Quantum Hall effect breakdown: can the bootstrap heating and inter-Landau-level scattering models be reconciled?

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Abstract

We study the breakdown of the integer quantum Hall effect in a series of n-type GaAs/AlGaAs heterostructures in which the current flows through a short and narrow constriction. The critical mean current density is strongly dependent on the constriction width and is \(\sim 10 \text{ A m}^{-1}\) for the narrowest (1 \(\mu\text{m}\)) constriction, in qualitative agreement with earlier work by Bliek et al. We compare our data with other breakdown measurements and with the bootstrap-electron-heating and inter-Landau-level-scattering models.

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1. Introduction

The breakdown of the dissipationless state of the integer quantum Hall effect (IQHE) \cite{1} is observed as a sharp increase in the longitudinal voltage drop, \(V_x\), when the current, \(I\), flowing down the length of the Hall bar exceeds a critical value, \(I_c\) \cite{2–7}. The effect can also be studied with \(I\) maintained at some constant and relatively large value and by sweeping the applied magnetic field, \(B\), away from the value \(B/\nu T_B\) that corresponds to integer filling factors, \(\nu = 2, 4, \ldots\) \cite{3,8}. An extensive review of IQHE breakdown has been published recently \cite{9}, but a complete understanding of the subject is still lacking. Komiyama and co-workers \cite{5,10} have proposed a bootstrap electron heating (BSEH) model to account for a common type of breakdown observed in many samples. This is a macroscopic model, which considers the energy gain and loss of electrons and the effect of the very strong dependence of the longitudinal conductivity, \(\sigma_{xx}\), on the electron temperature, \(T_e\). According to the model, the critical electric field, \(E_c\), for breakdown is given by

\[ v_c = E_c/B = (2h/\tau_e m)^{1/2}, \]

where \(v_c = E_c/B\) is the critical mean drift velocity and \(\tau_e \approx 1 \text{ns}/B(T)\) the electron–phonon energy relaxation time (see Eq. 2.12 of Ref. \cite{10}). However, a much larger value of \(E_c\) has been measured by Bliek et al. for the case of electron flow through a narrow (width \(w = 1 \mu\text{m}\)) and short (length \(l = 10 \mu\text{m}\)) channel \cite{11}. In this case, the critical mean current density, \(J_c = I_c/w \approx 30 \text{ A m}^{-1}\), through the channel corresponds...
to a value of \( E_c = J_c h/e^2 v \), which is more than an order of magnitude larger than that given by Eq. (1). This type of breakdown can be explained in terms of quasi-elastic-inter-Landau-level scattering (QUILLS) of electrons between adjacent Landau levels [12,13]. At \( v = 2 \), the critical electric field is given by

\[
f e E_c l_b \approx \hbar \omega_c,
\]

where \( l_b = \hbar/e B \) and \( f \) is a numerical factor \( \approx 3 \). This condition can be rewritten as \( E_c/B = f (\hbar \omega_c/m)^{1/2} \), giving a value of \( E_c \) that is \( \sim (\omega_c \tau_c)^{1/2} \approx 100 \) times larger than that given by Eq. (1).

Balaban et al. [14,15] have interpreted breakdown measurements in high-mobility samples in terms of QUILLS in the high-electric-field region close to the edge of the sample where the electric field is larger [16]. In addition, the remarkable series of voltage steps observed by Cage et al. at the onset of dissipation in the US resistance standard samples [17] has been attributed recently to a localised breakdown arising from QUILLS due to charged impurities located near the sample edge [13].

In this paper, we investigate breakdown in a series of Hall bars in which the current flows through a short (2 or 10 \( \mu \)m) and narrow constriction region of width varying between 1 and 50 \( \mu \)m. Our motivation was partly to confirm the early work by Bliek et al. [11]. In addition, by examining samples with different constriction widths, we have been able to study a range of breakdown behaviour from that characteristic of BSEH to that due to QUILLS.

2. Experimental results

Three GaAs/AlGaAs heterostructures: “type 1” with \( \mu = 20 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1} \) and \( N = 5.6 \times 10^{11} \text{ cm}^{-2} \), “type 2” with \( \mu = 70 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1} \) and \( N = 4 \times 10^{11} \text{ cm}^{-2} \), and “type 3” with \( \mu = 60 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1} \) and \( N = 5 \times 10^{11} \text{ cm}^{-2} \) were chosen for the investigation of the \( J_c(w) \) dependence. The design of type 1 and type 2 samples is shown schematically in the inset of Fig. 1. Type 3 heterostructure was fabricated into a standard Hall bar of width 100 \( \mu \)m. The wide sections of the Hall bar have a width of 200 \( \mu \)m and the width of the voltage probes is also 200 \( \mu \)m. The constriction region has a length of 10 or 2 \( \mu \)m and a width from 1 to 50 \( \mu \)m. The current is passed between probes 1 and 6 and the onset of dissipation in the vicinity of the constriction is detected by measuring the potential difference between probes 3 and 4, which gives the longitudinal voltage, \( V_x \). Fig. 1 shows the longitudinal voltage \( V_x \) as a function of current \( I \) at magnetic fields close to filling factor \( v = 2 \) and determination of \( I_c \). \( B \) varies from 11.4 T (right) to 12.4 T (left) in 0.2 T steps. The inset shows the geometry of the samples used.
with constrictions 50 and 100 μm (Fig. 2). Remarkably, samples with constrictions \( w \leq 10 \) μm show an absolute (difference < 0.1%) identity of \( V_x \) measured using opposite pairs of contacts. A similar identity at opposite edges was observed by Cage [17,18] and by us in p-type samples [19]. In both these experiments, the onset was accompanied by the presence of voltage steps.

Note that the \( J_c \) values for the wider constrictions (\( w > 40 \) μm) are \( \sim 1 \) A m\(^{-1}\), see inset, which is similar to that measured in many other experiments. This can be seen from Fig. 3, which plots \( J_c/\nu \) as a function of \( B \) for a wide range of samples: conventional Hall bars, Corbino-type samples and samples with constrictions, including our own samples. It can be seen that the empirical linear relation \( J_c = 0.05vB \), or equivalently, a critical group velocity \( v_c = 1300 \) m s\(^{-1}\) (cf. the spontaneous acoustic phonon emission model of the breakdown by Štředa [20]), provides a reasonable fit to many of the data points. This fit appears to be at least as good as that given by Eq. (1) for BSEH (see also Fig. 6 of Ref. [10]), and it is interesting to note that the BSEH formula gives the same result if we assume a \( B \)-independent scattering rate \( \tau_c \approx 2 \) ns (see Eq. (1)). Obviously, the representation of so many experimental results on a single figure (Fig. 3) is necessarily crude. It does not take into account such parameters as temperature or length of a sample. Nevertheless, it allows us to see the general behaviours of \( J_c(B) \) and \( J_c(w) \).

Closer inspection of Fig. 3 reveals that two sets of data are quite different from the general trend. The data of Bliek et al. [11] and some of our data, taken on samples with constrictions \( w = 1 \) μm, have \( J_c \) values typically 1 to 1 1/2 orders of magnitude higher than the general trend. They are more consistent with a QUILLS type of breakdown (Eq. (2)) than with the BSEH model. Interestingly, the data of Balaban et al. [14,15] on samples with constriction regions between 8 and 78 μm wide, which were interpreted using an edge-induced QUILLS model, have \( J_c \) values well below the general trend. A strong sublinear \( I_c(w) \) dependence was also observed.

3. Discussion and conclusions

As can be seen from Figs. 2 and 3, the breakdown current densities, \( J_c \), at \( v = 2 \) for our narrow constrictions (\( w = 1 \) and 10 μm) are considerably higher than the value \( J_c = 1 \) A m\(^{-1}\) that is characteristic of many of the data points in Fig. 3. In the inset of Fig. 2, we show that the variation of critical current with constriction width for \( w \) less than about 40 μm can be fitted reasonably well to the relation

\[
I_c = I_0 \ln(w/w_0),
\]  

(3)
setting $I_0 = 8 \mu A$ and $w_0 = 150 \mu m$. This relation for $I_c$ was used by Balaban et al. [14] assuming a Hall potential profile of the form derived by MacDonald et al. [16]. However, our values of $I_0$ and $w_0$ are substantially larger than those used in Ref. [14]. This point is evident from Fig. 3: Balaban et al.’s data points are well below the general trend and ours are well above. It is also interesting to note that the $I_c$ versus $w$ plot shown in Fig. 2 of Ref. [21] also shows a form similar to ours for $w < 40 \mu m$, although Kawaguchi et al. use their data in support of the BSEH model.

Despite the difference between our results and those of Ref. [14], it is clear that the high mean critical current densities, $J_c = 8 A \text{ m}^{-1}$, for our narrowest (1 $\mu m$) constriction are comparable with those required for QUILLS. For these samples, the dissipation is restricted to the region close to the constriction, i.e. at the critical current we measure a significant dissipative voltage $\sim 10 mV$ between probes 3 and 4 whilst the voltage drop between probes 2 and 3 and between probes 4 and 5 remains below the 1 $V$ level. It seems likely that the geometry of our samples inhibits the avalanche process required for BSEH: at $v = 2$, electrons injected into the upper unoccupied Landau level due to QUILLS in the constriction rapidly enter a region of much lower Hall electric field and are, therefore, unable to gain the energy required to excite more electrons from the lower Landau level.

At the critical current, the electrons pass through our $w = 1 \mu m$, $l = 10 \mu m$ constriction in a time $\sim 1$ ns. Kaya and co-workers [22] have found experimentally that a channel length $l > 100 \mu m$ is required for BSEH.

Komiya’s model is a bulk breakdown description based on the strong dependence of the bulk parameter, $\sigma_{xx}$, on electron temperature. Our results indicate that a local process can cause dissipation. A local mechanism due to inter-Landau-level scattering has recently been proposed by one of us [13] to explain the step-like QHBD observed in n-type devices by the NIST group [17] and on p-type samples by the current authors [19].

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References