Hybrid stable-chaotic states in coupled quantum well stadia

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Abstract

We use tunnel current spectroscopy to investigate the quantum states of two GaAs quantum wells coupled by a low (100 meV) (AlGa)As tunnel barrier. A high tilted magnetic field is used to generate strongly chaotic electron motion in the two wells which act as coupled chaotic ‘stadia’. The effect of the tunnel barrier on the dynamics of the system depends on the magnitude of the applied bias voltage \(\Delta V\). For \(\Delta V \gtrsim 375\) mV, the central potential barrier acts as a perturbation which modifies the trajectories of selected periodic orbits in the quantum well. Scattering off the central barrier also generates new periodic orbits involving multiple collisions on all three barriers. These orbits ‘scar’ distinct sets of eigenstates which generate periodic resonant peaks in the current–voltage characteristics of the device. When the device is biased such that the injected electrons just surmount the central barrier, our calculations reveal novel hybrid scarred states with both stable and chaotic characteristics.

Keywords: Quantum chaos; Resonant tunnelling; Semiconductor heterostructures

The link between the classical and quantum pictures of chaotic electron dynamics has been explored in recent experiments on GaAs/(AlGa)As resonant tunnelling diodes (RTDs) with wide (\(\sim 100\) nm) quantum wells (QWs) in high tilted magnetic fields \([1–6]\). The occurrence of unstable periodic orbits in a chaotic region of classical phase space leads to fluctuations in the density of states \([2]\) and wavefunction ‘scarring’ \([7]\), in which the probability density is concentrated along an orbital path. These effects can be directly related to distinct series of resonant features in the experimental current–voltage, \(I(V)\), characteristics of the RTDs \([2,6,8]\).

To illustrate this, in Fig. 1 we compare theoretical and experimental \(d^2I/dV^2\) versus \(V\) plots for the RTD described in Ref. [1] when a magnetic field \(B = 11.4\) T is applied at an angle \(\theta = 20^\circ\) to the normal to the well walls. Two distinct series of resonant peaks (labelled s and t in Fig. 1.) are observed in both the experimental (Fig. 1a) and theoretical (Fig. 1b) tunnelling characteristics. Detailed analysis of the tunnelling process \([2,8]\)
reveals that the low-frequency t-series of resonances originate from dominant transitions into a subset of states in the QW which are strongly scarred by unstable periodic traversing orbits in which the electron collides with the two confining barriers in turn (Fig. 1b left-hand (LH) inset). The higher-frequency s-resonances originate primarily from dominant transitions into states scarred by s-orbits in which the electron makes two successive collisions on the right-hand (RH) barrier for each collision on the LH barrier (Fig. 1b RH inset). During the orbital segment between successive collisions on the RH barrier, the electron passes close to the centre of the QW and moves almost parallel to the well walls. These orbits have also been investigated in recent magnetotunnelling studies of RTDs containing a δ-doping layer of ionized Si donors at the centre of the QW [8]. The s-resonances observed for $B = 11.4 \, \text{T}$ and $\theta = 20^\circ$ in the undoped RTD are not observed in the tunnelling characteristics of the δ-doped device because the underlying s-orbits graze the δ-doping layer and are therefore destroyed by ionized impurity scattering [8]. Quantitative analysis of the effects of ionized impurity scattering on the s-orbits and associated quantum states is complicated because the motion in the QW cannot be reduced to two-dimensions, as in the undoped case [2].

To avoid this difficulty, here we perturb s-orbits in a 120 nm wide QW by incorporating a low (100 meV) (AlGa)As barrier in the middle of the well. The potential energy of an electron at the conduction band edge varies with position along the $x$-axis perpendicular to the well walls, as shown in Fig. 2. The two AlAs barriers enclose a 120 nm wide QW which contains a 20 nm wide 100 meV high (Al$_{0.13}$Ga$_{0.87}$)As barrier at the centre. Under an applied bias voltage $V$, electrons tunnel from a two-dimensional electron gas (2DEG) in the LH (emitter) contact into the two coupled QWs formed by the three barriers. When a magnetic field $B$ is applied at an angle $\theta$ to the $x$-direction (see Fig. 2 inset) the in-plane energy of electrons in the 2DEG is quantized into Landau levels. Only the lowest
Landau level is populated and the total energy of an emitter electron is \( E_{2DEG} = \epsilon_n + \hbar B e \cos \theta/2m^* \), where \( \epsilon_n \) is the bound-state energy and \( m^* \) is the band-edge effective mass. Electrons in the coupled QWs are spatially confined by the barrier layers and by the magnetic field. This confinement quantizes the total energy of an electron in the QWs into discrete energy levels \( \epsilon_n (n = 1, 2, 3 \ldots) \). When \( V \) is increased, the emitter level scans the quantized energy spectrum of the QWs. Resonant tunnelling from the emitter into the \( n \)th bound state of the QWs occurs at the bias voltage for which \( E_{2DEG} = \epsilon_n \). The tunnel current flowing through the device is, therefore, strongly influenced by the energy level spectrum of the QWs and the associated wavefunctions which determine the tunnelling transition rates [2,6].

When \( B = 0 \) T, the electrons tunnel from the bound emitter state into the electric subbands of the coupled QWs, and execute regular, stable classical orbits. For \( V \gtrsim 375 \) mV, the electrons pass easily over the central barrier and perform periodic traversing orbits [1–3]. As the magnetic field is increased, the classical dynamics undergoes a transition from regular to strongly chaotic. For \( V \lesssim 375 \) mV, the electrons are strongly confined in either the LH or RH well so the current is then very small.

To investigate the tunnelling characteristics of the triple-barrier device we have calculated the rates of tunnelling transitions from the 2DEG into the two coupled QWs using a transfer-Hamiltonian formalism [2,6]. In this approach, resonant tunnelling is considered to be a sequential process in which the electron makes successive transitions first from the 2DEG into the eigenstates of the coupled QWs, and then from the coupled QWs into the RH collector contact. Because most of the applied bias voltage is dropped across the coupled QWs, the RH AlAs barrier is considerably more transparent to the tunnelling electrons than the LH barrier. Consequently, the occupancy of each state in the coupled QWs is almost zero, provided \( V \gtrsim 375 \) mV so that the electrons can easily surmount the low central potential barrier. To good approximation therefore, the current depends only on the transition rates through the LH barrier. The tunnelling rates into individual states are then used to calculate the total current flowing through the RTD. Nonparabolicity of the GaAs conduction band is included in our calculations using the method described in Ref. [6].

We now compare experimental and theoretical tunnelling characteristics of the triple barrier structure for \( B = 11.8 \) T and \( \theta = 20^\circ \) (Fig. 3). For bias voltages \( V \gtrsim 375 \) mV, the electrons which tunnel into the coupled QWs execute classical orbits which reach both of the AlAs barriers. In this voltage regime, the experimental \( \mathrm{d}^2I/\mathrm{d}V^2 \) versus \( V \) plot (Fig. 3a) reveals a single series of resonant peaks (labelled \( s \)) with a voltage spacing \( \Delta V \approx 25 \) mV. The corresponding theoretical derivative plot reveals a similar series of resonant peaks (labelled \( s' \) in Fig. 3b) for \( V > 500 \) mV. The calculated voltage spacing of these peaks (\( \Delta V = 21 \) mV) is in reasonable agreement with the measured value of 25 mV. Our calculations show that these resonances originate from dominant transitions into a series of individual states in which the probability density is strongly localized or ‘scarred’ [7] along a particular type of unstable periodic orbit in which the electron interacts with all three barriers. The shape of the scar pattern (shown in the RH inset to Fig. 3b) is similar to that produced by the \( s \)-orbit in the 120 nm wide single QW [1] (Fig. 1b RH inset). But for the coupled wells, the orbital path between successive collisions on the RH barrier is effectively shortened by collisions with the central barrier. As a consequence of the shorter period, the voltage spacings of the \( s' \) triple-barrier resonances (\( \Delta V = 21 \) mV) are larger than the double-barrier \( s \)-resonances (\( \Delta V = 12 \) mV).

Our calculations predict that for a small range of bias voltages close to 375 mV, the electrons tunnelling through the triple-barrier structure execute chaotic classical orbits in the LH well but emerge into stable trajectories in the RH well. Each time the electron crosses the central barrier it passes between regular and chaotic regions of phase space. This leads to the creation of a new class of ‘hybrid’ periodic orbits whose dynamical properties are considerably more complex than those studied in earlier double-barrier structures. The probability density plots of the corresponding eigenfunctions consist of an ordered and an irregular part (Fig. 3b LH inset). However, in the present experiments it is
not possible to associate particular resonant peaks with the hybrid states. At lower voltages the calculations show some weak resonances due to states localized in the LH well. But the theory assumes the electrons can easily escape through the collector barrier and therefore overestimates the current in this region. Experimentally, the tunnel current is very small (Fig. 3a) and no reproducible oscillatory structure is found at these low voltages.

Understanding the properties of ‘hybrid’ quantum states which relate to both stable and chaotic classical trajectories is a fundamental problem in quantum chaos. This problem has been highlighted in recent studies of excited multi-electron atoms [9]. In the atoms, transfer between chaotic and stable parts of phase space originates from atom-core scattering processes which are analogous to tunnelling through the central barrier in our coupled well structure. But in the coupled wells, the transitions between chaotic and stable orbits can be precisely controlled and modelled. Our experiments demonstrate the potential of triple-barrier structures for studying the quantum properties of coupled sub-systems with distinct dynamical properties. In future work, scaled field measurement techniques [4,5], which maintain fixed classical dynamics, may be helpful in providing evidence for the existence of hybrid quantum states.

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References