Time Consistency and Investment Incentives in Environmental Policy

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Abstract. We study environmental policy for a polluting firm that can invest in extra capacity. The optimal levels of allowed output as well as the tax rate are increasing in investment. With divisible investment, commitment always leads to the first best, under direct regulation and taxation. Time-consistent policy results in overinvestment with direct regulation and underinvestment with taxation. With indivisible investment and direct regulation, commitment leads to the first best. With taxation however, commitment may not lead to the first best and time consistency can lead to higher welfare. This remarkable result occurs because the firm can influence the tax rate.

JEL Classification: D62, Q28

Key words: Environmental policy, instruments, commitment, time consistency
1 Introduction

Many government policies are effective only if the government can credibly commit. Take for instance the Dutch government’s policy to contain aircraft noise from Schiphol airport in the 1990s. The airport was given an annual noise quota, but it chose to schedule so many flights that it had consumed this quota by the end of September. Enforcing the quota would imply shutting down the airport for the rest of the compliance year (until 31 October), which was clearly not a credible option. In the end, the government would tolerate Schiphol’s violation of the noise limits.

The example suggests that when the government cannot commit to regulation that directly or indirectly limits a firm’s size, the firm will grow faster than the government would like. The firm knows that the government will set more lenient regulation after the firm has invested in extra capacity. The firm may then invest just to obtain more lenient regulation.

In this paper we analyze the welfare effects of commitment and time consistency for the regulation of a polluting firm that can invest in extra capacity. Under commitment, the government first sets environmental policy and then the firm makes its investment decision. Under time consistency, the firm first makes its investment decision and then the government sets environmental policy. The government may announce a certain policy before the firm invests, but the firm knows this announcement is not credible and the government will reconsider its policy after the firm has made its investment decision. We analyze the policy instruments of direct regulation and taxation. This places our paper in the tradition of the price vs quantity literature that was started by Weitzman (1974) and has recently seen a surge in activity (Glazer and Janeba, 2004; Quirion, 2004; Tarui and Polasky, 2005).

We shall see that when the government can commit to strict output regulation, the firm will make the socially optimal investment decision. With time-consistent policy, however, the firm will get the government to allow more output by overinvesting, as in the Schiphol example. When the government can commit to a tax rate, it can

\[1\] As in the Schiphol example, the strict limit could still exist on paper, but the government may choose not to enforce it. This has the same effect as making the regulation more lenient.
implement the first best with divisible investment. Time-consistent policy now results in underinvestment. The firm will invest less so as to reduce the tax rate.

When the investment is of fixed size, however, the government can no longer reach the first best with commitment to a tax rate. The firm may invest when it should not, or it may not invest when it should. Whereas the former problem has not been noted before in the literature, the latter problem is well-known. Rose-Ackerman (1973) was the first to note that under marginal damage taxation a firm may exit the market although it makes a positive contribution to social welfare. This is because with increasing marginal damage, the tax bill exceeds environmental damage.

Given that with taxation and indivisible investment, neither commitment nor time consistency always yields the first best, it is unclear a priori which regime is best for social welfare. In fact, as we shall see, there are cases where time consistency actually improves upon commitment.

It is worthwhile to study this outcome more closely, since from its inception by Kydland and Prescott (1977) and Fischer (1980), the literature has almost unanimously found that with perfect information, commitment is always at least as good as time consistency. But this is not a universal truth. There are two sufficient conditions under which commitment is better than time consistency. The first condition is when commitment leads to the first best. In the present paper, this happens with divisible investment as well as with direct regulation and an indivisible investment.

Even when commitment does not implement the first best, it can still be better than time consistency. The second sufficient condition is that agents cannot influence time consistent policy. The government then has the option to “commit” to the time consistent policy. This condition holds in macroeconomic policy settings, where each individual agent is too small to influence government policy. This is the setting in which time consistency has mostly been analyzed so far. However, in the present paper, there is only one firm playing the government, and the firm can influence government policy.

The analysis has been applied to trade policy by Tornell (1991), Leahy and Neary (1999) and Miravete (2003), to utility price regulation by Levine et al. (2005) and to redistributive taxation by Michel and Paul (2002). Obviously, when information is revealed in the course of the game, the advantage of time-consistent policy is that the government can take this information into account (see Yao (1988), Malik (1991) and Tarui and Polasky (2005) in environmental policy).
Recently, Petrakis and Xepapadeas (2001), Arguedas and Hamoudi (2004) and Requate (2005) have also found that time consistency can improve upon commitment in environmental policy. We shall discuss these papers in Section 4.

Previous papers on time consistency in environmental policy have focused on investment in abatement technology, building on earlier work on innovation in abatement technology by Downing and White (1986) and Milliman and Prince (1989). Biglaiser et al. (1994) show that time-consistent quantity regulation does not achieve the social optimum. Firms may strategically over- or underinvest in their multi-period model where investment adds to the stock of abatement capital. Emission taxation does achieve the social optimum, because marginal environmental damage is constant. Thus, firms cannot influence the tax rate, because the government always sets it at the constant rate of marginal damage. Most other contributions assume, as does the present paper, that firms can only invest once and that marginal environmental damage is increasing.

With time-consistent emission taxation, a firm will overinvest in abatement in order to reduce the tax rate. With quantity regulation, on the other hand, the firm will underinvest in order to increase the allowed emission level. This has been shown by Kennedy and Laplante (1999) for a discrete choice between two abatement technologies and by Glazer and Janeba (2004) for continuous investment in abatement.

Since commitment to a tax rate may not be optimal, one may wonder why the government doesn’t commit to direct regulation or a nonlinear tax scheme instead. As for the former option, there may be other reasons, from which we abstract, why the government prefers taxation over direct regulation. For instance, there may be uncertainty over abatement cost, and the marginal environmental damage curve is relatively flat (Weitzmann, 1974), or the government can put the tax revenues to good use (e.g. Goulder et al., 1999). As for tax schemes, it remains an open question whether schemes that are more involved than simple linear tax or quantity instruments could be implemented in practice. Speaking more generally, the first-best commitment policy may look straightforward in the simple model of this paper. However, in reality there are many complications for the government to handle, including uncertainty

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3 Quirion (2004) combines the two motives.
and asymmetric information. It seems unlikely that the government can devise and implement exactly the optimal policy for every regulatory problem it has to deal with. Rather, as in the present paper, the government will have the choice between a number of approaches, none of which is ideal or always preferable.

The contribution of this paper is fivefold. First, we present a setting where time consistent policy may lead to higher welfare than commitment. Secondly we introduce two necessary conditions for this to happen in general. Thirdly, we point to recent papers in environmental economics that have also found that time consistency may be better than commitment, although the papers themselves may not have advertised this finding in particular. Fourthly, we analyze the case where the polluting firm can invest in extra production capacity, whereas the literature has always looked at investments in abatement. Finally, we introduce lumpy investment as the reason why commitment cannot implement the first best, whereas the literature has focused on imperfect information (Yao, 1988; Malik, 1991; Tarui and Polasky, 2005) or distributional concerns (Abrego and Perroni, 2002).

The rest of the paper is organized as follows. In Sections 2 and 3, we analyze direct regulation and taxation under commitment and time consistency. In Section 2, the size of the firm’s investment is treated as a continuous variable. In Section 3, we look at an investment of given size. Section 4 compares the welfare effects of commitment and time consistency, explaining why time consistency may be better for welfare. In Section 5, we look at possibilities for the government to reach the first best when this is not feasible with the instrument chosen and the prevailing policy regime. Section 6 concludes the paper.

2 Perfectly divisible investment

2.1 The model

We look at an activity by one firm which causes environmental damage to the rest of society. We will apply a partial equilibrium framework. The total and marginal
external costs of production \(q\) are given by \(EC(q)\) and \(MEC(q)\) respectively, with:

\[
EC(q) > 0 \quad \text{for } q > 0
\]
\[
MEC(q) \equiv EC'(q) > 0
\]
\[
MEC'(q) > 0
\]

Thus, marginal external cost \(MEC\) is increasing in production.\(^4\)

The firm needs to invest in productive capacity (e.g. build a factory) in order to produce. The more it invests, the lower its variable production cost. The investment itself is not polluting. The size of the investment is a continuous variable. Since a larger investment leads to higher investment cost, we can identify the size of the investment with the level of investment cost \(c\). Then the firm’s payoff or Net Private Benefit (net of private variable cost and gross of investment cost) is a function \(NPB(q,c)\) with:

\[
NPB_q > 0 \quad q \in [0, Q(c))
\]
\[
NPB'_c > 0
\]
\[
NPB_{qq} < 0 \quad NPB_{cc} > 0 \quad NPB_{qc} > 0
\]

Thus, marginal net private benefits of output are decreasing in \(q\). The investment increases private benefits as well as marginal private benefits of output throughout. Marginal net private benefits are positive for \(0 < q < Q(c)\). \(Q(c)\) is the profit-maximizing output quantity in the absence of government intervention.

We denote aggregate social benefits by \(AB\):

\[
AB(q,c) = NPB(q,c) - EC(q) - c
\]

Note that \(AB\) does not explicitly contain the consumer surplus. Consumer surplus may be constant, because the firm is competing in a perfectly competitive international market. If consumer surplus is not constant, we can interpret external cost \(EC\) as environmental cost minus consumer surplus.

\(^4\)In a more general setting, the firm would be able to reduce emissions per unit of output. Taking this into account would affect the analysis, if the output market were imperfectly competitive and the firm could invest in a reduction of marginal abatement cost. Then the government would no longer be able to reach the first best with environmental policy alone. Petrakis and Xepapadeas (1999) combine these two elements in an analysis of emission taxation. We discuss their findings in Section 4.
The first-best levels of output and investment, $q^{opt}$ and $c^{opt}$ respectively, follow from the maximization of $AB$ with respect to $q$ and $c$. From (3):

\[ NPB_q(q^{opt}, c^{opt}) = MEC(q^{opt}) \] \[ NPB_c(q^{opt}, c^{opt}) = 1 \]

The firm’s profits under output regulation (with allowed output equal to $\bar{q}$) and taxation at rate $t$ respectively are:

\[ \Pi(\bar{q}, c) = NPB(\bar{q}, c) - c \] \[ \Pi(q, c, t) = NPB(q, c) - tq - c \]

In all situations we will discuss, the firm is able to make a profit by investing.

### 2.2 Commitment

With commitment, the government first sets environmental policy (allowed output $q$ under direct regulation and output tax rate $t$ under taxation) and then the firm decides how much to invest and to produce.

Under direct regulation, the firm sets the investment at the level $c(\bar{q})$ that maximizes its profits (6), taking allowed output $\bar{q}$ as given:

\[ NPB_c(\bar{q}, c(\bar{q})) = 1 \]

Under taxation, the firm sets the output and investment levels that maximize its profits (7), taking the tax rate $t$ as given:

\[ NPB_q(q(t), c(t)) = t \] \[ NPB_c(q(t), c(t)) = 1 \]

We find:\textsuperscript{5}

**Proposition 1** With perfectly divisible investment and commitment, the government reaches its first best with output regulation as well as with output taxation.

\textsuperscript{5}All proofs are in the Appendix.
2.3 Time consistency

With time consistency, the firm first makes its investment decision. Then the government sets environmental policy (allowed output $q$ under direct regulation and output tax rate $t$ under taxation). Finally, the firm decides how much to produce.\(^6\)

Under direct regulation, the government sets the allowed output level that maximizes aggregate benefits (3), taking the investment level as given:

$$NPB_q[\bar{q}(c), c] = MEC[\bar{q}(c)]$$  \hspace{1cm} (11)

In stage one, the firm sets the investment level $\tilde{c}$ that maximizes its profits (6), taking into account that allowed output $\bar{q}(\tilde{c})$ depends on its investment level:

$$NPB_{\tilde{c}} + NPB_q \bar{q}'(c) - 1 = 0$$  \hspace{1cm} (12)

Proposition 2 With perfectly divisible investment and time consistency, the firm invests more than socially optimal under output regulation.

If the firm can influence the allowed output level, it will invest too much. This is because the firm’s investment increases its marginal benefit of output. This prompts the welfare-maximizing government to allow more output.

Under taxation, the government sets the tax rate that maximizes aggregate benefits (3), taking the investment level as given:\(^7\)

$$t(c) = NPB_q[q(c), c] = MEC[q(c)]$$  \hspace{1cm} (13)

In stage one, the firm sets the investment level $\hat{c}$ that maximizes its profits (6), taking into account that the tax rate $t(c)$ depends on its investment level:

$$NPB_{\hat{c}} - t'(c)q - 1 = 0$$  \hspace{1cm} (14)

\(^6\)Thus, while the government cannot commit to its policy before the firm makes its investment decision, it can do so before the firm sets its output level $q$. If the government would set the tax rate after the firm had set $q$, taxation would merely redistribute funds from the firm to the government and the tax rate would be indeterminate. Output regulation after the firm has already set its output level is clearly completely ineffective.

\(^7\)The first equality follows from (9).
**Proposition 3** With perfectly divisible investment and time consistency, the firm invests less than socially optimal under output taxation.

If the firm can influence the tax rate, it will invest too little. This is because the less the firm invests, the lower the tax rate on output will be.

### 3 Indivisible investment

#### 3.1 The model

We now assume that the investment is indivisible. It can be thought of as a plant of fixed size. External costs $EC$ are still given by (1). The payoff to the firm depends on whether the firm makes a certain investment. When the firm does not make the investment, its payoff is denoted by $NPB_0(q)$. When the firm makes the investment, its payoff is $NPB_1(q) - c$, with $c$ the investment cost. The investment itself is not polluting. Denoting marginal net private benefits by $MNPB$, we have:

\[
MNPB_j(q) = NPB'_j(q) > 0 \quad q \in [0, Q_j) \quad j = 0, 1
\]

\[
MNPB_1(q) > MNPB_0(q)
\]

\[
MNPB'_1(q) < 0
\]

Thus, marginal net private benefits are decreasing in $q$. The investment increases marginal private benefits throughout. Marginal private benefits are positive for $0 < q < Q_j, j = 0, 1$. $Q_j$ is the profit-maximizing output quantity in the absence of government intervention.

We denote aggregate social benefits without and with the investment by $AB_0$ and $AB_1$, respectively:

\[
AB_0(q) = NPB_0(q) - EC(q) \tag{15}
\]

\[
AB_1(q, c) = NPB_1(q) - c - EC(q) \tag{16}
\]

Figure 1 illustrates the setup. The curves for marginal net private benefits $MNPB_0$ and $MNPB_1$ and for marginal external cost $MEC$ are drawn as straight lines, and the investment shifts the $MNPB$ curve up in parallel fashion. However, our results do not rely on these assumptions. All we assume is that the $MEC$ curve is rising and the $MNPB$ curves are declining in $q$, and the investment shifts the $MNPB$ curve up.
Let us now determine the first-best outcome. Without investment, the optimal output level is $q_0^{opt}$ in Figure 1. With investment, it is $q_1^{opt}$. The increase in gross aggregate benefits due to the investment, given that allowed output levels before and after the investment are set optimally, is $DEFG$ in Figure 1. When investment cost is less than $DEFG$, the firm should invest and produce $q_1^{opt}$. When investment cost is higher than $DEFG$, the firm should not invest and produce $q_0^{opt}$.

Lemma 1 Define $q_1^{opt} > q_0^{opt}$ and $b_s$ by:

\[ MNPB_j(q_j^{opt}) = MEC(q_j^{opt}) \quad j = 0, 1 \]  
\[ b_s = NPB_1(q_1^{opt}) - NPB_0(q_0^{opt}) - EC(q_1^{opt}) + EC(q_0^{opt}) \]

The first best or socially optimal outcome with indivisible investment is:

1. When $c < b_s$: Investment and output at $q_1^{opt}$.
2. When $c > b_s$: No investment and output at $q_0^{opt}$.

3.2 Commitment

With commitment, the government first sets environmental policy and then the firm decides whether or not to invest and how much to produce.

Let us first look at output regulation. It turns out that, as with divisible investment, the government can always reach the first best in this case. Remember that in Figure 1, the area $DEFG$ denotes the increase in gross aggregate benefits from the investment. Suppose the government tries to implement the first best with output regulation. Then for $c > DEFG$, it would set allowed output at $q_0^{opt}$ and hope that the firm would not invest. Given $q_0^{opt}$, the firm’s benefits from the investment are $DHFG$. The firm will not invest, because when the investment costs exceed $DEFG$, they also exceed the increase in private benefits $DHFG < DEFG$. Thus, the first best is implemented.

For $c < DEFG$, the government would set allowed output at $q_1^{opt}$ and hope that the firm would invest. Given $q_1^{opt}$, the firm’s benefits from the investment are $DEJG$. The firm will invest, because when the investment costs are below $DEFG$, they are
Figure 1: Marginal net private benefits and marginal external cost with and without the investment

also below the increase in private benefits $DEJG > DEFG$. Again, the first best is implemented. Stated formally:

**Proposition 4** With indivisible investment and commitment, the government can reach the first best under output regulation.

Now let us look at the instrument of taxation. Given the firm’s investment decision, its choice of $q_j$ as a function of the tax rate $t$ is given implicitly by:

$$ t = MNPB_j [q_j(t)] \quad j = 0, 1 \quad (19) $$

Let us denote the optimal tax rate, in case the investment has (not) been made, by $t_1$ ($t_0$). From (17) and (19):

$$ t_j = MEC(q_j^{opt}) = MNPB_j(q_j^{opt}) \quad j = 0, 1 \quad (20) $$

Note that $t_1 > t_0$, because $q_1^{opt} > q_0^{opt}$ and $MEC' > 0$. 12
It turns out that, unlike with divisible investment, the government cannot always reach the first best using taxation. For low investment cost ($c < b_s$, as defined by (18)), first best implies investment and a tax rate of $t_1$. Given that the tax rate is $t_1$, the firm will produce $q_0(t_1)$ when it does not invest and $q_1(t_1) = q_{1}^{\text{opt}}$ when it does. In Figure 1, the increase in after-tax profits, upon which the firm bases its investment decision, is given by $DENG$. This is below the socially optimal threshold level of $DEFG$. Thus for investment cost between $DENG$ and $DEFG$, the government cannot reach the first best. The government would like the firm to invest at the first-best tax rate of $t_1$, but with the tax rate at $t_1$, the investment is not profitable for the firm.

For high investment cost ($c > b_s$), first best implies no investment and a tax rate of $t_0$. Given that the tax rate is $t_0$, the firm will produce $q_0(t_0) = q_{0}^{\text{opt}}$ when it does not invest and $q_1(t_0)$ when it does. In Figure 1, the increase in after-tax profits is given by $DMFG$. This exceeds the socially optimal threshold level of $DEFG$. Thus for investment cost between $DEFG$ and $DMFG$, the government cannot reach the first best. The government would like the firm to refrain from investing at the first-best tax rate of $t_0$, but with the tax rate at $t_0$, the investment is profitable for the firm.

Formally stated:

**Proposition 5** With indivisible investment and commitment, the government can reach the first best under taxation if and only if either $c < b_s^1$ or $c > b_s^0$, where:

\[
\begin{align*}
    b_s^0 & \equiv NPB_1[q_1(t_0)] - t_0 q_1(t_0) - NPB_0[q_0(t_0)] + t_0 q_0(t_0) \\
    b_s^1 & \equiv NPB_1[q_1(t_1)] - t_1 q_1(t_1) - NPB_0[q_0(t_1)] + t_1 q_0(t_1)
\end{align*}
\]

and $b_s^1 < b_s < b_s^0$, with $b_s$ defined by (18).

For investment cost between $b_s^1$ and $b_s^0$, we can determine the second-best tax rate. This is the tax rate that maximizes aggregate benefits, given that the government sets the tax rate before the firm’s investment decision. We find that the second-best tax rate just allows the investment for low investment cost and just prohibits it for high investment cost:
Proposition 6  With indivisible investment and commitment, the second-best tax rate is \( t^*(c) - \varepsilon \) for \( b_1 < c < c^d \) and \( t^*(c) + \varepsilon \) for \( c^d < c < b'_0 \), with \( b'_0 \) defined by (21), \( b'_1 \) by (22), \( c^d \) by:

\[
NPB_1[q_1(t^*(c^d))] - EC[q_1(t^*(c^d))] - c^d = NPB_0[q_0(t^*(c^d))] - EC[q_0(t^*(c^d))]
\]

and \( t^*(c) \) by

\[
NPB_1[q_1(t^*(c))] - t^*(c)q_1(t^*(c)) - c = NPB_0[q_0(t^*(c))] - t^*(c)q_0(t^*(c))
\]

where \( q_j(t), j = 0, 1, \) is given by (19).

3.3 Time consistency

With time consistency, the firm first decides whether or not to invest. Then the government sets environmental policy. Finally, the firm decides how much to produce.

First we look at output regulation. The government will set allowed output at \( q_{0}^{\text{opt}} \) when the firm does not invest and at \( q_{1}^{\text{opt}} \) when it does (see (17)). In Figure 1, the increase in private benefits from investment is then given by the area \( \text{DEFG} + q_{0}^{\text{opt}}FEq_{1}^{\text{opt}} \). Obviously this area exceeds the increase \( \text{DEFG} \) in gross aggregate benefits. Thus, for investment cost between \( \text{DEFG} \) and \( \text{DEFG} + q_{0}^{\text{opt}}FEq_{1}^{\text{opt}} \), the firm invests when this is not first best. The problem is that the firm is not confronted with the external cost of its investment decision. It receives the increase in private benefits, but does not have to pay for the increase in external cost. Stated formally:

Proposition 7  With indivisible investment and time consistency, the firm invests under output regulation if and only if \( c < b_r \), where \( b_r \) is defined by:

\[
b_r \equiv NPB_1(q_1^{\text{opt}}) - NPB_0(q_0^{\text{opt}})
\]

Since \( b_r > b_s \), defined by (18), the firm invests when this is not first best for \( b_s < c < b_r \).

With taxation, the tax rate will be \( t_0 \) when the firm does not invest and \( t_1 \) when it does (see (20)). In Figure 1, the increase in after-tax profits due to the investment is
$$DEt_1 - GFt_0 = DENG - t_1 NFt_0.$$ This increase is clearly smaller than the increase in gross aggregate benefits $DEFG$. Thus, for investment cost between $DENG - t_1 NFt_0$ and $DEFG$, the firm does not invest when it should. Intuitively, the first best could be implemented with a tax rate of $t_0$ for production below $q_0^{opt}$ and a tax rate according to marginal external cost for production between $q_0^{opt}$ and $q_1^{opt}$, reaching $t_1$ for $q_1^{opt}$. However, the firm must actually pay $t_1$ for all units of production when it has invested. Thus, the increase in the tax bill exceeds the increase in social cost.

**Proposition 8** With indivisible investment and time consistency, the firm invests under taxation if and only if $c < b_t$, where $b_t$ is defined by:

$$b_t \equiv NPB_1(q_1^{opt}) - t_1 q_1^{opt} - NPB_0(q_0^{opt}) + t_0 q_0^{opt} \tag{25}$$

Since $b_t < b_s$, defined by (18), the firm does not invest when this is first best for $b_t < c < b_s$.

### 4 Commitment versus time consistency

#### 4.1 Comparing commitment and time consistency

In this section, we compare the outcomes of commitment and time consistency on welfare. The analysis for taxation and direct regulation under divisible investment, as well as for direct regulation under indivisible investment, is straightforward. Commitment leads to the first best, and time consistency cannot improve upon that. With taxation and indivisible investment however, commitment does not always lead to the first best. Then time consistency may improve upon commitment. We shall now analyse this case in more detail.

The first best outcome is given by Lemma 1. From Proposition 5, the government can reach the first best under time consistency for $c < b_1^t$ and $c > b_0^t$. The second-best policy for $b_1^t < c < b_0^t$ is given by Proposition 6. Finally, under time consistency, the firm invests and produces $q_1^{opt}$ when investment cost is below $b_t$, as defined by (25), and produces $q_0^{opt}$ without investment when investment cost exceeds $b_t$.

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8In subsection 5.1, we discuss how the government can reach the first best with commitment.
Table 1: Taxation and indivisible investment: Comparing commitment and time consistency

<table>
<thead>
<tr>
<th></th>
<th>First best</th>
<th>Commitment</th>
<th>Time consistency</th>
<th>Better?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c &lt; b_t$</td>
<td>inv, $q_1^{opt}$</td>
<td>inv, $q_1^{opt}$</td>
<td>inv, $q_1^{opt}$</td>
<td>both first best</td>
</tr>
<tr>
<td>$b_t &lt; c &lt; b_t'$</td>
<td>inv, $q_1^{opt}$</td>
<td>inv, $q_1^{opt}$</td>
<td>no inv, $q_0^{opt}$</td>
<td>commitment ?</td>
</tr>
<tr>
<td>$b_t' &lt; c &lt; \min[c^d, b_s]$</td>
<td>inv, $q_1^{opt}$</td>
<td>inv, $q &gt; q_1^{opt}$</td>
<td>no inv, $q_0^{opt}$</td>
<td>time consistency</td>
</tr>
<tr>
<td>$b_s &lt; c &lt; c^d$</td>
<td>no inv, $q_0^{opt}$</td>
<td>inv, $q &gt; q_1^{opt}$</td>
<td>no inv, $q_0^{opt}$</td>
<td>time consistency</td>
</tr>
<tr>
<td>$c^d &lt; c &lt; b_s$</td>
<td>no inv, $q_0^{opt}$</td>
<td>no inv, $q &lt; q_0^{opt}$</td>
<td>no inv, $q_0^{opt}$</td>
<td>time consistency</td>
</tr>
<tr>
<td>$\max[c^d, b_s] &lt; c &lt; b_0'$</td>
<td>no inv, $q_0^{opt}$</td>
<td>no inv, $q &lt; q_0^{opt}$</td>
<td>no inv, $q_0^{opt}$</td>
<td>time consistency</td>
</tr>
<tr>
<td>$c &gt; b_0'$</td>
<td>no inv, $q_0^{opt}$</td>
<td>no inv, $q_0^{opt}$</td>
<td>no inv, $q_0^{opt}$</td>
<td>both first best</td>
</tr>
</tbody>
</table>

We know from Proposition 5 that $b_t' < b_s < b_0'$ and from Proposition 8 that $b_t < b_s$. In general, $c^d$ can be smaller or larger than $b_s$.\(^9\) Finally, it can be shown that:

**Lemma 2** \(b_t < b_t', \text{ with } b_t' \text{ given by } (22) \text{ and } b_t \text{ by } (25)\).

We are now ready to compare commitment and time consistency, as in Table 1. First, for investment cost below \(b_t\), it is optimal for the firm to invest and to produce \(q_1^{opt}\). Both commitment and time consistency implement the first best. For investment cost between \(b_t\) and \(b_t'\), commitment still implements the first best of investment and \(q_1^{opt}\). However, time consistency no longer implements the first best, because the firm does not invest. Under both regimes, the tax rate is \(t_1\) when the firm invests. But when the firm does not invest, the tax rate is \(t_0 < t_1\) under time consistency and \(t_1\) under commitment. Thus, not investing is more attractive under time consistency. This is unfortunate from a social welfare point of view, because it is socially optimal for the firm to invest. Hence commitment leads to a better outcome than time consistency.

In the next cost bracket, \(b_t' < c < \min[c^d, b_s]\), the first best is still investment and \(q_1^{opt}\) and time consistency again leads to no investment and \(q_0^{opt}\). However, commitment does not lead to the first best anymore. The government has to reduce its tax rate below \(t_1\) in order to get the firm to invest. As a result, the firm will produce more than the optimal amount \(q_1^{opt}\). With neither commitment nor time consistency implementing the first best, neither scenario dominates the other for the whole cost bracket. By

\(^9\)In the special case with linear marginal cost and benefit curves and a parallel shift in the MNPB curve, as in Figure 1, we have \(c^d = b_s = \frac{1}{4}(b_t' + b_0')\).
continuity, since commitment is better for the previous cost bracket, it will also be
good for investment cost in the low range of this bracket. And since, as we shall see
shortly, time consistency is better for the next cost bracket, it will also be better for
investment cost in the high range of this bracket.

Which is the next cost bracket depends on whether \( c^d \) is above or below \( b_s \). Let
us first look at the case \( c^d > b_s \), so that there is a cost bracket with \( b_s < c < c_d \).
Time consistent policy still leads to \( q_0^{opt} \) without investment and commitment still
leads to output higher than \( q_1^{opt} \) with investment. However, the first-best outcome has
now changed to \( q_0^{opt} \) without investment. Thus, time consistent policy implements the
first best and commitment does not. The firm does not invest under time consistency,
because the tax rate would rise from \( t_0 \) to \( t_1 \) if it did. The government cannot reproduce
this result under commitment, because it would have to keep the tax rate at \( t_0 \) if the
firm invested. Given a tax rate of \( t_0 \), the firm would invest. Then it is better to set the
tax rate at a higher level to keep overproduction to a minimum, given the investment.

The second possibility is that \( c^d < b_s \), so that there is a cost bracket with \( c^d < c < b_s \).
The first best is still investment and \( q_1^{opt} \), with time consistency again leading to no
investment and \( q_0^{opt} \). Commitment now leads to no investment and output below \( q_0^{opt} \).
Output is below \( q_0^{opt} \), because the government has to set the tax rate above \( t_0 \) in order
to discourage the investment. Although neither commitment nor time consistency
implements the first best, it can be seen that time consistency is actually better. In
both scenarios, the firm is discouraged from investing. Time consistency results in the
optimal output level \( q_0^{opt} \) given that the firm does not invest, but commitment results
in less output. Under time consistency, investment is discouraged because it will cause
the tax rate to rise from \( t_0 \) to \( t_1 \). Under commitment, the government is restricted to
setting the same tax rate whether or not the firm invests. It then has to set the tax
rate above \( t_0 \) to discourage investment.

In the penultimate cost bracket \( \max \left[ c^d, b_s \right] < c < b_0 \), time consistency implements
the first best of \( q_0^{opt} \) without investment. Commitment again leads to no investment
and production below \( q_0^{opt} \). Again, the government cannot reproduce the first best with
commitment, because the firm would invest given a tax rate of \( t_0 \). The government
then chooses to set the tax rate above $t_0$ which discourages the investment but also leads to output below the first best level of $q^{opt}_0$.

Finally, for investment cost above $b_0^t$, both commitment and time consistency implement the first best of $q^{opt}_0$ without investment.

4.2 Discussion

The result that time consistency can be better than commitment may come as a surprise. Until now, most analyses have found, or simply taken for granted, that commitment is always at least as good as time consistency. The exceptions are a working paper by Petrakis and Xepapadeas (2001), Arguedas and Hamoudi (2004) and Requate (2005). In this subsection, we discuss two necessary conditions for time consistency to be better than commitment and apply them to the game of the present paper as well as to the three papers just mentioned.

The first necessary condition for time consistency to improve upon commitment is that commitment does not implement the first best. In the present paper, commitment implements the first best under divisible investment, and with direct regulation under indivisible investment. With divisible investment, there is a single instrument (either direct regulation or taxation) to tackle a single distortion (pollution).

Indivisible investment introduces another distortion. However, output regulation still leads to the first best. The reason is that given the output level allowed by the government, the firm’s investment decision only affects its own profits. The firm makes the decision that maximizes its profits, but since this is the only effect of its decision, it also maximizes aggregate benefits.

This also explains why the government cannot always reach its first best with commitment under taxation. Given the tax rate, the firm’s investment decision not only affects its own profits, but also environmental damage and government revenue. When investment is perfectly divisible, the latter two effects cancel each other out, because the tax rate equals marginal environmental damage. However, with indivisible investment and increasing marginal environmental damage, this no longer holds.

The condition that the government cannot implement the first best is a necessary,
but not a sufficient condition for time consistency to be better than commitment. Indeed, in most games studied in the literature so far, commitment cannot implement the first best, but it is still better than time consistency. This is because the literature has mainly concentrated on games with many small agents playing the government.\textsuperscript{10} In these games, each agent considers himself so small that he cannot influence time-consistent policy. In this setting, when the government can commit, it can “commit” to the time-consistent policy and thereby reproduce the outcome of time consistency with commitment. Commitment must then be at least as good as time consistency.

The second necessary condition for time consistency to lead to higher welfare than commitment is thus that the regulated agent(s) can influence time-consistent policy. This is true in the present paper, because the government will increase the tax rate when the firm invests.

Let us now apply this analysis to other papers that conclude that time consistency may be better than commitment. In Petrakis and Xepapadeas’ (2001) time consistency scenario, the firms first invest in pollution abatement. Then the government sets the emission tax rate. With commitment, the following order of the actions is reversed. Finally, in both scenarios, the firms set their output levels in an imperfectly competitive market. In Arguedas and Hamoudi’s (2004) model, the firm can invest in a technology that reduces the damaging impact of its emissions on the environment. The regulator can inspect the firm and impose a fine if emissions exceed the standard. With time consistency, the firm first invests in technology and then the regulator sets the the standard and the probability of inspection. With commitment, the following order is reversed. Finally, in both scenarios, the firm sets its emission level and is potentially subjected to inspection and a fine. In Requate’s (2005) model, there is a single firm that can invest in R\&D effort to make it more likely that it will find a new technology. If it does, it can sell the technology to the polluting industry.\textsuperscript{11} Requate (2005) discusses several scenarios for the timing of environmental policy. The regulator sets

\textsuperscript{10}E.g. Marsiliani and Renström (2000) and Abrego and Perroni (2002) in environmental policy, where the time consistency problem arises from the government’s redistribution concerns.

\textsuperscript{11}Denicolò (1999) also discusses time-consistent and commitment policies in this setup, but he does not compare them.
environmental policy (either an emission tax or tradable permits) after the polluting firms have decided whether or not to adopt the new technology, after the outcome of R&D effort is known (but before the monopolist sets its price), or before the monopolist has made his R&D effort. In the latter case, the regulator either sets the same policy regardless of the R&D outcome, or makes policy dependent on that outcome.

The second necessary condition for time consistency to be better than commitment is met in all these games, because they are between the government and one or at most a few firms. With respect to the first condition, Petrakis and Xepapadeas’ (2001) government has one instrument (emission taxation) for two distortions (pollution and imperfect competition). Arguedas and Hamoudi’s (2004) regulator is equipped with a formula for the noncompliance fine that keeps her from reaching the first best. Requate’s (2005) regulator has one or at most two instruments (when policy depends on R&D success) for three distortions (pollution, too little output by the R&D firm and a gap between the social and private value of the innovation).

When commitment does not implement the first best and private agents can influence time-consistent policy, time consistency may be better than commitment. However, these are only necessary and not sufficient conditions for time consistency to be best. With taxation and indivisible investment in the present paper, for instance, commitment is better for investment cost c just above \( b_1 \) (see the discussion of Table 1). Arguedas and Hamoudi (2004) find that time consistency is always better for the functional forms selected. Requate (2005) finds that commitment to a tax scheme is better than all other policies (but not first best), but cannot rank commitment to a single tax rate or permit volume above policies set later on in the game.

In Petrakis and Xepapadeas (2001), time consistency is better for a monopoly, but not for a larger number of firms. In the same vein, Poyago-Theotoky and Teerasuwan-najak (2002) find that time consistency is better for a heterogeneous duopoly when the products are sufficiently differentiated. Petrakis and Xepapadeas (1999), using different functional forms, find that commitment always leads to higher welfare for a monopoly.
5 Remedies

5.1 Commitment

As we know from Proposition 5, commitment to a tax rate does not always yield the first best when the investment decision is discrete. We shall discuss two alternatives to reach the first best in this case. The first option is straightforward: Commit to direct regulation instead. We know from Proposition 4 that this always results in the first best. The second option is to commit to a tax scheme.

The most obvious scheme, already mentioned by Rose-Ackerman (1973), is to tax each unit of output at its marginal external cost. Another option is to charge output at the first-best tax rate when the firm makes the first-best (“good”) investment decision. The bad investment decision, on the other hand, should trigger a tax rate so high that the firm prefers the good investment decision. The latter scheme has the disadvantage that it is not time-consistent.

5.2 Time-consistent policy

In this subsection, we ask what the government can do if it cannot reach the first best with time-consistent policy (be it with taxation or with direct regulation). We examine the options of commitment, directly regulating the investment, a tax scheme and instrument choice.

The most obvious remedy is for the government to commit to a tax rate (in the case of divisible investment) or direct regulation. Whereas the present paper analyzes a one-shot game, the incentive to commit will be larger in a repeated-game setting, where the government would like to establish a reputation over a longer time horizon. However, a politician’s time horizon is limited by the possibility that she may lose office after the next elections.

The big question is whether commitment to a strict environmental policy is credible. It may for instance imply that the government does not allow extra production when the firm invests. The firm may be tempted to test the government by making the investment. The firm could further try to make commitment unpleasant for the
Delegation is often discussed in the literature as a way out of the time consistency problem. For instance, the policymaker could delegate monetary policy to a central banker who is relatively conservative on inflation (Rogoff, 1985). Analogously, under direct regulation, the government could delegate the implementation of environmental policy to the Department of the Environment or the Environmental Protection Agency. This agency will in general be staffed by more environmentally-minded people who are less inclined to allow more pollution to an investing firm.\textsuperscript{12} Delegation can also take place on another level: The voters could vote for a government that values the environment more than they do (analogous to Persson and Tabellini (1994) and Besley and Coate (2001)).

If the government cannot commit, it may be able to regulate the investment directly, because the firm needs a licence for the investment. However, licences are not needed for many investments in nontangibles like R&D. Licences are needed, for instance, for building a new plant. One might think that the government could then withhold the licence if the investment would reduce welfare. However, this will not always be effective, for the following reasons.

First, the policy will only be effective if the issuing of building and environmental licence is coordinated. In the Netherlands, for instance, there was no such coordination until March 1993 (Michiels (1998), p. 188). Before that date, a firm could receive a building licence in anticipation of an environmental licence. Thus, the firm could start building the plant without the environmental licence, thereby putting pressure on the environmental department to issue an environmental licence. Without the environmental licence, the investment in the plant would be wasted. The legislator recognized this problem and mandated the coordinated issue of building and environmental licence in the March 1993 Act on the Environment.

Secondly, when the firm needs government approval for its investment, it may acquire this approval on the basis of misleading information. The firm may understate the costs or overstate the benefits of the investment. By the time the government dis-

\textsuperscript{12}Alternatively, the policymaker could devise an appropriate incentive scheme for the environmental agency (cf. Melumad and Mookherjee, 1989).
covers the true costs and benefits, the investment has already been made and cannot be reversed. Finally, when a firm expects a tightening of environmental regulation in the future, it will precipitate investments subject to government approval in order to benefit from existing lenient regulation.

Another road toward the first best is to implement a time-consistent tax scheme instead of a tax rate. Taxing each unit of output at its marginal social cost, as discussed in subsection 5.1, is a time-consistent scheme that leads to the first best.

Finally, let us discuss the option of choosing between taxation and direct regulation. If the government has not committed to an instrument beforehand, it can tailor instrument choice to the problem at hand. With indivisible investment, the firm may invest when it should not under output regulation, and may not invest when it should under taxation. The government could then apply direct regulation to all cases where the firm should invest \((c < b_s)\) as in (18)) and taxation to all cases where the firm should not invest \((c > b_s)\).

With divisible investment, the government can implement the first best if it applies direct regulation in case the firm’s investment is less than the optimal amount \((c < c^{opt})\) as in (4) and (5)) and taxation if the firm invests too much \((c > c^{opt})\). The firm’s payoff is then increasing for \(c < c^{opt}\), dropping discretely at \(c = c^{opt}\) and decreasing for \(c > c^{opt}\). Thus the firm will invest the optimal amount \(c^{opt}\). In both cases, the threat to apply taxation is credible because the government is indifferent between taxation and output regulation, given the firm’s investment decision.

6 Conclusion

In this paper, we have studied environmental policy for a polluting firm that can invest in extra capacity. We have looked at commitment and time consistency, direct regulation of output and output taxation, and divisible and indivisible investment.

We found that the government can reach the welfare optimum under commitment and with divisible investment. With time-consistent policy, the first best can no longer be implemented. The firm overinvests with direct regulation, in order to achieve more allowed output. With taxation on the other hand, the firm underinvests in order to
obtain a lower tax rate.

Indivisible investment introduces another distortion. Since the government only has one instrument (either direct regulation or taxation) for two distortions (pollution and indivisible investment), we would expect that commitment cannot always implement the first best anymore. This is true for taxation, but with direct regulation the government can still reach the first best. The reason is that given allowed output, the firm’s investment decision only affects its own payoff.

With taxation, neither commitment nor time-consistency always results in the first best. Indeed, there are cases where time consistency is better for welfare than commitment. This result may come as a surprise, since the literature so far has overwhelmingly concluded that commitment is always at least as good as time consistency. This is because the focus has mainly been on macroeconomic applications, with many private agents playing against the government. Each agent is then too small to influence time-consistent policy. The government then has the option to “commit” to and thereby reproduce the time-consistent policy. By contrast, the present paper analyzes a game between a single firm and the government. The firm can influence government policy, and thus the government cannot reproduce the outcome of time-consistent policy with commitment.

We derived our conclusion that time consistency may improve upon commitment in a model with a very stark indivisibility: The firm can either make an investment of a given size, or not at all. More subtle restrictions on the continuity of investment size would complicate the analysis, but lead to the same conclusion. These include the cases where the firm can choose from a range of discrete investment projects, or there is an upper or lower bound to the investment size.

7 Appendix

Proof of Proposition 1. For output regulation, it follows from (4), (5) and (8) that \( c(q_{opt}) = c^{opt} \). For taxation with \( t^{opt} = MEC(q^{opt}) \), it follows from (4), (5), (9) and (10) that \( q(t^{opt}) = q^{opt} \) and \( c(t^{opt}) = c^{opt} \).

Proof of Proposition 2. The firm’s investment level is determined by (12), where
by total differentiation of (11) with respect to $c$:

$$q'(c) = \frac{NPB_{qc}}{MEC' - NPB_{qq}} > 0$$

The inequality follows from $NPB_{qc}, MEC' > 0$ and $NPB_{qq} < 0$ according to (1) and (2). Comparing (12) to (5), we see that the LHS of (12) is positive for $c = c^{opt}$ and $q = q^{opt}$, since $q'(c), NPB_q > 0$. Since $NPB_{cc} < 0$, $\hat{c} > c^{opt}$ is needed for (12) to hold.

Proof of Proposition 3. The firm’s investment level is determined by (14), where by total differentiation of (13) with respect to $c$:

$$t'(c) = \frac{MEC'NPB_{qc} - NPB_{qq}}{MEC' - NPB_{qq}} > 0$$

The inequality follows from $NPB_{qc}, MEC' > 0$ and $NPB_{qq} < 0$ according to (1) and (2). Comparing (14) to (5), we see that the LHS of (14) is positive for $c = c^{opt}$ and $q = q^{opt}$, since $t'(c), NPB_q > 0$. Since $NPB_{cc} < 0$, $\hat{c} < c^{opt}$ is needed for (12) to hold.

Proof of Proposition 4. There are two cases to consider. The first case is $c > b_s$, with $b_s$ given by (18). The first best is reached when the firm does not invest at the allowed output level of $q_0^{opt}$, defined by (17). Given that allowed output is $q_0^{opt}$, the firm will invest when the investment cost is below the increase in private benefits $b_s'$:

$$b_s' \equiv NPB_1(q_0^{opt}) - NPB_0(q_0^{opt})$$

(26)

It can be shown that $b_s' < b_s$, defined in (18):

$$NPB_1(q_0^{opt}) - NPB_0(q_0^{opt}) < NPB_1(q_1^{opt}) - NPB_0(q_0^{opt}) - EC(q_1^{opt}) + EC(q_0^{opt})$$

Rearranging gives:

$$NPB_1(q_0^{opt}) - EC(q_0^{opt}) < NPB_1(q_1^{opt}) - EC(q_1^{opt})$$

or $AB_1(q_0^{opt}) < AB_1(q_1^{opt})$, because $q_1^{opt}$ maximizes $AB_1$. Thus, the firm does not invest, because $c > b_s > b_s'$, and the first best is implemented.

The second case to consider is $c < b_s$. The first best is reached when the firm invests at the allowed output level of $q_1^{opt}$, defined by (17). Given that allowed output is $q_1^{opt}$,
the firm will invest when the investment cost is below the increase in private benefits \( b'_1 \):

\[
b'_1 = NPB_1(q_1^{opt}) - NPB_0(q_1^{opt})
\]  

(27)

It can be shown that \( b'_1 < b_s \), defined in (18):

\[
NPB_1(q_1^{opt}) - NPB_0(q_1^{opt}) > NPB_1(q_1^{opt}) - NPB_0(q_0^{opt}) - EC(q_1^{opt}) + EC(q_0^{opt})
\]

Rearranging gives:

\[
NPB_0(q_0^{opt}) - EC(q_0^{opt}) > NPB_0(q_1^{opt}) - EC(q_1^{opt})
\]

or \( AB_0(q_0^{opt}) > AB_0(q_1^{opt}) \), because \( q_0^{opt} \) maximizes \( AB_0 \). Thus, the firm invests, because \( c < b_s < b'_0 \), and the first best is implemented.

Proof of Proposition 5. Given that the tax rate is \( t_0 \), the firm will invest when \( c < b'_0 \) as defined by (21). It can be shown that \( b'_0 < b_s \) from (18):

\[
NPB_1[q_1(t_0)] - t_0q_1(t_0) - NPB_0[q_0(t_0)] + t_0q_0(t_0) >
\]

\[
> NPB_1[q_1(t_1)] - NPB_0[q_0(t_0)] - EC[q_1(t_1)] + EC[q_0(t_0)]
\]

Rearranging and introducing the term \( t_0q_1(t_1) \) yields:

\[
\{NPB_1[q_1(t_0)] - NPB_1[q_1(t_1)] - t_0[q_1(t_0) - q_1(t_1)]\} +
\]

\[
+ \{EC[q_1(t_1)] - EC[q_0(t_0)] - t_0[q_1(t_1) - q_0(t_0)]\} > 0
\]  

(28)

Both terms between curly brackets on the LHS of (28) are positive. The first term is positive by \( q_1(t_0) > q_1(t_1) \), \( MNPB_1[q_1(t_0)] = t_0 \) and \( MNPB'_1 < 0 \). The second term is positive by \( q_1(t_1) > q_0(t_0) \), \( MEC[q_0(t_0)] = t_0 \) and \( MEC' > 0 \). The first (second) term corresponds to the area \( EMU \) (\( FEU \)) in Figure 1.

At \( t_1 \), the firm will invest when \( c < b'_1 \) as defined by (22). It can be shown that \( b'_1 < b_s \) from (18):

\[
NPB_1[q_1(t_1)] - t_1q_1(t_1) - NPB_0[q_0(t_1)] + t_1q_0(t_1) <
\]

\[
< NPB_1[q_1(t_1)] - NPB_0[q_0(t_0)] - EC[q_1(t_1)] + EC[q_0(t_0)]
\]

26
Rearranging and introducing the term $t_1q_0(t_0)$ yields:

\[
\begin{align*}
\{t_1[q_0(t_0) - q_0(t_1)] + NPB_0[q_0(t_1)] - NPB_0[q_0(t_0)]\} + \\
+ \{t_1[q_1(t_1) - q_0(t_0)] + EC[q_0(t_0)] - EC[q_1(t_1)]\} > 0
\end{align*}
\tag{29}
\]

Both terms between curly brackets on the LHS of (29) are positive. The first term is positive by $q_0(t_0) > q_1(t_1)$, $NPB_0[q_0(t_1)] = t_1$ and $MNPB_0' < 0$. The second term is positive by $q_1(t_1) > q_0(t_0)$, $MEC[q_1(t_1)] = t_1$ and $MEC^* > 0$. The first (second) term corresponds to the area $ZEF$ ($NZF$) in Figure 1.

**Proof of Proposition 7.** When the firm has (not) invested, the government will set the output level at $q_1^{opt}$ ($q_0^{opt}$), defined by (17). The firm invests when $c < b_r$ as defined by (24). It follows from $EC(q_1^{opt}) > EC(q_0^{opt})$ that $b_r > b_s$ as defined by:

\[b_r \equiv NPB_1(q_1^{opt}) - NPB_0(q_0^{opt}) > NPB_1(q_1^{opt}) - NPB_1(q_0^{opt}) - EC(q_1^{opt}) + EC(q_0^{opt})\]

**Proof of Proposition 8.** When the firm does not invest, the tax rate will be $t_0$ and the firm will produce $q_0^{opt}$. When the firm invests, the tax rate will be $t_1$ and the firm will produce $q_1^{opt}$. The firm will invest when $c < b_s$, defined by (25). We shall now see that $b_t < b_s$:

\[NPB_1(q_1^{opt}) - t_1q_1^{opt} - NPB_0(q_0^{opt}) + t_0q_0^{opt} < NPB_1(q_1^{opt}) - NPB_0(q_0^{opt}) - EC(q_1^{opt}) + EC(q_0^{opt})\]

This can be rewritten as:

\[t_0q_0^{opt} - EC(q_0^{opt}) < t_1q_1^{opt} - EC(q_1^{opt})\]

The inequality follows from the fact that $t(q) q - EC(q)$, with $t(q) \equiv MEC(q)$, is increasing in $q$:

\[
\frac{d}{dq} [t(q) q - EC(q)] = MEC'(q) q > 0
\]

**Proof of Lemma 1.** Without investment, maximizing (15) with respect to $q$ yields $q_0^{opt}$ in (17) with $j = 0$. With investment, maximizing (16) with respect to $q$ yields $q_1^{opt}$ in (17) with $j = 1$. Note that $q_1^{opt} > q_0^{opt}$, because $MNPB_1(q) > MNPB_0(q)$, $MNPB_1' < 0$ and $MEC^* > 0$. From (15) and (16), the investment is socially beneficial when $c < b_s$ as defined in (18).
Proof of Lemma 2. From (22) and (25), we need to show that:

\[ NPB_1(q_{1,\text{opt}}) - t_1 q_{1,\text{opt}} - NPB_0(q_{0,\text{opt}}) + t_0 q_{0,\text{opt}} < NPB_1[q_1(t_1)] - t_1 q_1(t_1) - NPB_0[q_0(t_1)] + t_1 q_0(t_1) \]

which reduces to:

\[ NPB_0[q_0(t_1)] - t_1 q_0(t_1) < NPB_0[q_0(t_0)] - t_0 q_0(t_0) \]

The inequality follows from the fact that \( t_1 > t_0 \) and \( NPB_0[q_0(t)] - t_1 q_0(t) \), with \( q_0(t) \) defined by (19), is declining in \( t \):

\[ \frac{d}{dt} [NPB_0[q_0(t)] - t_1 q_0(t)] = -q_0(t) < 0 \]

References


