The empirical relationship between UK net corporate borrowing and stockbuilding

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The literatures on net corporate borrowing and stockbuilding by firms are both based on a simple linear-quadratic (L-Q) model, but they have been developed independently. This paper explores the possibility that a firm may jointly optimise its borrowing and stockbuilding decisions, where previous papers have imposed ‘decision rule decompositions’. The resulting model is estimated using UK corporate data and demonstrates that net corporate borrowing and stockbuilding are indeed used as substitutes in production smoothing.

1. Introduction

The literatures on stockbuilding and corporate borrowing by firms have run in parallel with each other for many years. The linear-quadratic (L-Q) model, originating with Holt et al. (1960) and Lovell (1961), has been the workhorse of papers on inventories, augmented by stockout avoidance (Blanchard, 1983; West, 1986) and cost shocks (Blinder, 1986; Miron and Zeldes, 1988; Eichenbaum, 1989), as surveyed by Blinder and Maccini (1991) and Ramey and West (1999). These have given rise to the ‘production smoothing’ class of inventory models, (cf. Miron and Zeldes, 1988; Guariglia and Schiantarelli, 1998; Guariglia, 2000). This model has also been used to explain firms’ adjustment of asset and liabilities, with the most liquid components of the balance sheet acting as financial buffer stocks (Cuthbertson and Taylor, 1987; Ireland and Wren-Lewis, 1992; Mizen, 1994). There are remarkable similarities between the cost functions (based on Holt et al., 1960, and Lovell, 1961) and intertemporal optimisation technology (derived by Sargent, 1977; Hansen and Sargent, 1981) in each literature yet the decision rule decomposition theorems imposed by previous authors have kept the literatures apart. Empirical studies have allowed financial constraints to limit the stockbuilding activity of firms, and borrowing has been specified as a function of stockbuilding, but there has been little attempt to allow inventories and borrowing decisions to be determined simultaneously. Previous models have optimised either inventories or borrowing while treating the other as an exogenous variable (see Callen et al. 1990; Ireland and Wren-Lewis,
Failure to consider the influence of financing and inventory investment jointly has been noted as a major unanswered question in this field (Lovell, 1994; Hubbard, 1998).

The empirical investigation of stockbuilding and firm level borrowing can be split into two distinct types of studies. Panel approaches such as Carpenter et al. (1994, 1998), Kashyap et al. (1994), Milne (1994), Gertler and Gilchrist (1994), Bernanke et al. (1996), Guariglia and Schiantarelli (1998), Guariglia (2000), Schiantarelli (1995), Hubbard (1998), and Small (2000) look into the question of how liquidity constraints have a differential impact on inventory decisions of firms that are constrained to a greater or lesser degree. While these papers make extensive references to the literature on corporate financing they typically do not jointly model the net corporate borrowing decision along with the inventory decision. The time series approaches, on the other hand, typically treat the stockbuilding decision as an exogenous factor in the decision to borrow, rather than a simultaneous one (cf. Kashyap et al., 1993). Much of the focus of these papers is on the relevance of bank lending channels through advances to corporates (Ireland and Wren-Lewis, 1992; Brigden and Mizen, 1999); the response from difference sources of funds (Kashyap et al., 1993; Chrystal and Mizen, 2000); and the most appropriate liquid asset for use as a financial buffer (Ireland and Wren-Lewis, 1992, and Mizen 1994).

We attempt to cut through these partial analyses by offering a jointly optimising model that describes the dynamic adjustment process of inventories and net corporate borrowing together. We have good reason to desire inventories and net borrowing to be jointly determined: recent evidence shows that there is a marked correspondence between the cyclical behaviour of the stock-to-output ratio and the net corporate borrowing of UK firms, (see Fig. 1a). Previous time series analysis would have treated one of these variables as an exogenous process, despite the evidence within the literature that they can both be treated as endogenous choice variables. We examine a model that allows the optimisation of both variables to be jointly determined by relaxing the decision rule decomposition theorem so that we can treat both variables as endogenous in an empirical investigation.2

The paper is organised as follows. In the next section we discuss the different modelling strategies used in the stockbuilding and buffer stock literature, and present our joint model of stockbuilding and net corporate borrowing. This is then estimated and tested on UK corporate sector data in Section 3. The implications of

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1 These papers have faced severe limitations from the econometric methods available at the time, and they have either specified their models in such a way as to remove one of the equations from the system or have found ingenious ways around the problem by drawing on fully specified structural models.

2 This requires only a small alteration to the cost function, that can be easily justified on economic grounds; as far as we are aware, this is the first attempt to integrate these two literatures at the optimisation stage, although we think that the extension is relatively minor. The optimisation solution has been employed previously in a slightly different context by Eichenbaum (1983) who considered the joint determination of inventories and quasi-fixed factors of production (labour). There was no attempt to discuss borrowing decisions in that paper.
the empirical findings are discussed and the performance of the model is evaluated. The final section concludes.

2. Optimising models of net corporate borrowing and stockholding

2.1 Previous approaches

The most straightforward L-Q model of stockbuilding is given by West (1986). This states that the firm faces the intertemporal optimisation problem of

\[
\max_{s_{t+1}} \sum_{i=1}^{T} D(P_{t+i}Z_{t+i} - L(Y_{t+i}, \Delta Y_{t+i}) - \Phi(s_{t+i}, Z_{t+i}))
\]

s.t. \( Y_t = \Delta s_t + Z_t \)

where \( E_{t-1} \) is the expectations operator based on information available up to time \( t - 1 \), \( D \) is a discount factor, \( P_{t+i} \) is the price level, \( Z_{t+i} \) is real sales, \( Y_{t+i} \) is real output, \( s_{t+i} \) is real stockbuilding and \( \Delta \) is the first difference operator. \( L(\cdot) \) is the adjustment cost or loss function facing the firm, and \( \Phi(\cdot) \) is the specific cost associated with stockholding and stockouts. The cost and stockholding costs are defined as

Fig. 1a. Hodrick-Prescott filtered stockholding and net corporate borrowing (both series are detrended using a standard Hodrick-Prescott filter with \( \lambda = 1600 \)) (Source: Bank of England (1999)) Notes: The stock/output ratio refers to the right hand scale (rhs) measured in £ billion and the private non-financial corporations’ (PNFCs) net recourse (to borrowing) refers to the left hand scale (lhs) in percentage points. The data used to construct the figure are taken from Chart 1.2 of the Bank of England’s Inflation Report, 1999
where $w_t$ is a vector of input prices, $\theta_i$ are parameters, and $\varepsilon_{it}$ for $i = 1, 2$ are shocks to production and sales. The variable $s_t^*$ is the target or optimal stock value and this is typically taken as a function of sales, $\lambda Z_t$. This has become the standard 'base' model. Variations on the choice of functional forms for $L(\cdot)$ and $\Phi(\cdot)$ are the basis for alternative models (see Galeotti et al., 1997, for a survey). For example, Ramey (1991) introduces nonlinear marginal adjustment costs to output by adding a cubic term, $Y_t^3$, to the loss function; Eichenbaum (1983) and Maccini (1984) add semi-fixed factors of production—labour and capital respectively—so that the cost function is a quadratic function of these variables in levels and first differences. This is generalised following Galeotti et al. (1997) to

$$\begin{align*}
L(w_t, K_t, Y_t, T_t, I_t) &= a_0 + a_1 K_t + a_2 Y_t + a_3 T_t + a_4 I_t \\
&+ \frac{1}{2} (a_5 K_t^2 + a_6 Y_t^2 + a_7 T_t^2 + a_8 I_t^2) + \lambda w_t K_t \\
&+ \alpha_{10} w_t Y_t + \alpha_{11} w_t T_t + \alpha_{12} w_t I_t + \alpha_{13} K_t Y_t + \alpha_{14} K_t I_t \\
&+ \alpha_{15} Y_t I_t + \varepsilon_1 K_t + \varepsilon_2 Y_t + \varepsilon_3 I_t
\end{align*}$$

where $K_t$ is capital, $T_t$ is technological change, and $I_t$ is fixed investment.

The first step towards linking stockbuilding and borrowing is made by Callen et al. (1990) and Guariglia (2000), who optimise subject to an explicitly defined financing constraint. In Guariglia (2000) there are two choice variables, the inventory level and borrowing, but the paper does not consider whether inventories and borrowing are substitutes. Instead, it focuses on the effect that financial variables may impose on the inventory accumulation decisions of firms with different degrees of financial constraint. The cost function is augmented by a term to reflect the cost of servicing outstanding borrowing, $b_{t-1}$. Guariglia solves the following model

$$\begin{align*}
\max_{t, Y_t} \sum_{i=1}^{T} D^i (Z_{t+i} + \Delta b_{t+i} - L) \\
s.t. \\
Y_t &= \Delta s_t + Z_t \\
L(w_t, Z_t, s_t, b_{t-1}, Y_t, \Delta Y_t) &= \frac{\theta_1}{2} (\Delta Y_t)^2 + \frac{\theta_2}{2} (Y_t)^2 + \theta_3 (w_t Y_t) \\
&+ \frac{\theta_4}{2} (s_t - s_t^*)^2 + a_0 + \frac{a_1}{2} b_{t-1} \left( b_{t-1} \right) b_{t-1} \\
\frac{b_t}{s_t} &\leq M_t
\end{align*}$$

These models are extensions of the standard 'base' model and are designed to capture the effects of financial constraints on inventory and borrowing decisions.
The fifth term in the loss function $L(\cdot)$ reflects the cost of servicing the borrowing of the previous period, where $a_0$ is the safe interest rate plus risk premium and $a_1$ is the interest rate component that increases with the outstanding debt-to-inventories ratio. The last equation defines some ceiling, $M_t$, on this ratio imposed by the financial sector.

Callen et al. (1990) assume that the interest cost of outstanding debt is $r_t b_t$ and solve the static nominal profit of the firm

$$\Pi = P_t Z_t - L(Y_t) - \Phi(s_t, Z_t, \sigma^2_{zt}) - r_t b_t - h_t s_t$$

(5)

where the $\sigma^2_{zt}$ is the variance of sales, and $h_t$ is the holding cost of stocks. By invoking the financing constraint, $\Delta b_t = L(Z_t) + r_t b_{t-1} + h_t s_t - P_t Z_t$, to equate new borrowing to costs net of revenue, they derive a static solution for stockbuilding, $s_t^* = f(Y_t, \sigma^2_{zt}, r_t b_{t-1}, h_t)$. The dynamic adjustment is then calculated around this steady-state value by minimisation of the following loss function

$$\min_{s_{t-1}} E_{t-1} \sum_{i=1}^{T} (a_1(s_{t+i} - s^*_{t+i})^2 + a_2(\Delta s_{t+i})^2 + a_3(\Delta^2 s_{t+i})^2)$$

(6)

This gives the adjustment path of stocks as a function of previous values and the future path of the steady state

$$s_{t+i+1} = \lambda_1 s_{t+i} + \lambda_2 s_{t+i-1} + (1 - \lambda_1 - \lambda_2) \sum_{i=1}^{T} \gamma_i s^*_{t+i}$$

(7)

with $\lambda_i$ equal to a nonlinear function of $\lambda_1$ and $\lambda_2$, the roots of the Euler equation derived from eq. (6).

The approach by Callen et al. (1990) is the closest to that of the buffer stock models of financial variables, which also optimise a loss function around a steady state value. Kanniainen and Tarkka (1986), Cuthbertson and Taylor (1987), and Mizen (1994) use a similar loss function for liquid assets

$$\min_{m_t} E_{t-1} \sum_{i=1}^{T} D'(a_1(m_{t+i} - m^*_{t+i})^2 + a_2(\Delta m_{t+i})^2)$$

(8)

where $m_{t+i}$ is the money holding of the firm, $m^*_{t+i}$ is the desired balance, and $\Delta m_{t+i}$ the adjustment to money balances in the period. A firm minimises the deviation from the desired balance subject to adjustment costs, the relative costs of which are defined by the ratio of the parameters $a_1$ and $a_2$. Ireland and Wren-Lewis (1992) have a similar process in mind when they consider the profit maximising firm that maximises dividends subject to a budget (borrowing) constraint

$$\max_{b_t} E_{t-1} \sum_{i=1}^{T} D'(a_1(d_{t+i})^2 - a_2(\Delta d_{t+i})^2 - a_3(b_{t+i} - b^*_{t+i})^2)$$

s.t. $\Delta b_t = e_t - d_t$

(9)
where $e_t$ is the earnings of the firm and $d_{t+i}$ is the dividend payment per period. Here $b_{t+i}$ is the firm’s buffer stock and in their empirical work is represented (separately) by bank advances, gross liquidity, and as net liquidity.

The generic solution to these buffer stock models (allowing for variations in the specific functional form of the loss function) is similar to eq. (7). The Euler equations derived from these intertemporal loss functions have a common forward-looking solution where the buffering asset or liability, $x_t$, makes partial adjustment to the lagged dependent variable and the (geometrically weighted) future steady state values based on the roots $\lambda_1$ and $\lambda_2$

$$x_{t+i+1} = \lambda_1 x_{t+i} + \lambda_2 x_{t+i-1} + (1 - \lambda_1 - \lambda_2) \sum_{i=1}^{T} \gamma_i x_{t+i}$$

As with the stockbuilding equations, minor variations are introduced to the loss function to allow for costs of adjustment associated with earnings, movement of quasi-fixed (asset) stocks etc (see Nickell, 1985; Mizen, 1994). The model developed below makes a further variation to the loss function, which allows joint maximisation of stocks and net corporate borrowing. The latter term is defined as the ratio of borrowing to liquid assets in logarithms, $(m4t - m4_t)$, which we label, $c_t$.

2.2 A joint optimising approach

As with the standard L-Q model, we assume that the firm produces a storable output that can be sold or held as stocks if demand falls, and conversely, that stocks can be run down if demand exceeds current production capacity. The firm is also assumed to have limited access to credit and can use its net corporate borrowing to purchase additional stock or fund overtime to increase its own production temporarily. Credit is constrained by banks to account for the imperfections that arise from adverse selection, moral hazard, agency, and monitoring costs, which means that $m4t \leq \bar{m}$, but since $m4_t$ is unconstrained, $c_t$ is also unconstrained. We assume that the firm faces a quadratic cost function in net corporate borrowing and stocks, in which it faces the costs of deviations from target and the cost of adjustment.

When the firm is subject to demand and supply shocks, both stocks and net corporate borrowing provide a degree of short-term flexibility to avoid more costly adjustment of production, employment, and investment. The key to the relationship between stockbuilding and net corporate borrowing lies in the way that we specify the decision process of the firm. Eichenbaum (1983) indicates that a significant ‘roadblock’ to interaction between decisions is to be found in the ‘decision rule decomposition’, which he proceeds to move out of the way. He

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3 This is common practice, see inter alia Eichenbaum (1983, 1989), Hall et al. (1989), Ireland and Wren-Lewis (1992), and Rossana (1998).
notes firm-level decisions will be completely independent if the matrices defining
the autocorrelation structure have zero elements on the off-diagonals. Non-zero
off-diagonal elements overcome the decomposition problem in his analysis and
ours. We specifically consider a firm that uses both net corporate borrowing and
stockholding as a means to smooth the costs of variable production, employment
and investment, treating stocks and net corporate borrowing as potential comple-
ments or substitutes in the smoothing process. We are interested in the sign and
magnitude of the off diagonal elements because our amendment to the loss func-
tion allows both stocks and net corporate borrowing to interact in the dynamic
structure to overcome shocks to production. Stocks can be run down to accom-
modate a positive demand shock or negative supply shock; alternatively, bank
borrowing can be used to pay for additional (external) capacity to overcome
production constraints. This suggests that they should be substitutes, since the
use of stockbuilding and borrowing to cover the same shocks to production is
inefficient. In our model, therefore, we specify a loss function as
\[
L(c_{t+i}, s_{t+i}, \Delta c_{t+i}, \Delta s_{t+i}) = E_{t+i}\{\theta_1(c_{t+i} - c^*_i)^2 + \theta_2(\Delta c_{t+i})^2 + \theta_3(s_{t+i} - s^*_i)^2 \\
+ \theta_4(\Delta s_{t+i})^2 + \theta_5(c_{t+i} - c^*_i)(s_{t+i} - s^*_i)\} 
\]
(10)
The first term reflects the cost of departures of net corporate borrowing (credit),
c_{t+i}, from its target, c^*_i, to be defined later; the second term reflects the adjustment
costs of altering net corporate borrowing \Delta c_{t+i}; the third and fourth terms are
equivalents for the deviation of stocks, s_{t+i}, from a target level, s^*_i, and the
adjustment of stocks \Delta s_{t+i}. Finally, the fifth term indicates that it is costly for a
firm to hold excess net corporate borrowing over target if it has excess stocks, or to
hold inadequate net corporate borrowing if it also has inadequate stocks. To a
degree this term compensates for the cost (arising from the quadratic terms) of
lower than desired stocks/borrowing if the other is above its desired level.\(^4\) We can
think of this term as an incentive to avoid a costly ‘belt and braces’ approach to
production smoothing. When short-term adjustments to production in the face of
demand and supply shocks are covered twice over by stocks and by net borrowing,
the firm is operating inefficiently since it could economise on either stocks or
borrowing and still avoid disruption to production. Equally, the term could be
seen as the cost that the firm incurs if an embarrassing ‘no belt, no braces’ outcome
occurs, when neither net borrowing nor stockbuilding are available in sufficient
quantity to smooth production. In this respect it is a cost that corresponds to a
‘stockout’ which cannot be overcome by resorting to borrowing. The coefficients \(\theta_i\)
represent the relative importance of each term in the cost function, our priors
suggest that \(\theta_i > 0\) for all \(i = 1, 2, 3, 4, 5\). The model is thus

\(^4\) A similar compensating multiplicative term has been used by amongst others Nickell (1985) and
\[
\max_{\xi_{t+1}} \sum_{i=1}^{T} D_i (Z_{t+i} + \Delta c_{t+i} - L) \\
\text{s.t.} \\
Y_t = \Delta s_t + Z_t
\]
\[
L(c_{t+1}, s_{t+1}, \Delta c_{t+1}, \Delta s_{t+1}) = E_{t-1} \left( \theta_1 (c_{t+i} - c_{t+i}')^2 + \theta_2 (\Delta c_{t+i})^2 \right.
\]
\[
+ \theta_3 (s_{t+i} - s_{t+i}')^2 + \theta_4 (\Delta s_{t+i})^2 \\
+ \theta_5 (c_{t+i} - c_{t+i}') (s_{t+i} - s_{t+i}') \right)
\]

Here the level of output is assumed to be predetermined according to quasi-fixed factors of production, but the firm has discretion over the amount of stockbuilding and borrowing required (up to the limit set by banks) once sales are known. The loss function is eq. (10).

Taking the derivative with respect to \( c_{t+i} \) gives the Euler equation
\[
E_{t-1} \left[ \theta_1 (c_{t+i} - c_{t+i})^2 + \theta_2 (\Delta c_{t+i})^2 \right.
\]
\[
+ \theta_3 (s_{t+i} - s_{t+i}')^2 + \theta_4 (\Delta s_{t+i})^2 \\
+ \theta_5 (c_{t+i} - c_{t+i}') (s_{t+i} - s_{t+i}') \right] = 0
\]
We can solve the second order difference equation using the historically given values of \( c_{t-i} \) and the transversality condition, which provides a terminal condition for the solution path. Sargent (1977) and Hansen and Sargent (1981) show that a sufficient condition for a solution is that the sequence \( \{c_{t+i}\}_{i=1}^{T} \) is of exponential order less than \( D^{-1/2} \). We factorise the equation to find the roots and rearranging to give solution paths for \( \{c_{t+i}\}_{i=1}^{T} \) and \( \{s_{t+i}\}_{i=1}^{T} \). Clearly these must be solved simultaneously since they each depends on the path of the other. The target values \( \{c_{t+i}\}_{i=1}^{T} \) and \( \{s_{t+i}\}_{i=1}^{T} \) depend on a vector of exogenous driving variables, \( y_{t+i} \).

We take \( y_{t+i} \) to be described by the following stochastic process,
\[
(I - \varphi_1 L - \varphi_2 L^2) y_t = \xi_t, \text{ where } \varphi_1 = (I + \varphi_1) \text{ and } \varphi_2 = -\varphi_1.
\]

The solutions to the equations are
\[
c_t = \lambda_1 c_{t-1} + (1 - \lambda_1) (1 - \lambda_1 D) \\
\times \left\{ \left[ \frac{I - (\lambda_1 D) \varphi_2 L}{(1 - \lambda_1 D)(1 - (\lambda_1 D) \varphi_2)} \right] \gamma_c y_{t-1} \\
+ \left[ \frac{1 - (\lambda_1 D) \varphi_1 + (\lambda_1 D)^2 \varphi_2 L + (\lambda_1 D) \varphi_3 L^2}{(1 - \lambda_1 D) \varphi_1 (1 - \lambda_1 D) \varphi_2 (1 - \lambda_1 D) \varphi_3) \right] \delta_{s_t} \right\} \tag{12a}
\]
\[
s_t = \lambda_1 s_{t-1} + (1 - \lambda_1) (1 - \lambda_1 D) \\
\times \left\{ \left[ \frac{I - (\lambda_1 D) \varphi_2 L}{(1 - \lambda_1 D)(1 - (\lambda_1 D) \varphi_2)} \right] \gamma_s y_t \\
+ \left[ \frac{1 - (\lambda_1 D) \varphi_1 + (\lambda_1 D)^2 \varphi_2 L + (\lambda_1 D) \varphi_3 L^2}{(1 - \lambda_1 D) \varphi_1 (1 - \lambda_1 D) \varphi_2 (1 - \lambda_1 D) \varphi_3) \right] \delta_{c_t} \right\} \tag{12b}
\]
where
\[
\gamma_c = \frac{-(\theta_1 \kappa_c + \theta_3 \kappa_s)}{\theta_2 D}, \quad \gamma_s = \frac{-(\theta_3 \kappa_s + \theta_5 \kappa_c)}{\theta_4 D}, \quad \delta_c = \frac{\theta_5}{\theta_2 D^2} \quad \text{and} \quad \delta_s = \frac{\theta_5}{\theta_4 D^2}
\]
We define

\[ 
\Psi_z = (1 - \lambda_{1z})(1 - \lambda_{1z}D) \left[ \prod_j (1 - \lambda_{1z}D\phi_j) \right]^{-1} 
\]

\[ 
\Psi_{yz} = (1 - \lambda_{1z})[(I - \lambda_{1z}D\phi_y)]^{-1} \quad \text{for} \quad x, z = c, s, x \neq z 
\]

\[ 
A_0 X_t = A_1 X_{t-1} + A_2 X_{t-2} + \Xi_t 
\]

\[ 
X_t = (c_t, s_t, y_t)' 
\]

\[ 
\Xi_t = (\xi_{ct}, \xi_{st}, \xi_{yt})' 
\]

\[ 
A_0 = \begin{pmatrix} 
1 & \delta_c \Psi_c & \gamma_c \Psi_{yc} \\
\delta_c \Psi_s & 1 & \gamma_s \Psi_{ys} \\
0 & 0 & I 
\end{pmatrix} 
\]

\[ 
A_1 = \begin{pmatrix} 
\lambda_{1c} & \delta_c \Psi_c (\lambda_{1c}D\phi_{1c} + (\lambda_{1c}D)^2\phi_{3c}) & -\lambda_{1s}D\gamma_c \Psi_{yc} \\
\delta_c \Psi_s (\lambda_{1c}D\phi_{1c} + (\lambda_{1c}D)^2\phi_{3c}) & \lambda_{1s} & -\lambda_{1s}D\gamma_s \Psi_{ys} \\
0 & 0 & (I + \varphi_y) 
\end{pmatrix} 
\]

\[ 
A_2 = \begin{pmatrix} 
0 & (\delta_c \Psi_c)(\lambda_{1c}D)\phi_{3c} & 0 \\
(\delta_c \Psi_s)(\lambda_{1c}D)\phi_{3c} & 0 & 0 \\
0 & 0 & -\varphi_y 
\end{pmatrix} 
\]

and the variance-covariance matrix is \( \Sigma \).

If we take the relation \( A_0 X_t = A_1 X_{t-1} + A_2 X_{t-2} + \Xi_t \) and define \( A_1 = A_0^{-1} A_1 \)
and \( A_2 = A_0^{-1} A_2 \) then \( X_t = A_1 X_{t-1} + A_2 X_{t-2} + A_0^{-1} \Xi_t \) and using the formula
\[ 
\Delta X_t = (A_0 - I + A_1)X_{t-1} - A_1 \Delta X_{t-1} + A_0^{-1} \Xi_t 
\] we can write the vector error correction format for this model as:

\[ 
\Delta X_t = B_1 X_{t-1} + B_2 \Delta X_{t-1} + \Xi_t 
\]

\[ 
B_1 = A_1 - I + A_2 
\]

\[ 
= \begin{pmatrix} 
\lambda_{1c} - 1 - \delta_c \Psi_c \Psi_s \delta_s \Phi_s & -\delta_c \Psi_c (\lambda_{1c} + \Phi_s) & \gamma_c (I + \varphi_y) \\
-\delta_c \Psi_s (\lambda_{1c} + \Phi_s) & \lambda_{1s} - 1 - \delta_s \Psi_c \Psi_s \delta_s \Phi_c & \gamma_s (I + \varphi_y) \\
0 & 0 & 0 
\end{pmatrix} 
\]

\[ 
B_2 = -A_2 = -(A_0^{-1} A_2) 
\]

where
\[ \Phi_z = (\lambda_{1z} D (\phi_{1x} + \phi_{3z}) + (\lambda_{1z} D)^2 \phi_{2z}), \]
\[ \Gamma_x = (1 + \lambda_{1x} D) \lambda_x \Psi_x + (1 + \lambda_{1x} D)^2 (\gamma_x \Psi_x \Psi_x \delta_x) \]
\[ \Gamma_y = (1 + \lambda_{1y} D) \lambda_y \Psi_y + (1 + \lambda_{1y} D)^2 (\gamma_y \Psi_y \Psi_y \delta_y) \]

for \( x, z = c, s, x \neq z \).

For stability we require \( \lambda_{1x}, \lambda_{1s} < 1 \). If the non-stationary variables in our equation are cointegrated we can decompose \( B_0 \) into the cointegrating matrix, \( \mathbf{\beta} \), and the loading matrix, \( \mathbf{\alpha} \), in conventional fashion \( B_0 = \mathbf{\Pi} = \mathbf{\alpha} \mathbf{\beta}' \). Exact identification of the system with \( r \) endogenous variables requires \( r^2 \) restrictions to be imposed on the \( \mathbf{\beta} \) matrix. These can be imposed trivially (for \( k = 7, r = 2 \)) by requiring unit coefficients on the endogenous variables and two further restrictions, one for each equation. Any further constraints on the parameter values can be tested using a \( \chi^2 (m) \) test of the \( m \) over-identifying restrictions.

### 3. Evidence from UK firms, 1978–98

We estimate our model for the Private Non-Financial Corporations (PNFC) sector of the UK over the sample period 1978(1)–1998(4) using quarterly data. All variables are in logarithms except interest rates. Net corporate borrowing \( (c_t) \) is measured as the logarithm of the ratio of real lending to firms by M4 institutions i.e. banks and building societies \( (m4lt) \) relative to PNFCs real M4 holdings \( (m4t) \). This makes our measure of net short term borrowing different to that of Gertler and Gilchrist (1994) because they include in their measure of short term debt funds raised in the commercial paper market by large US corporates. The commercial paper market is relatively underdeveloped in the UK compared to the US where it is the major non-bank source of external finance (see Kashyap et al., 1993). The functional relationship defining PNFC’s \( m4t \) and \( m4lt \) have been estimated as separate equations in a systems context by Brigden and Mizen (1999). This gives us some guidance over the likely set of exogenous variables and their influence, although we re-assess their likely effects on net corporate borrowing.

On the supply side, the credit view suggests that there are two channels that influence the supply of \( m4lt \), the ‘balance sheet channel’ and the ‘bank lending channel’. Increasing real financial wealth will tend to raise the available supply of \( m4lt \), according to the balance sheet channel (cf. Bernanke and Gertler, 1995, and Hubbard, 1998) and this will increase the ratio relative to \( m4t \), ceteris paribus. Recorded financial wealth of the sector is a good measure of the health of corporate balance sheets, on which the readiness of banks to lend depends. We use total financial assets of the sector deflated by the implicit price level to measure wealth, \( w_t \), which we use as the determinant of balance sheet effects; we expect a positive effect on \( c_t \) in the long run. The bank

\[^5\] Note that since much of gross financial wealth is comprised of M4 holdings, there may be a negative relationship between changes in wealth and net corporate borrowing for accounting reasons in the short run.
lending channel suggests that credit spreads will measure the extent to which banks wish to discourage recourse to bank financed lending. Jaffee and Russell (1976) and Stiglitz and Weiss (1981) have shown that banks may introduce equilibrium credit rationing because the interest rate is a poor discriminator between ‘good’ and ‘bad’ borrowers. But banks may nevertheless use the spread between credit and deposit rates \( c_s \) as a means of restricting bank lending availability on the supply side, and we would expect a negative relationship between \( c_s \) and \( c_t \).6

On the demand side, growing total financial wealth, \( w_t \), would increase the stock of \( m4 \) and \( m4l \). The elasticity of lending with respect to wealth exceeds the wealth elasticity of money, reflecting the relative importance of the balance sheet channel on net corporate borrowing, Brigden and Mizen (1999). It implies that as financial wealth rises with the business cycle we should expect the ratio of borrowing to deposits to rise with the same cycle. Also on the demand side, the spread of credit rates over deposits indicates that opportunity cost of borrowing and this will typically diminish the demand for net borrowing and increase the demand for \( m4 \), by the PNFC sector. The demand for \( m4l \), will depend on the availability of internal finance so that current retained earnings, \( r_{et} \), would tend to diminish net recourse to banks for short term financing purposes.7 The existence of agency costs, as pointed out by Bernanke and Gertler (1995), ensures that internal finance is always preferred to external finance because it is available at lower cost creating a hierarchy of finance, Myers and Majluf (1984). The effect of \( r_{et} \) on \( c_t \) is therefore expected to be negative.

Our variable \( s_t \) is defined as stockbuilding relative to output, and is measured as the stock level of manufacturer’s work-in-progress and finished goods relative to GDP in real terms.8 In the stockbuilding equation we would expect the stock-to-output ratio \( (sq_t) \) to be dependent primarily on the holding cost of stocks, \( h_{ct} \); as costs rise we expect stockbuilding to fall as a ratio of output. The most commonly cited indicator of stock holding costs is the HM-Treasury measure defined by Kelly (1984), Melliss (1986), and used previously by Callan et al. (1990). The variation in the stock-to-output ratio would also be determined by the business cycle. We proxy the cycle using the financial wealth variable \( (w_t) \), since wealth rises with

---

6 Other authors use different measures of bank dependence such as the dividend payout ratio, Fazzari et al., 1988; bond rating, Whited, 1992; size and age of the firm, Devereux and Schiantarelli, 1990, Carpenter, et al., 1994; Gertler and Gilchrist, 1994; and the coverage ratio, Guariglia, 2000. Many of these measures are firm specific and are therefore most readily applicable to panel studies.

7 A number of authors (Carpenter et al., 1994 and Small, 2000) use cash flow as a proxy for internal finance, calculated as the sum of depreciation and operating profits minus taxation; Callen et al. (1990) use undistributed income. Carpenter et al. (1998) use cash flow, the coverage ratio, and cash stocks as their proxies for internal finance.

8 In earlier versions of the model presented here, income was treated as an explanatory variable and stockbuilding was measured independently of its ratio to output. The overidentifying restriction imposing a unit coefficient on output was always accepted by a likelihood ratio test, however, so we use the stock-to-output ratio.
the business cycle but previous studies have often referred to sales. Stock-to-output would rise to meet demand as wealth and sales follow the cycle.\(^9\)

The descriptive statistics for net corporate borrowing and the stock-to-output ratio are given in Table 1. The table provides the number of observations, the sample period, the mean, standard deviation, skewness, excess kurtosis, minimum and maximum values of the data. A normality test is accepted for the net corporate borrowing variable but not for the stock-to-output ratio which has a bi-modal distribution due the pronounced downward trend in the data over the sample. We take careful steps to deal with the trend later in the paper.

\[\text{Table 1 Descriptive statistics}\]

<table>
<thead>
<tr>
<th>Sample size 88</th>
<th>Sample period 1977 (1)–1998 (4)</th>
</tr>
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<tbody>
<tr>
<td><strong>Net corporate borrowing</strong></td>
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<tr>
<td>Mean</td>
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<tr>
<td>Std. devn.</td>
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<tr>
<td>Skewness</td>
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<tr>
<td>Excess kurtosis</td>
<td>−0.508951</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.274710</td>
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<tr>
<td>Maximum</td>
<td>0.692990</td>
</tr>
<tr>
<td>Normality (\chi^2) (2)</td>
<td>1.3934 [0.4982]</td>
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<tr>
<td><strong>Stock-to-output</strong></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
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<tr>
<td>Std. devn.</td>
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<tr>
<td>Skewness</td>
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<tr>
<td>Excess kurtosis</td>
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</tr>
<tr>
<td>Minimum</td>
<td>−0.371704</td>
</tr>
<tr>
<td>Maximum</td>
<td>−0.055341</td>
</tr>
<tr>
<td>Normality (\chi^2) (2)</td>
<td>19.664 [0.0001]</td>
</tr>
</tbody>
</table>

*Note: p-value in brackets.*

3.1 The equilibrium stock-to-output ratio and net corporate borrowing

The interaction between stockbuilding and net corporate borrowing has been left to one side because the equilibrium relationship between each of the endogenous variables and their exogenous driving variables has been, and still is, poorly understood. The problem is less severe for net corporate borrowing, but it is apparent that a major consideration in the stockbuilding equation is the fact that the level of stockbuilding and its ratio to output, has been subject to a downward secular trend (see Figure 1b). Stockbuilding has been a good leading indicator of GDP growth on an annual basis, but has one of the most difficult components to forecast using

\(^9\) It is possible that in the short term the opposite effect will occur as larger-than-expected sales diminish stocks.
macroeconomic data primarily because of the difficulty of accounting for the pronounced secular decline in the stock-to-output ratio over the 1980s and early 1990s.

There have been many explanations offered for this empirical puzzle but determining whether the recent flattening since the mid-1990s is due to a change in trend or a cyclical variation around the trend poses the greatest difficulty of all. Various measures of the trend have been offered as potential explanations for the decline in the stock-to-output ratio (Bank of England, 1999b). First, the trend may be due to the relative decline in the manufacturing sector as a share of GDP. Historically, the manufacturing sector has been a major contributor to total stockholding, and the declining share of GDP may explain why stocks have fallen (see Fig. 1b). Second, improvements have been made in the stock management processes over the period, and these influences such as computerisation of stock control and just-in-time delivery may have reduced manufacturing industries holdings of work-in-progress significantly, as well as final goods inventories. Third, the retail sector has had a correspondingly larger share of GDP as manufacturing has declined, and since the mid-1990s in particular has had a growing stock-to-sales ratio. Whilst the distribution networks have gained from the same efficiencies as the manufacturers with computerisation, just-in-time delivery and electronic interchange of information, the larger retailers have increasingly absorbed the wholesaler’s function. This may have increased their stocks, especially if product variety has increased, and this could explain the flattening off of the ratio in the later 1990s.

In the light of the different explanations offered in the literature, we take several measures of the trend and examine various options to attempt to capture the declining ratio. Each has its drawbacks, but as we discover, the trends are all highly correlated with each other. If we use a time trend or a smooth transition in the just-in-time technology, following Rosanna (1998), this assumes a constant rate of decline or a uniform diffusion process, which relies heavily on the judgement of the modeller as to when the decline began, how fast it has proceeded and how long it might be expected to continue. Alternatively, the change in the
composition of GDP captures the structural changes in inventory composition and may reflect the falling ratio, but is likely to also capture many other influences on the structure of GDP such as the changing ratios of manufacturing output to total production, retail-to-wholesale stocks, and retail stocks-to-sales. Unless care is taken with this second group of measures, we may end up accounting for the declining ratio by use of a major component of the variable to be explained.

Preliminary testing of the variables in our system indicates that all variables are non-stationary over our sample period. The system we estimate is defined by the
\[ (k/C2) \text{vector } X_t \]
where
\[ X_t = \{ c_t, sq_t, w_t, ret_t, hc_t, cs_t, trend \} \]
and
\[ \Delta X_t = B_0 X_{t-1} + B_1 \Delta X_{t-1} + \Xi_t \]

Both \( B_0 \) and \( B_1 \) are matrices of coefficients to be estimated, and are expected to conform to the theoretical values as follows
\[ B_1 = \Lambda_1 - I + \Lambda_2 \]
\[ B_2 = -\Lambda_2 = -(A_0^{-1}A_2) \]

It is here that we will discover whether the coefficient estimates \( \delta \) and \( \epsilon \) give a positive value on \( \theta_5 \) in the cost function (10). In practical terms, the matrix \( B_0 \) describes the product of the feedback matrix and the matrix of coefficients that produces a cointegrating linear combination of non-stationary variables in levels. The first two rows represent the coefficients for the long-run equations of the endogenous variables, \( c_t \) and \( sq_t \), and the remaining rows the coefficients for the long-run equations of the exogenous variables \( \{ w_t, ret_t, hc_t, cs_t, trend \} \). The fact that the remaining rows have zero elements indicates that the theory does not predict more than two cointegrating relationships in the system. Using the Johansen procedure we show that using our data we can reject the null of not more than one vector, but cannot reject the null of not more than two, thus we can confirm empirically that there are in fact two cointegrating relationships between the variables of our system. In the subsequent analysis we restrict the rank to equal to two \( (r = 2) \).

The next step we take is to define these two equilibria that we have detected. Taking the lead from our theoretical model, we associate the two relationships with the two endogenous variables, net corporate borrowing and the stock-to-output ratio. These are exactly identified by imposing unit coefficients on the net borrowing and stock-building variables and \( r(r - 1) \) additional exact identifying

---

10. Tests were initially conducted using constant with trend, and subsequently using constant with no trend. The ADF statistics for all variables failed to reject the null of non-stationarity, although net corporate borrowing was close to rejecting the null when the trend was excluded. First differences of the variables comprehensively reject the null even at the 1% critical values.
restrictions. On theoretical grounds we do not expect the ‘holding cost of stocks’ variable to influence net corporate borrowing so we can identify the net corporate borrowing equation by excluding this variable (imposing a zero restriction on the coefficient of the holding cost of stocks variable) from this equation. Likewise we do not expect the credit-deposit spread to influence the stock-to-output ratio equation and again we can identify the stock-to-output equation by imposing a zero restriction on the coefficient of the credit-deposit spread variable in the stock-to-output equation. Using these two additional restrictions we can ensure that the equations are fully identified, allowing us to observe the coefficient standard errors, and we can then associate each equilibrium with one of the two endogenous variables. This exactly identified model can be further restricted by imposing m over-identifying restrictions on the parameter values of the system and jointly testing these using a likelihood ratio test.

In addition to the exact identification we can also examine the feedback coefficients of the system to determine whether in fact we have correctly assigned the equations to each variable. We would expect there to be strong negative feedback from the error correction term (the cointegrating residuals from the relationship associated with each endogenous variable) to the current difference in that endogenous variable. The feedback matrix is (standard errors in brackets):

\[
\begin{pmatrix}
-0.42339 & -0.02501 \\
0.6216 & 0.013730 \\
-0.02640 & -0.01734 \\
0.01947 & 0.00430
\end{pmatrix}
\]

This shows that the main diagonal is significant and negative as we would expect if the equations had been correctly assigned, but the off-diagonal elements are smaller in size and marginally significant in only one case, at the ten percent level. The feedback is much stronger in the first equation, which is the net corporate borrowing equation, and weaker in the second equation, the stock-to-output equation. Thus the feedback matrix confirms what we have found in the exact identification process and ratifies our assignment of cointegrating vectors to endogenous variables.

The results reported in Table 2 are the first two rows of the matrix from the estimates of eq. (14). Column (1) is the base around which our system is built. It contains one cointegrating relation for net corporate borrowing and another for stockbuilding, both of which are functions of the same small set of variables that are used in Brigden and Mizen (1999). At this point we focus on the nature of the equilibrium equations examining the interaction in the long run between the two endogenous variables, net corporate borrowing and the stock-to-output ratio, and the relationship with the exogenous variables \( \{ w_t, r_t, h_t, c_t, \text{trend} \} \).

Taking each of the exogenous variables in turn in the net corporate borrowing equation (column one) and then turning to the stock-to-output equation (column two) we interpret the elasticities with respect to each variable. The most noticeable result is that there is no interaction between stock-to-output and net corporate
borrowing since both equations have zero coefficients on the alternate endogenous variable. This implies that there is no long-run interaction between these variables but does not exclude the possibility that they may act as potential substitutes in short-term adjustment. Beginning with the net corporate borrowing equation we find that net financial wealth variable is an important explanatory variable, the interpretation of which is consistent with supply-side and demand-side influences. On the supply side, there is evidence consistent with a balance sheet channel. The positive impact of financial wealth on the ratio of real lending to real liquid assets implies that lending is positively associated with improving financial health. This result holds for two reasons: not only are banks less cautious about lending to financially healthy firms because higher net wealth implies a more successful firm with less likelihood of failure and hence default, but firms with higher net wealth also have access to greater collateral on which to borrow, offering security to the

<table>
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<th>(3)</th>
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<td>0.000</td>
<td>-1.000</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
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<td>$sqt$</td>
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<td>0.000</td>
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<tr>
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<td>(0.000)</td>
<td>(0.000)</td>
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<tr>
<td>Log Likelihood</td>
<td>779.45</td>
<td>780.14</td>
<td>792.69</td>
</tr>
<tr>
<td>LR test ~ $\chi^2(m)$</td>
<td>m = 5</td>
<td>0.265</td>
<td>m = 6</td>
</tr>
<tr>
<td></td>
<td>[0.998]</td>
<td>[0.835]</td>
<td>[0.778]</td>
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Notes: Sample period 1978(1)–1998(4). The trend in each model is (1) no trend; (2) time trend; (3) the ratio of manufacturing stocks to total stocks; (4) ratio of manufacturing work-in-progress to total stocks; (5) ratio of retail stocks to total stocks; and (6) retail stock to sales ratio.

Correlation matrix of trend measures (3)–(6):

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<th>(4)</th>
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<th>(6)</th>
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<td>(6)</td>
<td>-0.887</td>
<td>-0.876</td>
<td>0.807</td>
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</table>
lender. On the demand side firms increase their demand for net corporate borrowing as net financial wealth improves, since this indicates their own success and the improvement in cyclical factors that typically precede expansion. The wealth effect implies that a one percent increase in financial wealth leads to increase of 0.18% in net corporate borrowing. This figure suggests that although much of real financial wealth is held in liquid form, corporates increase their borrowing by slightly more than liquid assets as wealth increases in the long run.

Turning to the effect of retained earnings, the elasticity is expected to be negative. As internal sources of funds become available when retained earnings expand we expect to find the firm substituting from a relatively expensive form of external finance to a cheaper internal source of funds. This prediction arises from the

### Table 2 Continued

<table>
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<tr>
<td></td>
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Log likelihood 790.81 788.65 788.29
LR test $\chi^2(m)$ for trend $m=6$ 1.589 1.077 1.284

<table>
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<tr>
<td></td>
<td>0.953</td>
<td>0.983</td>
<td>0.973</td>
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Correlation between cointegration residuals (stock-to-output ratio) after using different trend options:

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<td>(3)</td>
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<td>(4)</td>
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<td>0.976</td>
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Estimation is by full information maximum likelihood. Individual columns are the restricted cointegrating vectors, pairs of columns represent the equilibria for the system. Overidentifying restrictions are imposed and tested using $\chi^2(m)$ test. Standard errors are in brackets, and $p$-values are in square brackets.
hierarchy of finance view (Myers and Majluf, 1984). Although we have no theoretical prior about the size of the coefficient, the unconstrained estimate of the coefficient on this variable was close to $-0.5$. It was possible to impose the value of $-0.5$ as an additional overidentifying restriction on the system and this was justified on the basis of earlier work in Brigden and Mizen (1999). In that earlier paper the coefficient estimates from the separate corporate borrowing and money equations (in logarithms) were $-0.5$ and $0.0$ respectively. Since the left hand side variable in the equation we are estimating in this paper is the sum of the left hand side variables in the two equations in Brigden and Mizen (1999), the estimated value of this parameter on the retained earnings coefficient should be the sum of the two coefficients in the earlier work, which is $-0.5$. The coefficient value, which was close to $-0.5$ when estimated with constraints being imposed, should in fact be $-0.5$ if the results reported here are to be consistent with the earlier study.

The holding cost of stocks variable does not appear in the net corporate borrowing equation since there is no economic reason for the cost of holding stocks to influence net corporate borrowing. Indeed this variable was excluded with a zero coefficient restriction in order to identify the equation at an earlier stage.

The last variable in the net corporate borrowing equation is the spread of credit over deposit rates. Our expectation is that the sign, under a bank-lending channel on the supply-side and an opportunity cost of borrowing from banks will be a negative on the demand-side. The estimated elasticity on the spread of credit over deposit rates is negative but the coefficient is not significantly different from zero in the long run. We expect the long-run level of net corporate borrowing to be influenced by the spread of credit over deposit rates because it is the opportunity cost of borrowing versus reducing deposits, but when the margin that banks earn between rates on credit and deposits is fairly constant this variable may not be significant at all. The effect, such as there is, will be captured by the constant term.

The stock-to-output ratio is driven by two variables, real financial wealth and the holding cost of stocks. In earlier versions of the stockbuilding equation the output variable was included as an explanatory variable, but the coefficient value was consistently close to unity and the later versions of the system were based on a stock-to-output ratio. The interpretation of the equation must take this matter into account, since the endogenous variable in our equation includes output, which is the normal scalar for a stockbuilding equation. The output effect is endogenised in the stock-to-output ratio, and the impact of the wealth variable explains how the ratio varies as financial wealth rises and falls. This can be given a demand-side interpretation consistent with the role that wealth plays in the net corporate borrowing equation: net wealth gives a further indication of the impact of the cycle. The one-for-one response of the stock-to-output ratio to wealth is the result of a freely estimated coefficient that was subsequently restricted to equal unity by an overidentifying restriction. This restriction could not be rejected, although it has no particular theoretical significance.
Neither retained earnings nor the credit-deposit spread variables have a theoretical or empirical role to play in the stock-to-output equation, but the holding cost of stocks variable is a significant variable in the stock-to-output equation because it represents the after-tax holding cost of increasing the ratio. We would expect to find that increases in the cost of holding stock would induce stockbuilding to decline in relation to output. Empirically this is what we find.

The graphical output for column (1) in Fig. 2 shows that the fitted model for each variable tracks the actual net corporate lending and the stock-to-output series closely and the cross-plots lie in the neighbourhood of the 45° line. These features suggest that the small group of explanatory variables are able to explain the long-run behaviour of the two endogenous variables in the system very well.

The dimension of the stock-to-output equation that might be considered the most challenging is the downward secular trend. We take a number of different approaches based on our earlier discussion, including (separately) a simple linear time trend, two measures of the decline of manufacturing industry using the share of total stocks held by manufacturing industry and the share of work-in-progress held by manufacturing industry, and the rising share of total stocks held by the retail sector and their increasing stock-to-sales ratio. The results for these models are presented in Table 2 by columns (2)–(6), and the pattern is consistent
irrespective of the measure chosen. The reason for this is demonstrated in the notes to Table 2 where we present the correlation matrix for the trend measures, where there is strong positive correlation between measures of manufacturing stocks, and negative correlation between these and the retail stocks, confirming the time paths in Figure 1a. It is unsurprising, therefore, that they all tell the same story.

First, all of the restrictions that were selected for the previous model, column (1), are imposed without rejection in columns (2)–(6) according to a likelihood ratio test. The remaining unconstrained variables take on magnitudes very close to their former values in column (1), since the coefficient on wealth remains close to 0.18 (values lie in the range 0.17–0.21), and the credit spread remains insignificant. Second, a further restriction is placed on the trend in the net corporate borrowing equation. Column (2) introduces a linear time trend, which is significant \( p \text{ value} = 0.0027 \), and indicates a modest rate of decline at 2.4% per annum. Column (3) includes the ratio of manufacturing stocks as a proportion of the total, which has the correct sign (positive) but is only on the margins of significance \( p \text{ value} = 0.0969 \). If the decline has been due to the shrinkage in the manufacturing sector, or the increased efficiency in the stockholding practices of that sector, one reason for the marginal significance may be the relative unimportance of some components of manufacturing stocks, such as finished goods or input materials and fuels. There is evidence to suggest that it is the work-in-progress component of stocks that has fallen more dramatically than other components of manufacturing stocks. Column (4) shows the impact of manufacturing work-in-progress relative to total stocks and the coefficient has the expected positive sign, and a slightly lower \( p \) value \( p \text{ value} = 0.0719 \). Columns (5) and (6) refer to the impact of the growing retail stocks as a proportion of the total and the increasing stock-to-sales ratio. Both coefficients have the correct (negative) sign, but the magnitudes and significance are quite different. The former has a \( p \) value of 0.1471 while the stock-to-sales variable is strongly significant \( p \text{ value} = 0.0069 \), suggesting that the recent increase in retail stockbuilding may have been coincident with the decline in total stockholding. Amalgamation of retailing and wholesaling functions may have simultaneously raised the stock-to-sales ratio of the retail sector while reducing the total stockholding of sectors as a whole. The notes to Table 2 include a correlation matrix for different residual series from the six cointegrating equations for the stock-to-output ratio. There is a remarkably high degree of correlation between these residual series even with the model excluding the trend measure, which implies that the short-run dynamics are unlikely to be affected by the choice of trend measure. The next step is to examine the structure and performance of the dynamic model in order to determine the nature of the interaction between the stock-to-output ratio and net corporate borrowing.

3.2 Dynamic adjustment and interaction

The estimates of the long-run equilibria in Table 2 were generated in the context of an unrestricted vector error correction (VECM) model. Having established the fact
that there are \( r = 2 \) cointegrating relationships among the \( k = 7 \) variables, we can write a conditional VECM in terms of the \( r \) cointegrating relationships and dynamic terms, treating the other other variables in the system as exogenous, i.e. driven by marginal processes that exclude the cointegrating relationships above. Taking eq. (14), we can write the conditional model as

\[
\Delta x_t = b_0 X_{t-1} + b_1 \Delta X_{t-1} + \Xi_t
\]  

(15)

where \( x_t \) represents the endogenous variables in \( X_t \), and in this case comprises the first two elements \( \{c_t, sq_t\} \). The matrices \( b_0 \) and \( b_1 \) are the first two rows of \( B_0 \) and \( B_1 \) respectively. The structural VECM is derived by imposing further identifying restrictions \( C_0 \) on eq. (15)

\[
C_0 \Delta x_t = C_0 b_0 X_{t-1} + C_0 b_1 \Delta X_{t-1} + C_0 \Xi_t
\]  

(16)

There are a number of different identification procedures that could be employed but we impose restrictions on the contemporaneous dynamic terms directly, imposing the restrictions on the LHS term, through the structure of \( C_0 \), (i.e. \( C_0 \Delta x_t \)). We choose this approach since it is the easiest to interpret and gives a clear indication of whether net corporate borrowing and stockbuilding influence each other by revealing whether the coefficients \( \delta_1 \) and \( \delta_2 \) in eq. (14), which in turn establishes the sign of \( \theta_5 \) in the cost function (10).

The dynamic system is reported in Table 3, and both the equilibrium correction terms enter each equation. From the resulting signs, magnitudes and significance we deduce that a disequilibrium in stock-to-output ratio (net corporate borrowing) has a small but significant negative effect on contemporaneous change in net corporate borrowing (stock-to-output ratio). These suggest that there are non-negative off-diagonal terms in matrix \( b_1 \), although the net corporate borrowing disequilibrium is only significant at the 10% level. Own disequilibria are negative and significant confirming that the system is dynamically stable.

Other coefficient values confirm our priors. In the stock-to-output equation, the response to the cost of stockholding implies that as the cost accelerates the stock-to-output ratio falls. This implies that firms economise on the ratio—perhaps by introducing more efficient stock management measures or just-in-time technology—in order to avoid the escalating costs of keeping goods in stock. As internal finance (retained earnings) increases firms lower their ratio of stocks-to-output, possibly because they can afford to purchase additional stock or increased production if required from internal funds. A change in the net financial wealth reduces stock-to-output as we would expect since changing financial wealth is our measure of the business cycle and as business increases firms respond in the short term by reducing stocks to meet demand.

In the net corporate borrowing equation, there is a strong positive response to net borrowing in the previous period, which suggests that the decision to borrow is smoothed over several quarters. The holding cost of stocks variable reduces the recourse to net borrowing. At first this is somewhat puzzling, since we do not expect there to be any influence from the cost of holding stocks on net corporate borrowing. However, there is some evidence suggests that there can be a positive
relation in the short term between the level of stocks and the level of borrowing if distress borrowing is used to fund the temporary increase in stocks. When the holding cost of stocks increases, stocks are expected to fall, and thus we may have captured the reverse of the above effect, as falling stocks lowers net corporate borrowing temporarily either by reducing distress borrowing or increasing deposits. In any event these effects are very small. As wealth rises, net corporate borrowing falls, which is a feature of the fact that the wealth variable includes a significant M4 component and as it rises the net corporate borrowing falls by definition. As retained earnings increase so net borrowing declines because firms substitute away from the relatively costly external finance towards internal finance.

The dynamic system yields satisfactory diagnostics and the exclusion of insignificant variables to yield a parsimonious model is not rejected by the likelihood ratio tests of the relevant over-identifying restrictions. The dynamic response is given in Fig. 3. A goodness of fit of 68% is impressive for a dynamic system and the first panel shows the graphic fitted values against actual changes for changes to stockholding and then changes to net corporate borrowing. The lower panel

Table 3 Dynamic vector equilibrium correction model

<table>
<thead>
<tr>
<th>Sample 1978 (2) to 1998 (4); T = 83</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta s_t = -0.163 + 0.064 \Delta c_{t-1} - 0.017(\text{ECM } s_t)<em>{t-1} - 0.021(\text{ECM } c_t)</em>{t-1}$</td>
</tr>
<tr>
<td>$(-3.270)$ $2.557$ $(-4.839)$ $(-1.739)$</td>
</tr>
<tr>
<td>$-0.0013 \Delta^2 h_t - 0.018 \Delta r_t - 0.055 \Delta w_t$</td>
</tr>
<tr>
<td>$(-2.675)$ $(-2.802)$ $(-1.670)$</td>
</tr>
<tr>
<td>AR 1 – 10: F(10, 60) = 1.17 [0.33]; Normality $\chi^2(2) = 3.21 [0.20]$;</td>
</tr>
<tr>
<td>ARCH: F(4, 60) = 1.08 [0.37]; $\chi^2$: F(24, 45) = 0.410 [0.99]</td>
</tr>
<tr>
<td>$\Delta c_t = 0.038 + 0.551 \Delta c_{t-1} - 0.035(\text{ECM } s_t)<em>{t-1} - 0.157(\text{ECM } c_t)</em>{t-1}$</td>
</tr>
<tr>
<td>$(0.206)$ $(5.848)$ $(-2.549)$ $(-3.445)$</td>
</tr>
<tr>
<td>$-0.004 \Delta h_{c_{t-1}} - 0.235 \Delta w_t - 0.057 \Delta r_{c_{t-1}}$</td>
</tr>
<tr>
<td>$(-2.47)$ $(-2.011)$ $(-2.392)$</td>
</tr>
<tr>
<td>AR 1–10: F(10, 60) = 1.87 [0.07]; Normality: $\chi^2(2) = 0.306 [0.86]$;</td>
</tr>
<tr>
<td>ARCH: F(4, 60) = 1.381 [0.25]; $\chi^2$: F(24, 45) = 1.406 [0.15]</td>
</tr>
</tbody>
</table>

Diagnostics

| $R^2$ (LR) = 0.687 |
| Log likelihood = 723.2 |
| LR test of over-identifying restrictions: $\chi^2(11) = 9.039 [0.618]$ |
| Vector AR 1–10: F(40,110) = 1.00 [0.48] |
| Vector normality: $\chi^2(4) = 3.25 [0.52]$ |
| Vector $\chi^2$: F(72, 147) = 1.04 [0.41] |

Notes: The numbers in round brackets beneath the estimated coefficients are t-values, and the numbers in square brackets after diagnostic statistics are p-values.
Fig. 3. Dynamic VECM graphics Notes: the top panel reproduces the fitted and actual values for the dynamic equations for stock-to-output ratio and the net corporate borrowing equations. The bottom panel shows the residual sum of squares and 1-step ahead residuals and standard errors for stock-to-output ratio and the net corporate borrowing equations, the log likelihood over $T$, the encompassing test LR(11) and 5% critical value, and a 1-step ahead and forecast Chow test with 5% critical values.
reports the recursive tests. The first four graphs show, in sequence, the residual sum of squares for each of the equations and the 1-step ahead residuals with standard error bands. The next four graphs show the log likelihood for the system, which declines steadily, a recursive encompassing test which does not exceed the 5% critical value at any point, and the 1-step ahead and forecast Chow tests recursively that do not exceed their 5% critical values. At only one point in late 1988 do the residuals violate the standard error bands, but these Chow tests do not indicate a structural break at that point.

The main point of the dynamic system is to examine the sign and significance of stock-to-output and net corporate borrowing disequilibria in the dynamic equations. We expect to find that these terms have negative and significant coefficients in their own equations since this is a basic feature of dynamic stability, but the sign and significance of these same terms in the other equation reveals the features about the cost function (10) which are vital to our analysis. In both equations we find negative coefficients on the off-diagonal terms on the equilibrium correction terms in each equation. Despite the fact that the coefficients are small, they are significant. The fact that the coefficients on the error correction terms are all negative is consistent with the reduction of net liquidity (stocks) when stocks (net liquidity) exceed their desired levels but it is not sufficient to sign \( \delta_c \) and \( \delta_s \), since these appear in matrix \( B_0 \), which represents the \( \Pi \) matrix, \( \Pi = x_\beta \). Retrieving the elements of the \( \Pi \) matrix we find that the sign of the elements conforms to the sign predicted by eq. (14). This in turn confirms that the coefficients \( \delta_c \) and \( \delta_s \) are positive in eq. (14), which implies \( \theta_2 \) is positive in the cost function (10). A further test which imposes the restrictions that the off-diagonal terms equal zero can be rejected according to a Wald test. The statistic for the zero restrictions is distributed with a \( \chi^2(2) \) distribution and the statistic with p-value in brackets is 8.9993 [0.0111].

These results validate our theoretical model, showing that UK PNFCs are behaving efficiently by treating stock-to-output and net corporate borrowing as substitutes by UK PNFCs, as suggested by our theoretical model. The implication of this finding is that the decision rule decomposition theorem is overturned, and is unduly restrictive. Firms use both stockbuilding (relative to output) and net borrowing to buffer supply and demand shocks. A production smoothing model should allow both variables to act as buffers for an efficient firm.

4. Conclusions

In this paper we have noted that the two literatures on stockbuilding and corporate borrowing have developed in parallel, using the same cost functions and optimisation techniques to derive similar production smoothing models for inventories and liquid assets. While interactions have been investigated by allowing stockbuilding levels and borrowing constraints to enter equations for borrowing and stockbuilding respectively, there has been no attempt to consider how firms might choose to
jointly optimise the two decisions. This has been due to the decision rule decomposition theorem, which separates the two decisions.

In this paper we make an empirical point that shows a simple alteration to the cost function allowing the firm to use stock-to-output and net corporate borrowing as substitutes in the face of production shocks receives support from UK data. Our empirical investigation makes two observations. First, the stock-to-output ratio and net corporate borrowing decisions of UK private non financial corporations (PNFCs) vary in relation to real financial wealth, retained earnings (internal finance), stockholding costs, the credit-deposit spread, and a trend. These long-run equilibria are independent of each other, in that borrowing does not depend on stock-to-output in the long run or vice versa. Second, the dynamic analysis shows that these two variables do act as substitutes in the short run. Hence, overborrowing does tend to diminish stockholding and vice versa, and an increase in borrowing is contemporaneously associated with a fall in stockbuilding. This means that we can confirm that the two variables are treated as substitutes. The implication of these findings is that it is inappropriate to treat one or the other of these variables as exogenous. They interact with each other and are jointly optimised by the efficient firm.

Acknowledgements

The author would like to thank Mark Astley, Hasan Bakhshi, John Butler, Alec Chrystal, Danny Gabay and two anonymous referees for comments and suggestions.

References


