Chaos in quantum wells and analogous optical systems


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Abstract

The quantized states of a 60 nm wide potential well in a large tilted magnetic field are investigated using scaled field resonant tunnelling spectroscopy. In contrast to previous experiments on this type of system, the tunnelling characteristics are measured by changing both the magnetic field strength $B$ and the applied bias voltage $V$ such that $V/B^2$ is approximately constant. This ensures that the classical phase space for electrons in the potential well has the same mixed stable-chaotic character for all fields. As a consequence of this scaling, each closed orbit in the potential well produces many periodic resonant peaks in plots of $d^2I/dB^2$ versus $B$. This type of scaled field experiment can be used to probe quantum states corresponding to dynamical regimes which are inaccessible to fixed field resonant tunnelling studies. We also consider analogies between the electron orbits and light rays in gradient refractive index lenses.

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The classical motion of electrons in a wide quantum well (QW) is strongly chaotic in the presence of a large tilted magnetic field [1–13]. Although most electron orbits are highly irregular [1], a variety of unstable but periodic orbits also exist. Each periodic orbit with period $T_p$, produces regular fluctuations in the density of quantized energy levels [5] and also wavefunction ‘scarring’, a concentration of probability density along the orbital path [5,6]. Wavefunction scarring occurs in a subset of states whose energy spacing is $\Delta e = h/T_p$ [1]. When the QW is incorporated in a resonant tunnelling diode (RTD), wavefunction scarring produces series of quasi-periodic resonant peaks in the current–voltage characteristics $I(V)$. For given $B$, the periodic orbits accessible to the tunnelling electrons often change rapidly as $V$ is increased [3]. This greaty complicates the interpretation of the experimental data and can make it impossible to relate the resonant peaks to particular orbits. Here, we show that this problem can be overcome by changing $V$ and $B$ simultaneously so that $V/B^2$ is approximately constant. This ensures that
the electrons tunnel into a fixed set of orbits which generate long series of periodic resonant peaks in plots of current $I$ versus $B$, and derivative plots.

The GaAs/(Al$_{0.4}$Ga$_{0.6}$)As RTD investigated contains a QW of width $w = 60$ nm [3]. A magnetic field is applied at an angle $\theta$ to the growth ($x$-) direction (Fig. 1). Under an applied bias voltage, a two-dimensional electron gas (2DEG) accumulates in the undoped spacer layer adjacent to the left-hand (LH) barrier. Current flows when electrons tunnel from the emitter 2DEG into the quantized energy levels of the QW. This current was measured using a scaled field technique in which both $V$ and $B$ are changed simultaneously.

To explain this technique, we first consider the Hamiltonian for electron motion in the QW. In the gauge given by the vector potential $A = (0, xB \sin \theta - zB \cos \theta, 0)$ (co-ordinates shown in Fig. 1) the Hamiltonian for motion in the QW, $|x| < \frac{1}{2}w$, is [5,6]

$$H = \varepsilon = \frac{p_x^2}{2m^*(V)} + \frac{(ex \sin \theta - ez \cos \theta)^2}{2m^*(V)}$$

$$+ eF(\frac{1}{2}w - x),$$

where $F \propto V$ is the electric field in the QW, $p_x$, $p_z$ are canonical momentum components and $m^*(V) = 0.067m_e(1 + zK(V))$ is a voltage-dependent effective mass which, due to conduction band non-parabolicity, increases with the mean kinetic energy $K(V)$ of an electron in the QW. The parameter $z = 2$ eV$^{-1}$ [6]. The Hamiltonian in Eq. (1) depends on the $x$- and $z$-co-ordinates and explicitly on the electric and magnetic fields. However, if we define a scaled

$$H' = \frac{m^*(V)H}{B^2} = \varepsilon'$$

$$= \frac{p_x^2}{2m^*(V)} + \frac{(ex \sin \theta - ez \cos \theta)^2}{2m^*(V)}$$

$$+ e\beta(\frac{1}{2}w - x),$$

(2)

then $F$ and $B$ only appear through the parameter $\beta = m^*(V)F/B^2$. If both $V$ and $B$ are changed so as to keep $\beta$ constant, then the scaled Hamiltonian is invariant. Since $\varepsilon$ is proportional to $F$, if $\beta$ is constant the scaled energy $\varepsilon' = m^*(V)\varepsilon/B^2$ is also constant and the tunnelling electrons will be injected into exactly the same scaled classical phase space for all $B$ and $V$ values.

Fig. 2 shows a Poincaré section (slice through the classical phase space) calculated for $\beta = 5.97 \times 10^{-27}$ C m and $\theta = 40^\circ$.
injected into periodic orbits with starting velocities close to this ring [1]. We find that two distinct types of periodic orbit control the tunnelling characteristics of the RTD. These are shown in inset of Fig. 3(a) projected onto one of the electron wavefunctions that originates from quantizing the two orbits.

Recent theoretical work [5] has shown that resonant peaks in the tunnel current occur whenever the classical action of the corresponding periodic classical orbits satisfies the Bohr–Sommerfeld-like quantization condition

$$S_P = \oint p \cdot dr = B \oint p' \cdot dr = BS'_P = (n + \phi)\hbar,$$

where $dr$ is a line element along the orbit and $\phi \approx 1$ is a constant phase factor. For each periodic orbit of the scaled Hamiltonian $H'$, the scaled action $S'_P$ is constant [7]. Therefore, the above quantization condition shows that when the tunnel current is plotted as a function of $B$ for fixed $\beta$, each periodic orbit will produce a series of periodic resonant peaks with spacing $\Delta B = h/S'_P$.

To demonstrate this, Fig. 3(a) shows the $d^2I/dB^2$ versus $B$ curve calculated for the RTD with $\beta = 5.97 \times 10^{-27}$ C m and $\theta = 40^\circ$. The tunnelling transition rates into the quasi-bound states of the QW are calculated using a transfer-Hamiltonian approach [5,6]. The series of periodic resonant peaks ($\Delta B = 0.2$ T) originates from tunnelling into states which originate from quantizing the two periodic orbits that are accessible to the tunnelling electrons. The inset in Fig. 3(a) shows the probability distribution of one of the states (in the $x$–$z$ plane) with the two orbits overlaid. Since the actions of these orbits are almost identical, both orbits are quantized at the same $B$ values. Similar resonant peaks observed in the corresponding experimental data (Fig. 3(b)) provide clear experimental evidence for the existence of quantized states corresponding to these two orbits. Since $V$ and $B$ are scaled to keep $\beta$ constant, the orbits exist for all $B$ values and therefore generate many resonant peaks in $I(B)$ and its derivative plots.

In the absence of scattering, the scaled Hamiltonian for electrons in the QW generates an infinite number of periodic orbits. But, due to LO phonon emission, electrons in a real device can only complete a few of the shortest orbits before scattering. Quantized states corresponding to longer periodic orbits cannot be resolved in our tunnelling experiments. But, as we now explain, it might be possible to detect analogous electromagnetic modes in a particular type of optical cavity.

In the QW, the electron follows regular helical-like trajectories between collisions with the well walls. The chaos is generated by the collisions, which interrupt the regular helical motion at irregular times. As shown in Fig. 4(a), light rays also follow helical paths in gradient index (GRIN) optical fibres and lenses, in which the refractive index decreases approximately parabolically with distance from the centre of the fibre [14]. We have investigated the ray and wave properties of a GRIN lens whose surfaces are silvered to produce specular reflection. Just as in the RTD, when the planar ends of the lens are cut at an oblique angle relative to its axis, the ray dynamics are chaotic (Fig. 4(b)).

Rays which pass through the axis of the lens (meridional rays [14]) follow planar sinusoidal trajectories within the lens. But specular reflection at the silvered surfaces generates two-dimensional chaotic ray paths (Fig. 4(c)). To investigate how the onset
of chaos affects electromagnetic waves in the lens, we have solved Maxwell’s equations for polarized modes in which the electric field $E$ is normal to the two-dimensional (meridional) chaotic ray paths [15]. We find that unstable periodic ray paths often scar the electric field intensity (Fig. 5, inset). The spectrum of angular frequencies $\omega$ of the eigenmodes (vertical lines in Fig. 5) is highly irregular. These eigenmodes have a pronounced effect on plots of the transmission coefficient $T$ versus $\omega$ for laser light which enters and leaves the GRIN lens through semi-silvered ends. In particular, each eigenmode of the lens generates a very sharp resonance in $T(\omega)$ (Fig. 5). Due to the long coherence length of laser light, experimental measurements of $T(\omega)$ should be able to detect spectral fluctuations and scarring effects associated with very long periodic ray paths which are far beyond the resolution limits of the RTD [15].

References