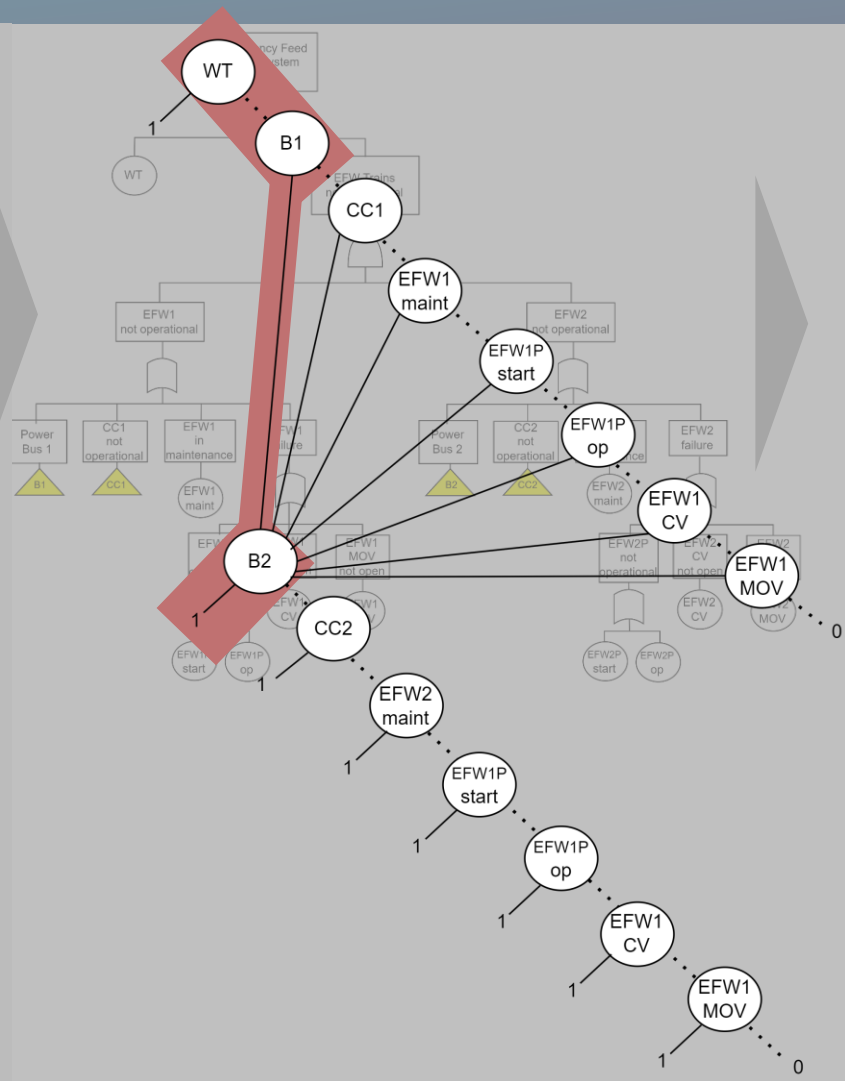
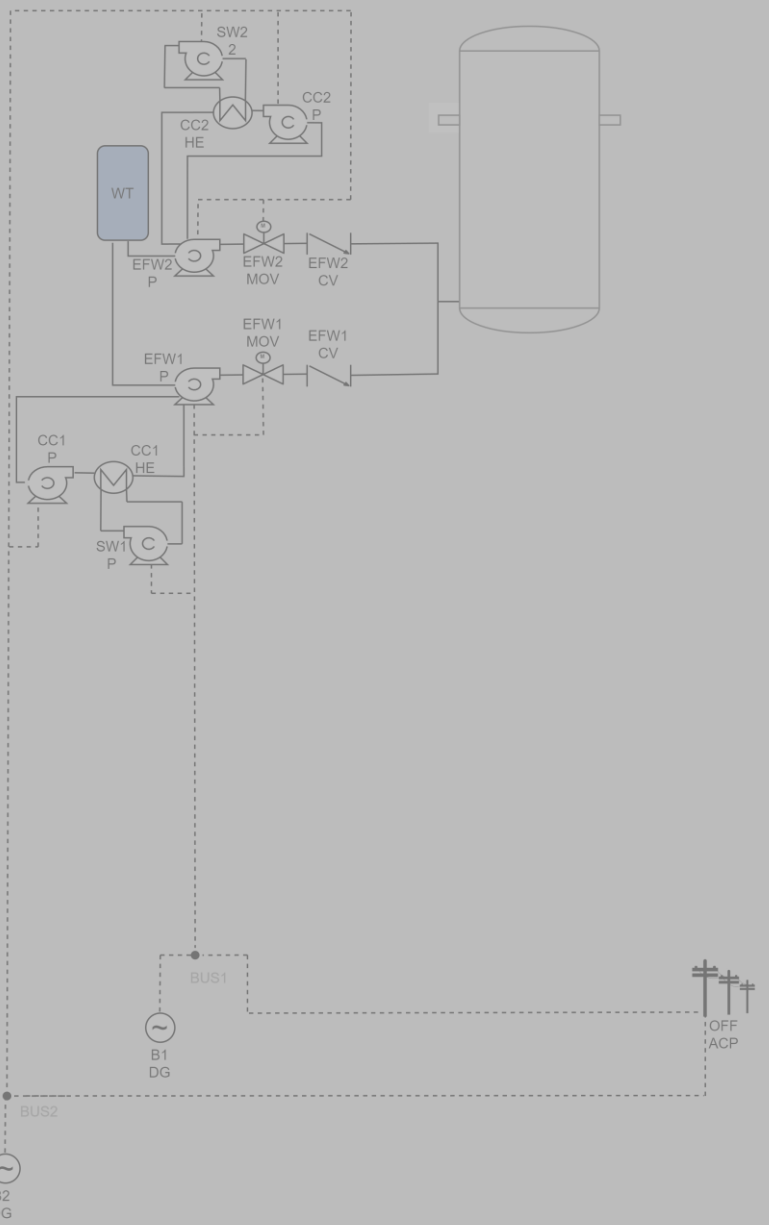
# D<sup>2</sup>T<sup>2</sup>: Components Dependency



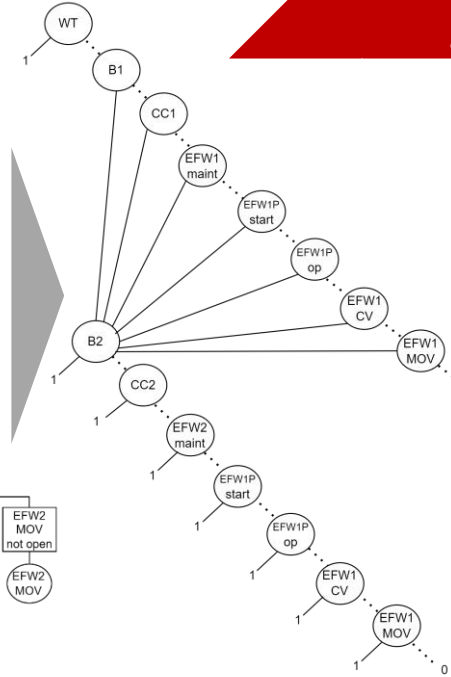
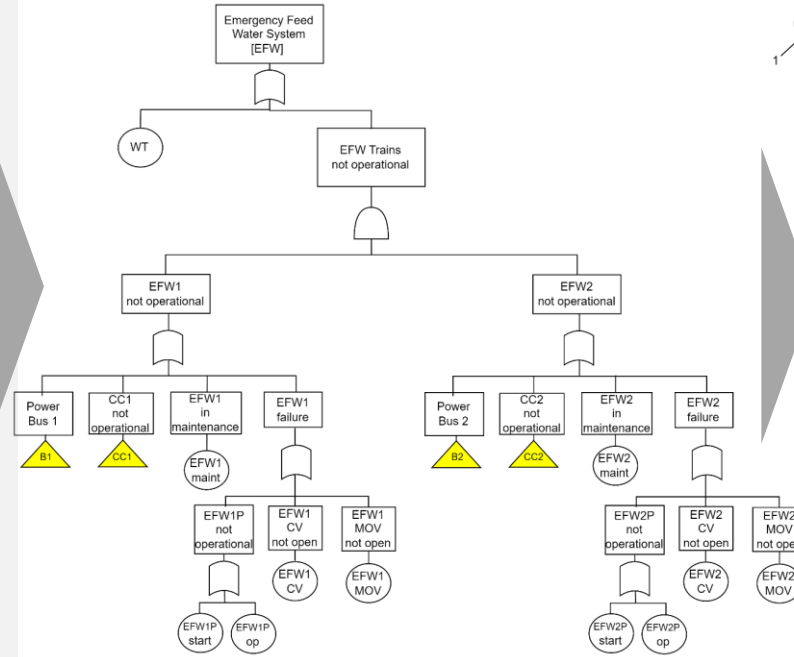
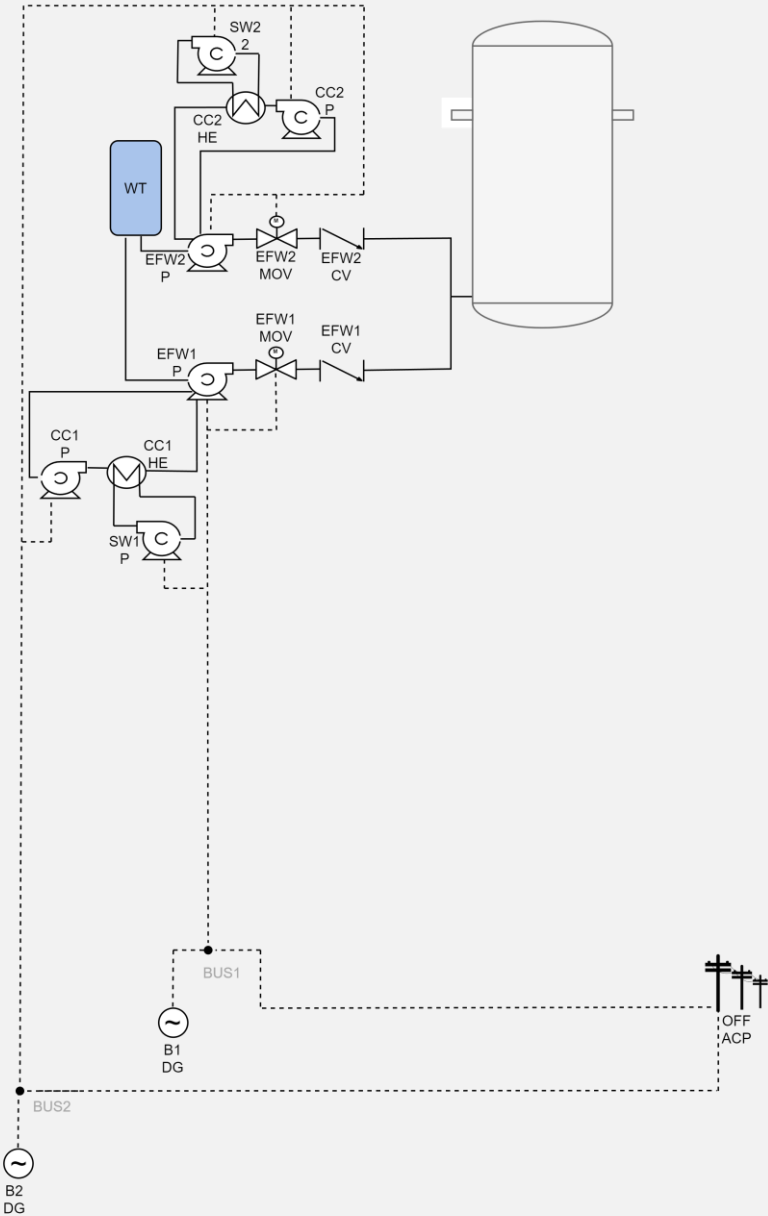
*COMPONENTS*

# D<sup>2</sup>T<sup>2</sup>: Components Dependency

## COMPONENTS



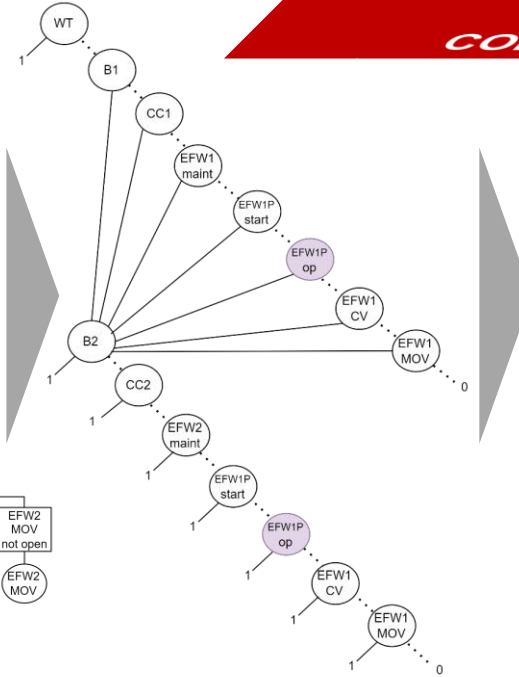
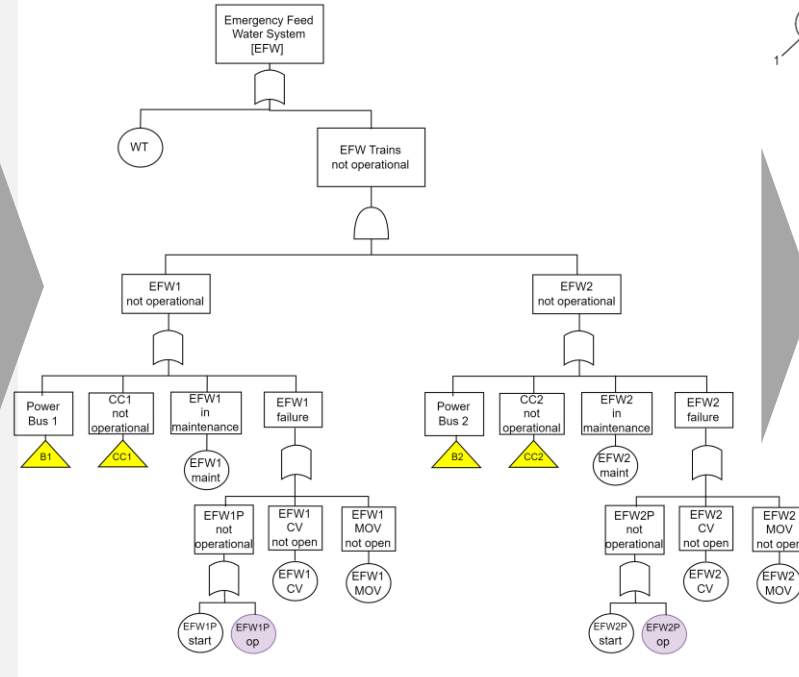
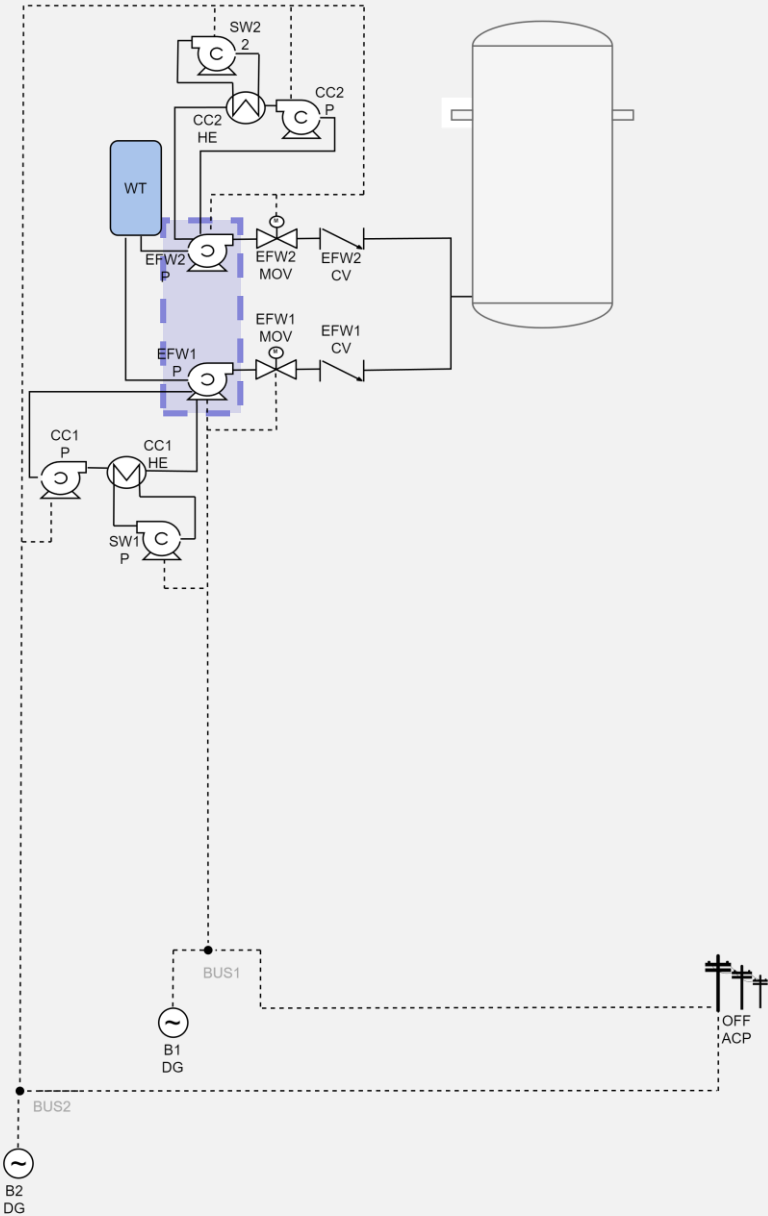
# D<sup>2</sup>T<sup>2</sup>: Components Dependency



**COMPONENTS**

**TOP EVENT  
PROBABILITY  
=  
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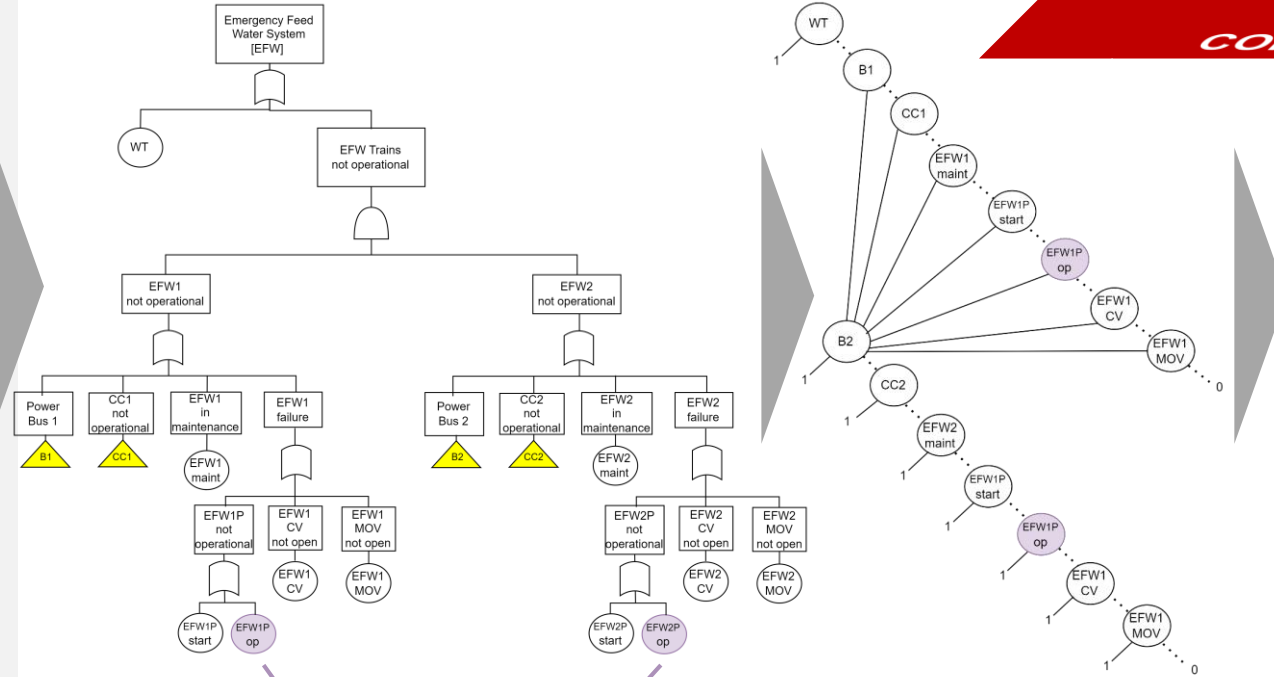
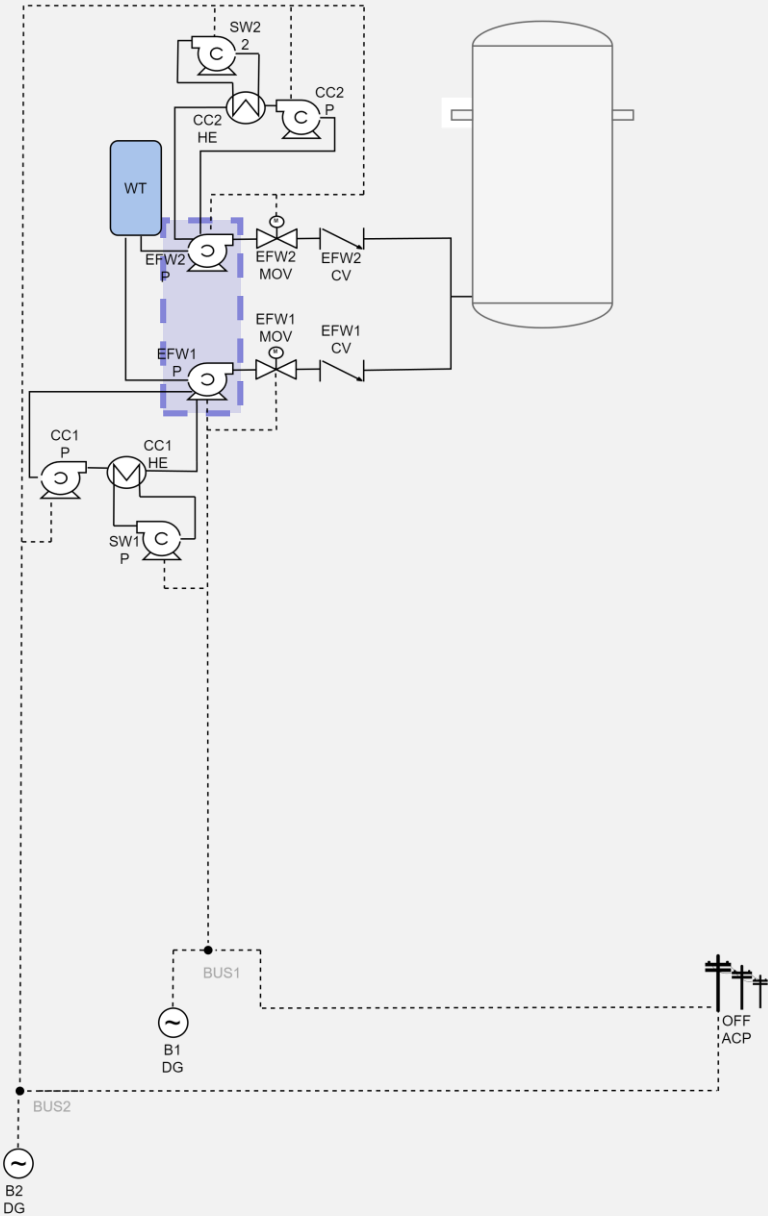
# D<sup>2</sup>T<sup>2</sup>: Components Dependency



**COMPONENTS**

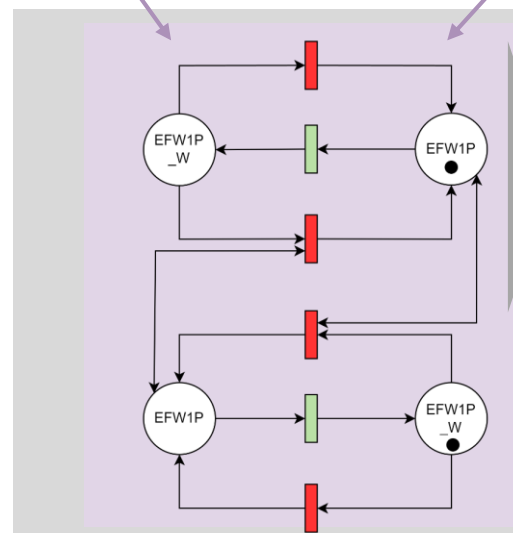
**TOP EVENT PROBABILITY = ?**

# D<sup>2</sup>T<sup>2</sup>: Components Dependency



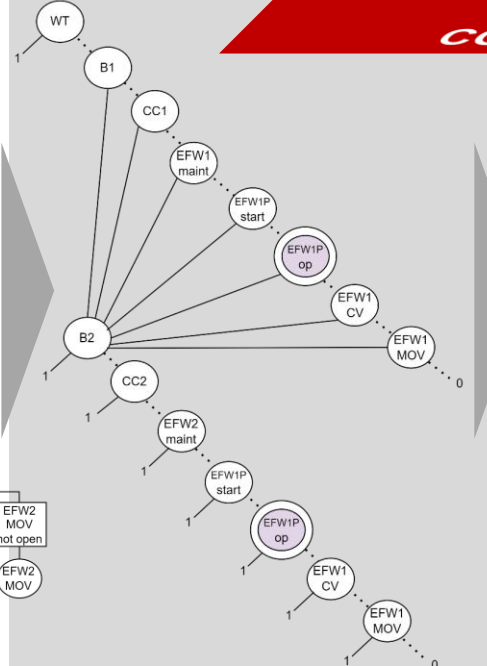
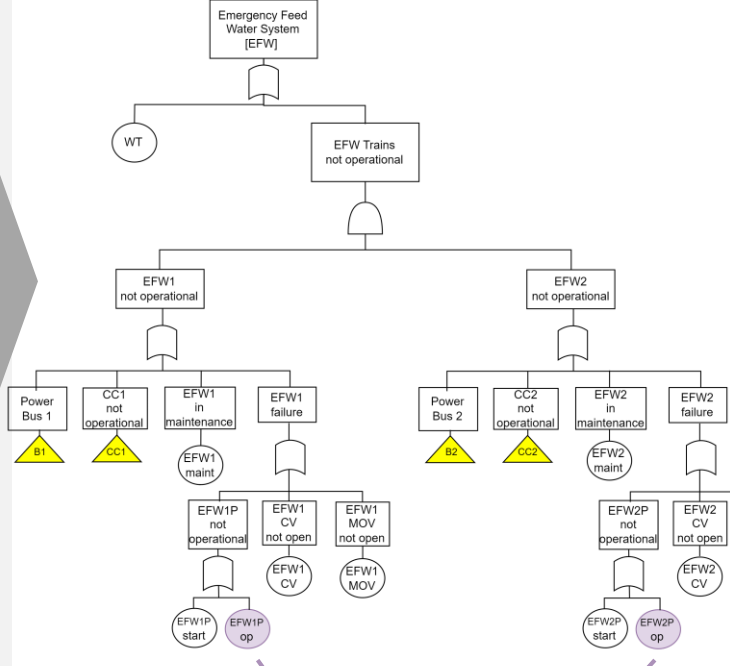
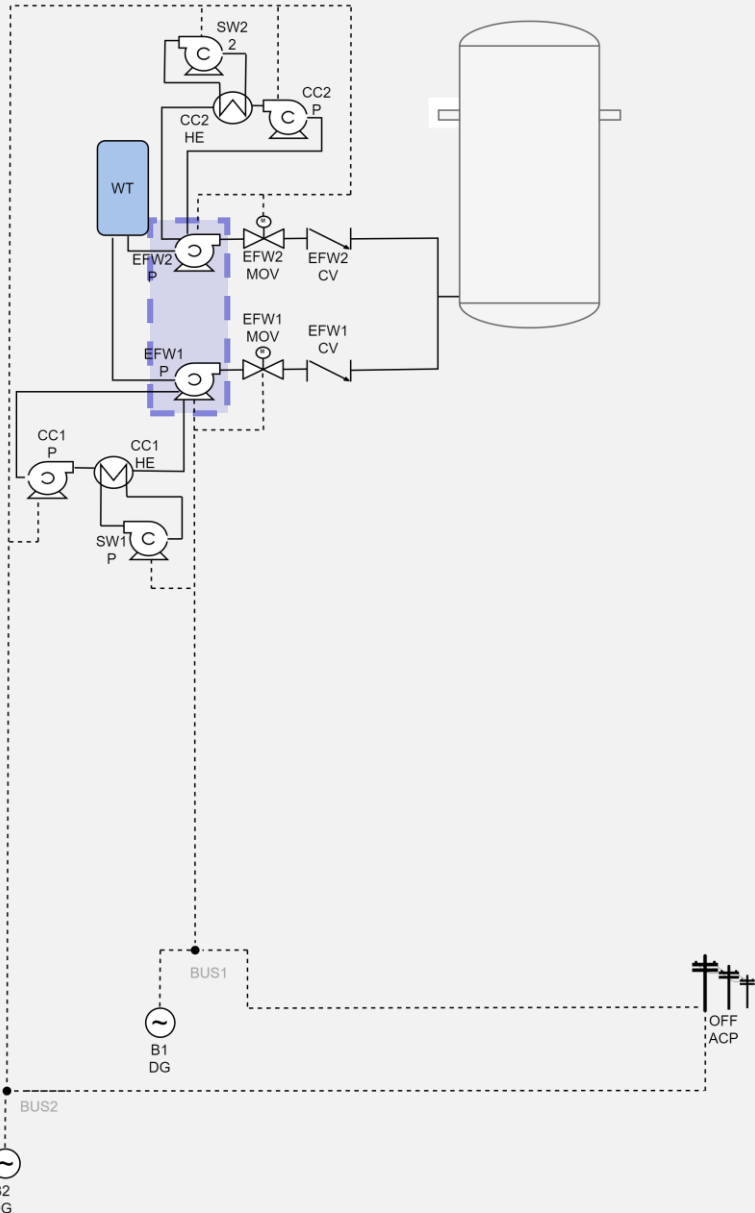
**COMPONENTS**

**TOP EVENT PROBABILITY**  
= ?



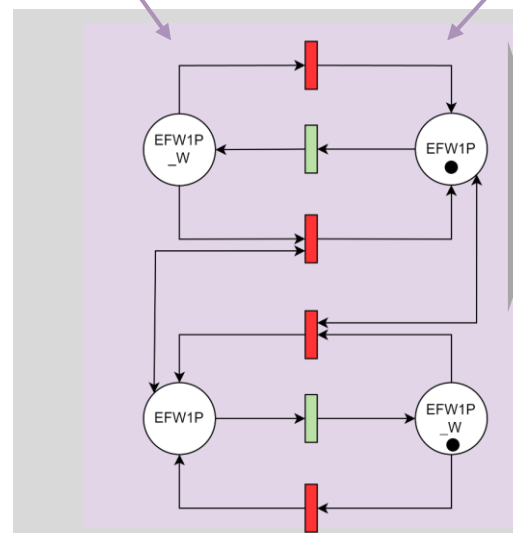
Joint Event	Probability
$\overline{EFW1Pop}, \overline{EFW2Pop}$	$9.9 \cdot 10^{-1}$
$\overline{EFW1Pop}, EFW2Pop$	$1.80 \cdot 10^{-4}$
$EFW1Pop, \overline{EFW2Pop}$	$1.79 \cdot 10^{-4}$
$EFW1Pop, EFW2Pop$	$2.96 \cdot 10^{-8}$

# D<sup>2</sup>T<sup>2</sup>: Components Dependency



COMPONENTS

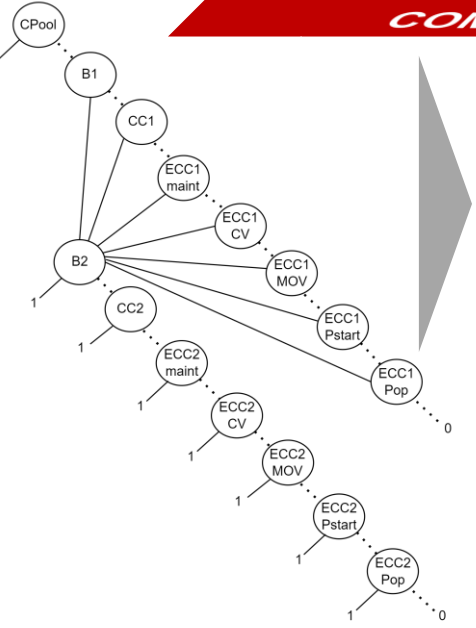
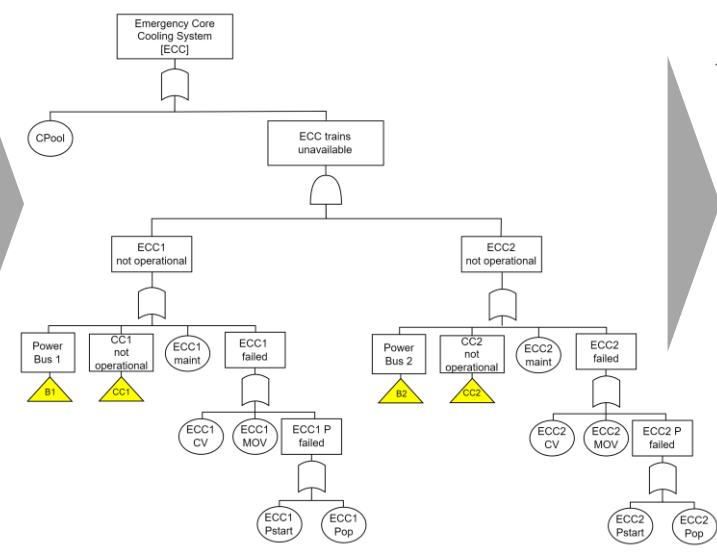
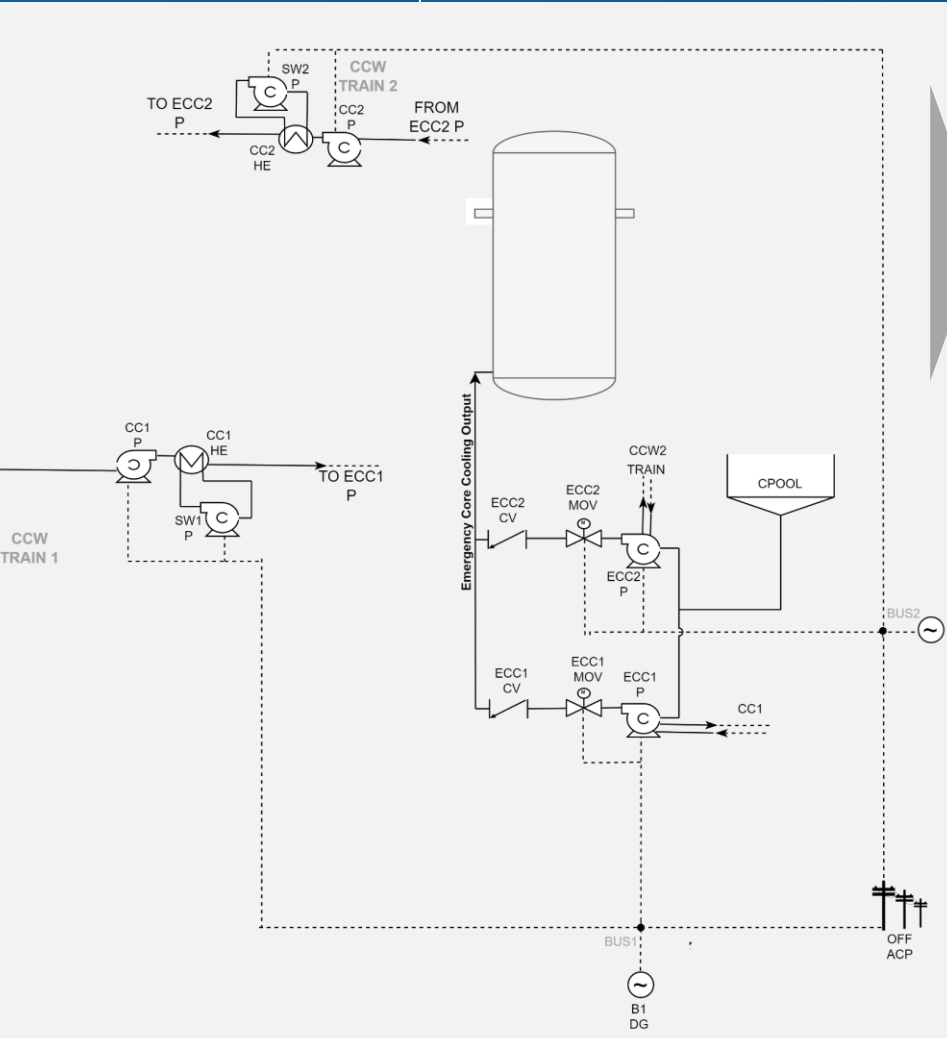
TOP EVENT  
PROBABILITY  
=  
0.0040



Joint Event	Probability
$\overline{EFW1Pop}, \overline{EFW2Pop}$	$9.9 \cdot 10^{-1}$
$\overline{EFW1Pop}, EFW2Pop$	$1.80 \cdot 10^{-4}$
$EFW1Pop, \overline{EFW2Pop}$	$1.79 \cdot 10^{-4}$
$EFW1Pop, EFW2Pop$	$2.96 \cdot 10^{-8}$

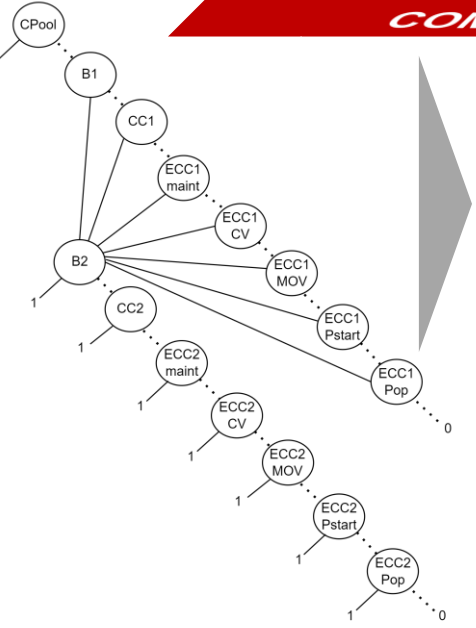
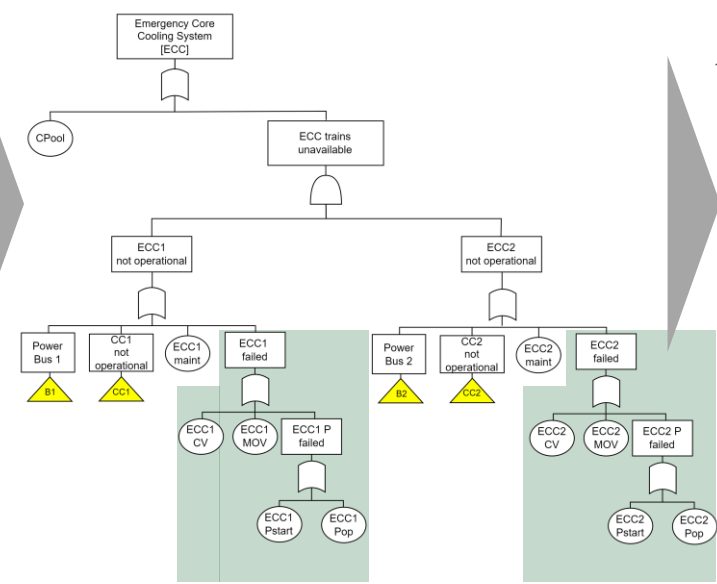
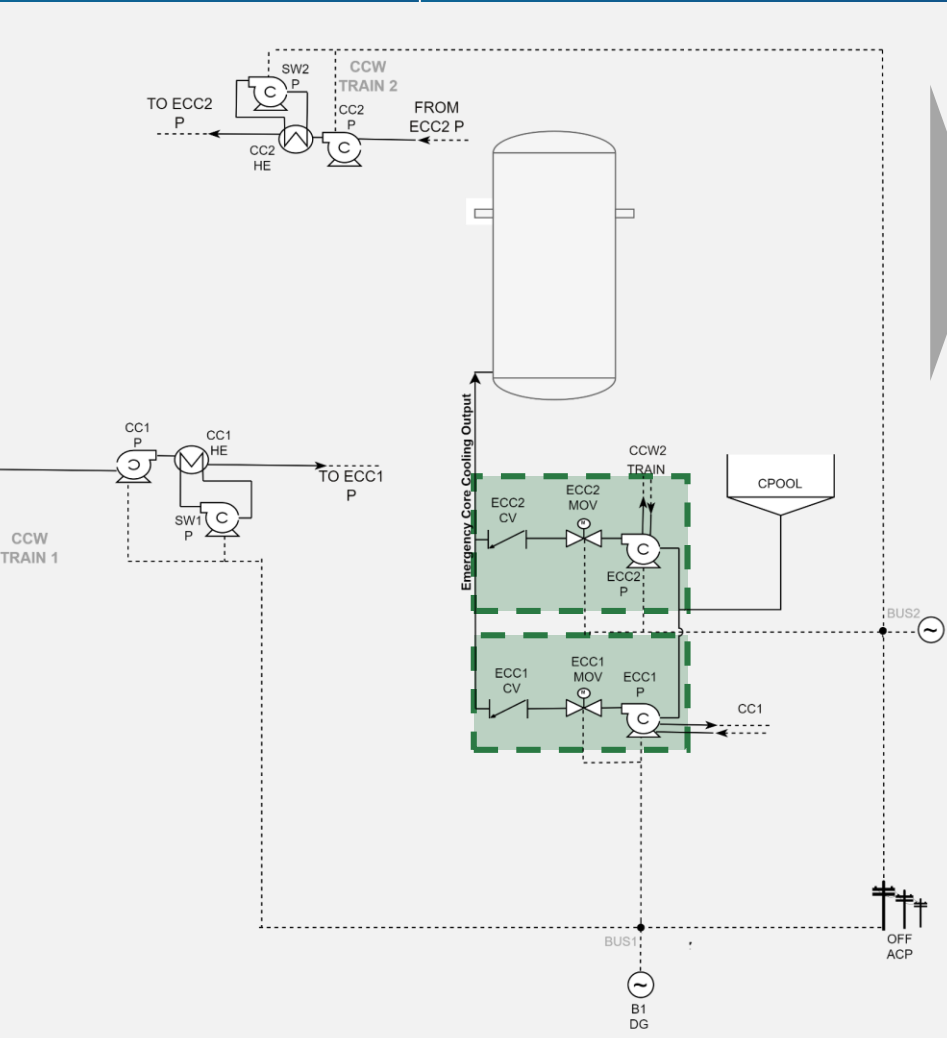


# D<sup>2</sup>T<sup>2</sup>: Trains Dependency



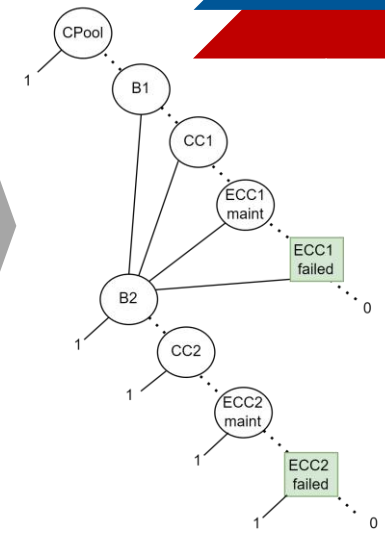
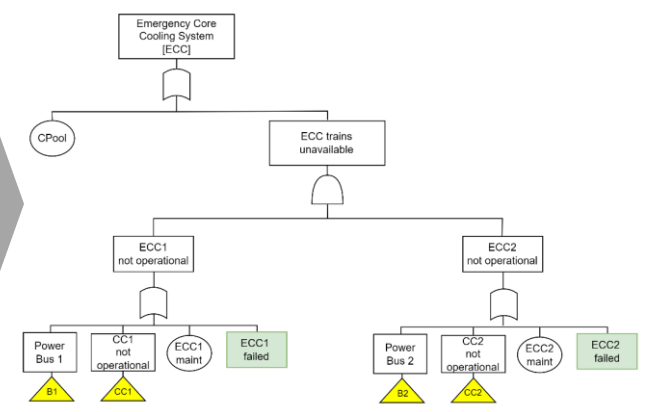
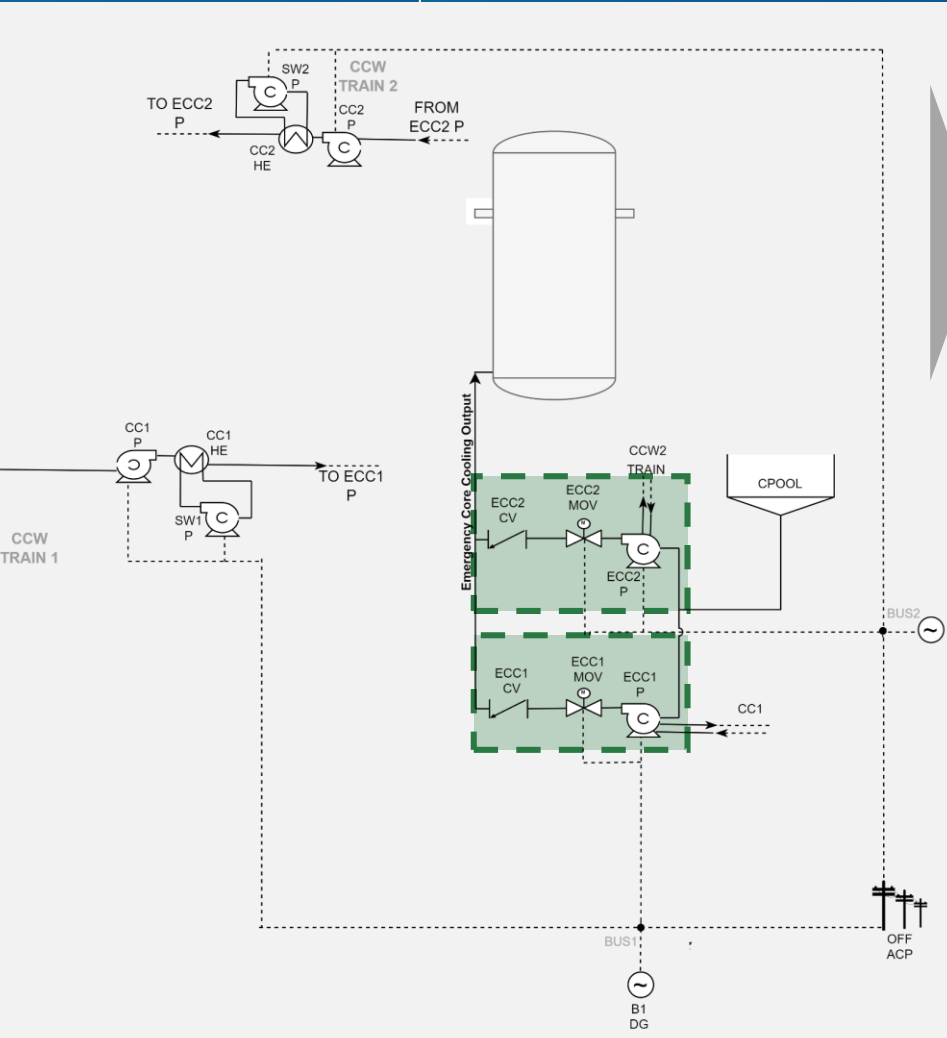
**TOP EVENT  
PROBABILITY  
=**  
**0.00541457**

# D<sup>2</sup>T<sup>2</sup>: Trains Dependency



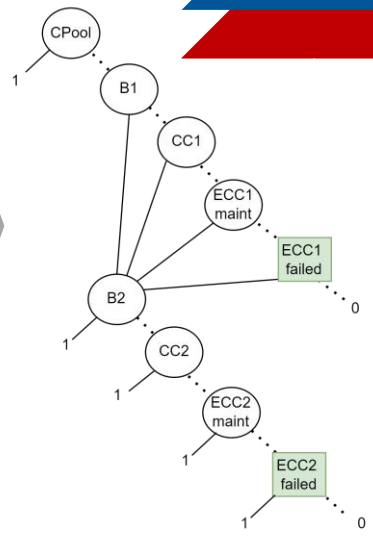
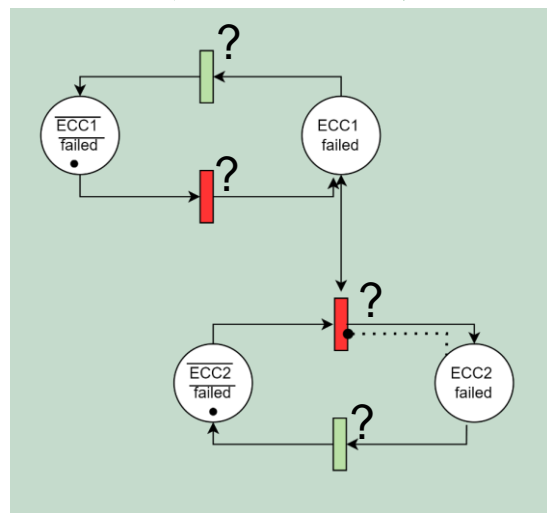
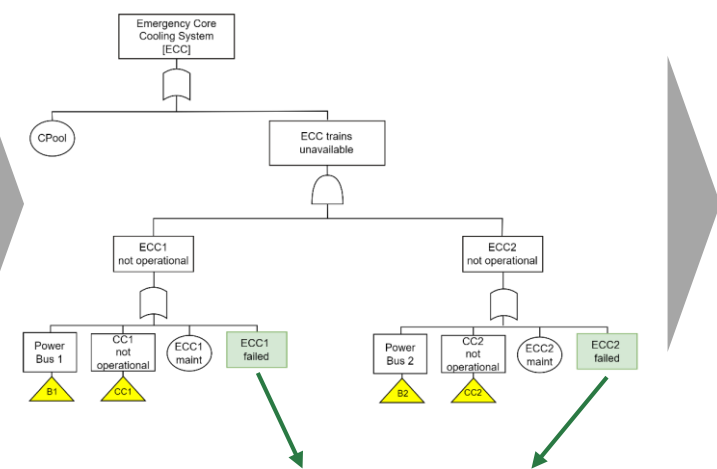
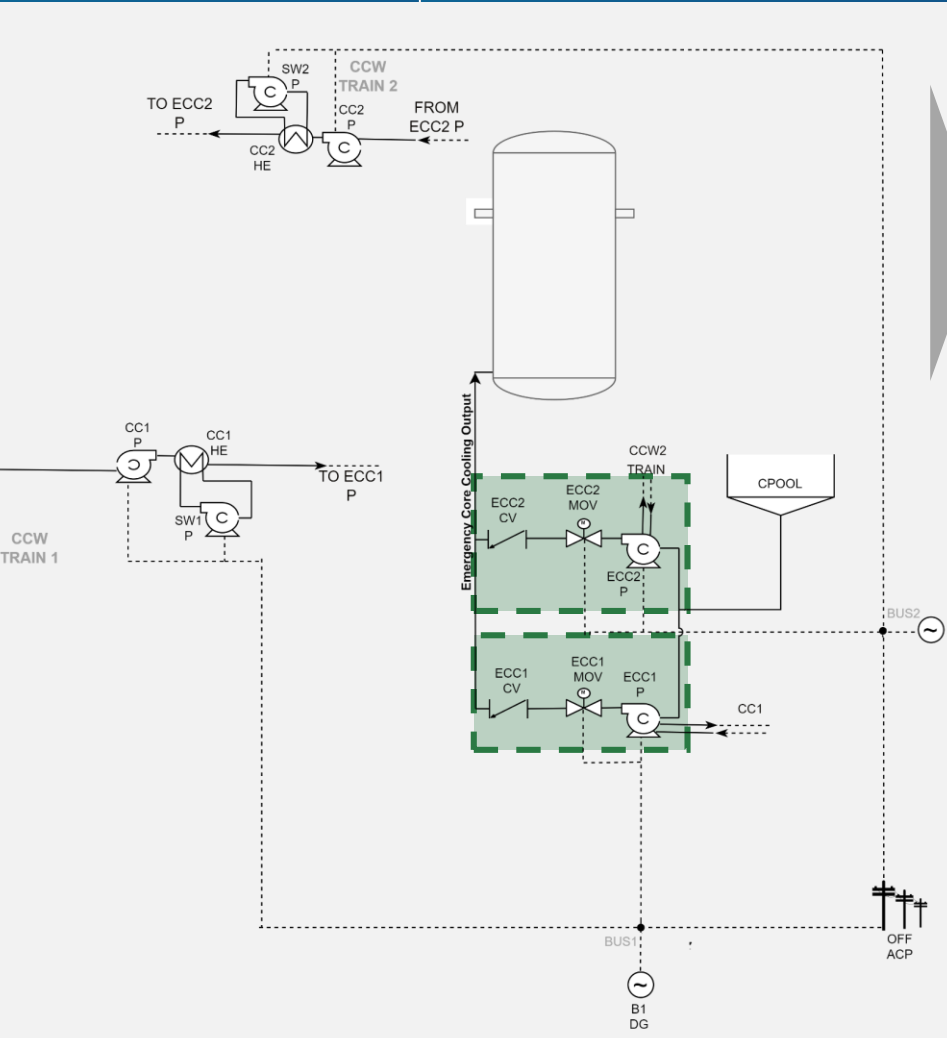
**TOP EVENT PROBABILITY**  
= ?

# D<sup>2</sup>T<sup>2</sup>: Trains Dependency



**TOP EVENT PROBABILITY**  
= ?

# D<sup>2</sup>T<sup>2</sup>: Trains Dependency

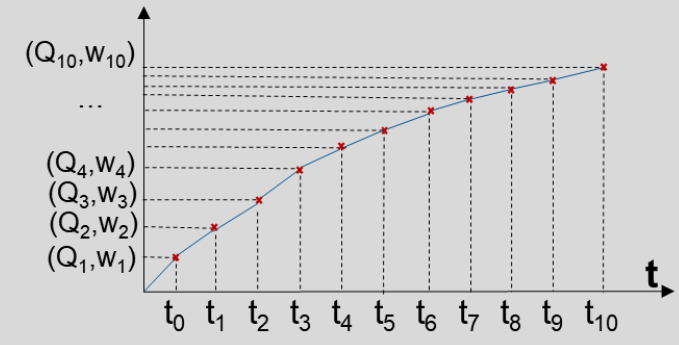
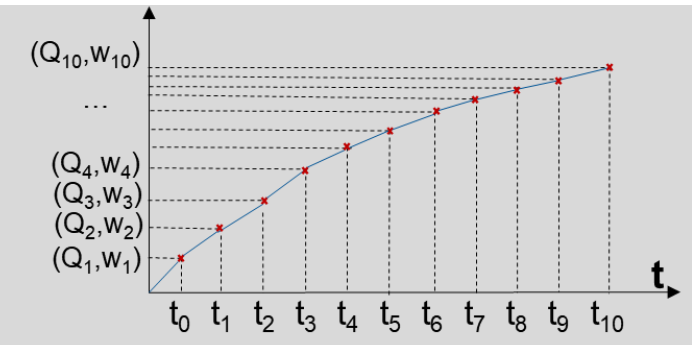
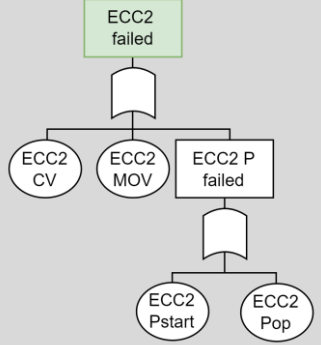
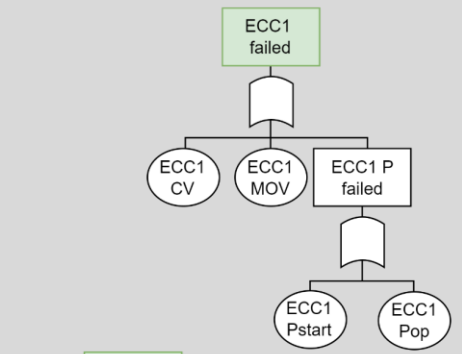
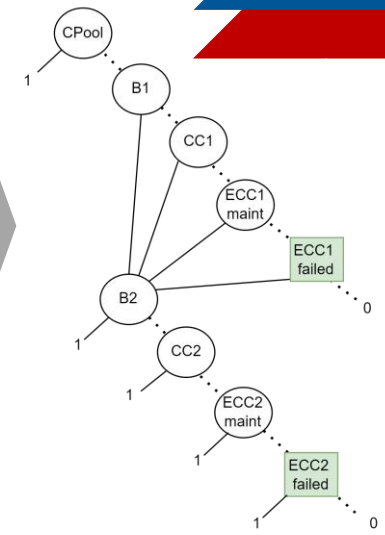
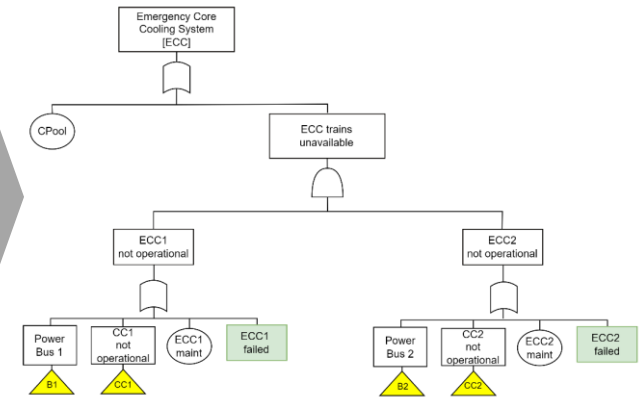
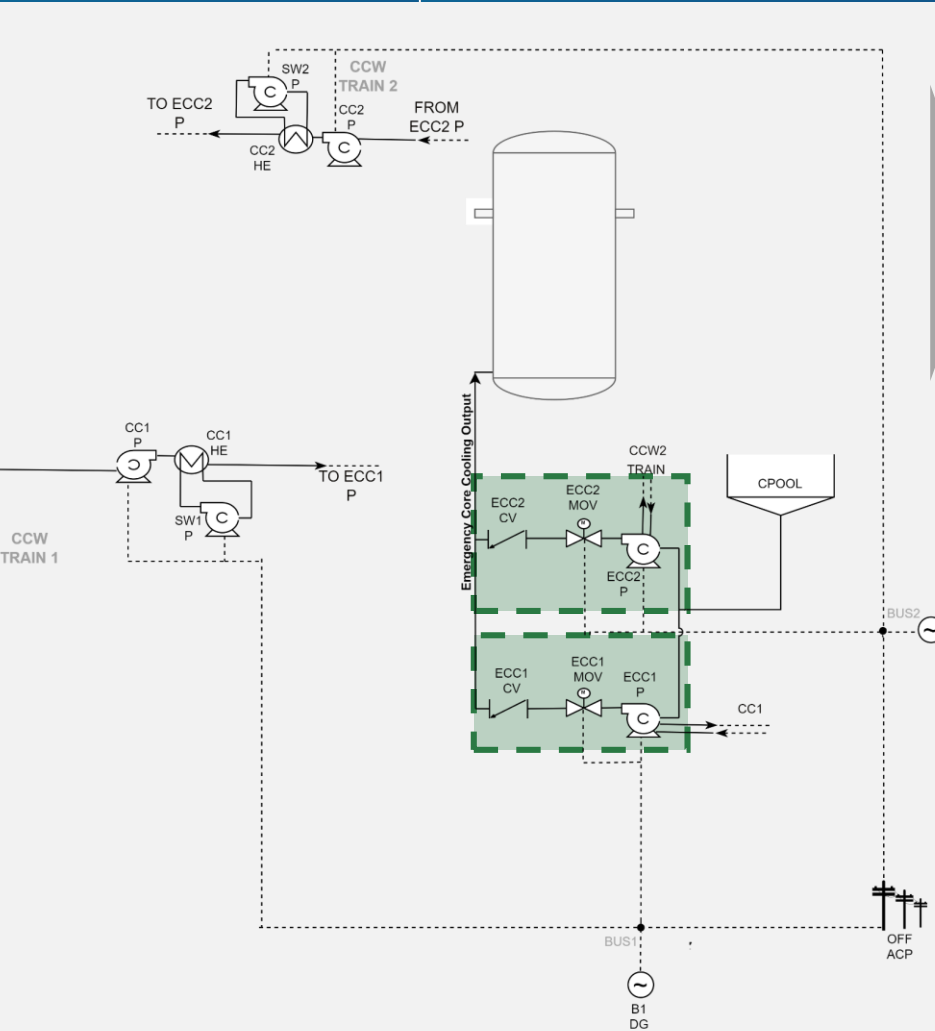


**TOP EVENT PROBABILITY**  
= ?

# D<sup>2</sup>T<sup>2</sup>: Trains Dependency



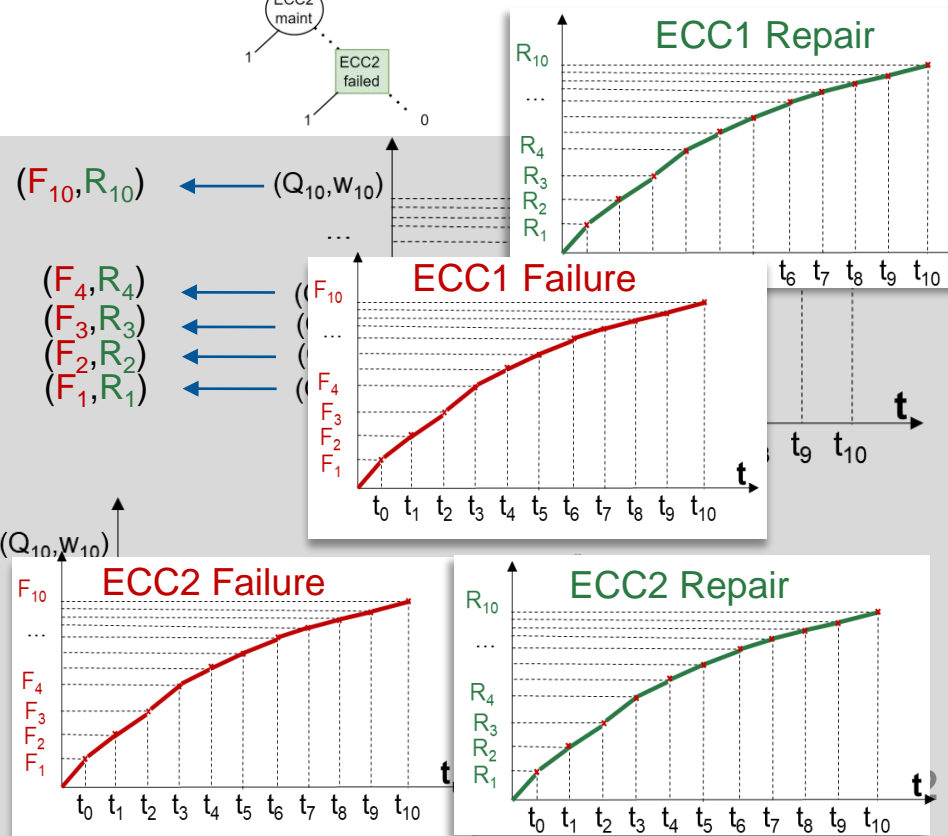
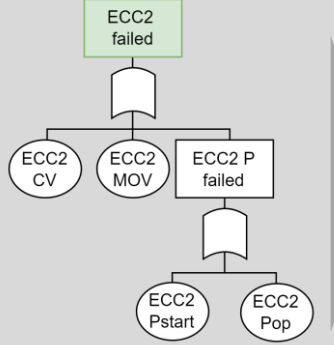
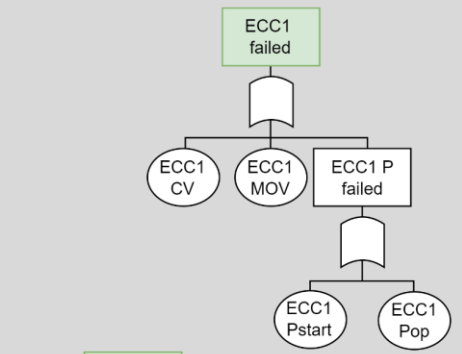
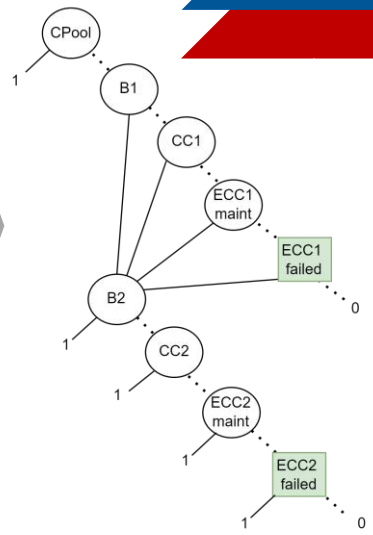
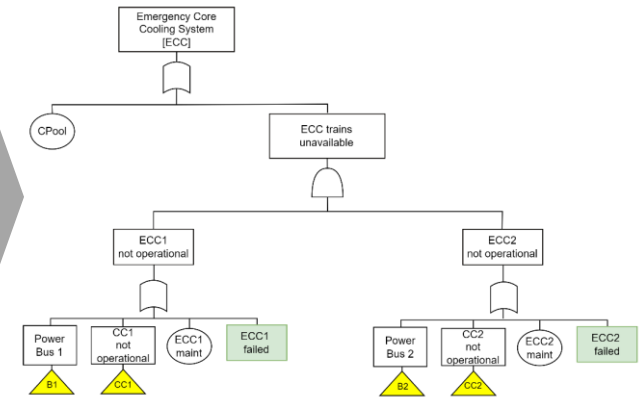
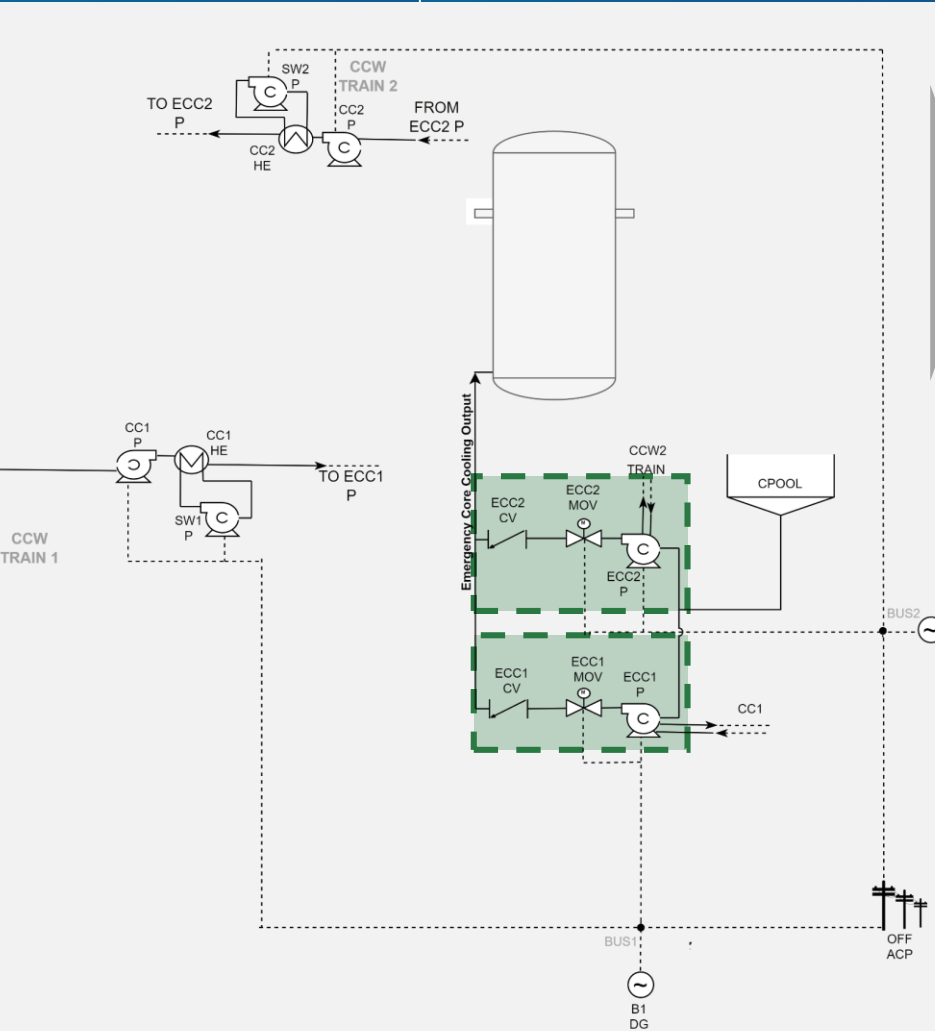
**TOP EVENT PROBABILITY = ?**



# D<sup>2</sup>T<sup>2</sup>: Trains Dependency



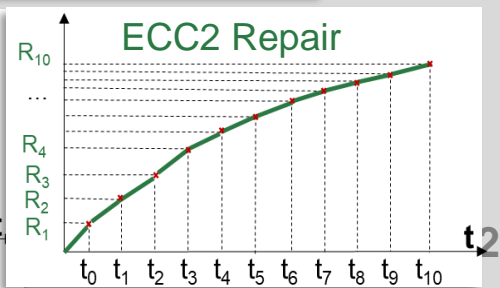
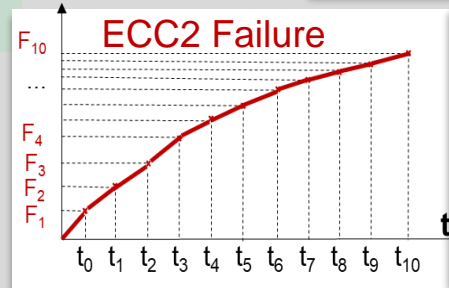
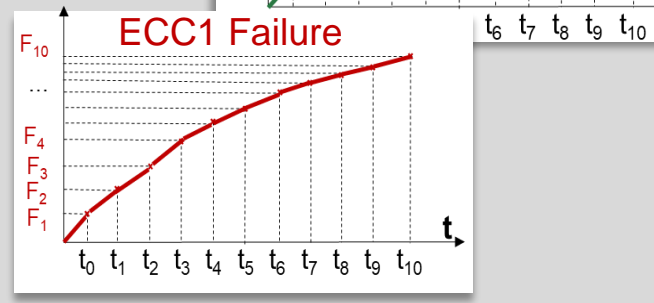
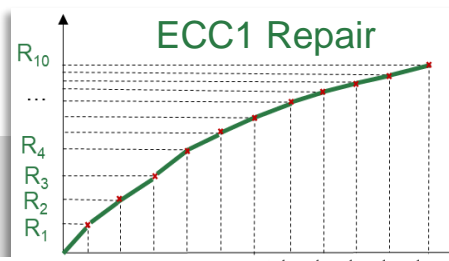
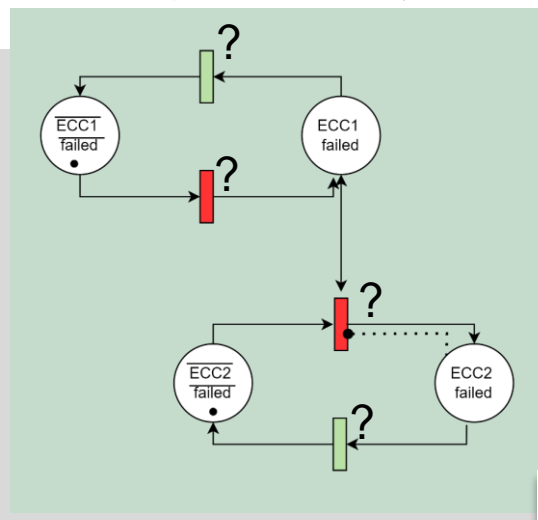
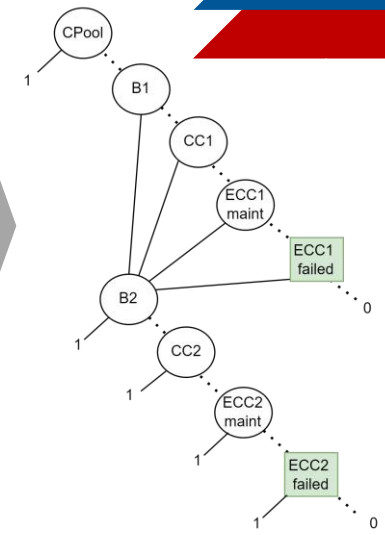
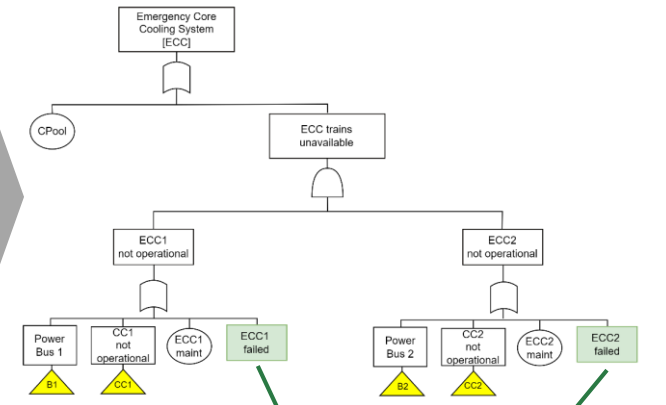
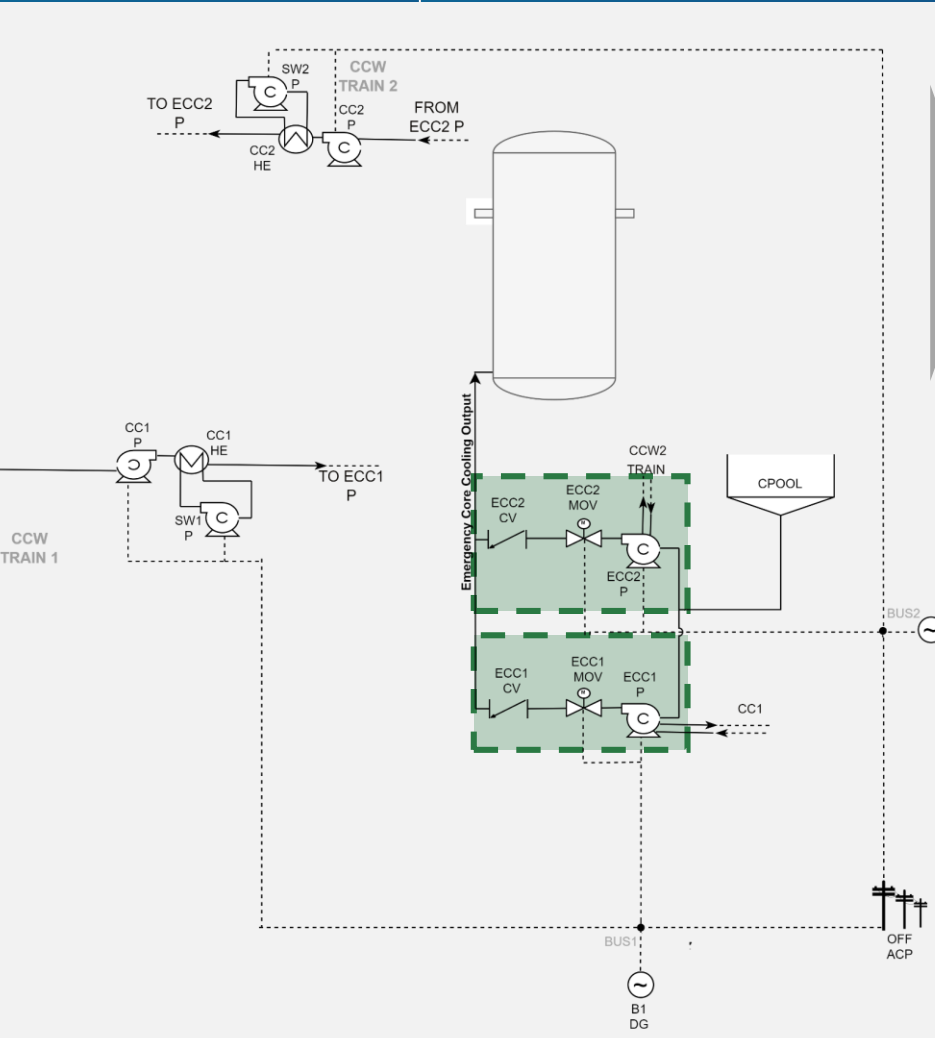
**TOP EVENT PROBABILITY = ?**



# D<sup>2</sup>T<sup>2</sup>: Trains Dependency

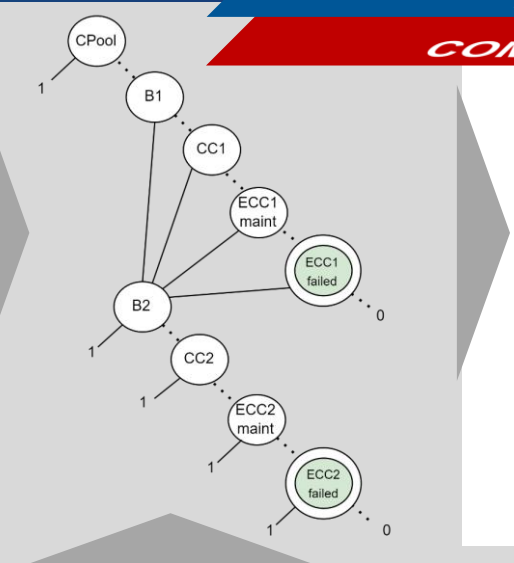
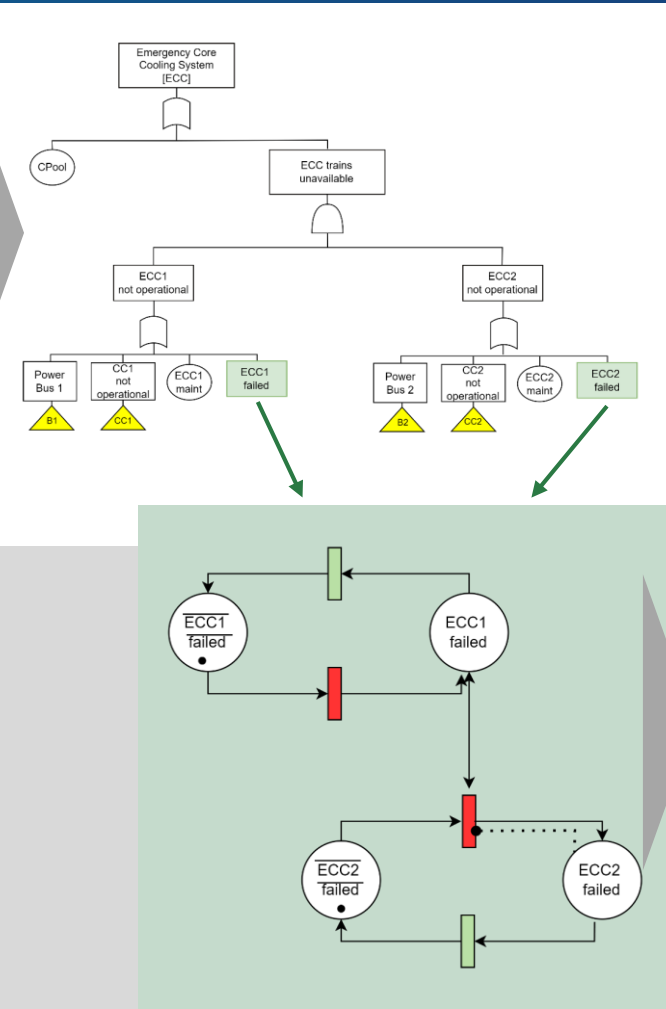
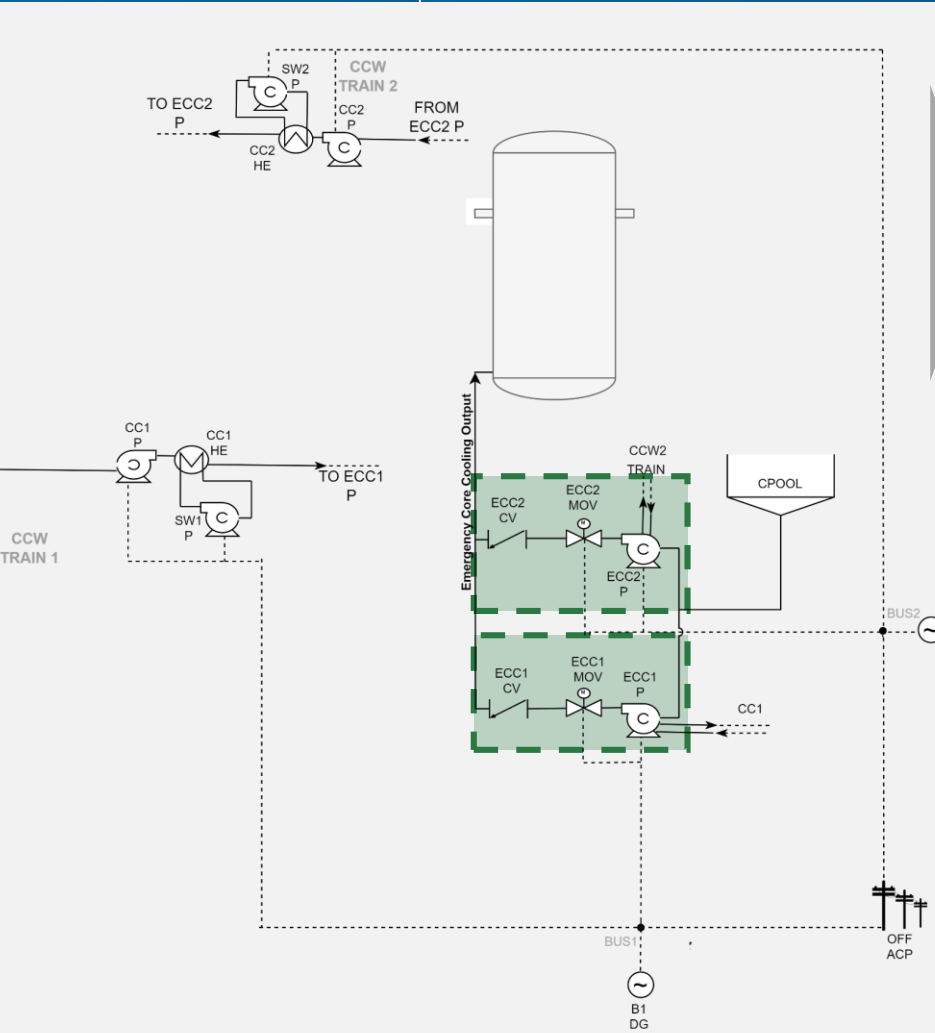


TOP EVENT  
PROBABILITY  
= ?



# D<sup>2</sup>T<sup>2</sup>: Trains Dependency

TRAINS  
COMPONENTS



**TOP EVENT PROBABILITY = 0.0035**

Joint Event	Probability
$\overline{ECC1 fail}, \overline{ECC2 fail}$	$9.996 \cdot 10^{-1}$
$\overline{ECC1 fail}, ECC2 fail$	$1.26 \cdot 10^{-5}$
$ECC1 fail, \overline{ECC2 fail}$	$5.20 \cdot 10^{-2}$
$ECC1 fail, ECC2 fail$	$1.39 \cdot 10^{-5}$

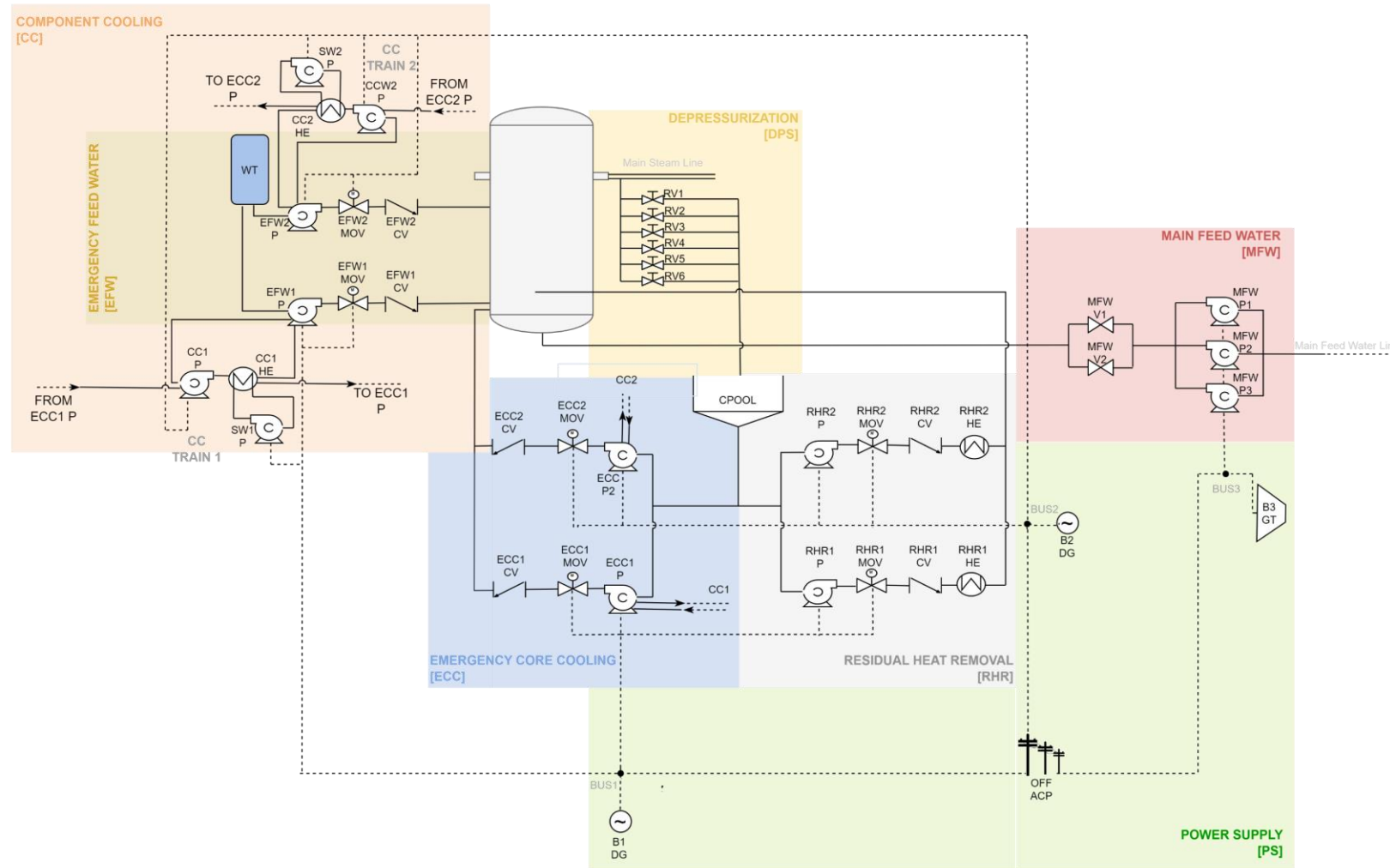


# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency

SUBSYSTEMS

TRAINS

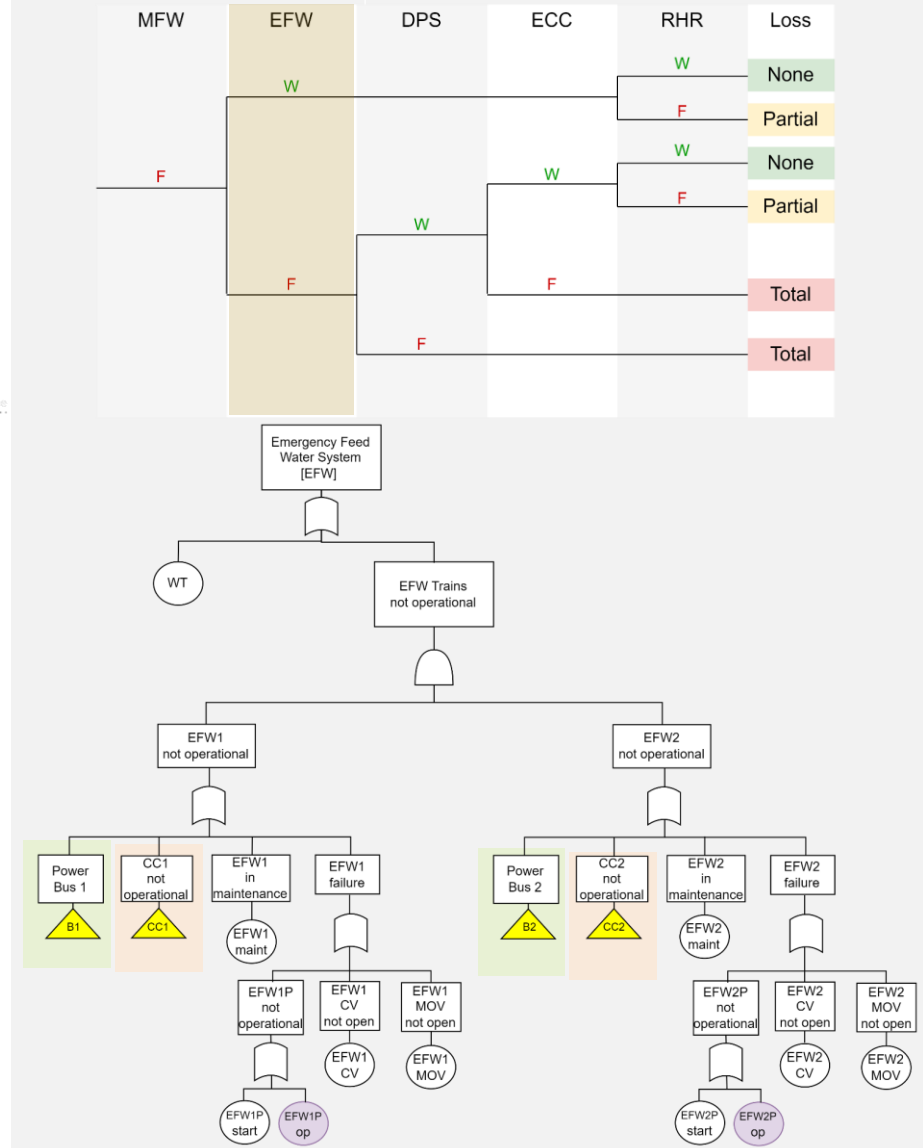
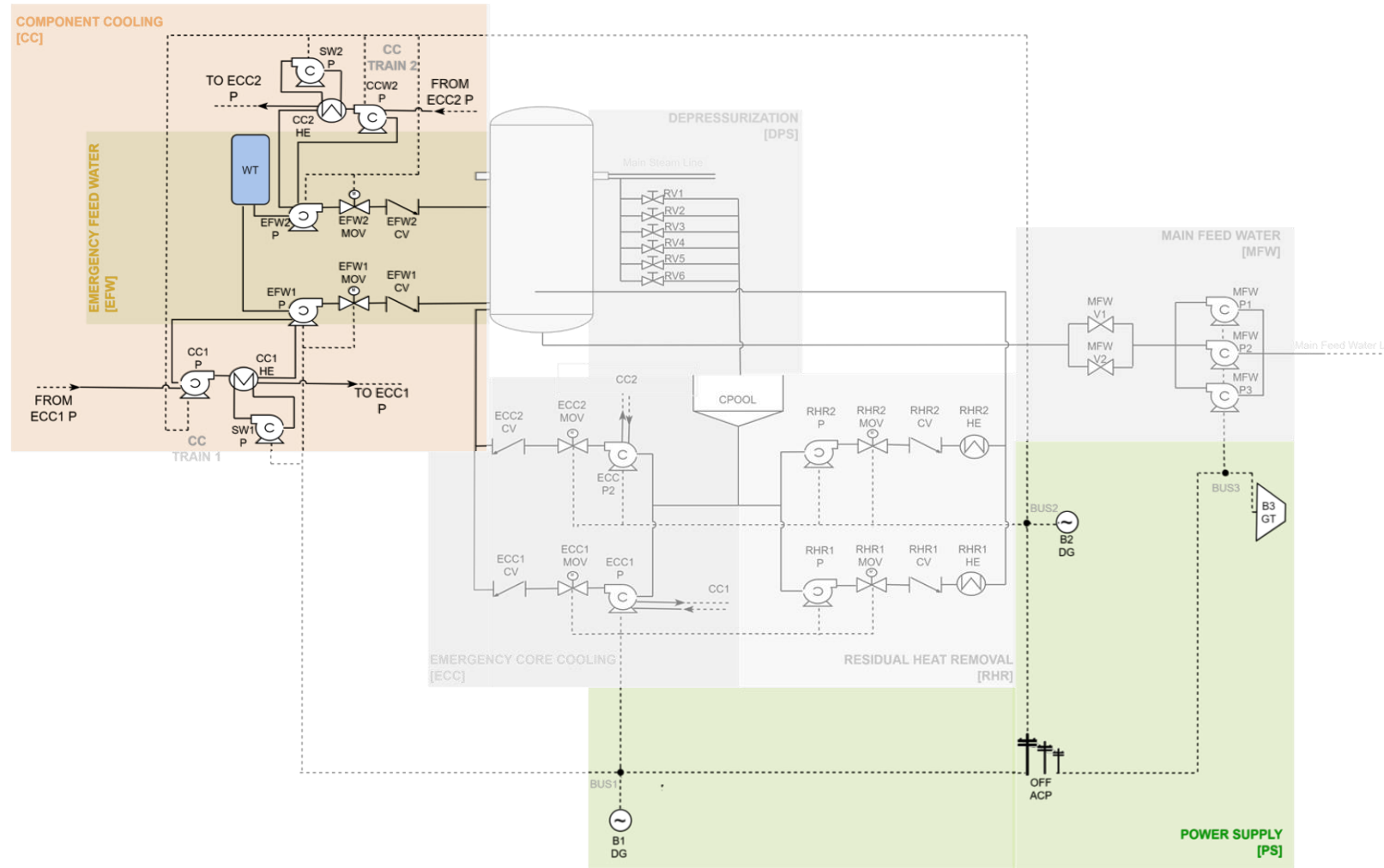
COMPONENTS



MFW	EFW	DPS	ECC	RHR	Loss
	W			W	None
				F	Partial
			W	W	None
F				F	Partial
	F	W			Total
		F	F		Total

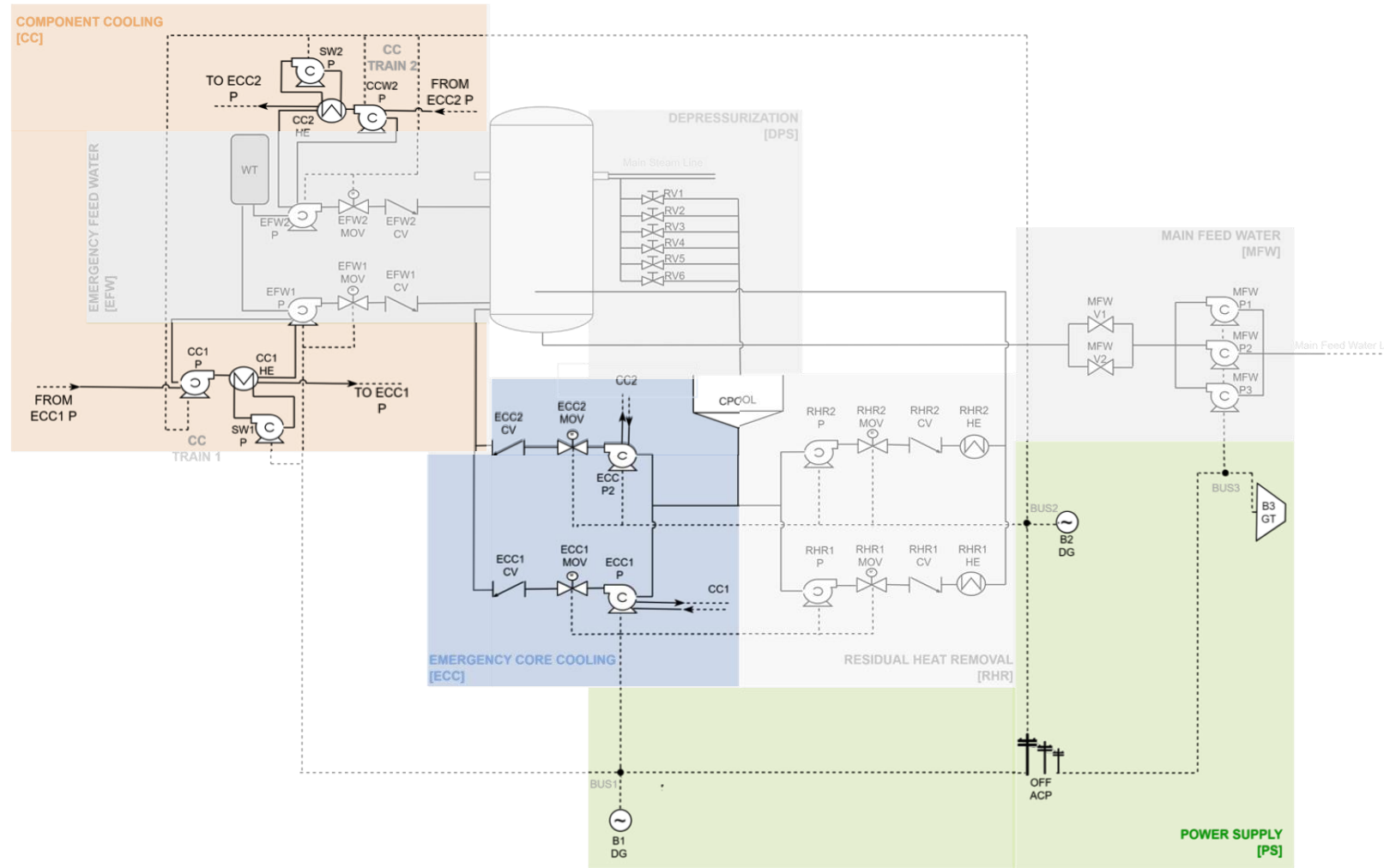
# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency

**SUBSYSTEMS**  
**TRAINS**  
**COMPONENTS**

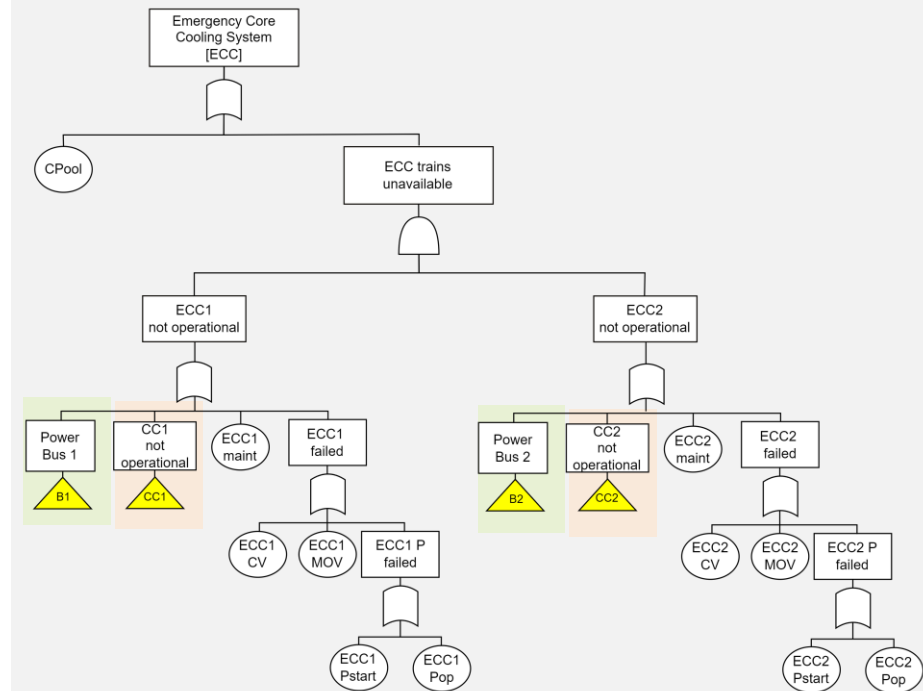


# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency

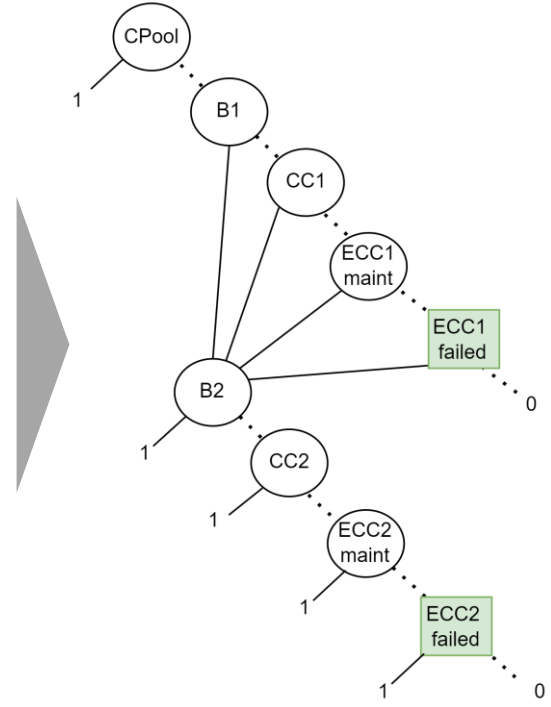
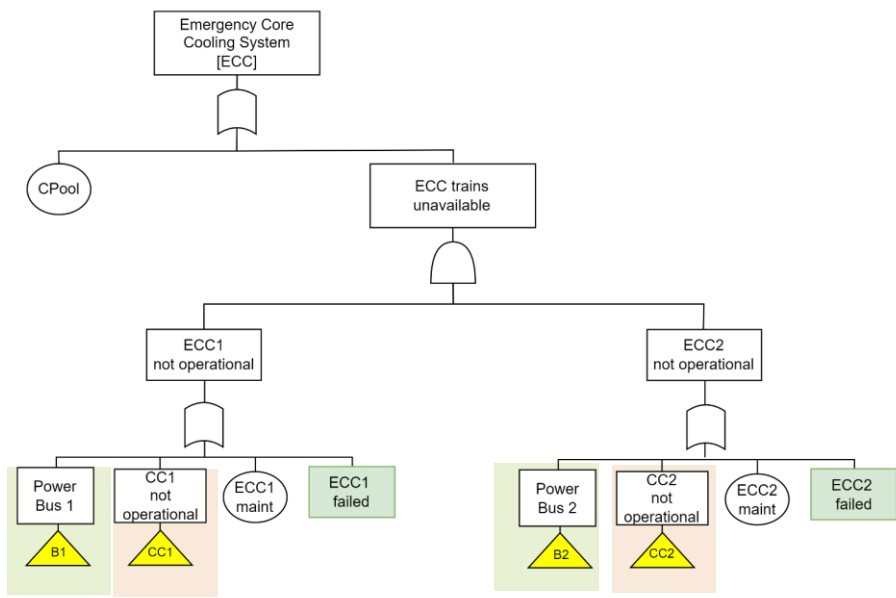
**SUBSYSTEMS**  
**TRAINS**  
**COMPONENTS**



MFW	EFW	DPS	ECC	RHR	Loss
	W		W	W	None
				F	Partial
			W	W	None
F				F	Partial
	F	W			Total
		F	F		Total



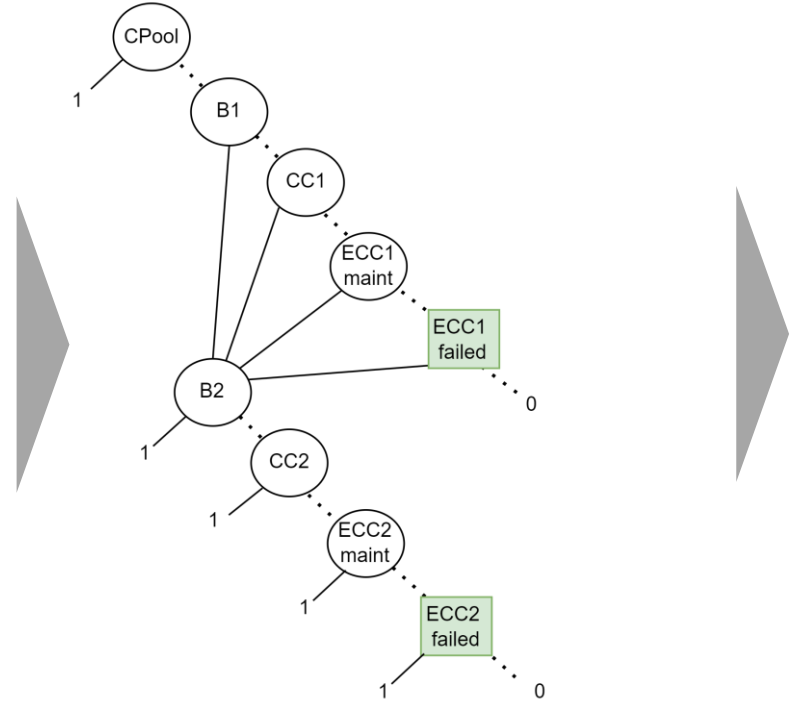
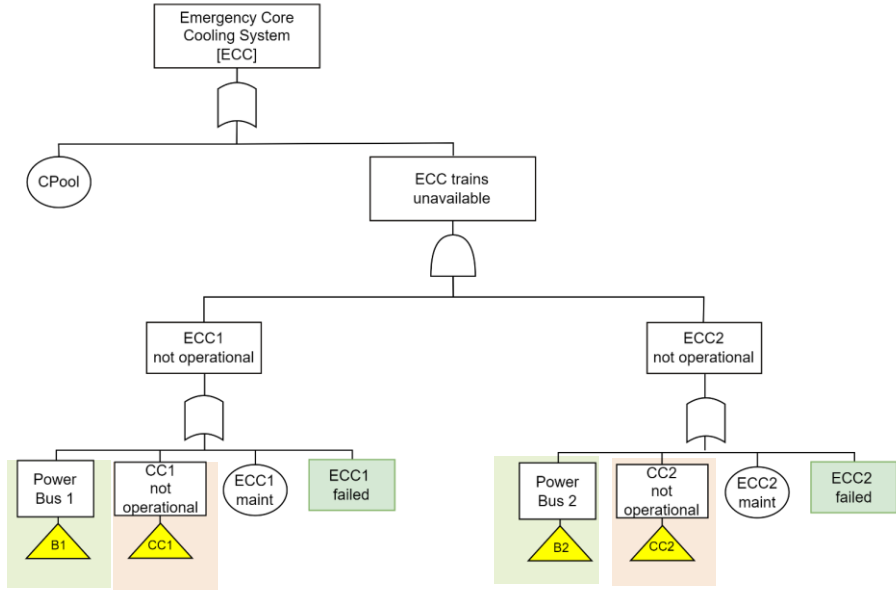
# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



$$P_{top} = \sum_i^n path_i = 0.0035$$

$$\vec{P}_{top} = \{p_{ss0}, p_{ss1}, \dots, p_{ssn}\}$$

# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency

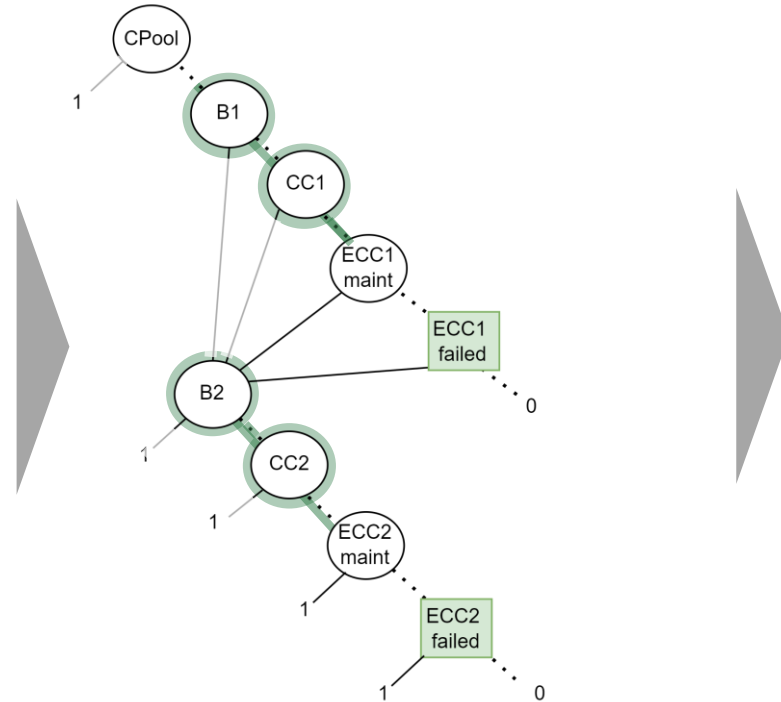
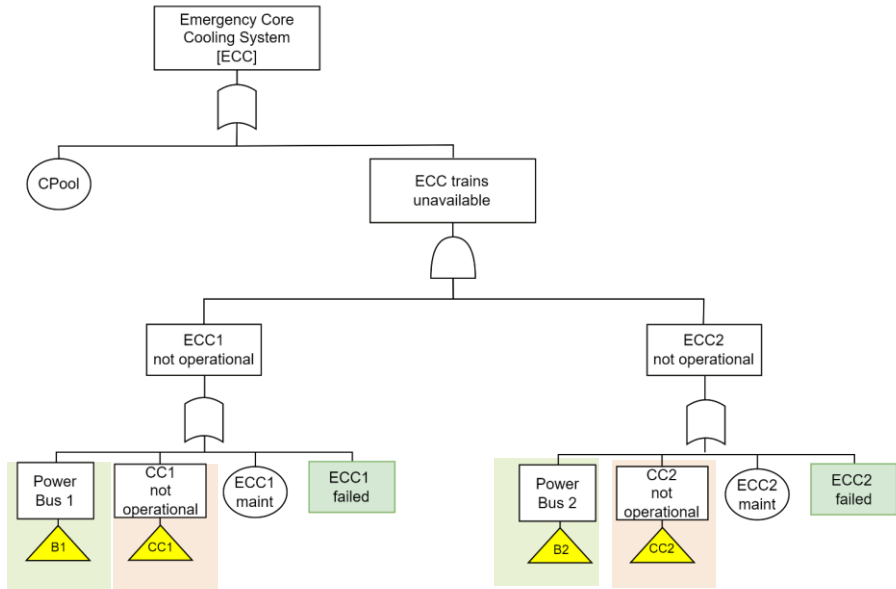


- B1, CC1, B2, CC2
- B1, CC1, B2,  $\overline{CC2}$
- B1, CC1,  $\overline{B2}$ , CC2
- B1, CC1,  $\overline{B2}$ ,  $\overline{CC2}$
- B1,  $\overline{CC1}$ , B2, CC2
- B1,  $\overline{CC1}$ , B2,  $\overline{CC2}$
- B1,  $\overline{CC1}$ ,  $\overline{B2}$ , CC2
- B1,  $\overline{CC1}$ ,  $\overline{B2}$ ,  $\overline{CC2}$
- $\overline{B1}$ , CC1, B2, CC2
- $\overline{B1}$ , CC1, B2,  $\overline{CC2}$
- $\overline{B1}$ , CC1,  $\overline{B2}$ , CC2
- $\overline{B1}$ , CC1,  $\overline{B2}$ ,  $\overline{CC2}$
- $\overline{B1}$ ,  $\overline{CC1}$ , B2, CC2
- $\overline{B1}$ ,  $\overline{CC1}$ , B2,  $\overline{CC2}$
- $\overline{B1}$ ,  $\overline{CC1}$ ,  $\overline{B2}$ , CC2
- $\overline{B1}$ ,  $\overline{CC1}$ ,  $\overline{B2}$ ,  $\overline{CC2}$

$P_{top}$

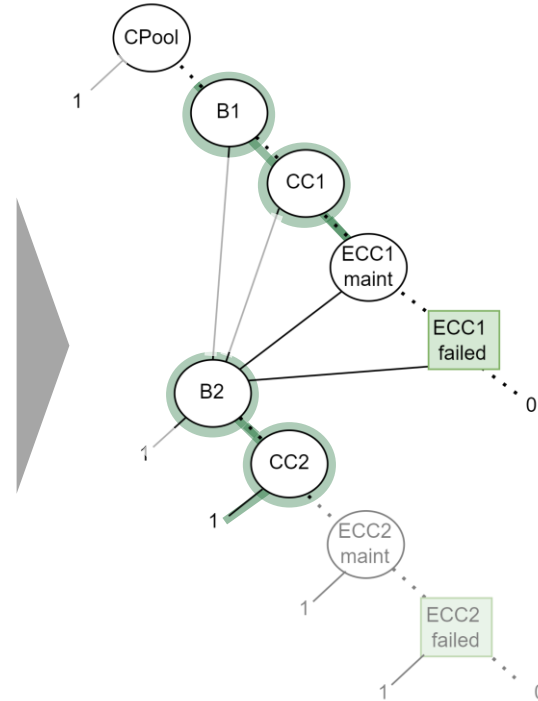
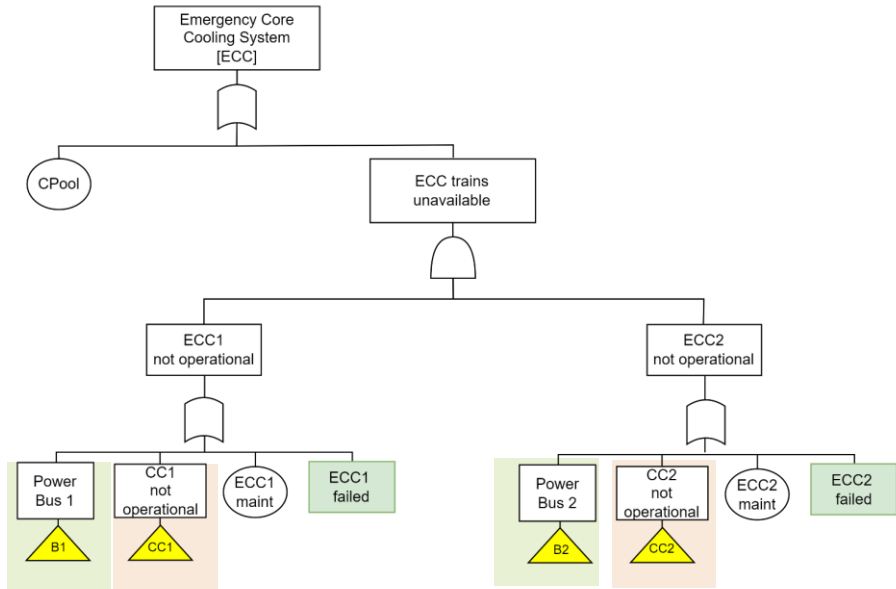


# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



- B1, CC1, B2, CC2
  - B1, CC1, B2,  $\overline{CC2}$
  - B1, CC1,  $\overline{B2}$ , CC2
  - B1, CC1,  $\overline{B2}$ ,  $\overline{CC2}$
  - B1,  $\overline{CC1}$ , B2, CC2
  - B1,  $\overline{CC1}$ , B2,  $\overline{CC2}$
  - B1,  $\overline{CC1}$ ,  $\overline{B2}$ , CC2
  - B1,  $\overline{CC1}$ ,  $\overline{B2}$ ,  $\overline{CC2}$
  - $\overline{B1}$ , CC1, B2, CC2
  - $\overline{B1}$ , CC1, B2,  $\overline{CC2}$
  - $\overline{B1}$ , CC1,  $\overline{B2}$ , CC2
  - $\overline{B1}$ , CC1,  $\overline{B2}$ ,  $\overline{CC2}$
  - $\overline{B1}$ ,  $\overline{CC1}$ , B2, CC2
  - $\overline{B1}$ ,  $\overline{CC1}$ , B2,  $\overline{CC2}$
  - $\overline{B1}$ ,  $\overline{CC1}$ ,  $\overline{B2}$ , CC2
  - $\overline{B1}$ ,  $\overline{CC1}$ ,  $\overline{B2}$ ,  $\overline{CC2}$
- $P_{top}$
- $\overline{B1}, \overline{CC1}, \overline{B2}, \overline{CC2}$   $5.62 \cdot 10^{-04}$

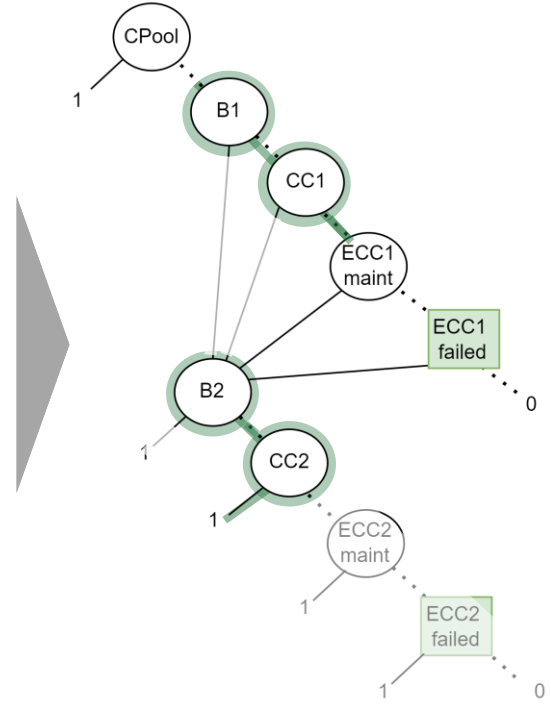
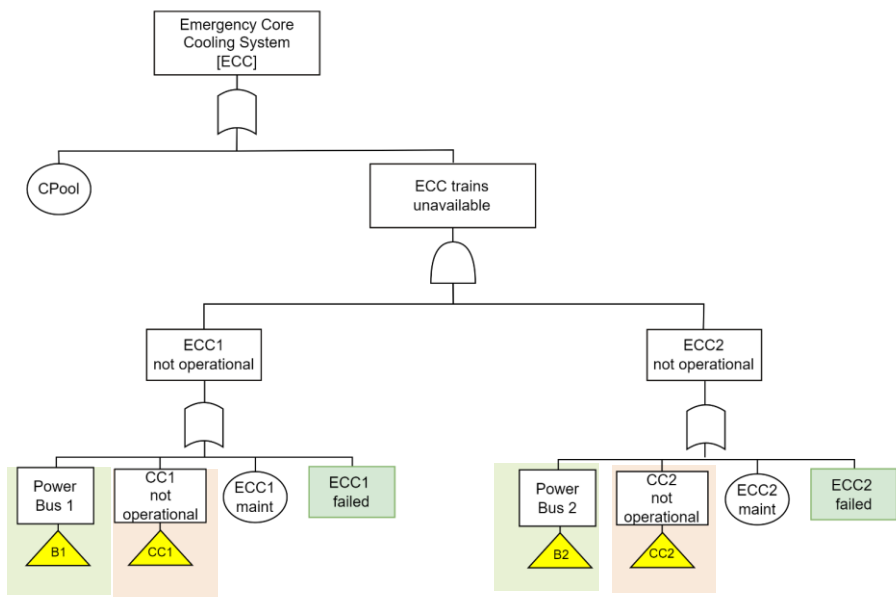
# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



- B1, CC1, B2, CC2
  - B1, CC1, B2,  $\overline{CC2}$
  - B1, CC1,  $\overline{B2}$ , CC2
  - B1, CC1,  $\overline{B2}$ ,  $\overline{CC2}$
  - B1,  $\overline{CC1}$ , B2, CC2
  - B1,  $\overline{CC1}$ , B2,  $\overline{CC2}$
  - B1,  $\overline{CC1}$ ,  $\overline{B2}$ , CC2
  - B1,  $\overline{CC1}$ ,  $\overline{B2}$ ,  $\overline{CC2}$
  - $\overline{B1}$ , CC1, B2, CC2
  - $\overline{B1}$ , CC1, B2,  $\overline{CC2}$
  - $\overline{B1}$ , CC1,  $\overline{B2}$ , CC2
  - $\overline{B1}$ , CC1,  $\overline{B2}$ ,  $\overline{CC2}$
  - $\overline{B1}$ ,  $\overline{CC1}$ , B2, CC2
  - $\overline{B1}$ ,  $\overline{CC1}$ , B2,  $\overline{CC2}$
  - $\overline{B1}$ ,  $\overline{CC1}$ ,  $\overline{B2}$ , CC2
  - $\overline{B1}$ ,  $\overline{CC1}$ ,  $\overline{B2}$ ,  $\overline{CC2}$
- $P_{top}$

→  $\overline{B1}, \overline{CC1}, \overline{B2}, CC2$   $3.30 \cdot 10^{-02}$   
 $B1, CC1, B2, CC2$   $5.62 \cdot 10^{-04}$

# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



$B1, CC1, B2, CC2$	1
$B1, CC1, B2, \overline{CC2}$	0
$B1, CC1, \overline{B2}, CC2$	1
$B1, CC1, \overline{B2}, \overline{CC2}$	$3.30 \cdot 10^{-02}$
$B1, \overline{CC1}, B2, CC2$	0
$B1, \overline{CC1}, B2, \overline{CC2}$	0
$B1, \overline{CC1}, \overline{B2}, CC2$	0
$B1, \overline{CC1}, \overline{B2}, \overline{CC2}$	0
$\overline{B1}, CC1, B2, CC2$	1
$\overline{B1}, CC1, B2, \overline{CC2}$	0
$\overline{B1}, CC1, \overline{B2}, CC2$	1
$\overline{B1}, CC1, \overline{B2}, \overline{CC2}$	$3.30 \cdot 10^{-02}$
$\overline{B1}, \overline{CC1}, B2, CC2$	$3.30 \cdot 10^{-02}$
$\overline{B1}, \overline{CC1}, B2, \overline{CC2}$	0
$\overline{B1}, \overline{CC1}, \overline{B2}, CC2$	$3.30 \cdot 10^{-02}$
$\overline{B1}, \overline{CC1}, \overline{B2}, \overline{CC2}$	0
$B1, CC1, B2, CC2$	$5.62 \cdot 10^{-04}$

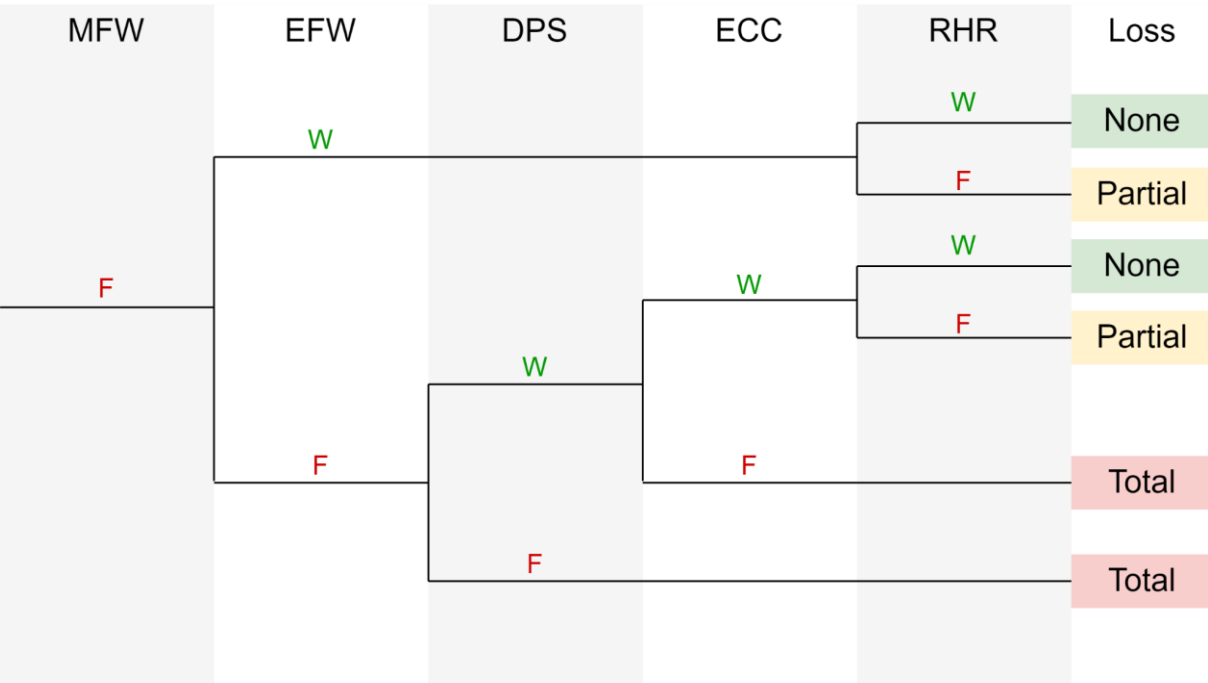
$P_{top}$







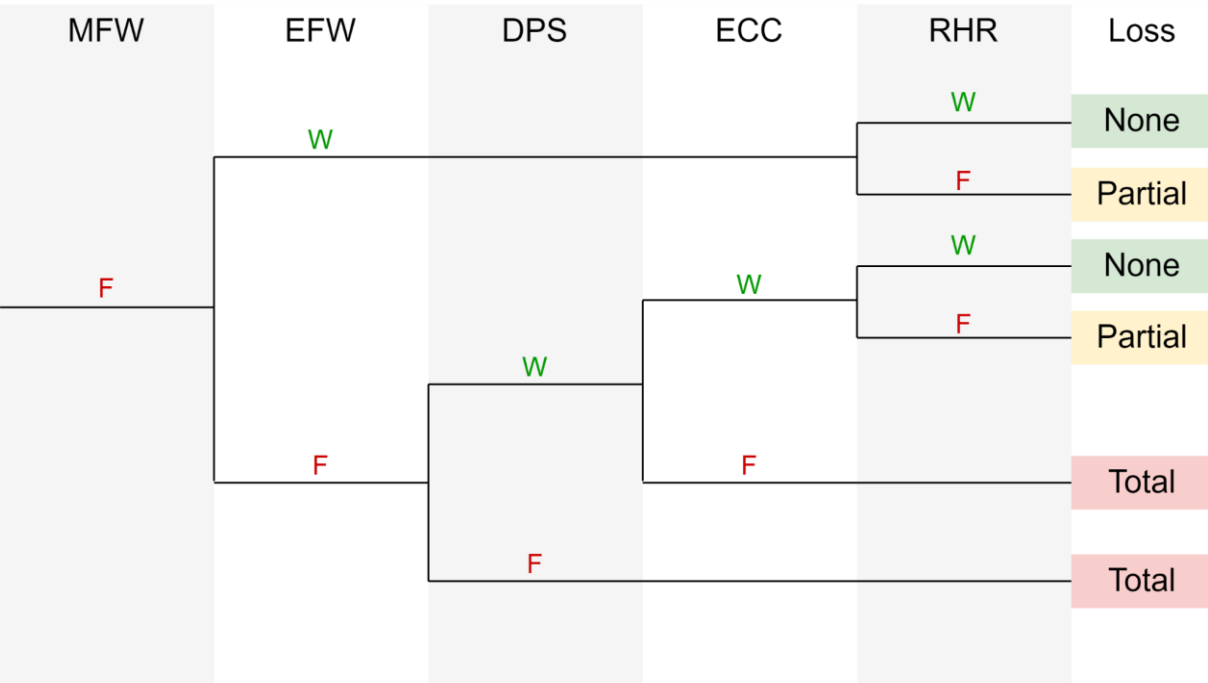
# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



$$\vec{W}_{none1} = \vec{W}_{MFW} \cdot \vec{P}_{EFW} \cdot \vec{P}_{RHR} \cdot \vec{P}_{SS}$$



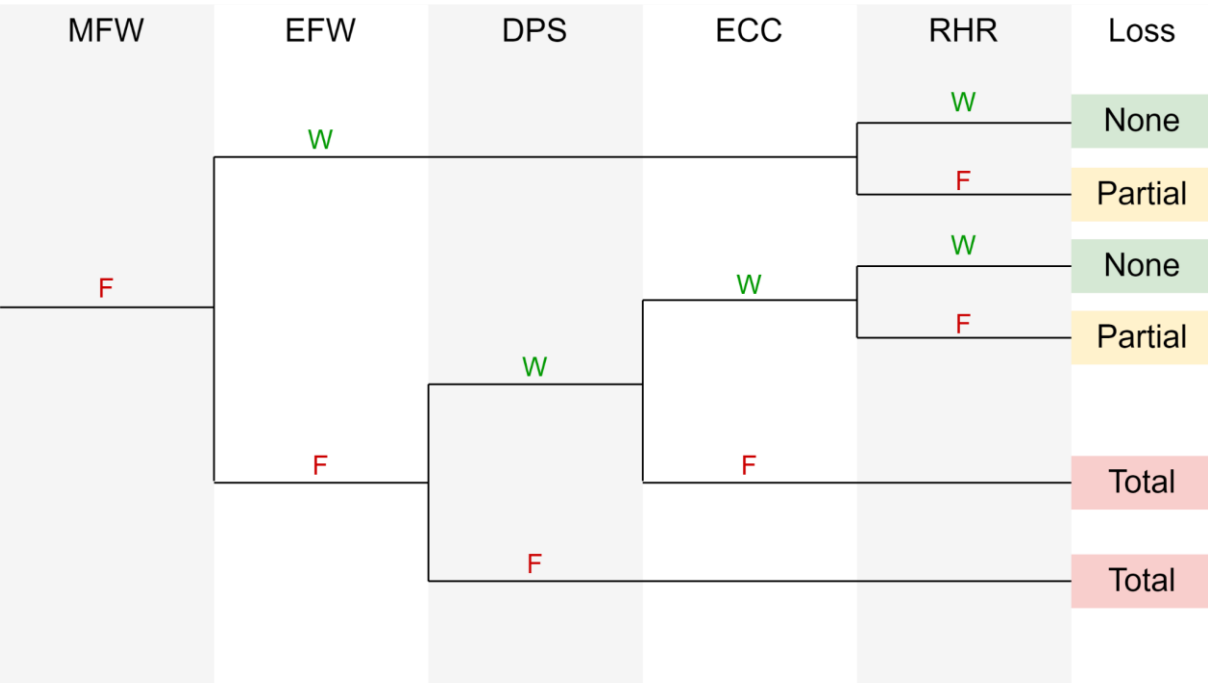
# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



$$\vec{W}_{partial1} = \vec{W}_{MFW} \cdot \vec{P}_{EFW} \cdot \vec{P}_{RHR} \cdot \vec{P}_{SS}$$



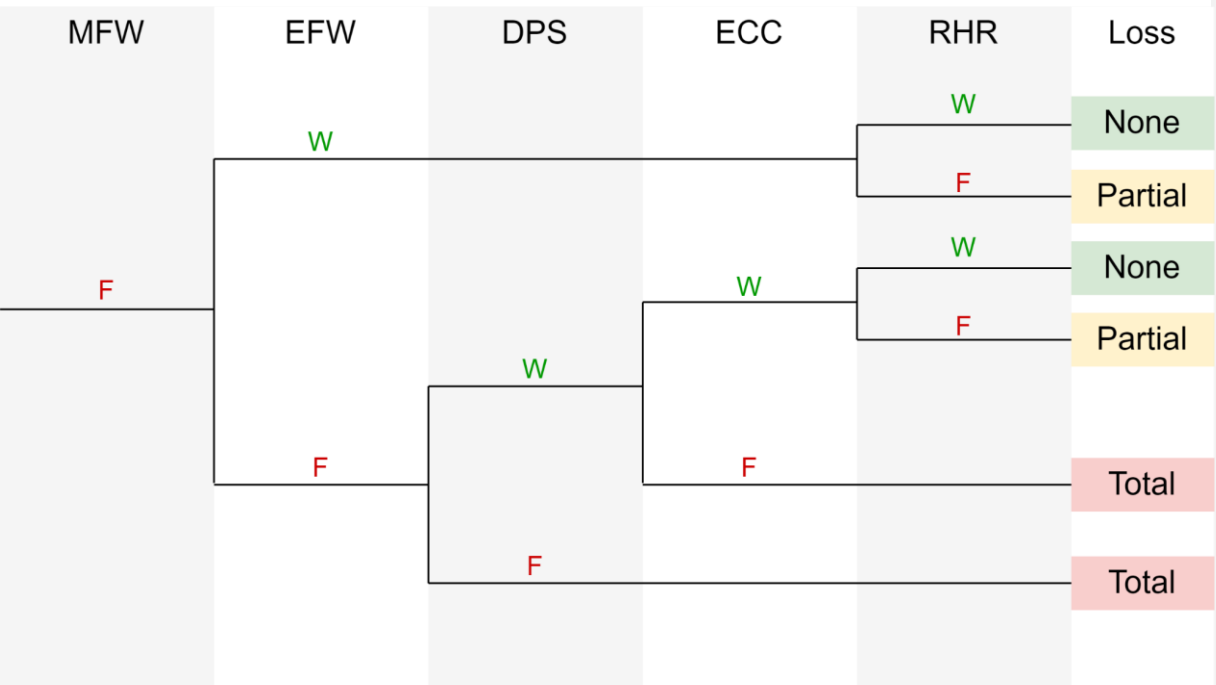
# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



$$\vec{W}_{none2} = \vec{W}_{MFW} \cdot \vec{P}_{EFW} \cdot \vec{P}_{DPS} \cdot \vec{P}_{ECC} \cdot \vec{P}_{RHR} \cdot \vec{P}_{SS}$$



# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



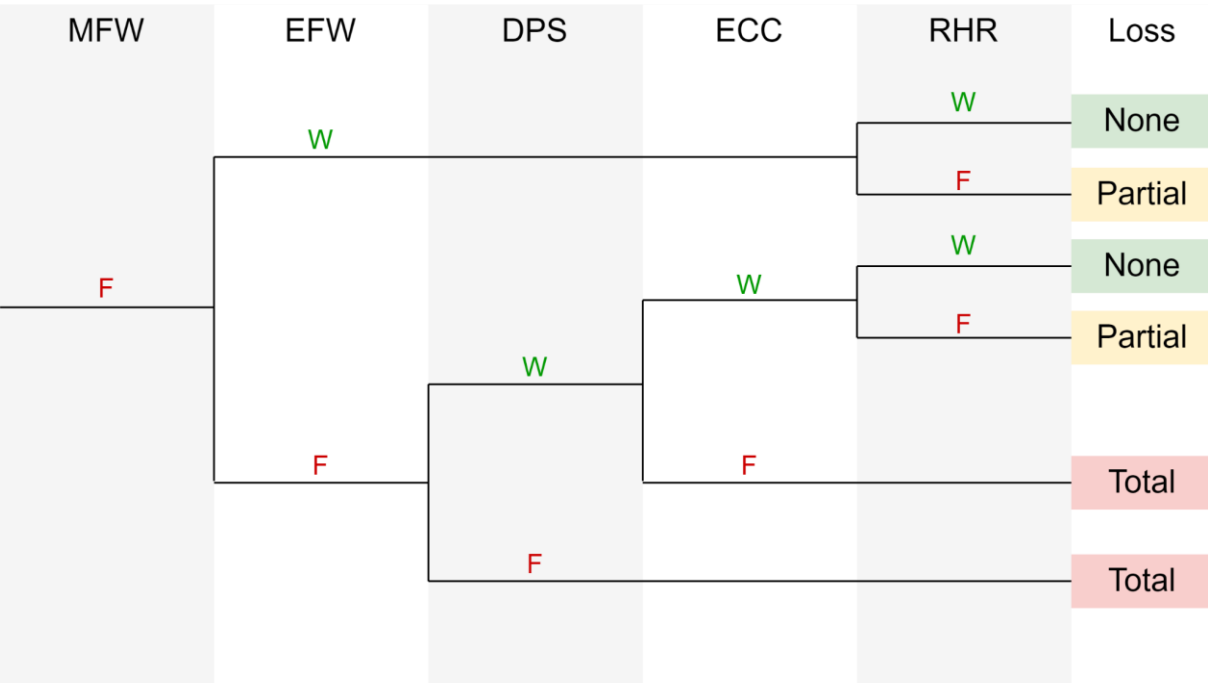
$$\vec{W}_{none} = \vec{W}_{none1} + \vec{W}_{none2}$$

$$\vec{W}_{partial} = \vec{W}_{partial1} + \vec{W}_{partial2}$$

$$\vec{W}_{total} = \vec{W}_{total1} + \vec{W}_{total2}$$



# D<sup>2</sup>T<sup>2</sup>: Subsystems Dependency



$$W_{none} = \sum_i \overline{W}^i_{none} = 2.371 \cdot 10^{-6} \text{ h}^{-1}$$

$$W_{partial} = \sum_i \overline{W}^i_{partial} = 9.977 \cdot 10^{-9} \text{ h}^{-1}$$

$$W_{total} = \sum_i \overline{W}^i_{total} = 5.345 \cdot 10^{-9} \text{ h}^{-1}$$



University of  
**Nottingham**

UK | CHINA | MALAYSIA

# Summing Up

Conclusions

- Umbrella methodology integrating flexible modelling techniques within traditional system safety methodologies
- Retains modelling framework familiarity, intuitivity and efficiency while enhancing accuracy
- High potential for modularization
- Dependencies included at any level of system safety modelling
- Algorithms and computational tools available (*NxGen Tool*)
- Removing hidden assumptions



## REFERENCES

- Andrews, John, and Silvia Tolo. "*Dynamic and dependent tree theory (D2T2): A framework for the analysis of fault trees with dependent basic events.*" *Reliability Engineering & System Safety* 230 (2023): 108959.
- Tolo, Silvia, and John Andrews. "*Fault Tree analysis including component dependencies.*" *IEEE Transactions on Reliability* (2023).
- Tolo, Silvia, and John Andrews. "*An integrated modelling framework for complex systems safety analysis.*" *Quality and Reliability Engineering International* 38.8 (2022): 4330-4350.

# Thank you

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