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The Safety and Reliability Society

2024 Webinar Programme

19th March 2024

**Modelling
dependencies in
complex systems:
Dynamic and
Dependent Tree Theory
(D²T²)**

Dr Silvia Tolo



Foundation



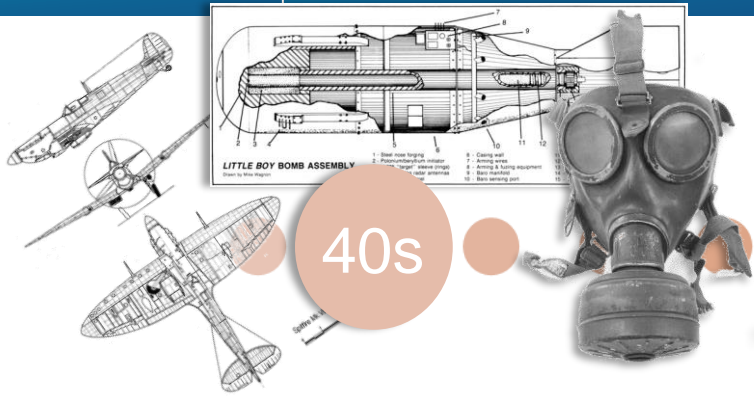
The answer to an old question?

**SYSTEM
RELIABILITY**





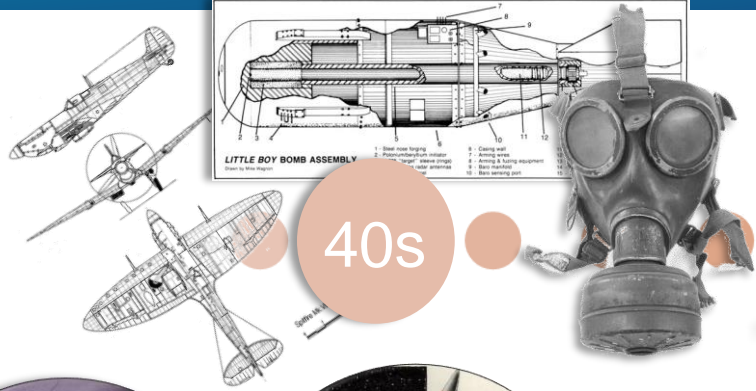
The answer to an old question?



**SYSTEM
RELIABILITY**



The answer to an old question?



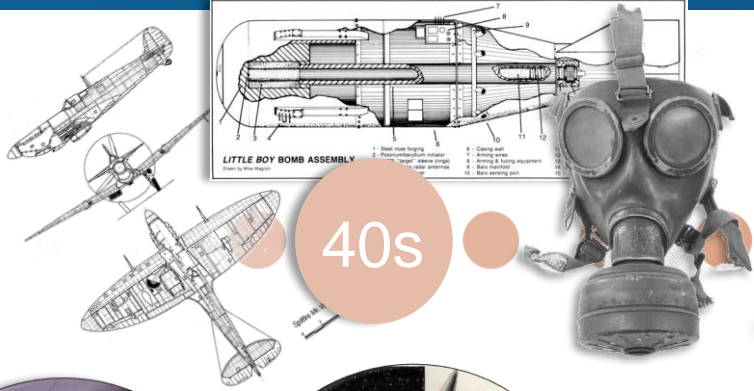
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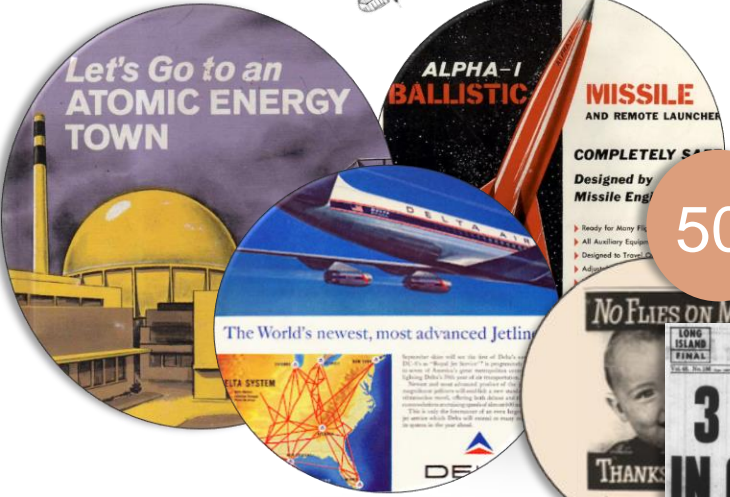
50s



The answer to an old question?

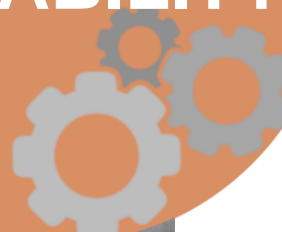


40s



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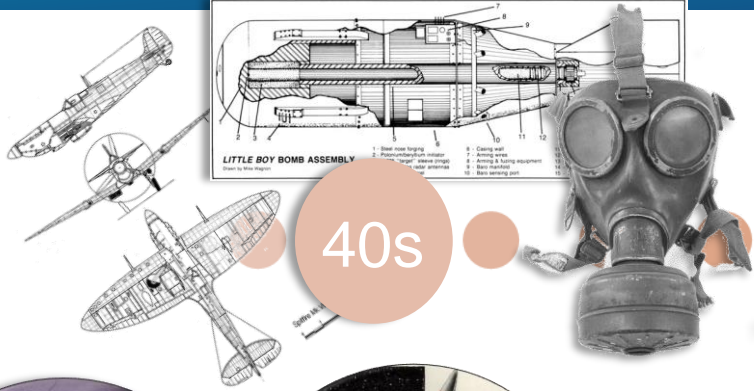
SYSTEM RELIABILITY



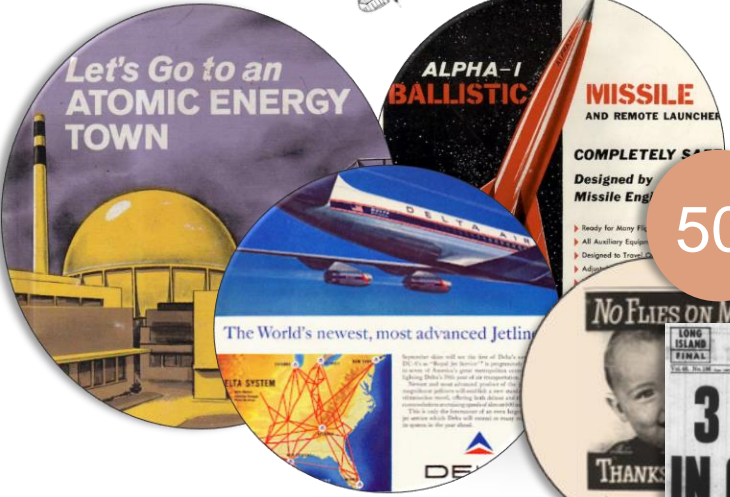
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The answer to an old question?



40s



50s

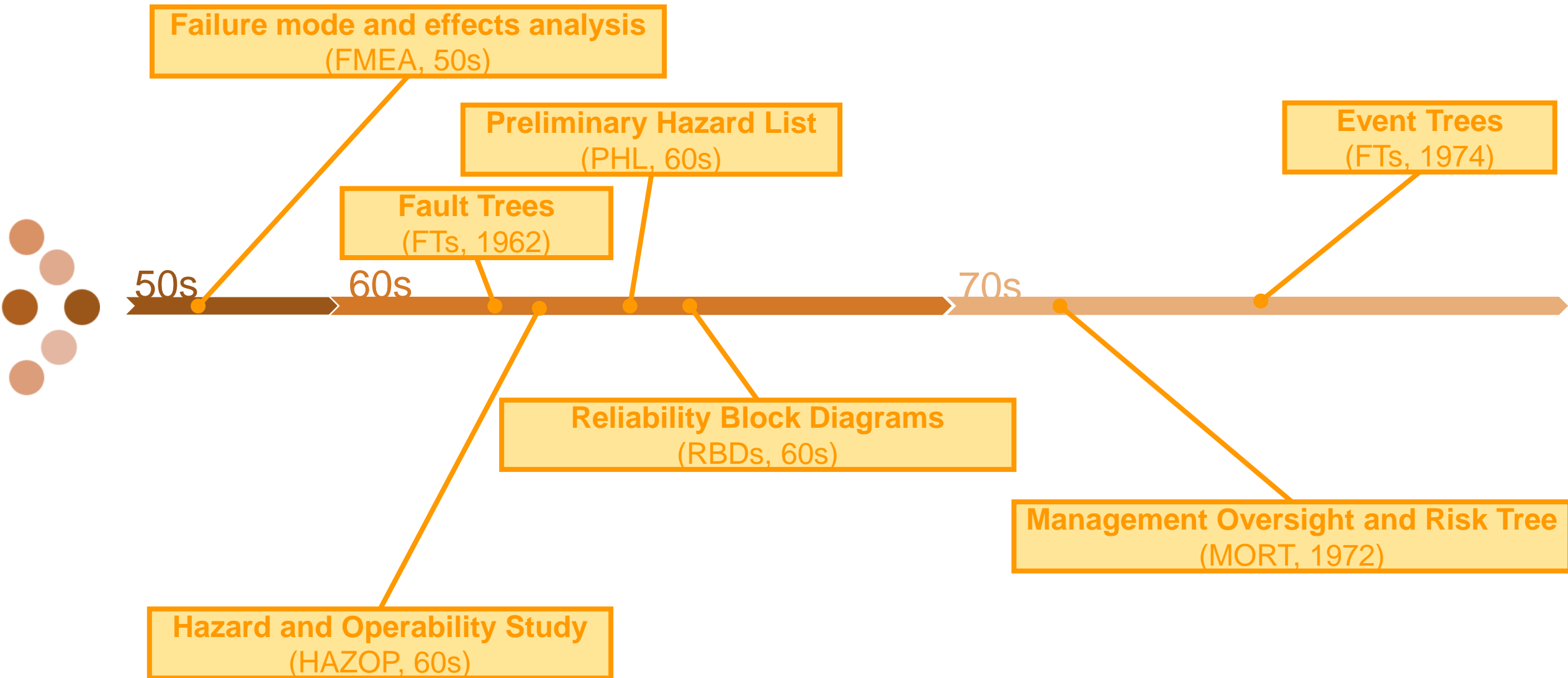
SYSTEM RELIABILITY



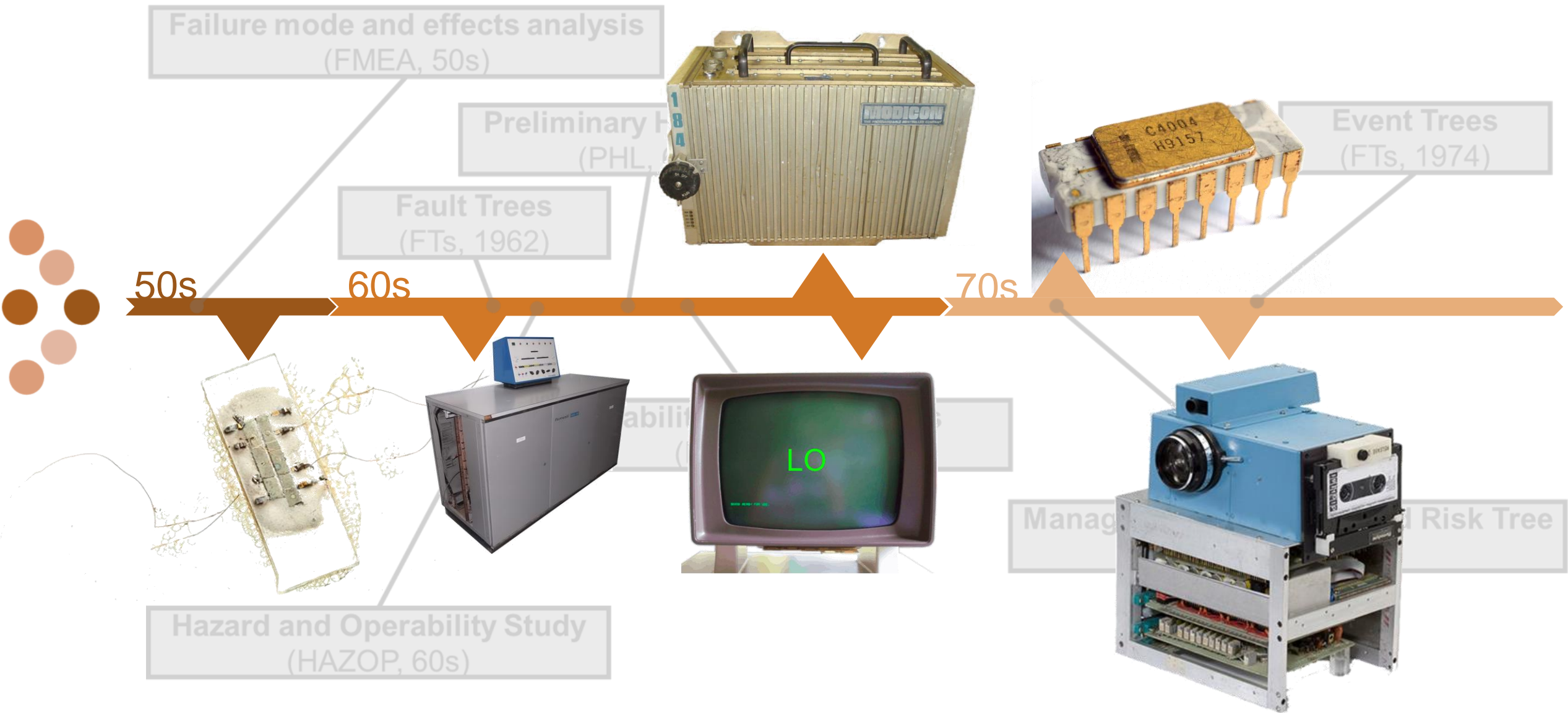
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The answer to an old question?



The answer to an old question?





LARGE-SCALE SYSTEMS

- High level of complexity
- Enormous number of components
- Low probability values
- Multiple stakeholders



LARGE-SCALE SYSTEMS

- High level of complexity
- Enormous number of components
- Low probability values
- Multiple stakeholders

EFFICIENCY ↑



Different systems...different tools?

TRADITIONAL
TECHNIQUES

EFFICIENCY



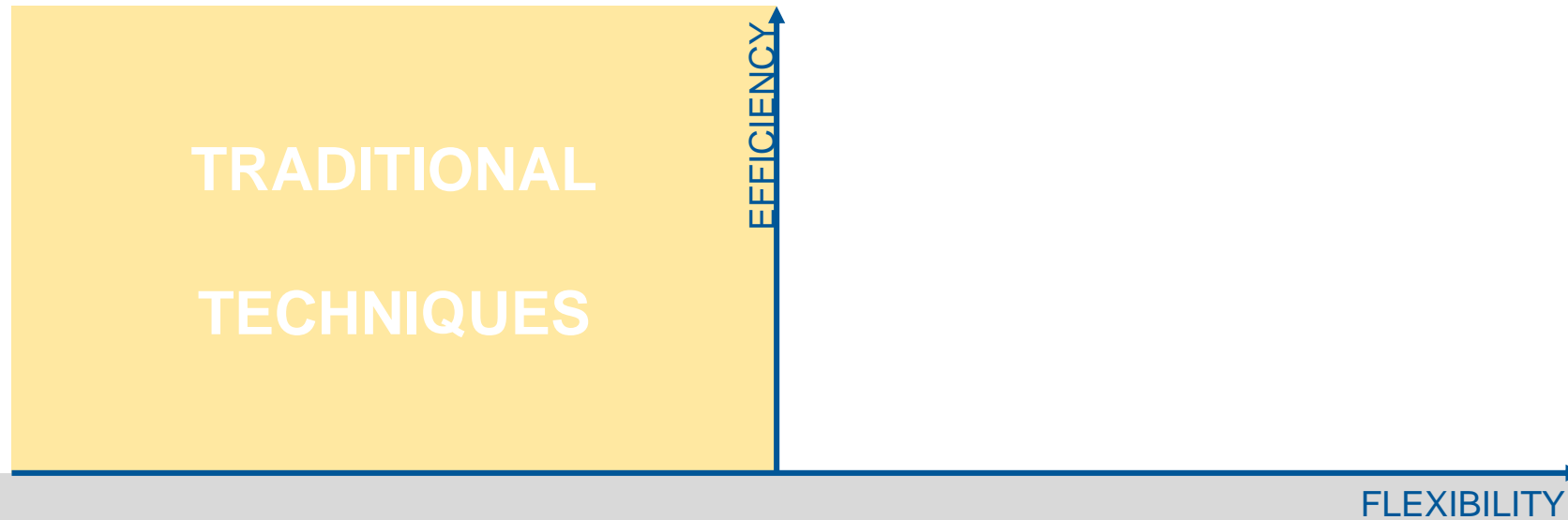
TRADITIONAL
TECHNIQUES

EFFICIENCY ↑

HIGH LEVEL OF AUTOMATION AND CONTROL TECHNOLOGY

- systems un-negligibly dynamic
- human-technology interface
- increasingly complex maintenance strategies

Dependencies between
failure events

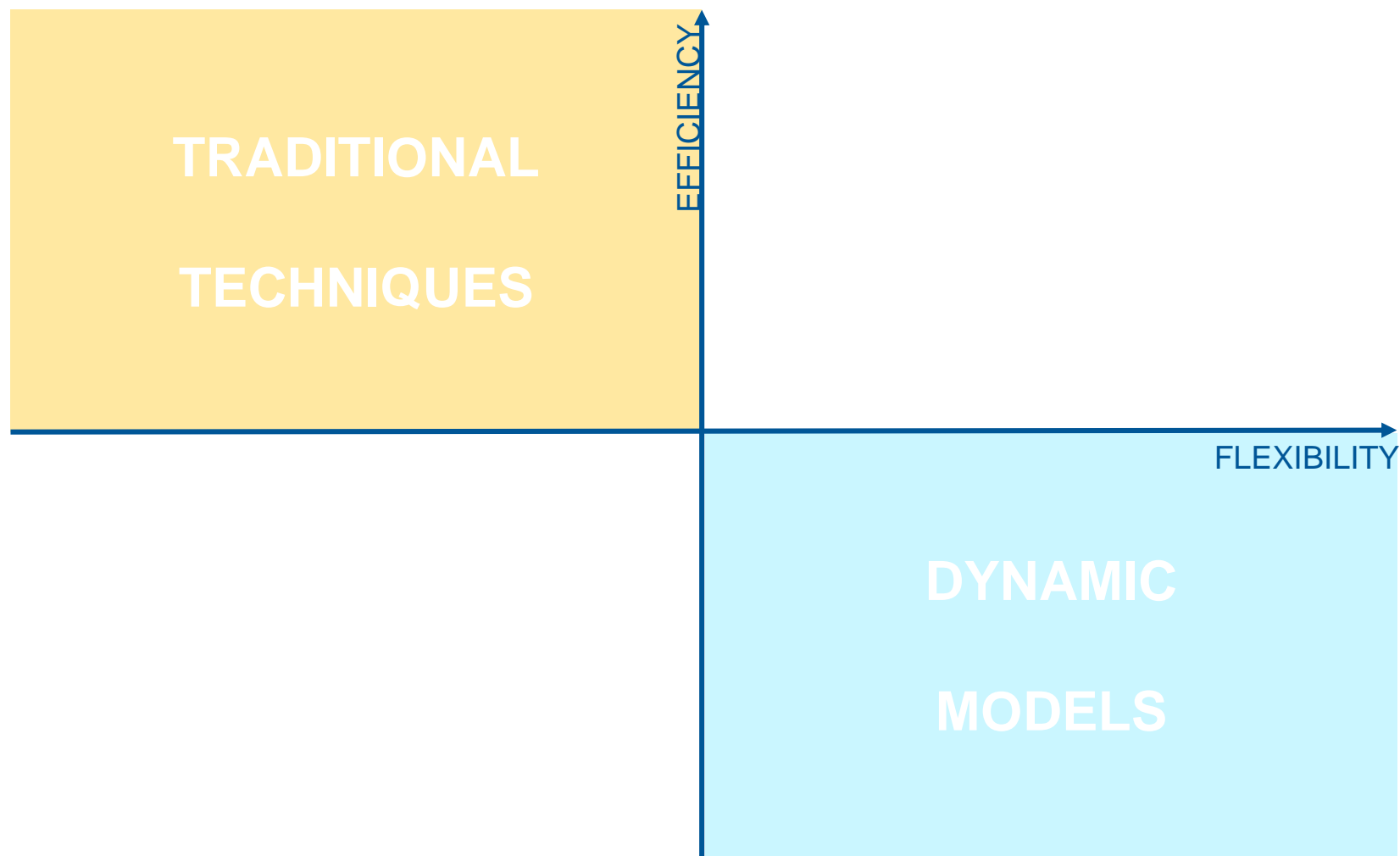


HIGH LEVEL OF AUTOMATION AND CONTROL TECHNOLOGY

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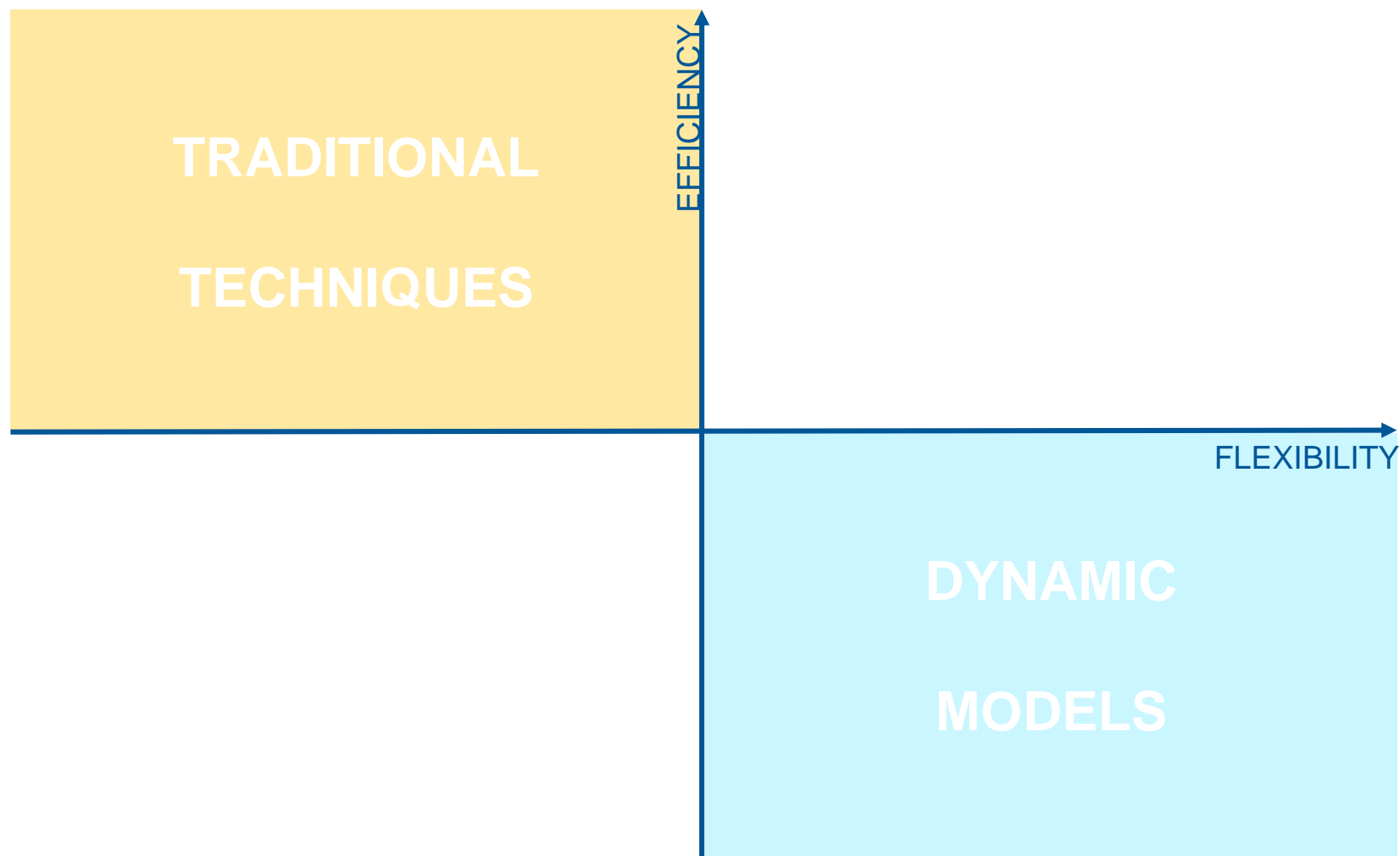


Different systems...different tools?





Different systems...different tools?



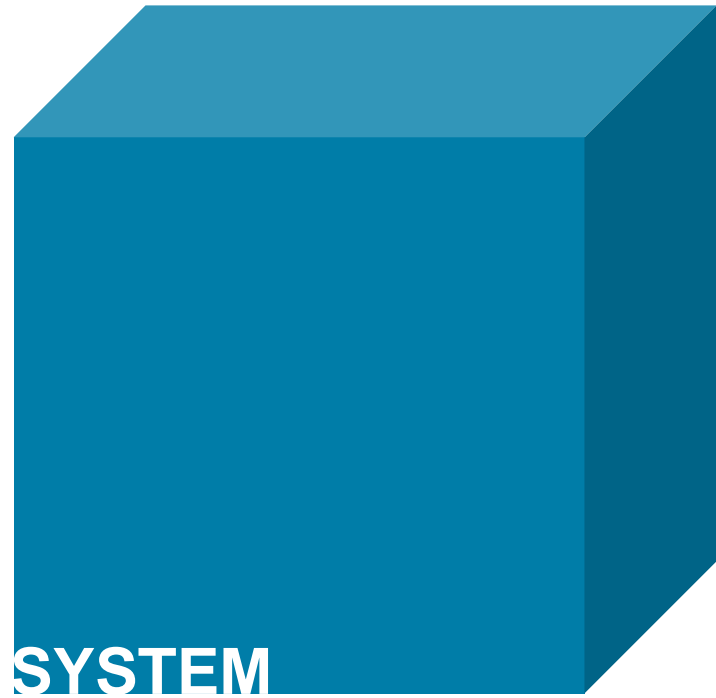


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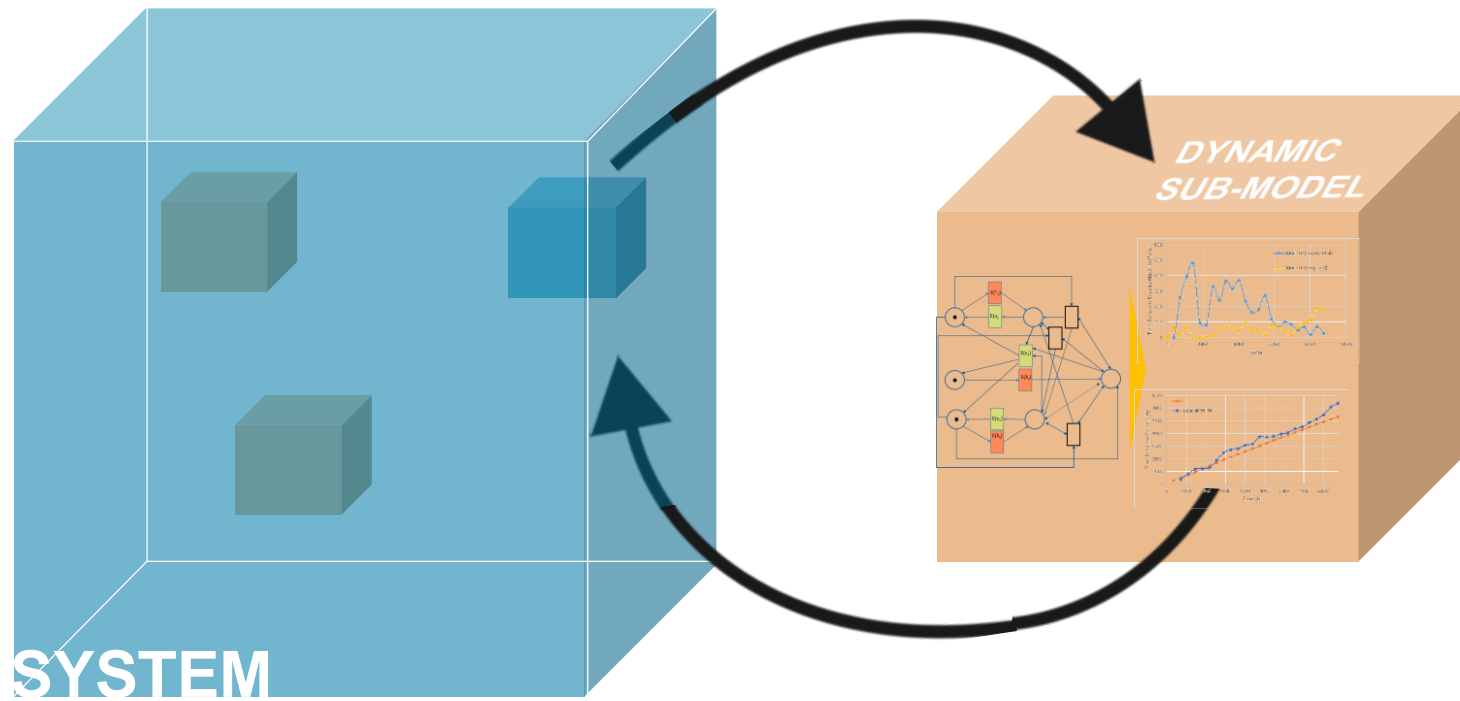
D²T²

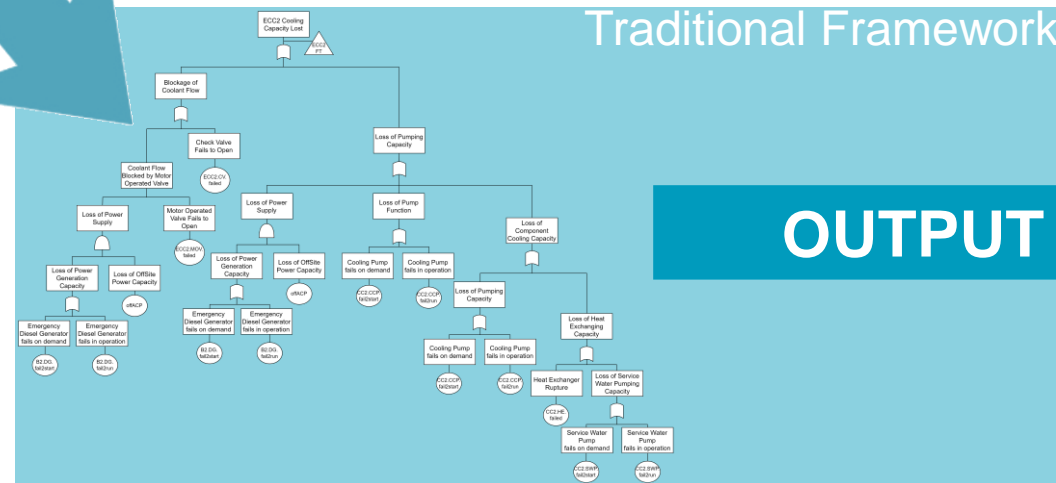
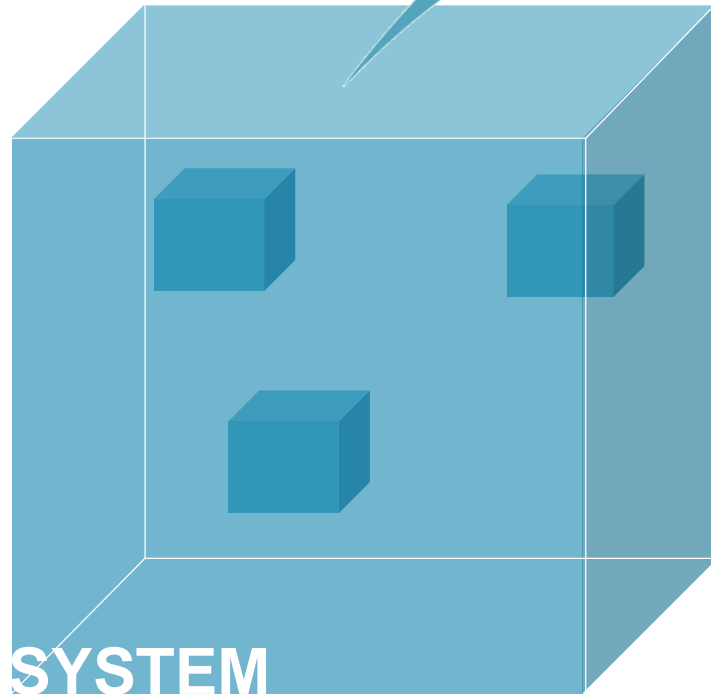
Dynamic and Dependent Tree Theory





D²T²: the general idea





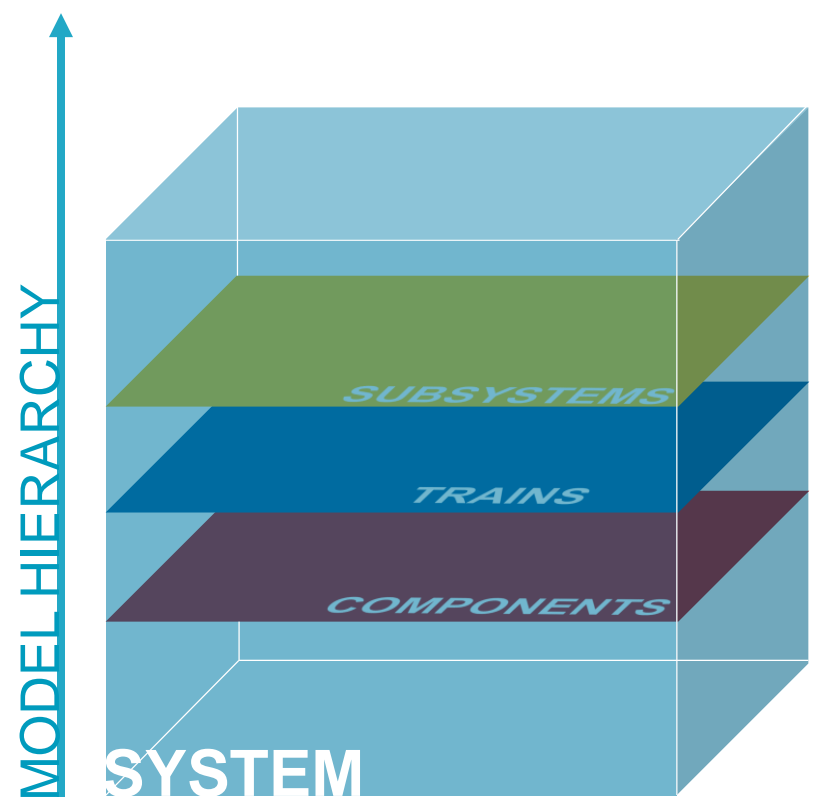
→ Tailored use of 'expensive' dynamic models

→ Preserves effectiveness of traditional techniques

→ Enhances modelling accuracy and flexibility

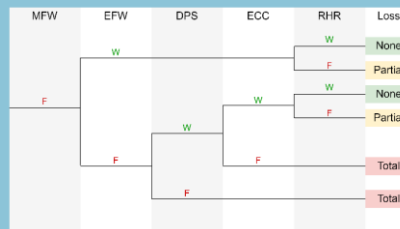
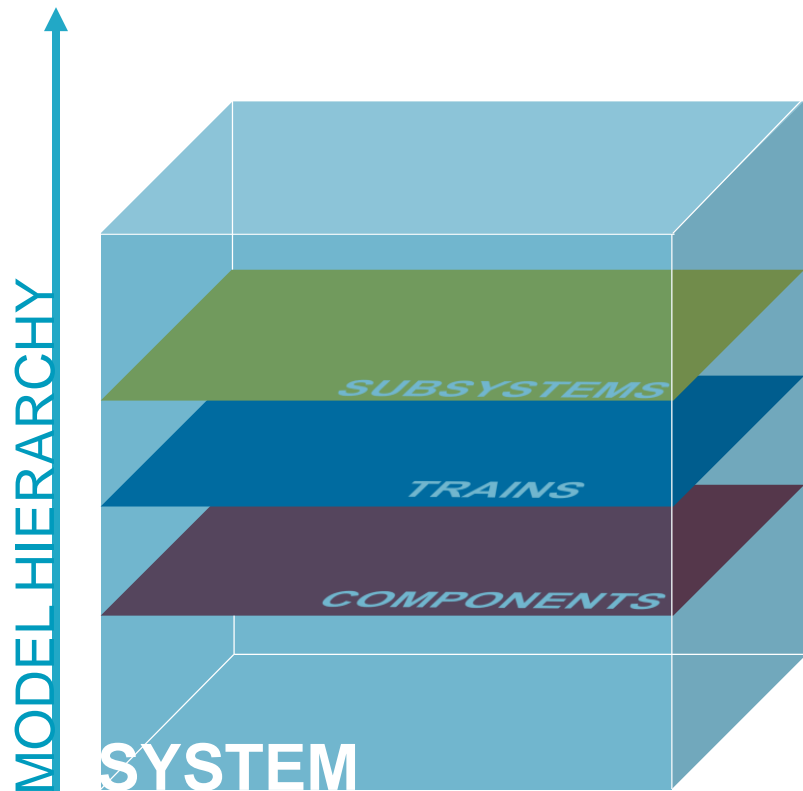


D²T² across modelling





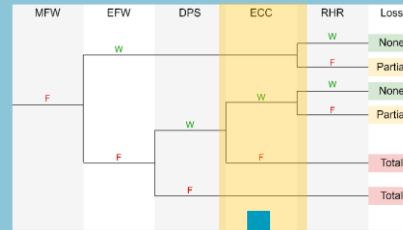
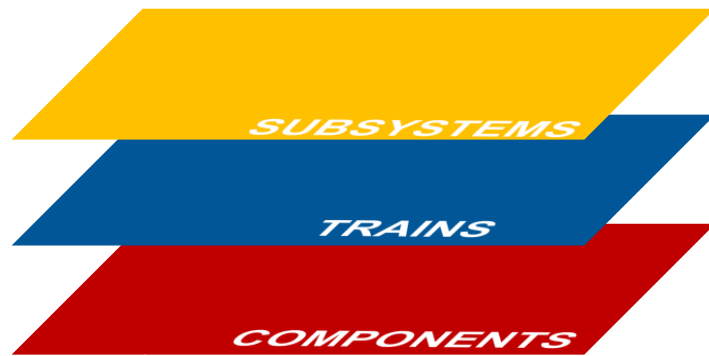
D²T² across the Modelling Layers



Event Tree [ET]:

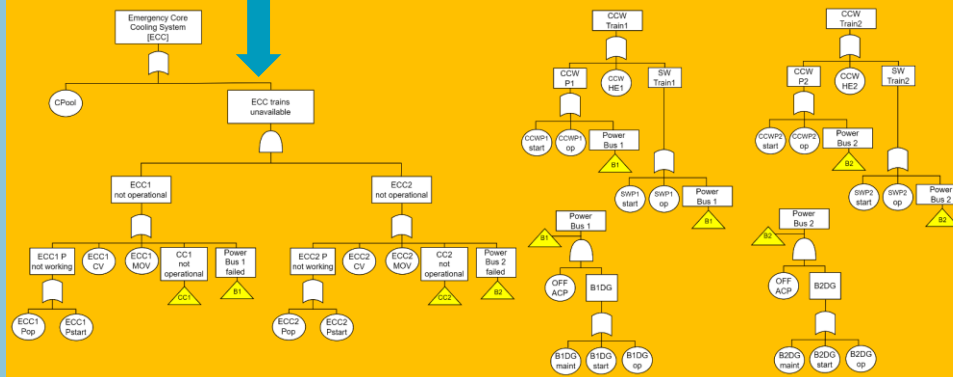
- Accident Sequence
- Subsystems interaction

MODEL HIERARCHY



Event Tree [ET]:

- Accident Sequence
- Subsystems interaction



Fault Tree [FT]:

- Sub-system failure

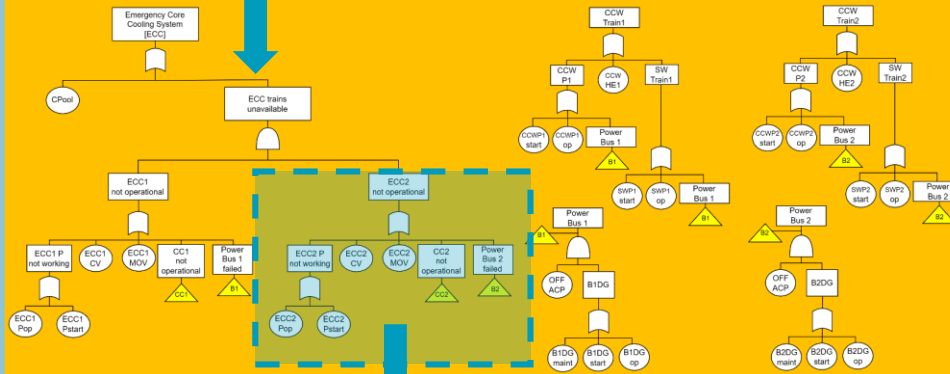
MODEL HIERARCHY ↑



MFW	EFW	DPS	ECC	RHR	Loss
	W		W	W	None
			F	F	Partial
F			W	W	None
			F	F	Partial
	F				Total
		F			Total

Event Tree [ET]:

- Accident Sequence
- Subsystems interaction

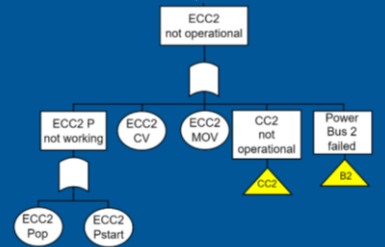


Fault Tree [FT]:

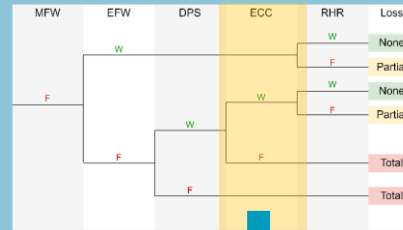
- Sub-system failure

Sub-Tree [FT]:

- Trains of identical components
- Redundancy



MODEL HIERARCHY

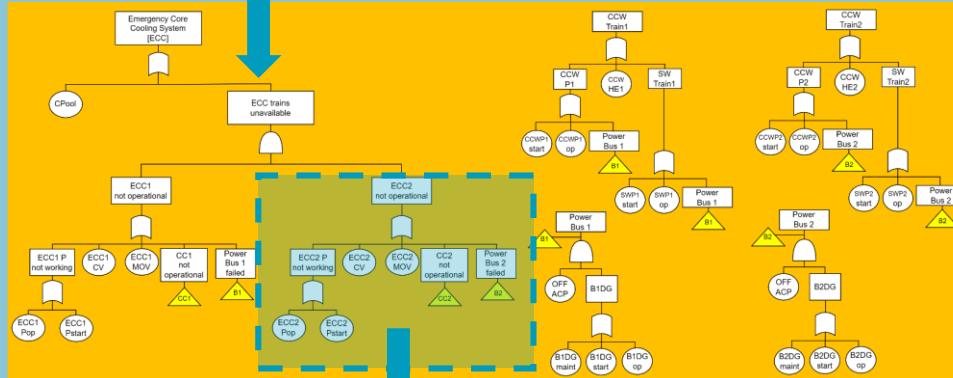


Event Tree [ET]:

- Accident Sequence
- Subsystems interaction

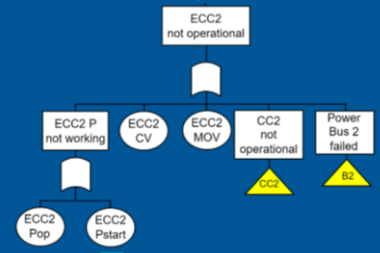
Fault Tree [FT]:

- Sub-system failure



Sub-Tree [FT]:

- Trains of identical components
- Redundancy



Basic Event [BE]:

- Component Failure Mechanism
- Reliability Metrics



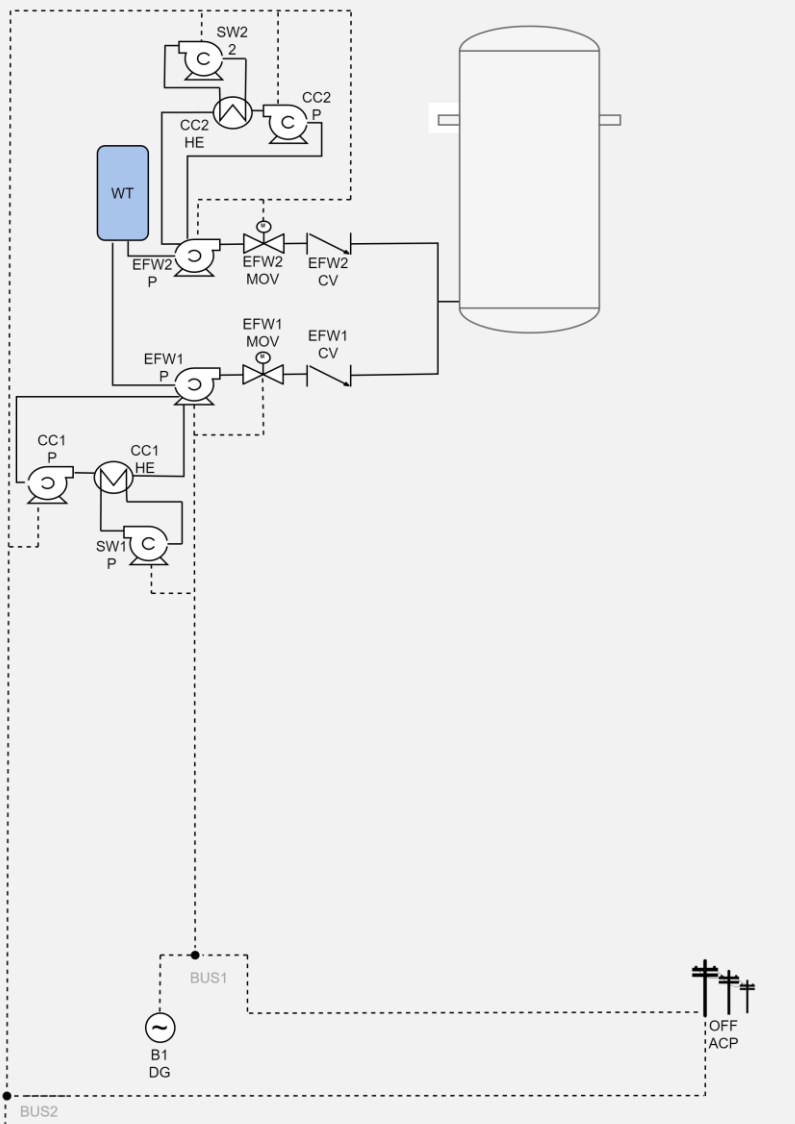


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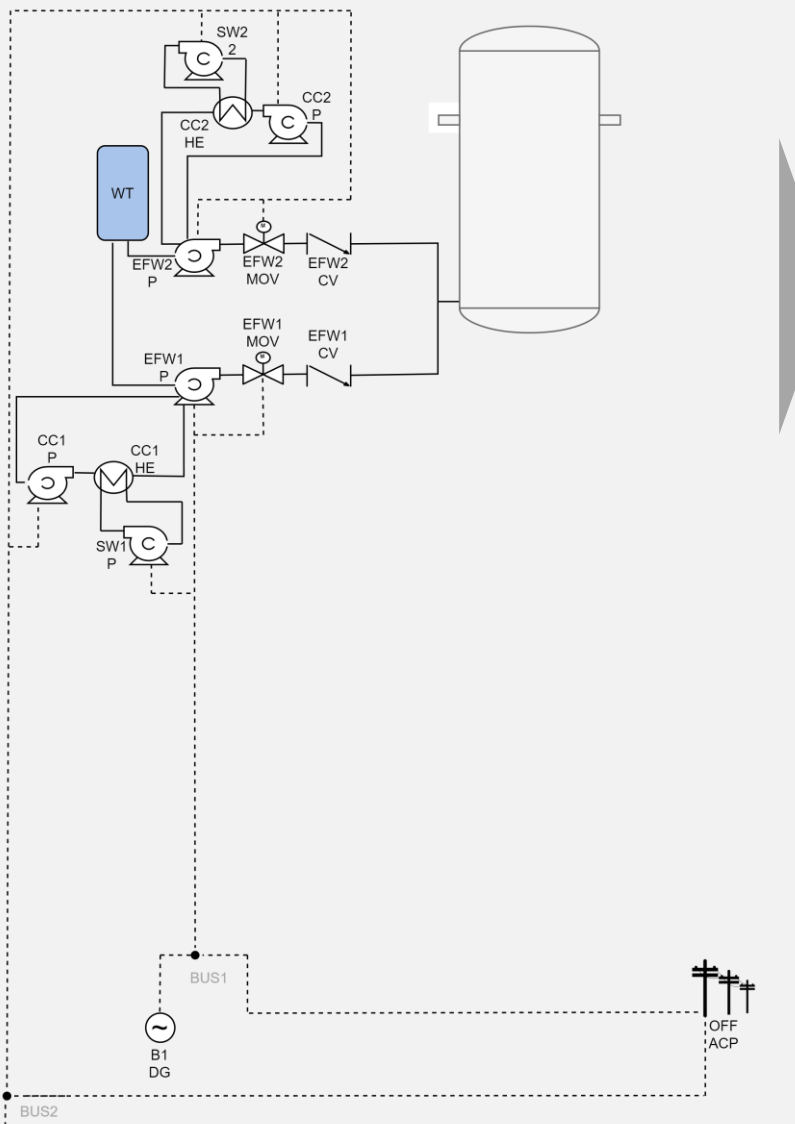
Hands on

D²T² application

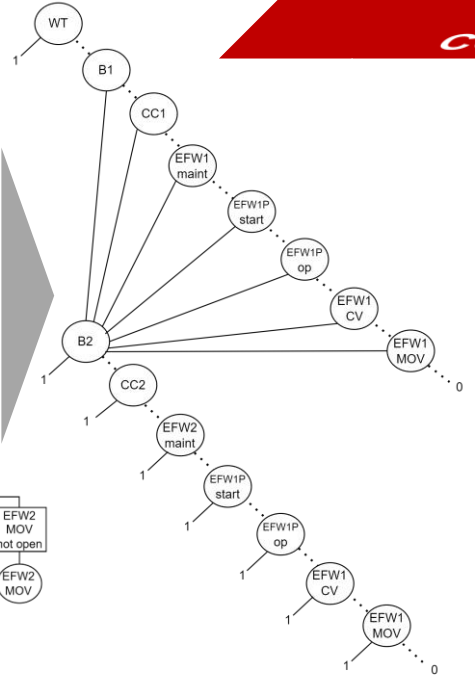
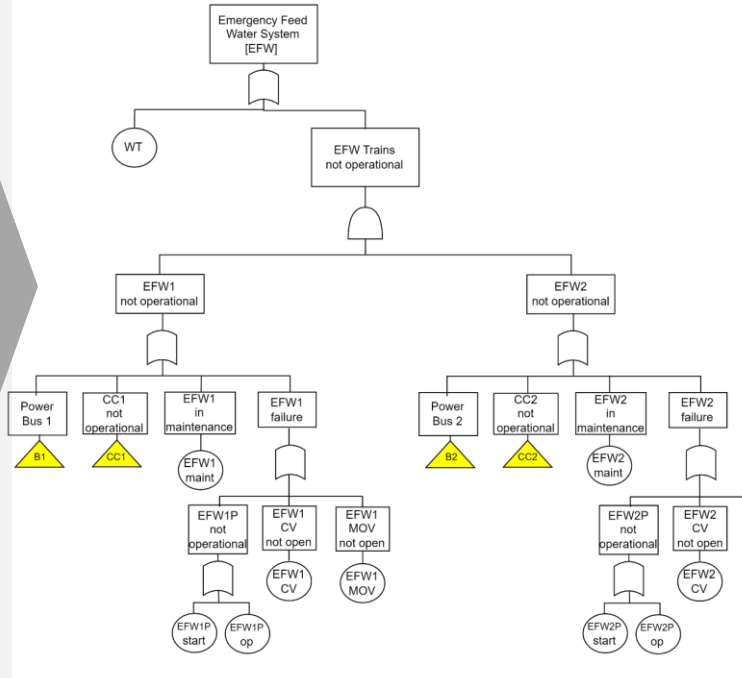


**BWR EMERGENCY
FEED WATER SYSTEM**

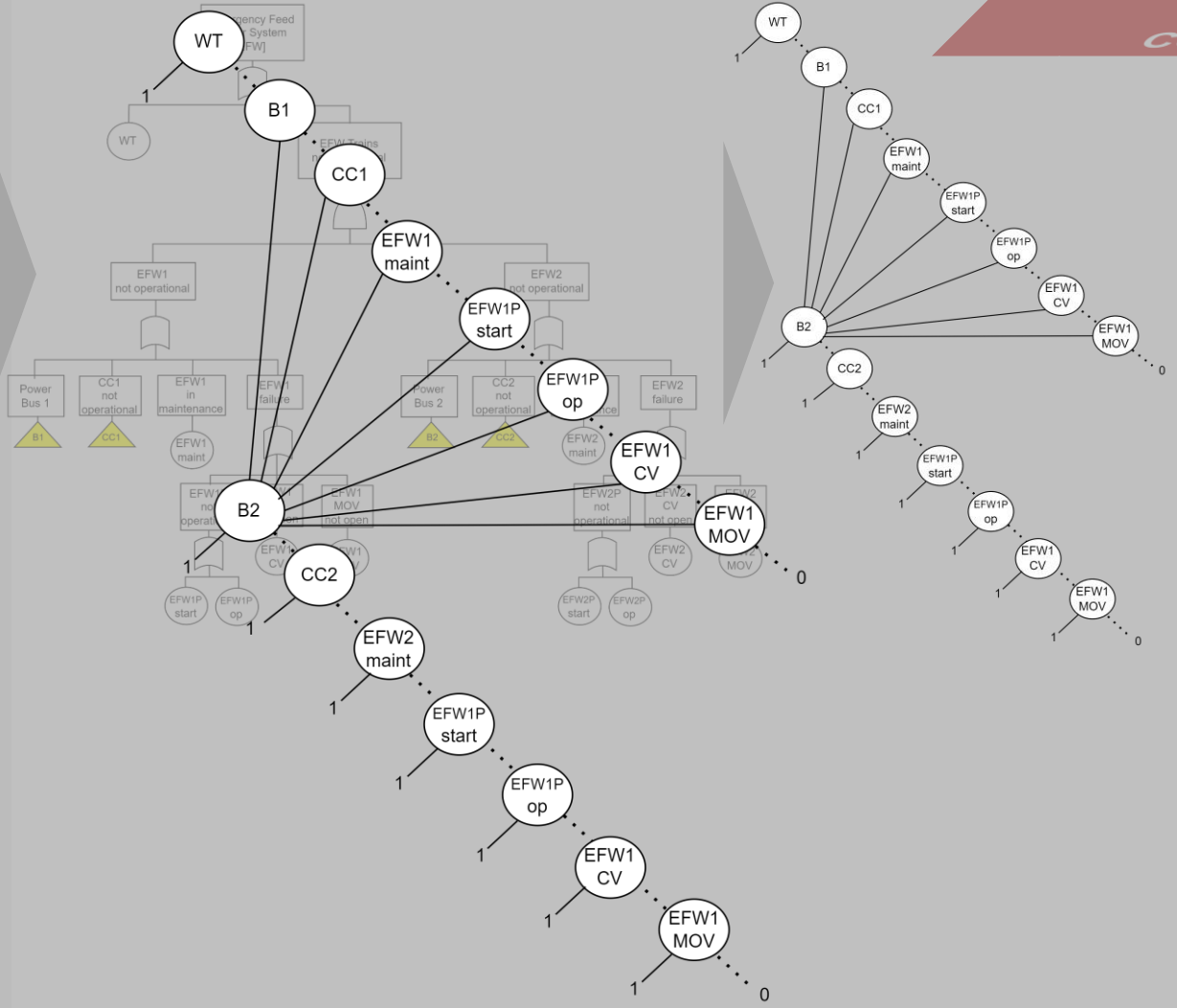
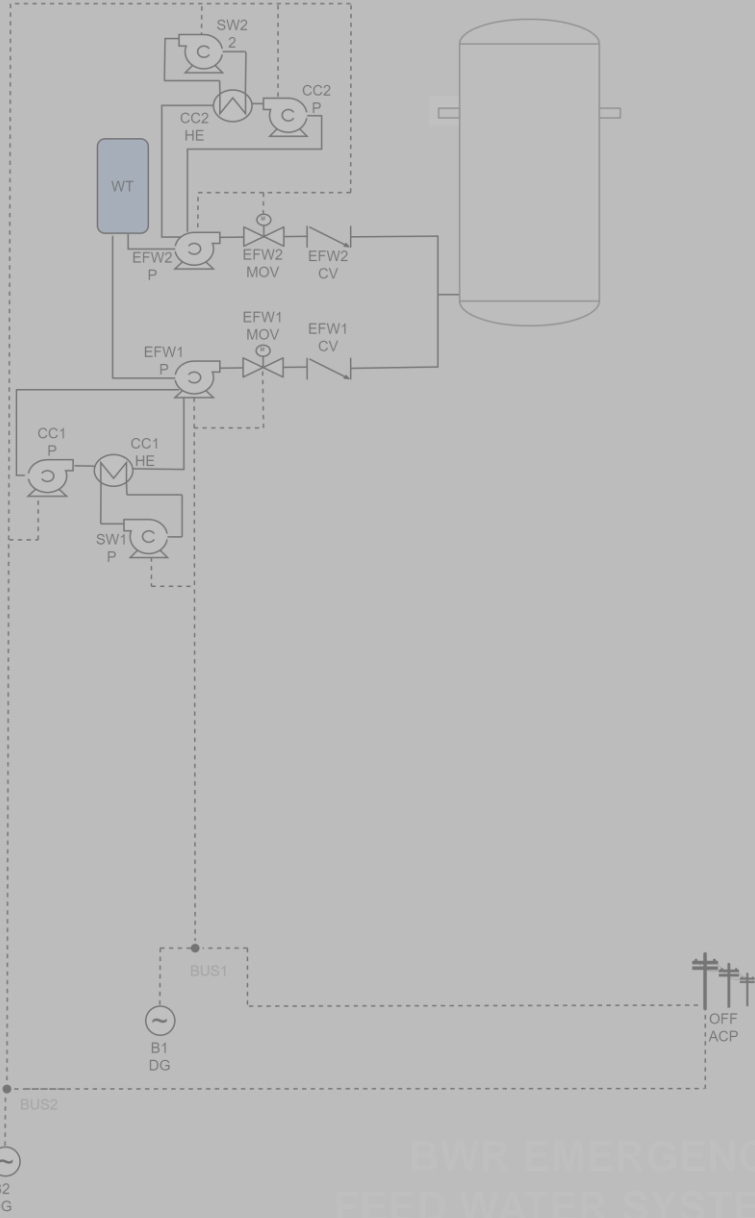
D²T²: Basic Event Dependency



BWR EMERGENCY FEED WATER SYSTEM



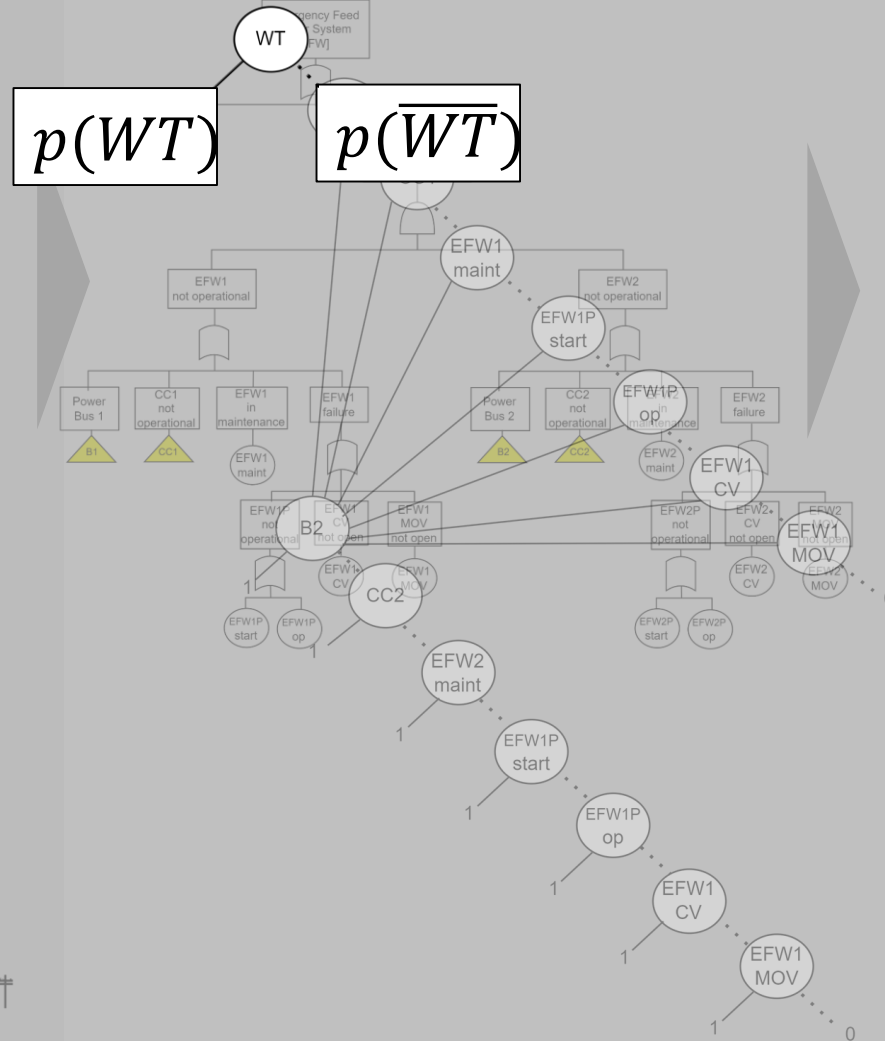
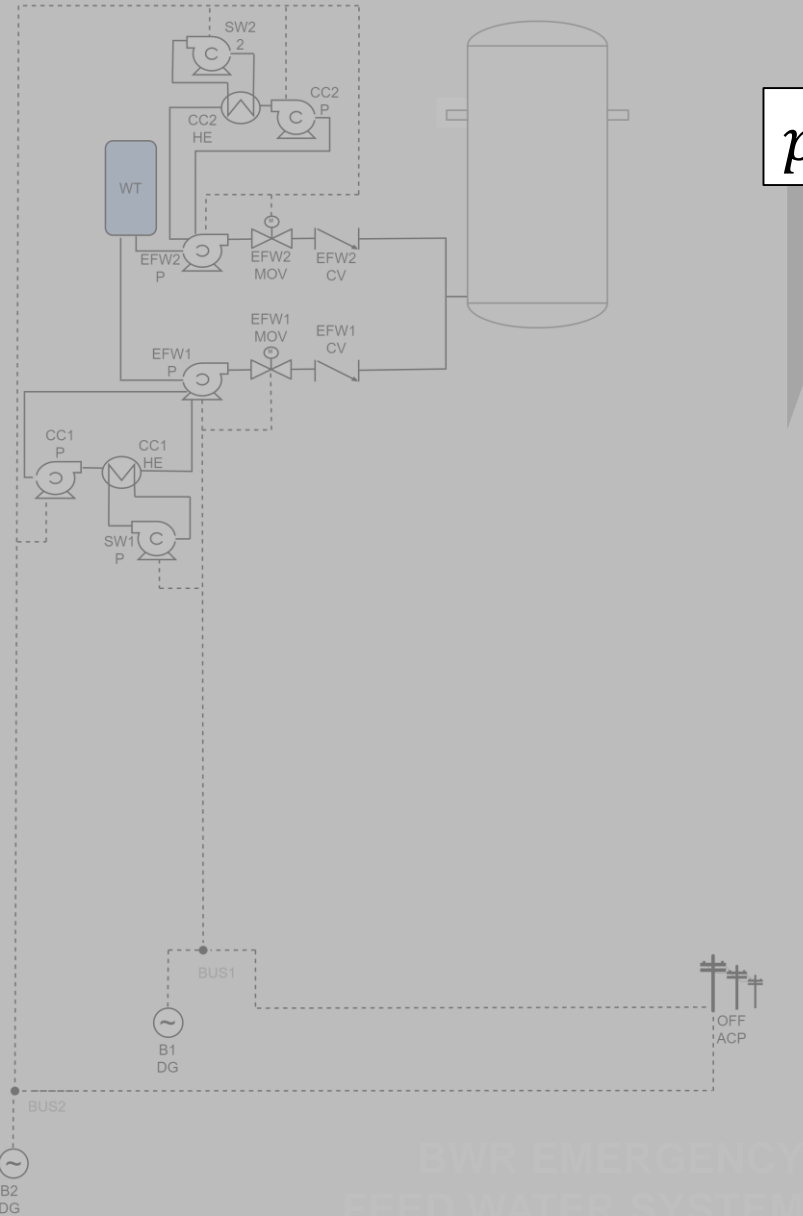
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BWR EMERGENCY FEED WATER SYSTEM

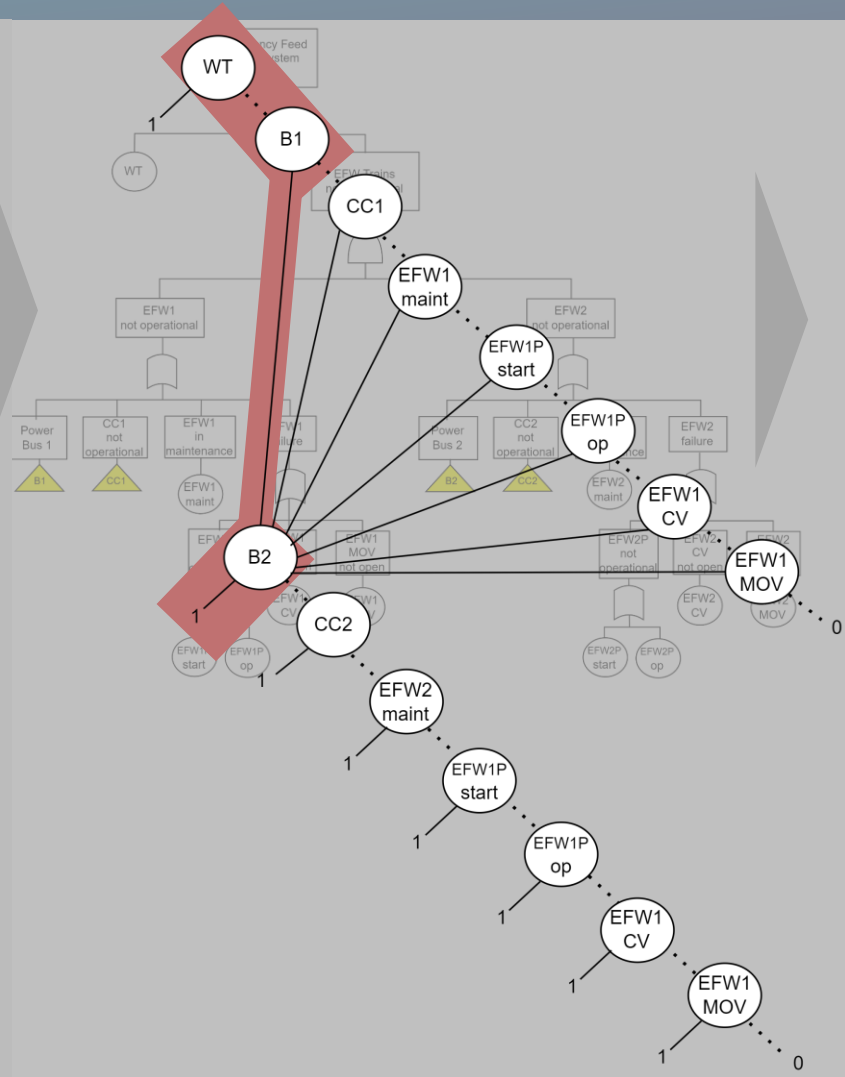
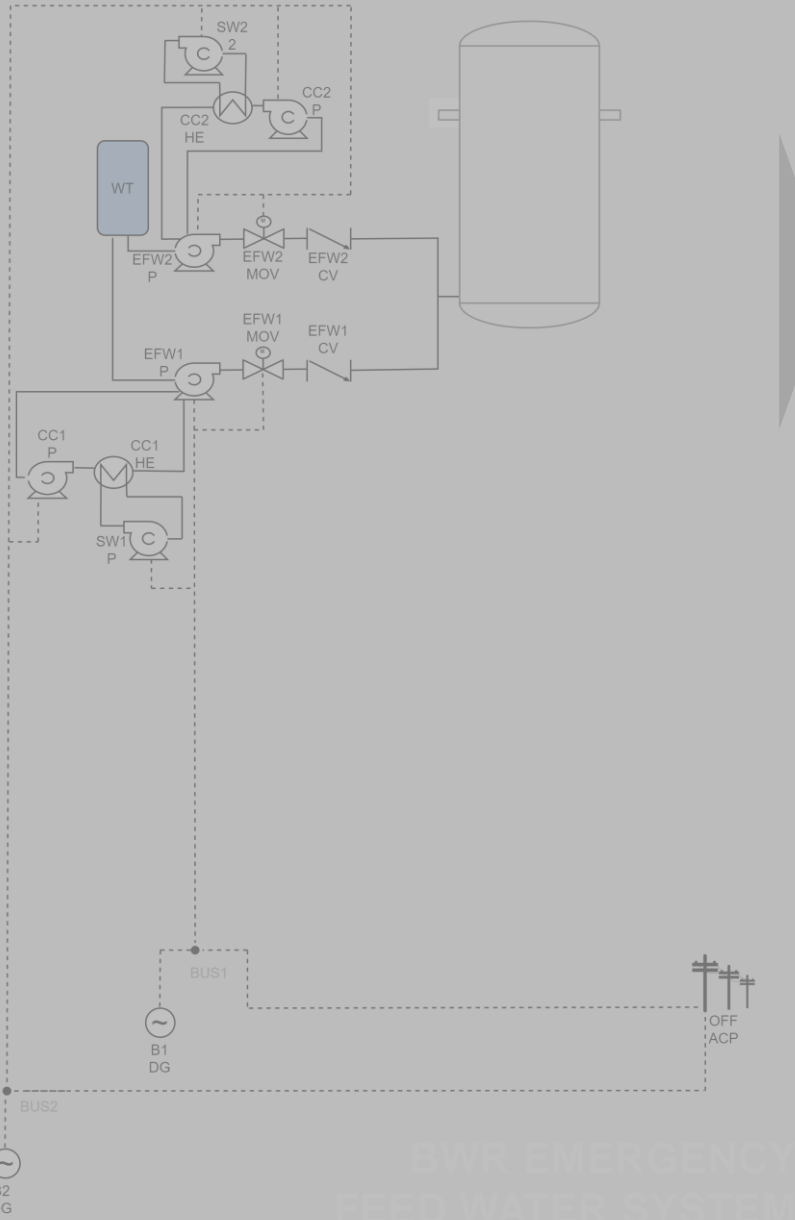
D²T²: Basic Event Dependency

COMPONENTS



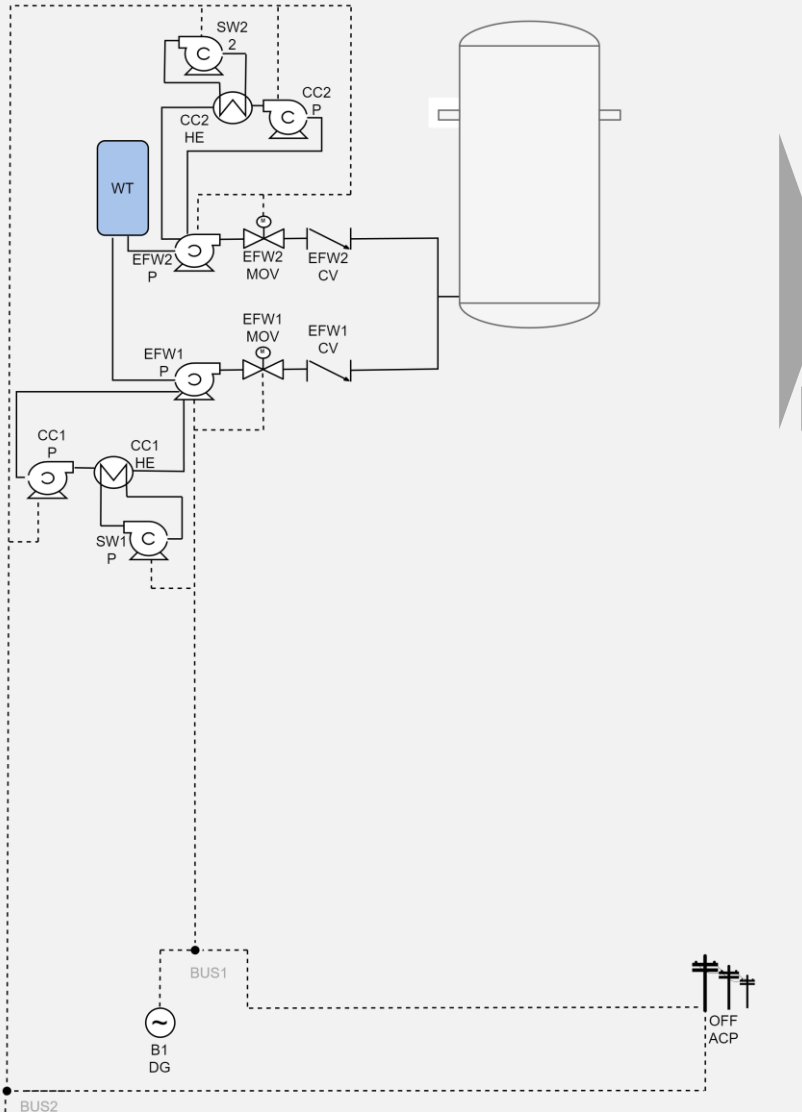
BWR EMERGENCY
COOLED WATER SYSTEM

D²T²: Basic Event Dependency

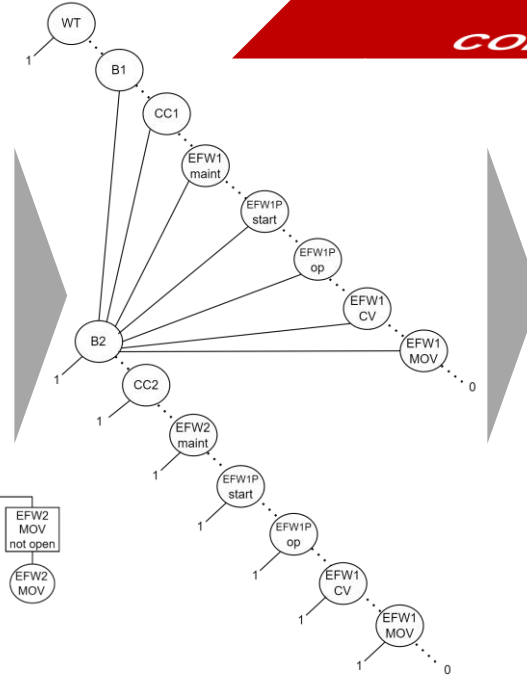
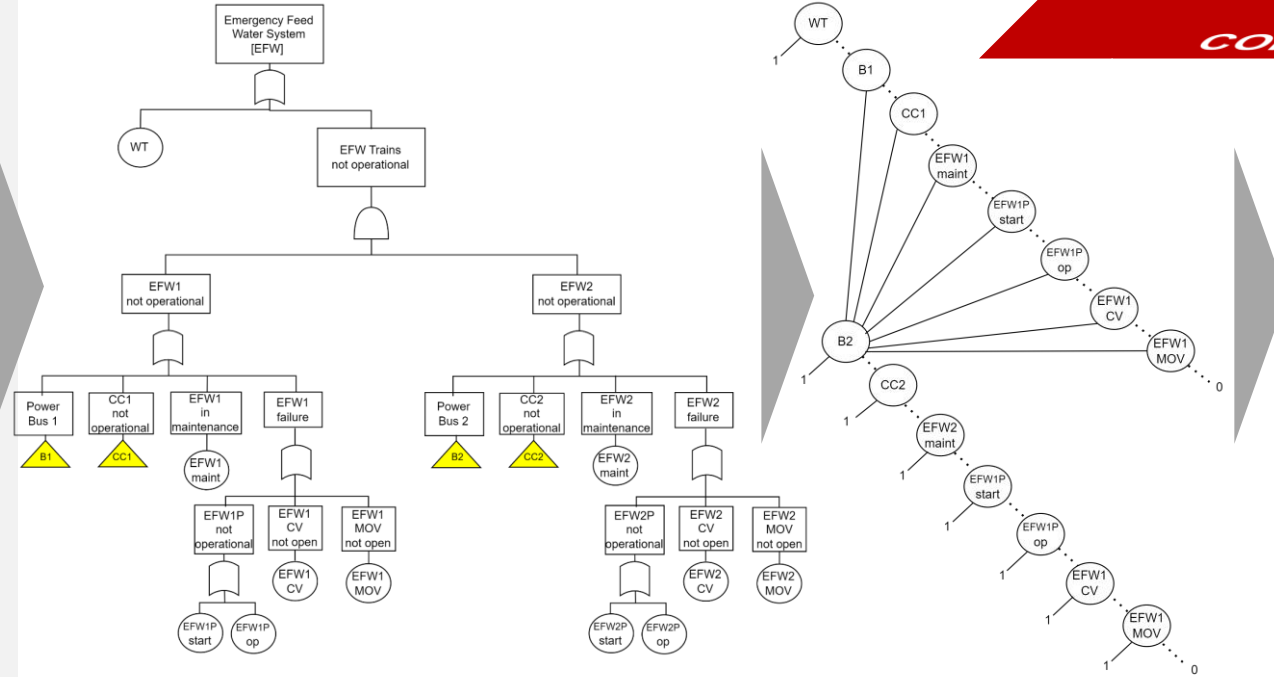


BWR EMERGENCY CORE-COOLED WATER SYSTEM

D²T²: Basic Event Dependency

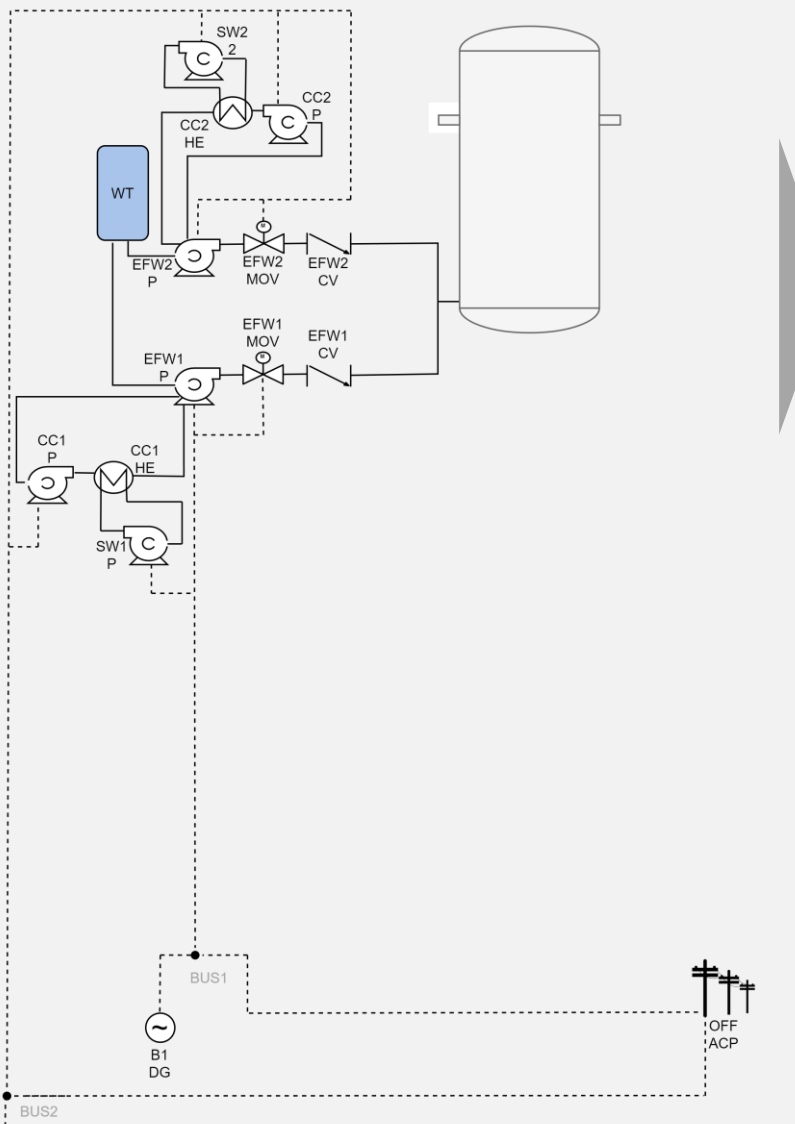


BWR EMERGENCY FEED WATER SYSTEM

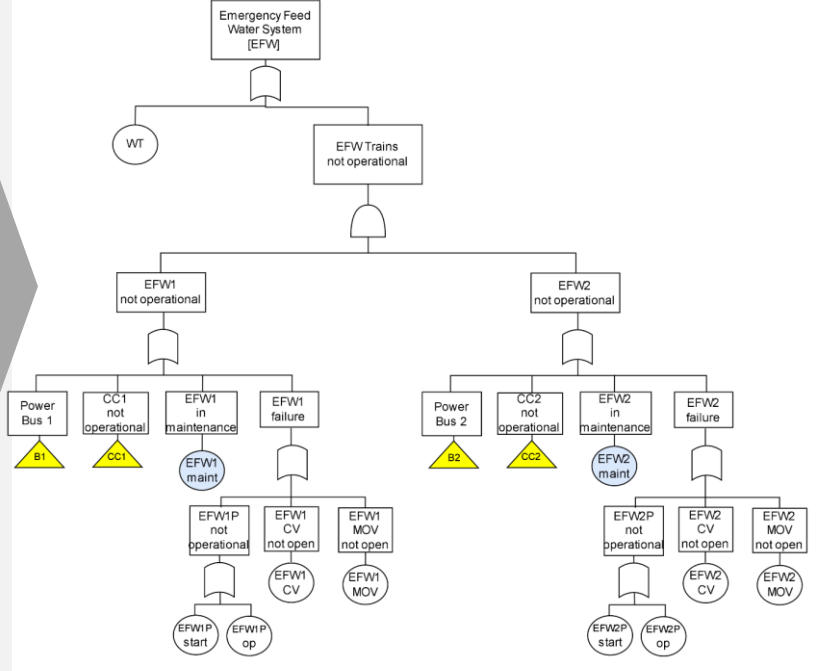


$P(\text{TOP})_{\text{FT}} = 3.7e-03$

D²T²: Basic Event Dependency



BWR EMERGENCY FEED WATER SYSTEM

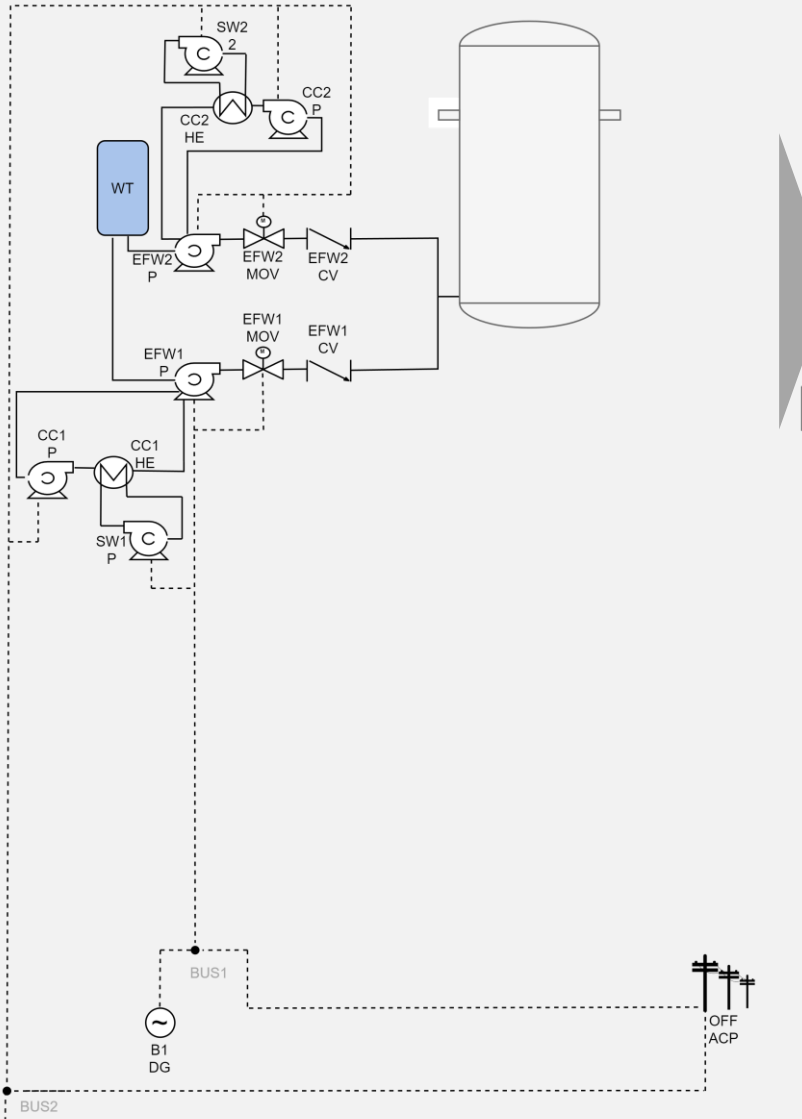


P(TOP) FT = 3.7e-03

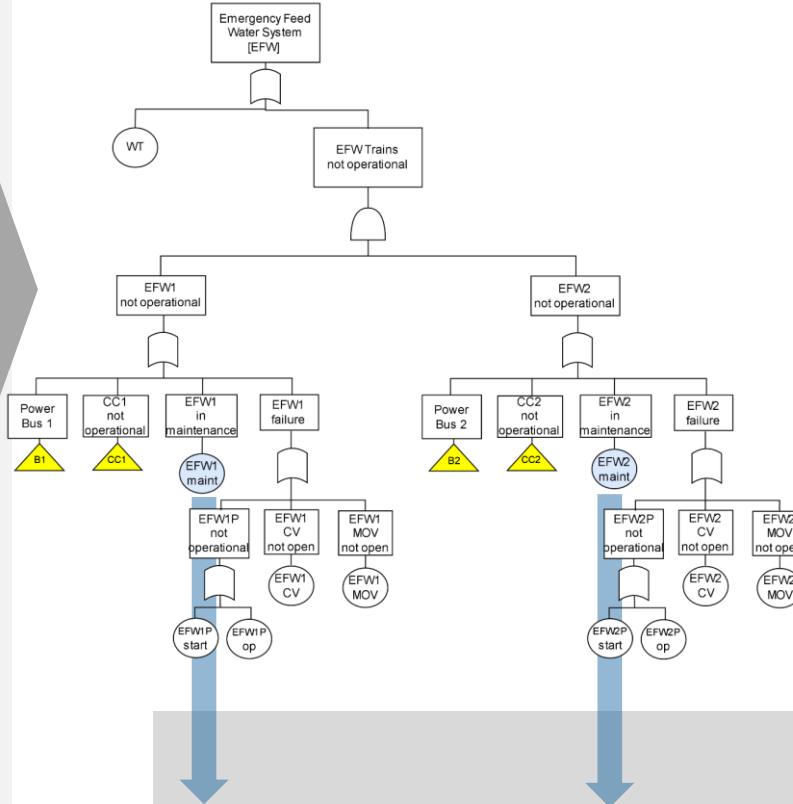
D²T²: Basic Event Dependency



P(TOP)^{FT} = 3.7e-03



BWR EMERGENCY FEED WATER SYSTEM

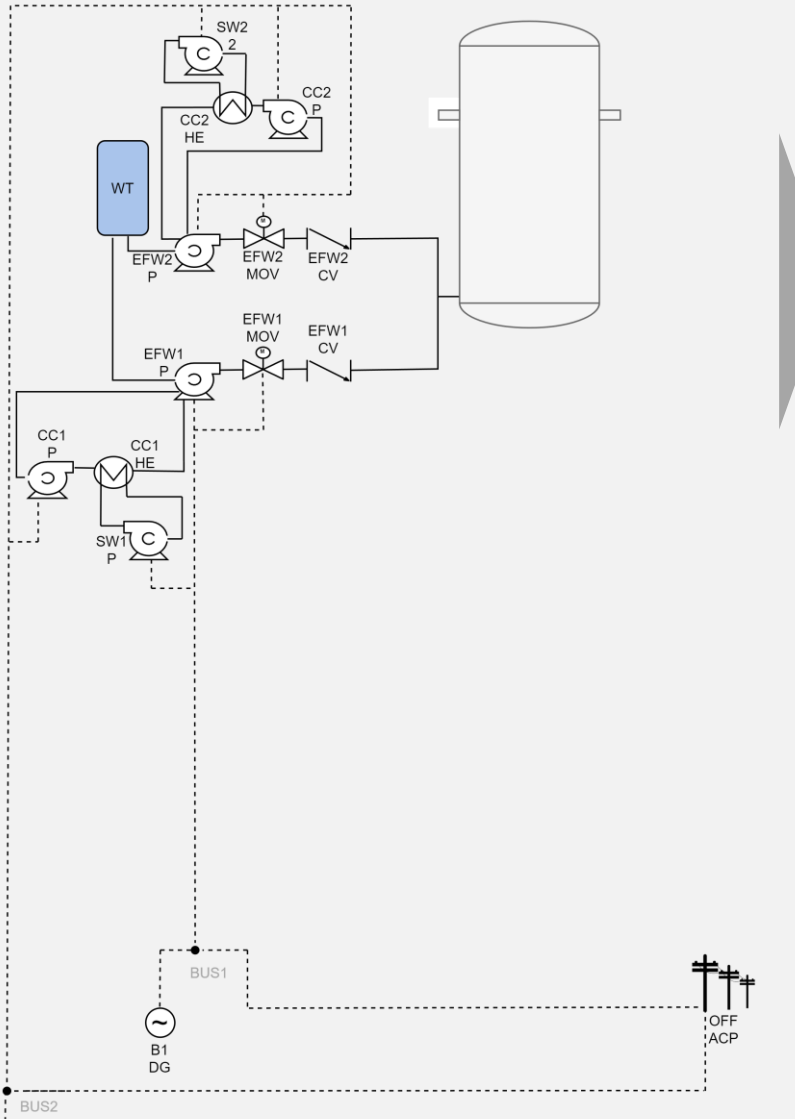


Maintenance:

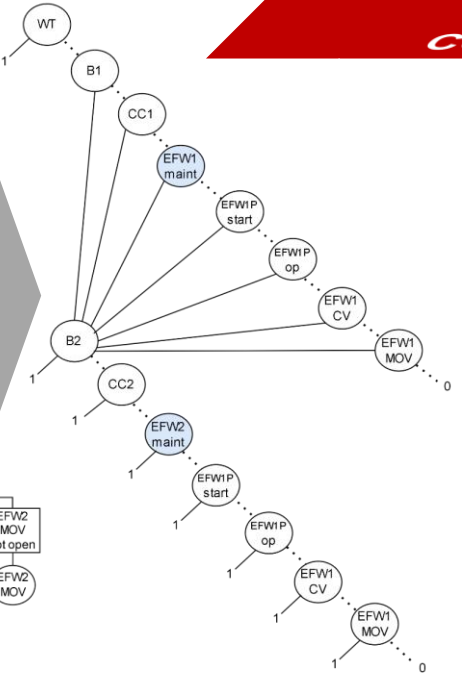
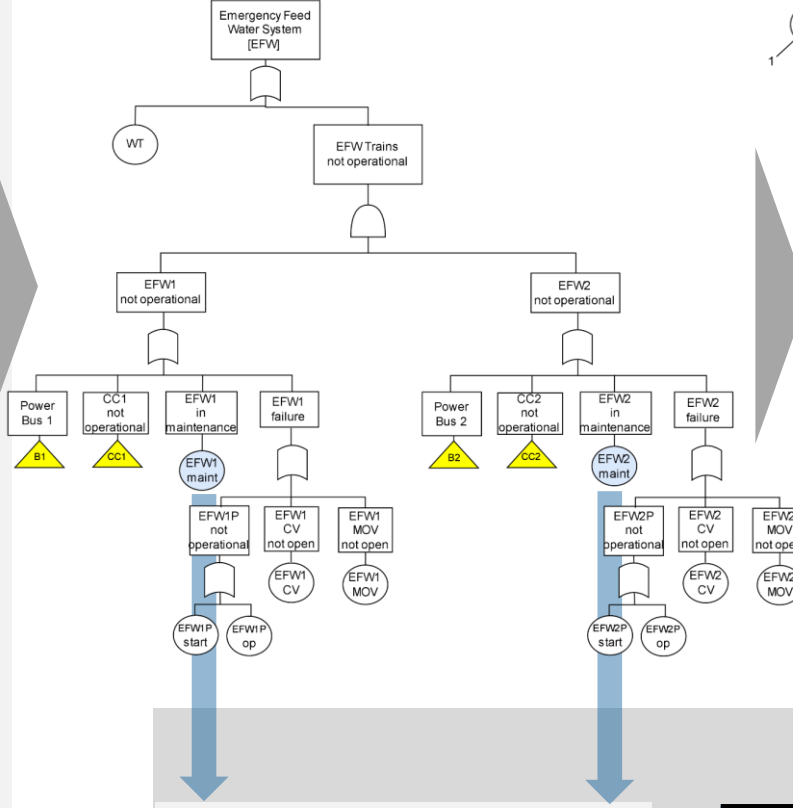
- Mutually exclusive
- 60h every three months

Joint Event	Probability
$\overline{EFW1maint}, \overline{EFW2maint}$	0.9452
$\overline{EFW1maint}, EFW2maint$	0.0274
$EFW1maint, \overline{EFW2maint}$	0.0274
$EFW1maint, EFW2maint$	0

D²T²: Basic Event Dependency



BWR EMERGENCY FEED WATER SYSTEM



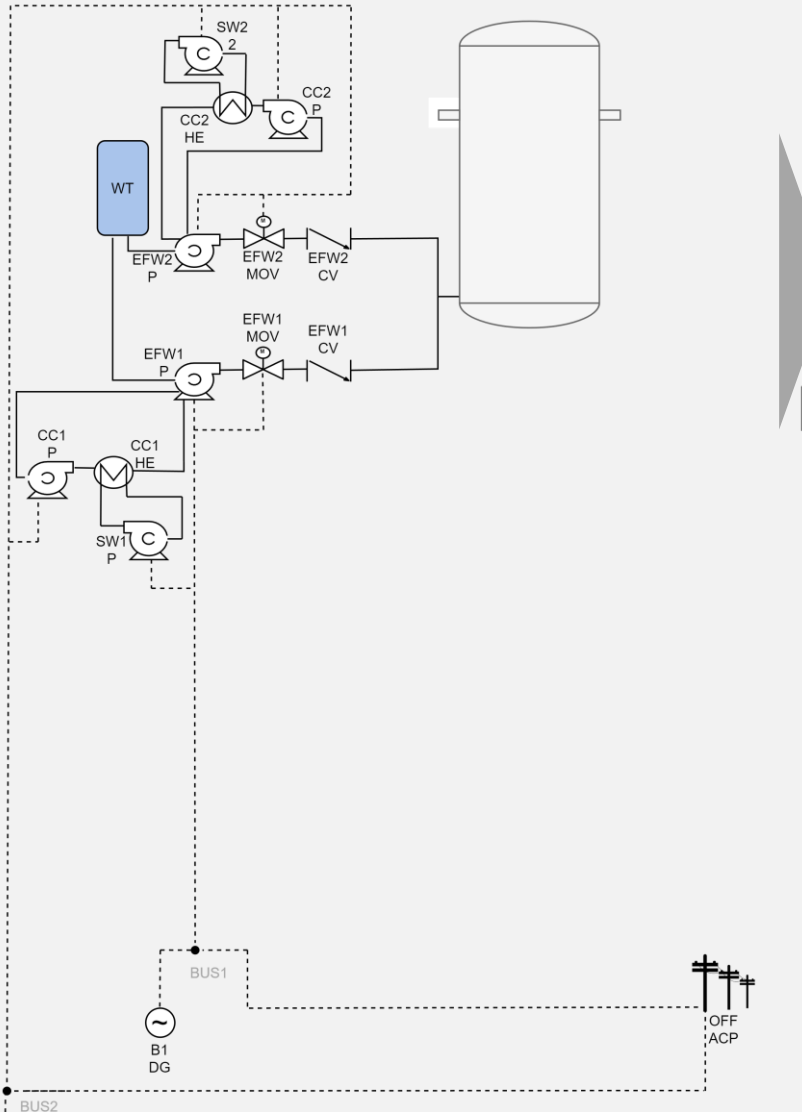
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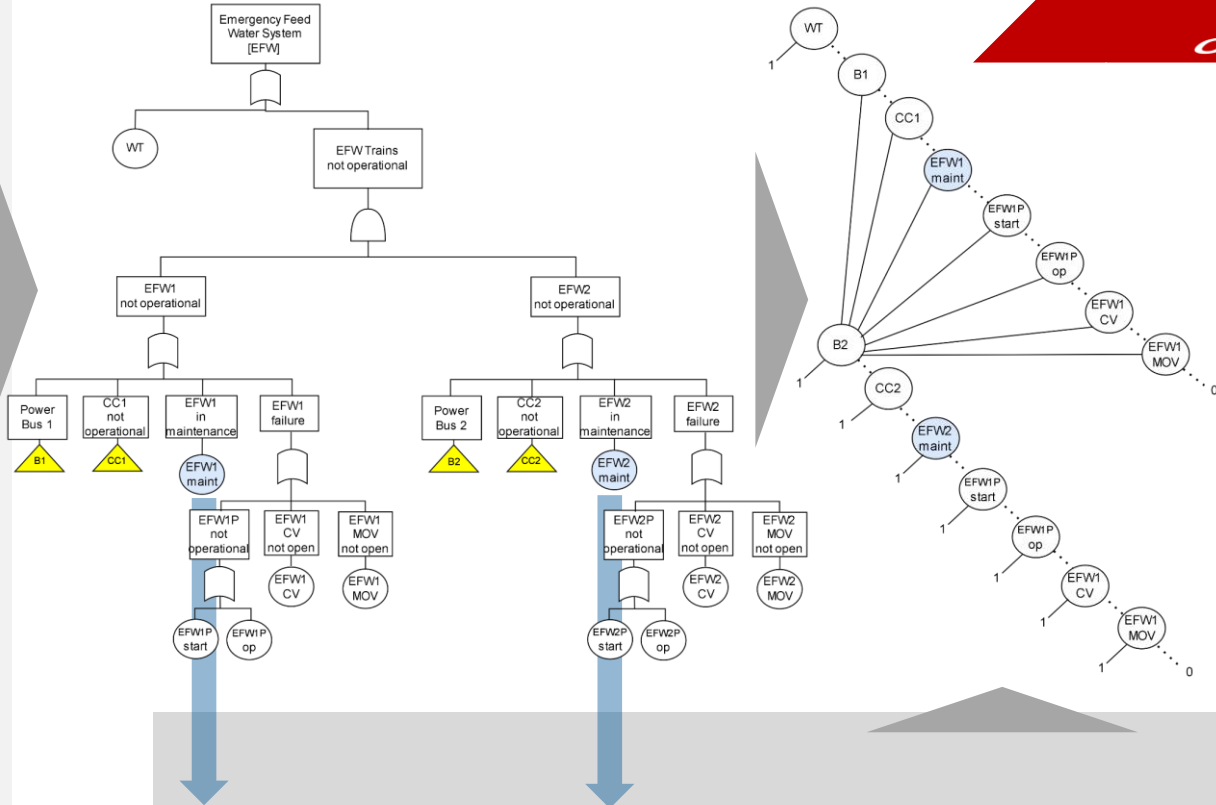
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D²T²: Basic Event Dependency



BWR EMERGENCY FEED WATER SYSTEM



$P(TOP)_{FT} = 3.7e-03$
 $P(TOP)_{D2T2} = 2.3e-03$

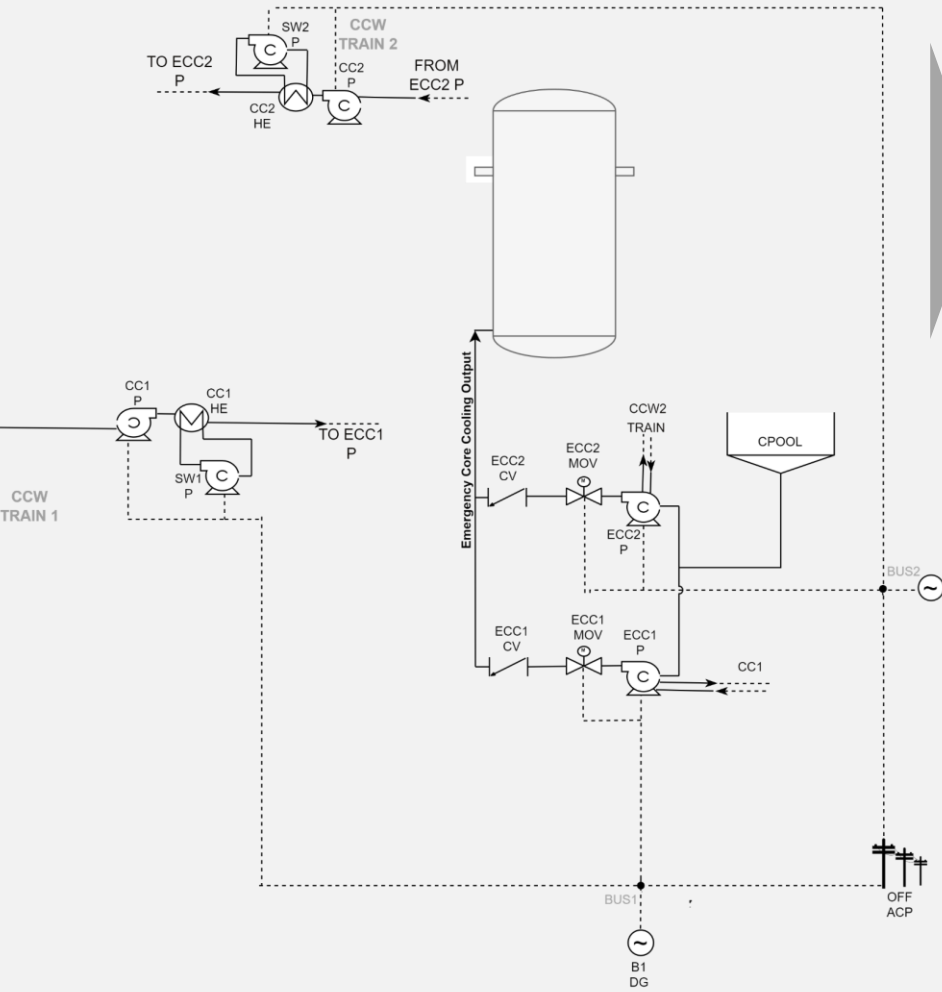
Maintenance:

- Mutually exclusive
- 60h every three months

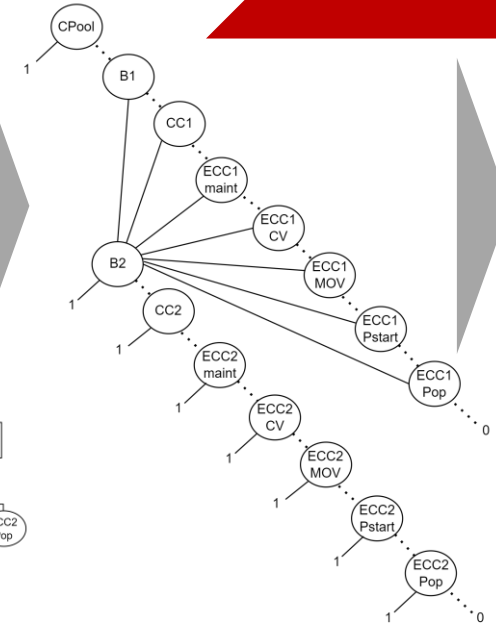
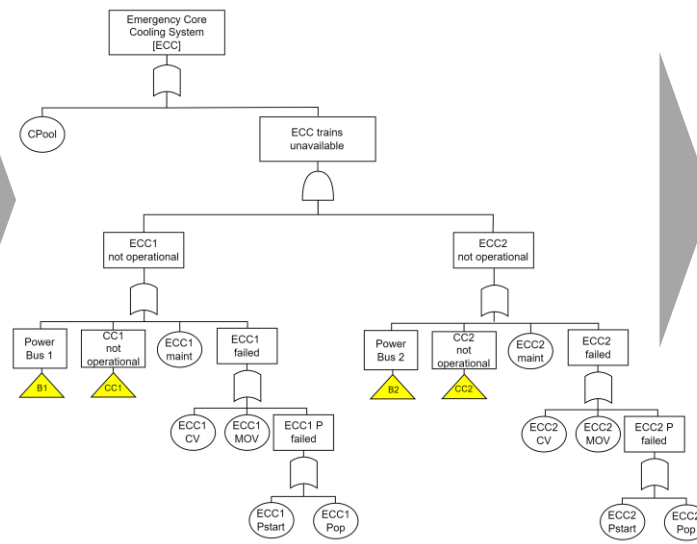
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D²T²: Intermediate Events Dependency

TRAINS

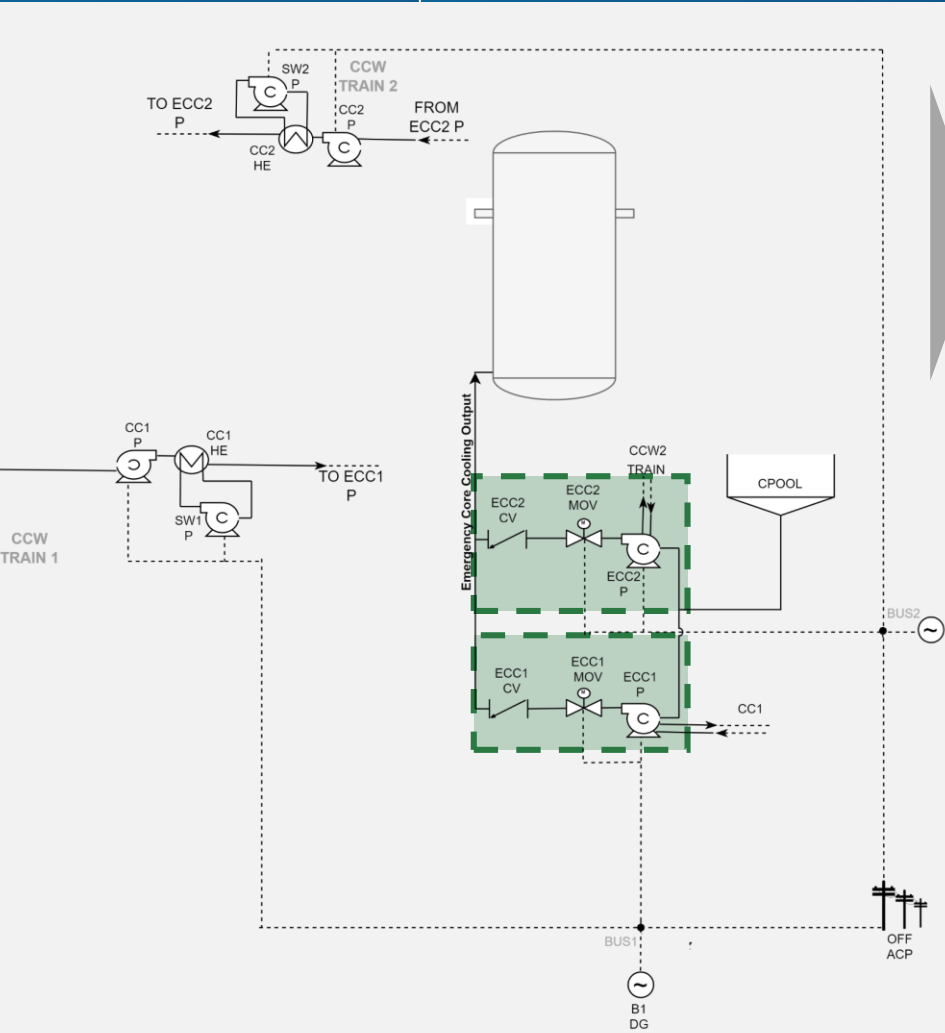


BWR EMERGENCY CORE COOLING SYSTEM

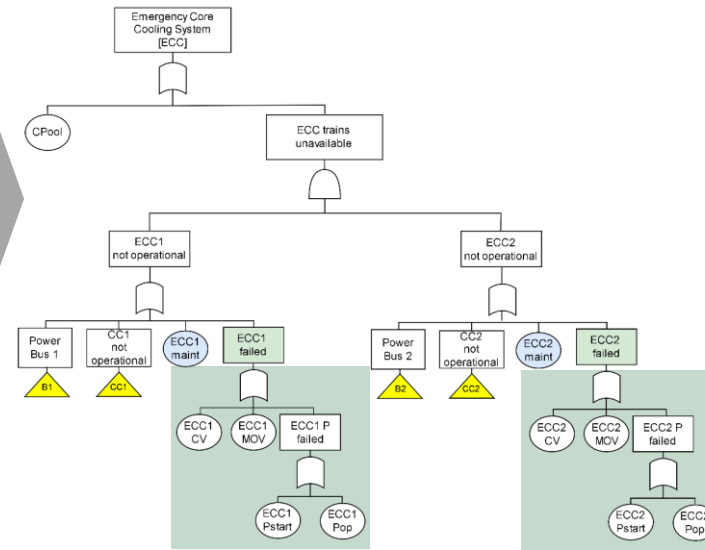


$P(\text{TOP})_{FT} = 5.53e-03$

$P(TOP)_{FT} = 5.53e-03$



BWR EMERGENCY CORE COOLING SYSTEM

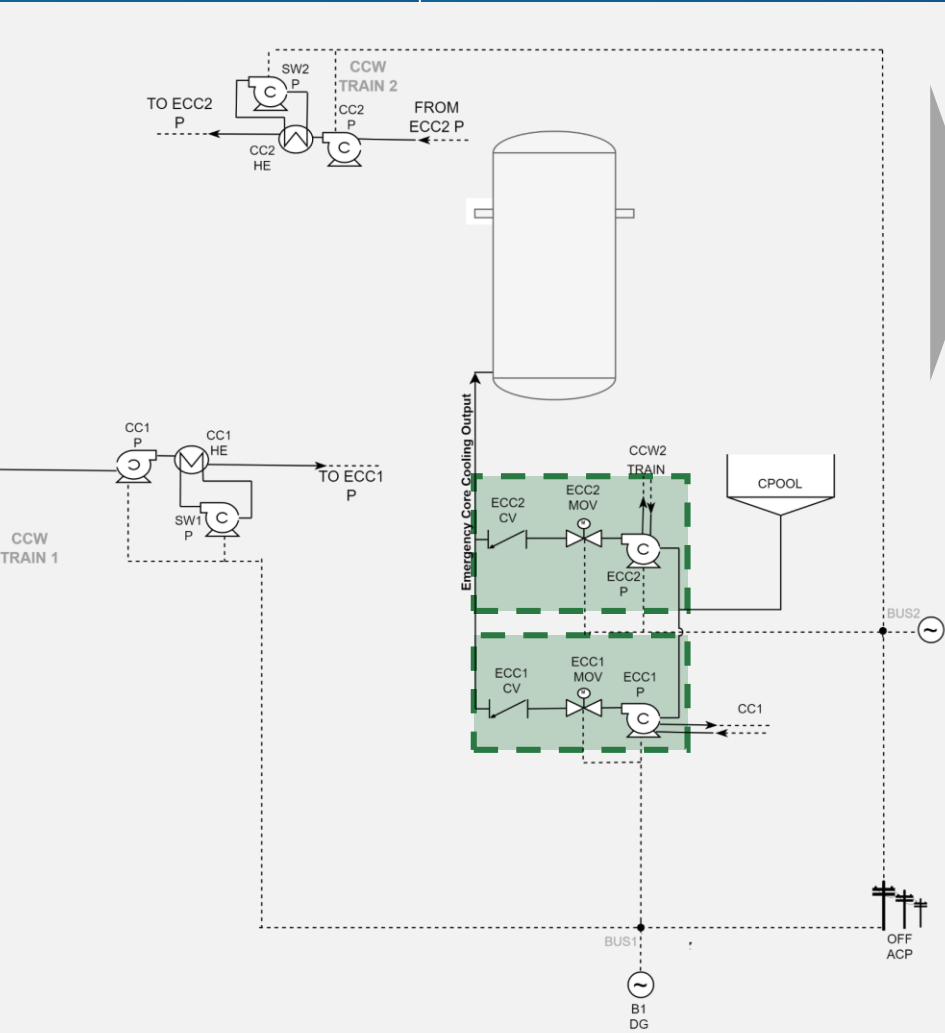


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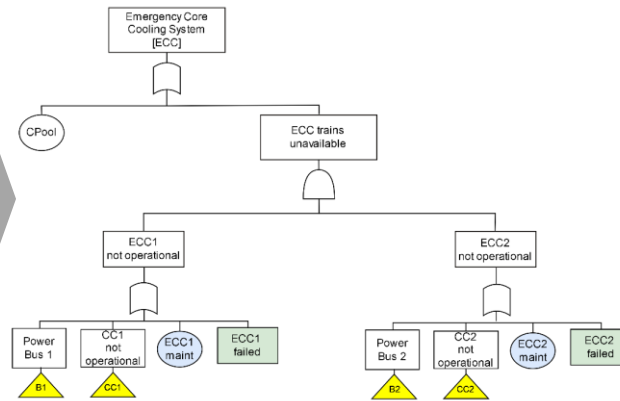
D²T²: Intermediate Events Dependency

TRAINS

$P(TOP)_{FT} = 5.53e-03$



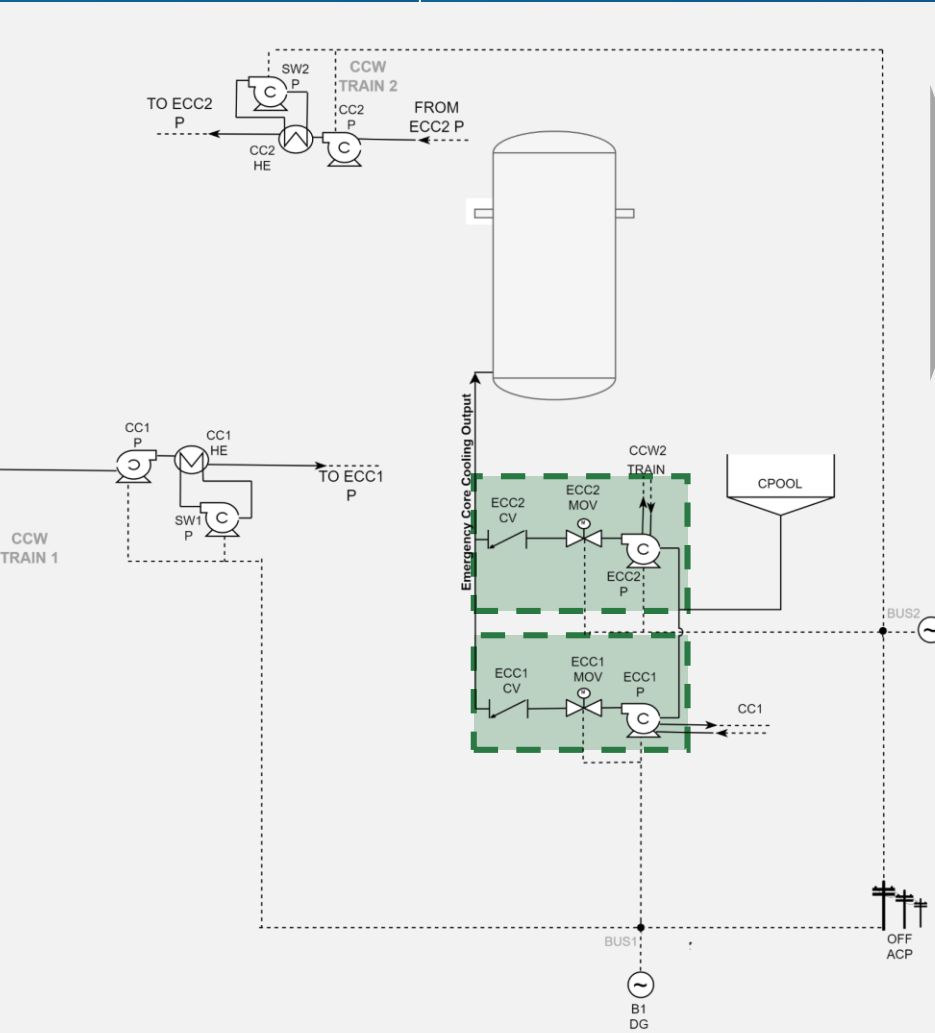
BWR EMERGENCY CORE COOLING SYSTEM



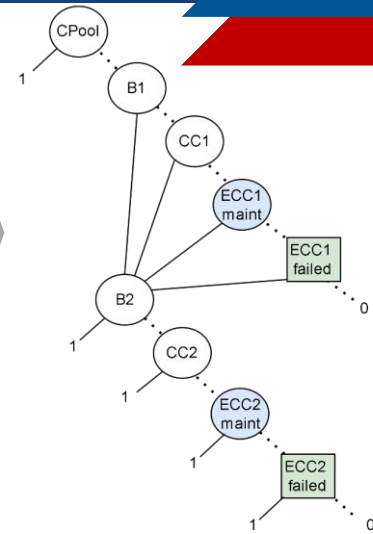
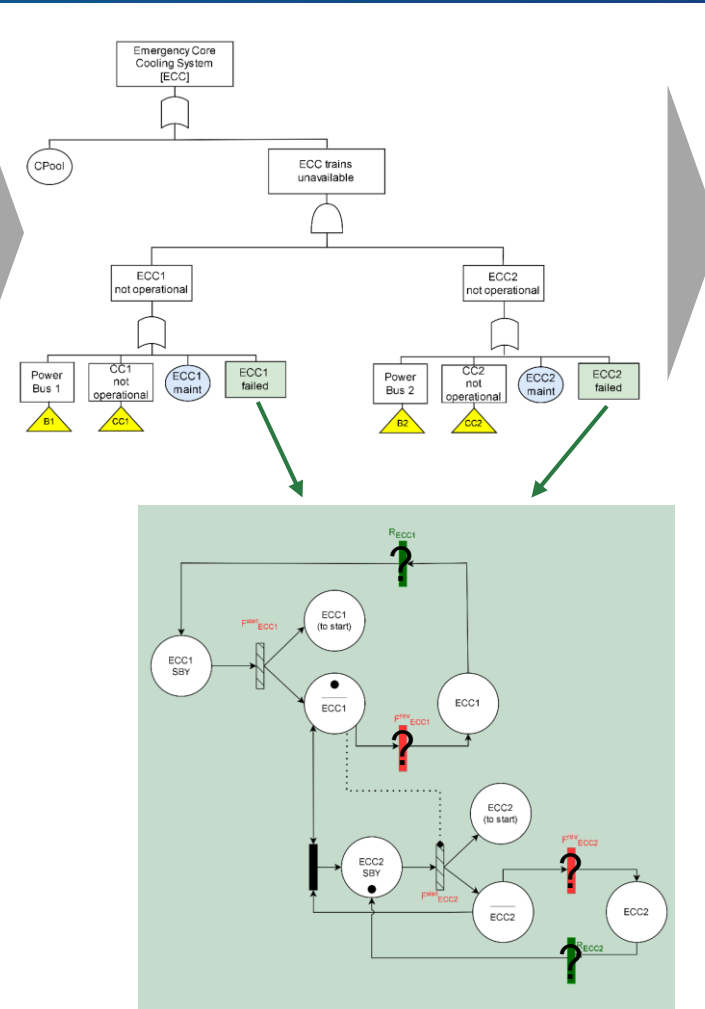
BWR EMERGENCY CORE COOLING SYSTEM

D²T²: Intermediate Events Dependency

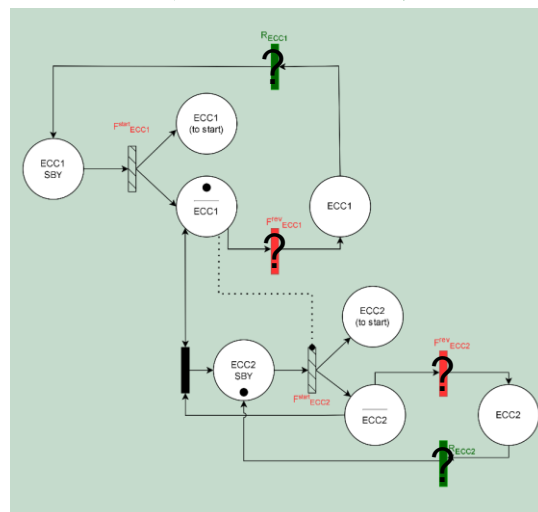
TRAINS



BWR EMERGENCY CORE COOLING SYSTEM

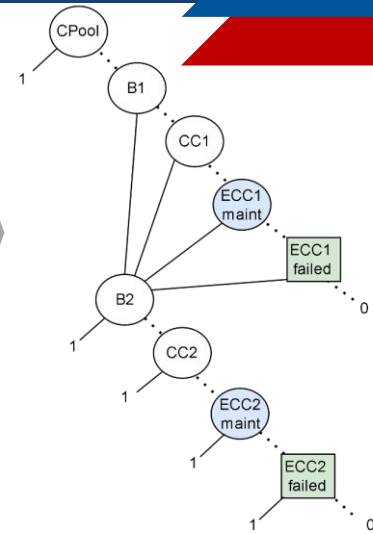
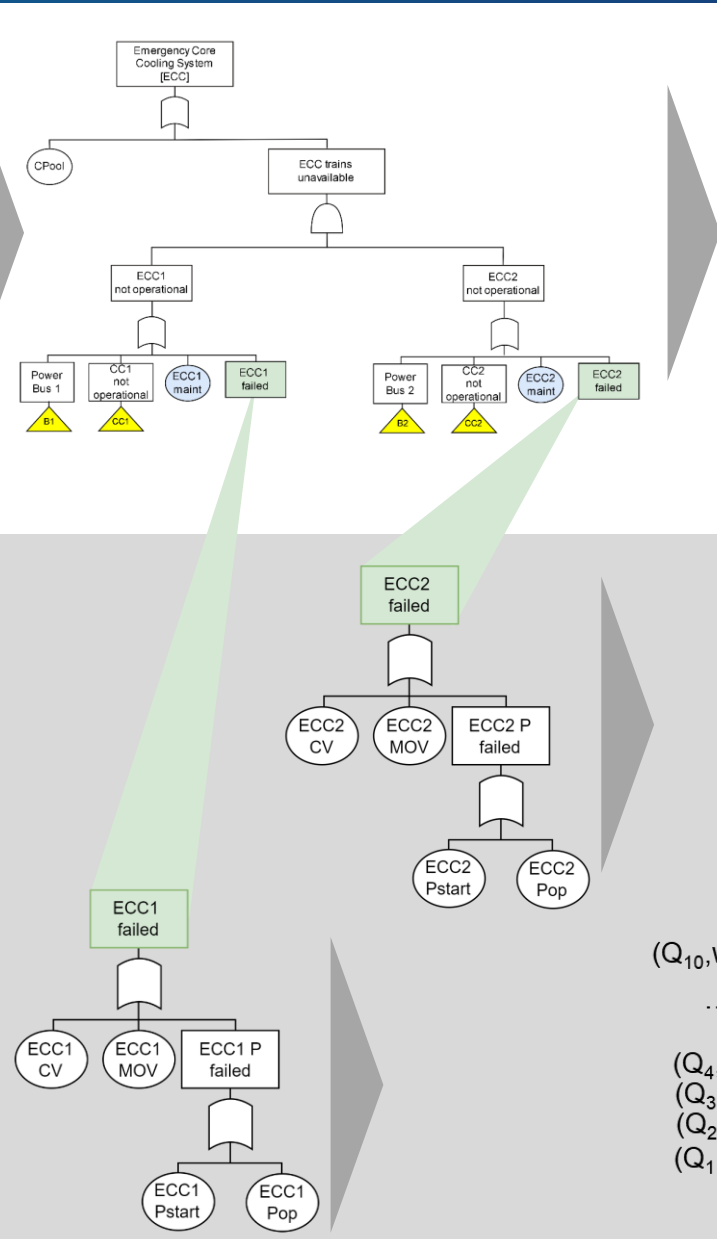
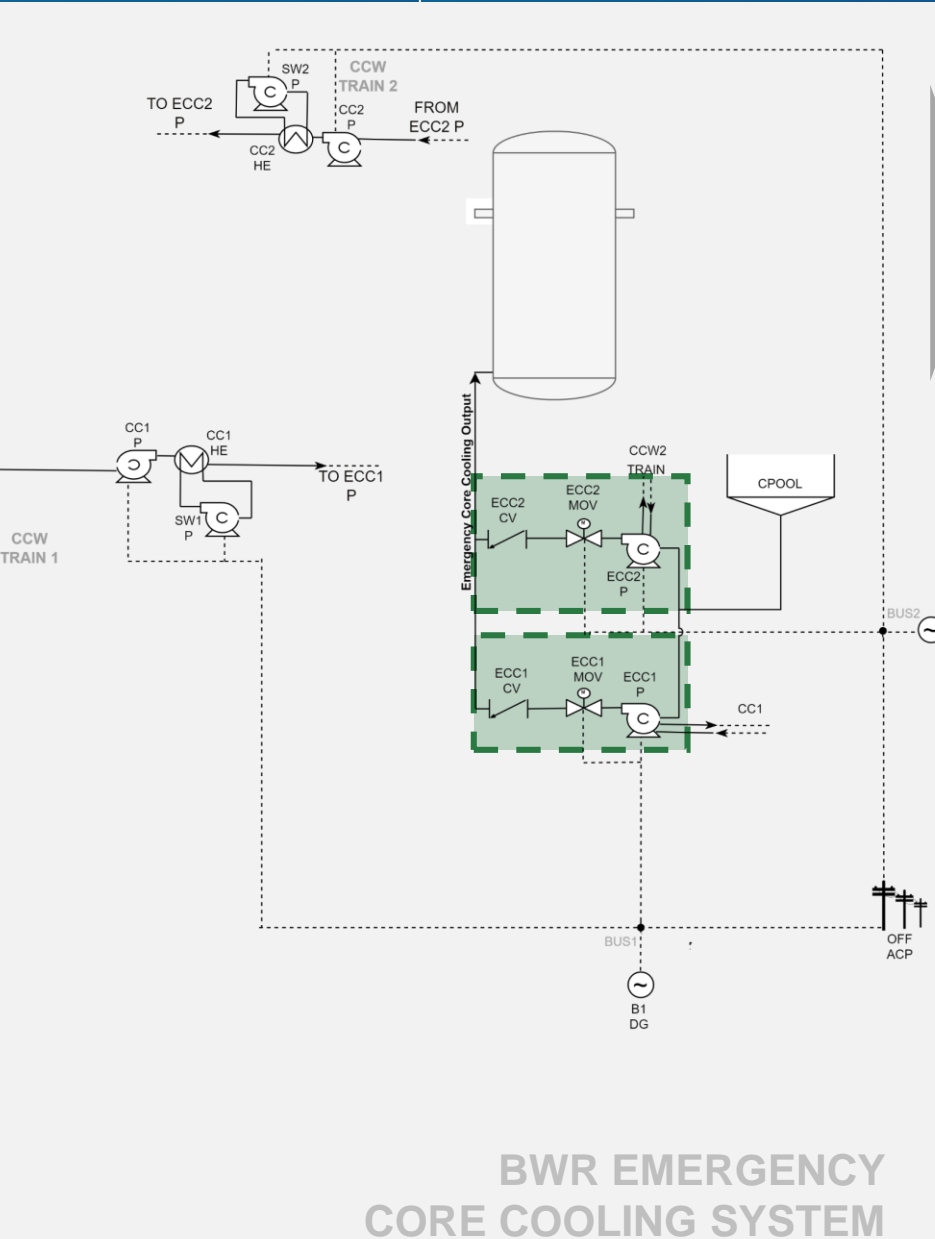


$P(\text{TOP})_{FT} = 5.53e-03$

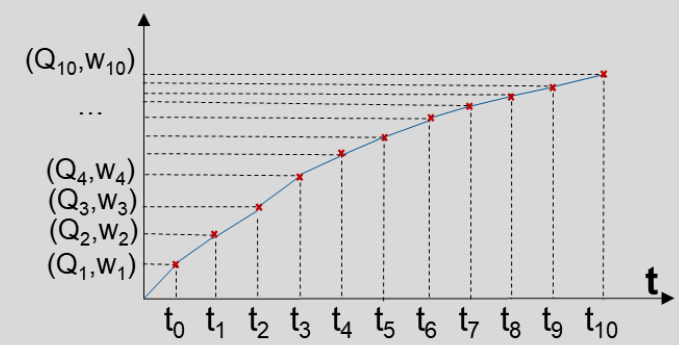
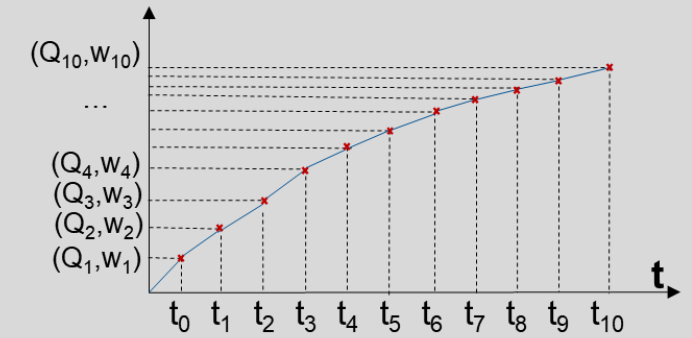


D²T²: Intermediate Events Dependency

TRAINS



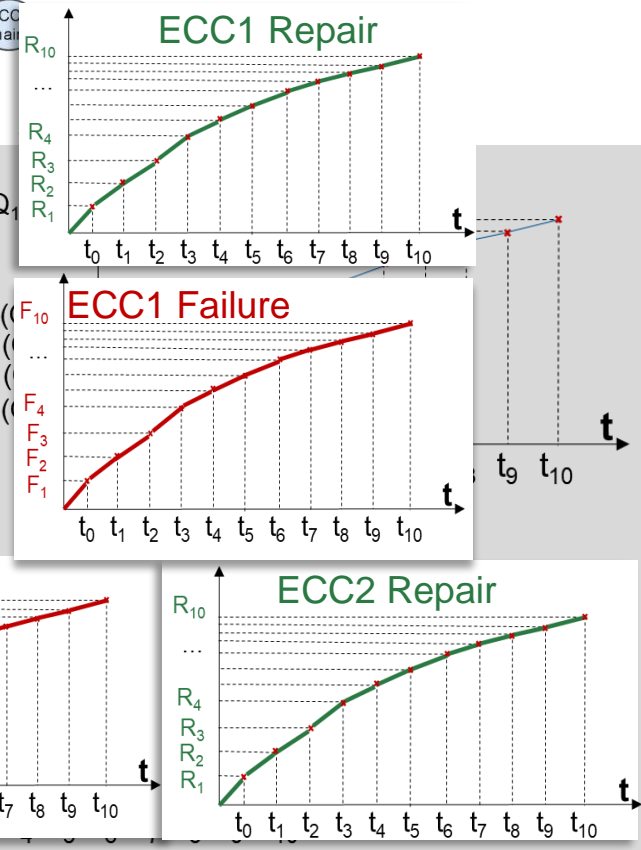
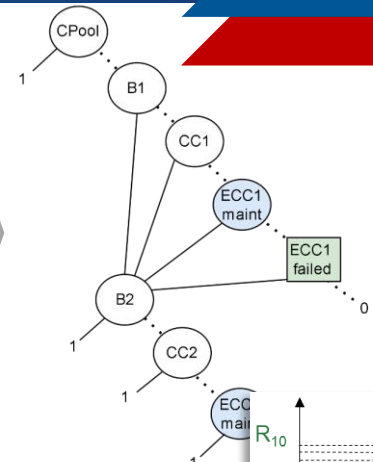
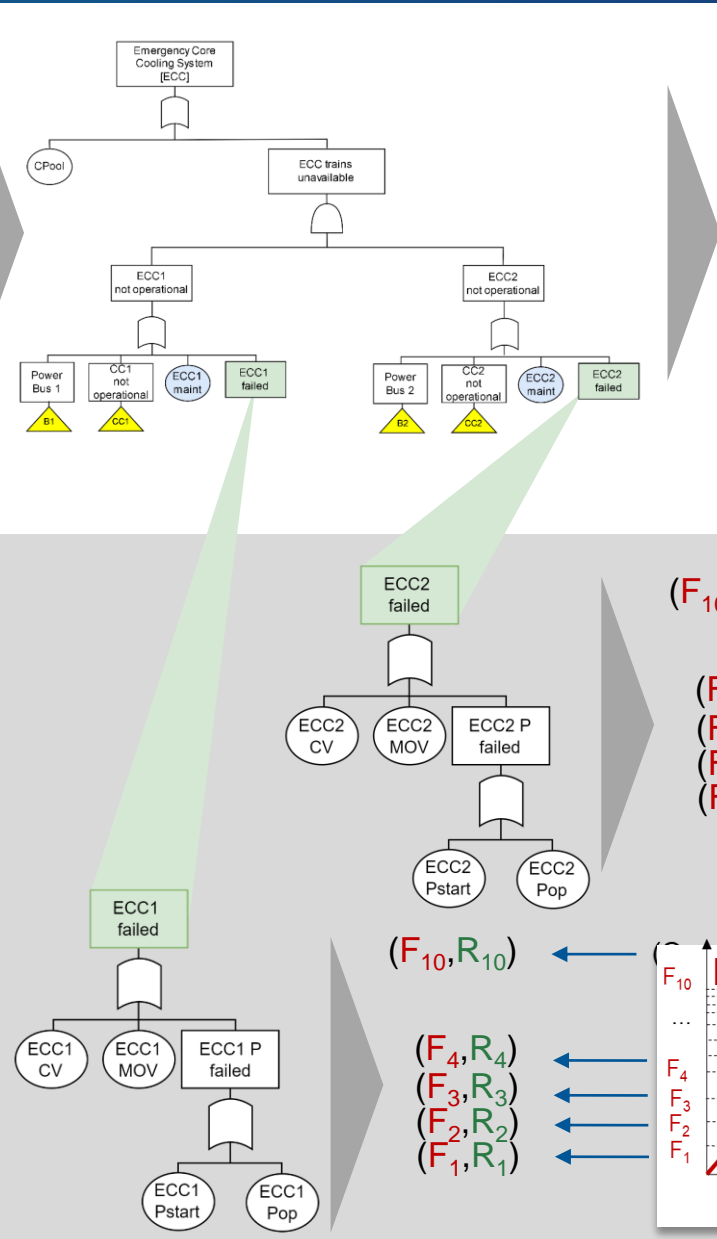
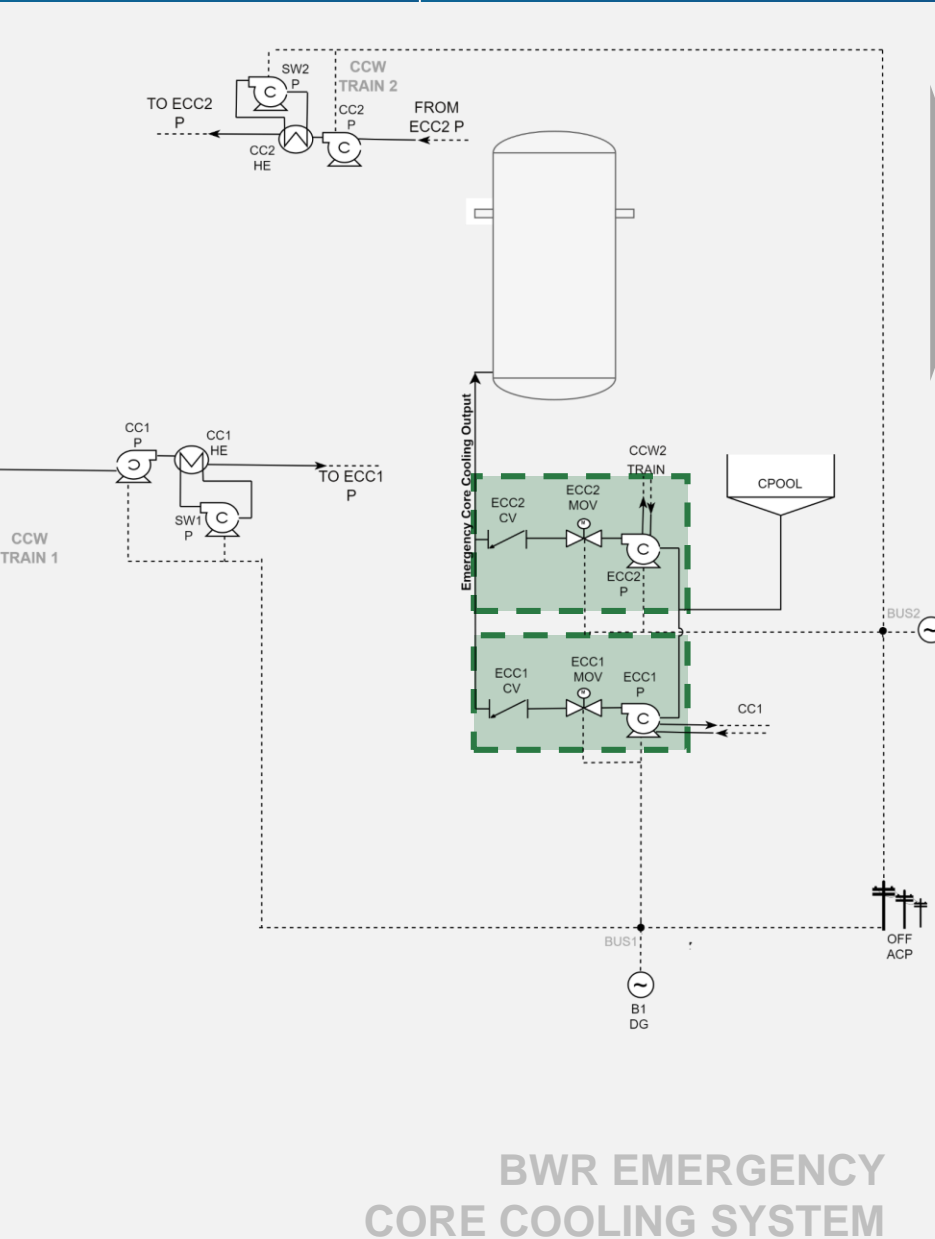
P(TOP) FT = 5.53e-03



D²T²: Intermediate Events Dependency

TRAINS

$$P(\text{TOP})_{FT} = 5.53e-03$$

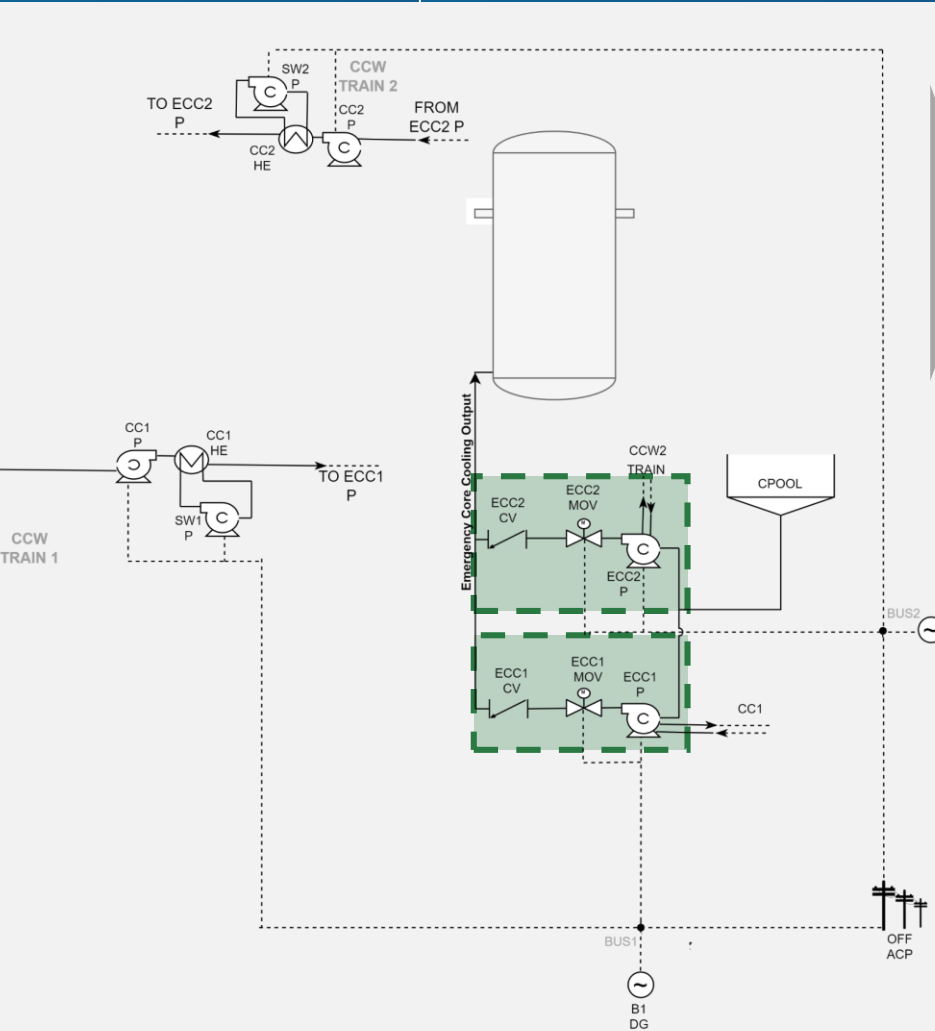


- (F_{10}, R_{10})
- (F_4, R_4)
- (F_3, R_3)
- (F_2, R_2)
- (F_1, R_1)

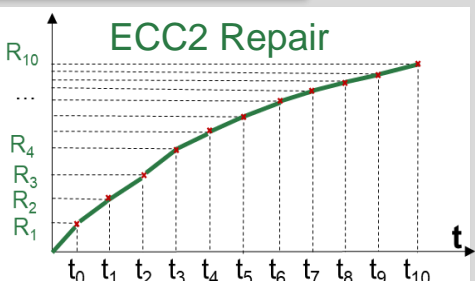
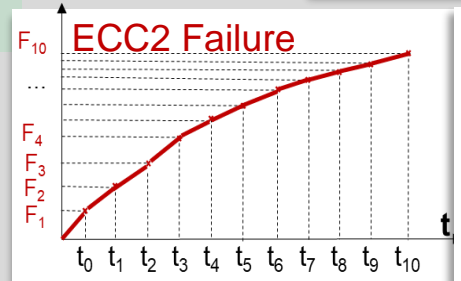
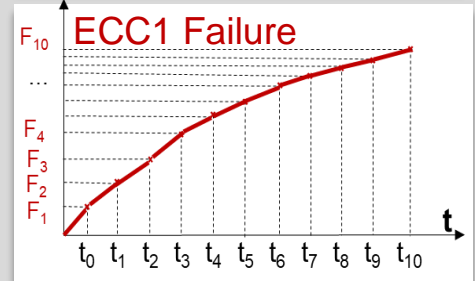
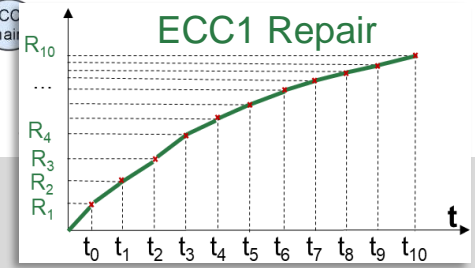
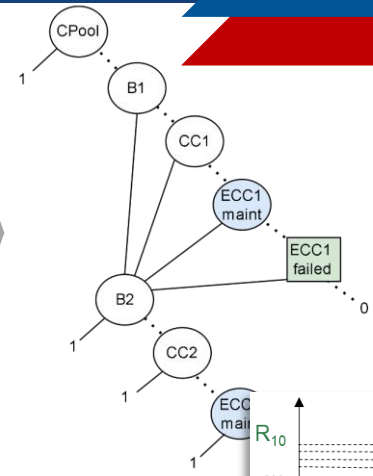
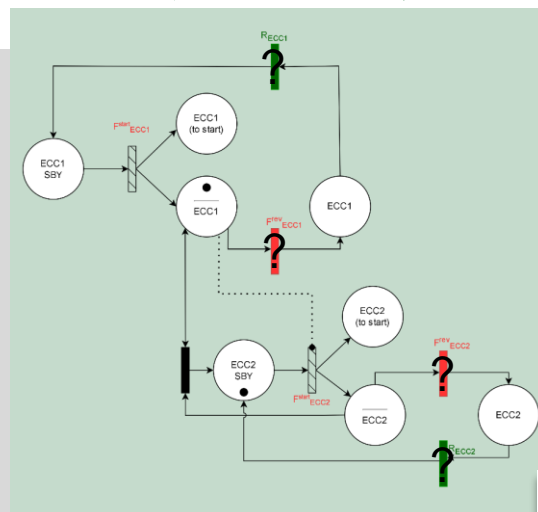
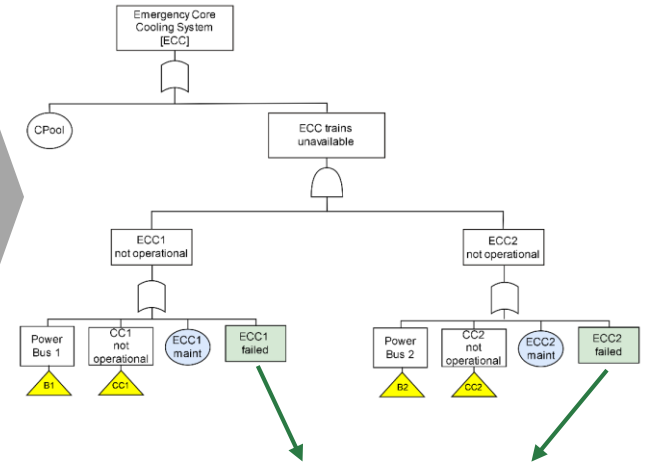
D²T²: Intermediate Events Dependency

TRAINS

$$P(\text{TOP})_{FT} = 5.53e-03$$

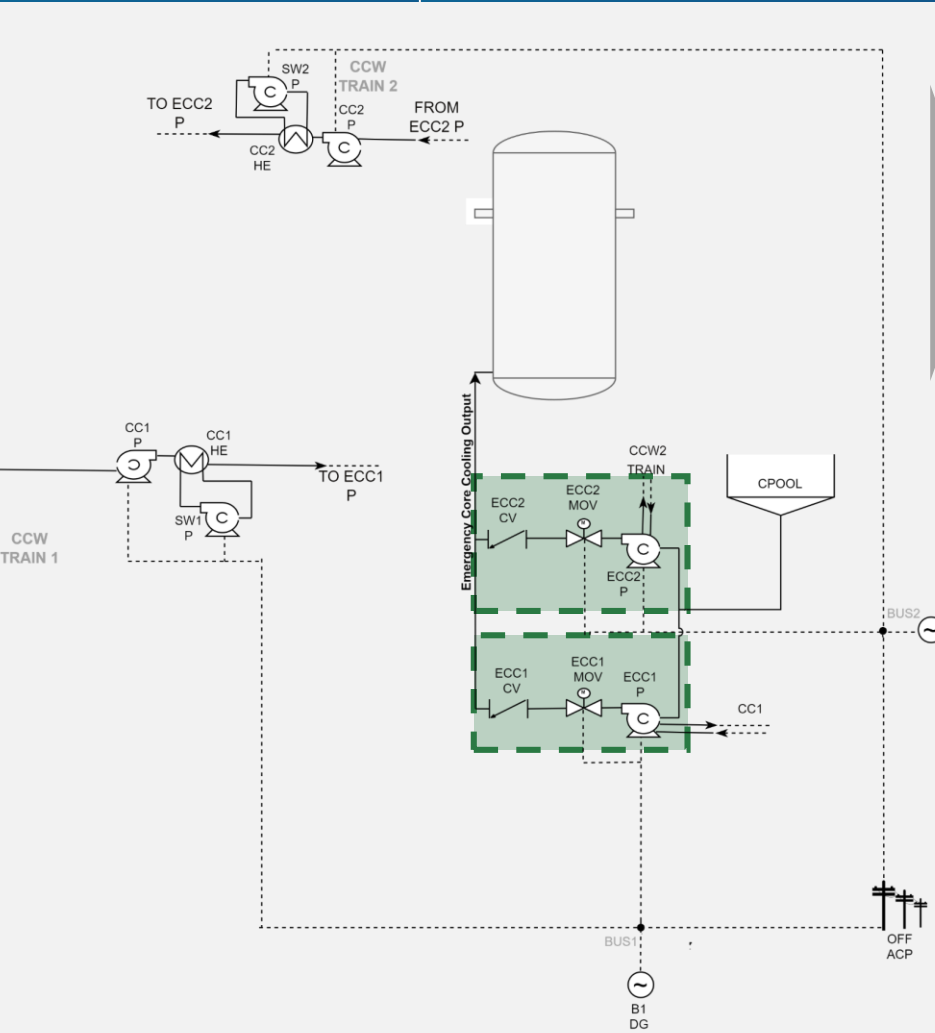


BWR EMERGENCY CORE COOLING SYSTEM

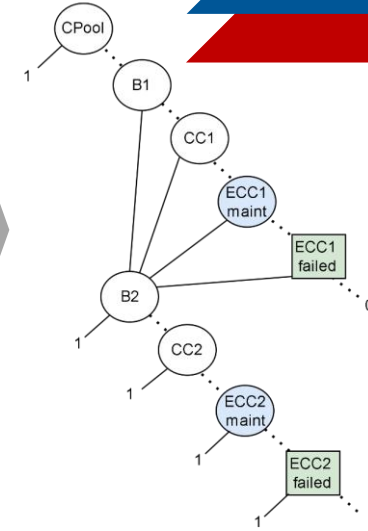
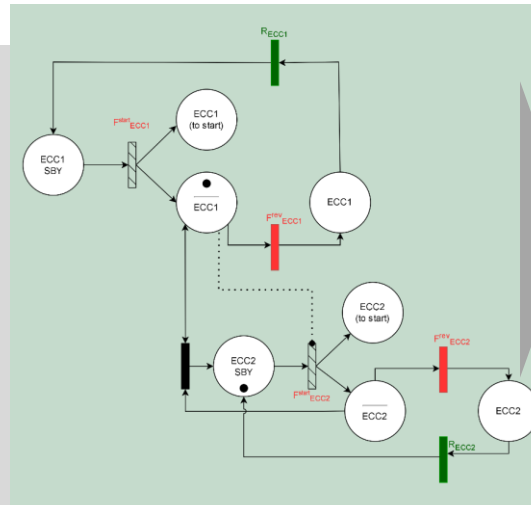


D²T²: Intermediate Events Dependency

TRAINS



BWR EMERGENCY CORE COOLING SYSTEM

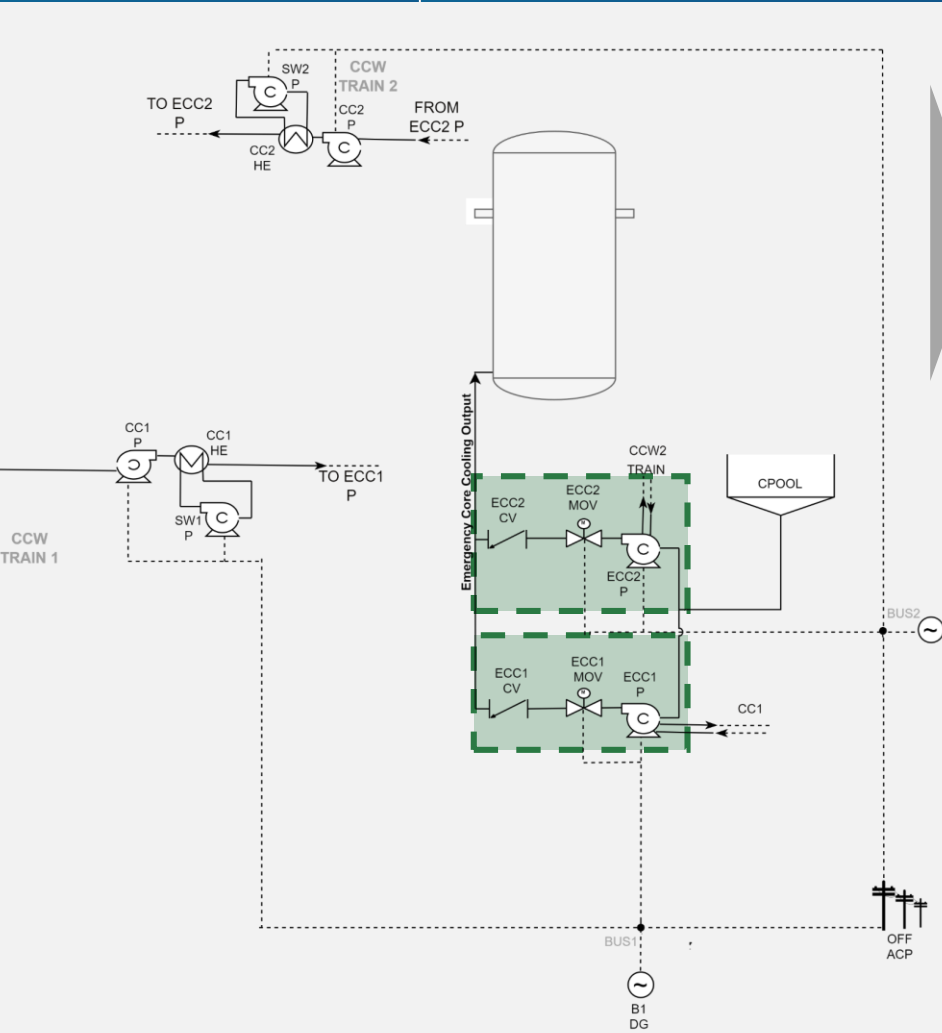


$P(\text{TOP})_{FT} = 5.53e-03$

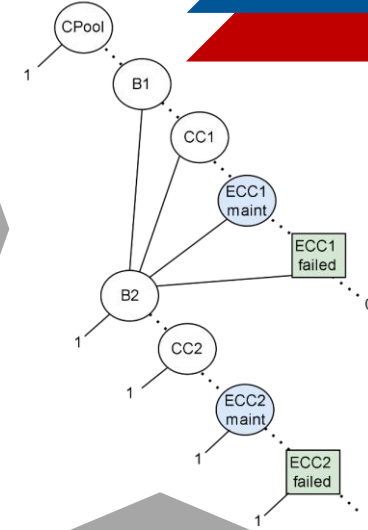
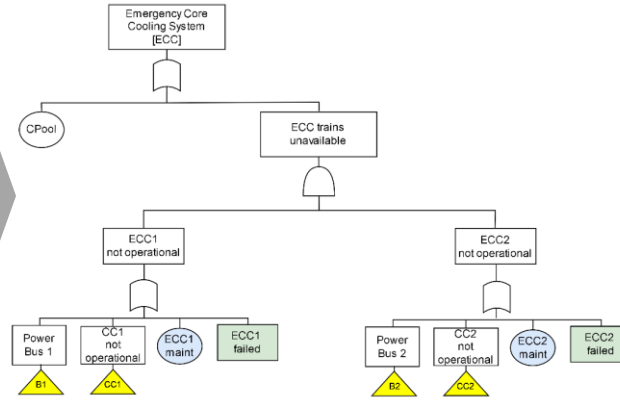
Joint Event	Probability
$\overline{ECC1 fail}, \overline{ECC2 fail}$	$9.97 \cdot 10^{-1}$
$ECC1 fail, \overline{ECC2 fail}$	$2.90 \cdot 10^{-3}$
$\overline{ECC1 fail}, ECC2 fail$	$9.95 \cdot 10^{-5}$
$ECC1 fail, ECC2 fail$	$8.62 \cdot 10^{-6}$

D²T²: Intermediate Events Dependency

TRAINS



BWR EMERGENCY CORE COOLING SYSTEM



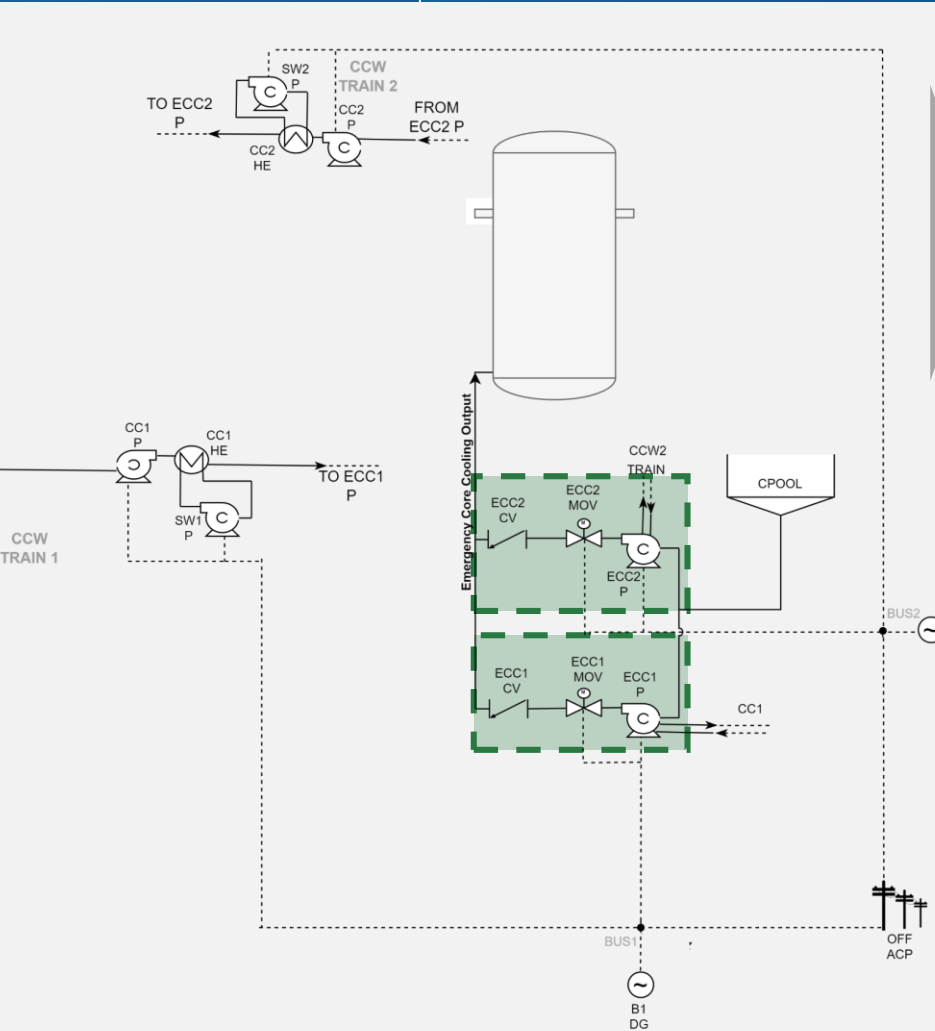
P(TOP) FT = 5.53e-03

Joint Event	Probability
$\overline{ECC1fail}, \overline{ECC2fail}$	$9.97 \cdot 10^{-1}$
$\overline{ECC1fail}, ECC2fail$	$2.90 \cdot 10^{-3}$
$ECC1fail, \overline{ECC2fail}$	$9.95 \cdot 10^{-5}$
$ECC1fail, ECC2fail$	$8.62 \cdot 10^{-6}$

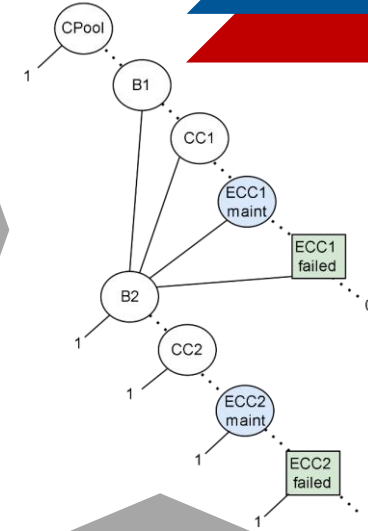
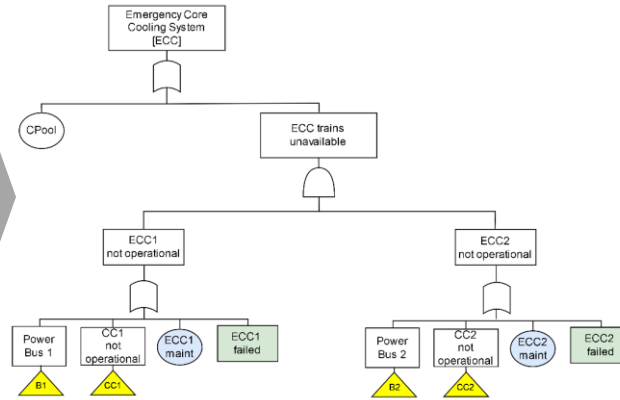
Joint Event	Probability
$\overline{ECC1maint}, \overline{ECC2maint}$	0.9452
$\overline{ECC1maint}, ECC2maint$	0.0274
$ECC1maint, \overline{ECC2maint}$	0.0274
$ECC1maint, ECC2maint$	0

D²T²: Intermediate Events Dependency

TRAINS



BWR EMERGENCY CORE COOLING SYSTEM



P(TOP) FT = 5.53e-03

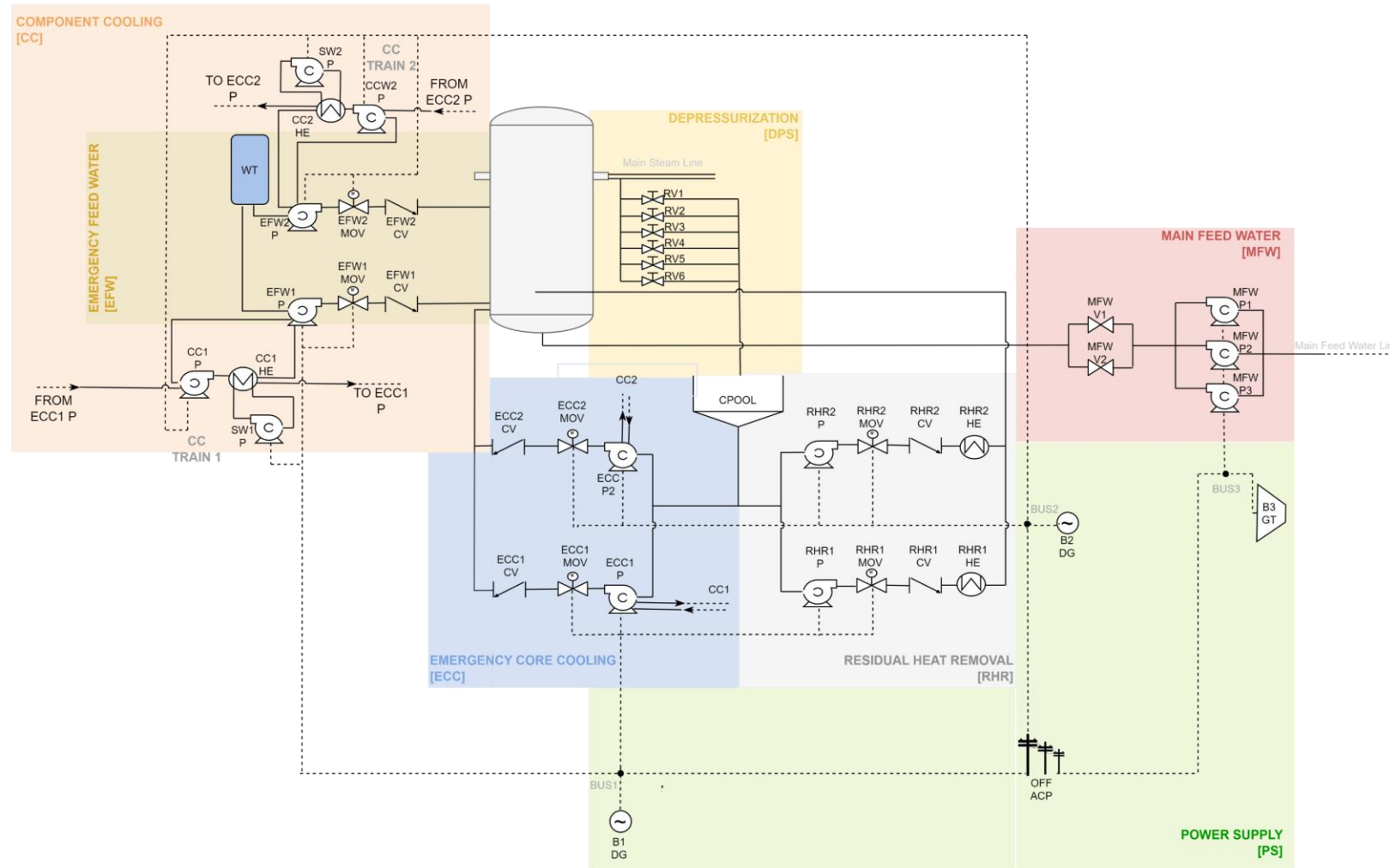
P(TOP) D²T² = 1.65e-03

Joint Event	Probability
$\overline{ECC1fail}, \overline{ECC2fail}$	$9.97 \cdot 10^{-1}$
$\overline{ECC1fail}, ECC2fail$	$2.90 \cdot 10^{-3}$
$ECC1fail, \overline{ECC2fail}$	$9.95 \cdot 10^{-5}$
$ECC1fail, ECC2fail$	$8.62 \cdot 10^{-6}$

Joint Event	Probability
$\overline{ECC1maint}, \overline{ECC2maint}$	0.9452
$\overline{ECC1maint}, ECC2maint$	0.0274
$ECC1maint, \overline{ECC2maint}$	0.0274
$ECC1maint, ECC2maint$	0

D²T²: Fault Trees Dependency

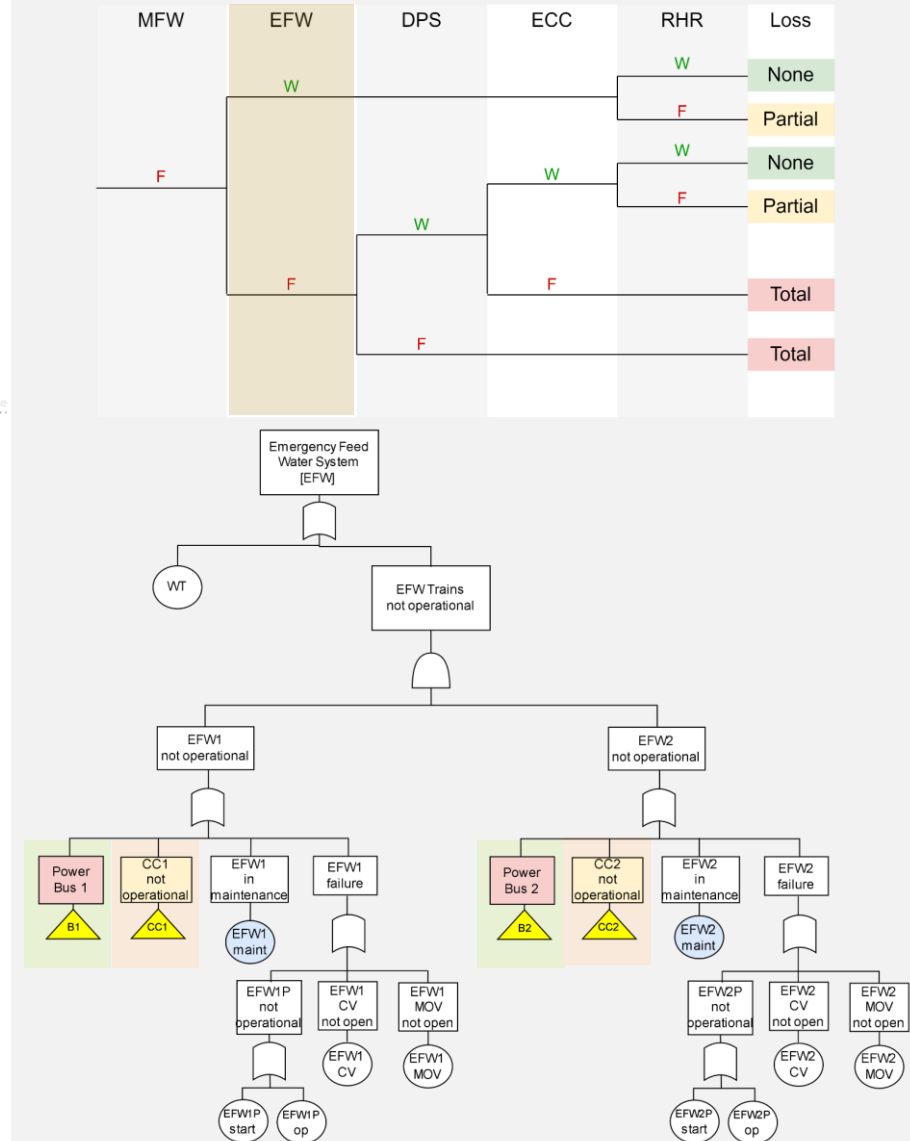
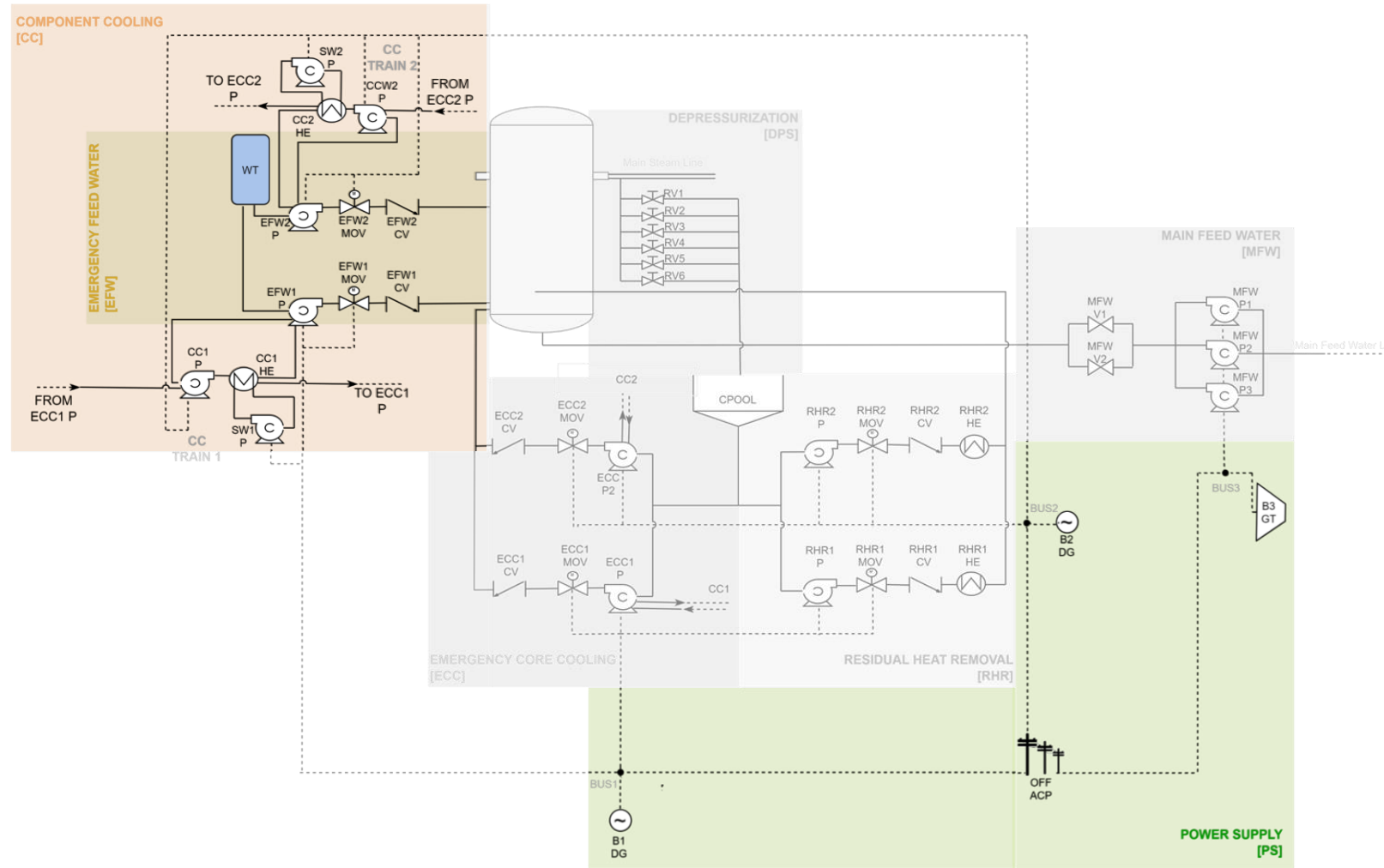
SUBSYSTEMS

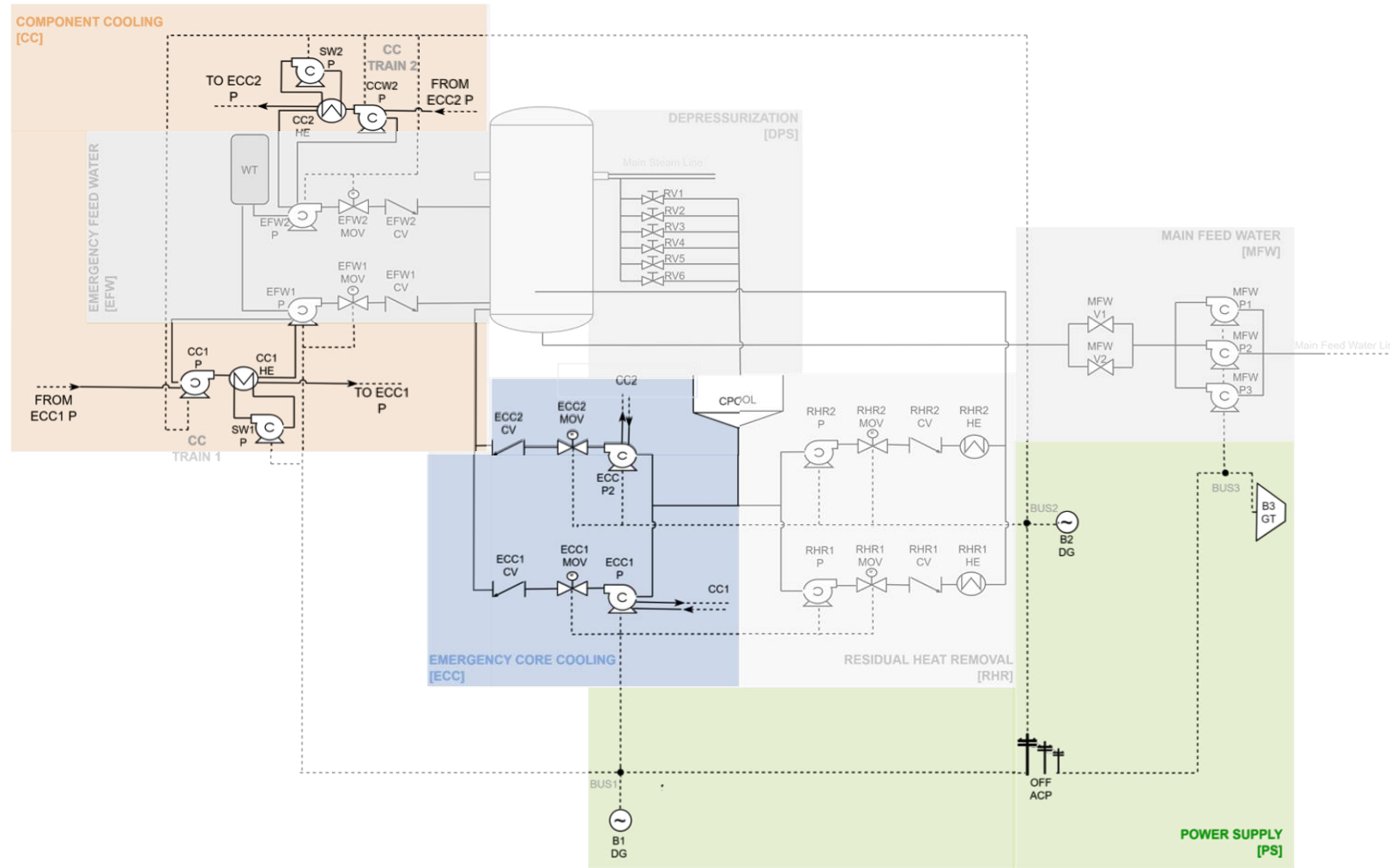


MFW	EFW	DPS	ECC	RHR	Loss
	W			W	None
				F	Partial
			W	W	None
				F	Partial
F					Total
	F	W			Total
			F		Total

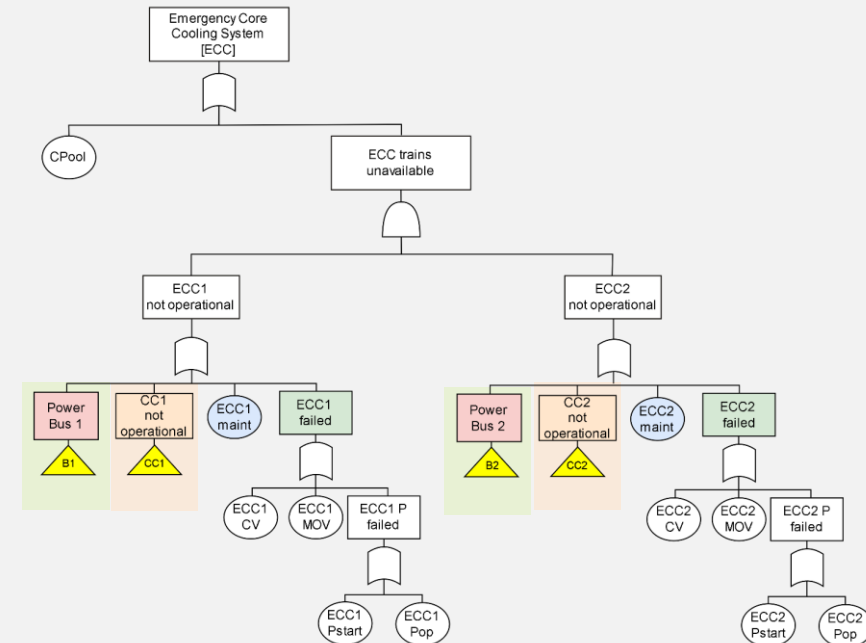
D²T²: Fault Trees Dependency

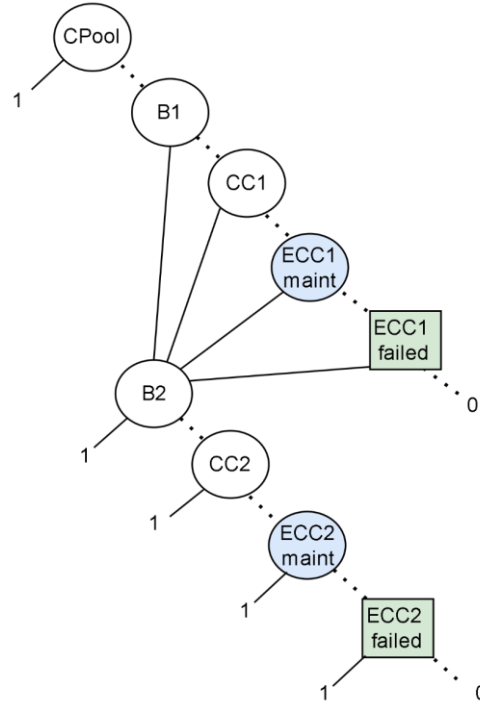
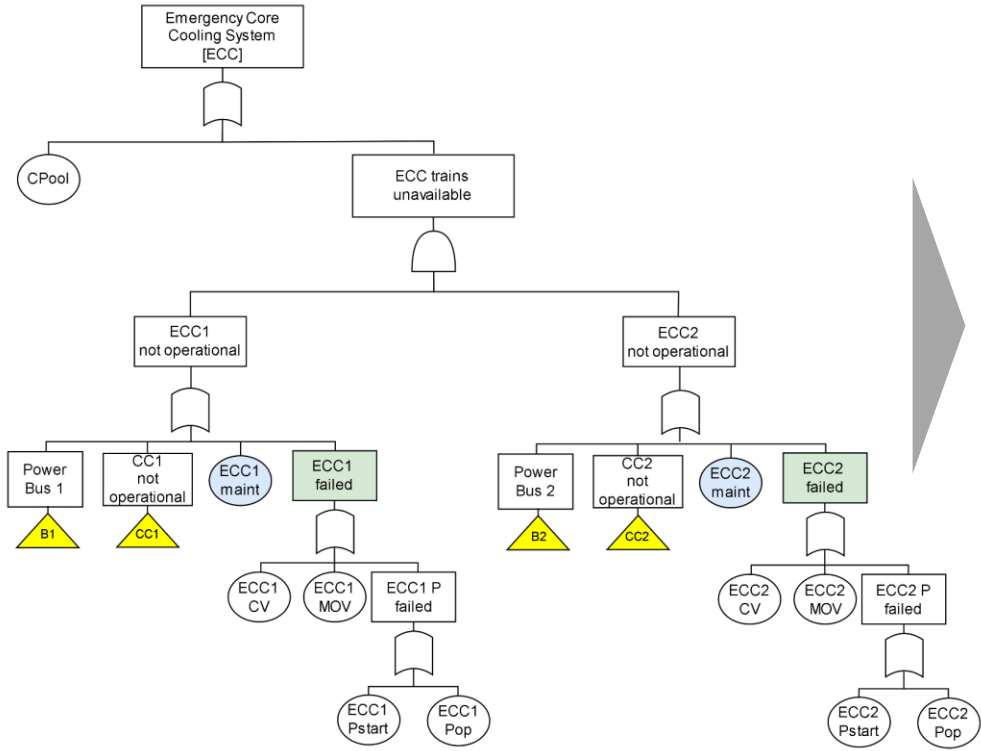
SUBSYSTEMS





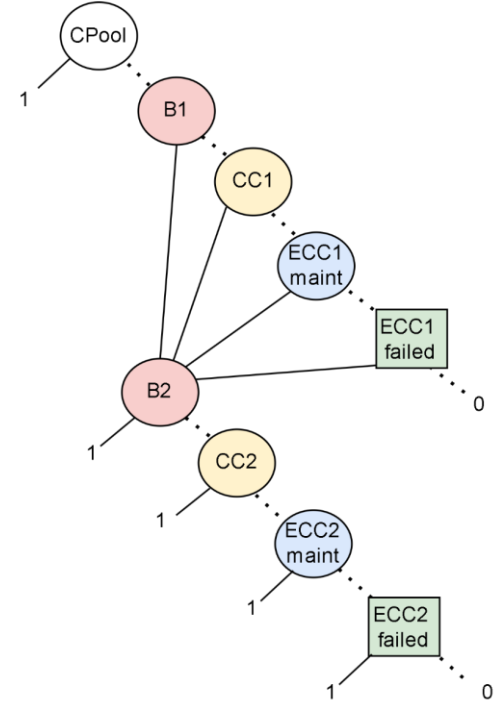
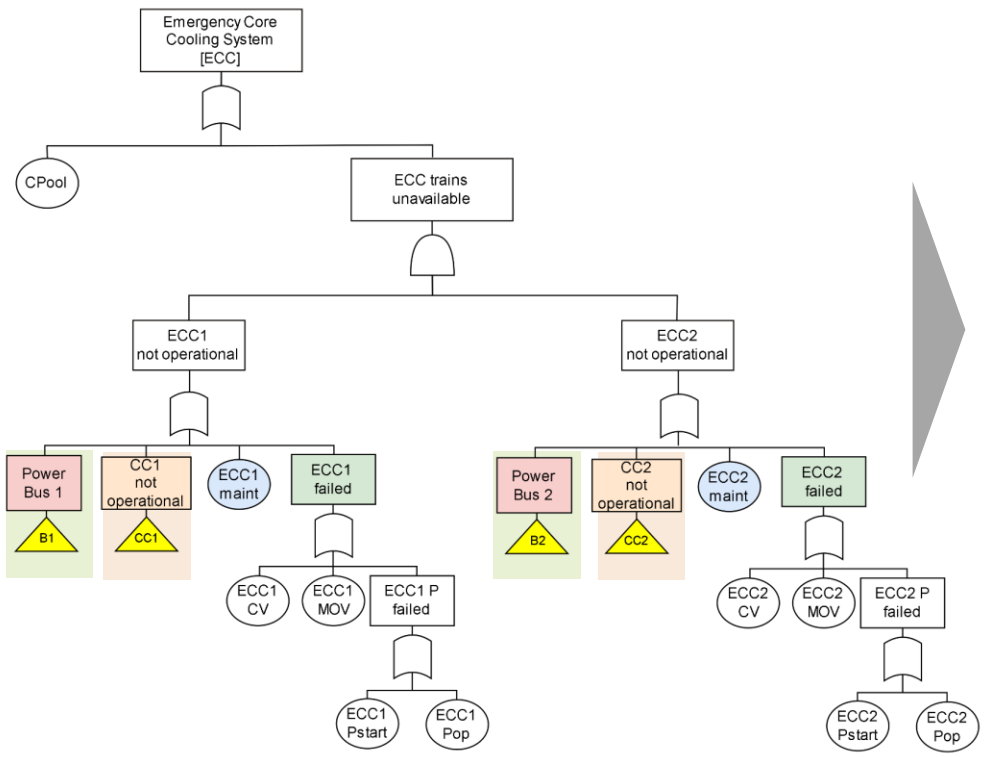
MFW	EFW	DPS	ECC	RHR	Loss
	W		W	W	None
			W	F	Partial
			W	W	None
F			F	F	Partial
	F	W			Total
		F			Total





$$P^{D2T}_{top} = \sum_i^n path_i = 1.65 \cdot 10^{-03}$$

D²T²: Fault Trees Dependency

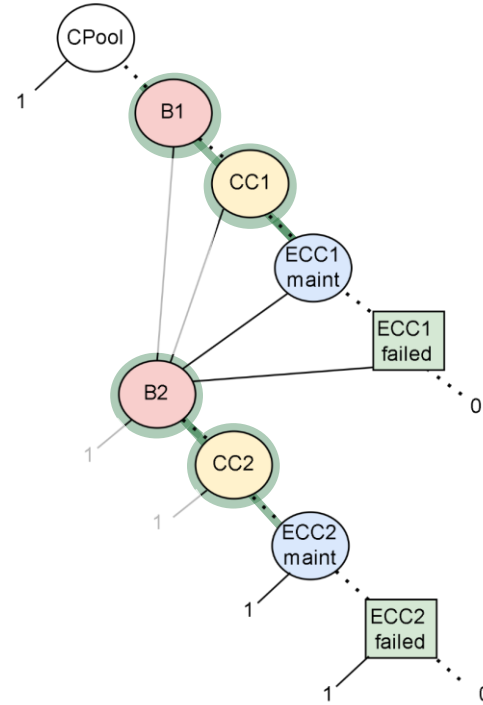
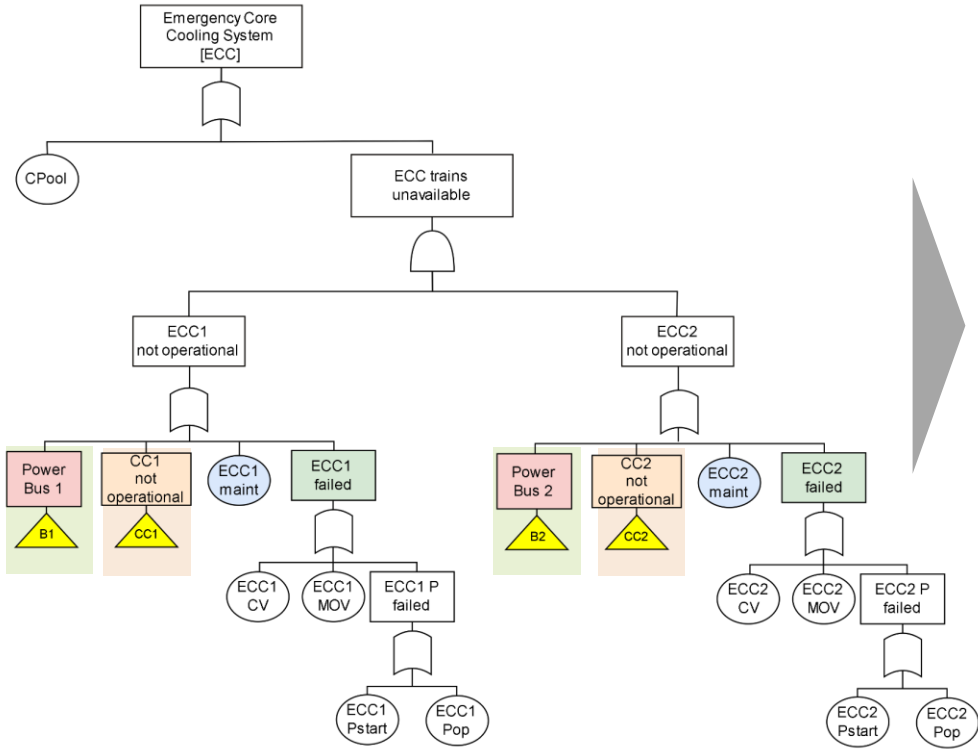


$$p_{D2T_{top}} = \sum_i^n path_i = 1.65 \cdot 10^{-03}$$

$$p_{D2T_{top}} \rightarrow \begin{cases} p_{ss0} \\ p_{ss1} \\ p_{ss2} \\ \dots \\ p_{ssn} \end{cases}$$

D²T²: Fault Trees Dependency

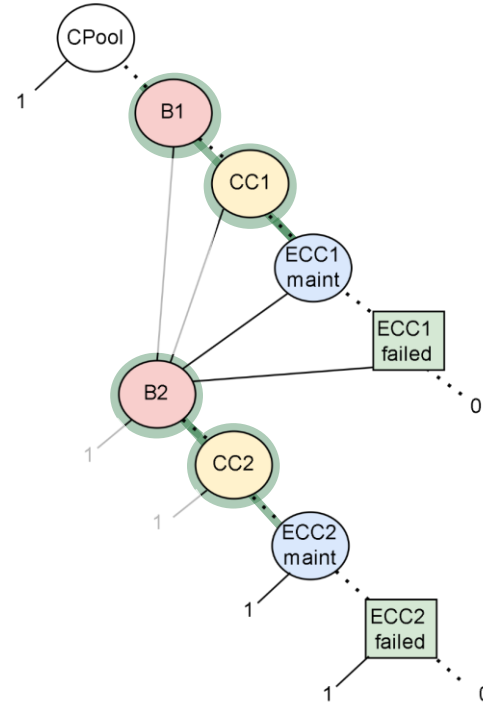
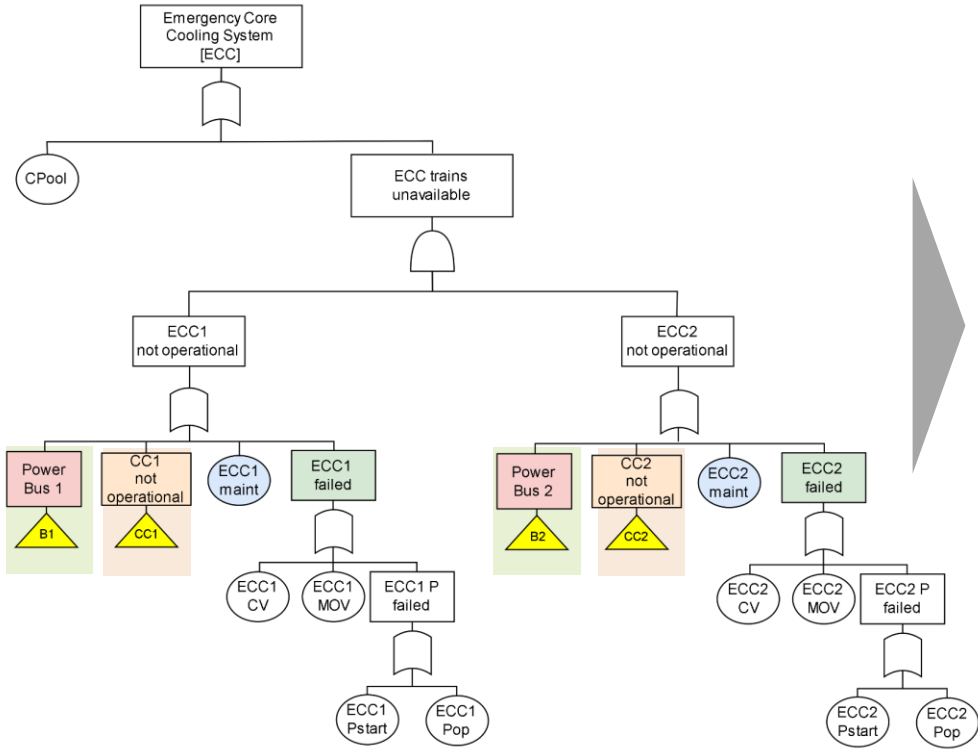
SUBSYSTEMS



SS0	B1, CC1, B2, CC2
SS1	B1, CC1, B2, $\overline{CC2}$
SS2	B1, CC1, $\overline{B2}$, CC2
SS3	B1, CC1, $\overline{B2}$, $\overline{CC2}$
SS4	B1, $\overline{CC1}$, B2, CC2
SS5	B1, $\overline{CC1}$, B2, $\overline{CC2}$
SS6	B1, $\overline{CC1}$, $\overline{B2}$, CC2
SS7	B1, $\overline{CC1}$, B2, $\overline{CC2}$
SS8	$\overline{B1}$, CC1, B2, CC2
SS9	$\overline{B1}$, CC1, B2, $\overline{CC2}$
SS10	$\overline{B1}$, CC1, $\overline{B2}$, CC2
SS11	$\overline{B1}$, CC1, $\overline{B2}$, $\overline{CC2}$
SS12	$\overline{B1}$, $\overline{CC1}$, B2, CC2
SS13	$\overline{B1}$, $\overline{CC1}$, B2, $\overline{CC2}$
SS14	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, CC2
SS15	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, $\overline{CC2}$

D²T²: Fault Trees Dependency

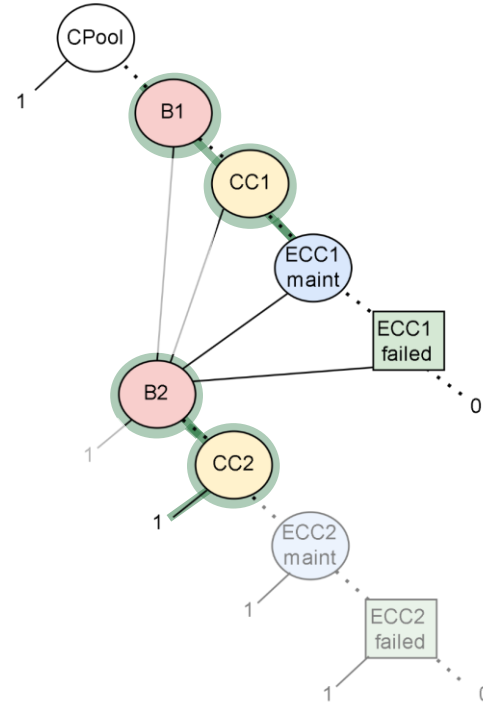
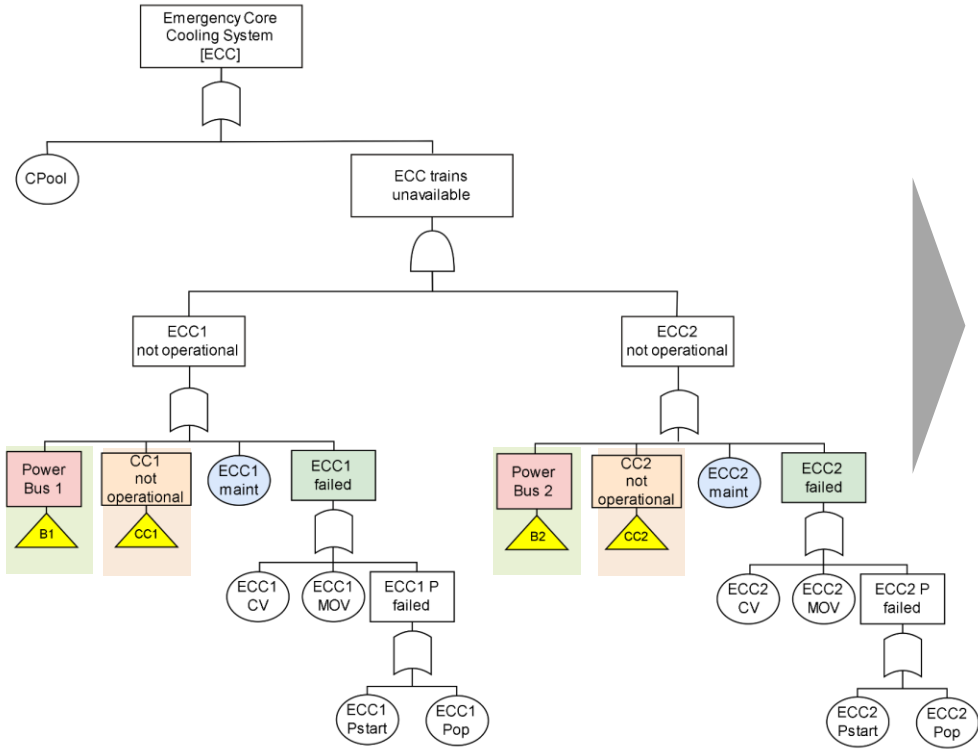
SUBSYSTEMS



SS0	B1, CC1, B2, CC2	
SS1	B1, CC1, B2, $\overline{CC2}$	
SS2	B1, CC1, $\overline{B2}$, CC2	
SS3	B1, CC1, $\overline{B2}$, $\overline{CC2}$	
SS4	B1, $\overline{CC1}$, B2, CC2	
SS5	B1, $\overline{CC1}$, B2, $\overline{CC2}$	
SS6	B1, $\overline{CC1}$, $\overline{B2}$, CC2	
SS7	B1, $\overline{CC1}$, B2, $\overline{CC2}$	
SS8	$\overline{B1}$, CC1, B2, CC2	
SS9	$\overline{B1}$, CC1, B2, $\overline{CC2}$	
SS10	$\overline{B1}$, CC1, $\overline{B2}$, CC2	
SS11	$\overline{B1}$, CC1, $\overline{B2}$, $\overline{CC2}$	
SS12	$\overline{B1}$, $\overline{CC1}$, B2, CC2	
SS13	$\overline{B1}$, $\overline{CC1}$, B2, $\overline{CC2}$	
SS14	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, CC2	
SS15	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, $\overline{CC2}$	$5.62 \cdot 10^{-04}$

D²T²: Fault Trees Dependency

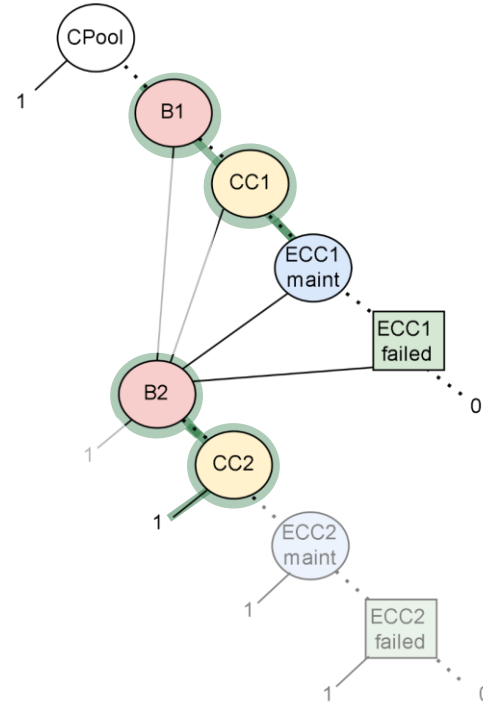
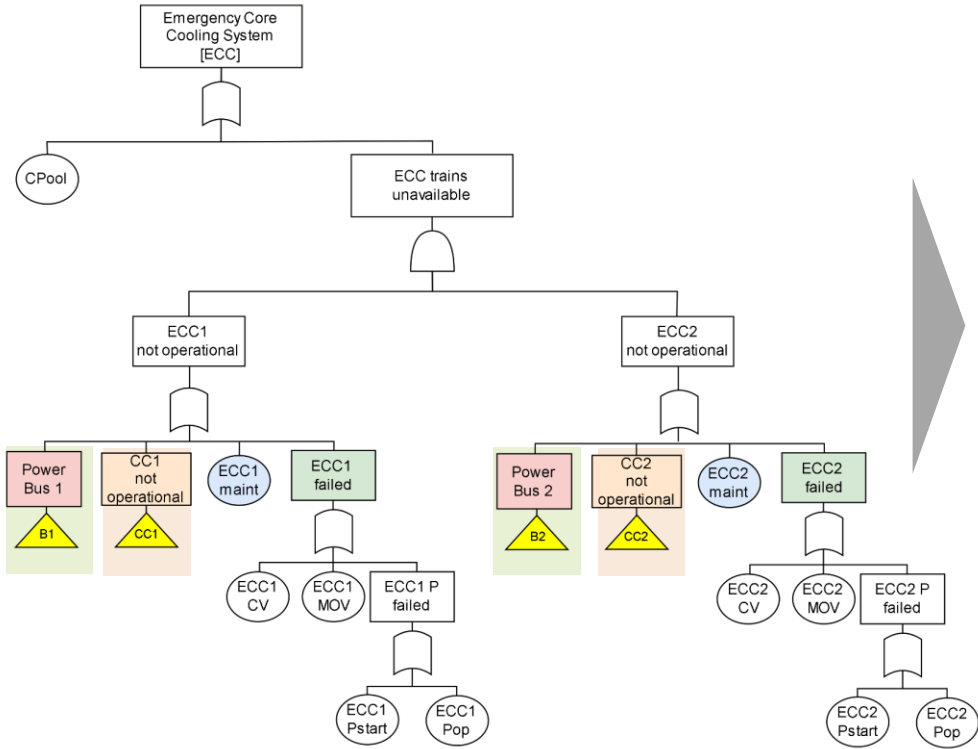
SUBSYSTEMS



SS0	B1, CC1, B2, CC2	
SS1	B1, CC1, B2, $\overline{CC2}$	
SS2	B1, CC1, $\overline{B2}$, CC2	
SS3	B1, CC1, $\overline{B2}$, $\overline{CC2}$	
SS4	B1, $\overline{CC1}$, B2, CC2	
SS5	B1, $\overline{CC1}$, B2, $\overline{CC2}$	
SS6	B1, $\overline{CC1}$, $\overline{B2}$, CC2	
SS7	B1, $\overline{CC1}$, $\overline{B2}$, $\overline{CC2}$	
SS8	$\overline{B1}$, CC1, B2, CC2	
SS9	$\overline{B1}$, CC1, B2, $\overline{CC2}$	
SS10	$\overline{B1}$, CC1, $\overline{B2}$, CC2	
SS11	$\overline{B1}$, CC1, $\overline{B2}$, $\overline{CC2}$	
SS12	$\overline{B1}$, $\overline{CC1}$, B2, CC2	
SS13	$\overline{B1}$, $\overline{CC1}$, B2, $\overline{CC2}$	
SS14	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, CC2	
SS15	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, $\overline{CC2}$	$5.62 \cdot 10^{-04}$

D²T²: Fault Trees Dependency

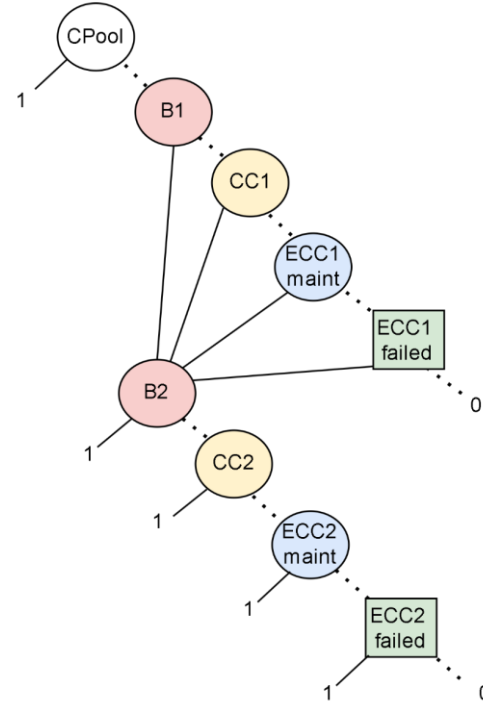
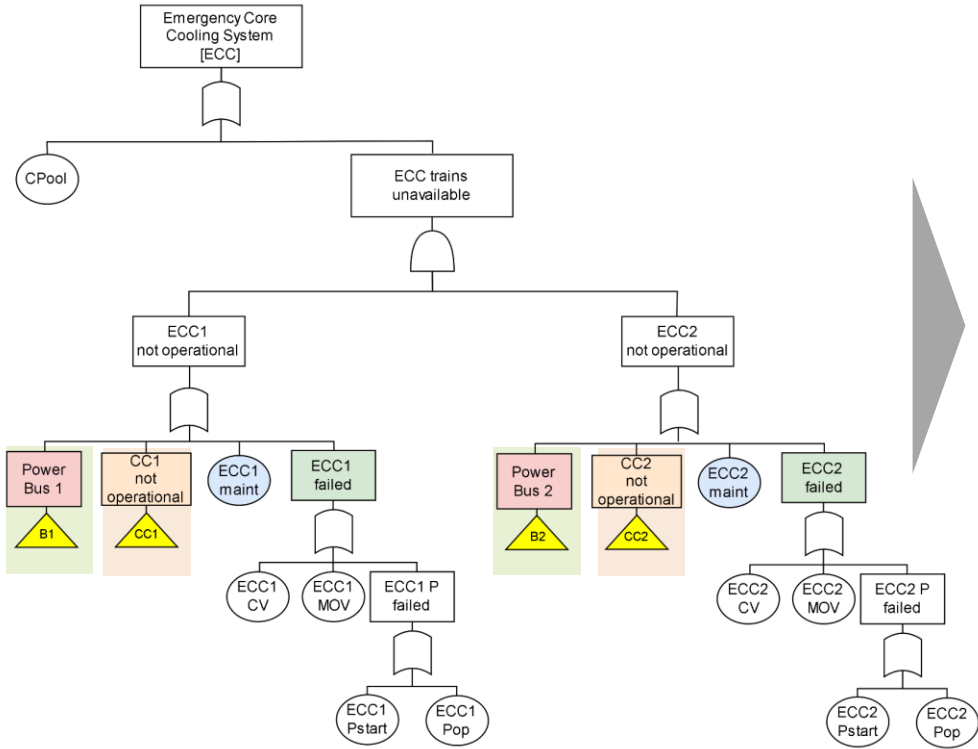
SUBSYSTEMS



SS0	B1, CC1, B2, CC2	
SS1	B1, CC1, B2, $\overline{CC2}$	
SS2	B1, CC1, $\overline{B2}$, CC2	
SS3	B1, CC1, $\overline{B2}$, $\overline{CC2}$	
SS4	B1, $\overline{CC1}$, B2, CC2	
SS5	B1, $\overline{CC1}$, B2, $\overline{CC2}$	
SS6	B1, $\overline{CC1}$, $\overline{B2}$, CC2	
SS7	B1, $\overline{CC1}$, $\overline{B2}$, $\overline{CC2}$	
SS8	$\overline{B1}$, CC1, B2, CC2	
SS9	$\overline{B1}$, CC1, B2, $\overline{CC2}$	
SS10	$\overline{B1}$, CC1, $\overline{B2}$, CC2	
SS11	$\overline{B1}$, CC1, $\overline{B2}$, $\overline{CC2}$	
SS12	$\overline{B1}$, $\overline{CC1}$, B2, CC2	
SS13	$\overline{B1}$, $\overline{CC1}$, B2, $\overline{CC2}$	
SS14	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, CC2	$3.30 \cdot 10^{-02}$
SS15	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, $\overline{CC2}$	$5.62 \cdot 10^{-04}$

D²T²: Fault Trees Dependency

SUBSYSTEMS



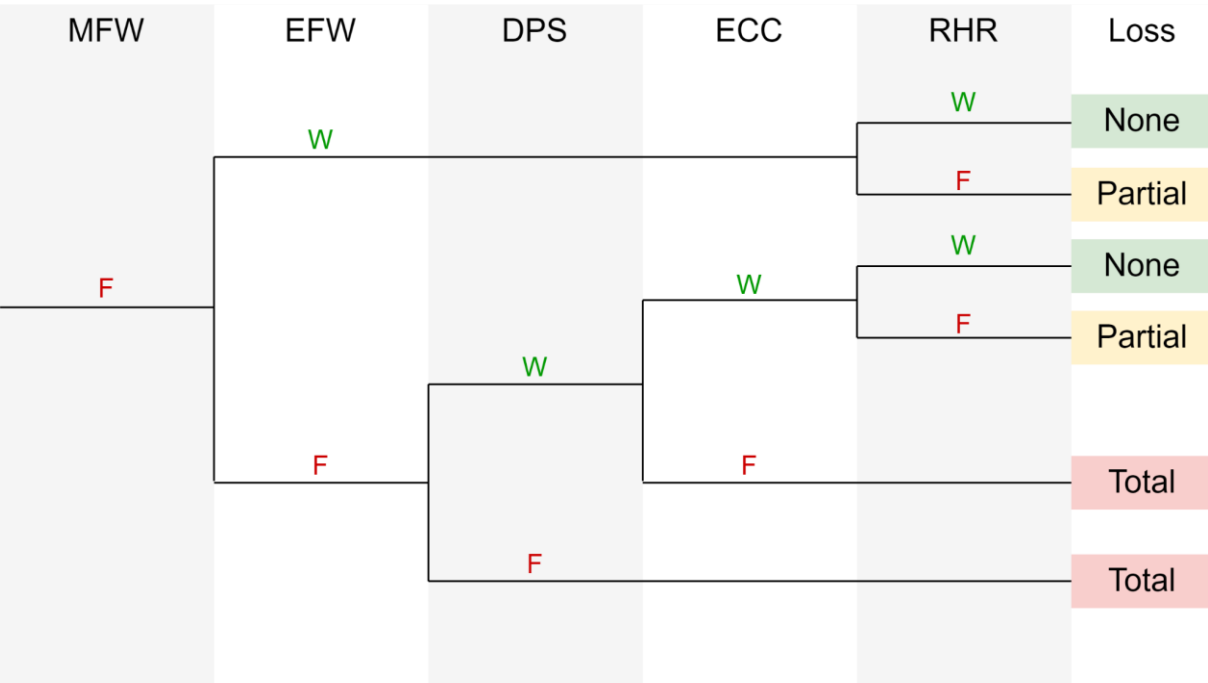
SS0	B1, CC1, B2, CC2	1
SS1	B1, CC1, B2, $\overline{CC2}$	0
SS2	B1, CC1, $\overline{B2}$, CC2	1
SS3	B1, CC1, $\overline{B2}$, $\overline{CC2}$	$3.30 \cdot 10^{-02}$
SS4	B1, $\overline{CC1}$, B2, CC2	0
SS5	B1, $\overline{CC1}$, B2, $\overline{CC2}$	0
SS6	B1, $\overline{CC1}$, $\overline{B2}$, CC2	0
SS7	B1, $\overline{CC1}$, B2, $\overline{CC2}$	0
SS8	$\overline{B1}$, CC1, B2, CC2	1
SS9	$\overline{B1}$, CC1, B2, $\overline{CC2}$	0
SS10	$\overline{B1}$, CC1, $\overline{B2}$, CC2	1
SS11	$\overline{B1}$, CC1, $\overline{B2}$, $\overline{CC2}$	$3.30 \cdot 10^{-02}$
SS12	$\overline{B1}$, $\overline{CC1}$, B2, CC2	$3.30 \cdot 10^{-02}$
SS13	$\overline{B1}$, $\overline{CC1}$, B2, $\overline{CC2}$	0
SS14	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, CC2	$3.30 \cdot 10^{-02}$
SS15	$\overline{B1}$, $\overline{CC1}$, $\overline{B2}$, $\overline{CC2}$	$5.62 \cdot 10^{-04}$

$\overrightarrow{pD2T}_{top}$



D²T²: Fault Trees Dependency

SUBSYSTEMS

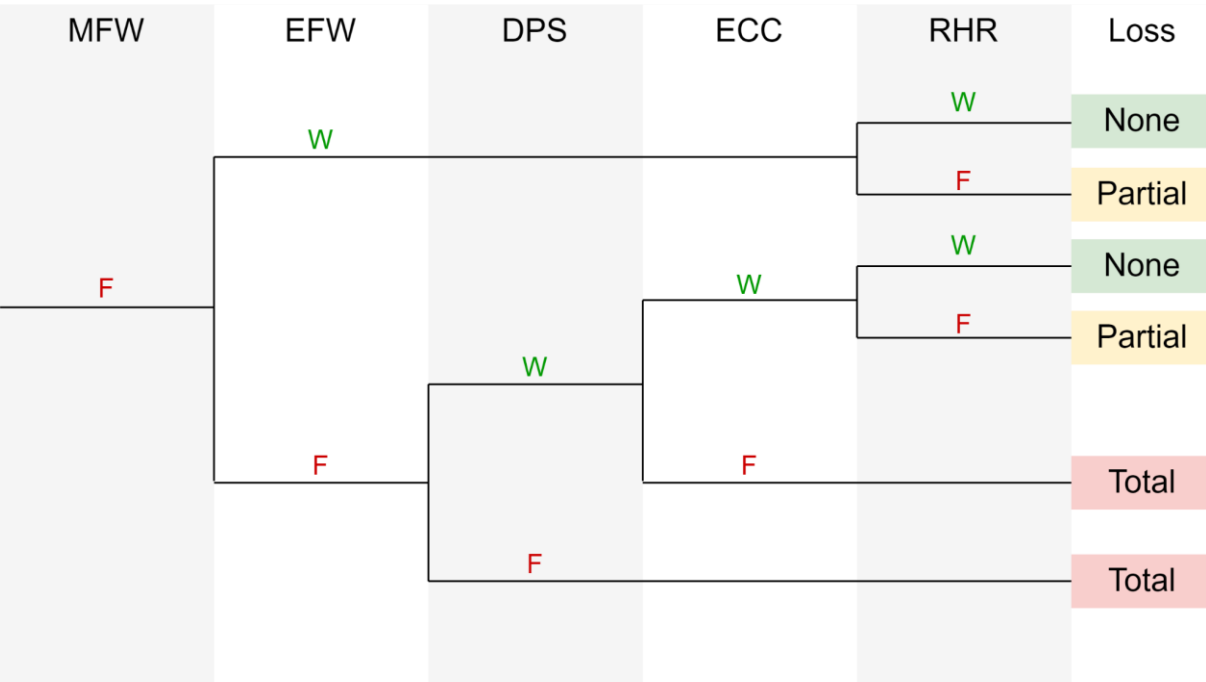


$$\vec{W}_{\text{none1}} = \vec{W}_{\text{MFW}} \odot \vec{P}_{\text{EFW}} \odot \vec{P}_{\text{RHR}} \odot \vec{P}_{\text{SS}}$$



D²T²: Fault Trees Dependency

SUBSYSTEMS

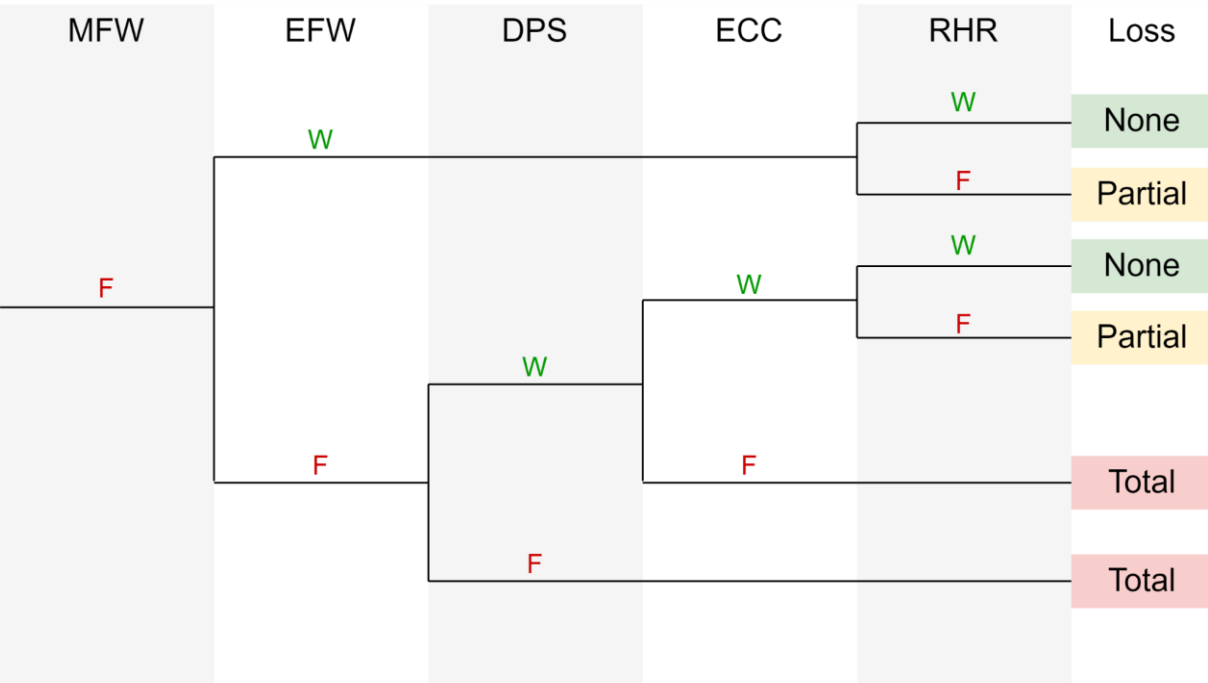


$$\vec{W}_{\text{partial1}} = \vec{W}_{\text{MFW}} \odot \vec{P}_{\text{EFW}} \odot \vec{P}_{\text{RHR}} \odot \vec{P}_{\text{SS}}$$



D²T²: Fault Trees Dependency

SUBSYSTEMS

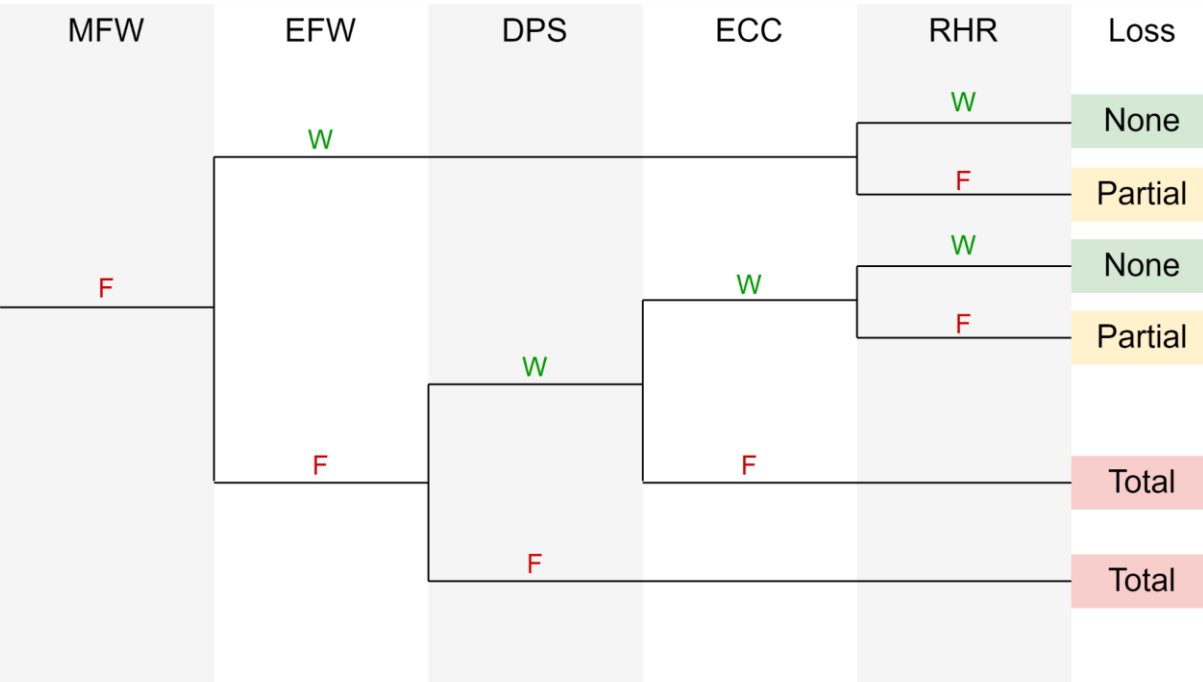


$$\vec{W}_{\text{none2}} = \vec{W}_{\text{MFW}} \odot \vec{P}_{\text{EFW}} \odot \vec{P}_{\text{DPS}} \odot \vec{P}_{\text{ECC}} \odot \vec{P}_{\text{RHR}} \odot \vec{P}_{\text{SS}}$$



D²T²: Fault Trees Dependency

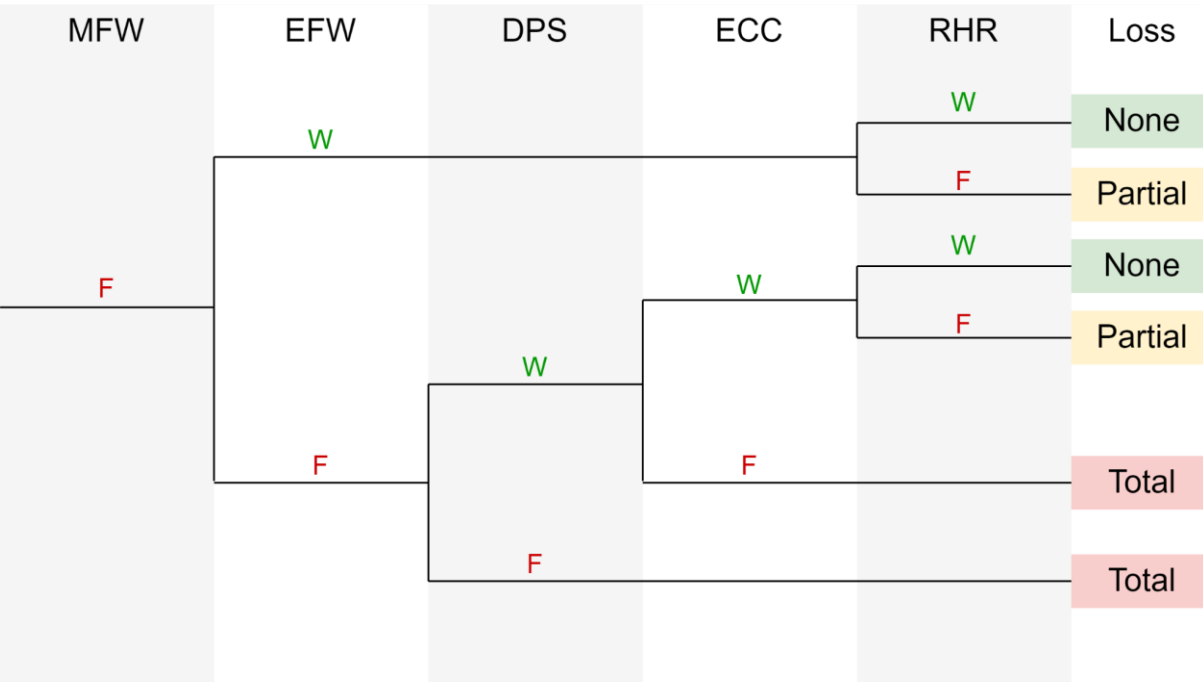
SUBSYSTEMS



$$\vec{W}_{\text{none}} = \vec{W}_{\text{none1}} + \vec{W}_{\text{none2}}$$

$$\vec{W}_{\text{partial}} = \vec{W}_{\text{partial1}} + \vec{W}_{\text{partial2}}$$

$$\vec{W}_{\text{total}} = \vec{W}_{\text{total1}} + \vec{W}_{\text{total2}}$$



$$W_{\text{none}} = \sum_i \overline{W}^i_{\text{none}}$$

$$W_{\text{partial}} = \sum_i \overline{W}^i_{\text{partial}}$$

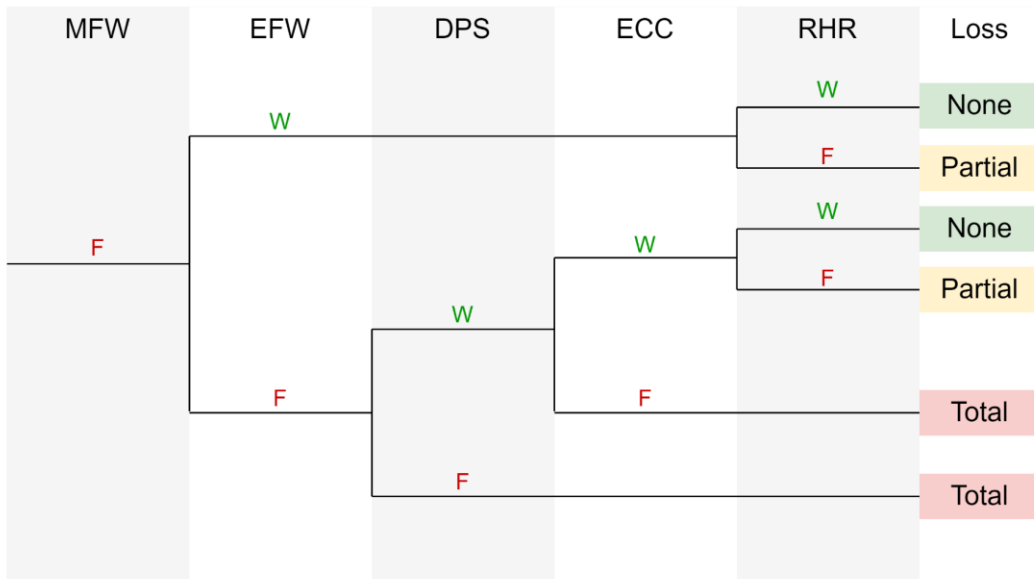
$$W_{\text{total}} = \sum_i \overline{W}^i_{\text{total}}$$

How does it compare?

More realistic modelling of:

- Redundancy
- Maintenance strategy
- Reliance on shared resources

System	FT	D ² T ²
EFW	3.70e-03	2.30e-03
ECC	5.53e-03	1.65e-03



Loss Magnitude	FT/ET [frequency]	D ² T ² [frequency]
None	1.87e-06 h ⁻¹	3.57e-06 h ⁻¹
Partial	4.84e-09 h ⁻¹	1.73e-09 h ⁻¹
Total	1.07e-10 h ⁻¹	2.32e-09 h ⁻¹



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Summing Up

Conclusions



- Umbrella methodology integrating flexible modelling techniques within traditional system safety methodologies
- Retains modelling framework familiarity, intuitivity and efficiency while enhancing accuracy
- High potential for modularization and model self-implementation
- Dependencies included at any level of system safety modelling
- Algorithms and computational tools (almost) available (*NxGen Tool*)
- Removing hidden assumptions



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REFERENCES

- Andrews, John, and Silvia Tolo. "*Dynamic and dependent tree theory (D2T2): A framework for the analysis of fault trees with dependent basic events.*" *Reliability Engineering & System Safety* 230 (2023): 108959.
- Tolo, Silvia, and John Andrews. "*Fault Tree analysis including component dependencies.*" *IEEE Transactions on Reliability* (2023).
- Tolo, Silvia, and John Andrews. "*An integrated modelling framework for complex systems safety analysis.*" *Quality and Reliability Engineering International* 38.8 (2022): 4330-4350.



For more info and newsletter
visit the [NexGen Project Website](#)

Thank you

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