Estimating Monetary Reaction Functions at Near Zero Interest Rates: An Example Using Japanese Data\textsuperscript{1}

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August 2006

Abstract: The importance of truncated distributions for bias in estimation of regression coefficients has been well understood by econometricians, but the relevance of truncation when estimating policy reaction functions has not been fully appreciated. Due to the emergence of low interest rates and the proximity of a zero lower bound (ZLB) on interest rates, coefficient estimates can be biased upwards. This paper illustrates the importance of measuring and correcting estimates for this bias using Japan’s unique experience of prolonged low inflation/deflation. While we would expect the monetary policy reaction function in Japan to differ from other countries in the G4, we show the bias from truncation of the interest rate distribution is significant and needs to be taken into account.

Key words: Monetary policy, Reaction functions, Zero lower bounds, Japan, Tobit.

JEL classifications: E42, E52

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1 Introduction

The importance of allowing for truncated distributions when estimating regression coefficients has been carefully researched by Goldberger (1981), Greene (1981), Amemiya (1984) and Chung and Goldberger (1984). The relevance of this literature has been widely understood in applications ranging from demand for telephone services to labour supply decisions. But the necessity of accounting for truncation needs to be reiterated for monetary economists attempting to estimate policy rules during recent episodes - most notably in Japan, the US and Switzerland - where rates of interest controlled by policymakers have reached low and near zero values. The estimation of monetary reaction functions using a dependent variable constrained by a zero lower bound (ZLB) without suitable adjustment will result in upwardly biased coefficient estimates. This represents a case where the policymaker is faced with regime change - very low interest rates in proximity to zero - that alters the reaction function by constraining the further downward movement of rates and putting a lower bound on rates (c.f. Adam et al., 2005 for an example of changing parameters of reaction functions with regime change when there is not a zero constraint). Our paper illustrates the extent of the bias using Japanese data.

Japan is the first country to have faced a lower bound on the nominal interest rate in practice and we might expect to observe bias in the coefficient estimates where the influence of truncation is not taken into account. The Japanese experience has attracted considerable interest in theory and policy literatures (c.f. Fuhrer and Madigan 1997; Ahearne et al. 2002; Jinushi et al. 2002; Eggertsson and Woodford, 2003; Ito and Mishkin, 2004; Kuttner and Posen, 2005 among others). Clarida et al. (1998), Bernanke and Gertler (1999), and Kuttner and Posen (2005) have uncovered much greater inflation response coefficients in monetary reaction functions for Japan, where the estimated response to inflation lie in the range 2.04 - 2.97, than estimated for other industrialised countries. Although it is expected that monetary policy responses will differ in Japan compared to the experiences of other G4 economies for institutional and economic reasons, the proximity of the interest rate to a zero lower bound is a relevant consideration since it introduces the possibility that these estimates are significantly biased upwards as we demonstrate here.

We make two points: we show by simulation that an estimator that does not allow for the truncation will typically generate upwardly biased coefficients when the regression variables are non-normally distributed (as is the case for our Japanese data), and we show that estimated coefficients are significantly biased due to the truncation effect of the ZLB. Our paper measures and corrects the biased estimates of the monetary reaction function to allow for the truncation of the dependent variable by using the Tobit model for Japan. The coefficient estimates in the literature of the response
to inflation in Japan exceed estimates in other G4 countries, but where they include a ZLB these are upwardly biased. We remind empirical monetary economists that estimates for samples including a ZLB episode should take truncation fully into account.

2 The Zero Lower Bound and Estimator Bias

A typically specified reaction function such as the modified Taylor rule is written as follows;

\[ r_t = (1 - \rho)\alpha_{IV} + (1 - \rho)\beta_{IV}\pi_{t+n} + (1 - \rho)\gamma_{IV}\hat{y}_t + \rho r_{t-1} + \varepsilon_t \]  

(1)

where \( r_t \) is the monthly call rate, \( \pi_{t+n} \) is the monthly inflation rate at the \( n^{th} \)-period horizon, and \( \hat{y}_t = y_t - y_t^* \) is the output gap, with \( y_t^* \) derived as potential output using an HP filter (c.f. Clarida et al. (1998))\(^2\). Estimated coefficients are derived from instrumental variable methods to control for endogeneity bias, \( \beta_{IV} \) and \( \gamma_{IV} \), are the marginal responses to inflation and the output gap. These can be estimated with and without smoothing (where \( 1 > \rho \geq 0 \), or \( \rho = 0 \) respectively) although estimates of \( \beta_{IV} \) and \( \gamma_{IV} \) are not significantly different in each case.

Estimation of (1) using Japanese data by Clarida et al. (1998), Bernanke and Gertler (1999), and Kuttner and Posen (2005) among others has uncovered that the inflation responsiveness coefficient \( \beta_{IV} \) is generally larger than estimates for other G4 countries. Although this is not always due to the ZLB, since some authors estimate reactions prior to the ZLB episode, authors such as Kohn (1996), Orphanides and Wieland (1999) and Yates (2003) have argued that a higher response to inflation is rational for any central bank facing nominal interest rates that approach the region of the ZLB. The proximity to the ZLB makes policymakers more responsive to inflation because the forward-looking policymaker properly recognizes the costs of implementing policy under the zero bound and takes precautionary measures to reduce the probability of deflation before rates reach zero. While there may be good reasons to think that the Bank of Japan would react more strongly than other central banks to inflation as the operational rate approaches zero, we can demonstrate that ignoring the truncation of

\(^2\)We follow the literature in using a Taylor rule. In using the Taylor rule we are able to compare our results with others reported in the literature, but in doing so we do not imply that the Bank of Japan was, or should have been, following a Taylor rule for policy making purposes during our sample. Policy by the Bank of Japan has undergone significant changes during the sample, not least when facing the approach of the ZLB, and this has created an apparent break in the volatility of interest rates. We are not primarily interested in the economic interpretation of the coefficients but the issue of potential bias in estimation in the reaction function. The change in volatility of interest rates might potentially affect our measure of bias but we investigate this matter later in the paper and estimates of the bias are unaffected.
the dependent variable through the impact of the ZLB results in upwardly biased estimates of the coefficient on inflation in equation (1). This might give a misleading impression of the extent of the reaction when close to the ZLB.

2.1 Estimator Bias

We denote by \( r^*_t \) the desired interest rate from the monetary reaction function. In the presence of a ZLB constraint the nominal interest rate cannot fall below zero but if the desired rate falls below zero the observed interest rate \( r_t \) will exceed the desired interest rate \( r^*_t \), and will be censored at zero:

\[
\begin{align*}
  r^*_t &= \alpha^* + \beta^* \pi_{t+n} + \gamma^* y_t + \epsilon_t \\
  r_t &= 0 \quad \text{if } r^*_t \leq 0 \\
  r_t &= r^*_t \quad \text{if } r^*_t > 0.
\end{align*}
\]

In the above Tobit model, the interpretation of the inflation rate coefficient \( \beta^* \) measures how much the desired interest rate \( r^*_t \) (not the observed interest rate \( r_t \)) increases with a unit increase in the future inflation rate. The marginal effect (denoted by \( \beta^*_a \)) of inflation rate on the observed interest rate \( r_t \) is given by

\[
\beta^*_a = \frac{\partial E(r_t \mid \pi_{t+n}, y_t)}{\partial \pi_{t+n}} = \beta^* \times \Pr(r^*_t > 0),
\]

which is the primary object of interest in most cases as well as in our paper. Hence, the adjusted coefficient \( \beta^*_a \) must be smaller than the unadjusted estimate of the coefficient \( \beta^* \). The more observations are censored, the smaller the adjusted coefficient becomes.

One can consider a situation in which a researcher that fails to take account of the truncation may erroneously run the following regression in order to estimate \( \beta_{IV} \), the marginal effect of \( \pi_{t+n} \) on the observed interest rate \( r_t \):

\[
  r_t = \alpha_{IV} + \beta_{IV} \pi_{t+n} + \gamma_{IV} y_t + \epsilon_t.
\]

The above regression (4) is obviously misspecified and the size of the bias in the inflation response coefficient \( \beta_{IV} \) will depend on how much it will deviate from the correct marginal effect \( \beta^*_a \) (i.e. the bias term is defined as \( \beta_{IV} - \beta^*_a \)). Hence, by examining the relationship between \( \beta_{IV} \) in (4) and \( \beta^*_a \) in (3) we can determine the extent of the bias.

Under multinormal distributional assumptions Green (1981) and Goldberger (1981) proved that

\[
\beta_{IV} = \beta^* \times \Pr(r^*_t > 0),
\]

which implies that \( \beta_{IV} = \beta^*_a \). This result is interesting because it shows that even though the Taylor rule regression in (4) is misspecified, it can actually
provide the correct marginal effect of $\pi_{t+n}$ on the observed interest rate $r_t$ under the multi-normality condition. However, this interesting result holds only if $r_t^*, \pi_t$, and $\tilde{y}_t$ are multinormally distributed, and this condition does not hold in our Japanese data set. Figures 2(a)-2(c) show the histograms and Jarque-Bera test results for $r_t, \pi_t$ and $\tilde{y}_t$ respectively, demonstrating that the normality condition is emphatically rejected for $r_t$ and $\pi_t$. Similar rejections of the normality condition can be shown for countries facing near zero lower bounds e.g. the US and Switzerland.

Chung and Goldberger (1984) generalised the result in (5) under considerably weaker distributional assumptions. They showed even without normality that
\[ \beta_{IV} = \beta^* \times \left( \sigma_{r^*r}/\sigma_{r^*}^2 \right) \] (6)
where $\sigma_{r^*r} = cov(r_t^*, r_t)$ and $\sigma_{r^*}^2 = var(r_t^*)$.\footnote{It can be shown that under normality $\sigma_{r^*r}/\sigma_{r^*}^2 = Pr(r_t^* > 0)$ so that (6) specialises to (5).} The relationship between $\beta_{IV}$ and $\beta_a^*$ without normality is obtained using (3) and (6) as follows:
\[ \beta_{IV} = \beta_a^* \times \delta \]
where $\delta = \sigma_{r^*r}/\sigma_{r^*}^2$. Therefore, the direction and the relative size of the bias term ($\beta_{IV} - \beta_a^*$) is determined by the multiplier $\delta$; an upward bias will result for $\delta > 1$ and a downward bias for $\delta < 1$. In the next section, we access by simulation the size of the multiplier $\delta$ using a data generating processes similar to our Japanese data.

2.2 Monte Carlo Simulation

The extent of the bias can be verified by conducting several Monte Carlo simulations. First, we check the validity of $\delta = 1$ in the normal case using the following data generating process:
\begin{align*}
r_t^* &= 0.9 + 2.2\pi_{t+n} + 0.1\tilde{y}_t + \varepsilon_t \\
r_t &= 0 \quad \text{if} \quad r_t^* \leq 0 \\
r_t &= r_t^* \quad \text{if} \quad r_t^* > 0 \\
t &= 1, 2, ..., T
\end{align*}
where $\pi_t = 0.95\pi_{t-1} + \eta_t$, $\eta_t \sim N(0,1)$, $\tilde{y}_t \sim N(0,1)$ and $\varepsilon_t \sim 0.1N(0,1)$. The values for the coefficients $\alpha, \beta$ and $\gamma$ are obtained from preliminary estimation using the Japanese data and we set $T = 297$, the number of observations in our data set. The results based on 10,000 replications are reported in the first column of Table 1. As predicted by the theory, we find $\delta = 1$ and hence there is no bias in the IV estimator: $\beta_{IV} = \beta_a^*$. Therefore, even though misspecified, the regression using $r_t$ as the dependent variable provides the correct marginal effect.
Second, we move onto the non-normal case. For this we replace $\pi_{t+n}$ in the data generating process in (7) with the actual inflation rate and $\tilde{y}_t$ with the actual output gap series in our Japanese data set. The results are reported in the second column of Table 1. The marginal effect $\beta_{IV}$ from the now misspecified regression is 2.09 while the corrected marginal effect $\beta^*_u$ is 1.83. This is because $\sigma_{r^*_t}/\sigma^2_{r_t}$ is greater than $Pr(r^*_t > 0)$, making $\delta > 1$. Hence, the result indicates that the conventionally obtained marginal effect of future inflation rate on the nominal interest rate may have been over-estimated because the truncation issue has not been properly addressed and the estimator is upwardly-biased. In the next section we estimate the relationship using a Tobit estimator that corrects for the truncation of the dependent variable.

3 Data and Results

To estimate the central bank reaction function we use a sample of monthly data from 1979/04 till 2003/12, giving 297 observations. The starting date corresponds to the point where the interbank lending rate became the chief operating instrument of monetary policy in Japan and all capital controls were finally abandoned. The operating rate is the overnight call rate, while the inflation rate is the annualized change in the Consumer Price Index and monthly output gap are constructed using the Industrial Production Index detrended using the Hodrick-Prescott (HP) filter. The time series plot of the overnight call rate is given in Figure 1 where it is easily seen that the interest rate fell below 0.5% in 1995 and further decreased to near zero values in 1999 as part of a zero interest rate policy.4

Following Kuttner and Posen (2005), we use $n = 12$ and first two lags of inflation rate and output gap as instruments as the base line case, but we check the robustness of our results by using a longer horizon, $n = 18$, and a larger instrument set including the first six lags of inflation rate and output gap. The results are reported in Table 2. The results indicate estimates of the response to inflation, $\beta$, are positive with a coefficient significantly greater than Taylor’s suggested value of 1.5 in all but one case. The magnitude of the coefficient varies with the horizon, but it is not significantly different when the instrument set is altered. The response to the output gap is not significant for any specification in line with earlier results reported by

4The Bank of Japan introduced the zero interest rate policy (ZIRP) in April 1999. This involved a decision to lower the interest rate in the current period to zero, but also to commit to a zero interest rate so long as deflation continued to exist. The Bank of Japan also implemented a policy of quantitative easing whereby it flooded the financial markets with liquidity as part of its policy to deal with deflation. These policies are consistent with equation (2) provided that the response to inflation (deflation) dominates the response to the output gap i.e. $\beta^* > \gamma^*$ since a deflation would generate a negative desired interest rate, which would restricted the actual rate to zero.
Clarida et al. (1998), Bernanke and Gertler (1999) and Kuttner and Posen (2005). These estimates provide the baseline IV estimates against which we can compare the corrected estimates ($\beta^*_a$ and $\gamma^*_a$) allowing for truncation of the dependent variable.

To correct for the truncation of the distribution in the lower tail we adjust for the probability that the interest rate falls below some threshold value. The reason to use a threshold value is that interestingly in the Japanese case, the observed interest rate was never actually equal to zero even under the zero interest rate policy (ZIRP). The lower bound was near zero but not equal to zero; the rate was very low at 0.5 or less from 1995. Therefore, we investigate the fact that Japan effectively experienced zero lower bound (ZLB) by treating the nominal interest rate as effectively zero lower bounded if it is below a certain threshold value $c$ which is near zero, considering four values for $c = 0.001, 0.01, 0.1$ and 0.5. Although the interest rate is not actually constrained by zero in any of these cases the proximity of the bound is such that the central bank would have been aware that it had little room for further policy loosening, and the effective point at which the bound was influential may well have occurred at 0.5 rather than 0.001. In Table 3, we compute the relative frequency (proportion of sample) where the nominal interest rate is smaller than $c$, and since the proportion increases with the distance of $c$ from zero we expect the extent of the recorded bias to increase with $c$. These probabilities are used to scale the Tobit estimators to give adjusted coefficients that allow for the truncation of the distribution of the nominal interest rate.

The Tobit estimation results are given in Table 4 for both large and small instrument sets. After adjustment for the probability that the interest rate falls below the threshold value the adjusted coefficients ($\beta^*_a$ and $\gamma^*_a$) give the corrected response to inflation and output gaps allowing for truncation of the distribution. The results in Table 4 demonstrate that the usual IV estimates ($\beta_{IV}$ and $\gamma_{IV}$ as in Table 2) reported in much of the literature are generally biased upwards. For example, considering $h = 12$ and the small IV set case, the usual IV estimate $\beta_{IV}$ is 2.14 while the Tobit-corrected estimate $\beta^*_a$ is in the range of 1.65-1.89, depending on different values of $c$. When either a longer forward horizon is used ($h = 18$) or a larger IV set is used, the results are qualitatively the same; i.e. there is a sizable bias in the conventionally reported IV estimates.

It is expected that for a larger value of $c$, more observations are censored and the correcting factor, $\Pr(r_t \leq c)$ becomes larger, implying that the size of the bias becomes greater and this expectation is confirmed in Table 4; the bias increases with the value of $c$. The 0.5 threshold has a larger adjustment (because $\Pr(r_t \leq c)$ is larger), resulting in a significantly smaller adjusted coefficient than for other threshold values at the same horizon. The adjustment necessarily brings the coefficients closer to Taylor’s original coefficient value, but they are still significantly different from 1.50, which
suggests that the response to inflation is significantly stronger in Japan than elsewhere as previous authors have discovered in episodes prior to the ZLB. Importantly however, there is significant bias in the unadjusted estimates due to the impact of the ZLB, meaning that the degree to which policy rates adjust to inflation is not as great as has been commonly reported in the recent literature. The coefficients on the output gap by contrast are small and significantly different from zero, except where the threshold is set to the value of 0.5, the bias here is smaller and typically not significant.

There may be good reasons to be sceptical about whether a Taylor rule accurately captures policymaking in Japan. The commitment to maintain a zero interest rate so long as deflation exists and a shift towards quantitative easing under the ZIRP cast some doubt on whether a Taylor rule is an appropriate specification for Japanese monetary policy in our sample. Also, there appears to have been a marked reduction in volatility of interest rates within the sample period (see Figure 1). However, our main point - that the ZLB can bias estimates because the interest rate distribution is truncated - is illustrated clearly by our results notwithstanding these qualifications to the economic interpretation of the Taylor rule. The point generalises to many other cases such as the United States and Switzerland where interest rates were very low and distributions were non-normal; here policies were pursued that were more consistent with a Taylor rule and estimated coefficients from policy reaction functions could be potentially biased by truncation in those cases also.

Adam et al. (2005) offer a narrative approach that deals with discrete changes in regime by estimating reaction functions for sub-samples of data (in their case the UK). This is a useful exercise provided a clear break in policy can be identified and the subsamples for regimes are sufficiently long. Here we have a break in regime in April 1999 which gives only 56 observations to estimate the reaction function under the ZIRP, but for most of the period there is no variation in the interest rate (it is essentially zero as the ZIRP implies).

There are other economic objections to Taylor rules. A significant contribution by Kuttner and Posen (2005) argues that the unreliability of output gap data for Japan causes doubts about the validity of Taylor rule estimates irrespective of regime change and Kim, Osborn and Sensier (2005) and Kesriyeli, Osborn and Sensier (2006) highlight the potential for nonlinearities in policy reaction functions, although their analysis is not directly applied to the case of Japan.

We model the impact on the bias correction multiplier, \( \delta \), allowing for a break in variance of the error term in equation (7), which is modelled as \( \sigma_t^2 = \text{var}(\varepsilon_t) = \sigma_1^2[t < \tau T] + \sigma_2^2[t > \tau T] \) where \( \tau \) can take various values to indicate a break in instability at different points in the sample. We set \( \sigma_1^2 = 1 \) and \( \sigma_2^2 = 0.1 \) and after 10,000 replications we find the bias is unaffected by breaks represented by different values of \( \tau \). Results available on request.
4 Conclusion

Japan’s unique experience of prolonged low inflation, deflation and the reality of a zero lower bound (ZLB) on the nominal interest rate would lead us to expect estimates of the monetary reaction function to differ from those in other countries in the G4. Estimates by Clarida et al. (1998), Bernanke and Gertler (1999), and Kuttner and Posen (2005) confirm that the response to inflation was much stronger than other countries. Most recent studies find similar coefficient estimates but ignore the important effect of truncation on estimated coefficients due to the ZLB. Our paper shows that ignoring the lower bound on interest rates biases the estimates upwards, but correcting estimates to allow for the truncation of the interest rate distribution gives lower responses albeit estimates of the response to inflation that are still significantly different from the value of 1.50 suggested by Taylor (1993). These results are not biased however, and give a true picture of the response to inflation under low and near zero interest rates.

References


